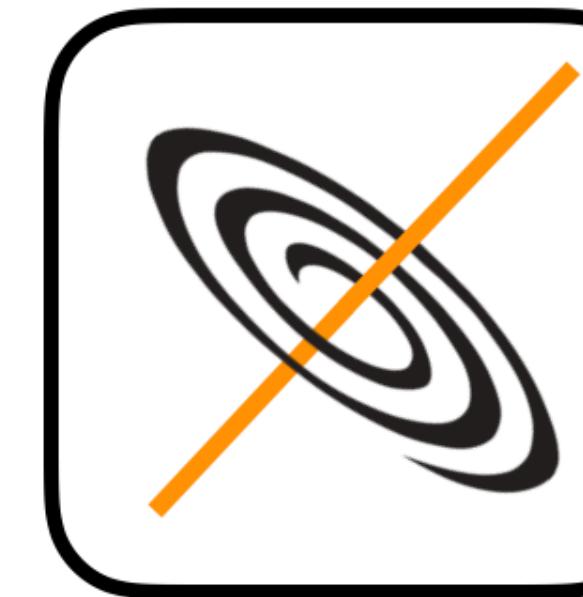


Numerical modelling of radiative and accelerative processes with the JetSeT code



JetSeT

Jets SED modeler and fitting Tool

Andrea Tramacere

<https://jetset.readthedocs.io/en/latest/>
<https://github.com/andreatramacere/jetset>
<https://www.facebook.com/jetsetastro/>

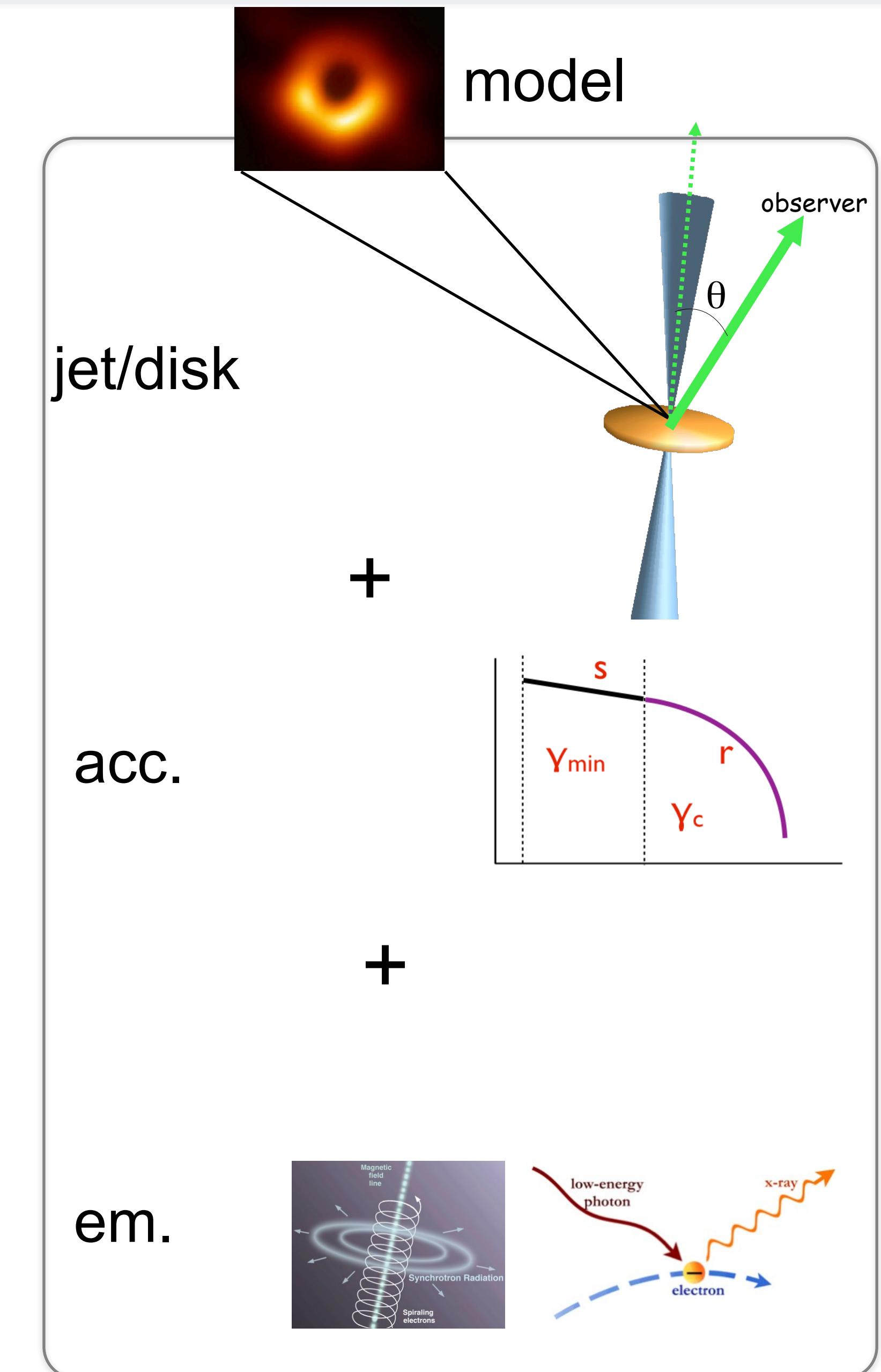
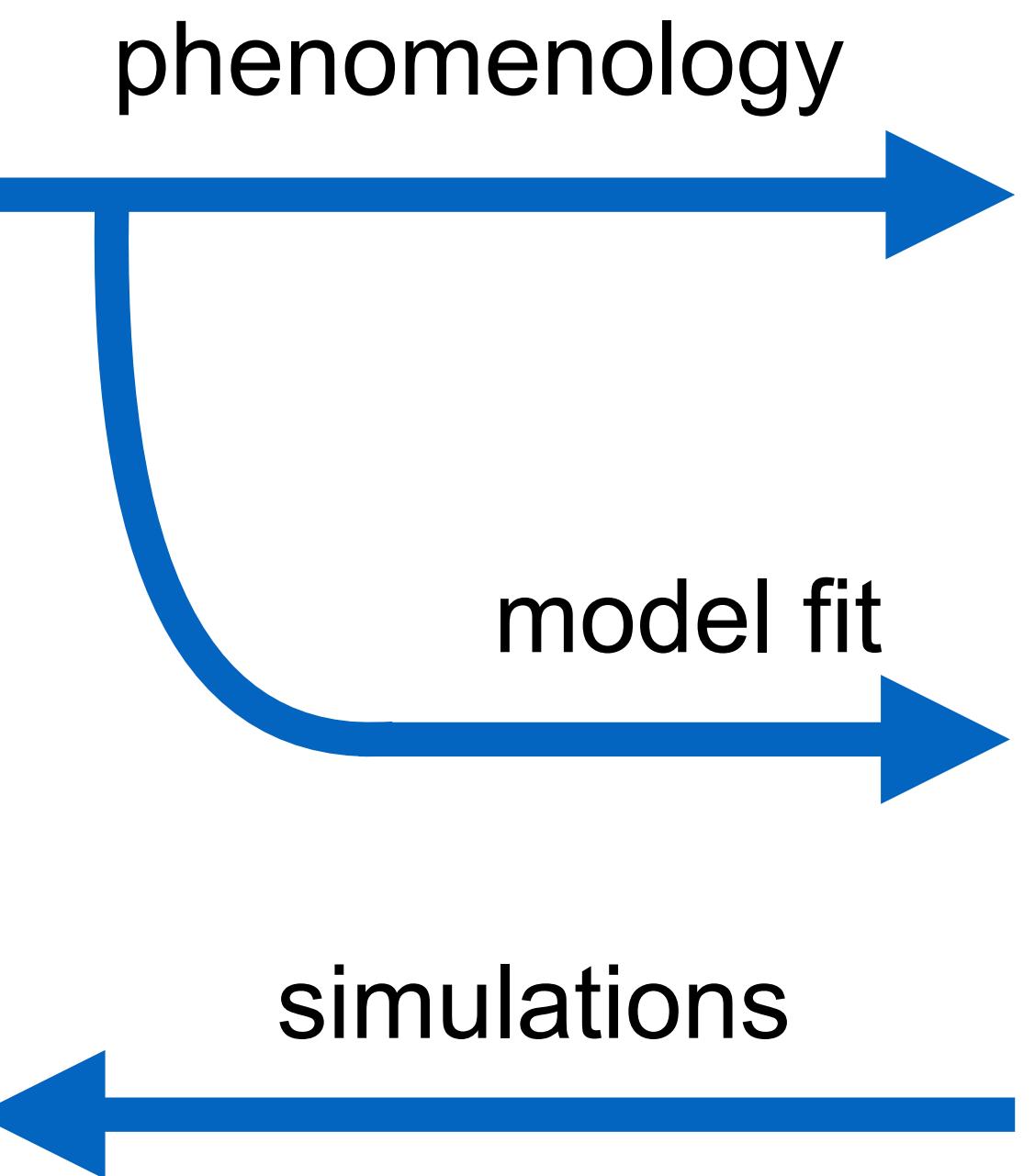
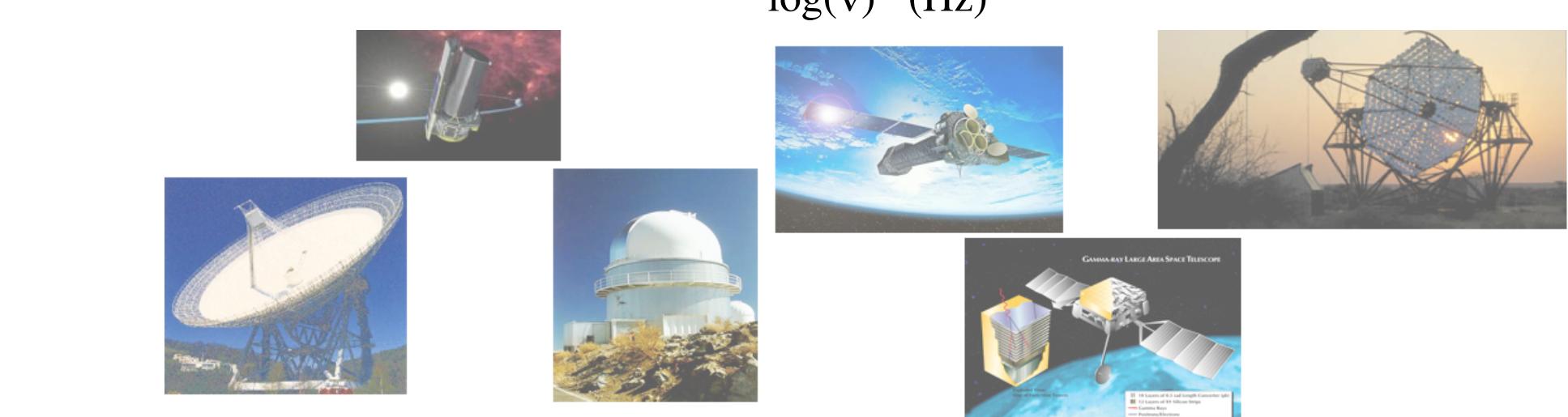
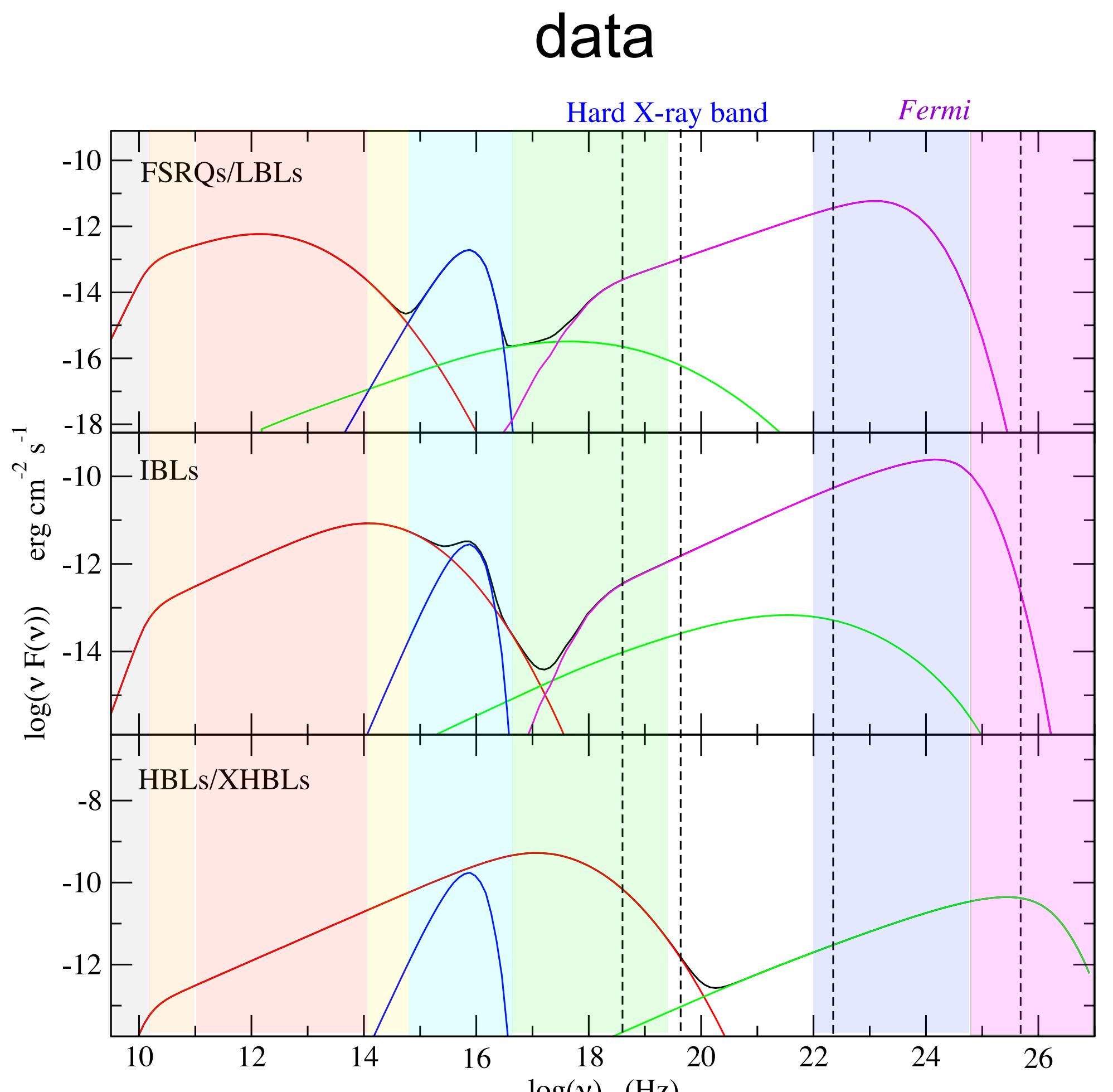
Geneva CTA Group
Feb 2021 zoom lectures

- Theoretical background
- definition of complex radiative **models** SSC/EC IC against CMB/BLR/DT, plus analytical and template models
- handling observed **data** (grouping, definition of data sets, etc...)
- **constraining** of the model in the pre-fitting stage, based on accurate and already published **phenomenological trends**
- **fitting of multiwavelength SEDs** using both **frequentist** approach (iminuit/scipy) and Bayesian **MCMC** sampling (emcee)
- Textbooks
 - Radiative Processes in Astrophysics, Ribicky & Lightman, John Wiley & Sons, 1991
 - High Energy Radiation from Black Holes: Gamma Rays, Cosmic Rays, and Neutrinos, Dermer & Menon, Princeton University Press 2009
 - Bayesian Reasoning in Data Analysis: A Critical Introduction, D'Agostini G., World Scientific, 2003

https://github.com/andreatramacere/Linnaeus_JetSeT_Lesson

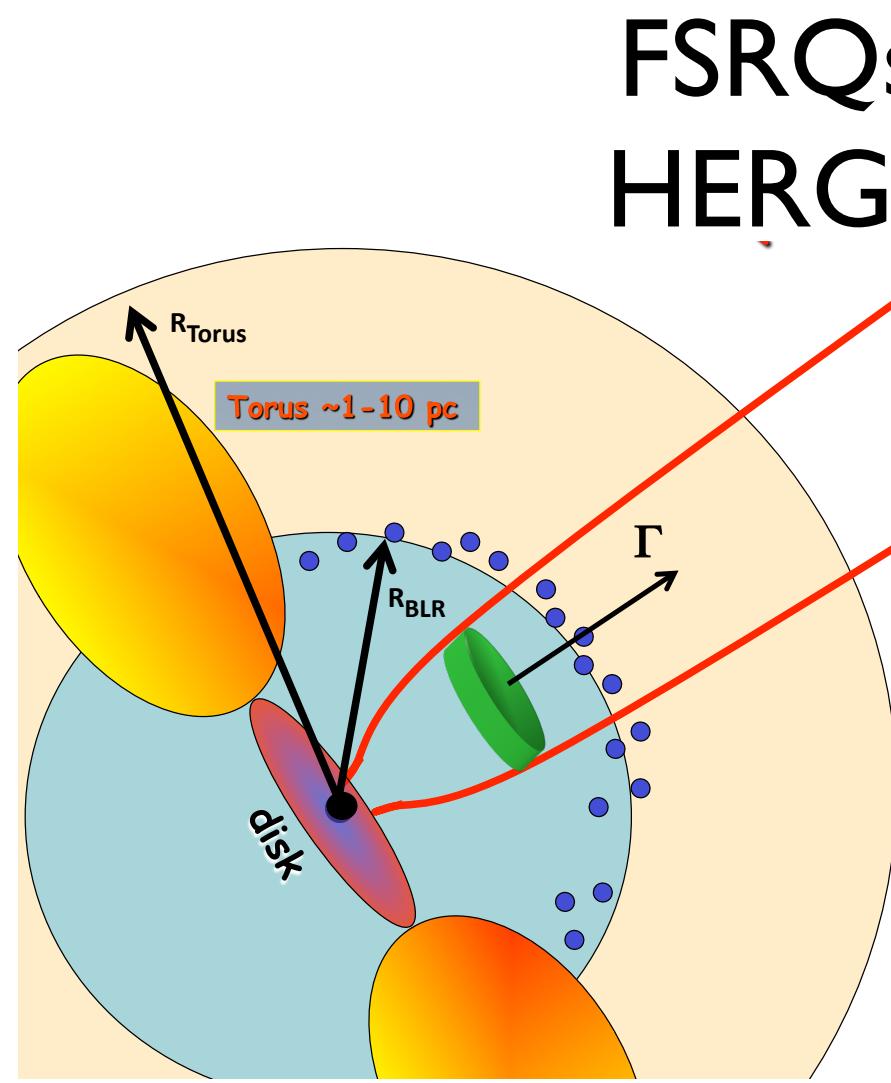
- Tutorial 1: basic operation with jet models
- Tutorial 2: phenomenological trends for synchrotron and SSC emission
- Tutorial 3: phenomenological trends for EC emission
- Tutorial 4: composite models and application to EBL (not covered in these slides)
- Tutorial 5: constraining of the model in the pre-fitting stage, based on accurate and already published phenomenological trends and fitting of multiwavelength SEDs using both frequentist approach ([iminuit/scipy](#)) and Bayesian MCMC sampling ([emcee](#))

Blazars in a nutshell



standard picture: acceleration/cooling balance

(Ghisellini,Fossati,Celotti 99-16)



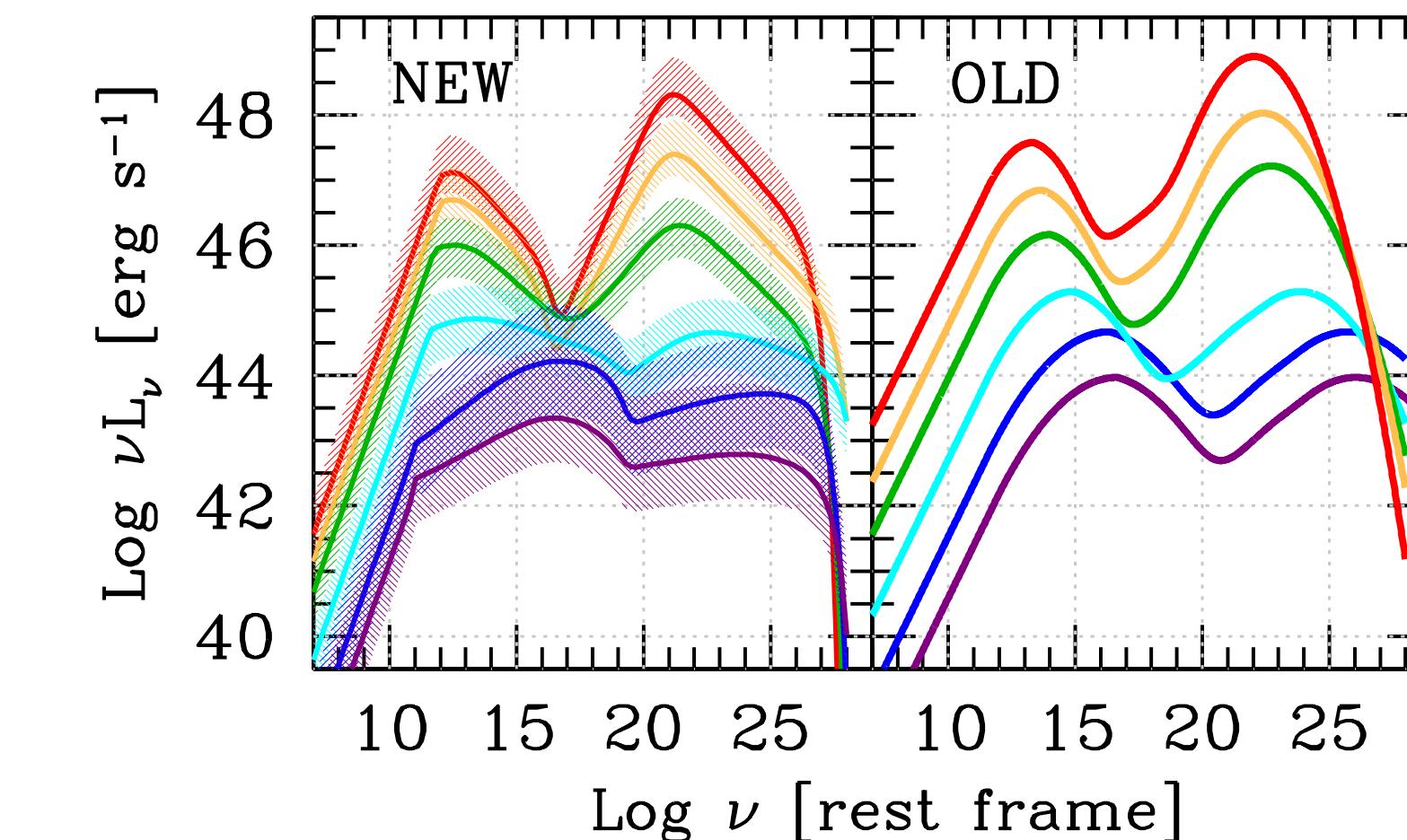
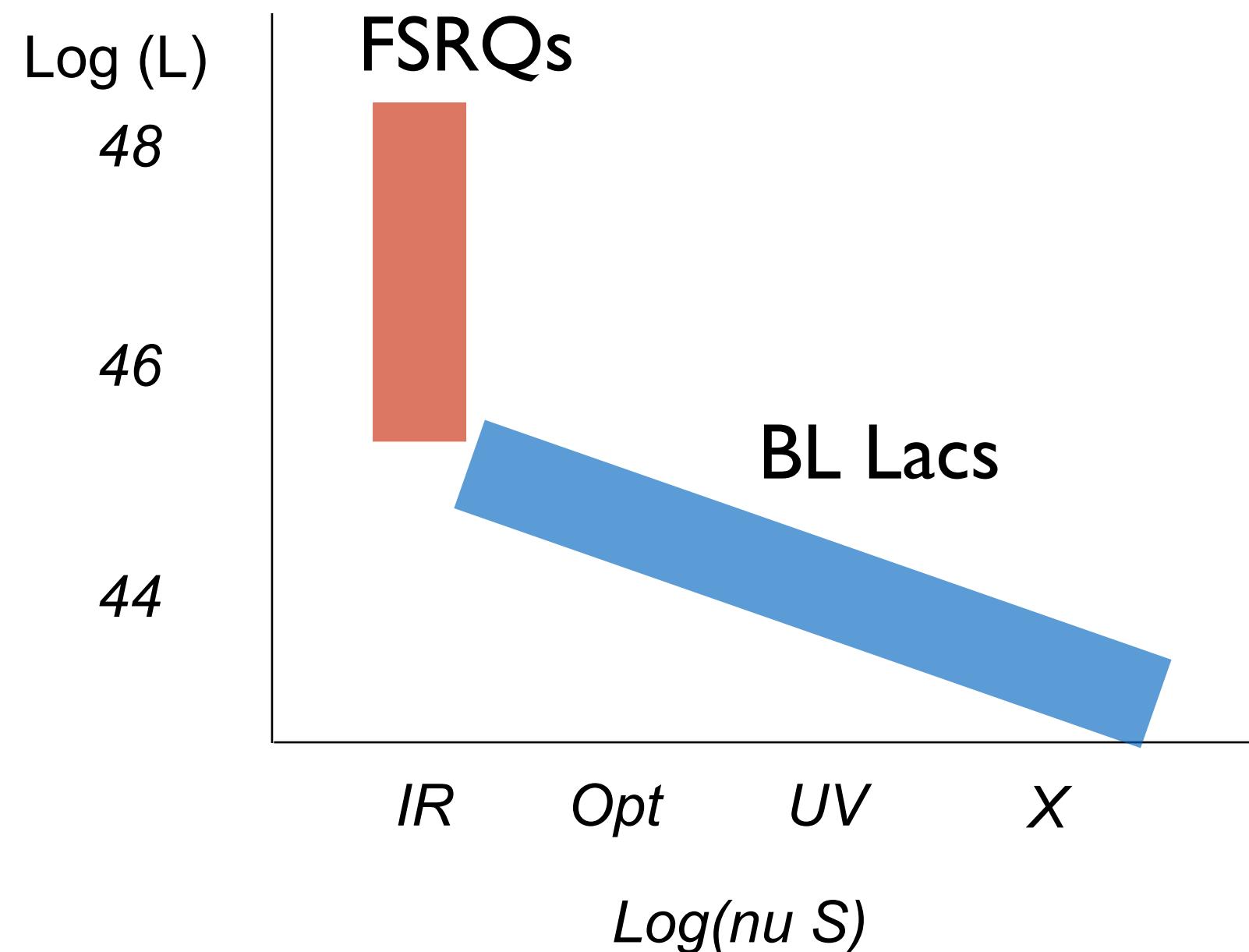
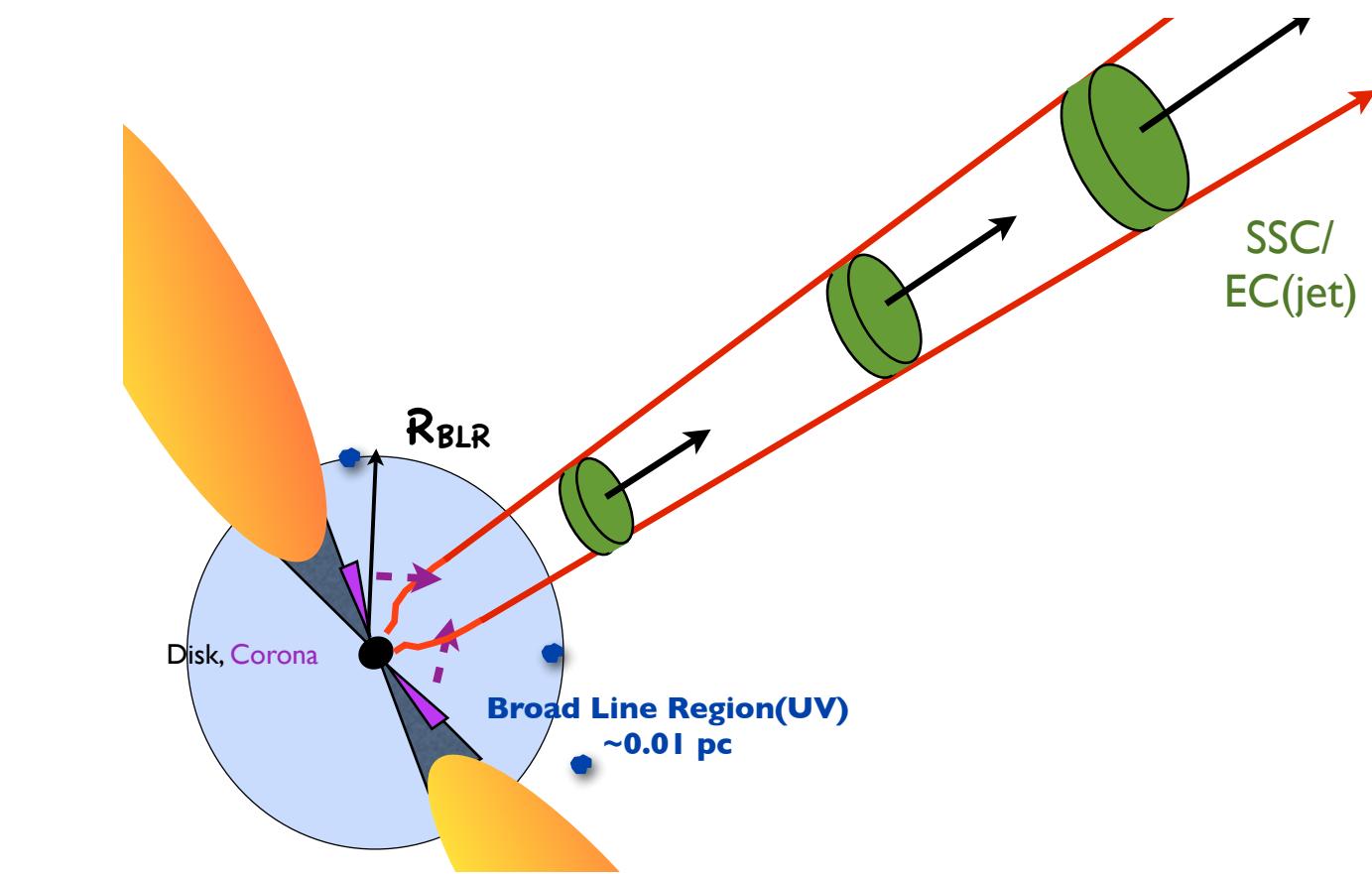
rad eff.
 $L_d > 10^{-2} L_{EDD}$

$$L_{Edd} \approx 1.4 \times 10^{44} \frac{M_{BH}}{10^6 M_\odot} \text{ erg s}^{-1}$$

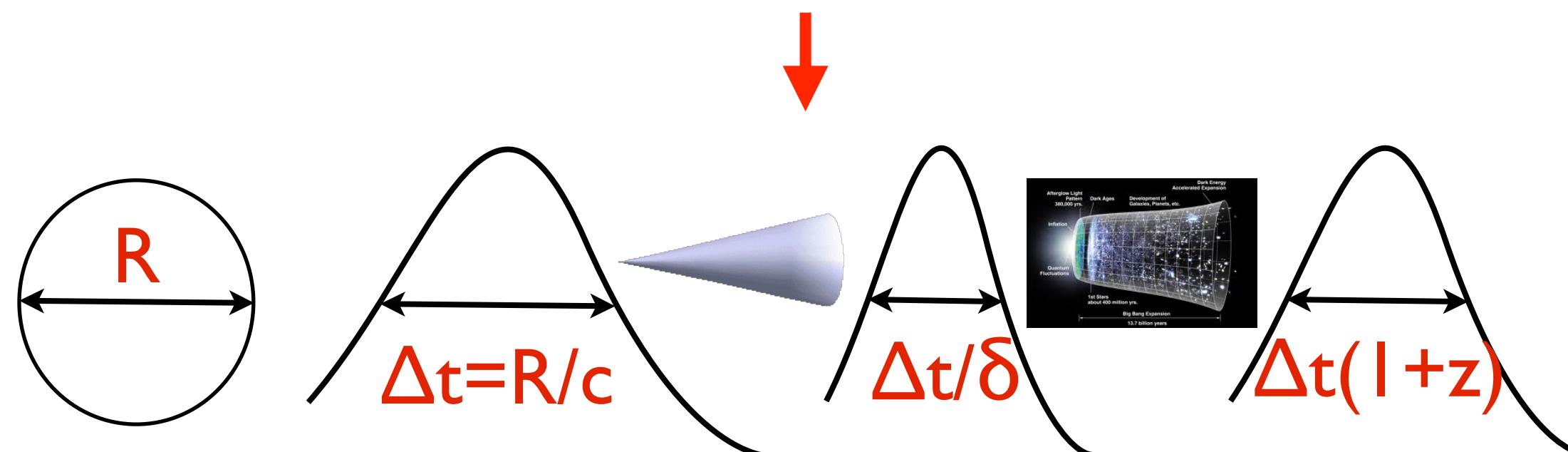
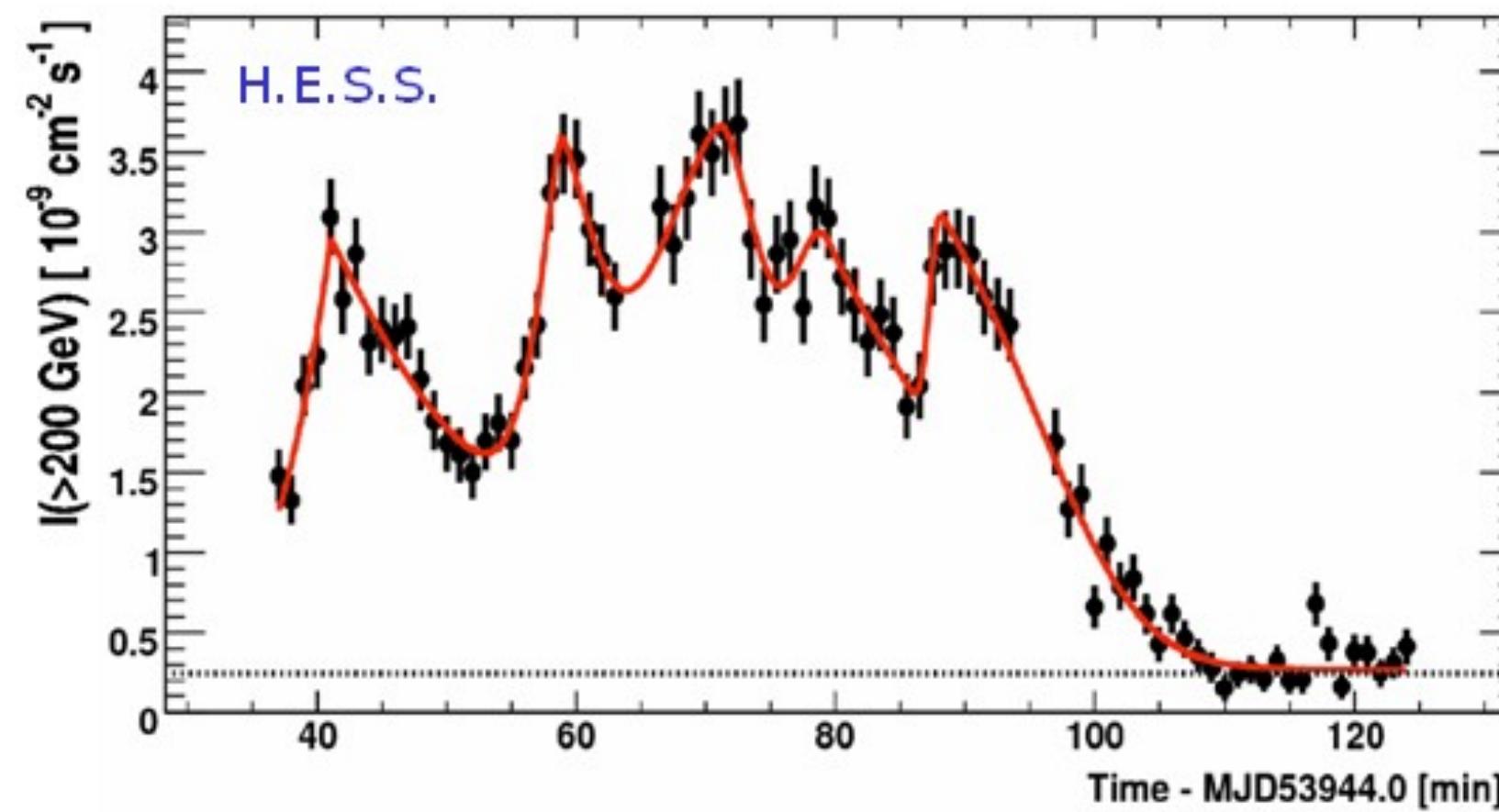
$$\left. \begin{aligned} U_{ext} &\simeq \frac{L_d}{R_{ext}^2 c} \\ R_{ext} &\simeq L_d^{1/2} \end{aligned} \right\} \rightarrow \begin{aligned} &\sim 0.1 \text{ erg/cm}^3 \text{ BLR} \\ &\sim 0.01 \text{ erg/cm}^3 \text{ DT} \end{aligned}$$

BL Lacs
LERGs

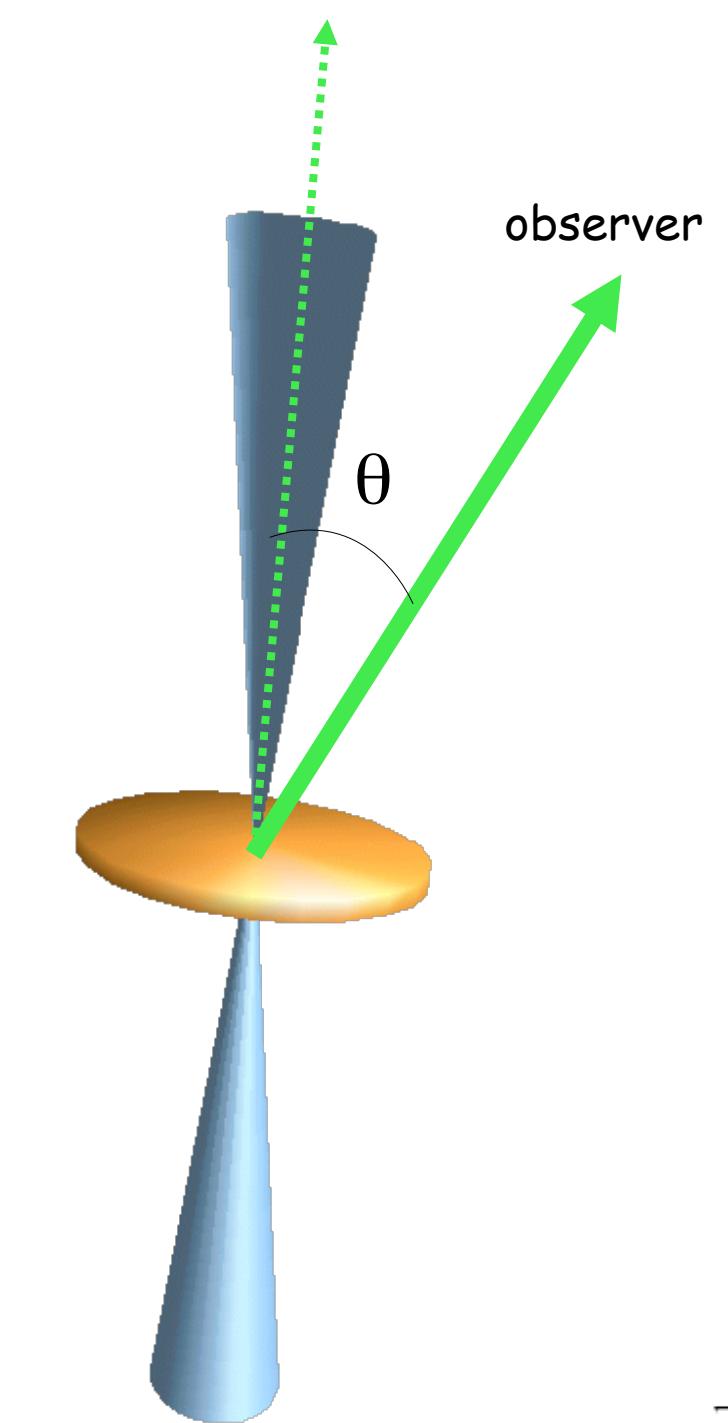
rad ineff.
 $L_d < 10^{-2} L_{EDD}$



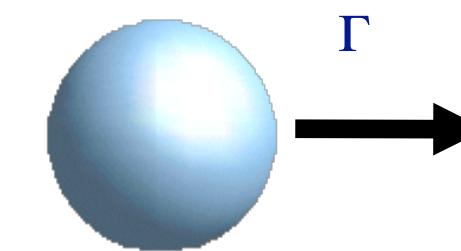
Beamed Emission



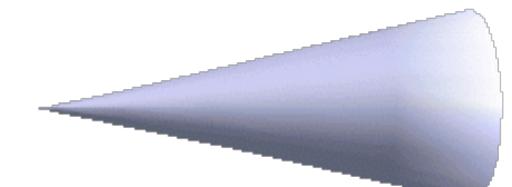
$$R \leq c \Delta t \delta / (1+z)$$



rest frame :
isotropic emission

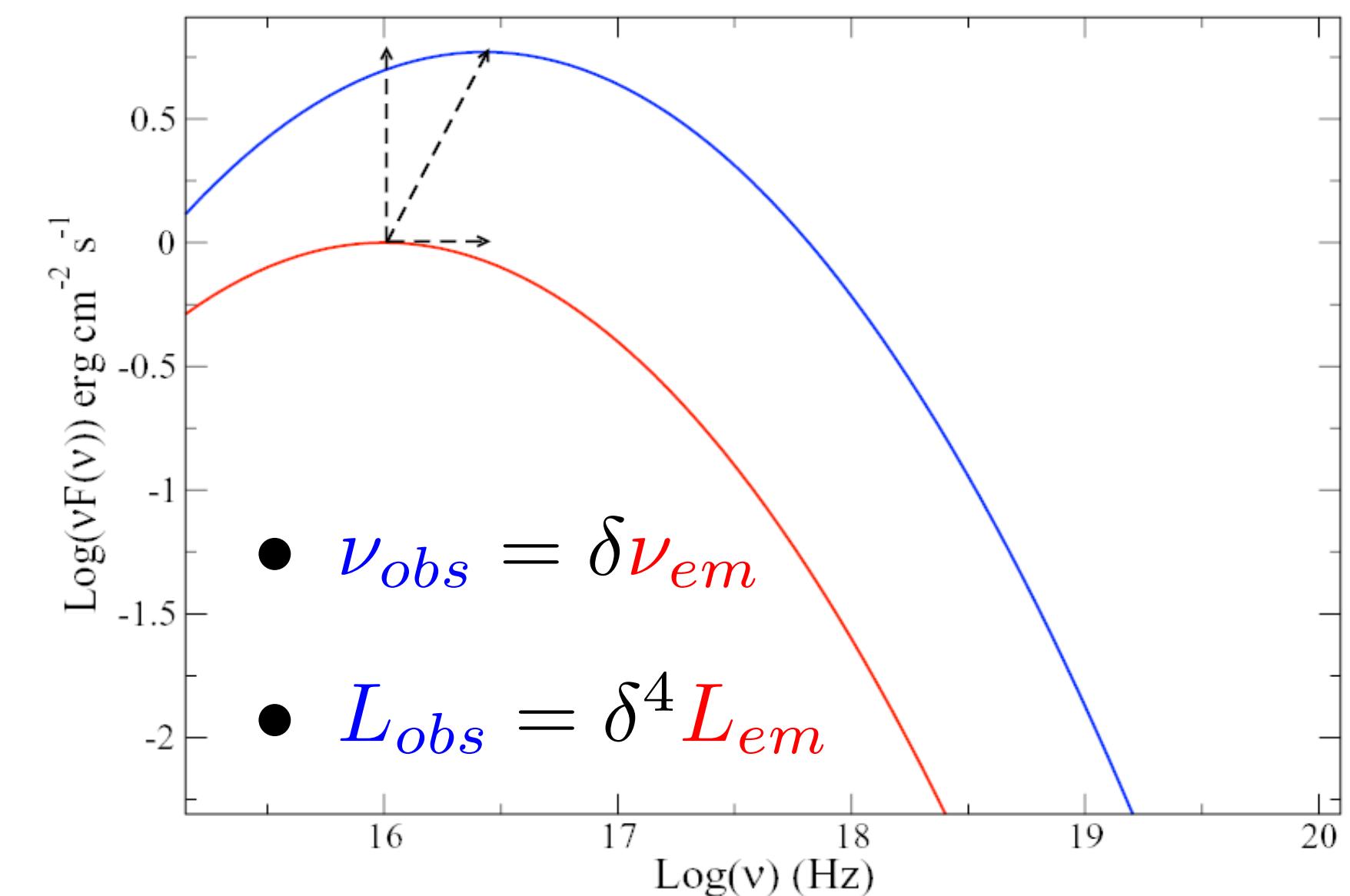


Observer frame: beamed

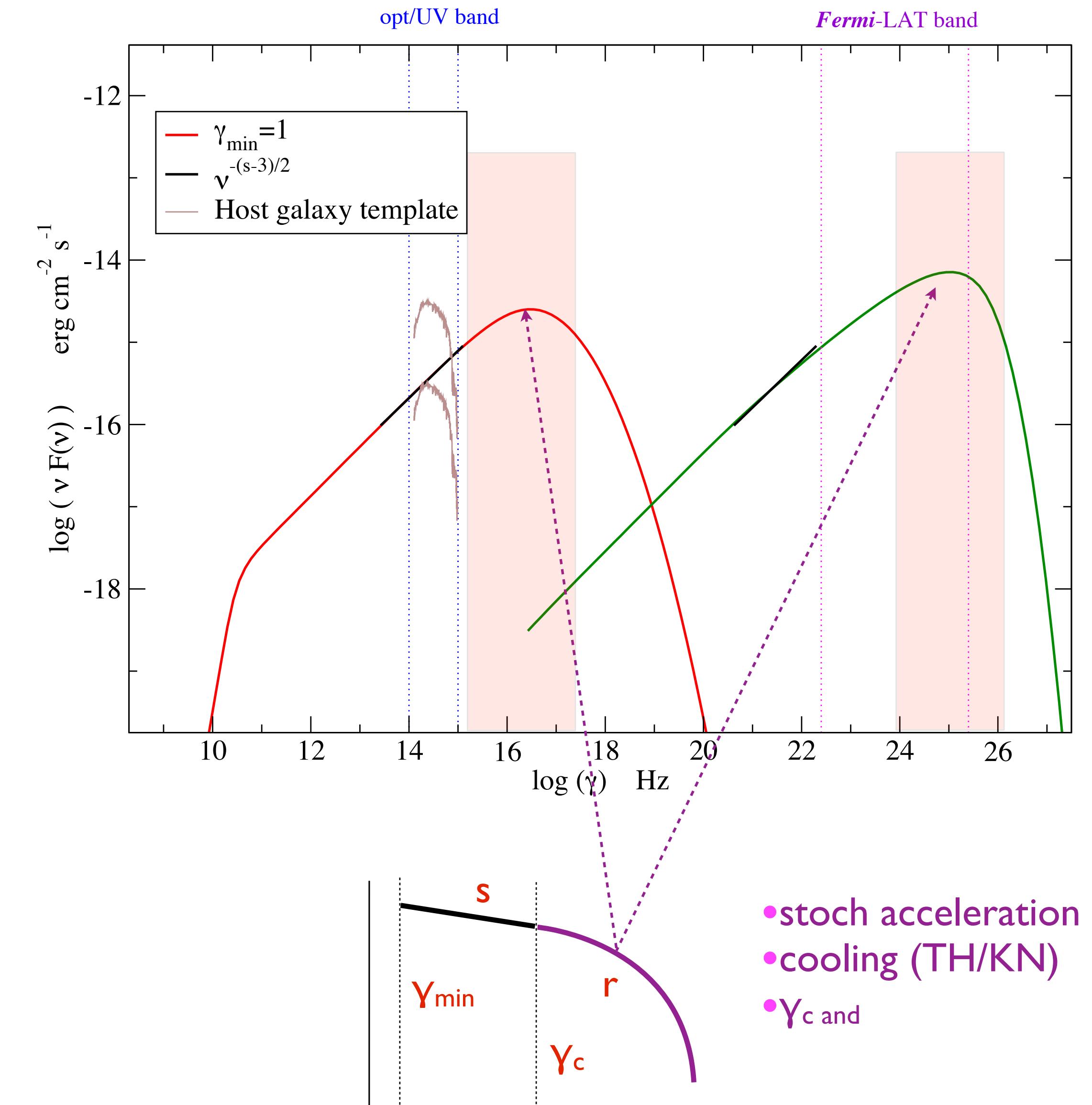
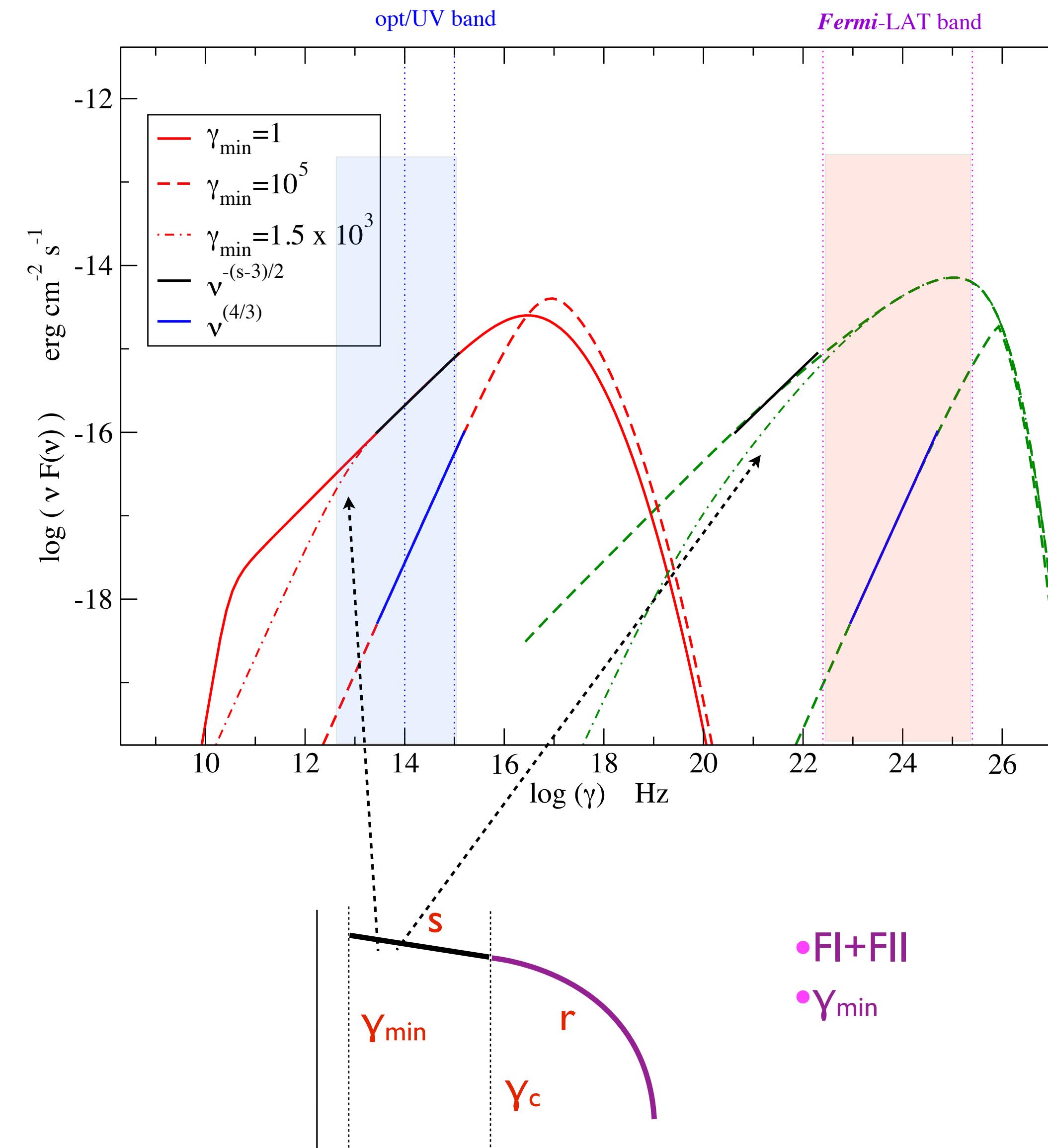


