Fundamentos de Algoritmos e Estrutura de Dados – Aula 05 - Grafos

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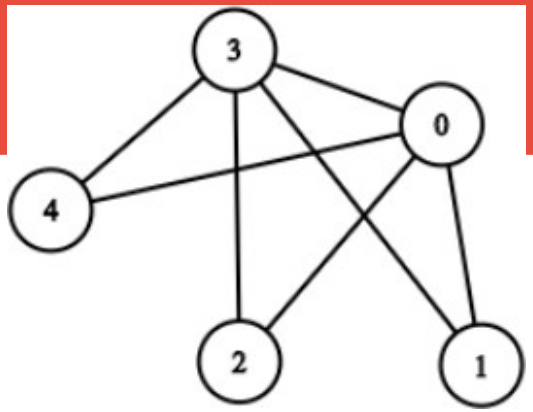
Plano de Aula

- **Grafos**
  - **Busca Profundidade**
  - **Busca Largura**
  - **Busca A***
- **Dijkstra**

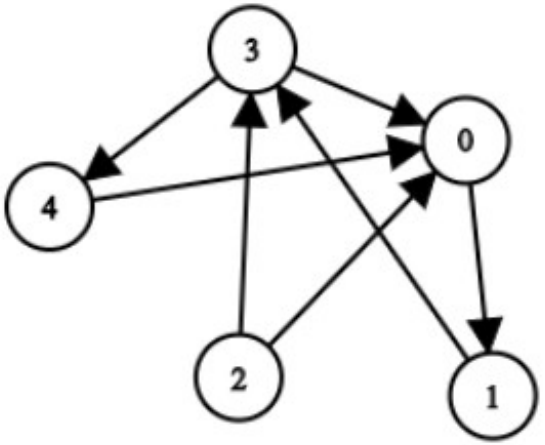
Grafos – Conceitos Básicos e Aplicações

Grafos