Beaming factor:

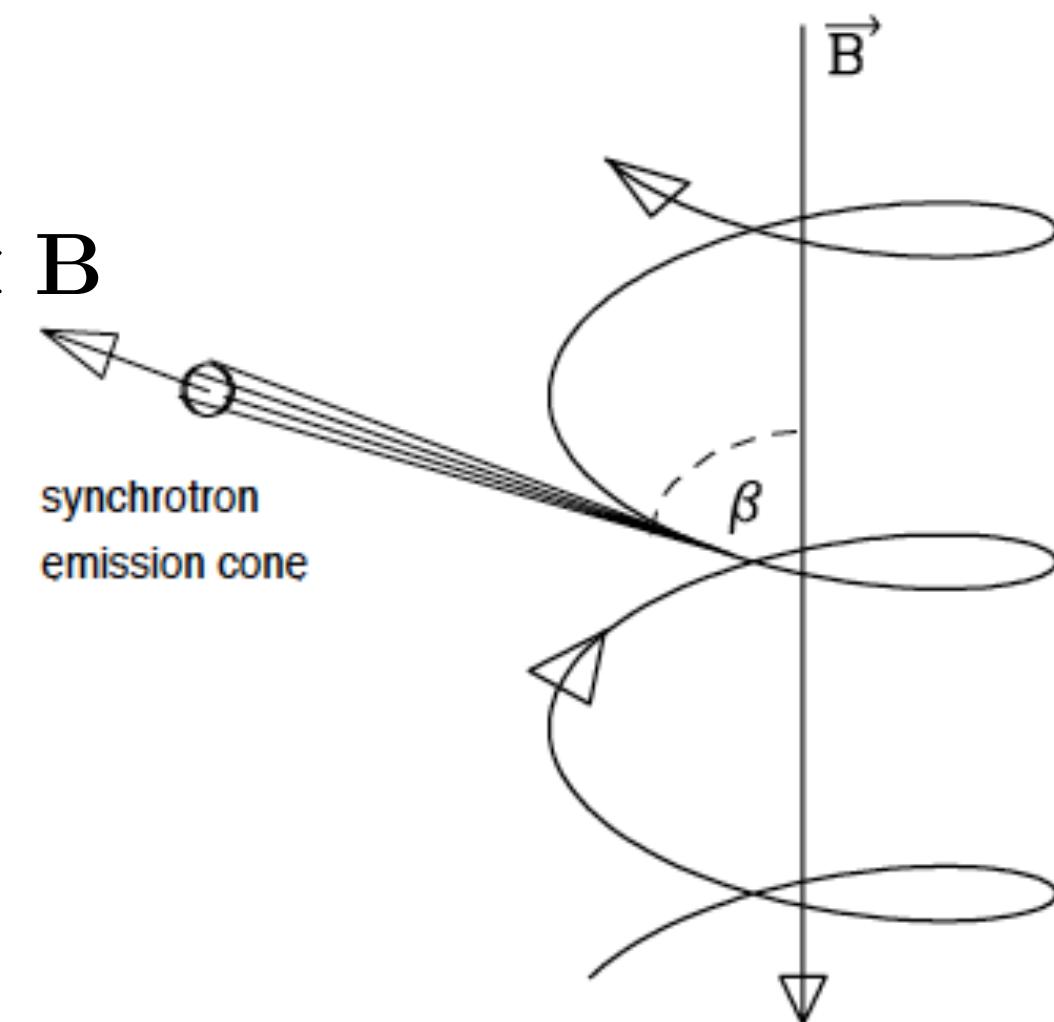
- $\delta = \frac{1}{\Gamma(1-\beta \cos(\theta))}$
- $\theta = 1/\Gamma$



SED shaping and constraining the electron distribution



$$m\gamma \frac{d(\mathbf{v})}{dt} = \frac{q}{c} \mathbf{v} \times \mathbf{B}$$



$$a_{\parallel} = 0$$

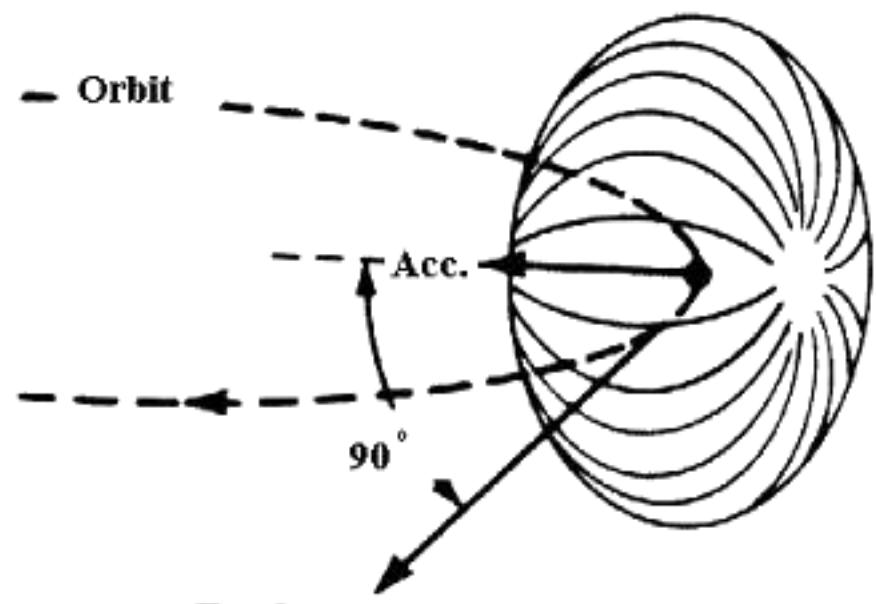
$$a_{\perp} = \frac{evB\sin\alpha}{\gamma m_e c}$$

$$\nu_B = \frac{eB}{2\pi\gamma mc} = \frac{\nu_L}{\gamma}$$

total emitted power

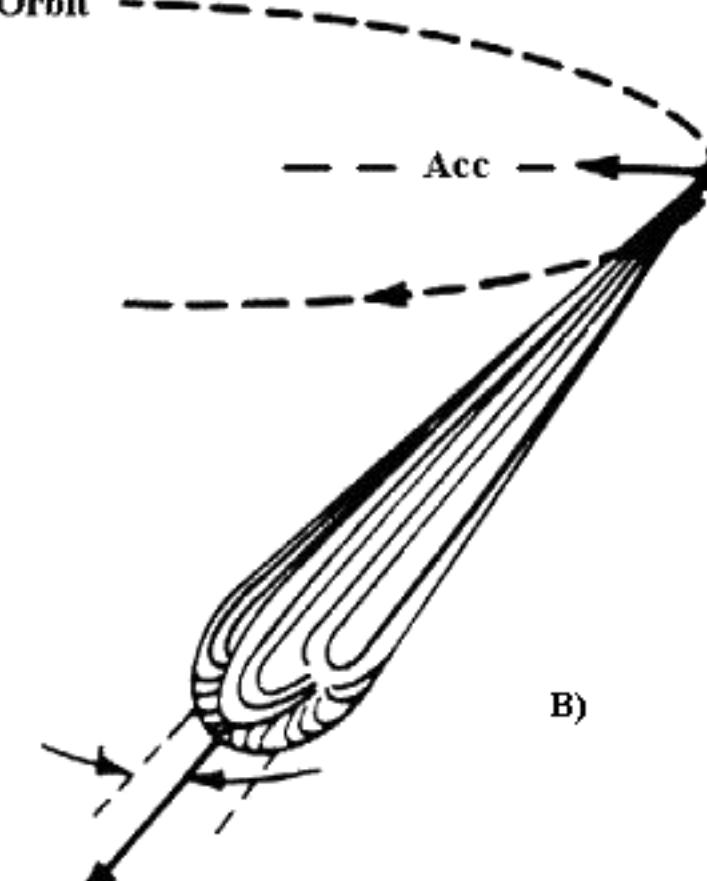
$$P_e = P'_e = \frac{2e^2}{3c^3} a'^2 = \frac{2e^2}{3c^3} [a'^2_{\parallel} + a'^2_{\perp}]$$

$$P_S = \frac{2e^4}{3m_e c^3} B^2 \gamma^2 \beta^2 \sin^2 \alpha$$



A)

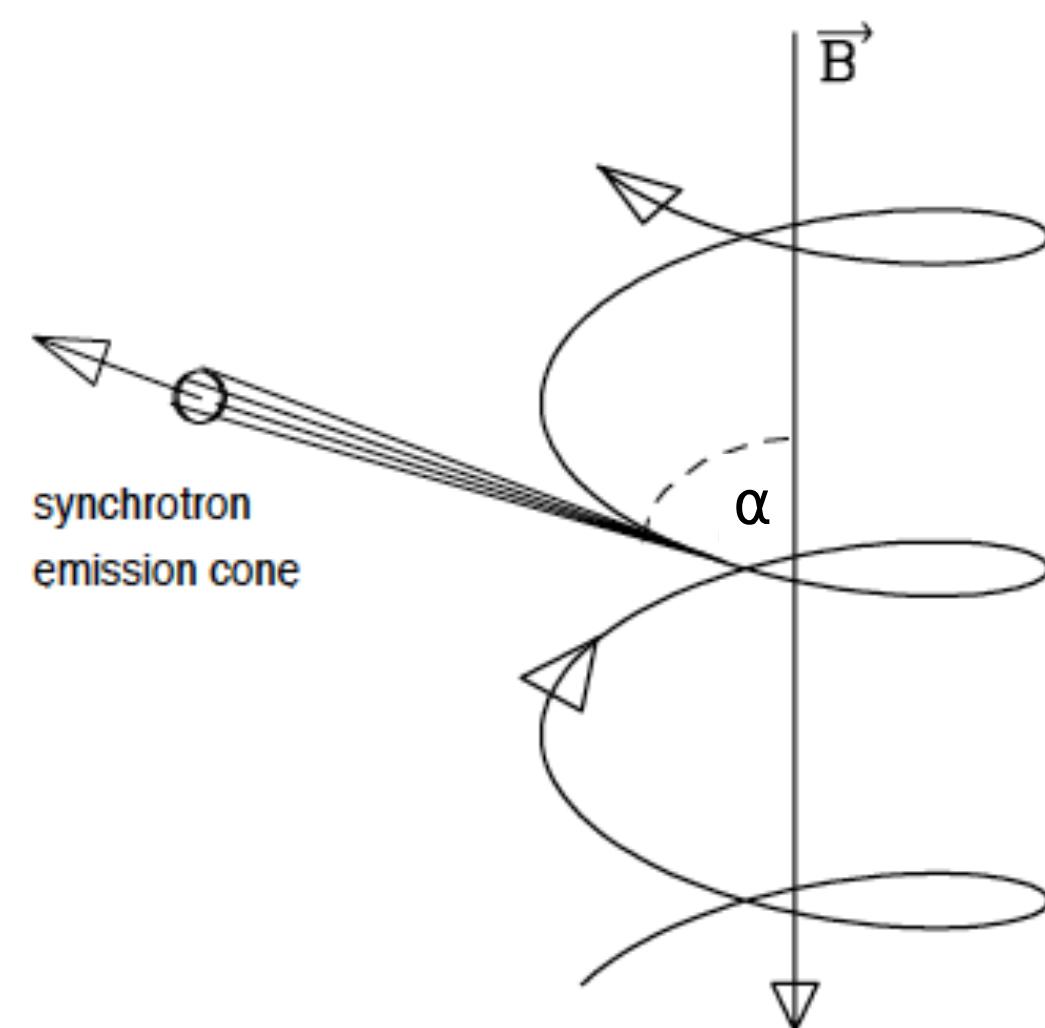
$$\Delta \phi = \frac{1}{\gamma}$$



B)

synchrotron basics

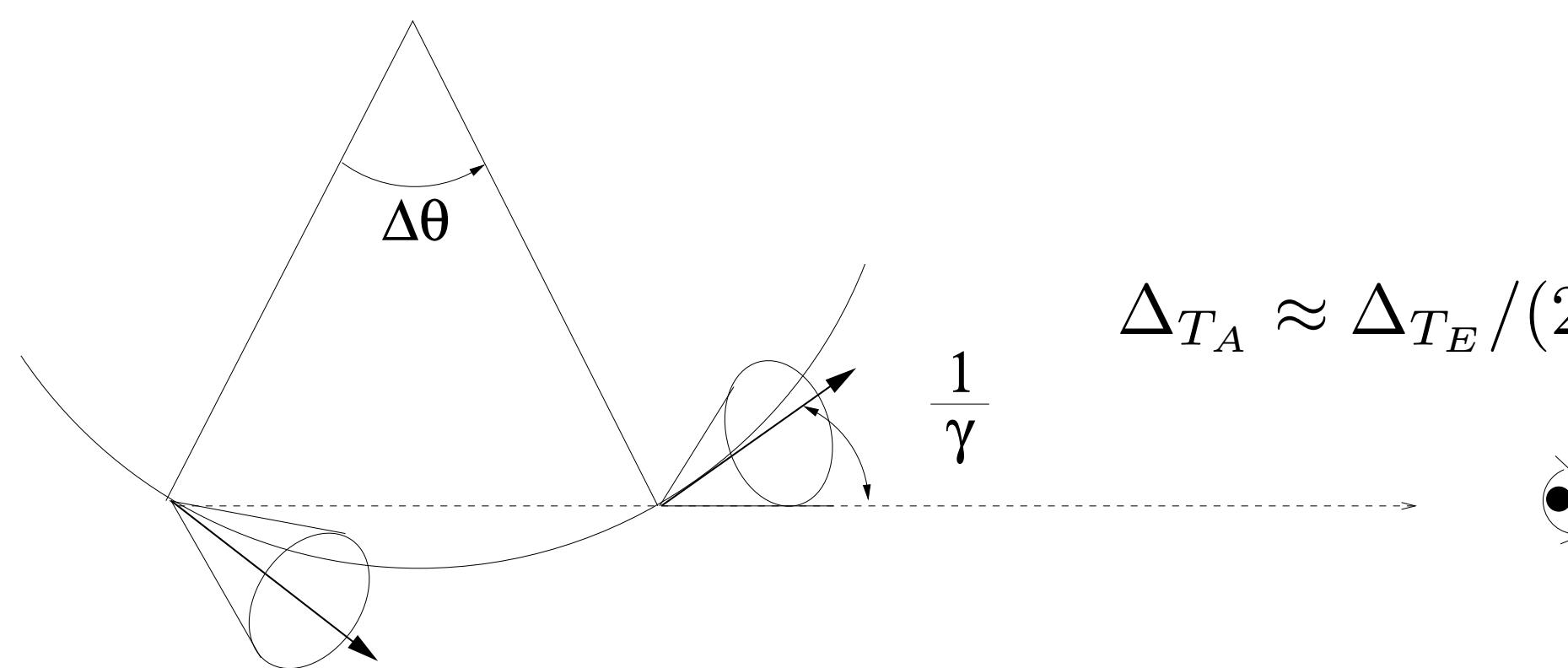
single particle



emitted spectrum

$$P_e(\nu, \gamma) = \frac{\sqrt{3}e^3 B \sin \alpha}{2m_e c^2} F(x)$$

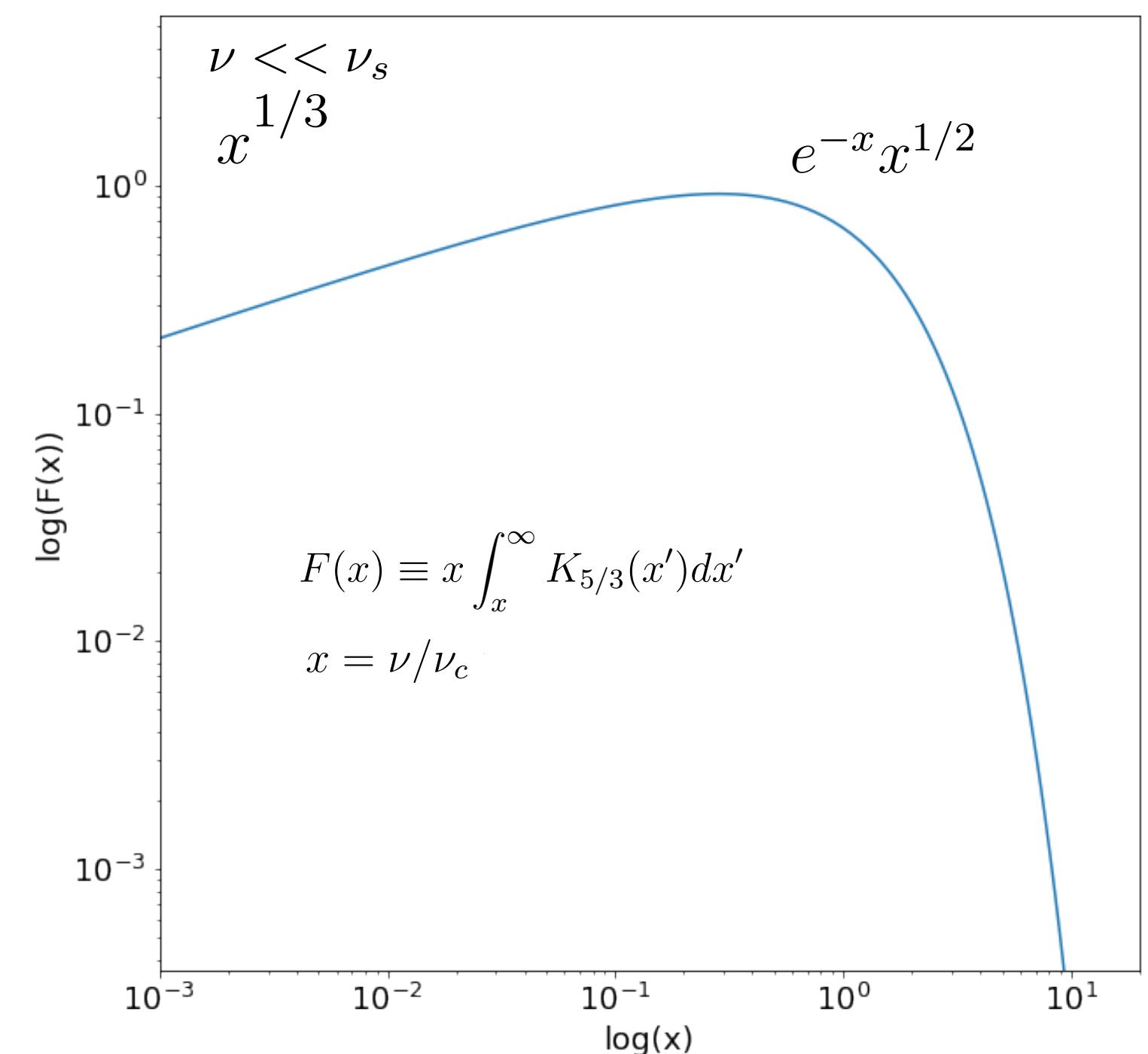
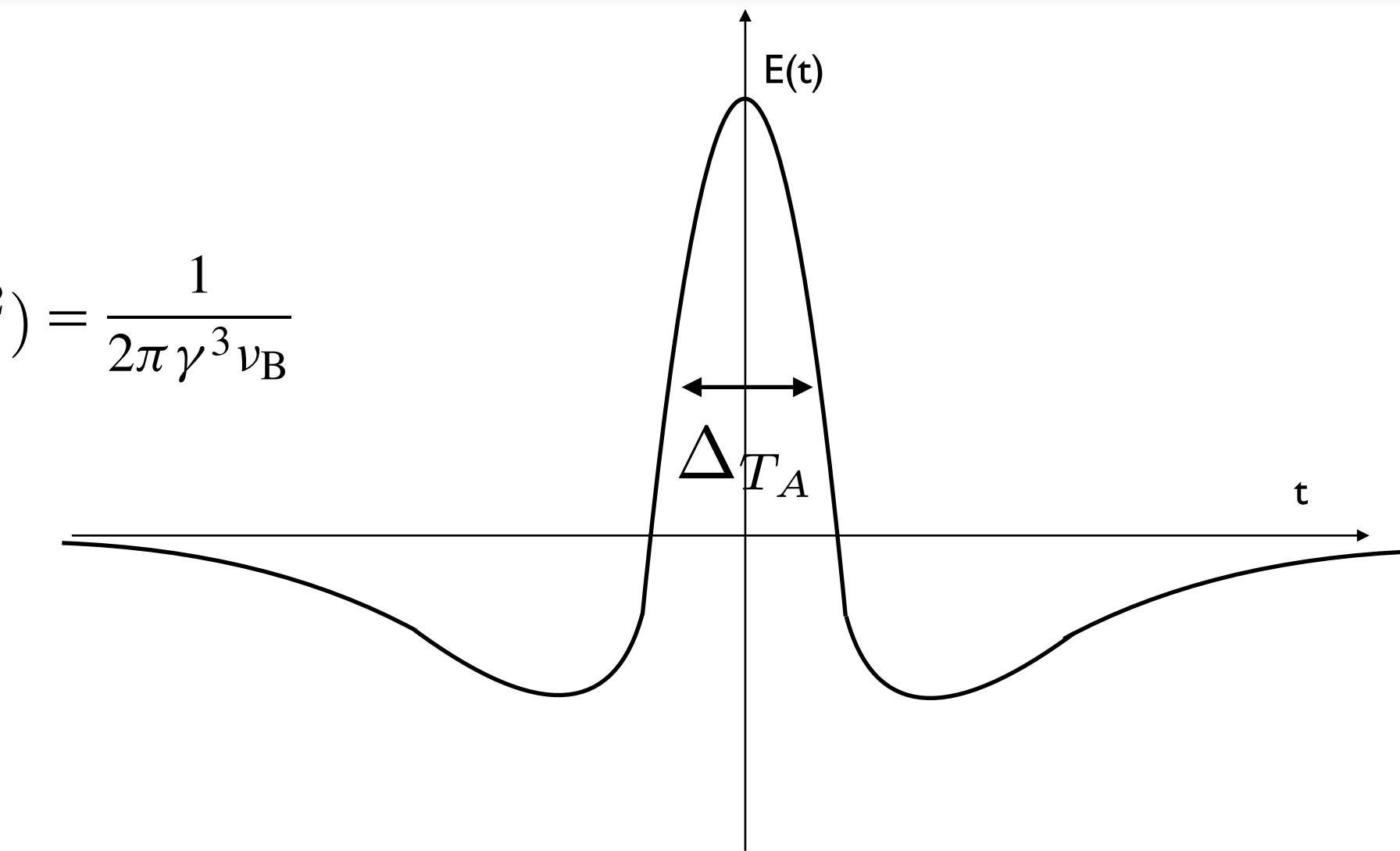
$$\nu_c = \frac{3}{2} \nu_B \gamma^3 \sin \alpha = \frac{3\gamma^2 e B \sin \alpha}{4\pi m_e c}$$



power spectrum

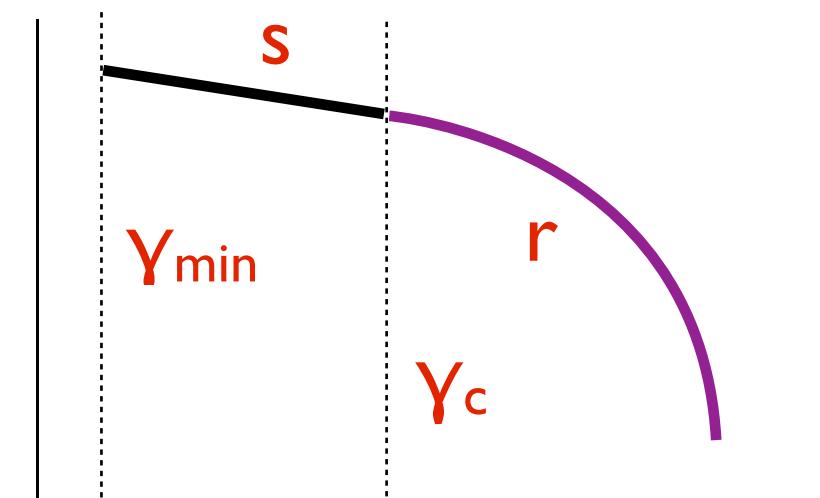
$$\Delta_{T_A} \nu_T = \frac{1}{2\pi} \rightarrow \nu_T = \gamma^3 \nu_B$$

$$\nu_s \propto 10^6 B \gamma^2 \sin(\alpha) \text{ Hz}$$



Tutorial 2

$$N(\gamma) \propto \gamma^{-s}$$



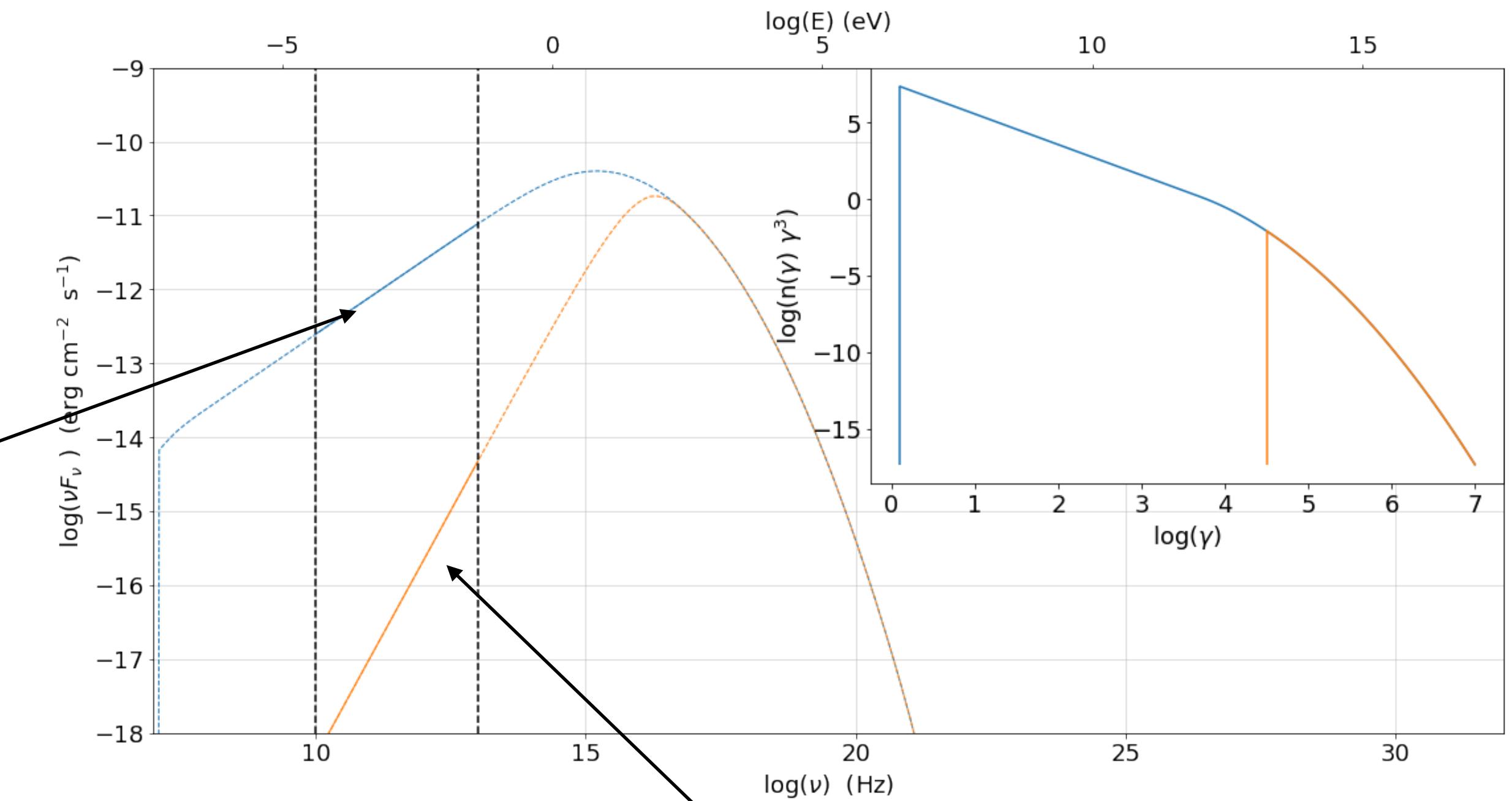
$$j_\nu^S(\nu) = \frac{1}{4\pi} \int_{\gamma_{min}}^{\gamma_{max}} P(\nu, \gamma) N(\gamma) d\gamma \propto \nu^{-\frac{s-1}{2}}$$

F_ν	refers to spectral index => SED =	$\frac{\nu F_\nu}{\nu I_\nu}$
I_ν		$\frac{\nu F_\nu}{\nu L_\nu}$
L_ν		

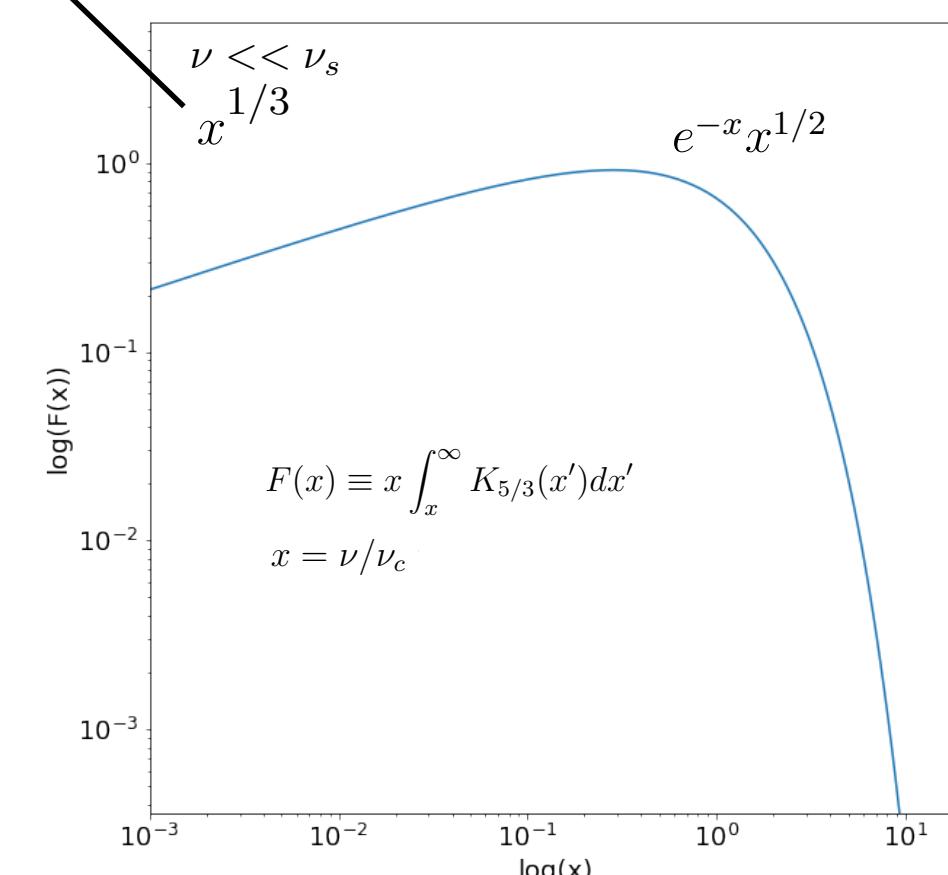
δ -approx relations

$$S_p^{Sync} \sim \frac{dN(\gamma)}{d\gamma} \gamma_{3p}^3 B^2 \delta^4 \quad \nu_p^{Sync} \sim 3.2 \times 10^6 (\gamma_{3p})^2 B \delta$$

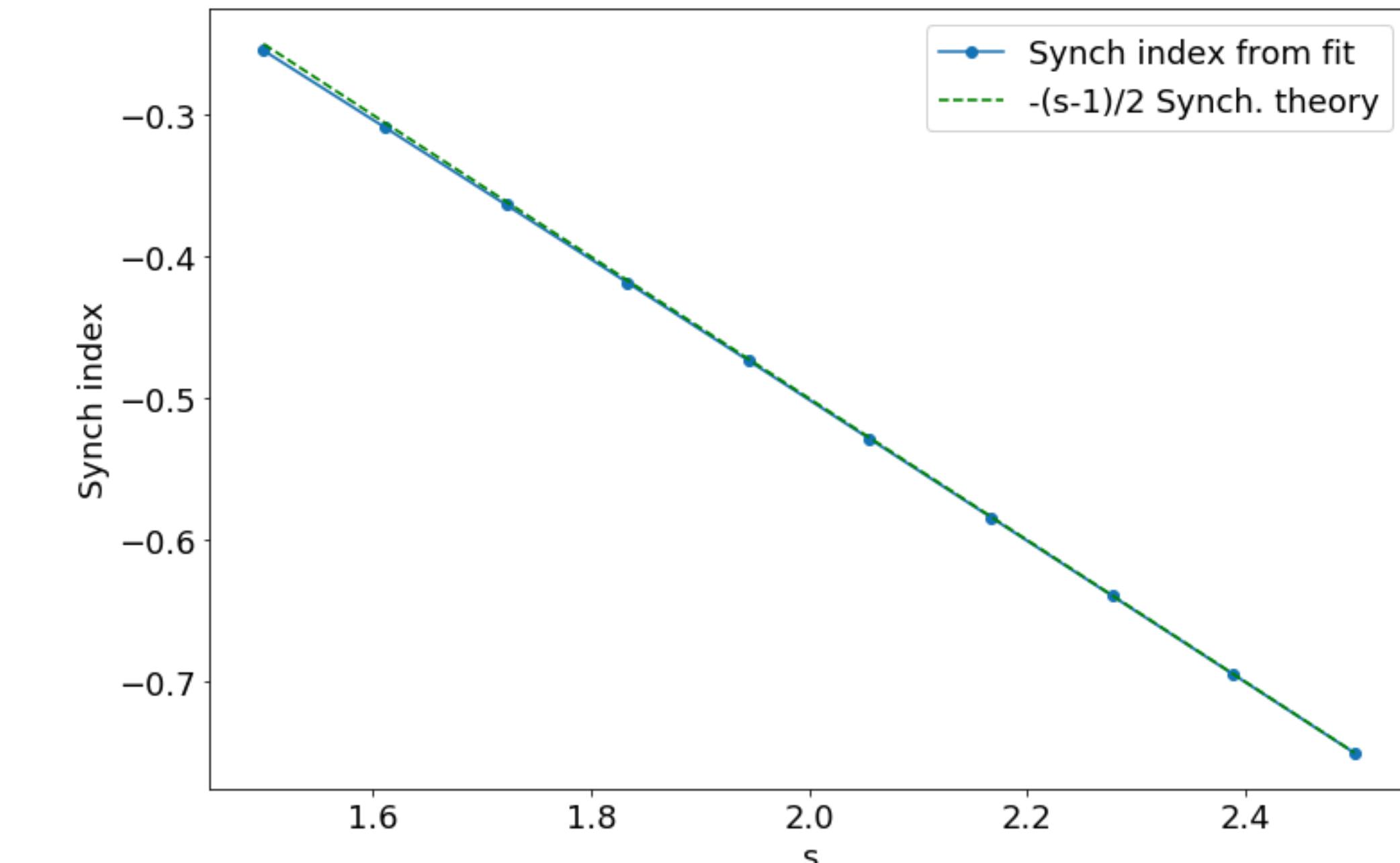
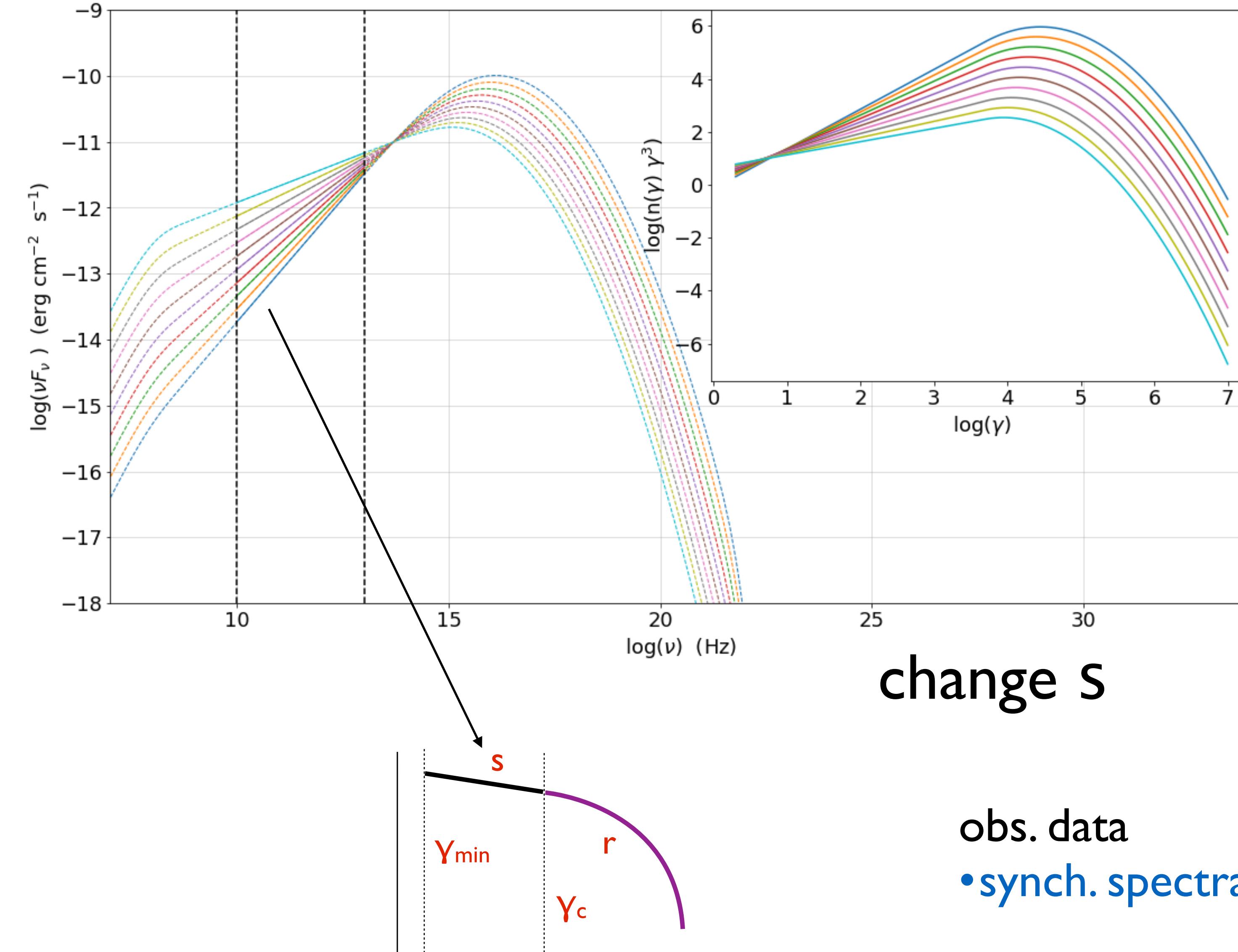
$$\text{SED} \propto N(\gamma) \gamma^3$$



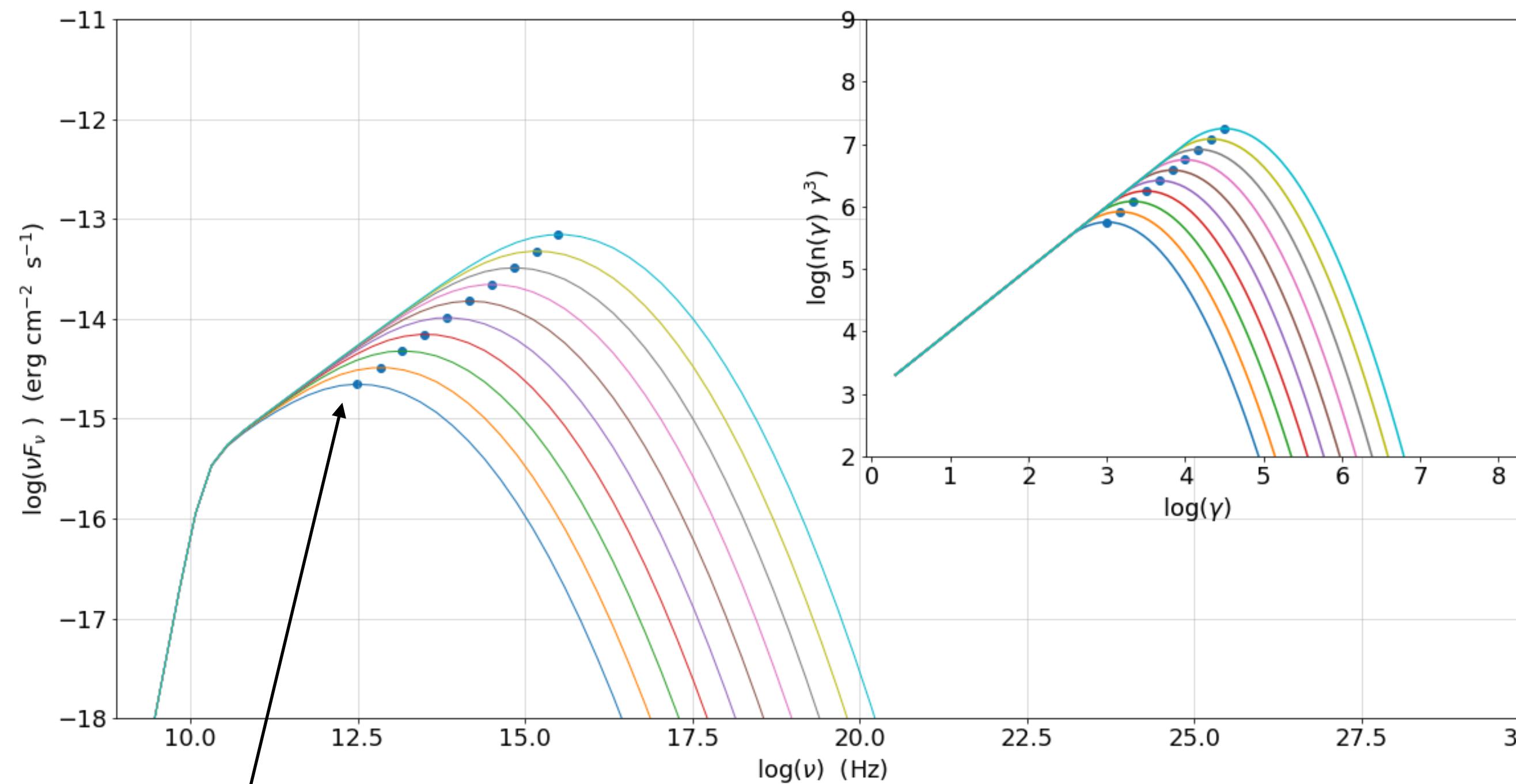
4/3 in SED corresponds
to 1/3 in spectral index



Tutorial 2

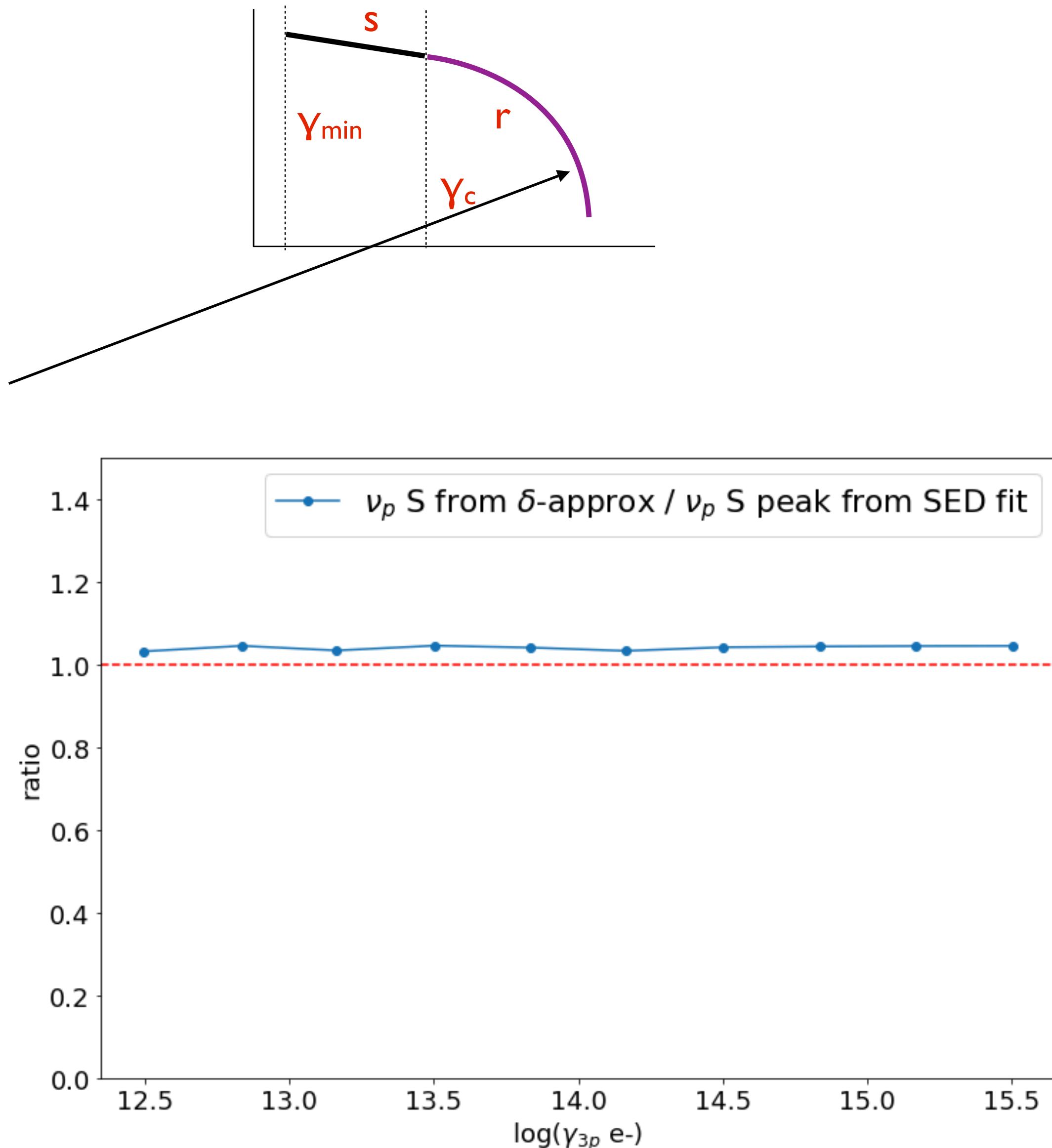


$$j_\nu^S(\nu) = \frac{1}{4\pi} \int_{\gamma_{min}}^{\gamma_{max}} P(\nu, \gamma) N(\gamma) d\gamma \propto \nu^{-\frac{s-1}{2}}$$

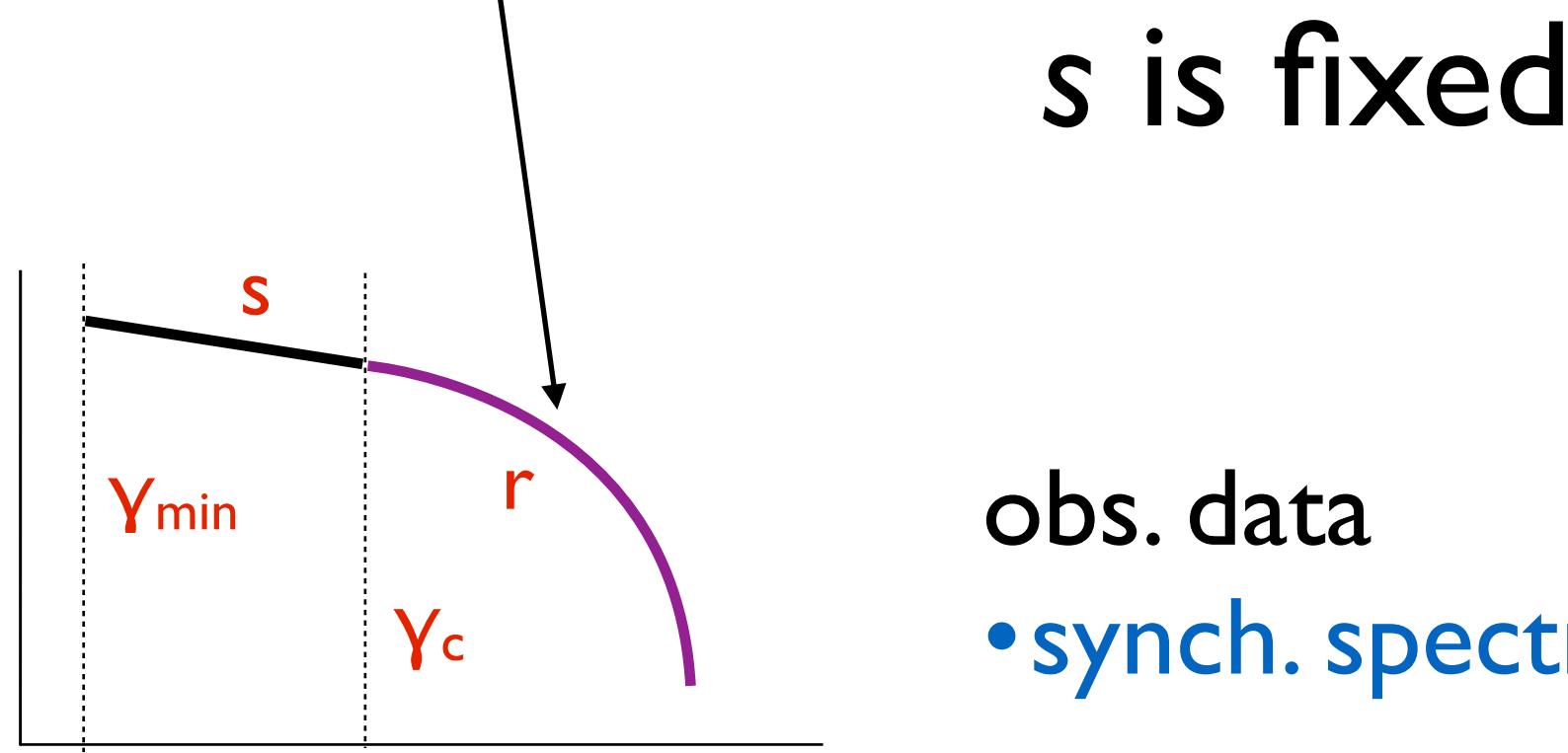
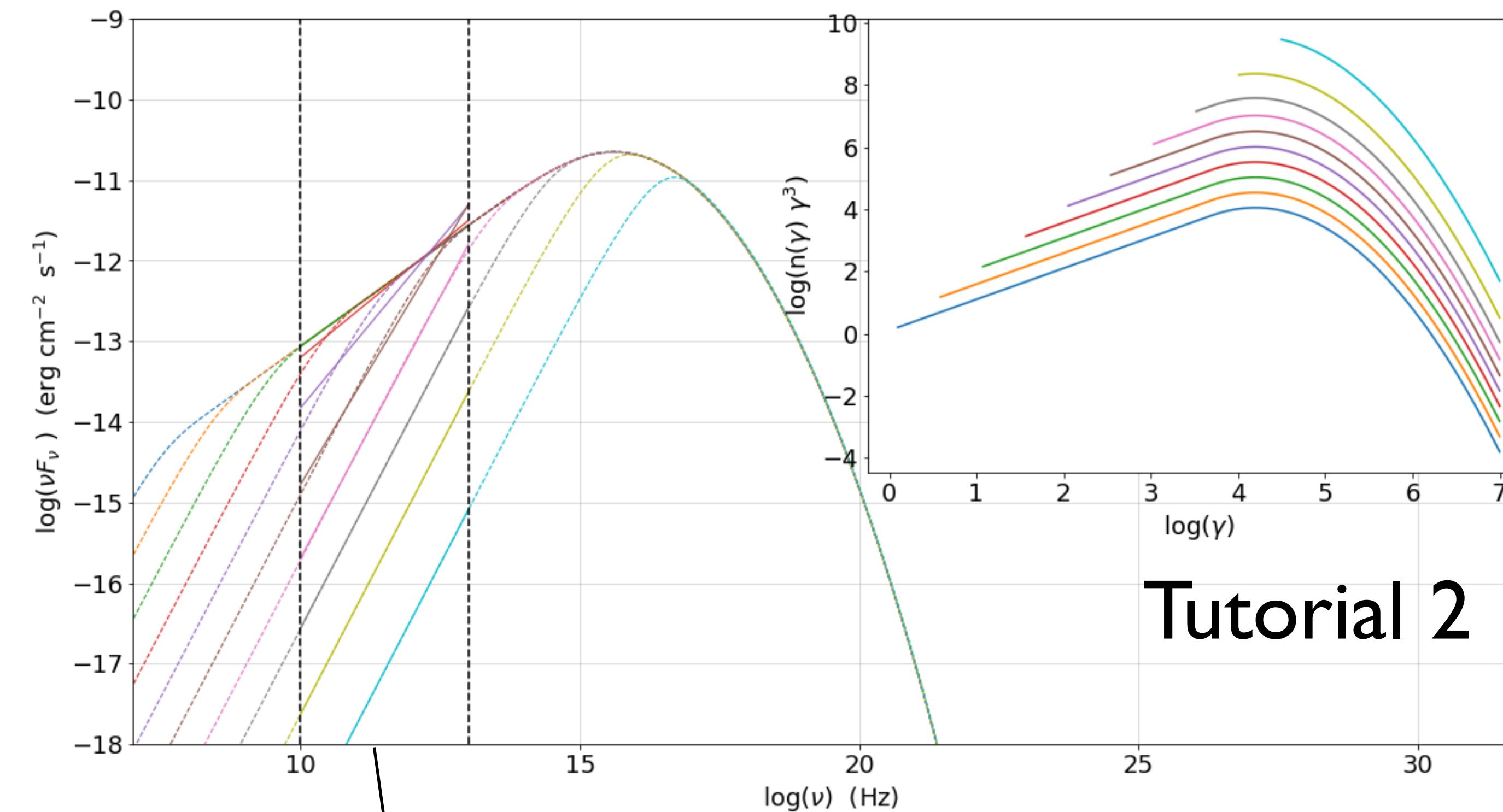
change γ_c

obs. data \longrightarrow model

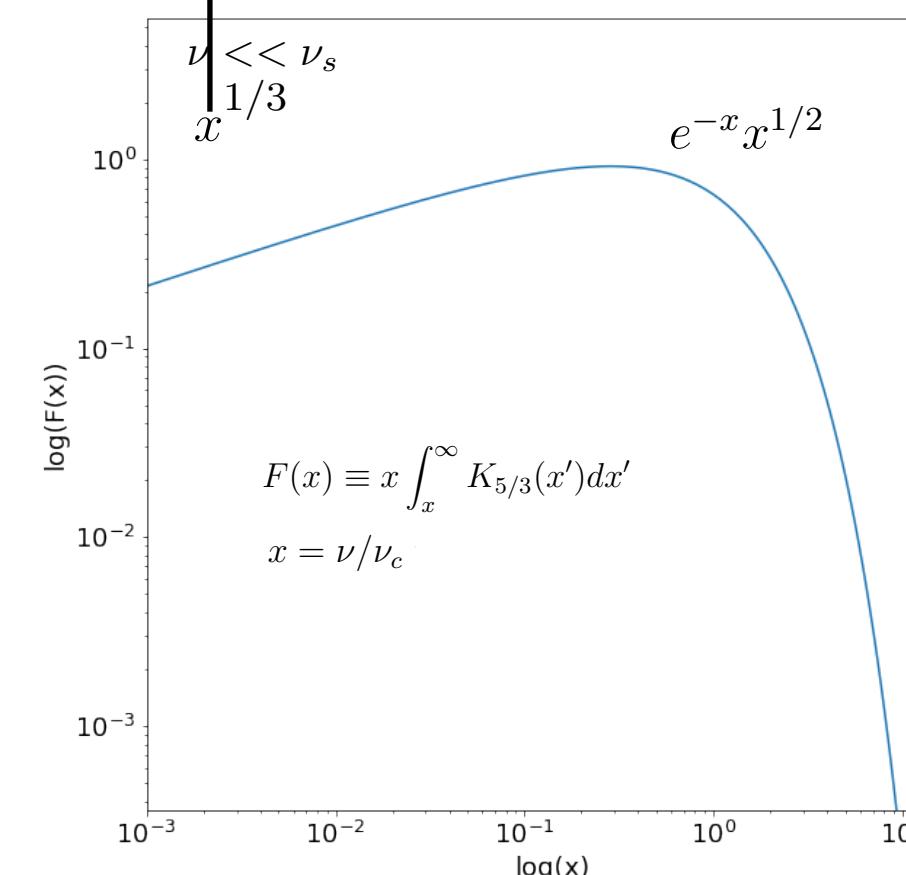
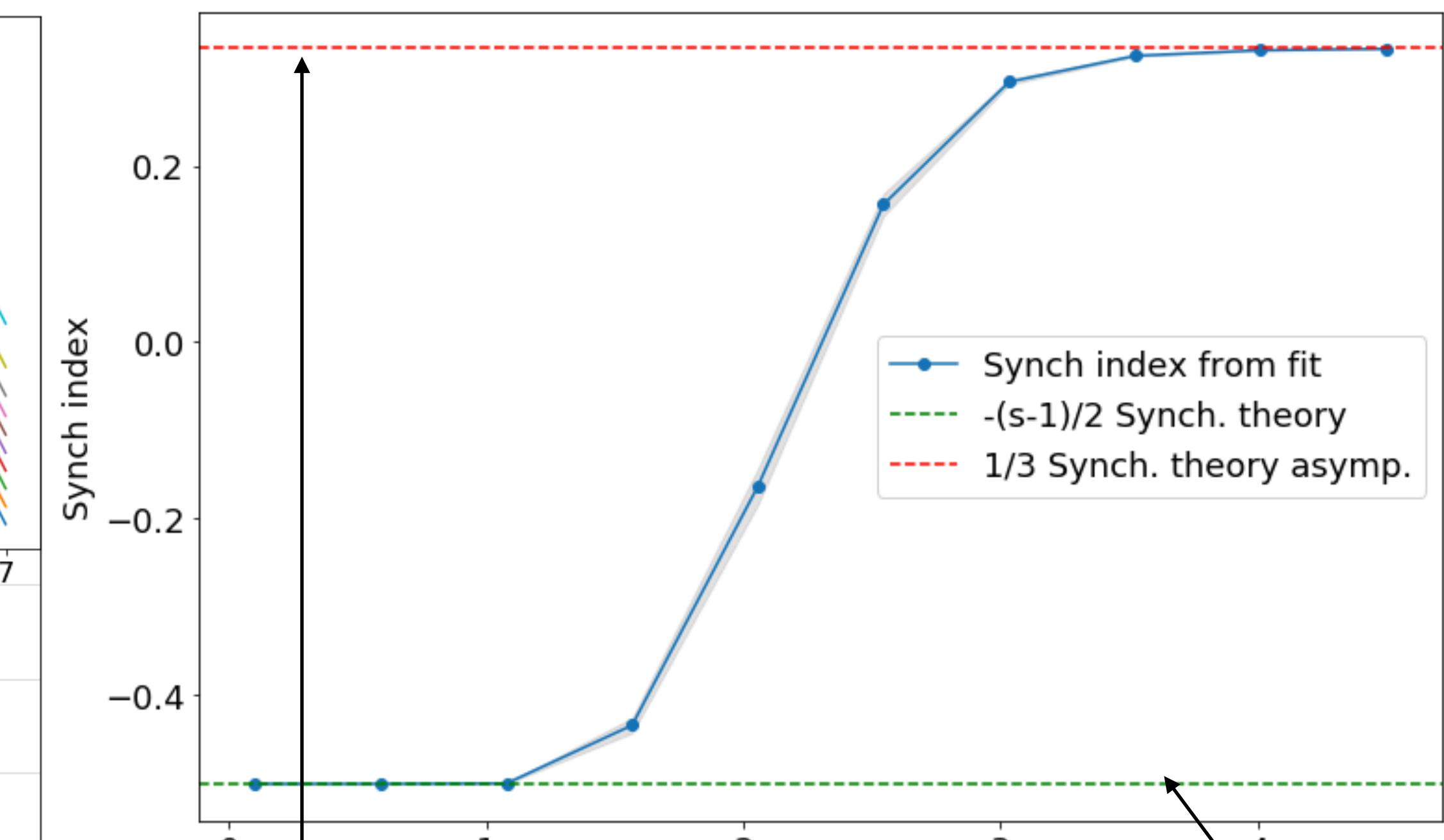
- ν_p^s
- γ_p^s



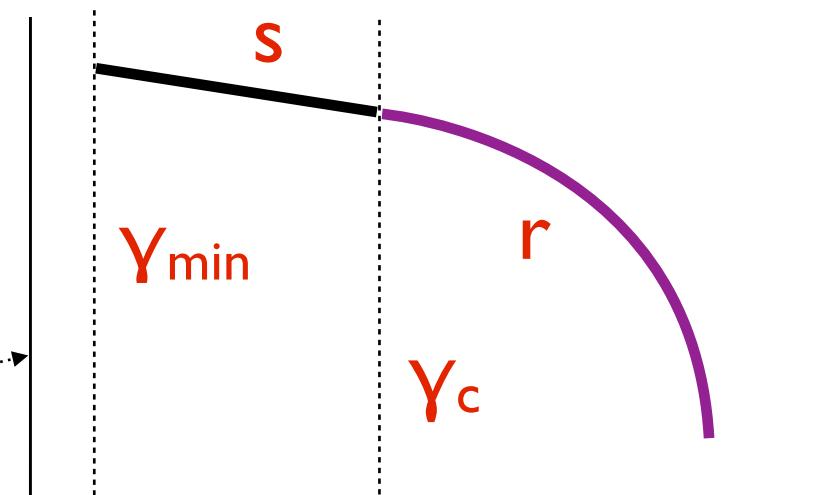
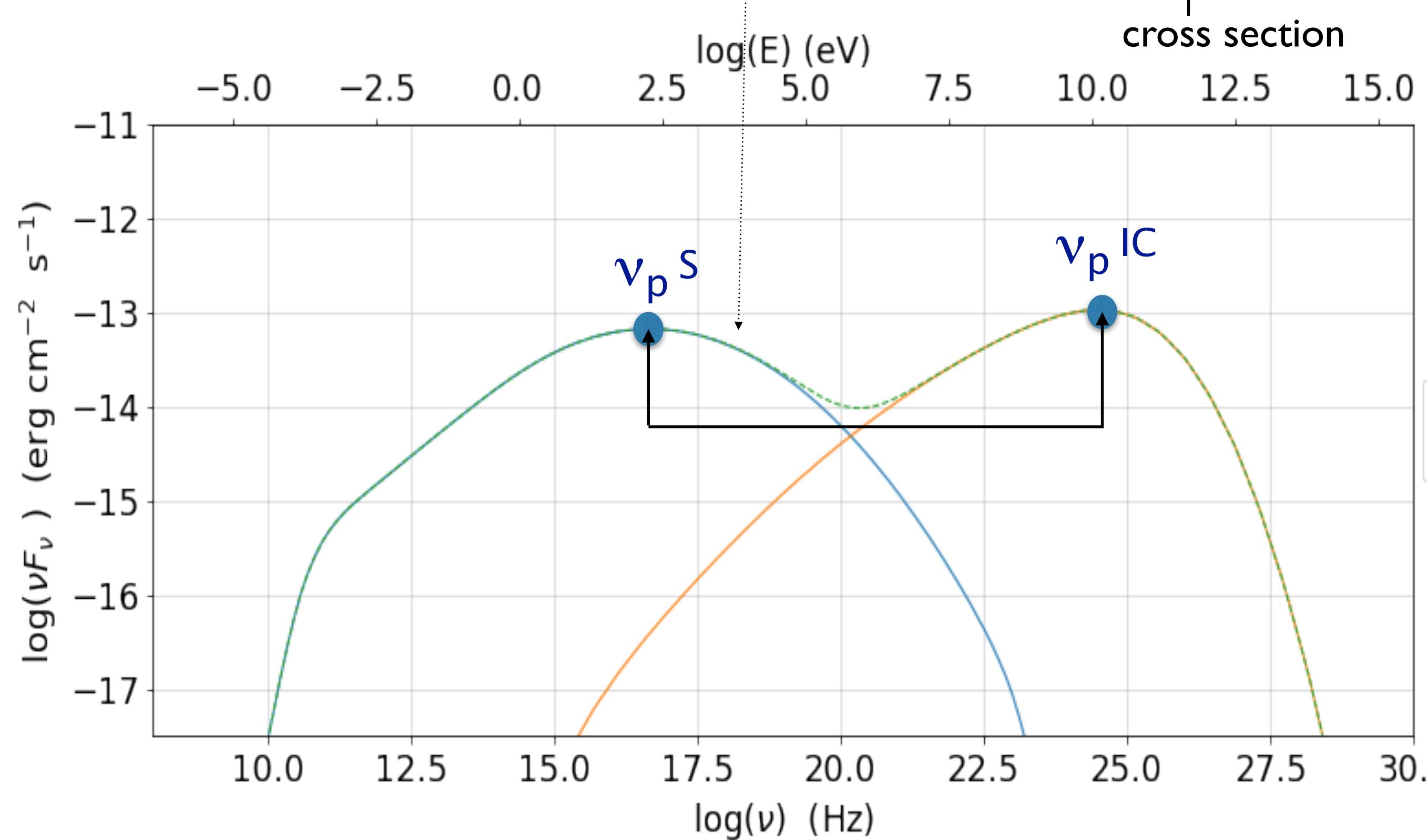
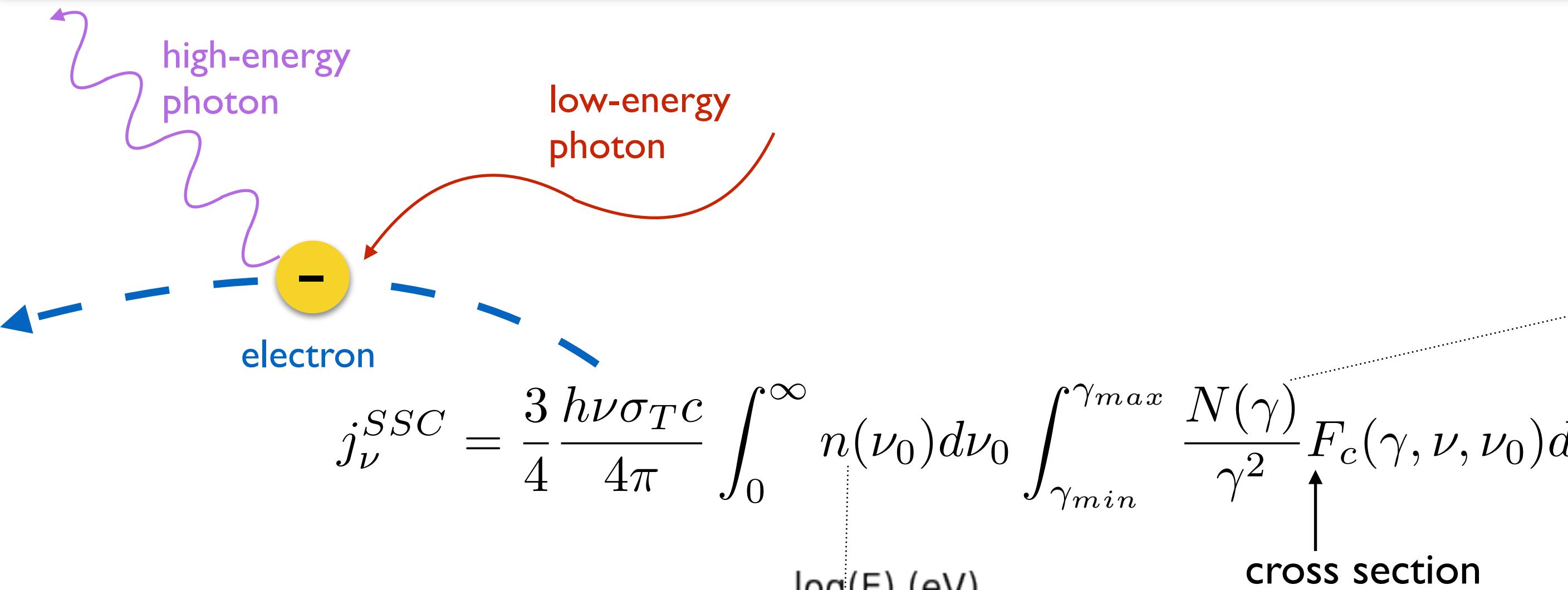
Synchrotron emission Estimate of γ_{min} from spectral index



obs. data → model
 • synch. spectral index
 • γ_{min}



IC emission basics



Tutorial 2

soft photon energy
in the e- rest frame

$$\epsilon' = \frac{h\nu'}{m_e c^2}$$

$$\epsilon' \ll m_e c^2$$

TH regime

$$\nu_p^{IC} / \nu_p^{SSC} \sim (4/3) \gamma_p^2$$

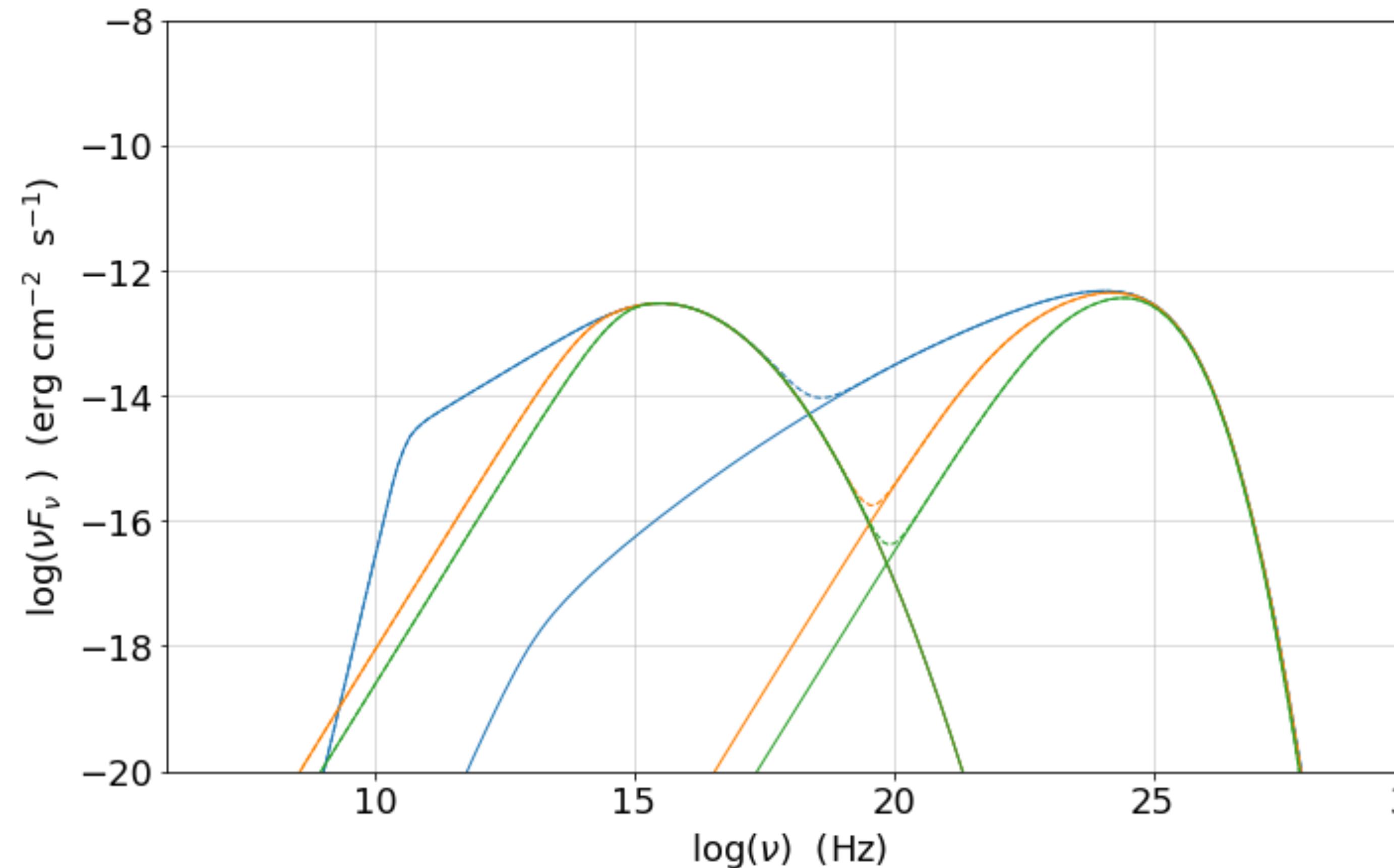
$$\epsilon' \geq m_e c^2$$

KN regime

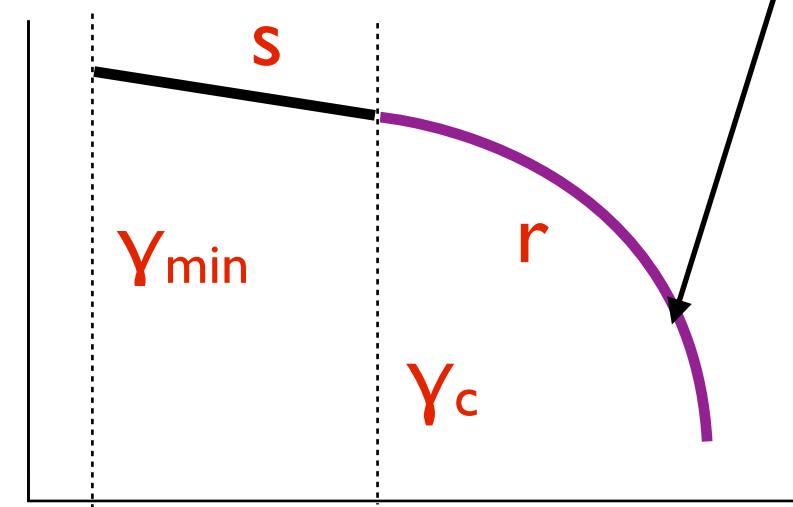
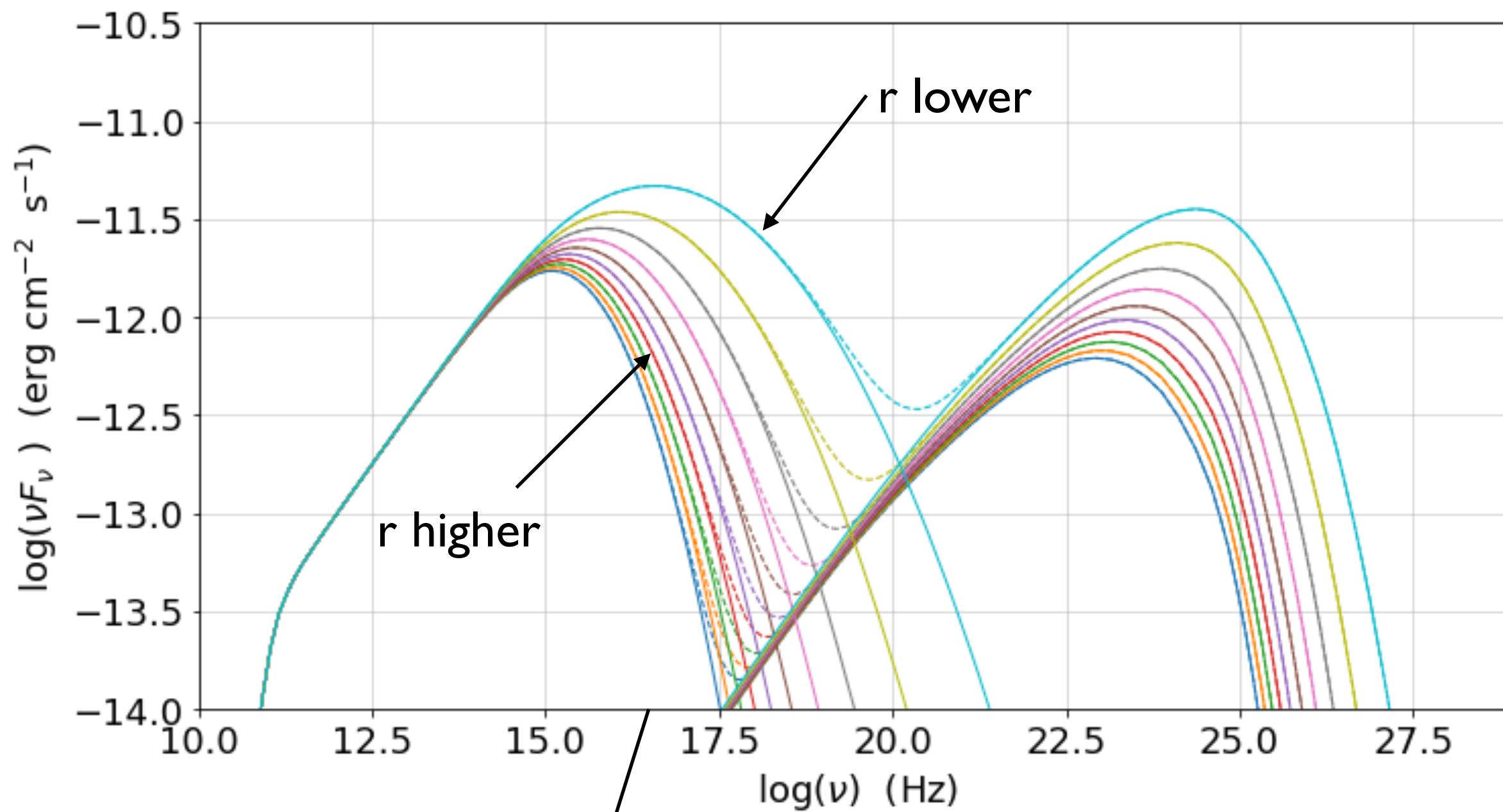
$$\nu_p^{IC} / \nu_p^{SSC} \sim \gamma_p$$

$$h\nu_p^{IC} \sim m_e c^2 \gamma_p$$

Adding the IC emission SSC case



IC emission TH/KN regime and peak curvature



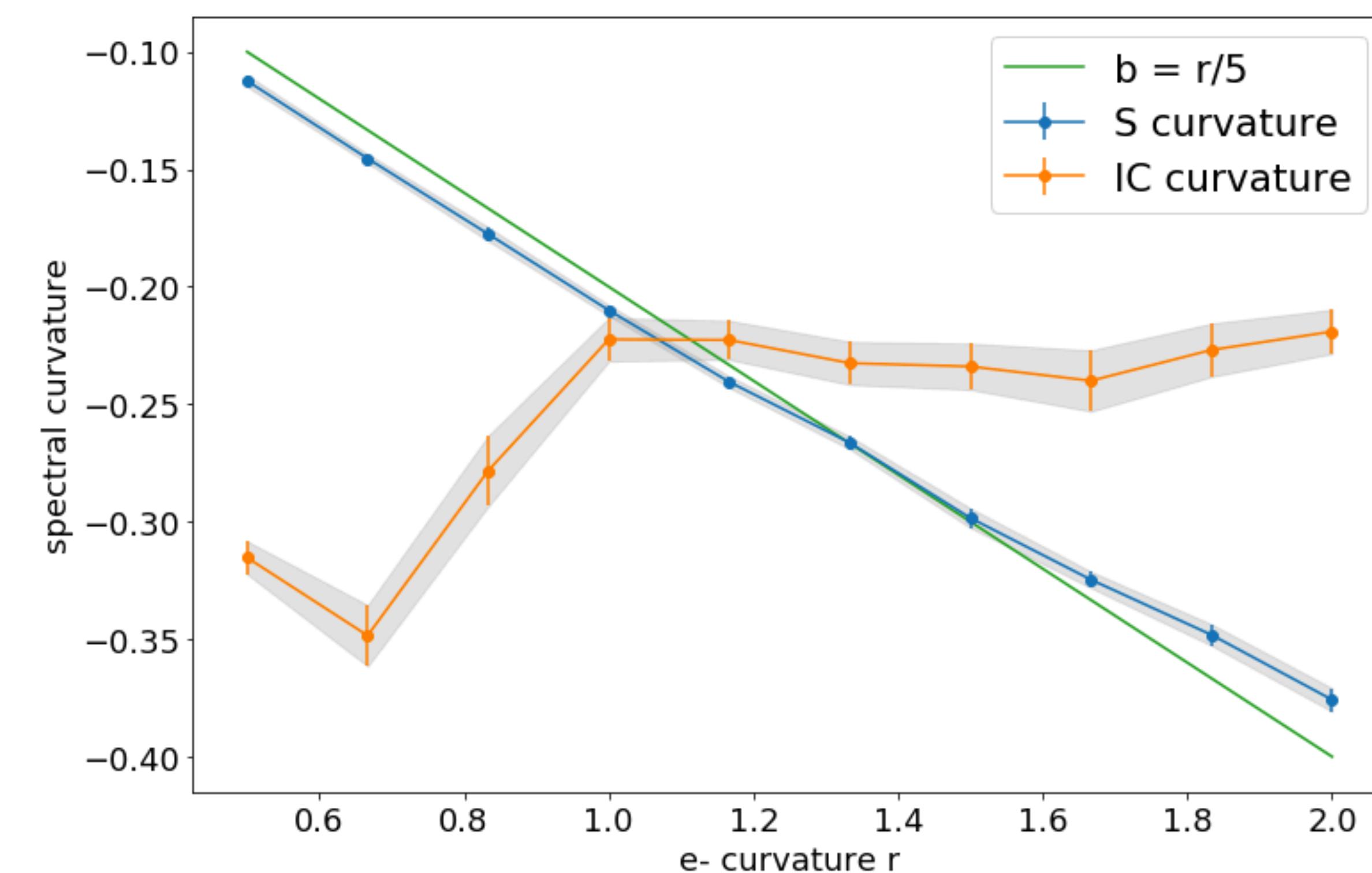
obs. data
• spectral curvature



model
• KN regime
• r

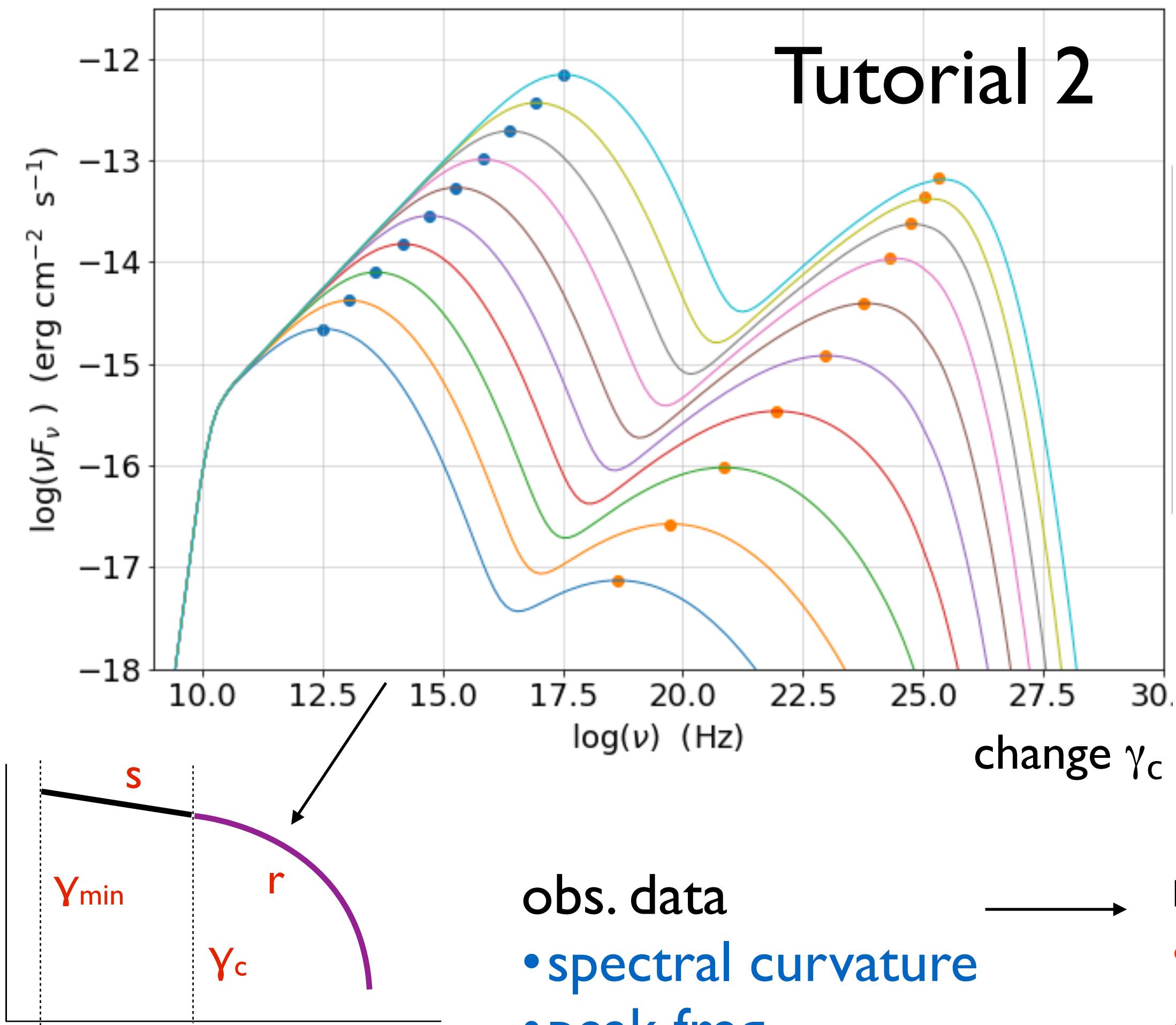
change r

Tutorial 2



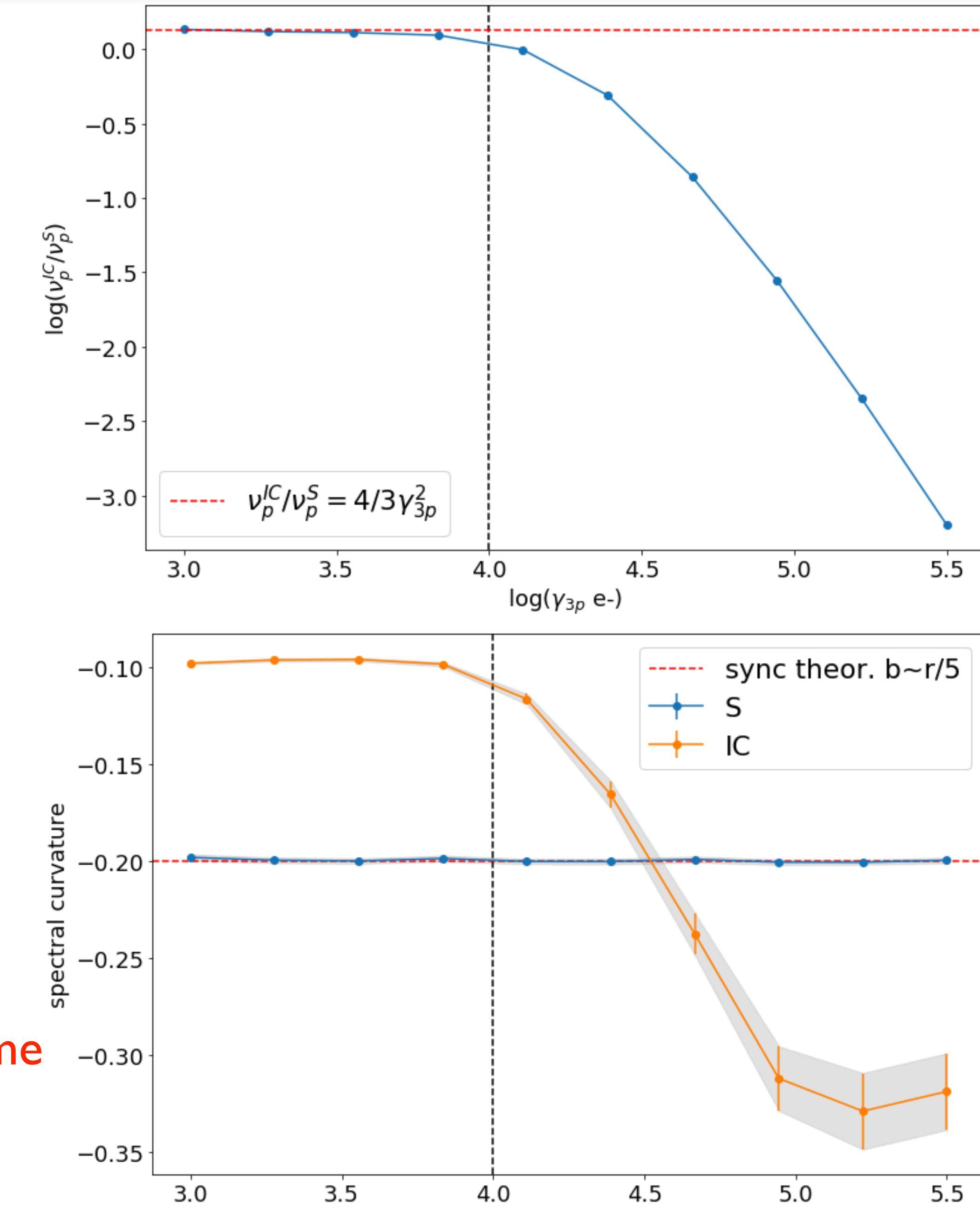
IC emission TH/KN regime and peak freq.

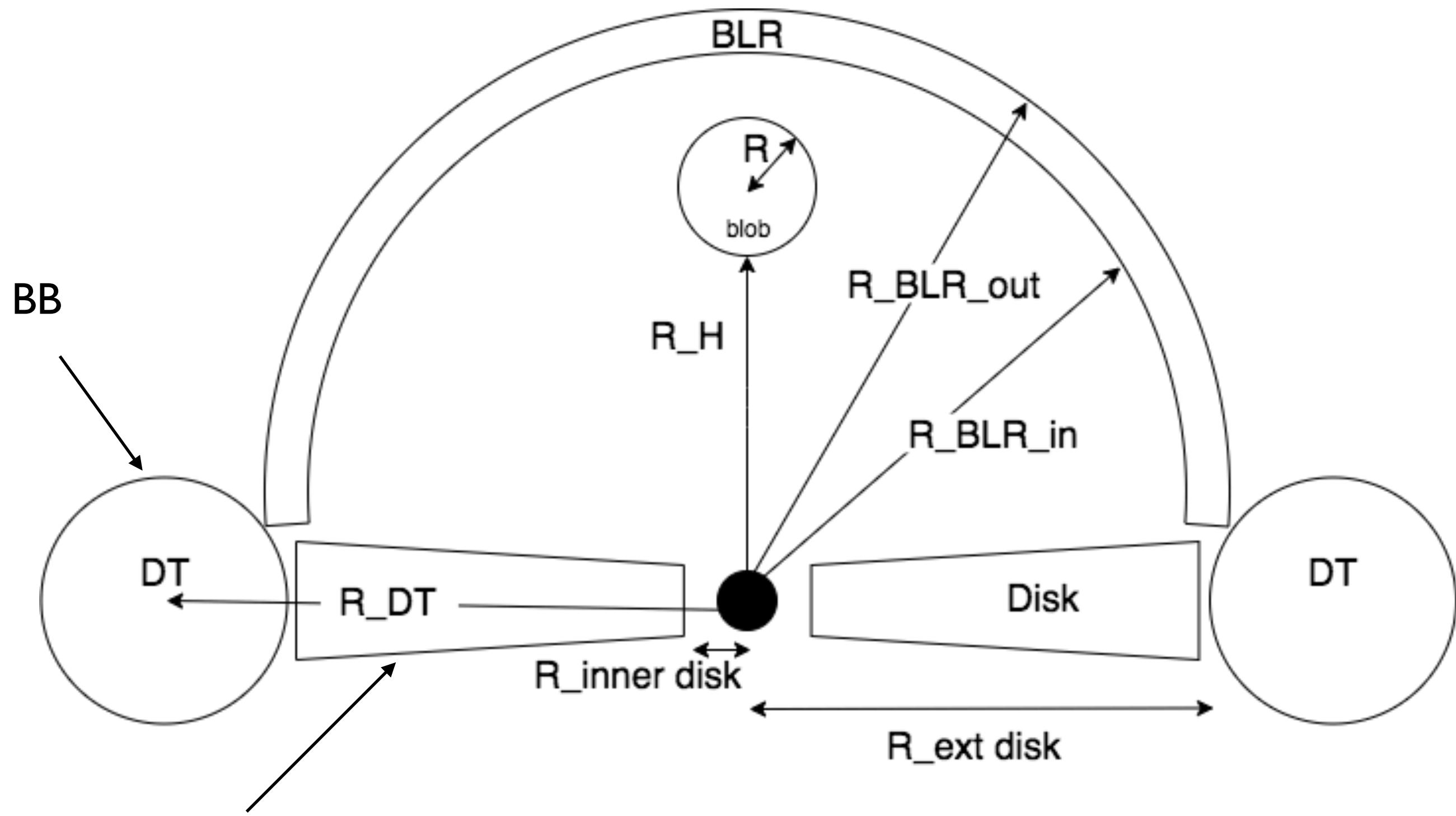
- $v_p^{IC} / v_p^S \sim (4/3) \gamma_p^2 \equiv \gamma_{3p}^2$ is true only in TH regime



- obs. data
- spectral curvature
 - peak freq.

- model
- KN regime
 - γ_p, γ_c





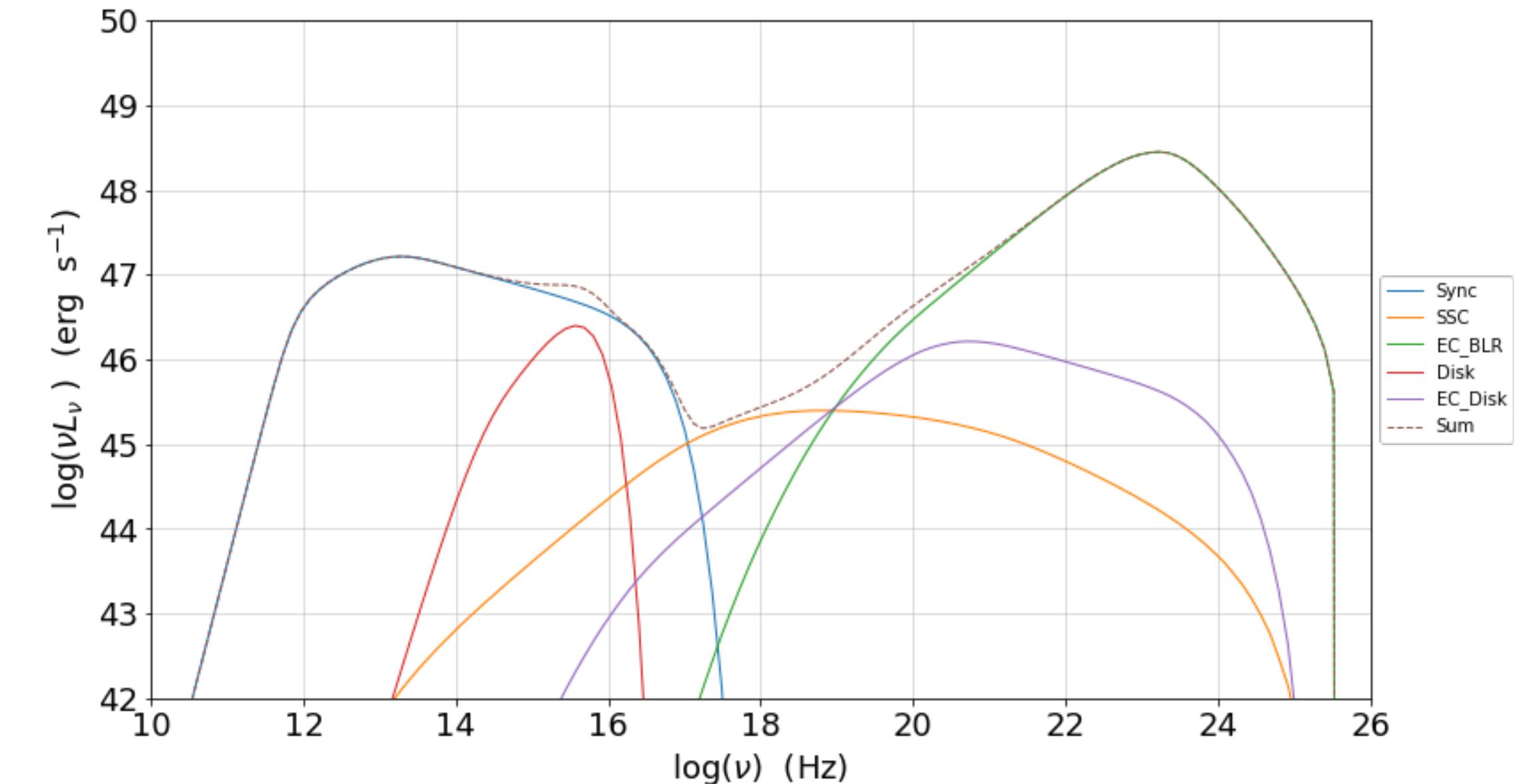
multi T BB $T(R) \approx T_*(R/R_*)^{-3/4}$

$$I_{v'} = \frac{1}{4\pi} \int d\Omega' \delta^3 I_{v=(v'/\Gamma)} \\ = \Gamma \tau \frac{L_{\text{nuc}}}{4\pi R^2} f_{v=(v'/\Gamma)}(T_{\text{ext}})$$

$$u'_{ext} \simeq \Gamma^2 u_{ext} \\ L_{ERC} \simeq \Gamma^6 U_{ext}$$

$$\eta = \frac{\dot{\gamma}_{IC}}{\dot{\gamma}_{sync}} = \frac{U_{ph}}{U_B}$$

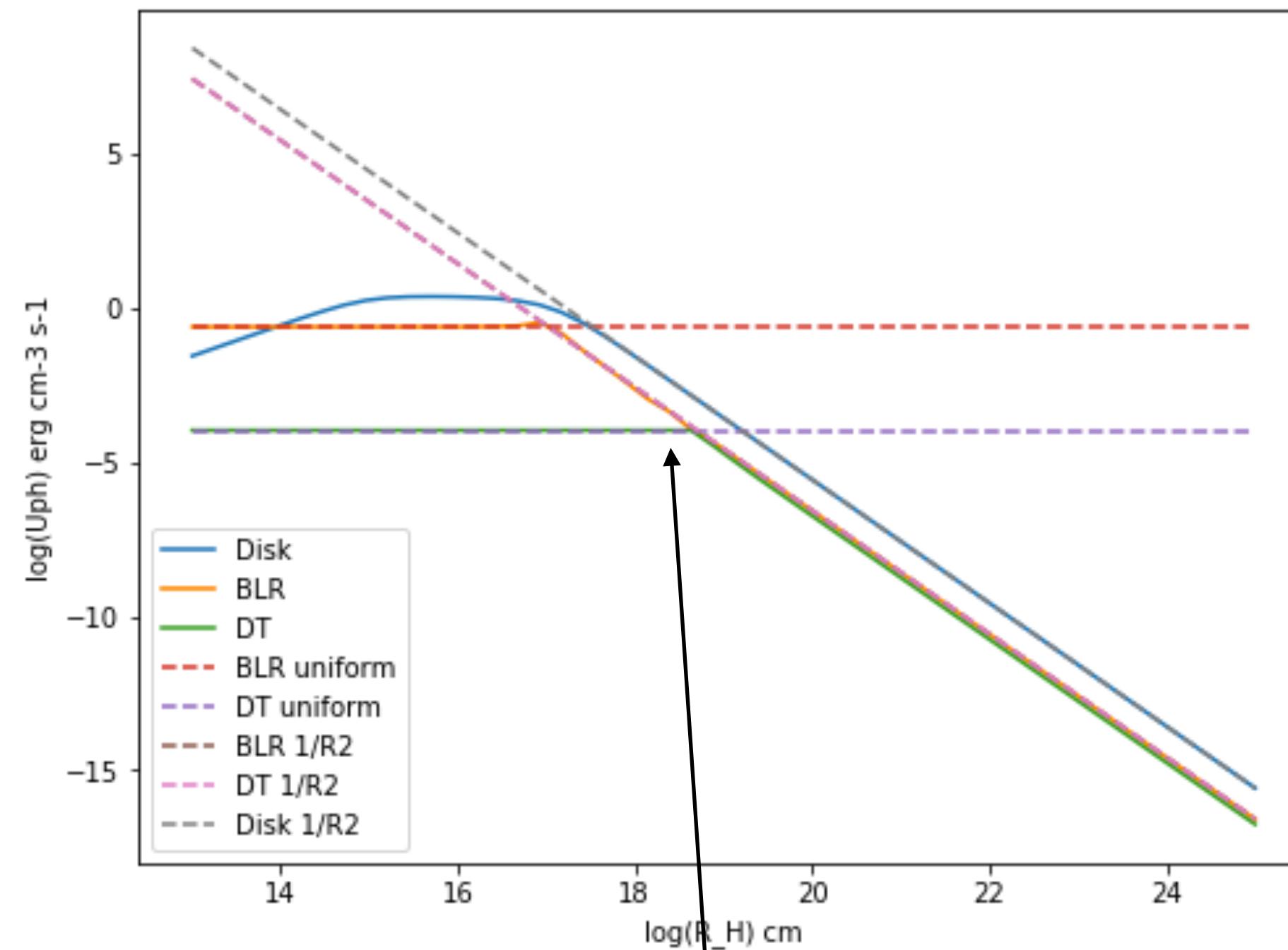
$$\epsilon^{-3} I_\epsilon \text{ and } \epsilon^{-2} j(\epsilon, \Omega) \\ \frac{u(\epsilon, \Omega)}{\epsilon^3} = \frac{u'(\epsilon', \Omega')}{\epsilon'^3} = i n v.$$



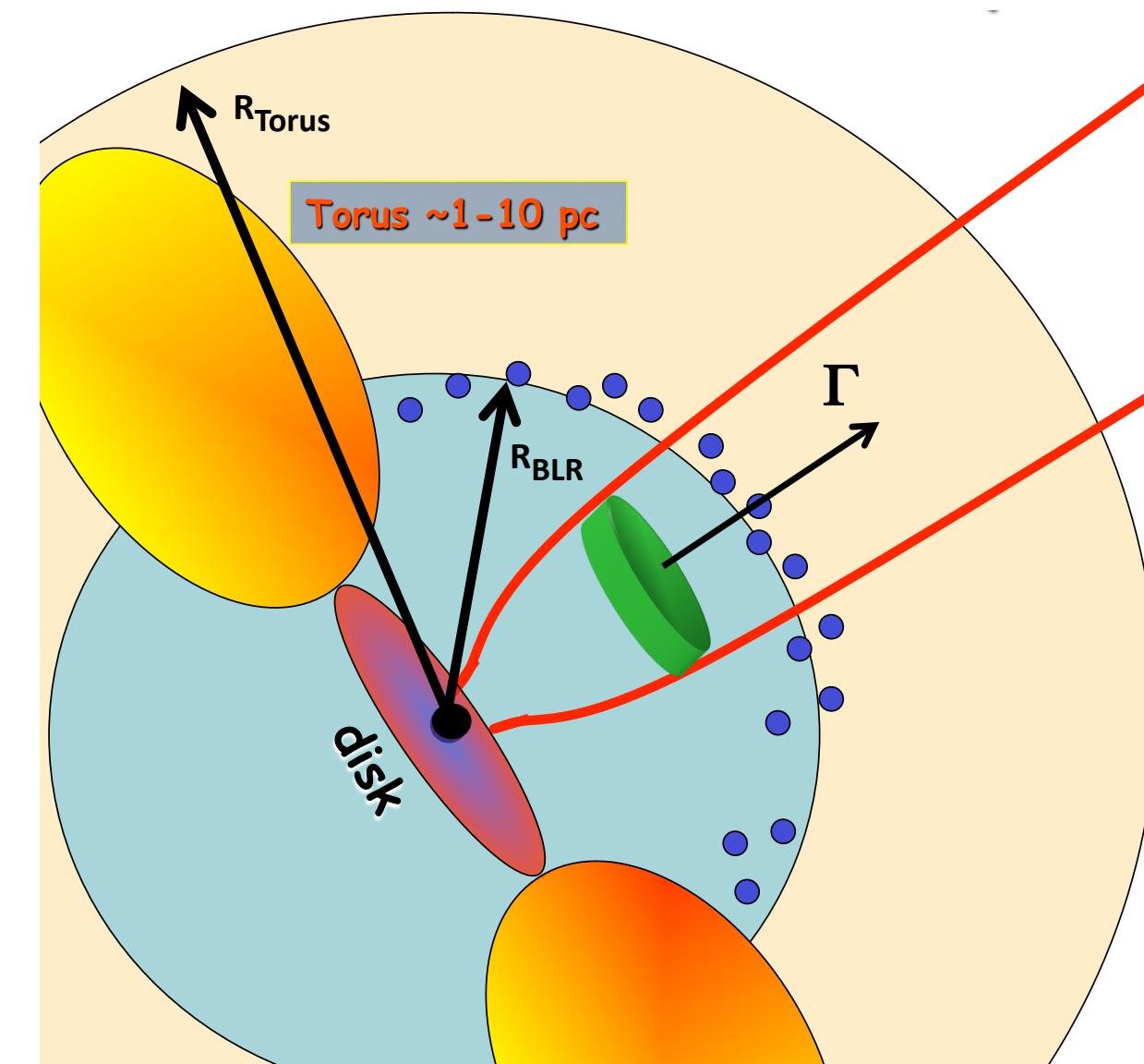
Transformation of the radiative fields

External Compton Scenario

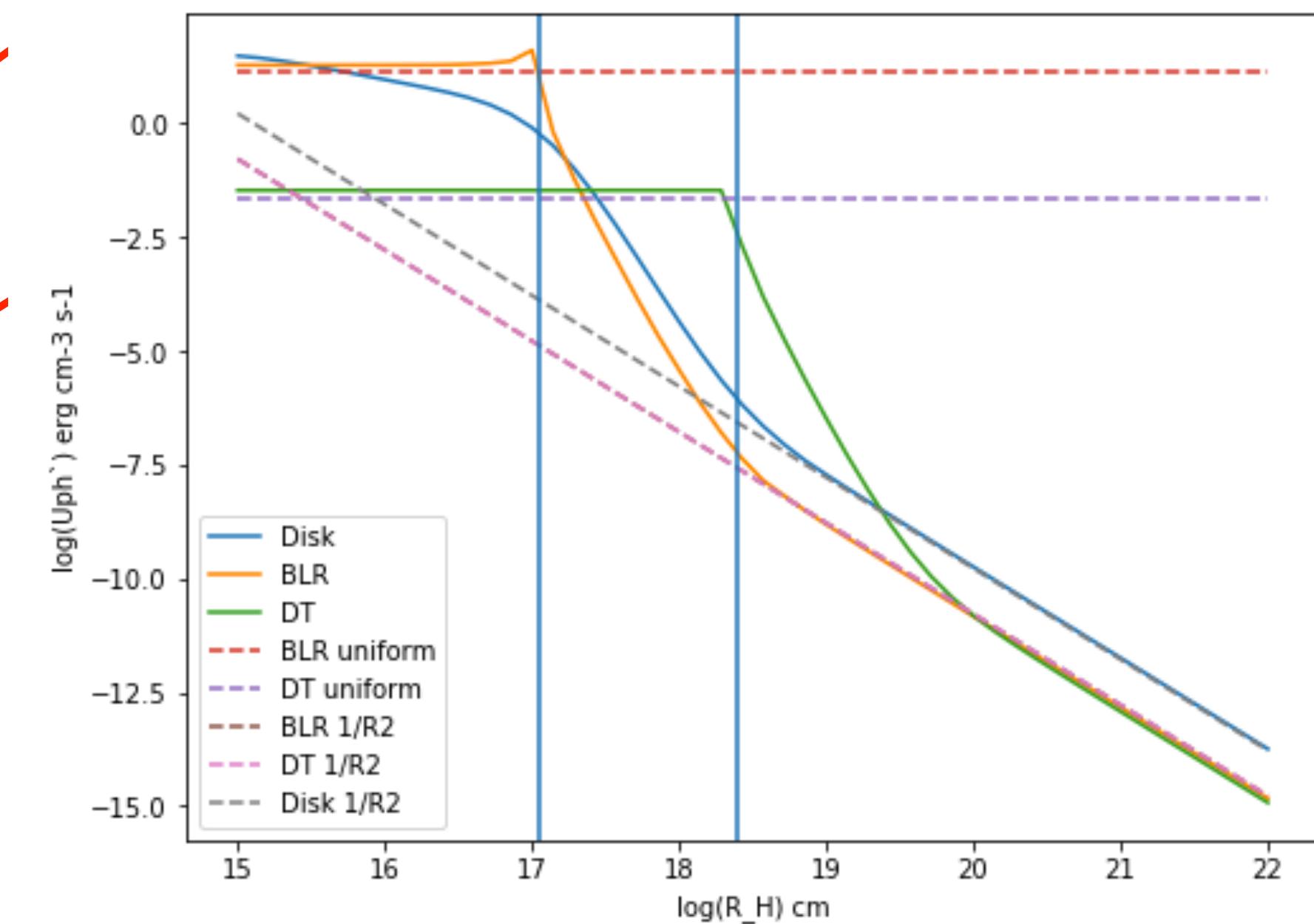
external photon fields in the disk
rest frame



constant within
the sphere



external photon fields in the blob
rest frame

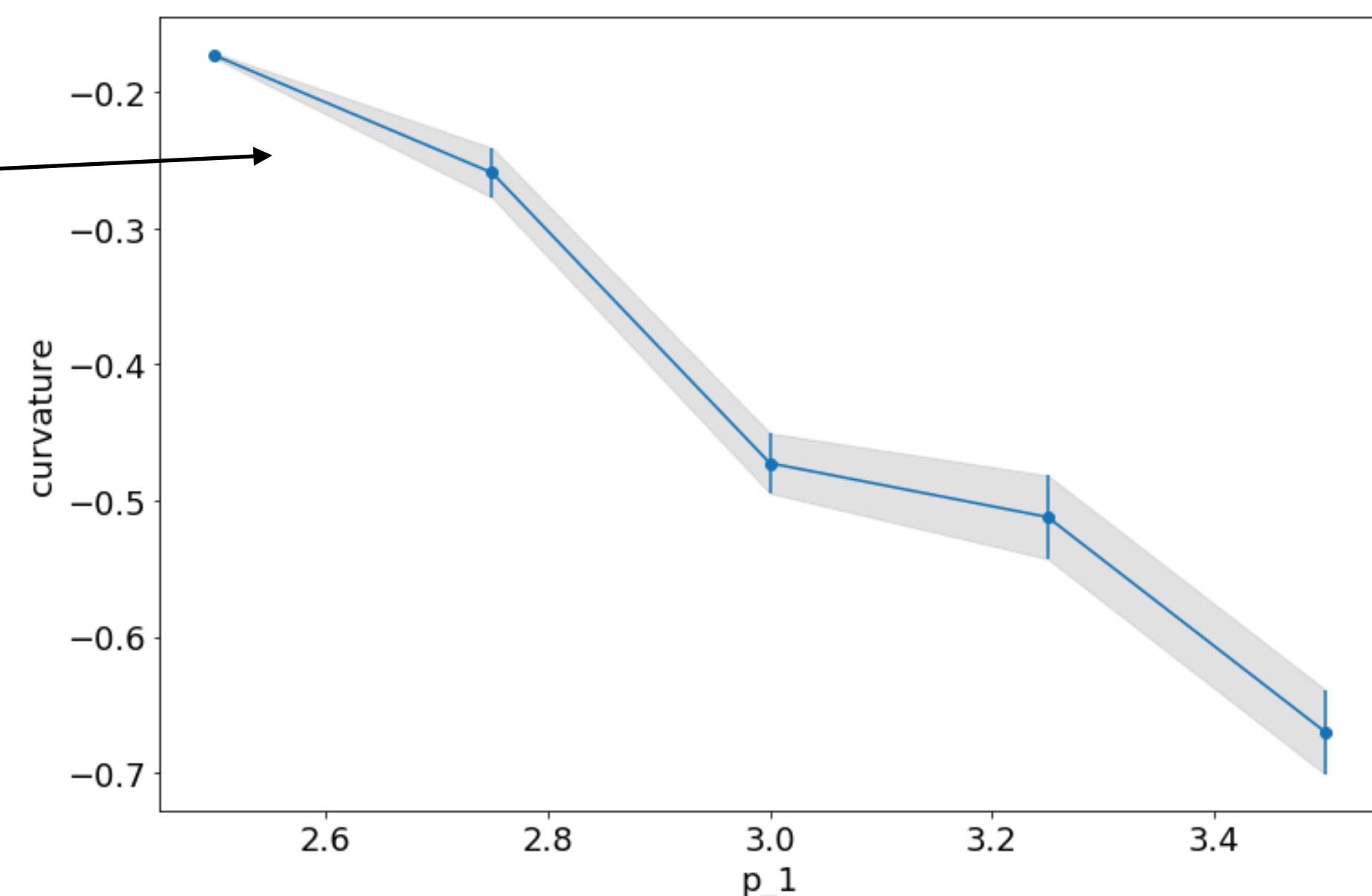
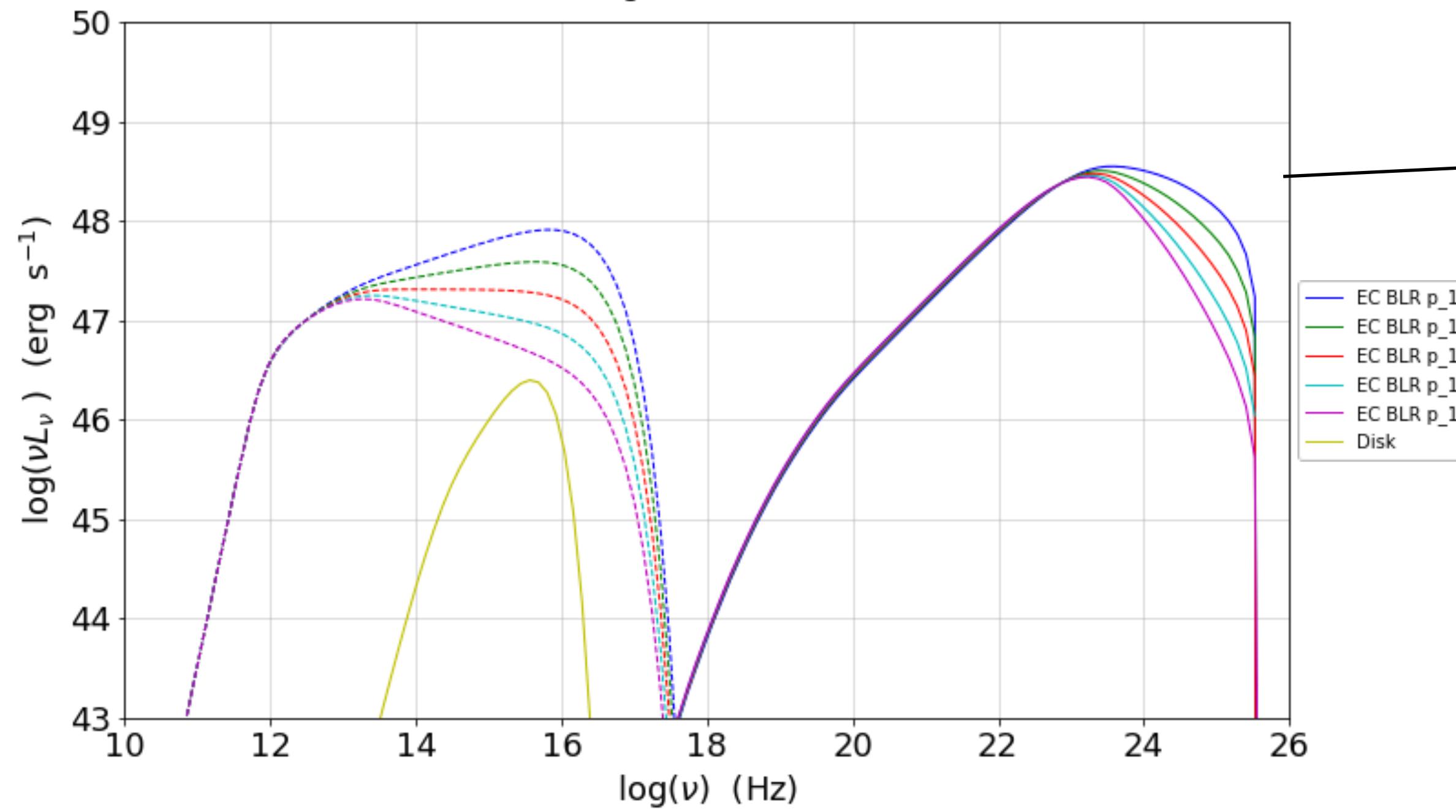
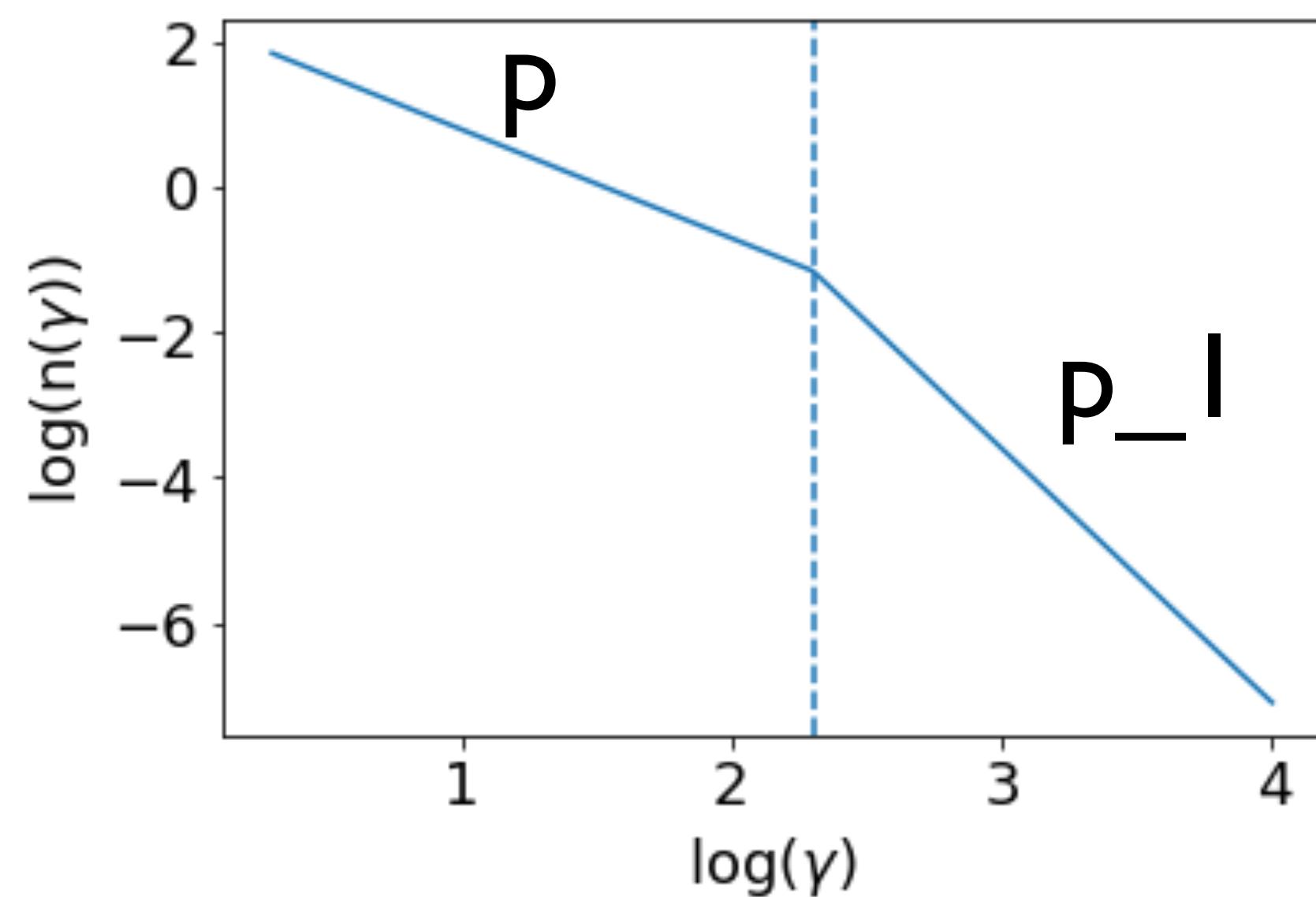
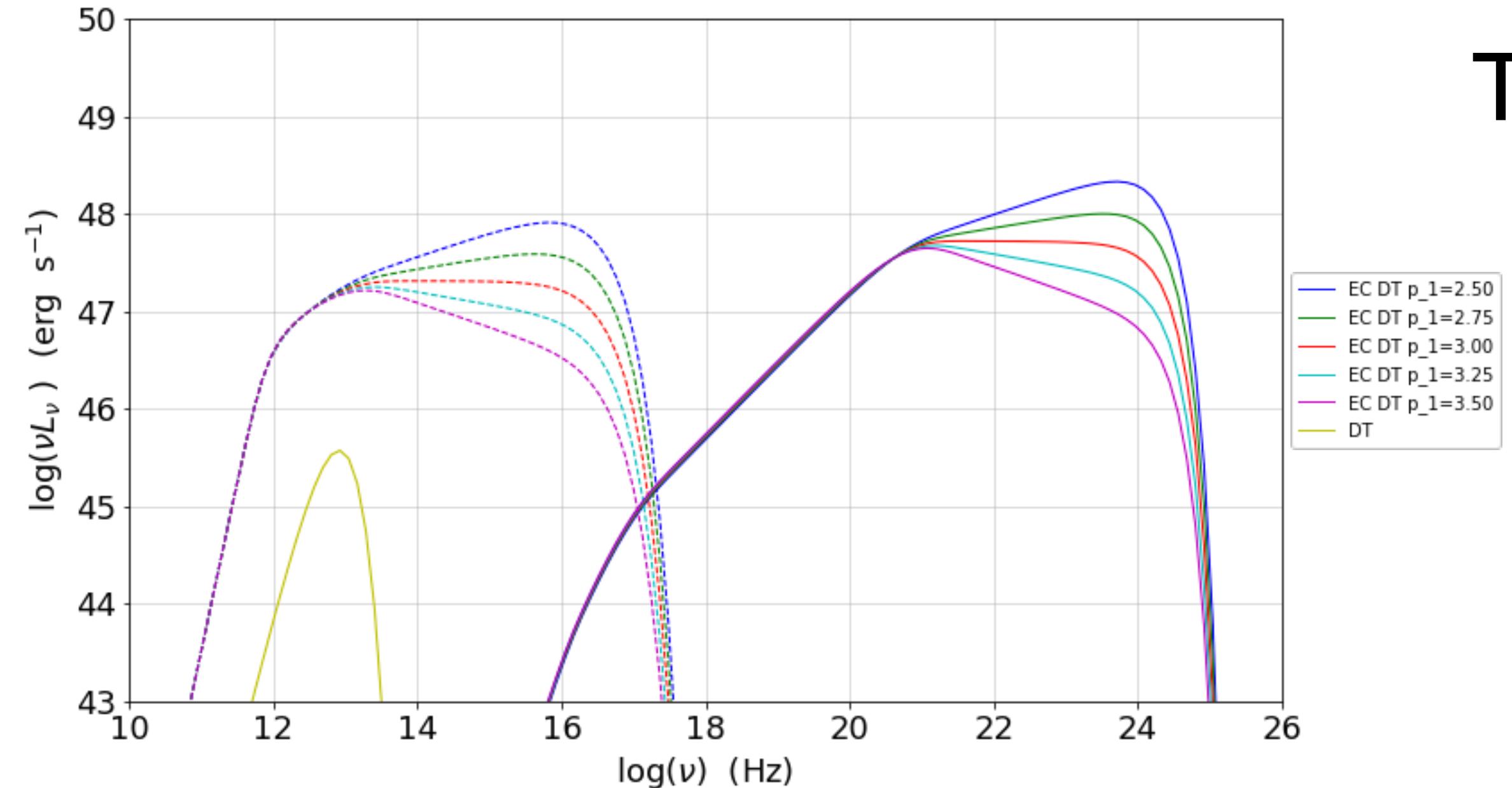


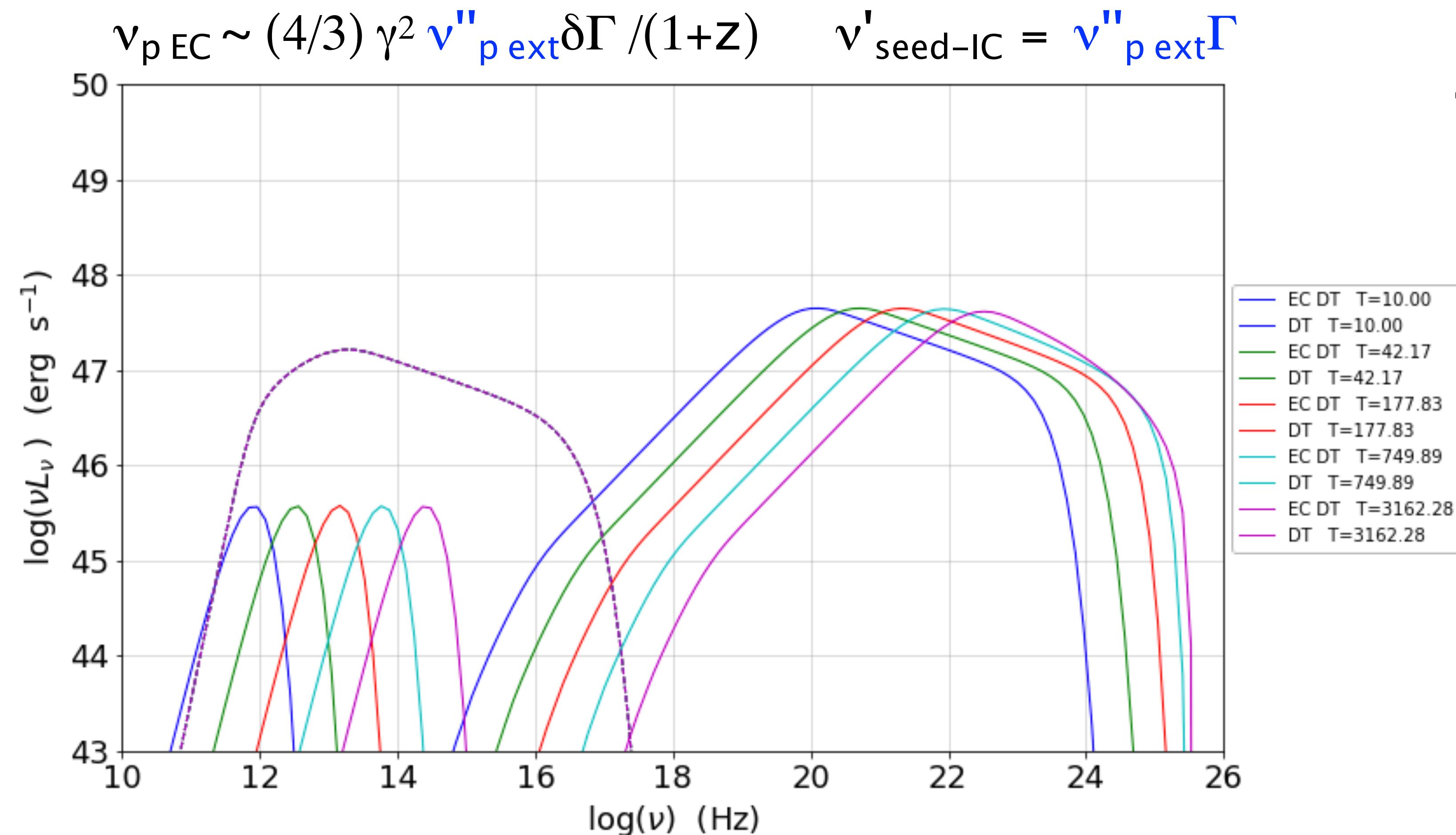
$$u'_{ext} \simeq \Gamma^2 u_{ext}$$

Tutorial 3

External Compton Scenario and TH/KN

Tutorial 3





obs. data

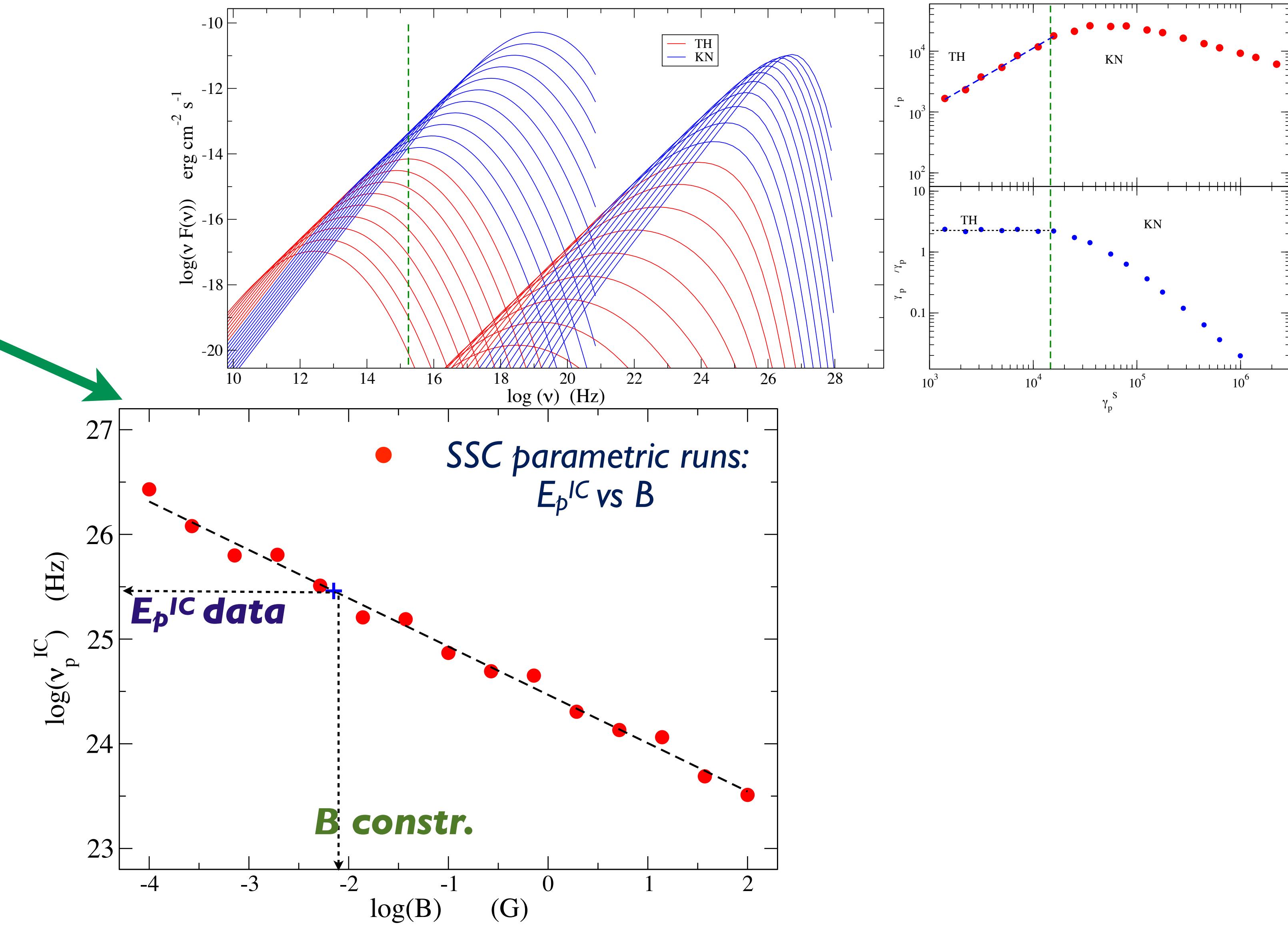
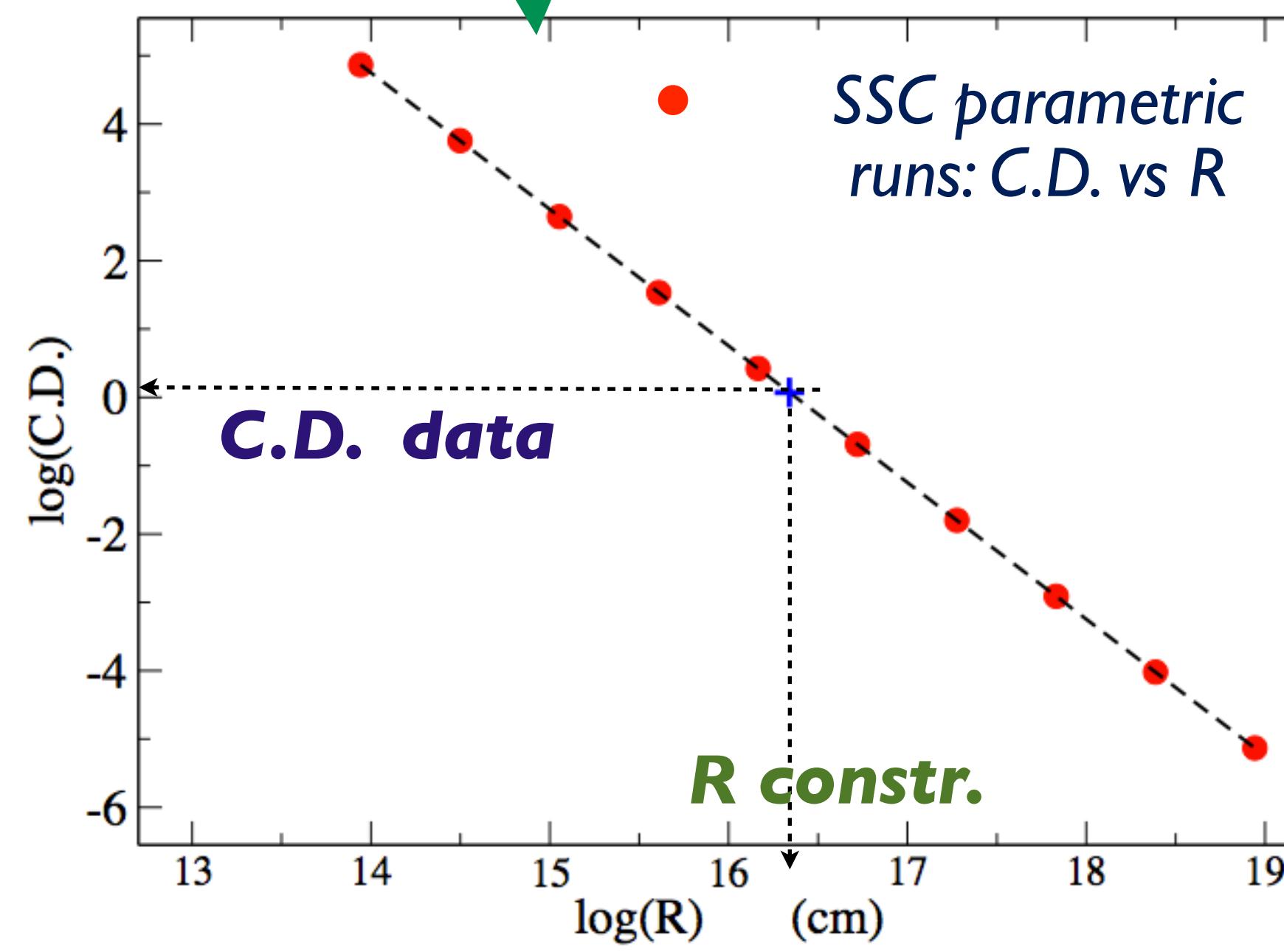
→ model

•peak freq.

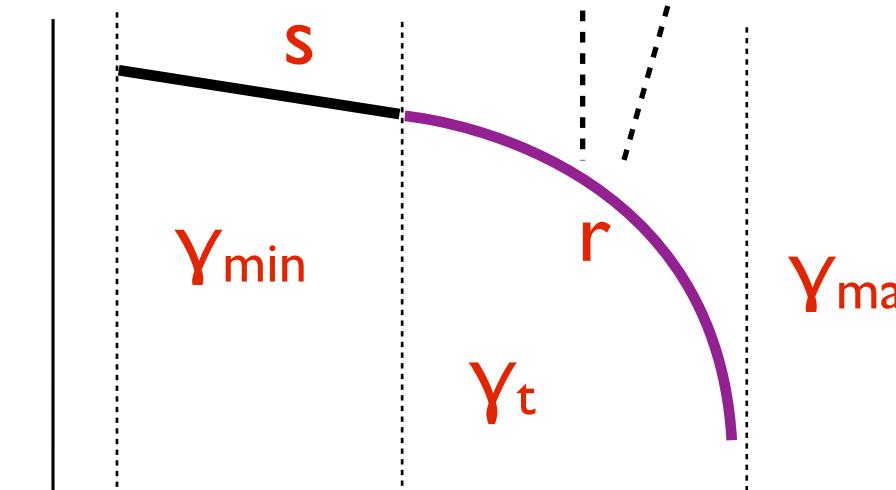
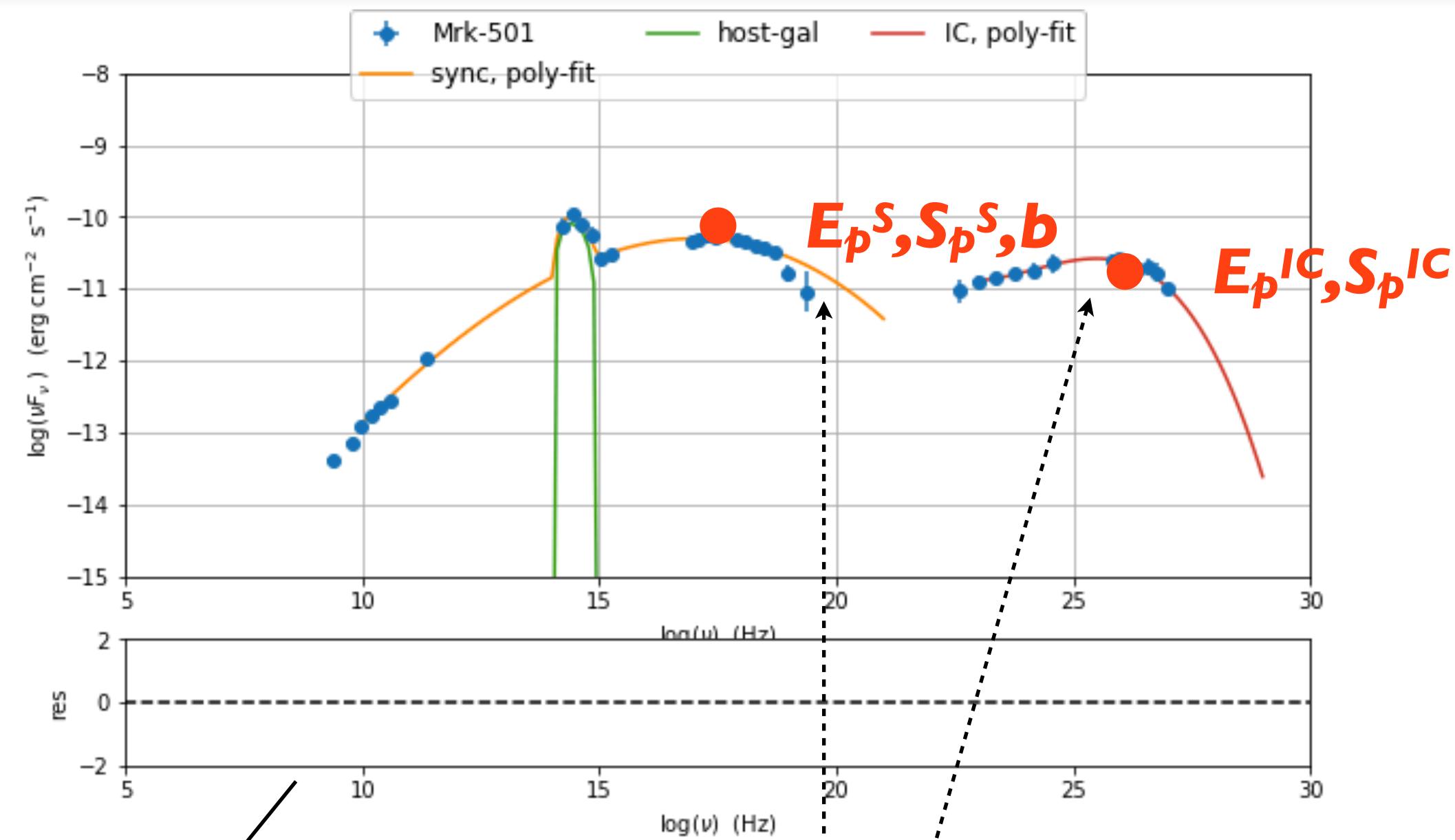
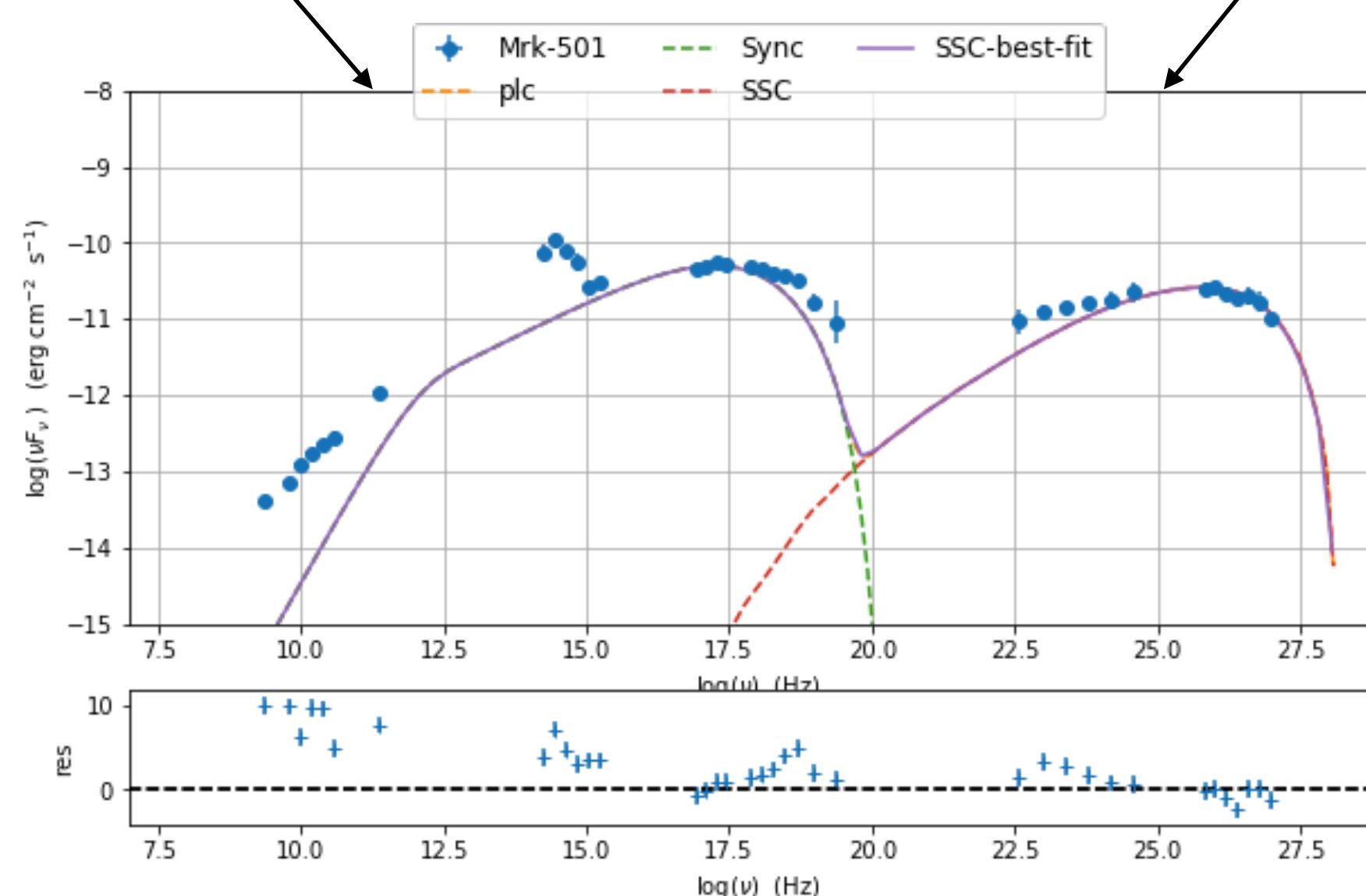
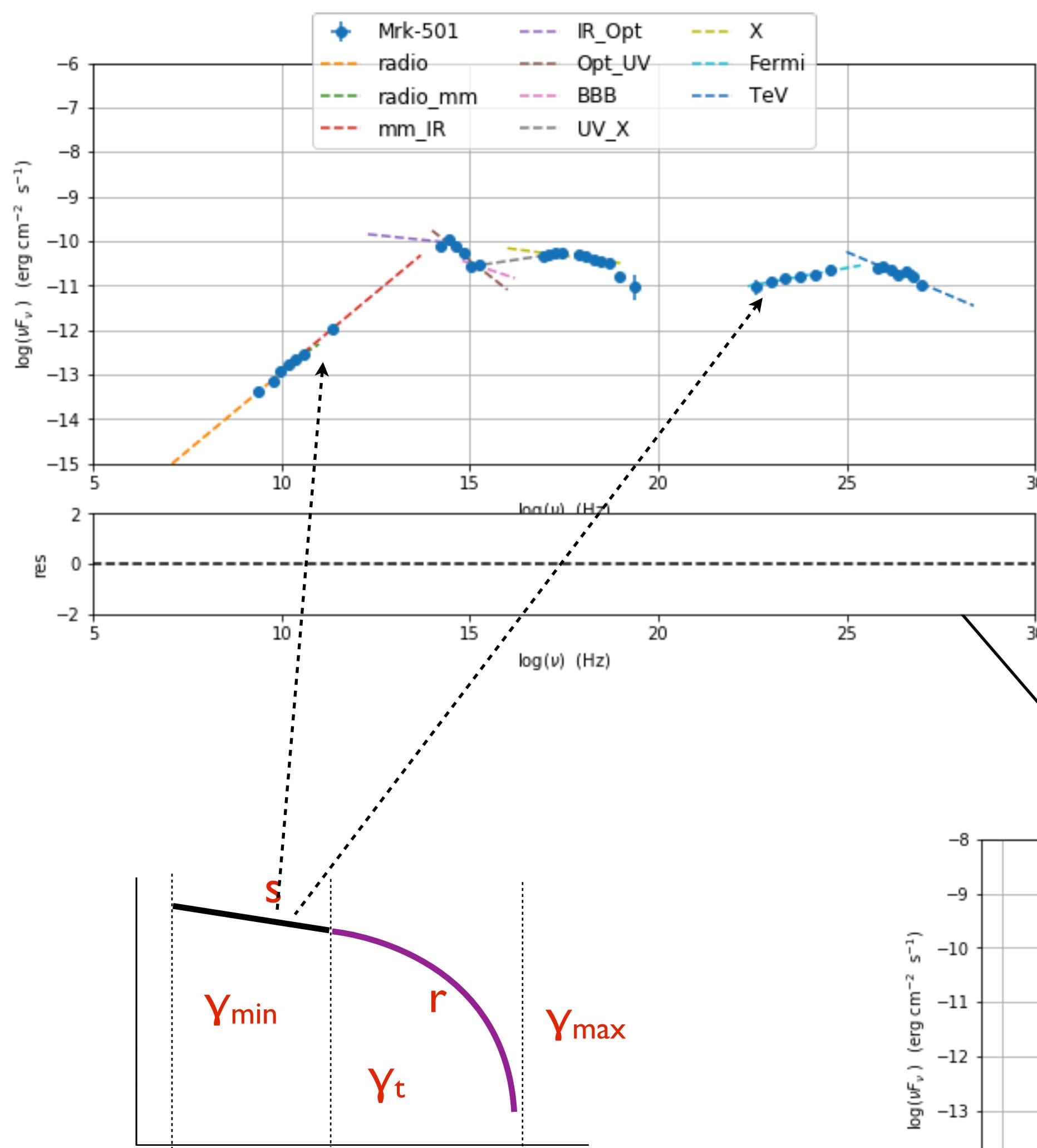
•T seed ext.

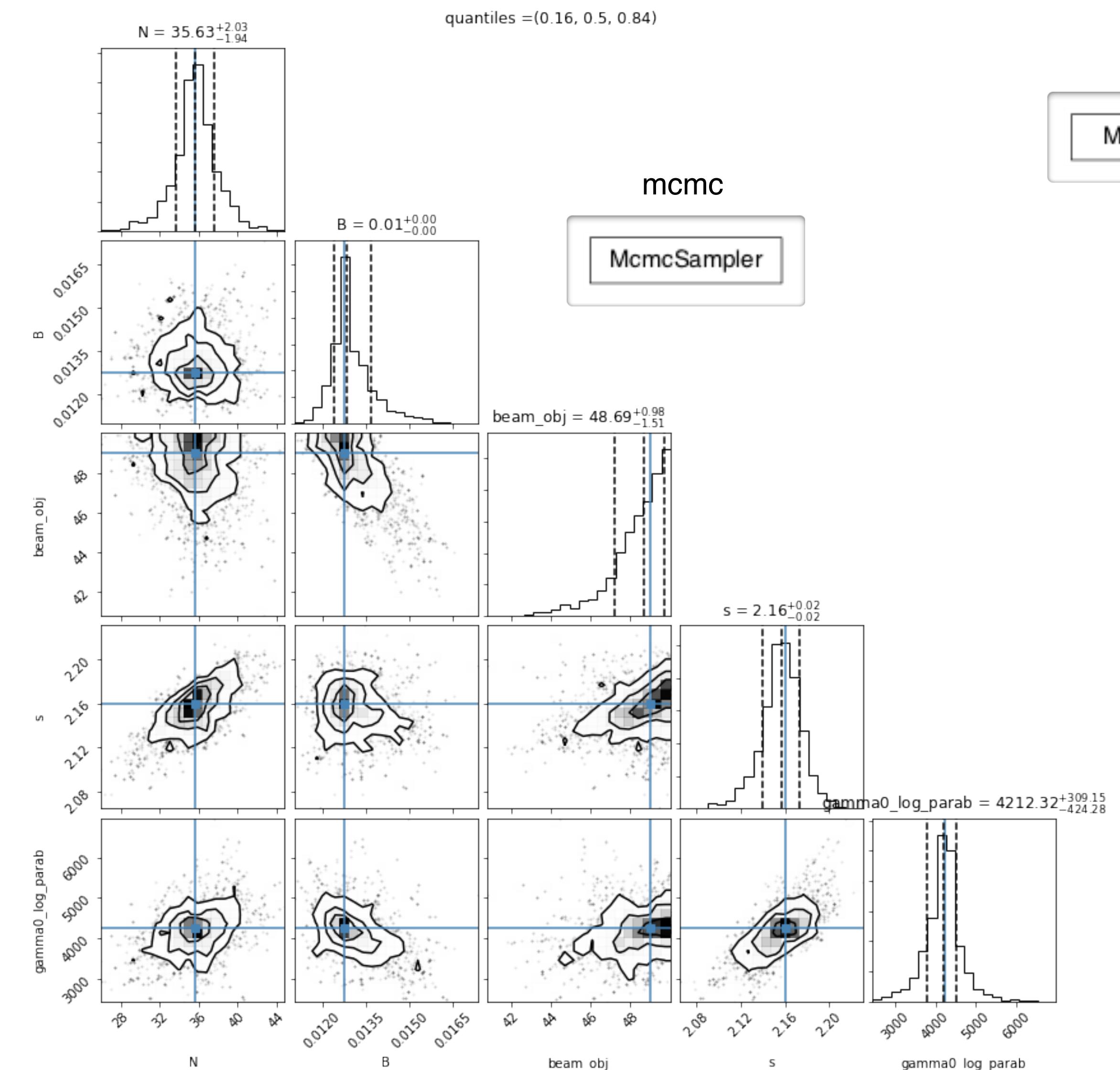
SED shaping and model constraining

- ✓ $\Gamma \Rightarrow s$
- ✓ $b \Rightarrow r$
- ✓ $E_p^s, r, s \Rightarrow \gamma_0$
- ✓ $t_{var}, \delta \Rightarrow R$ u.l.
- ✓ $N \Rightarrow$ best S_p^s match
- $B \Rightarrow$ best E_p^{IC}, S_p^{IC} match
- $R \Rightarrow CD$



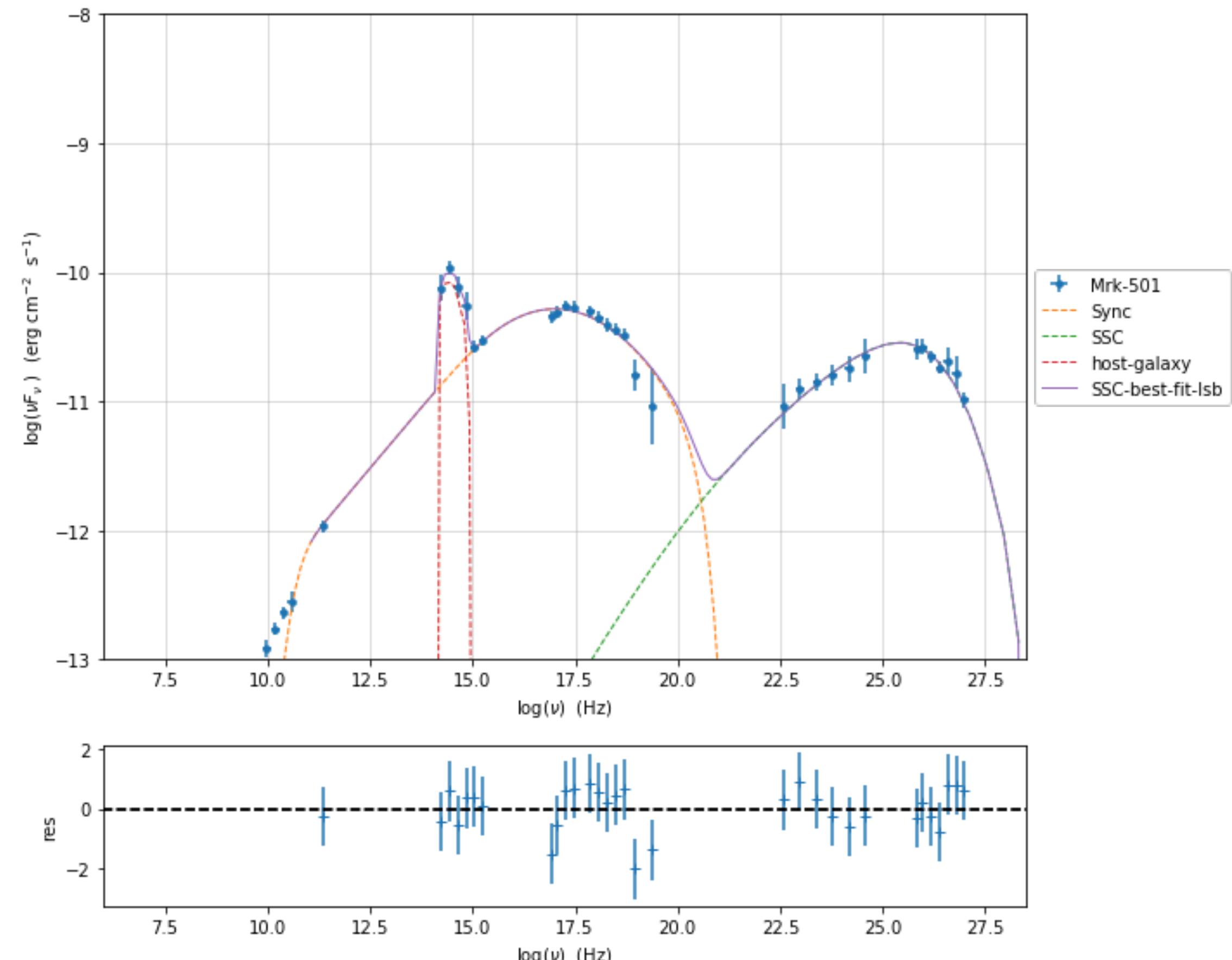
Tutorial 5





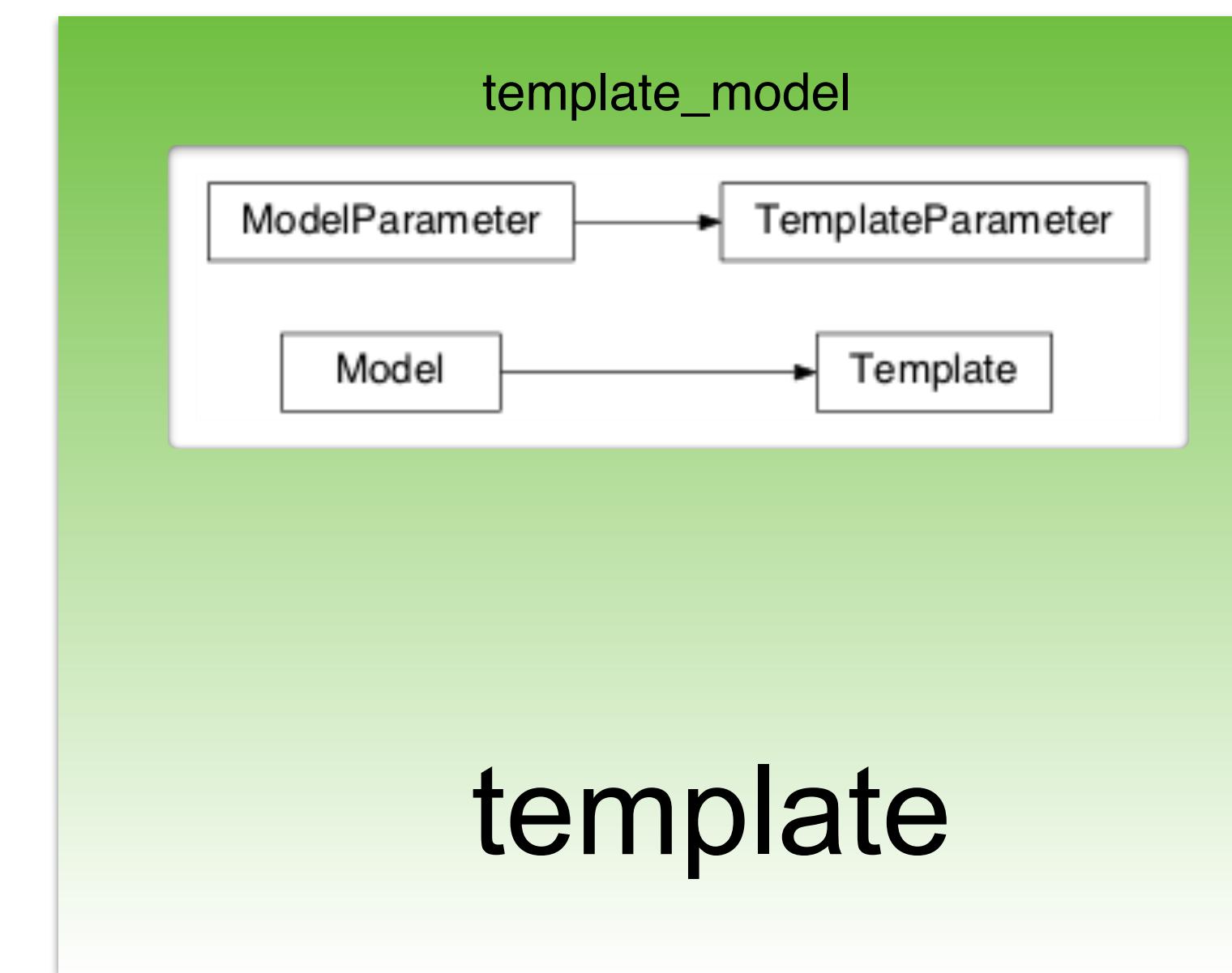
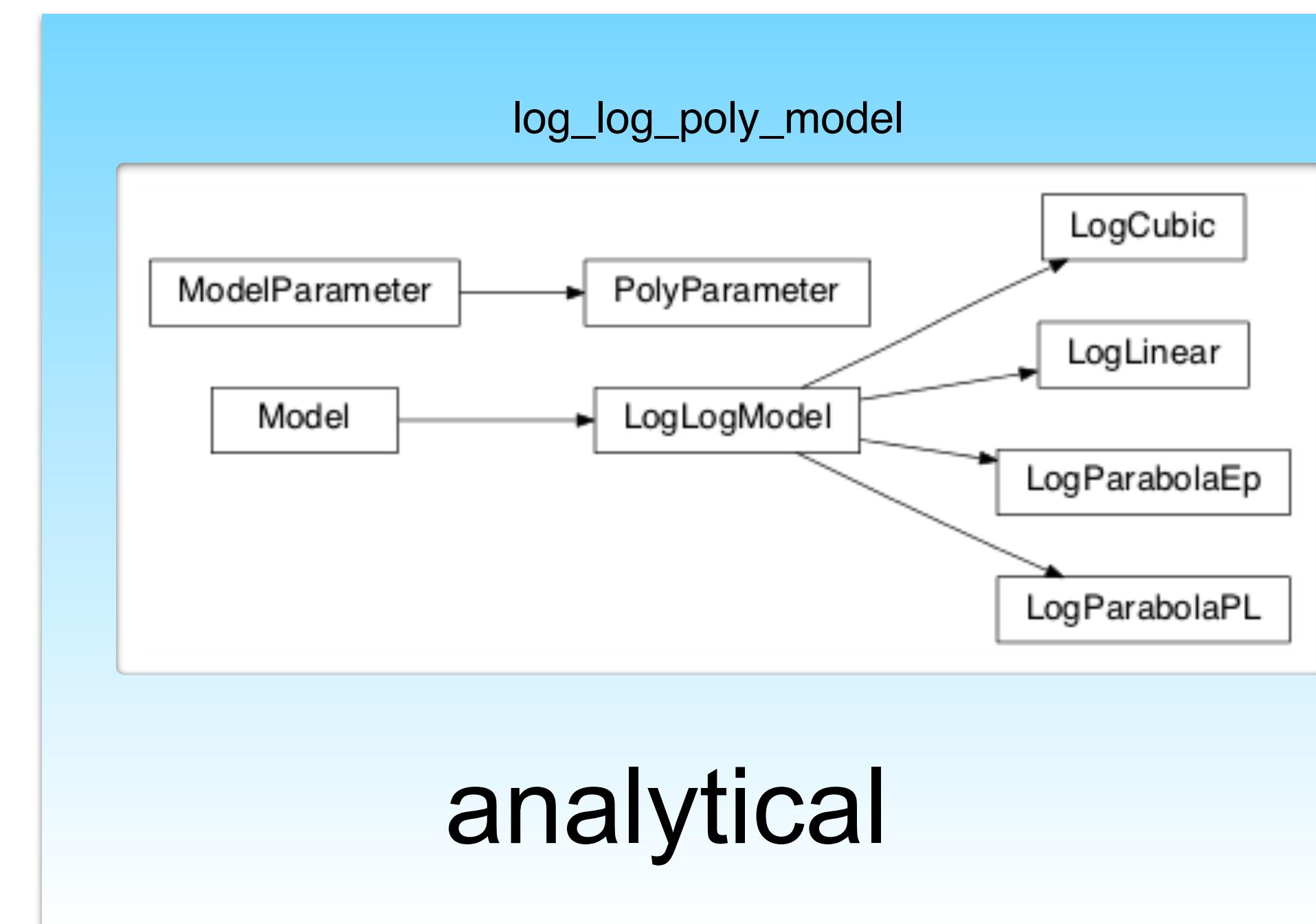
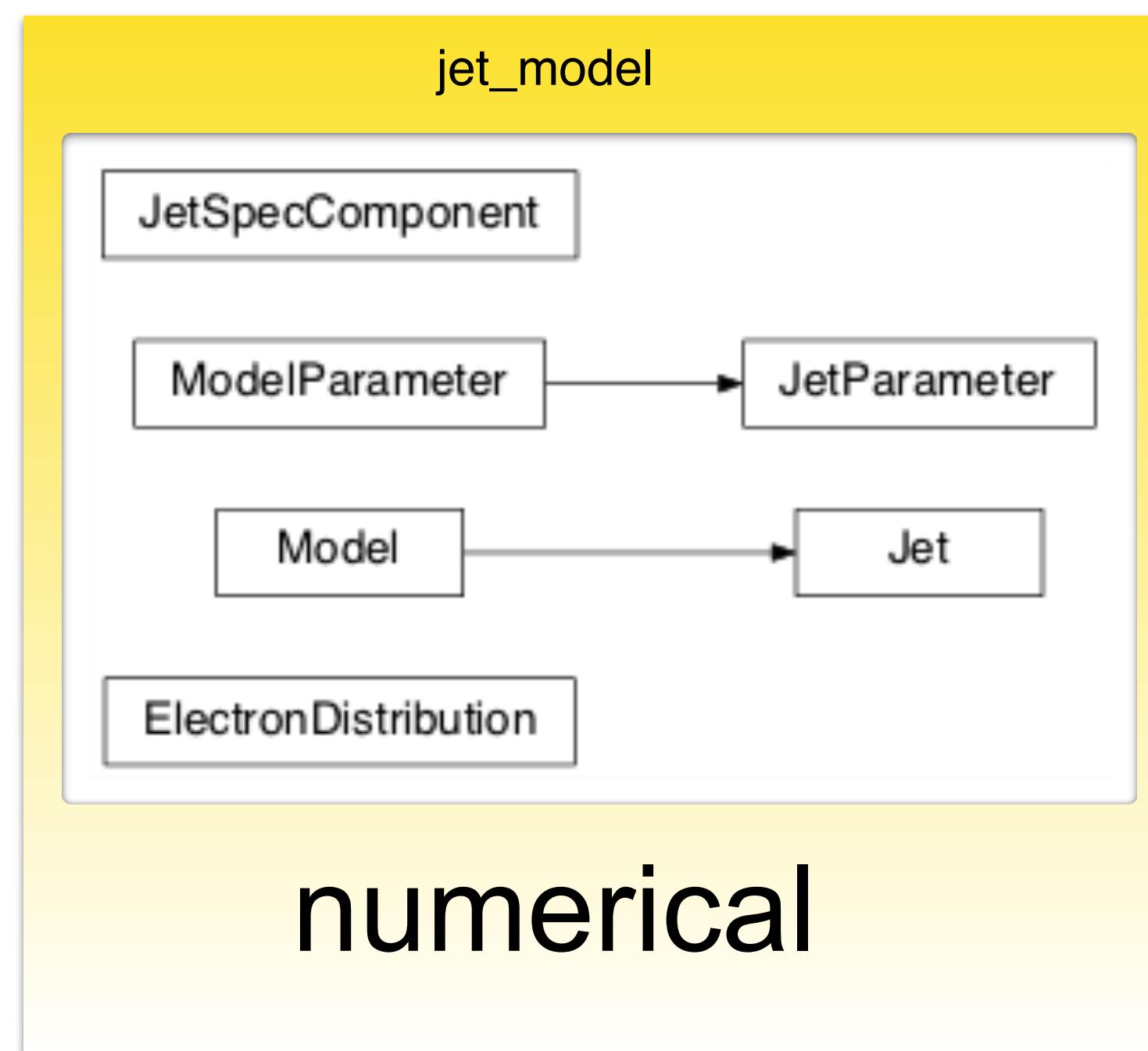
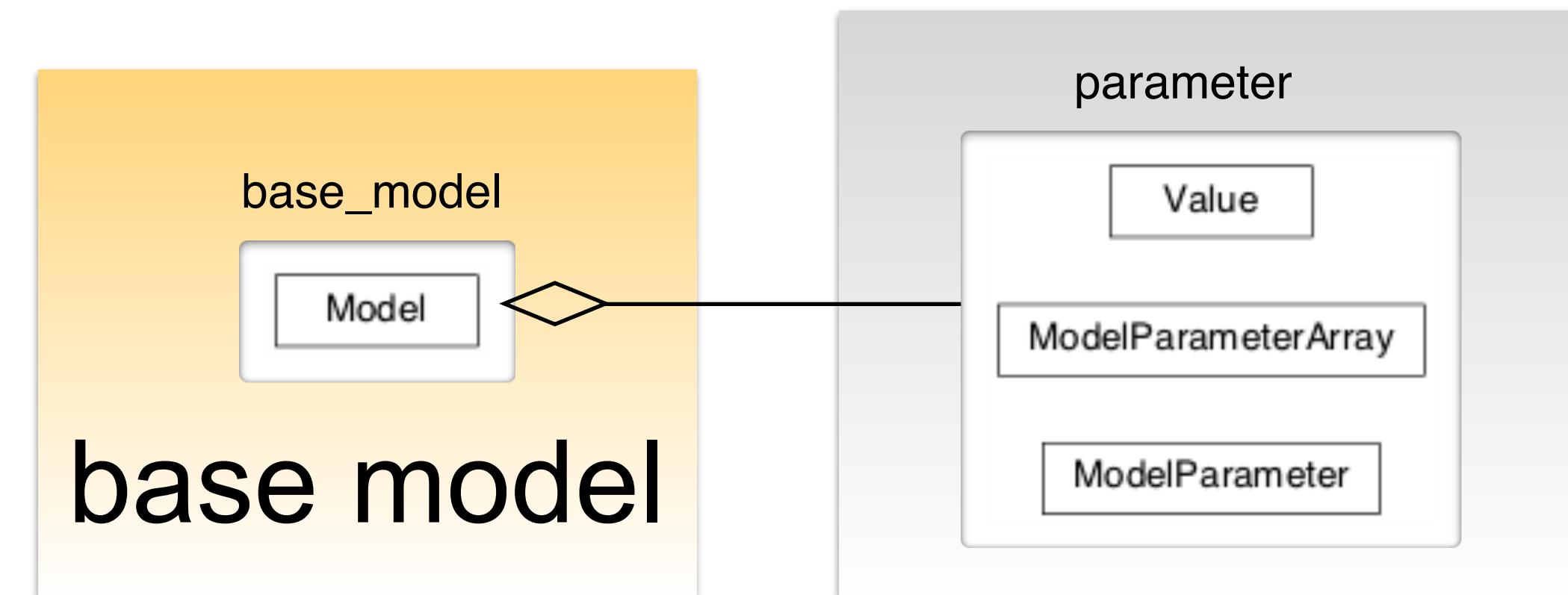
$$p(\text{parameters given data}) \propto p(\text{data given parameters}) \times (\text{parameters})$$

Posterior = Likelihood \times Prior

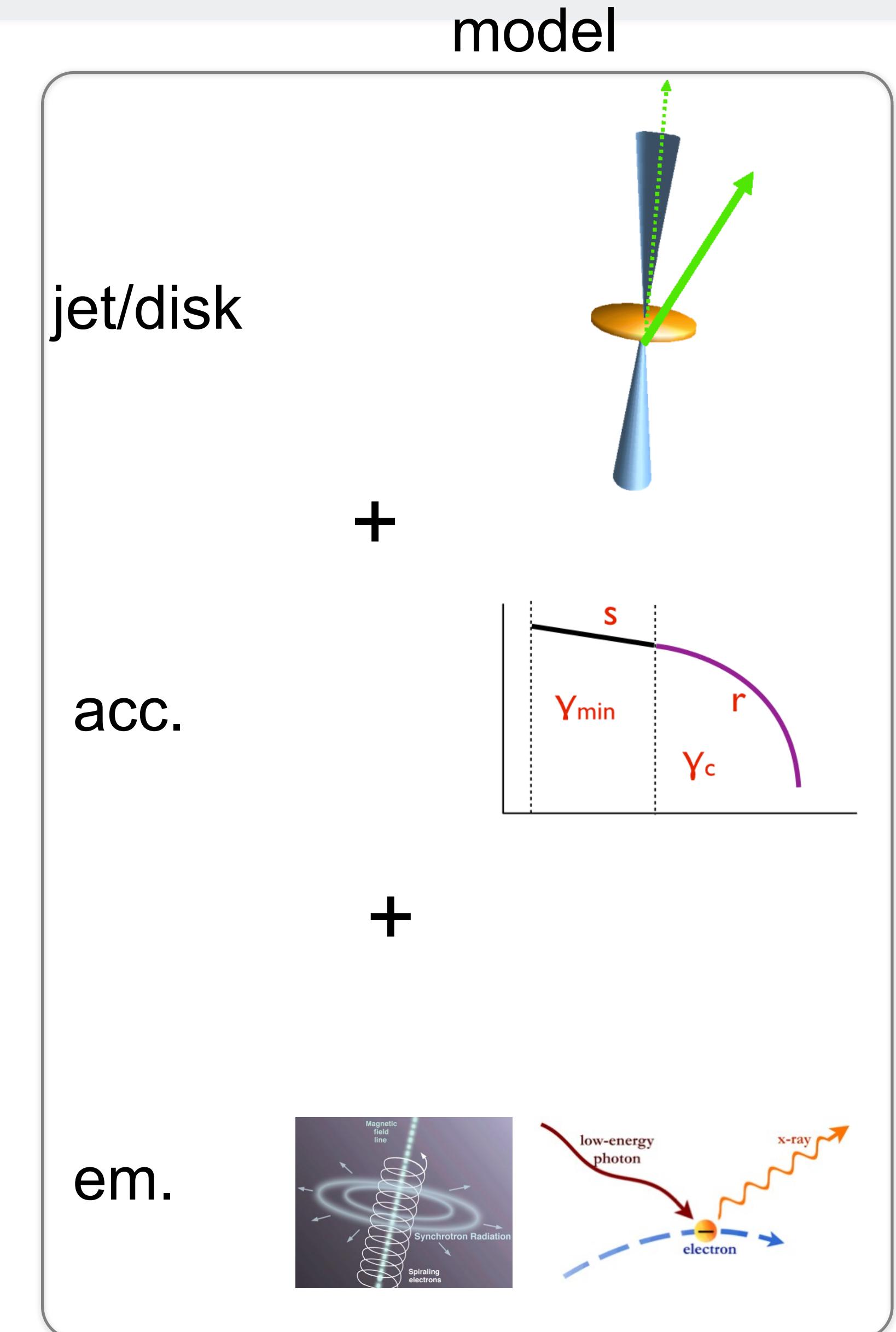
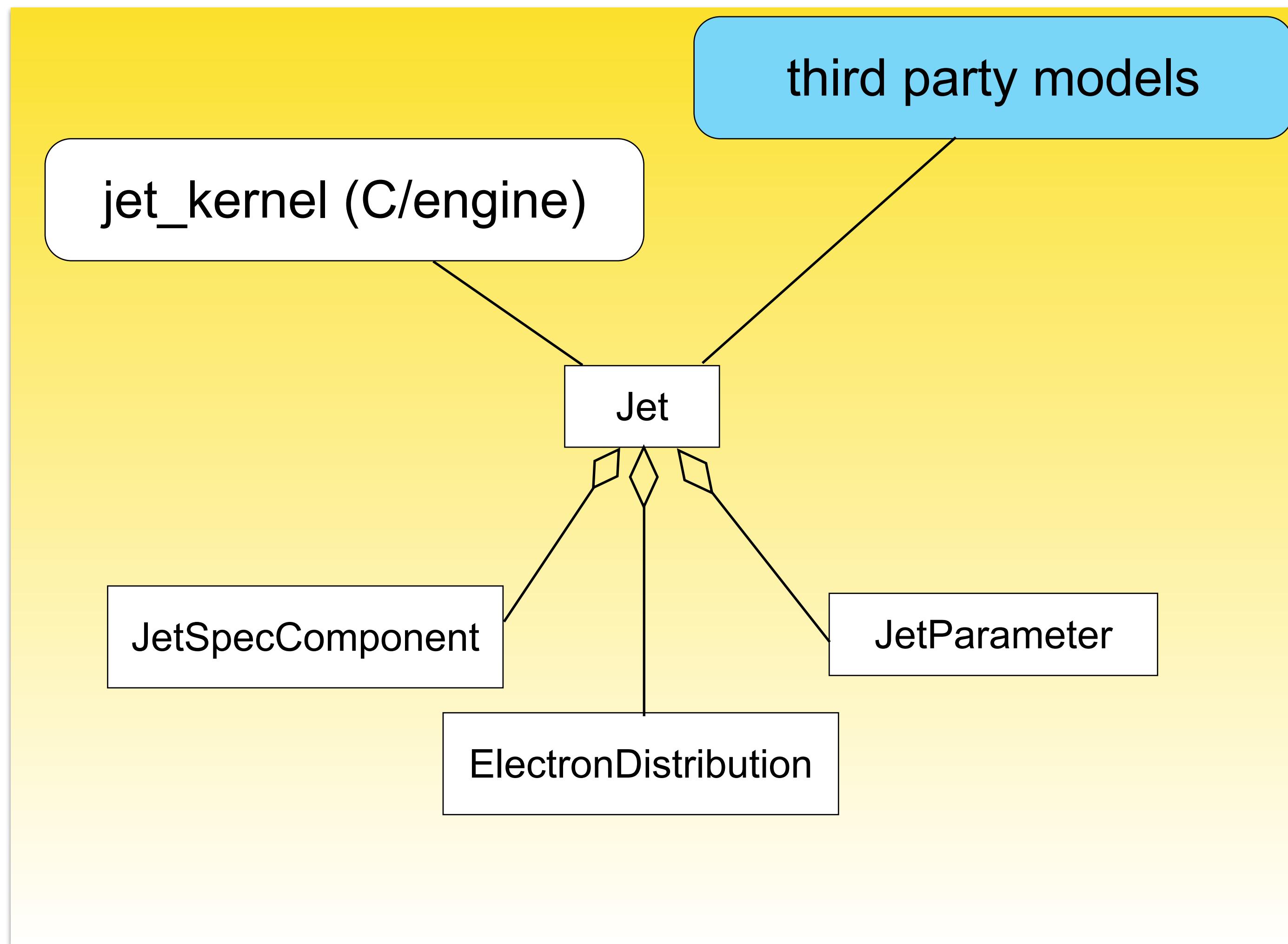


backup slides

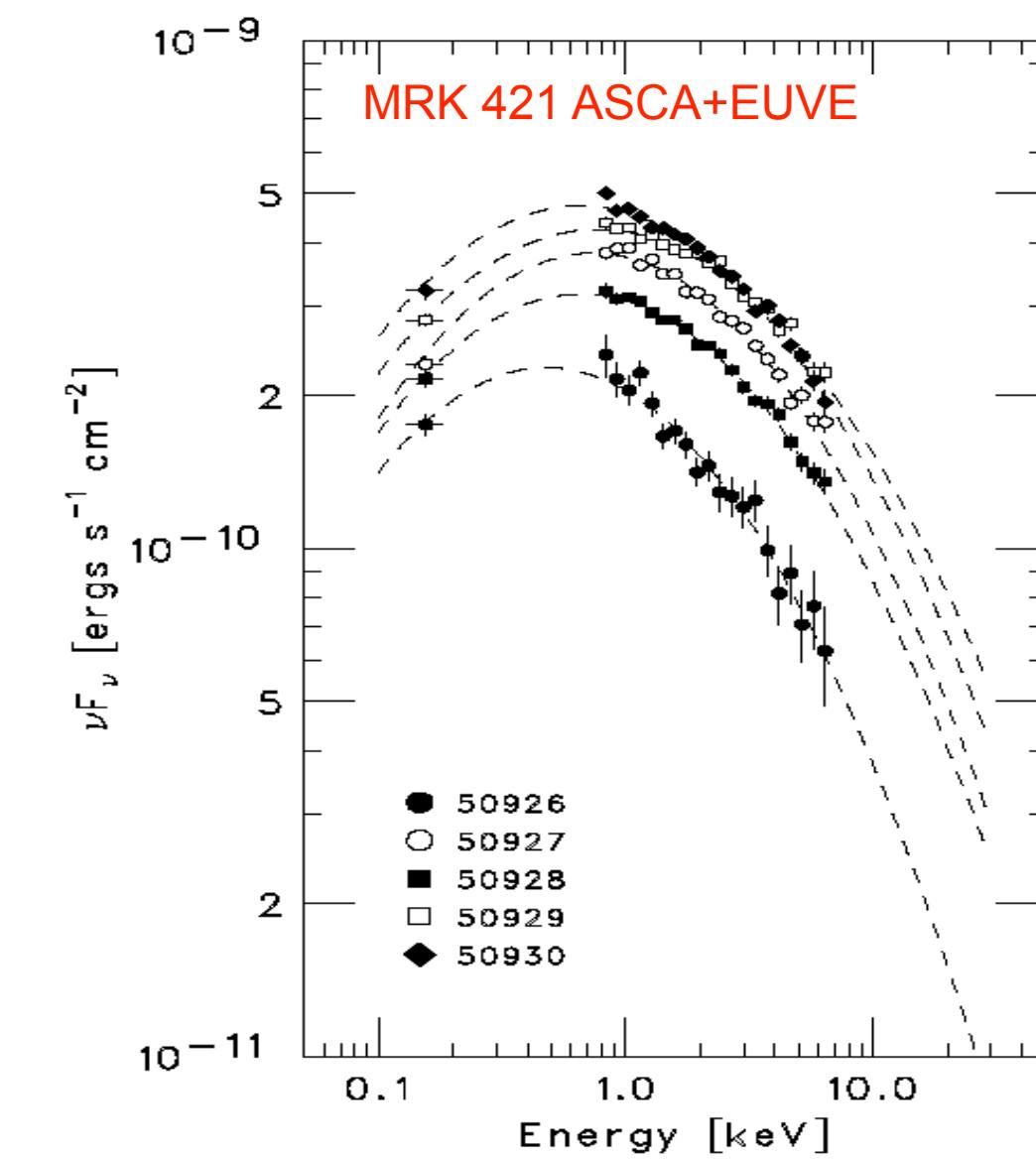
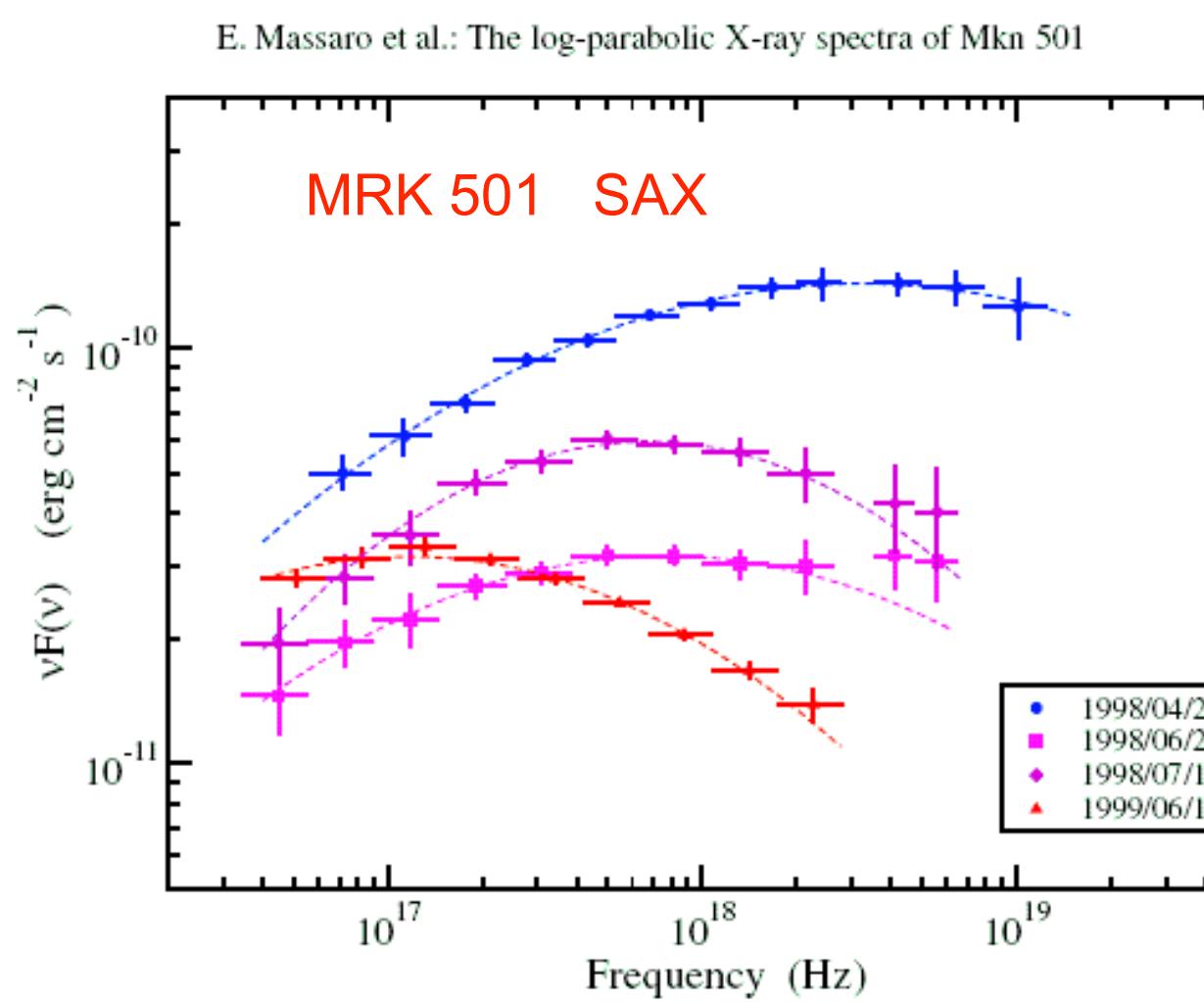
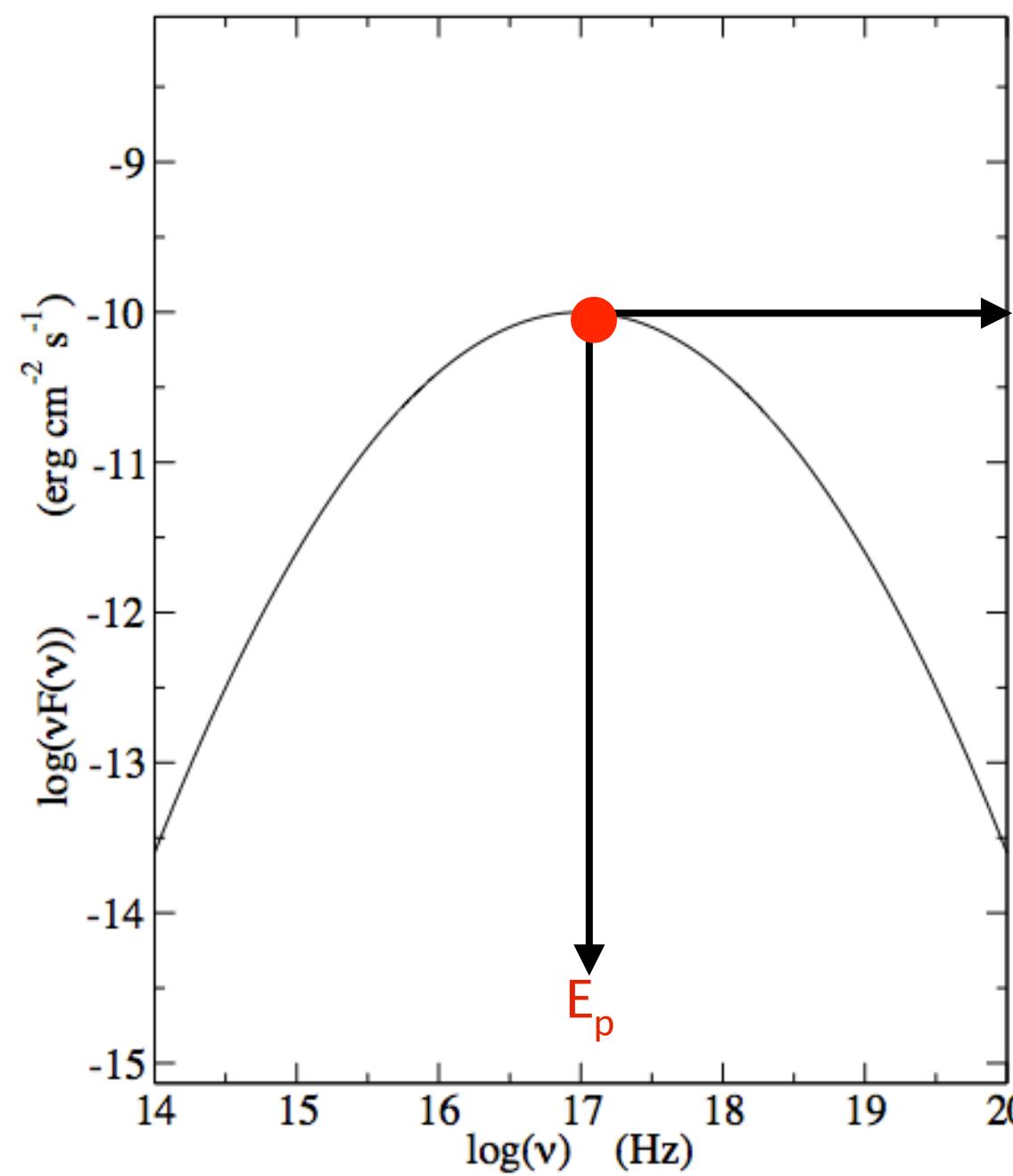
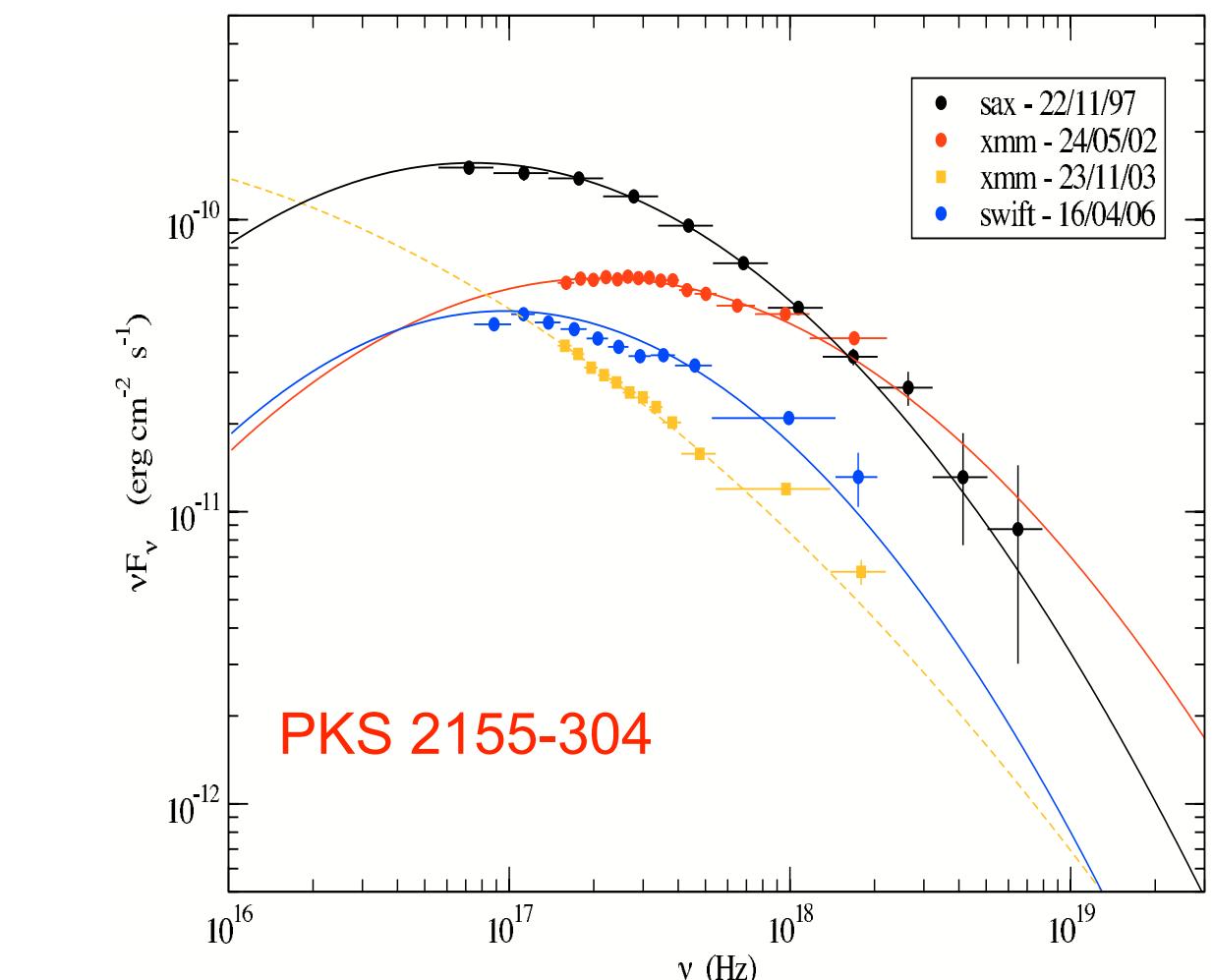
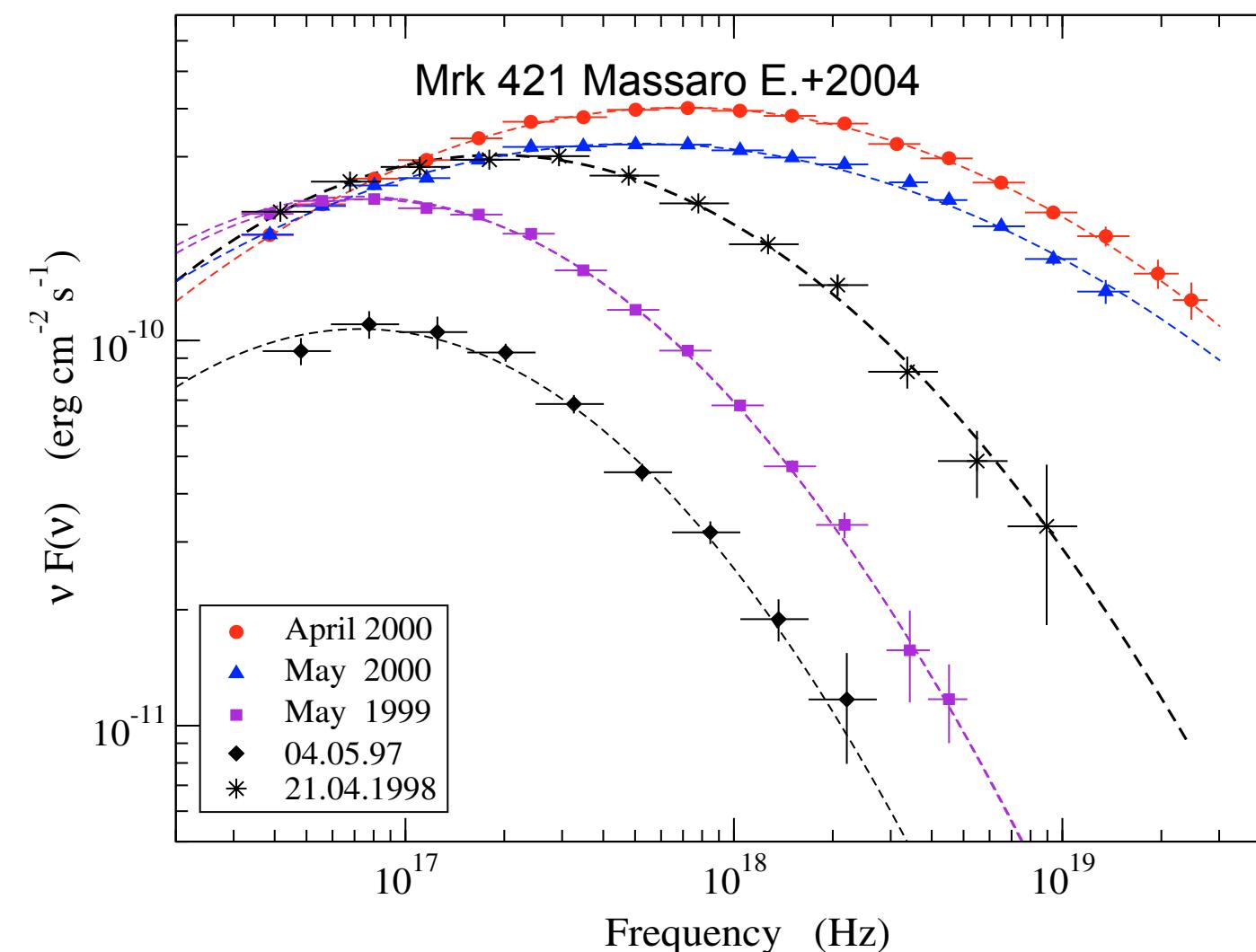
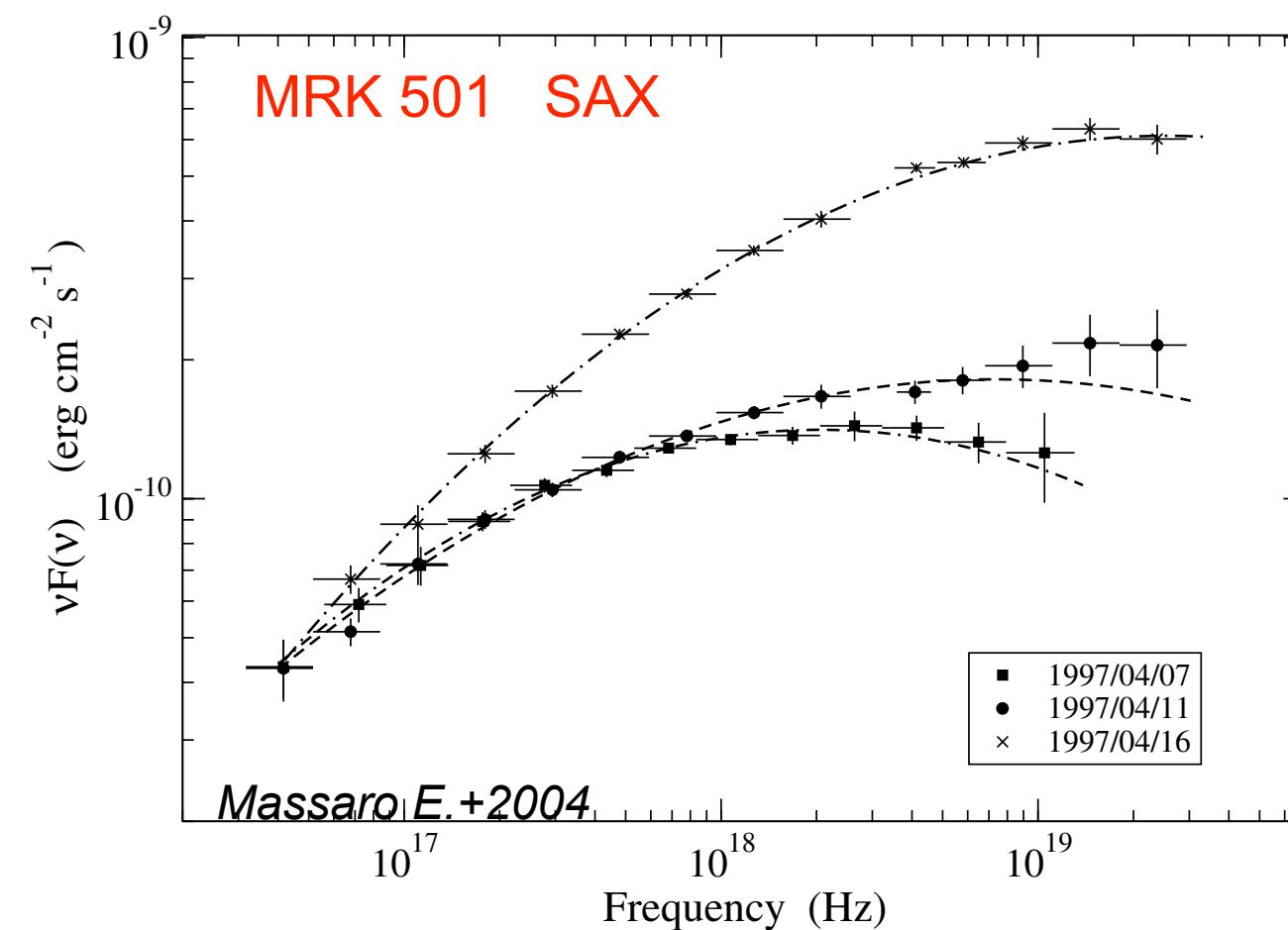
models



jet model definition

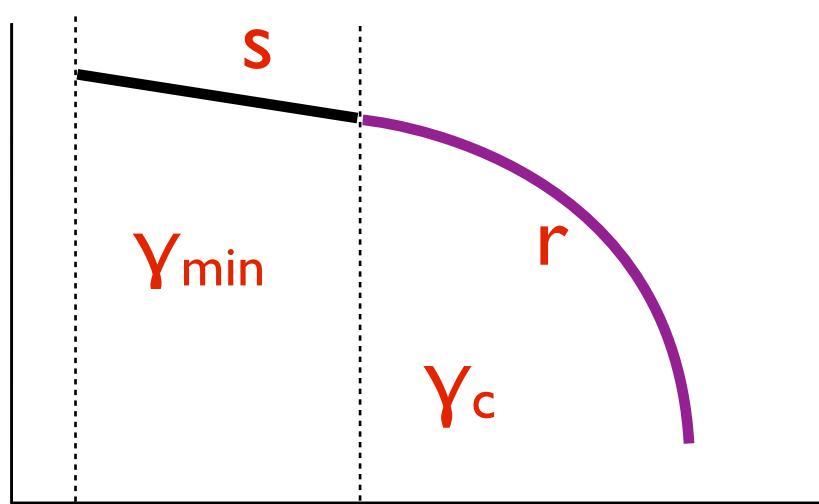


X-ray SPECTRAL DISTRIBUTION OF HBLs (stochastic acceleration)

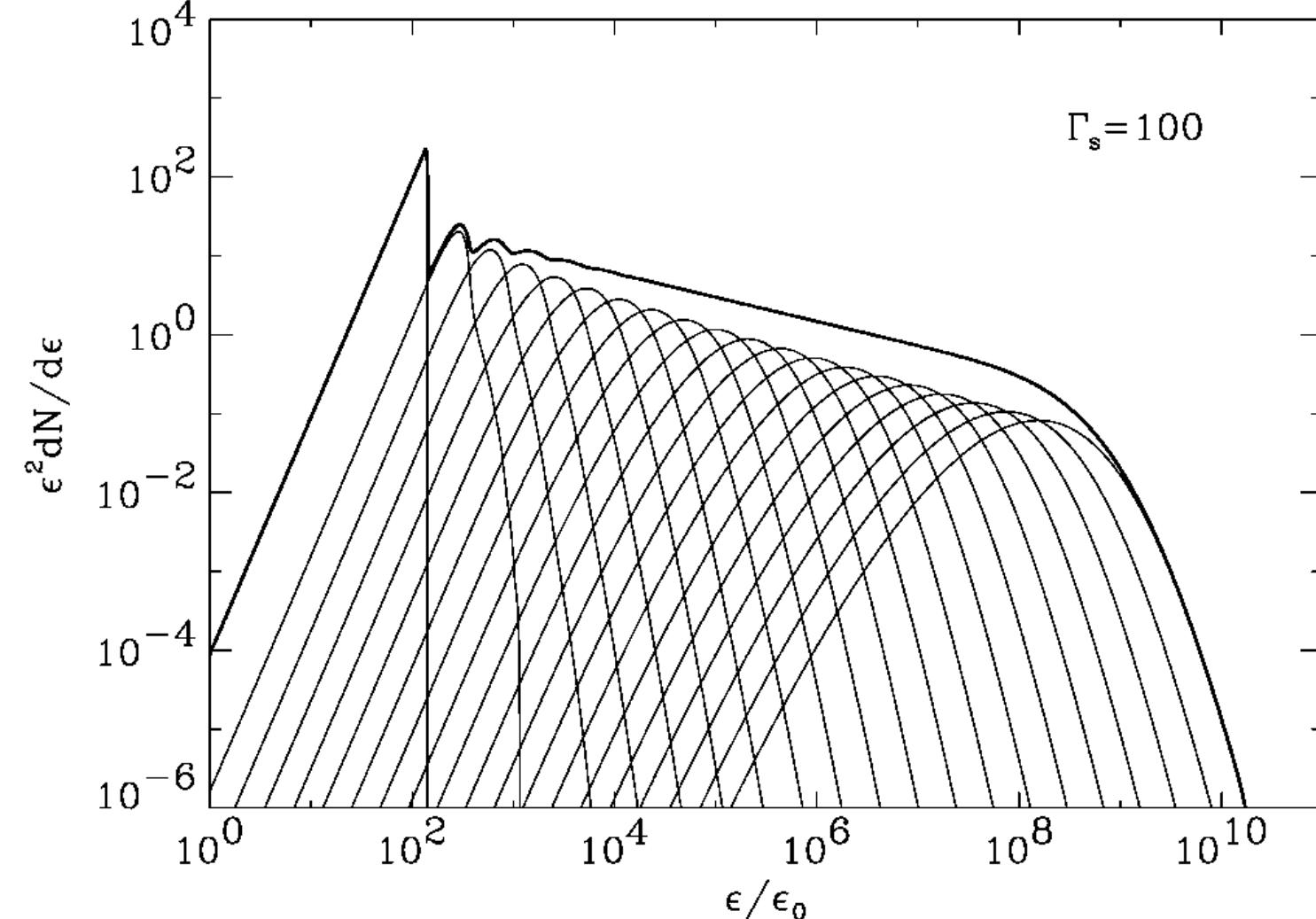


LP+PL spectra

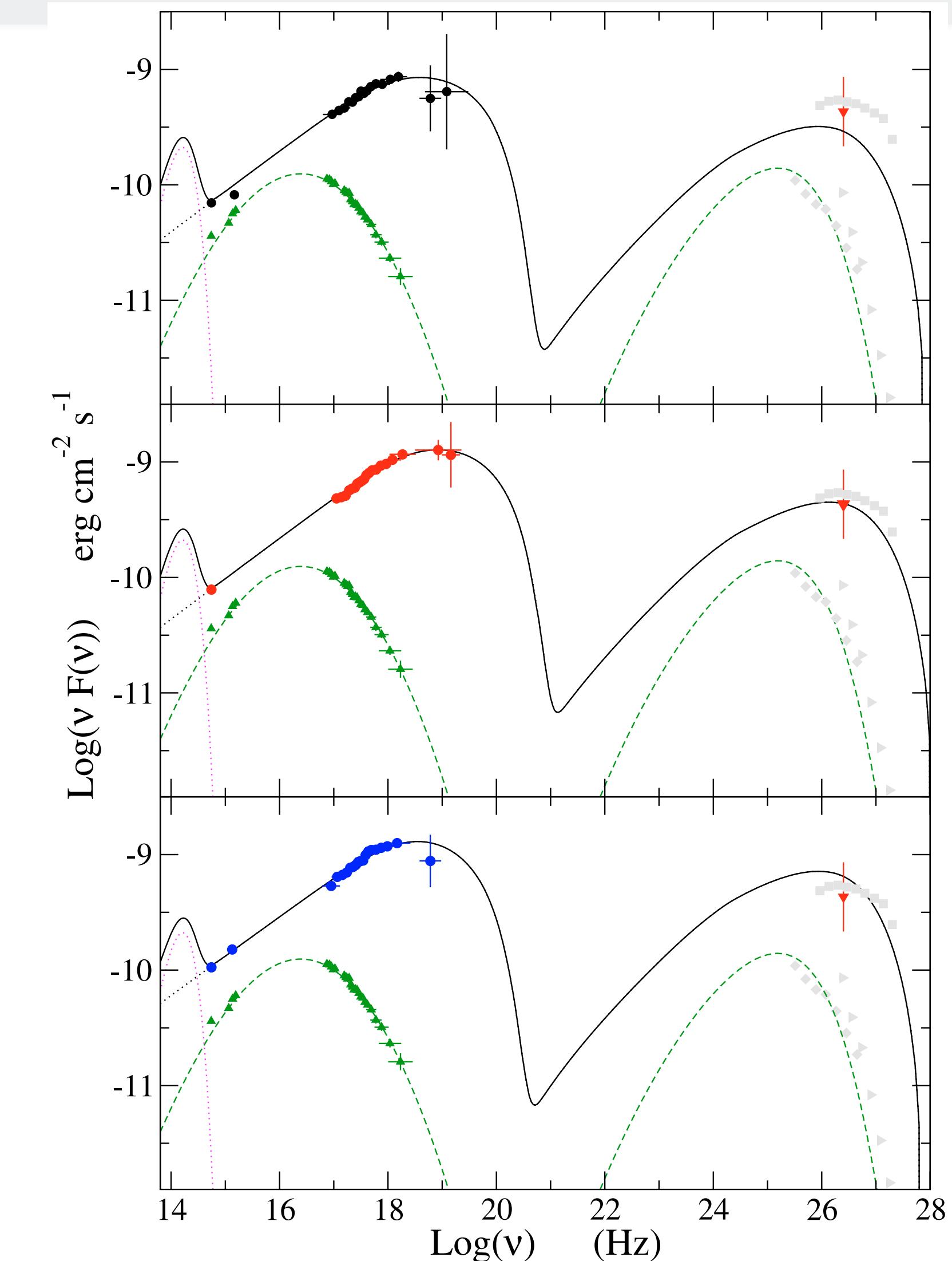
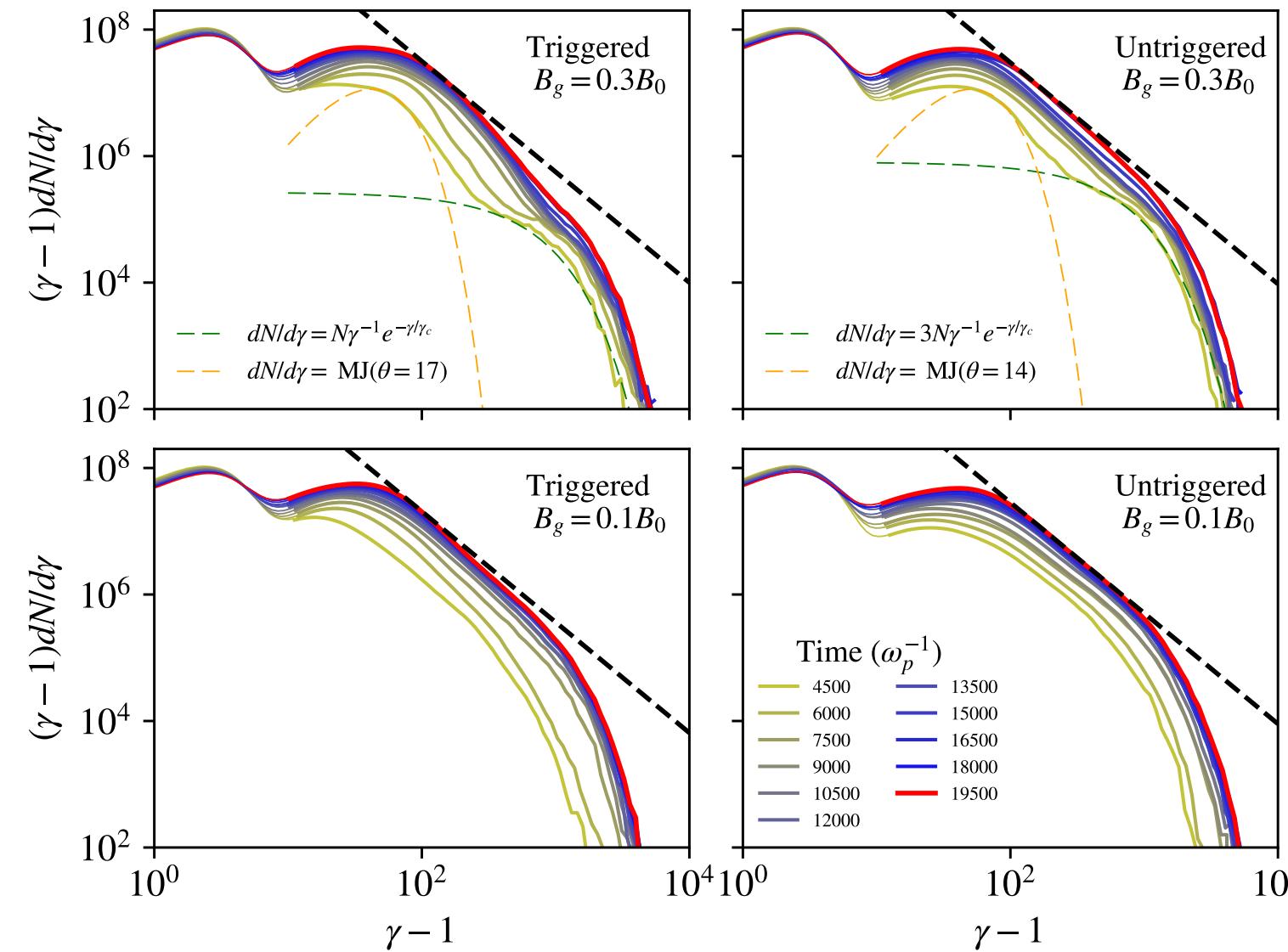
Synch index~[1.6-1.7]=> $s\sim[2.2-2.4]$



Lemoine,Pelletier 2003 FI multip.

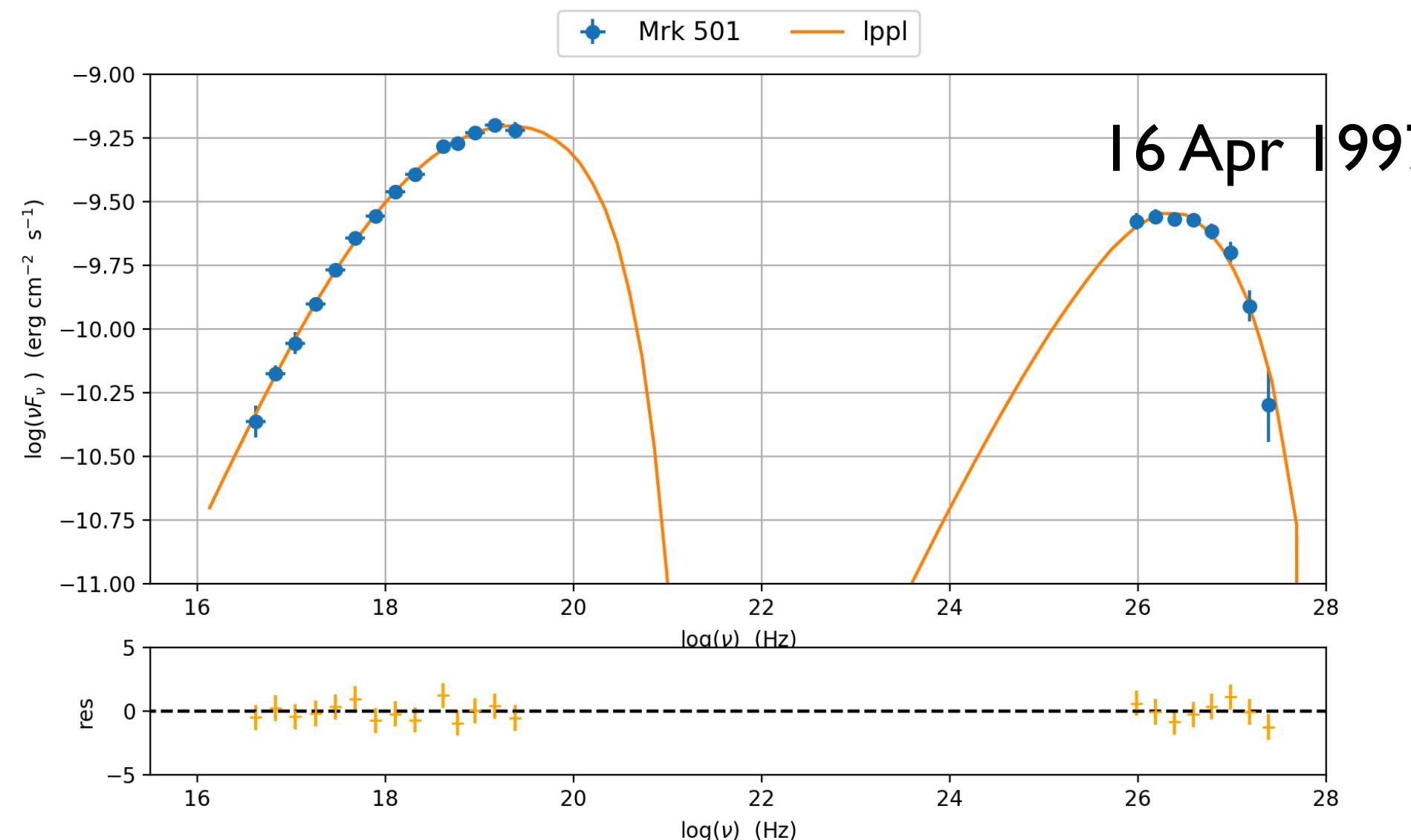
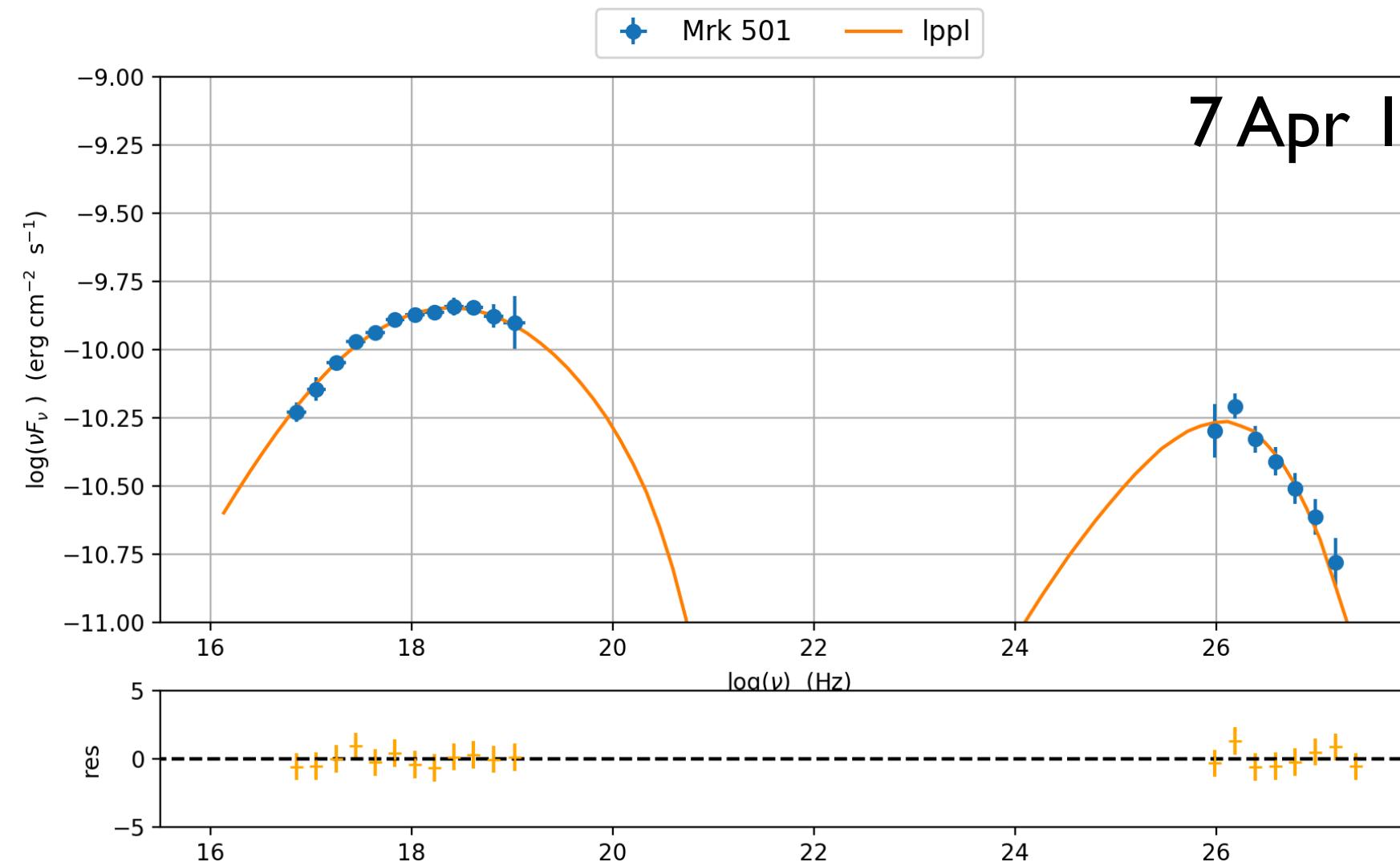


Ball+ 2019 magnetic reconnection (PIC)



Mrk 501 1997 Flare

Massaro & Tramacere +2006



$$s = 1 + \frac{t_{acc}}{2t_{esc}}$$

s **r**
 γ_c

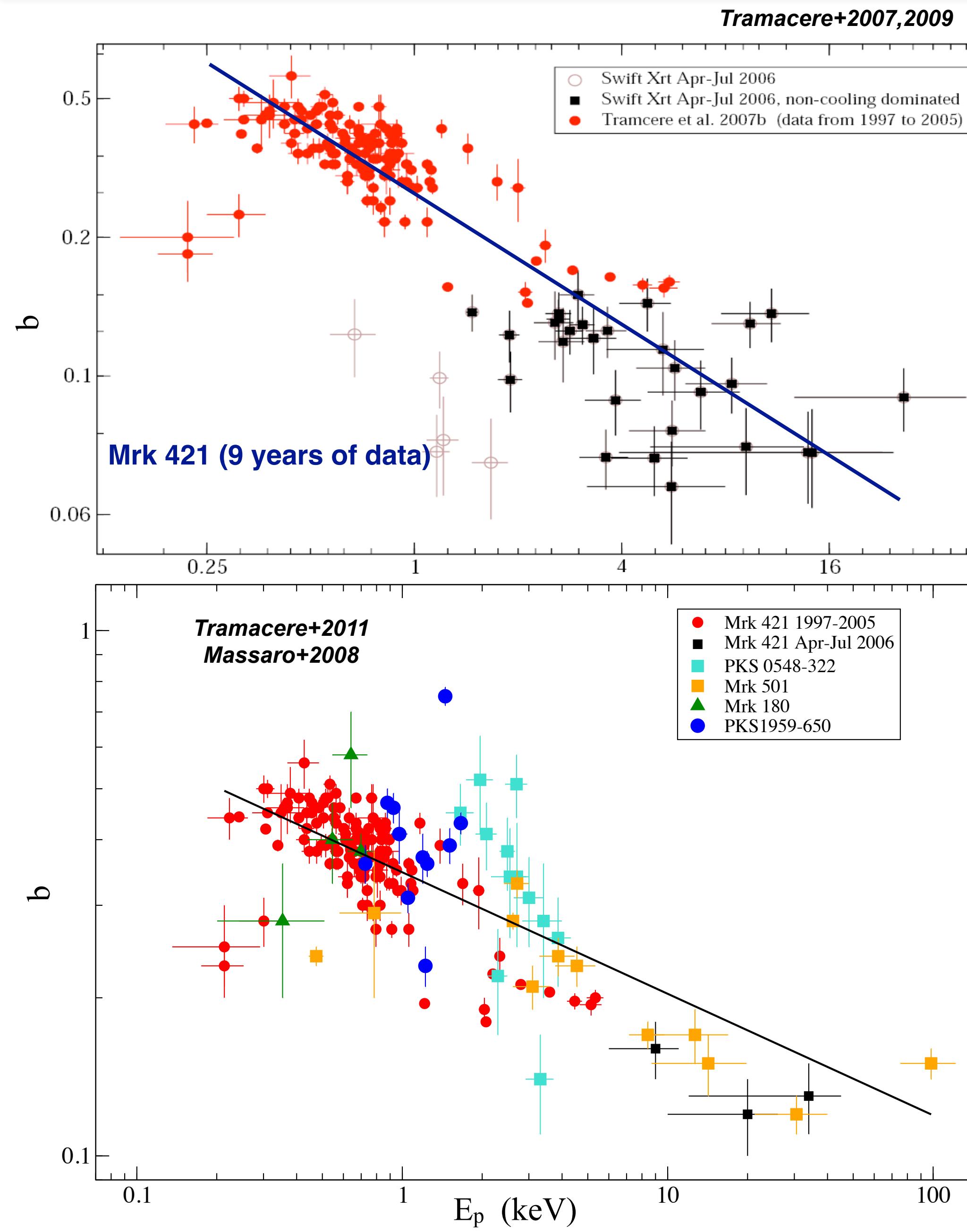
best fit pars

best-fit parameters:

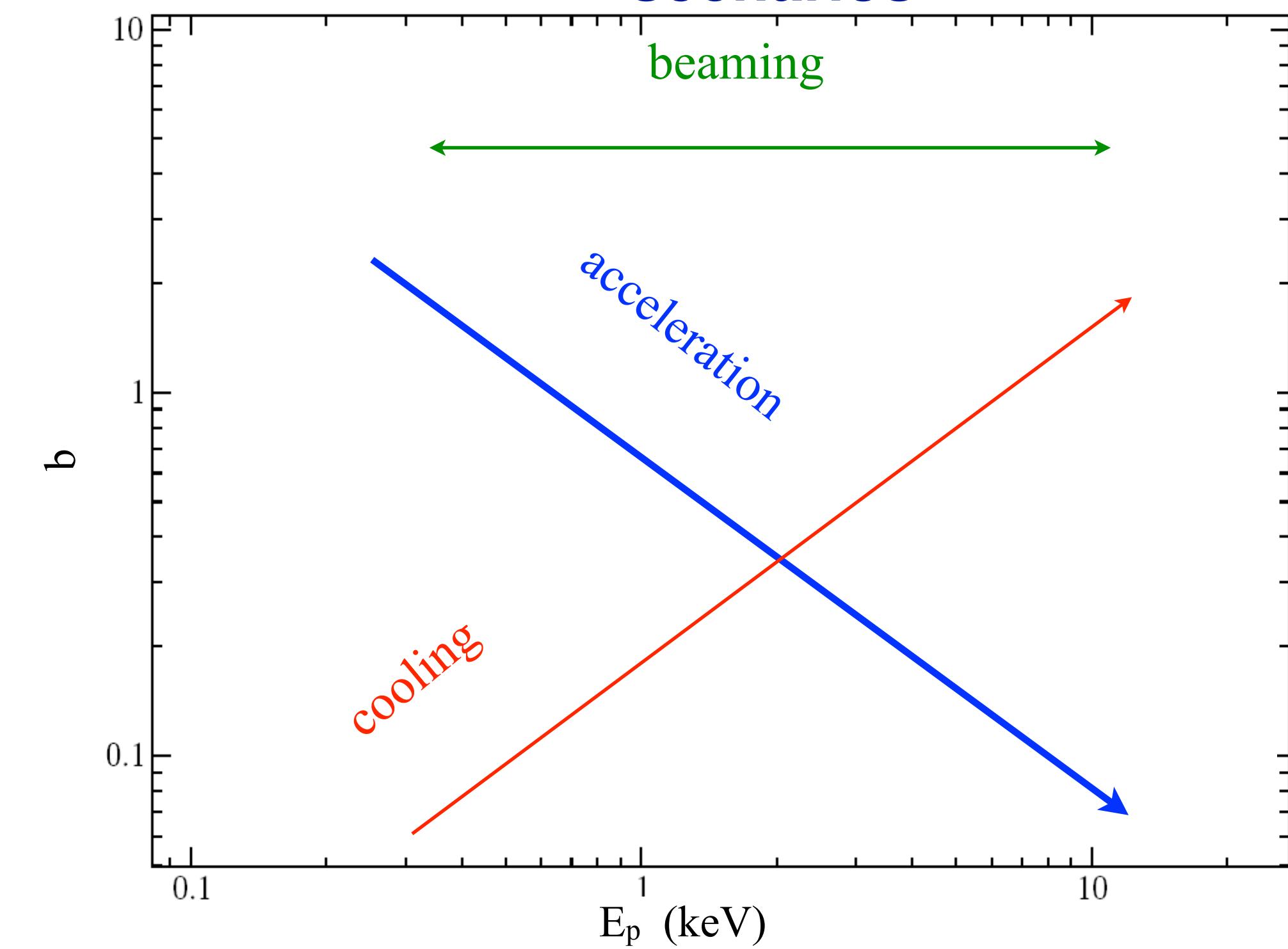
Name	best-fit value	best-fit err +
B	+1.072178e-01	+5.436622e-03
N	+4.585348e+00	+4.756569e-01
R	Frozen	Frozen
beam_obj	+2.450884e+01	+7.642113e-01
gamma0_log_parab	+6.609649e+04	+7.427709e+03
gmax	+1.860044e+14	+5.881595e+14
gmin	+1.404527e+03	+2.198648e+02
r	+7.513452e-01	+5.059815e-02
s	+1.638026e+00	+3.170384e-02
z_cosm	Frozen	Frozen

best-fit parameters:

Name	best-fit value	best-fit err +
B	+3.065207e-01	+1.159567e-02
N	+1.079944e+02	+7.375385e+00
R	Frozen	Frozen
beam_obj	+2.722013e+01	+5.889626e-01
gamma0_log_parab	+6.493888e+04	+5.410315e+03
gmax	+1.902146e+06	+2.216666e+02
gmin	+3.003970e+02	+5.686711e+01
r	+6.778727e-01	+3.526656e-02
s	+1.321307e+00	+1.844825e-02
z_cosm	Frozen	Frozen



Ep-vs-b, different scenarios

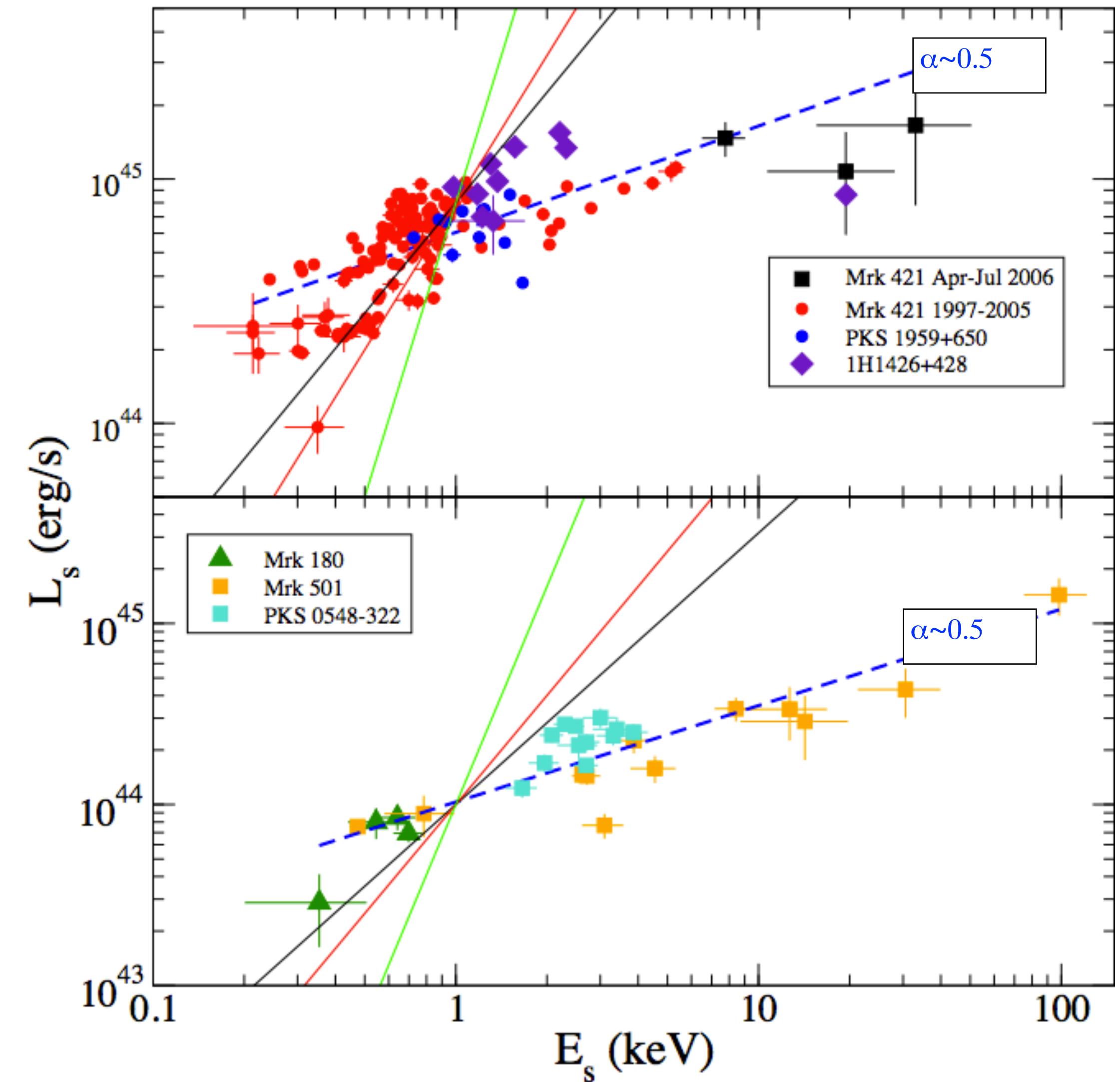


11 years of data:

**PKS 0548-322, 1H1426+418,
Mrk 501, 1ES1959+650, PKS2155-34**

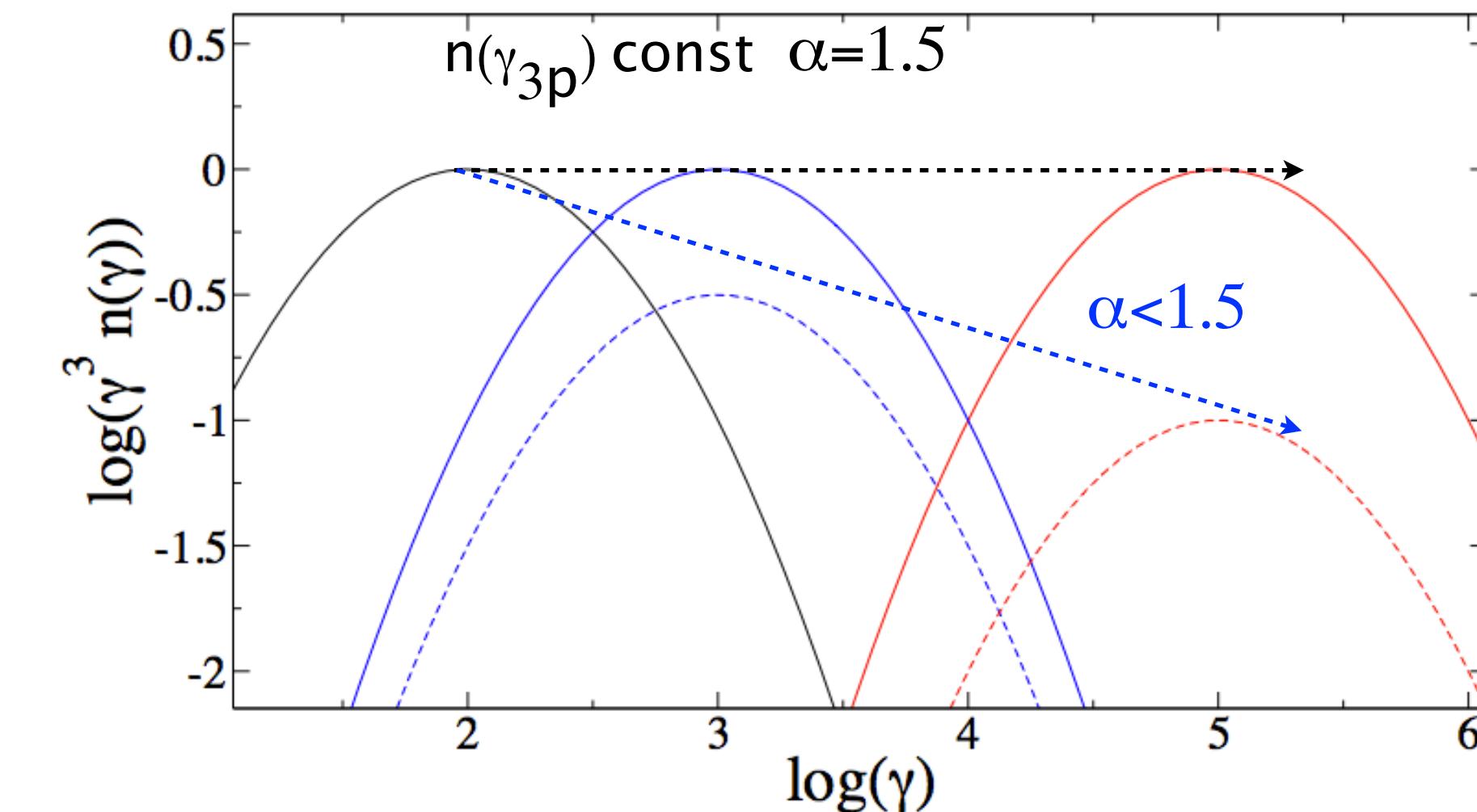
Long term (overall 13 years of data) Ep-vs-b trends hint for an acceleration dominated scenario

long-trend main drivers



• $\gamma_{3p} \uparrow$ and $n(\gamma_{3p}) \downarrow \Rightarrow \alpha < 1.5$
acceleration+energy conservation

• $B \rightarrow \alpha = 2.0$, incompatible as
• $\delta \rightarrow \alpha = 4$ long-trend main driver

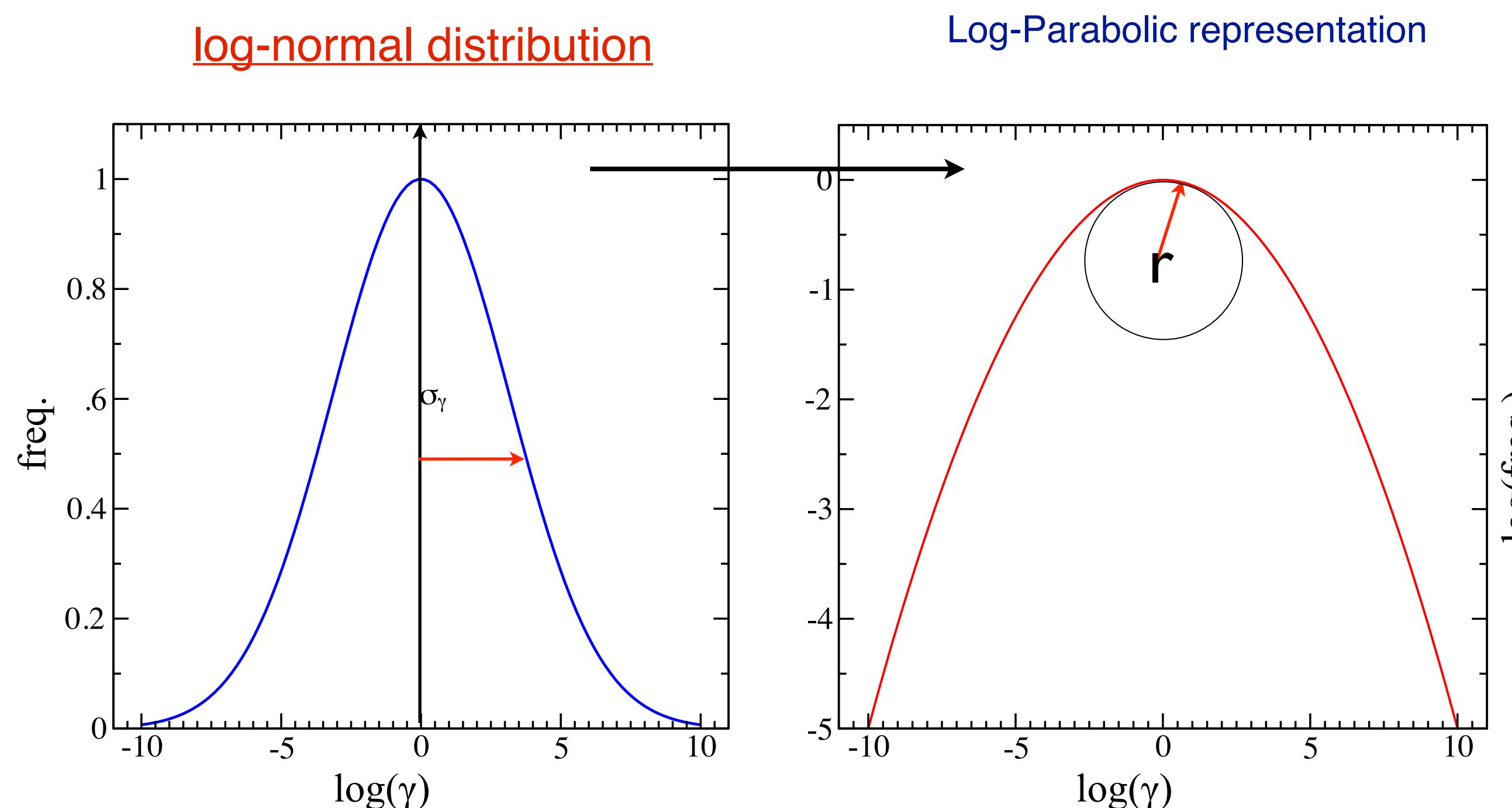


The origin of the log-parabolic shape: statistical derivation



$$\gamma_{n_s} = \gamma_0 \prod_{i=1}^{n_s} \varepsilon_i$$

C.L. Theorem
multipl. case



$$\log(n(\gamma)) \propto \frac{(\log \gamma - \mu)^2}{2\sigma_\gamma^2} \propto r [\log(\gamma) - \mu]^2$$

$$\sigma_y^2 = \sigma^2(\log(\gamma)) \approx n_s \left(\frac{\sigma_\varepsilon}{\bar{\varepsilon}} \right)$$

curvature = $\frac{1}{\sigma_\gamma} = 2r$

$$r = \frac{c_e}{2n_s \left(\frac{\sigma_\varepsilon}{\bar{\varepsilon}} \right)^2}$$

The origin of the log-parabolic shape: diffusion equation approach

$$\frac{\partial n(\gamma, t)}{\partial t} = \frac{\partial}{\partial \gamma} \left\{ -[S(\gamma, t) + D_A(\gamma, t)]n(\gamma, t) + D_p(\gamma, t)\frac{\partial n(\gamma, t)}{\partial \gamma} \right\} - \frac{n(\gamma, t)}{T_{esc}(\gamma)} + Q(\gamma, t)$$

CL theorem

analytical solution for:

$D_p \sim \gamma^q$, $q=2$
“hard-sphere” case

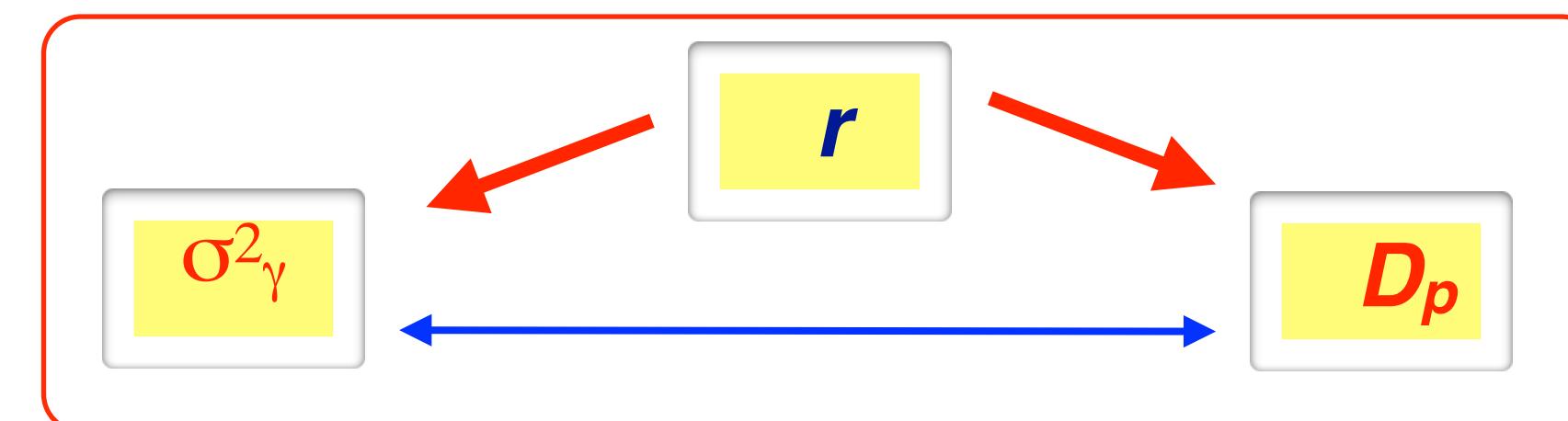
Melrose 1968, Kardashedv 1962

$$n(\gamma, t) = \frac{N_0}{\gamma \sqrt{4\pi D_{p0} t}} \exp \left\{ -\frac{[\ln(\gamma/\gamma_0) - (A_{p0} - D_{p0})t]^2}{4D_{p0}t} \right\}$$

$$\log(n(\gamma)) \propto \frac{(\log \gamma - \mu)^2}{2\sigma_\gamma^2} \propto r [\log(\gamma) - \mu]^2$$

$$\propto \frac{1}{D_{p0}t} \rightarrow D_{p0} \propto \left(\frac{\sigma_\varepsilon}{\bar{\varepsilon}} \right)^2$$

The curvature r is inversely proportional to $t \Rightarrow n_s$ and $D_p \Rightarrow \sigma_\varepsilon$



Temporal evolution

Tramacere +2011

injection term

$$L_{inj} = \frac{4}{3}\pi R^3 \int \gamma_{inj} m_e c^2 Q(\gamma_{inj}, t) d\gamma_{inj} \quad (\text{erg/s})$$

systematic term

$$S(\gamma, t) = -C(\gamma, t) + A(\gamma, t)$$

cooling term

$$C(\gamma) = |\dot{\gamma}_{\text{synch}}| + |\dot{\gamma}_{\text{IC}}|$$

syst. acc. term

$$A(\gamma) = A_{p0}\gamma, t_A = \frac{1}{A_0}$$

$$\frac{\partial n(\gamma, t)}{\partial t} = \frac{\partial}{\partial \gamma} \left\{ -[S(\gamma, t) + D_A(\gamma, t)]n(\gamma, t) + D_p(\gamma, t)\frac{\partial n(\gamma, t)}{\partial \gamma} \right\} - \frac{n(\gamma, t)}{T_{\text{esc}}(\gamma)} + Q(\gamma, t)$$

Turbulent magnetic field

$$W(k) = \frac{\delta B(k_0^2)}{8\pi} \left(\frac{k}{k_0}\right)^{-q}$$

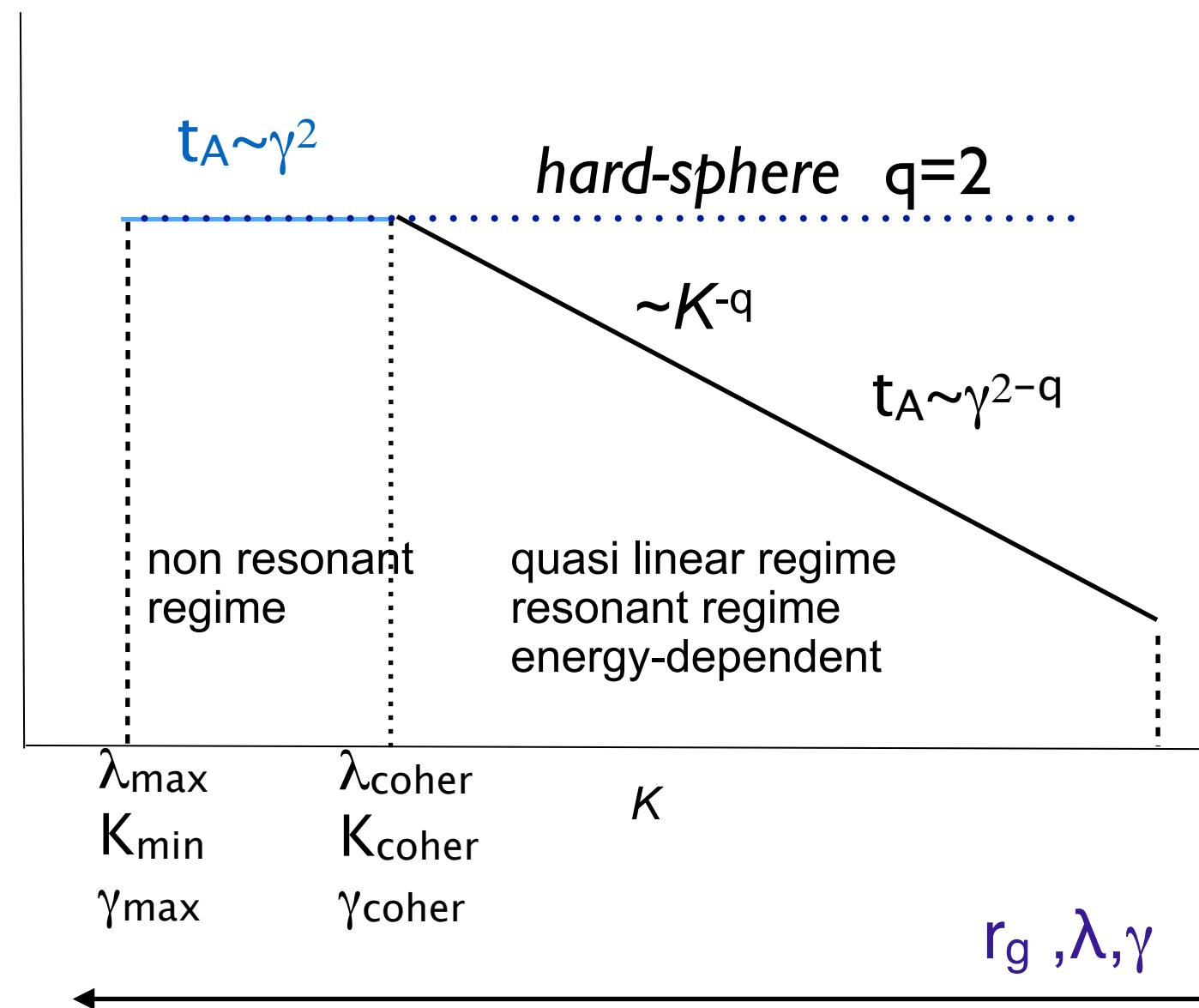
momentum diffusion term

$$D_p \approx \beta_A^2 \left(\frac{\delta B}{B_0}\right)^2 \left(\frac{\rho_g}{\lambda_{\max}}\right)^{q-1} \frac{p^2 c^2}{\rho_g c}$$

set-up of the accelerator

$$t_D = \frac{1}{D_{p0}} \left(\frac{\gamma}{\gamma_0} \right)^{2-q}$$

$$t_{DA} = \frac{1}{2D_{p0}} \left(\frac{\gamma}{\gamma_0} \right)^{2-q}$$

 $W(k)$ 

e-folding time

observed values

$$E_{p1}/E_{p2} \sim 5$$

$$\Delta t \sim \text{few ks}$$

values compatible with
Tammi & Duffy 2009

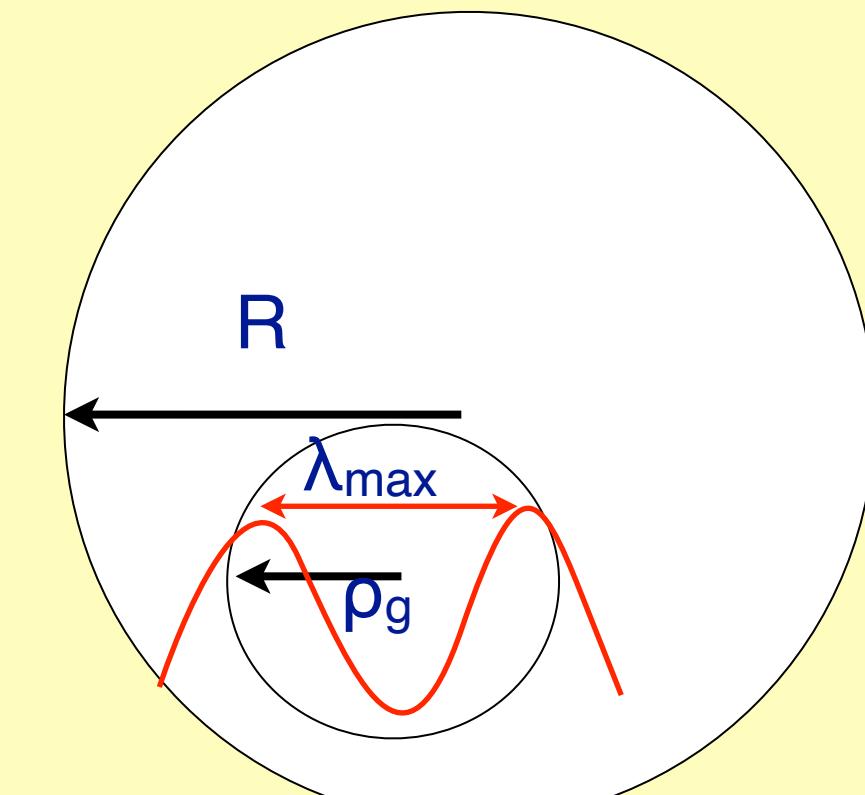
$$t_{DA} \sim < 5 \text{ ks}$$

$$t_D \sim < 10 \text{ ks}$$

set-up of the accelerator

- $R \sim 10^{13}-10^{15} \text{ cm}$
- $\delta B/B \ll 1$, $B \sim [0.01-1.0] \text{ G}$
- $\beta_A \sim 0.1-0.5$
- $\lambda_{\max} < R \Rightarrow \sim 10^{[9-15]} \text{ cm}$
- $\rho_g < \lambda_{\max} \Rightarrow \gamma_{\max} \sim 10^{7.5}$

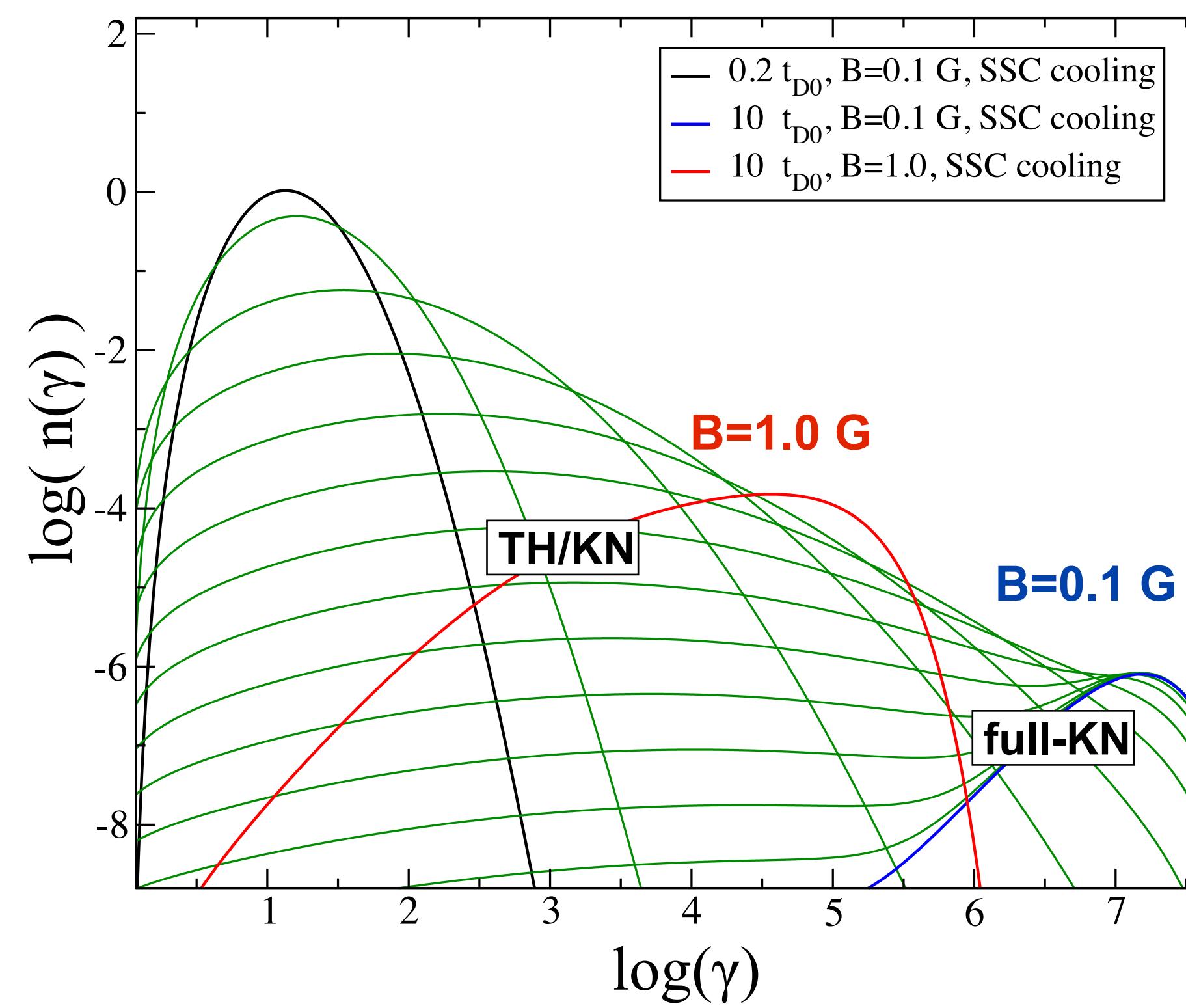
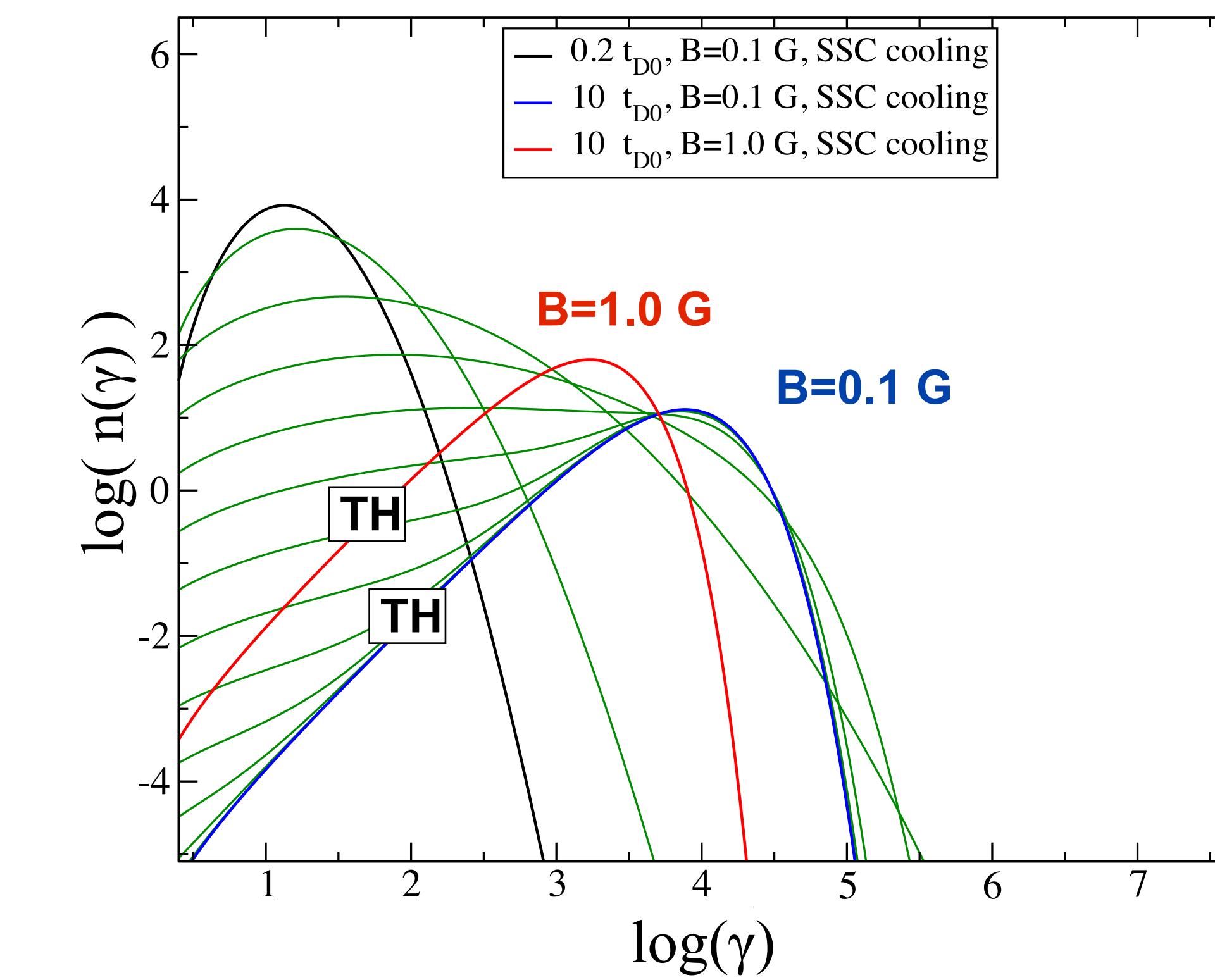
$$\rightarrow t_D \sim < 10^4 \text{ ks}$$



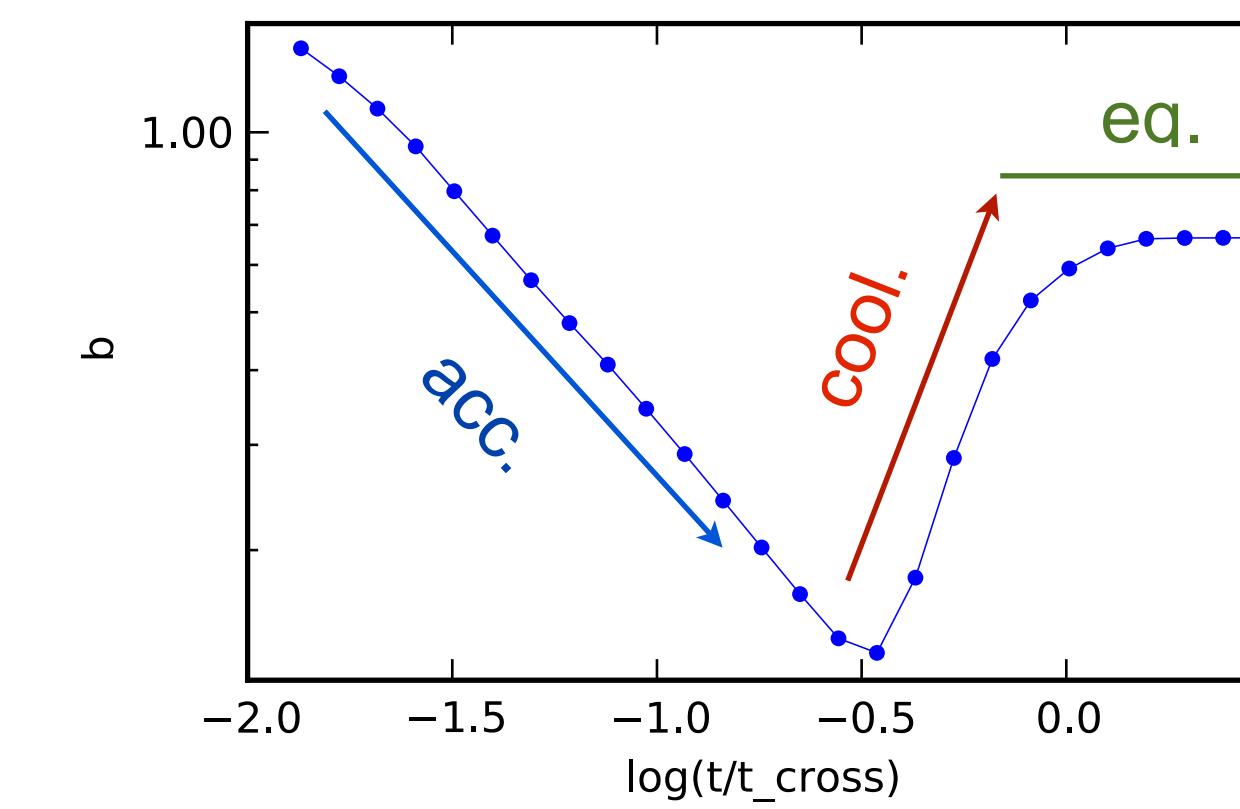
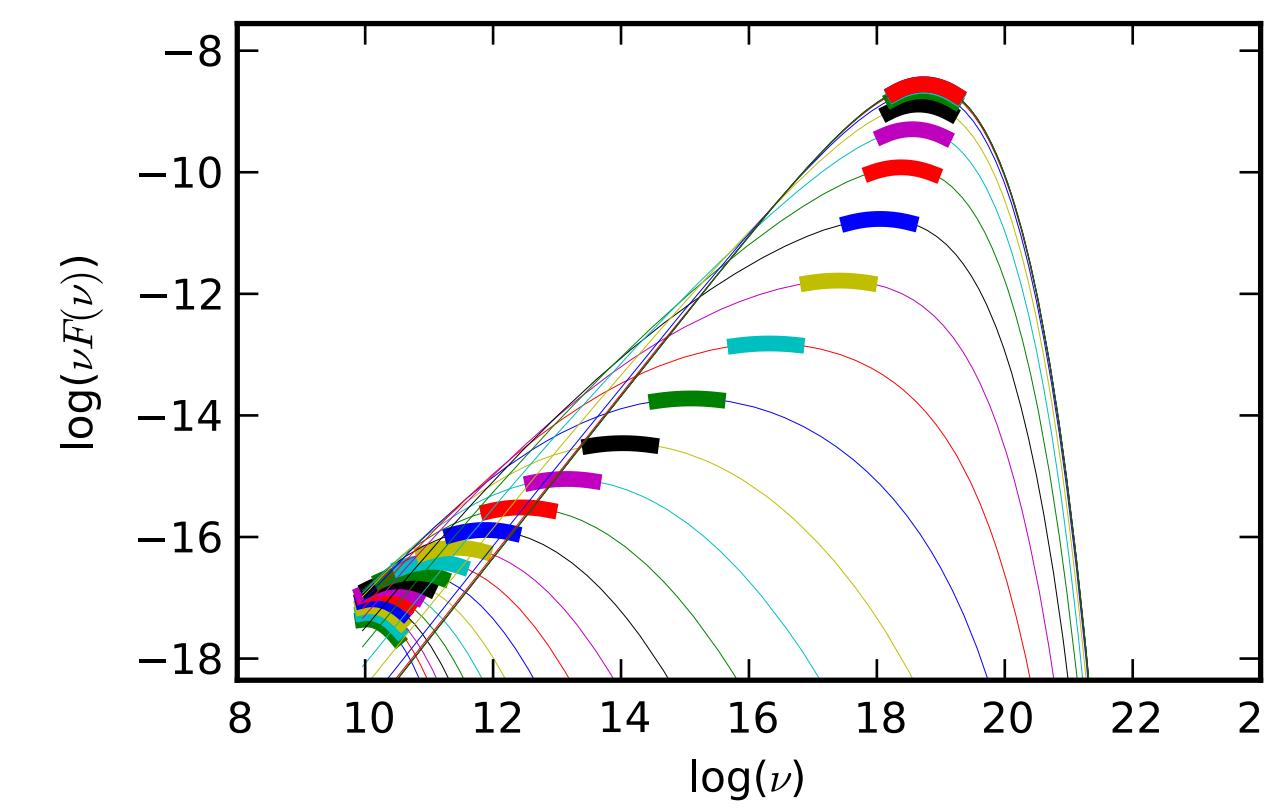
$$D_p \approx \beta_A^2 \left(\frac{\delta B}{B_0} \right)^2 \left(\frac{\rho_g}{\lambda_{\max}} \right)^{q-1} \frac{p^2 c^2}{\rho_g c}$$

$$\rho_g = pc/qB$$

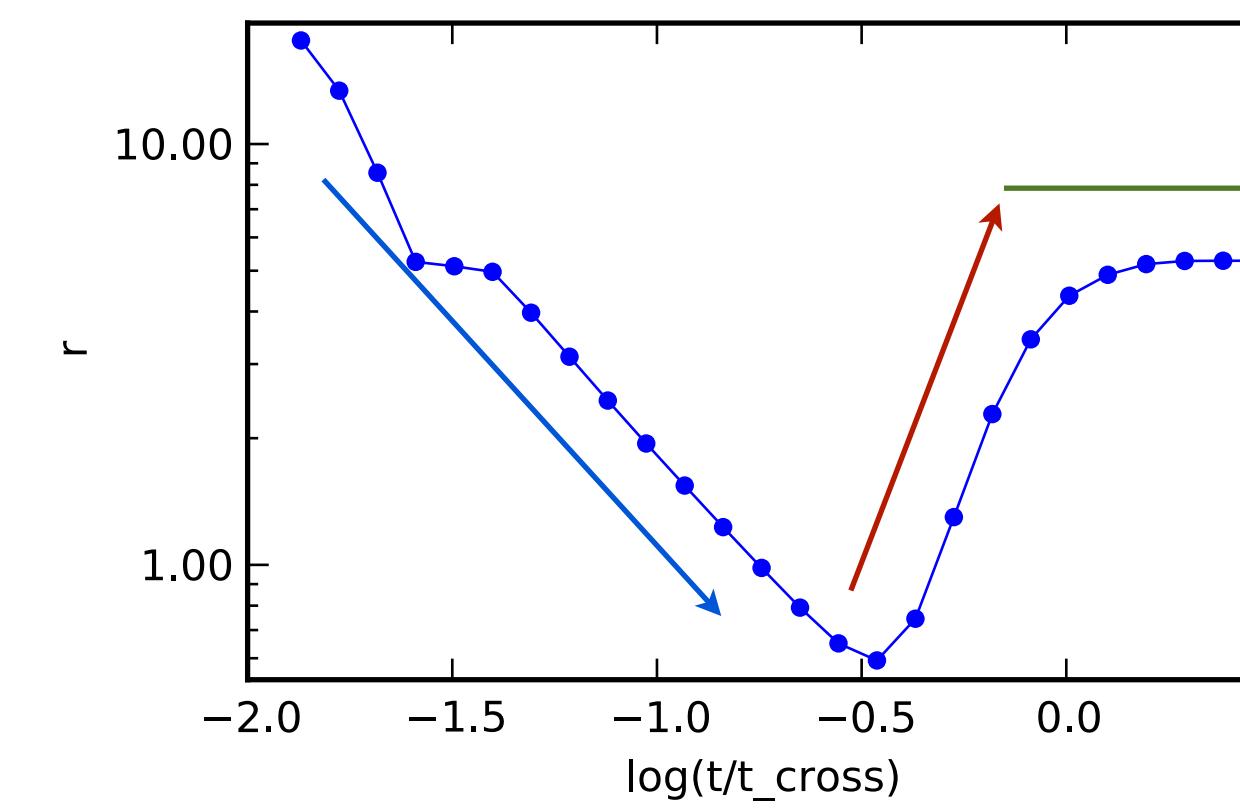
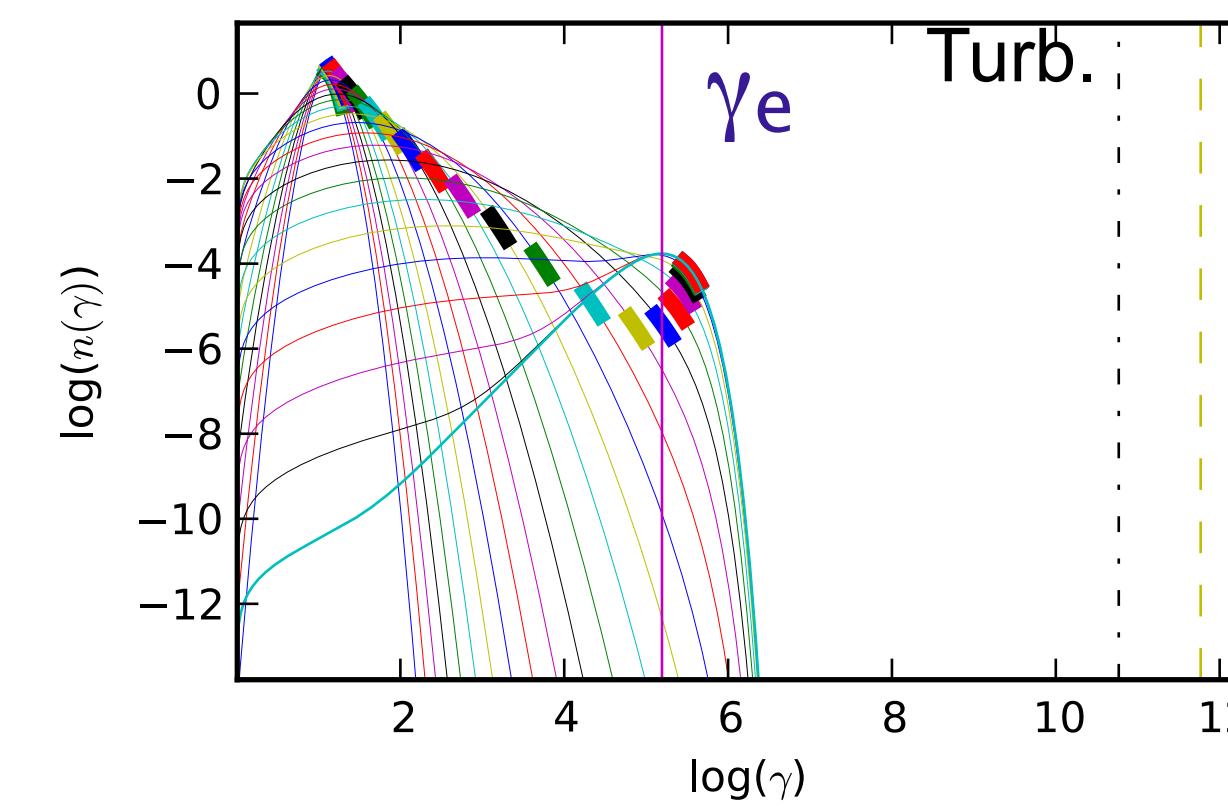
IC cooling and equilibrium

 $R = 1 \times 10^{15} \text{ cm}$  $R = 5 \times 10^{13} \text{ cm}$ 

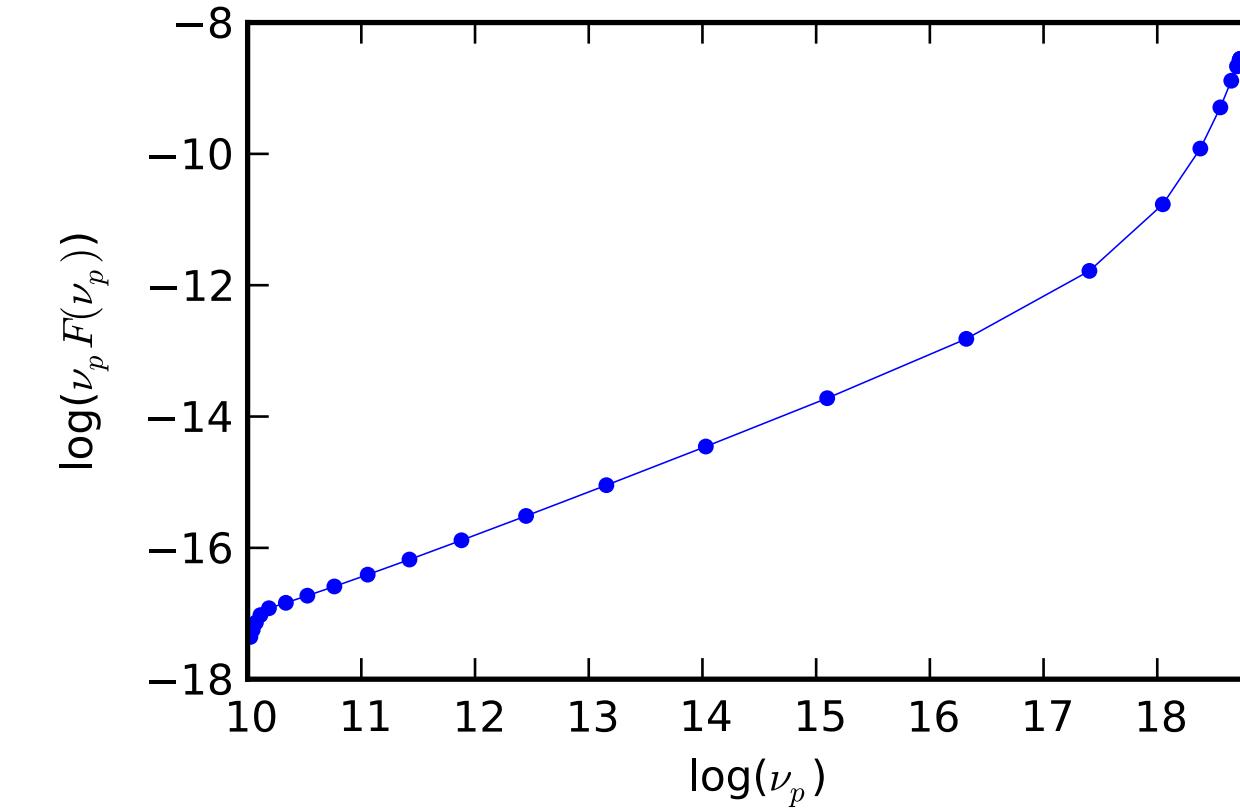
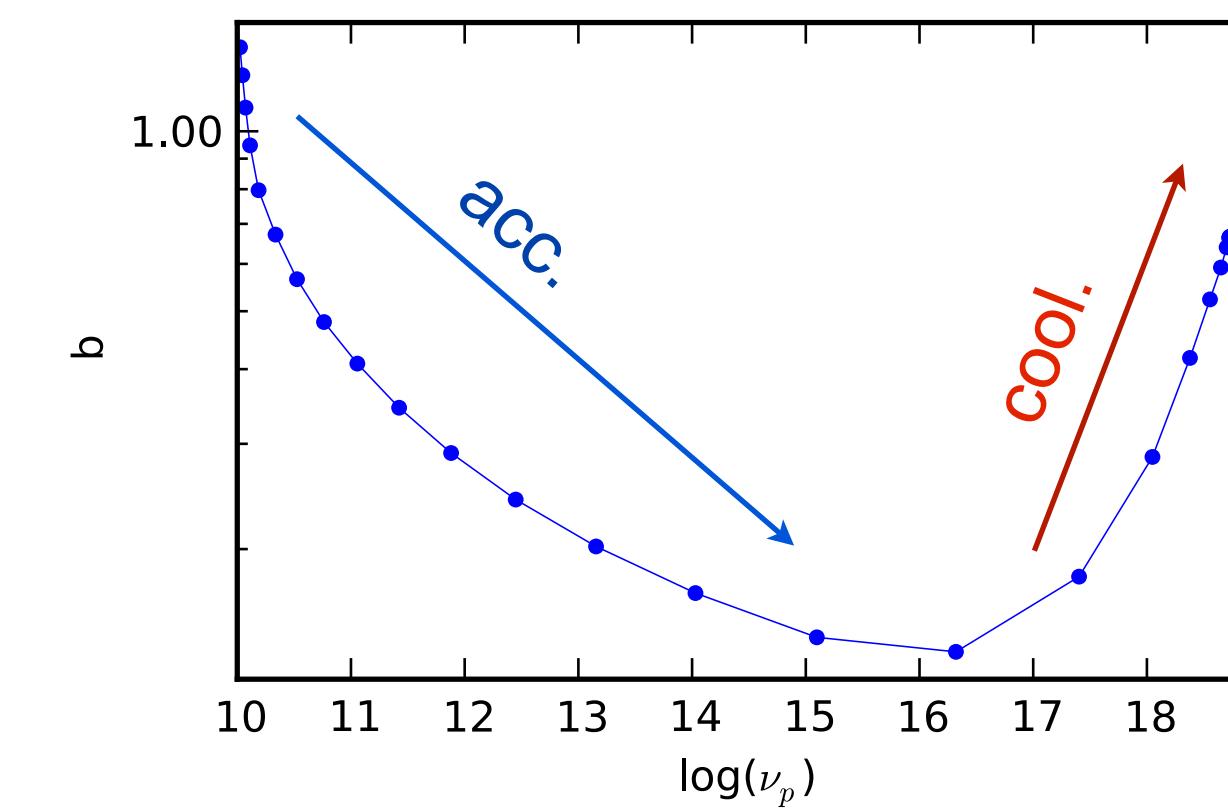
acceleration-vs-equil.



Synchrotron

Tramacere +20 II

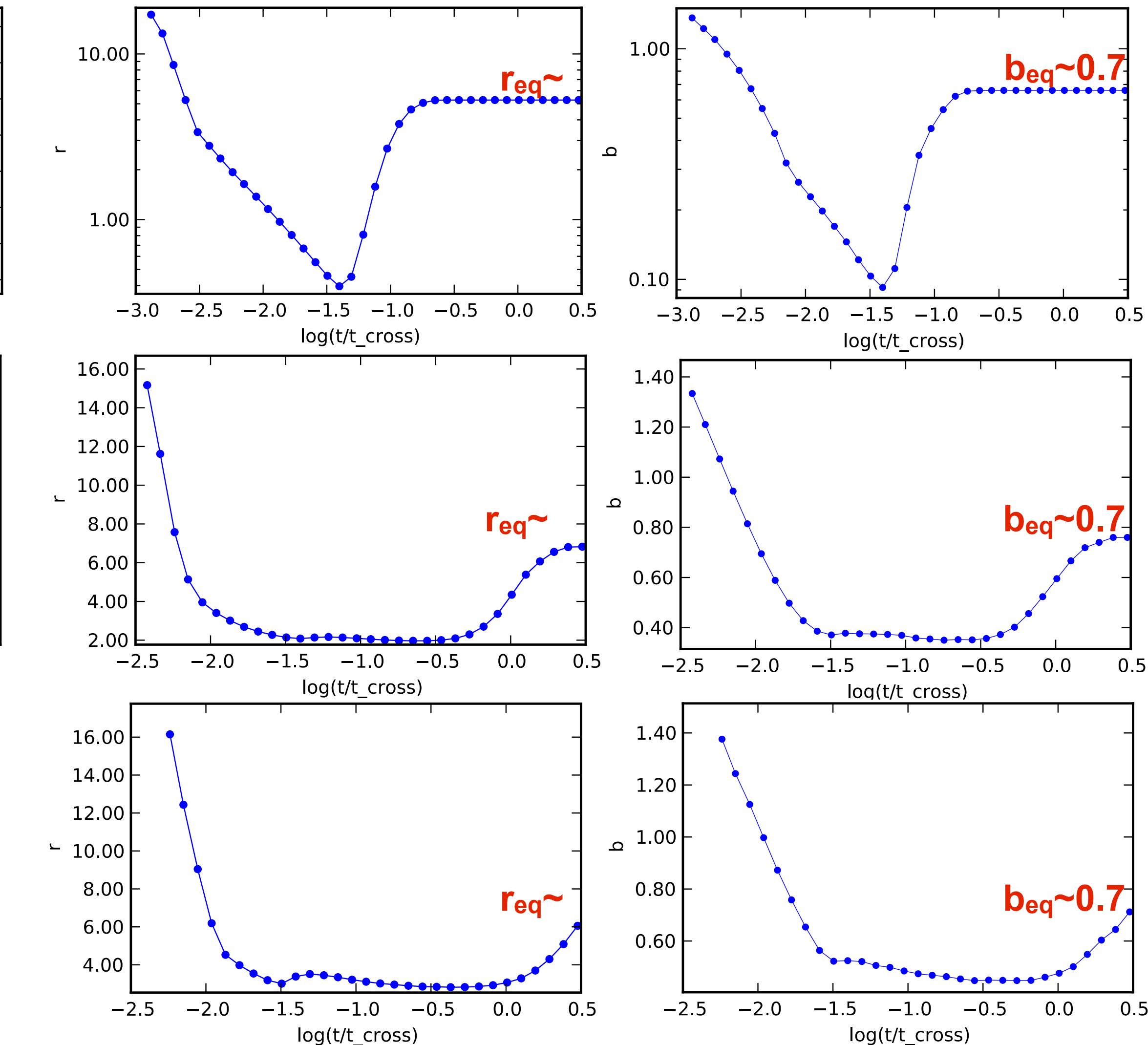
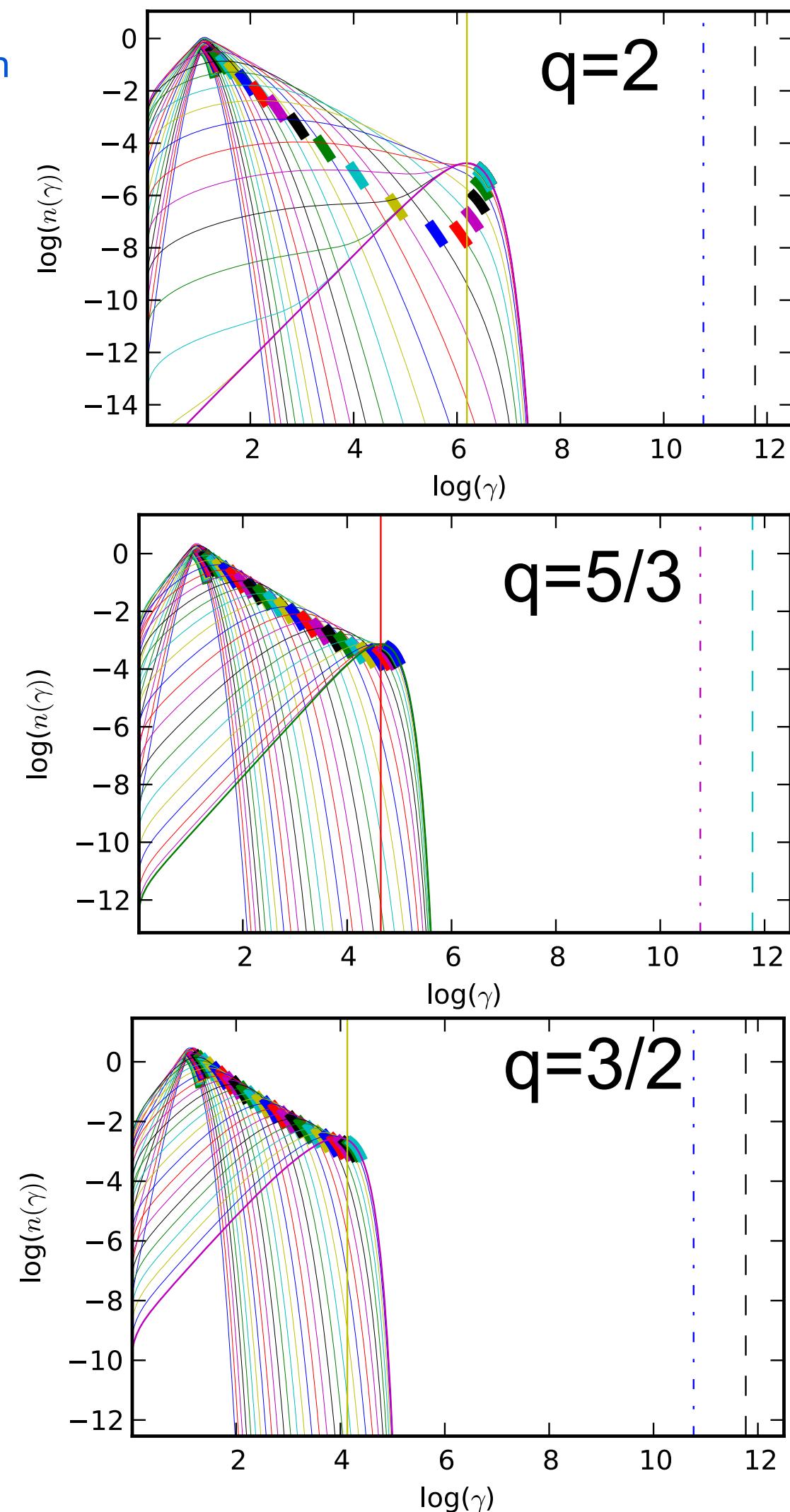
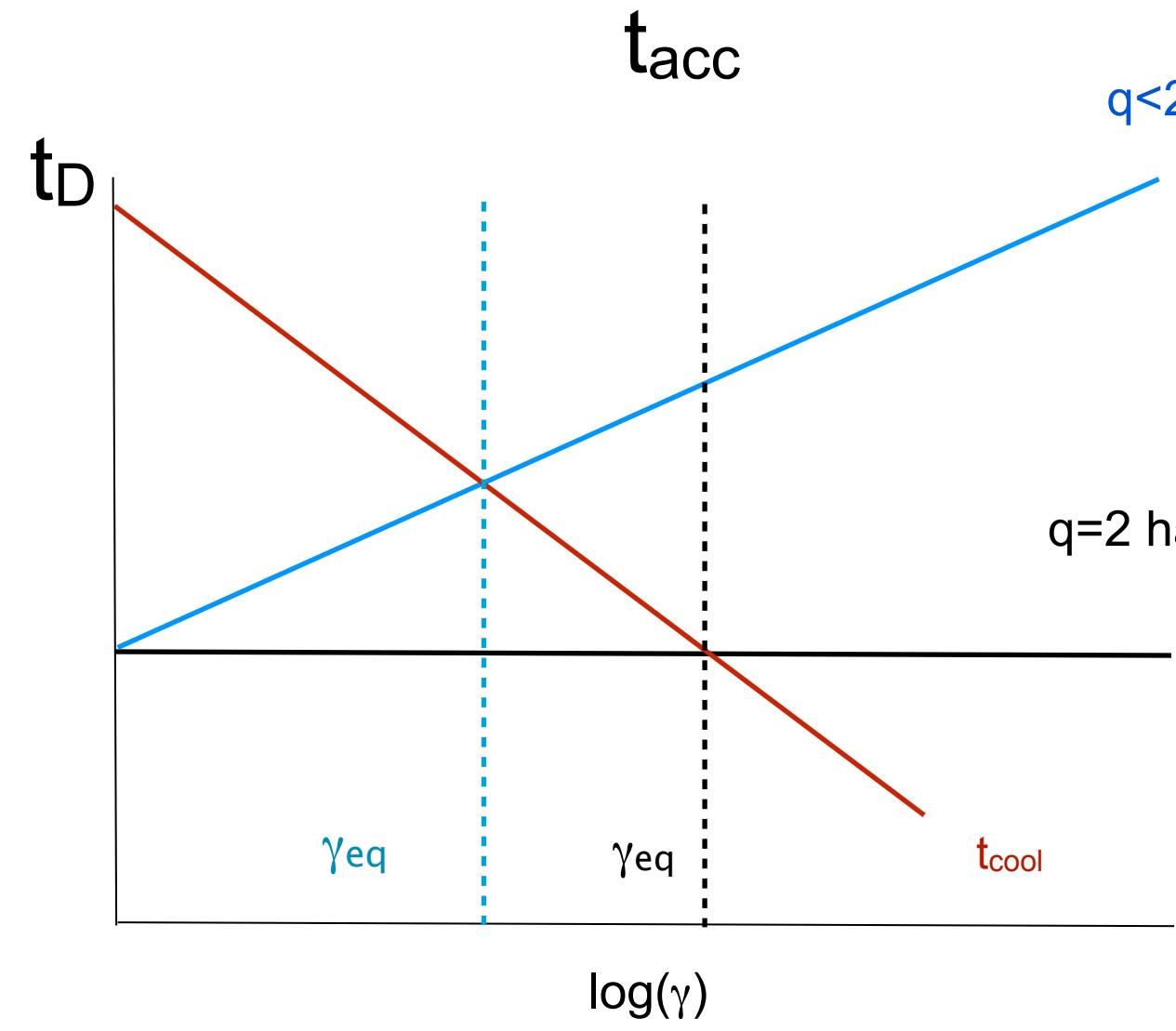
electrons



Synchrotron

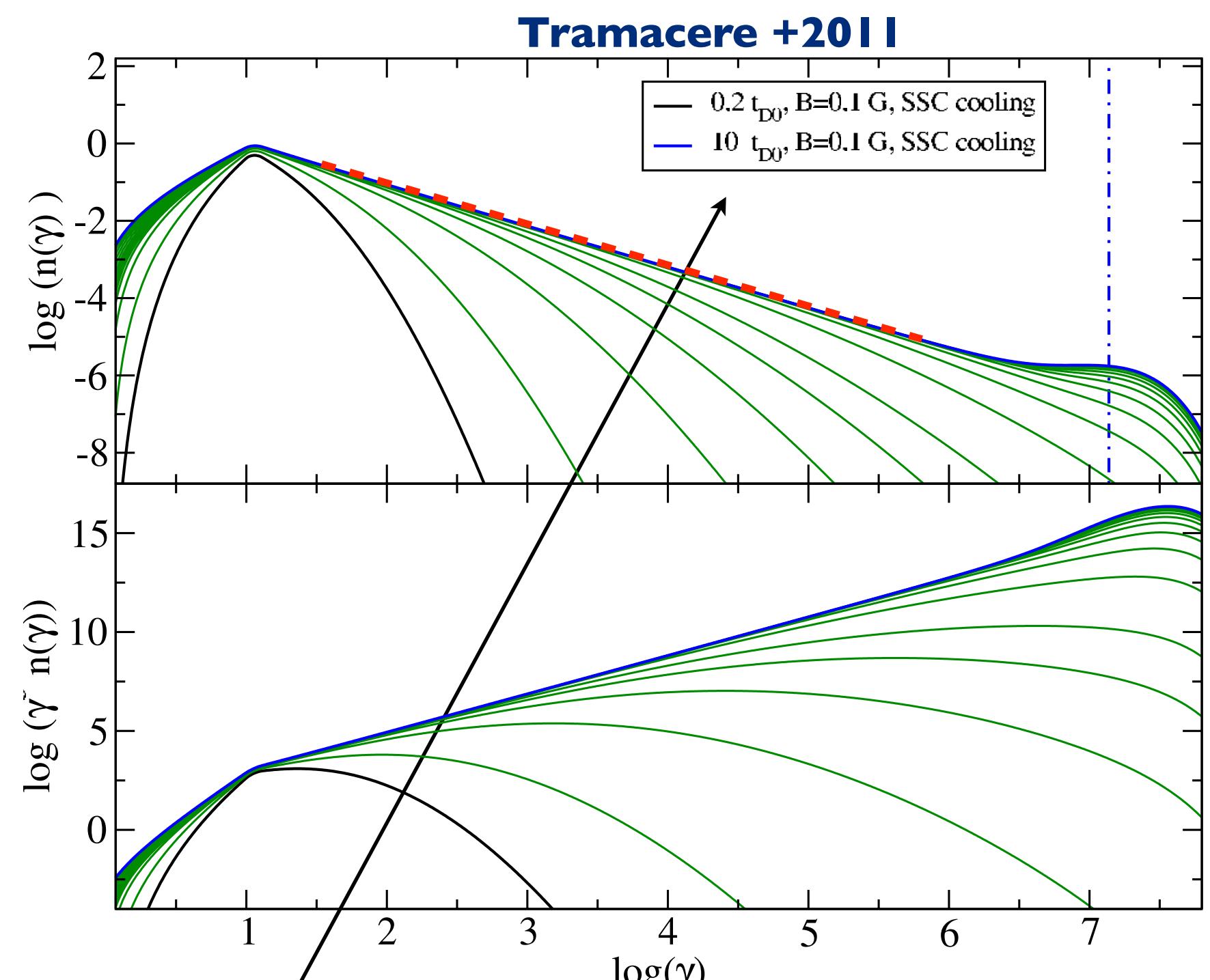
effect of the turbulence index q and cooling

$B=1.0 \text{ G}$, $t_{D0}=10^3$, $R=5\times 10^{15} \text{ cm}$



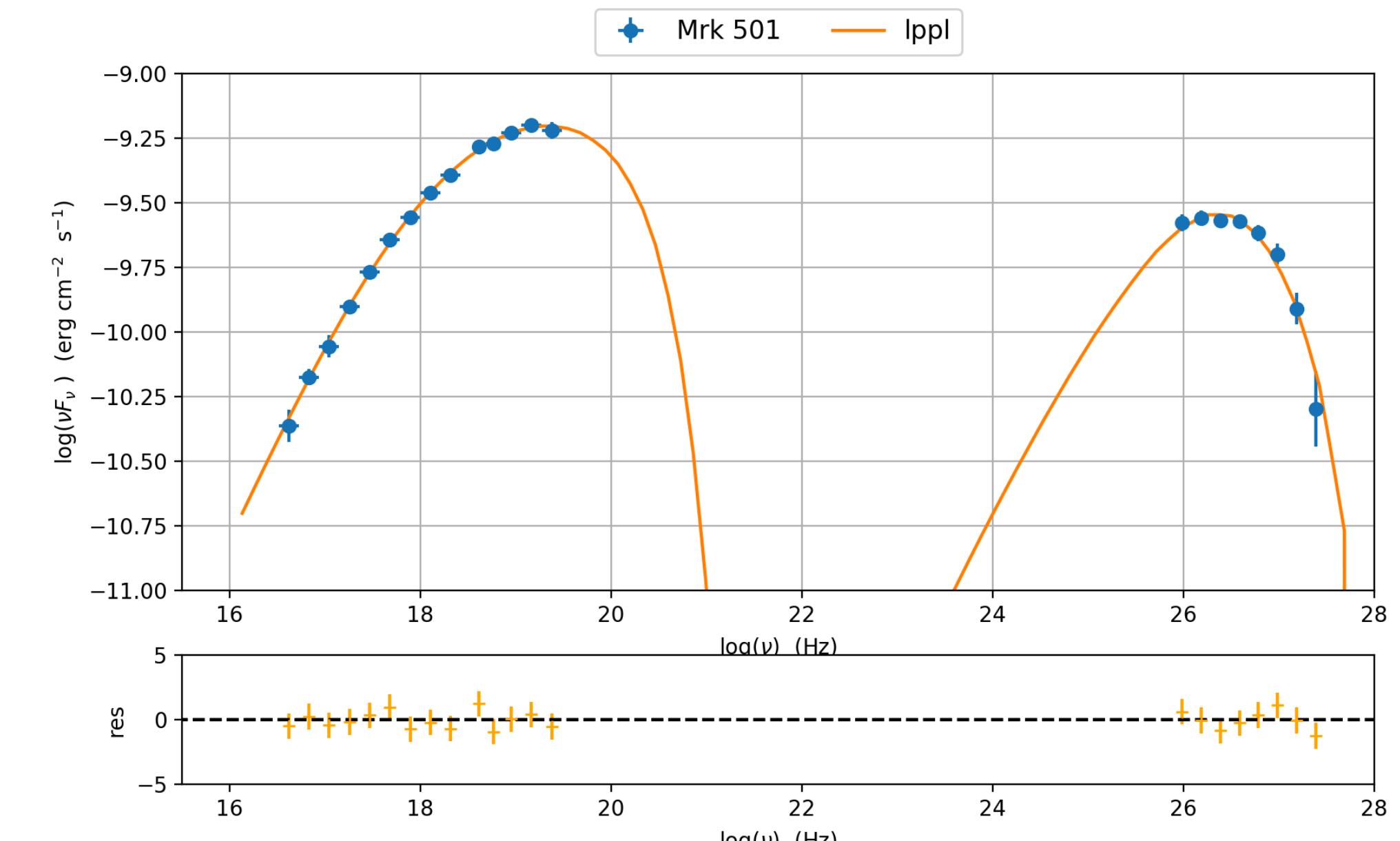
Pile-up and hard spectra

$q=2$, $R=10^{15}$ cm, $B=0.1$ G, $t_{\text{inj}}=t_D=10^4$ s



$$s \text{ in agreement with } s = 1 + \frac{t_{acc}}{2t_{esc}}$$

Mrk 501 1997



Massaro & Tramacere +2006

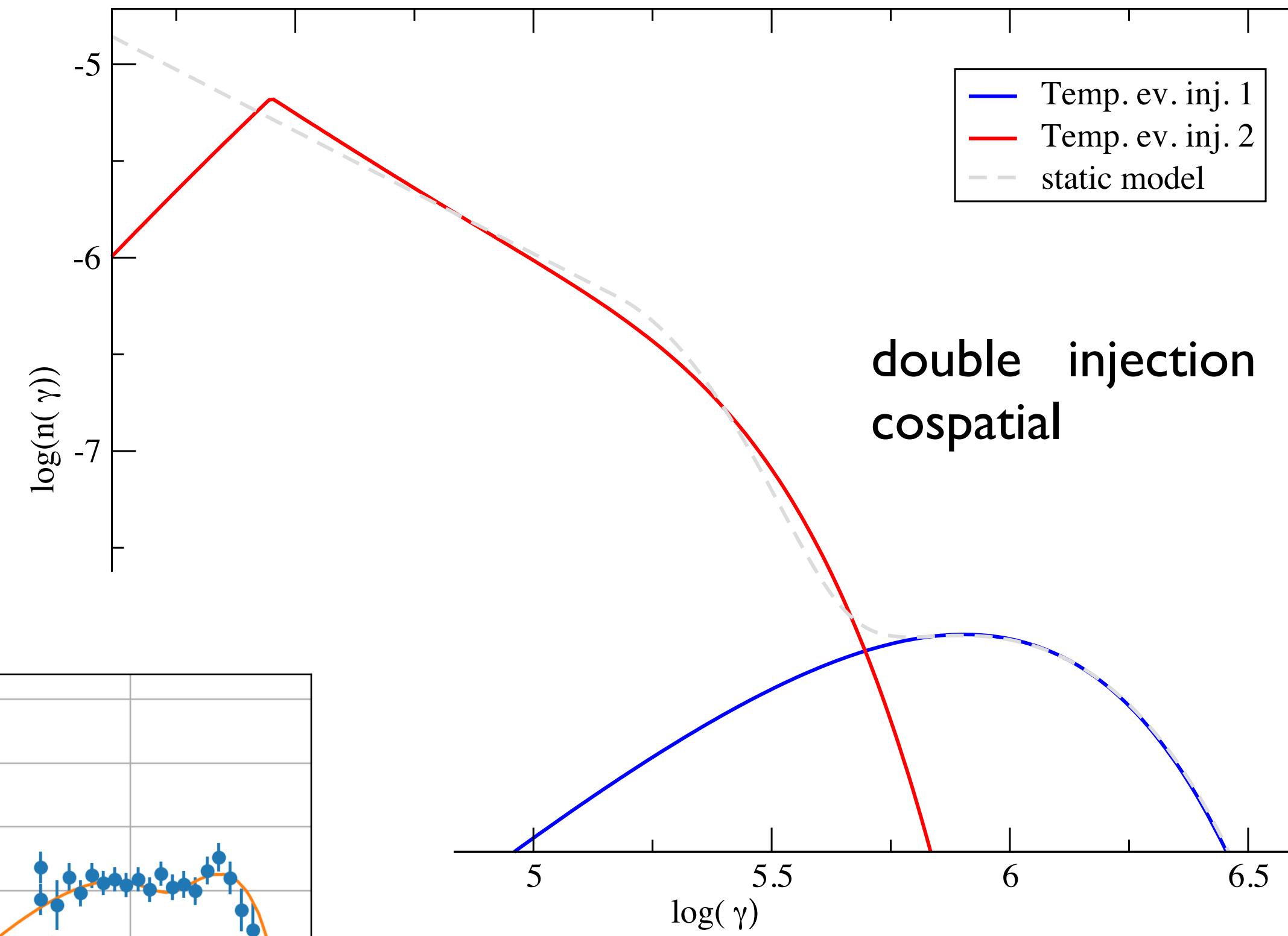
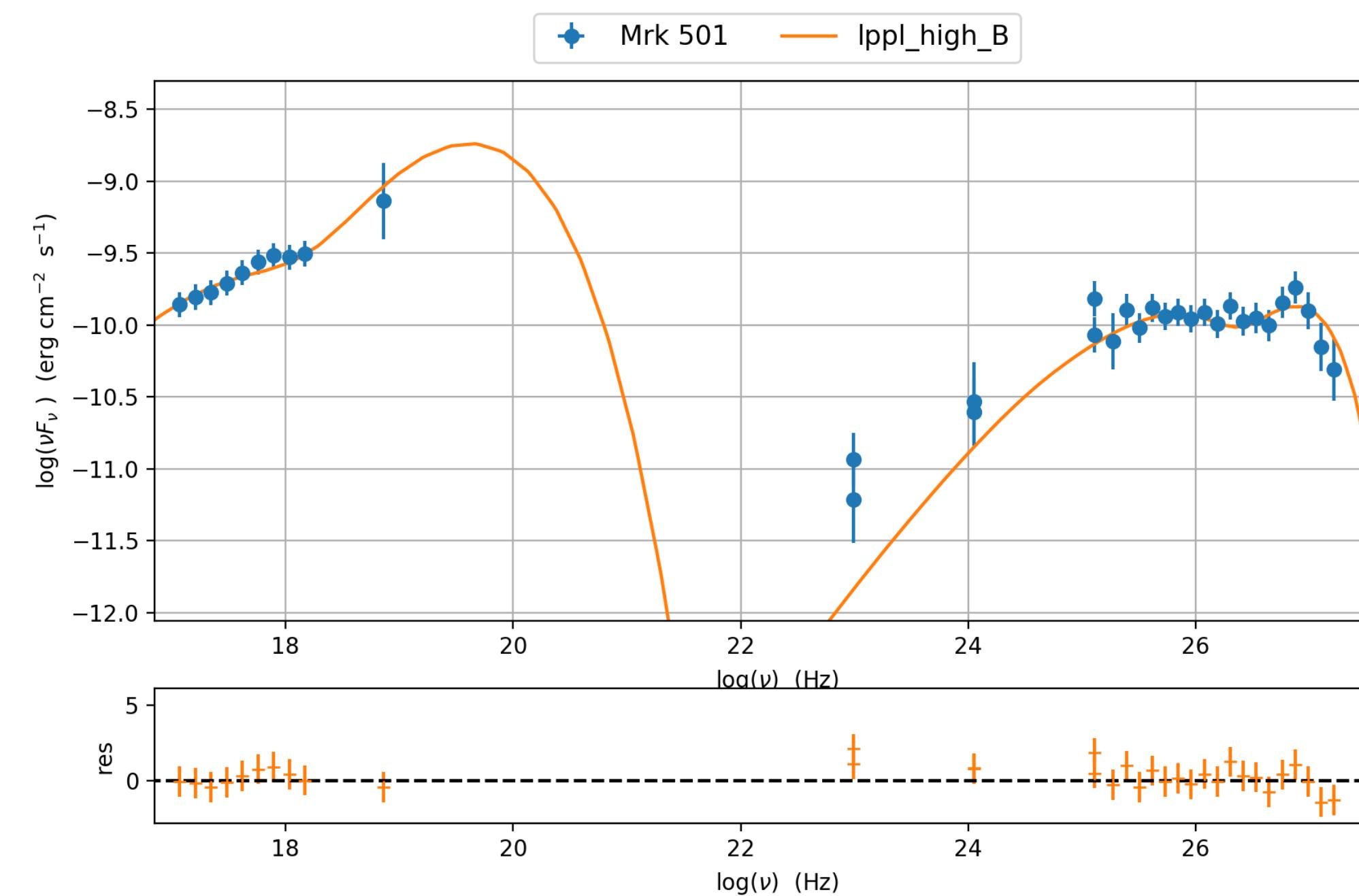
$s \sim 1.6$

$r \sim 0.7-0.8 \ll r_{eq} \sim 6$

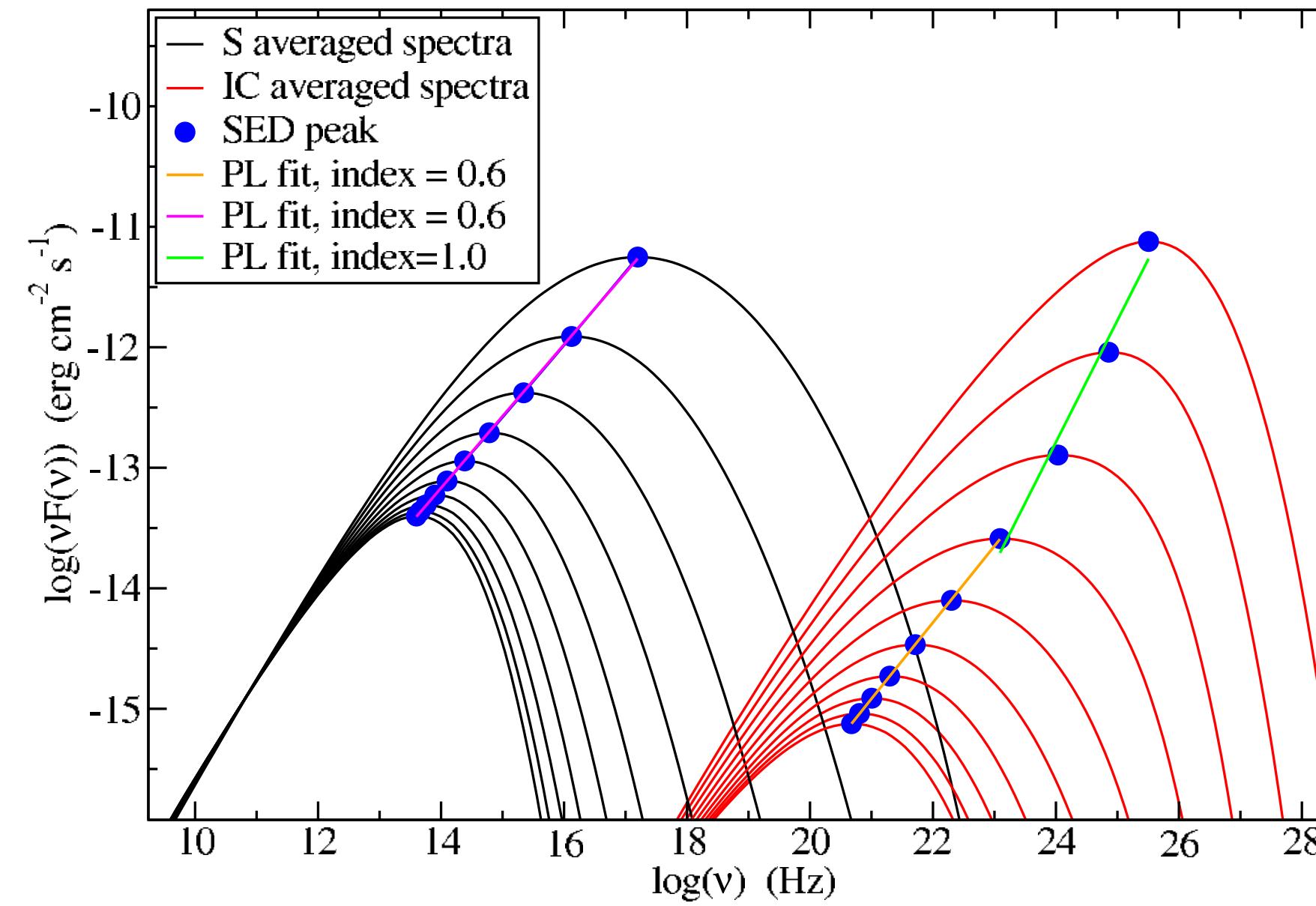
$s \ll s_{FI} \sim 2.3$

Pile-up and hard spectra

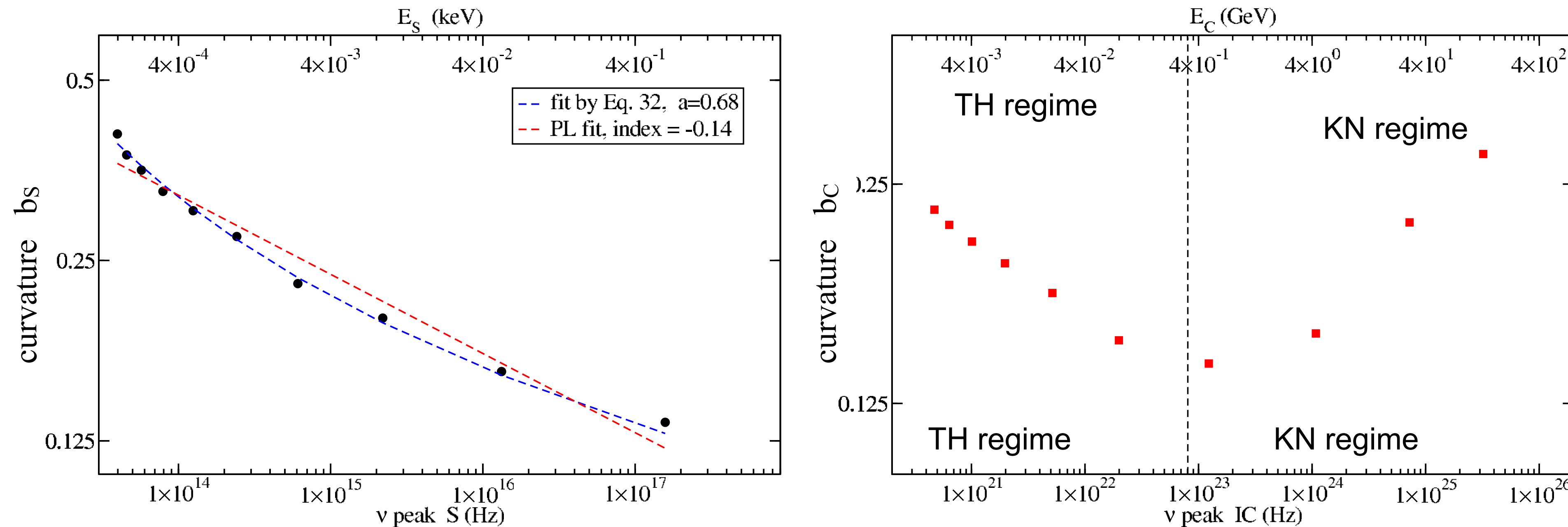
Mrk 501 2014 Flare
MAGIC paper
A&A 637, A86 (2020)



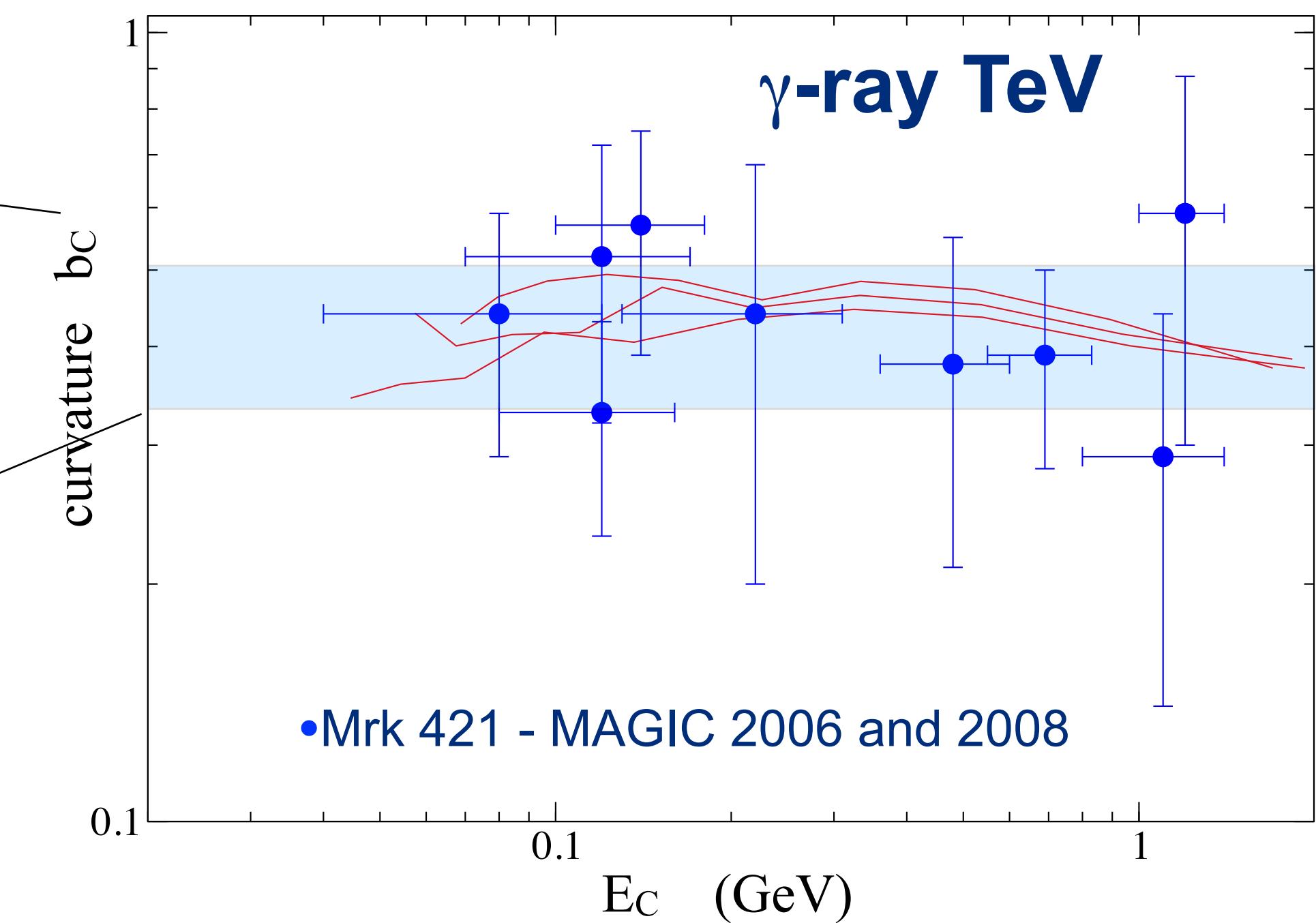
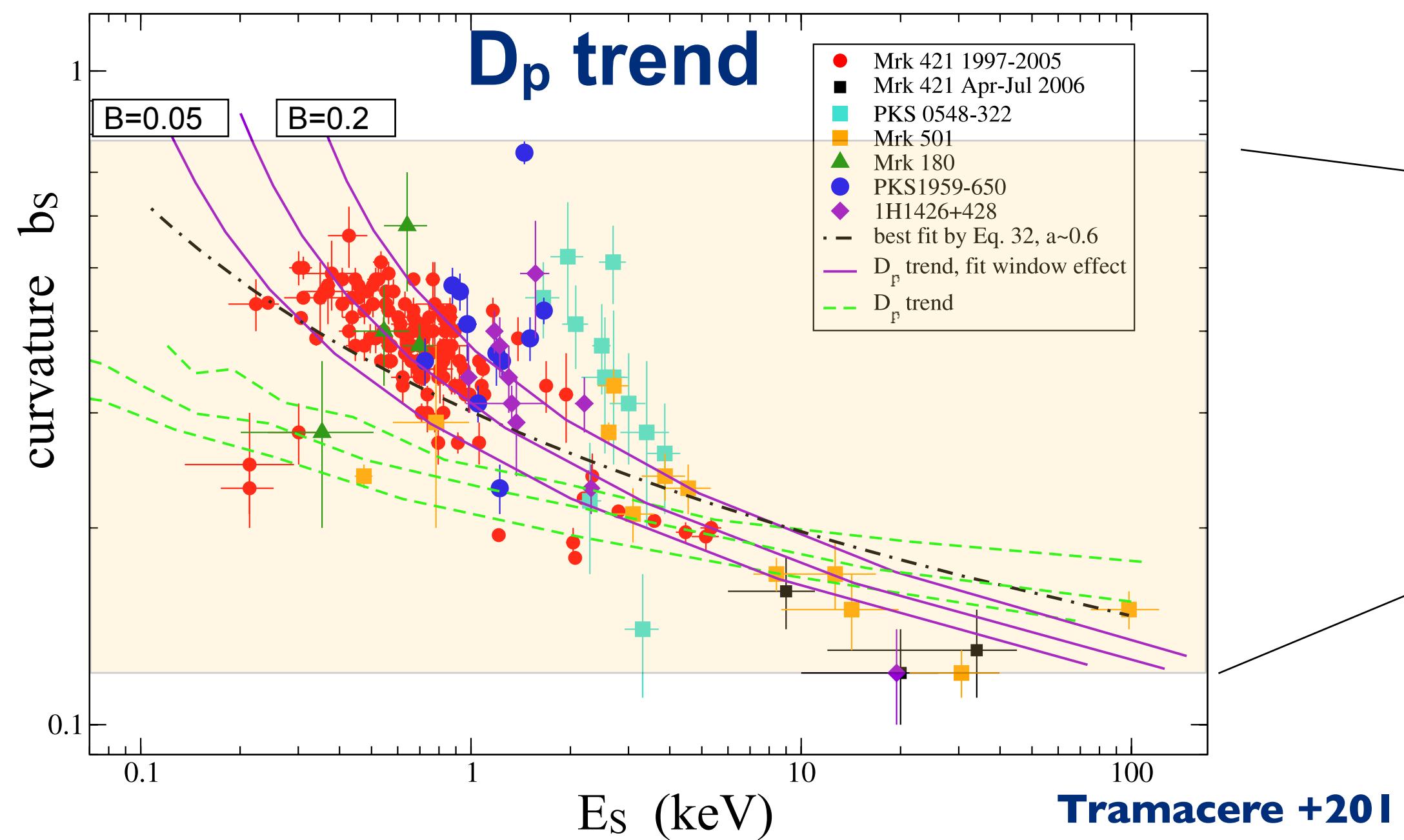
S vs IC



Tramacere +2011



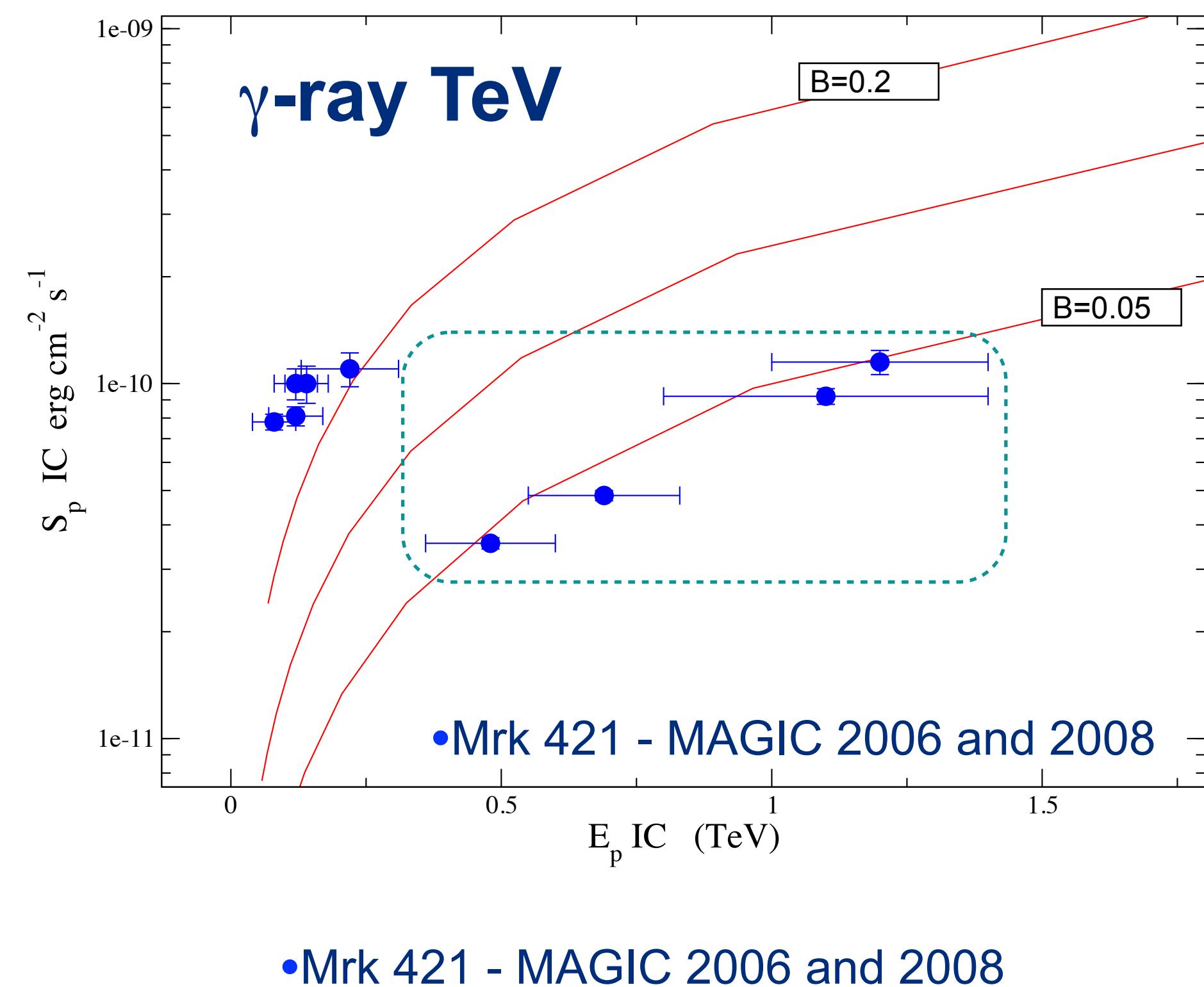
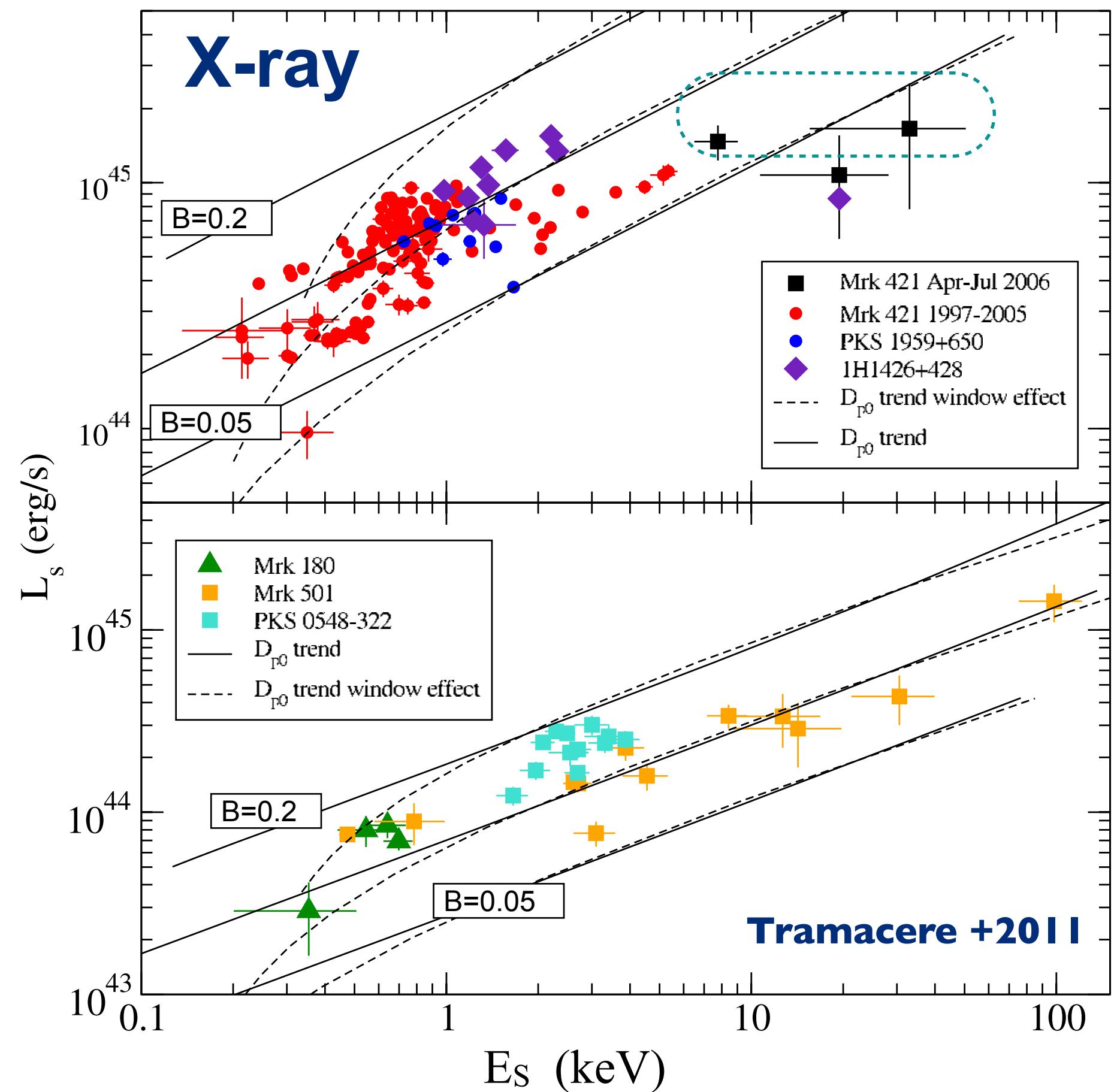
	Acceleration dominated	Equilibrium
curvature trend	curvature decreasing trend $b-E_p$	curvature stable or increasing ($r \sim 7, b \sim 1.3$)
spectral shape	LPPL or LP	PL+exp-cutoff or Maxwellian



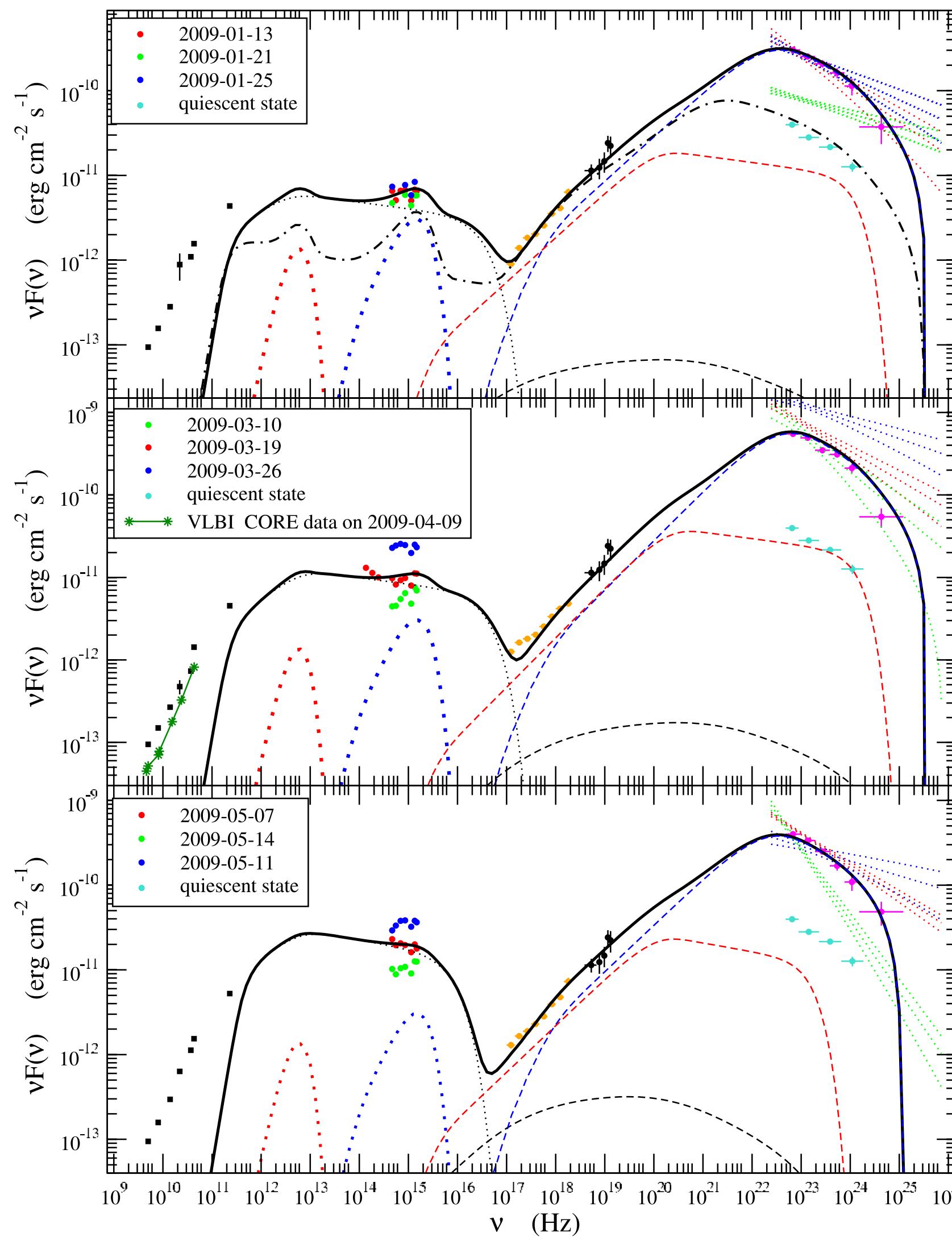
- data span **13 years**, both flaring and quiescent states
- We are able to reproduce these long-term behaviours, by changing the value of only one parameter (D_p)
- for $q=2$, curvature values imply distribution far from the equilibrium ($b \sim [1.0-0.7]$)
- More data needed at GeV/TeV, curvature seems to be cooling-dominated
- Similar trend observed in GRBs (Massaro & Grindlay 2001)

$L_{\text{inj}} (E_s - b_s \text{ trend})$ (erg s $^{-1}$)	5×10^{39}
$L_{\text{inj}} (E_s - L_s \text{ trend})$ (erg s $^{-1}$)	$5 \times 10^{38}, 5 \times 10^{39}$
q	2
t_A (s)	1.2×10^3
$t_{D_0} = 1/D_{P0}$ (s)	$[1.5 \times 10^4, 1.5 \times 10^5]$
T_{inj} (s)	10^4
T_{esc} (R/c)	2.0

E_s - L_s X-ray trend and γ -ray predictions



- the E_s - S_s (E_s - L_s) relation follows naturally from that between E_s and b_s
- the low L_{inj} objects (Mrk 501 vs Mrk 421) reach a larger E_s , compatibly with larger γ_{eq}
- Mrk 421 MAGIC data on 2006 match very well the Synchrotron prediction with simultaneous X-ray data
- the average index of the trend $L_s \propto E_s^\alpha$ with $\alpha \sim 0.6$, is compatible with the data, and with a scenario in which a typical constant energy ($L_{\text{inj}} \times t_{\text{ini}}$) is injected for any flare (jet-feeding problem), whilst the peak dynamic is ruled by the turbulence in the magnetic field.

PSK 1510-089 Abdo+ 2010**Dammando+ 2012**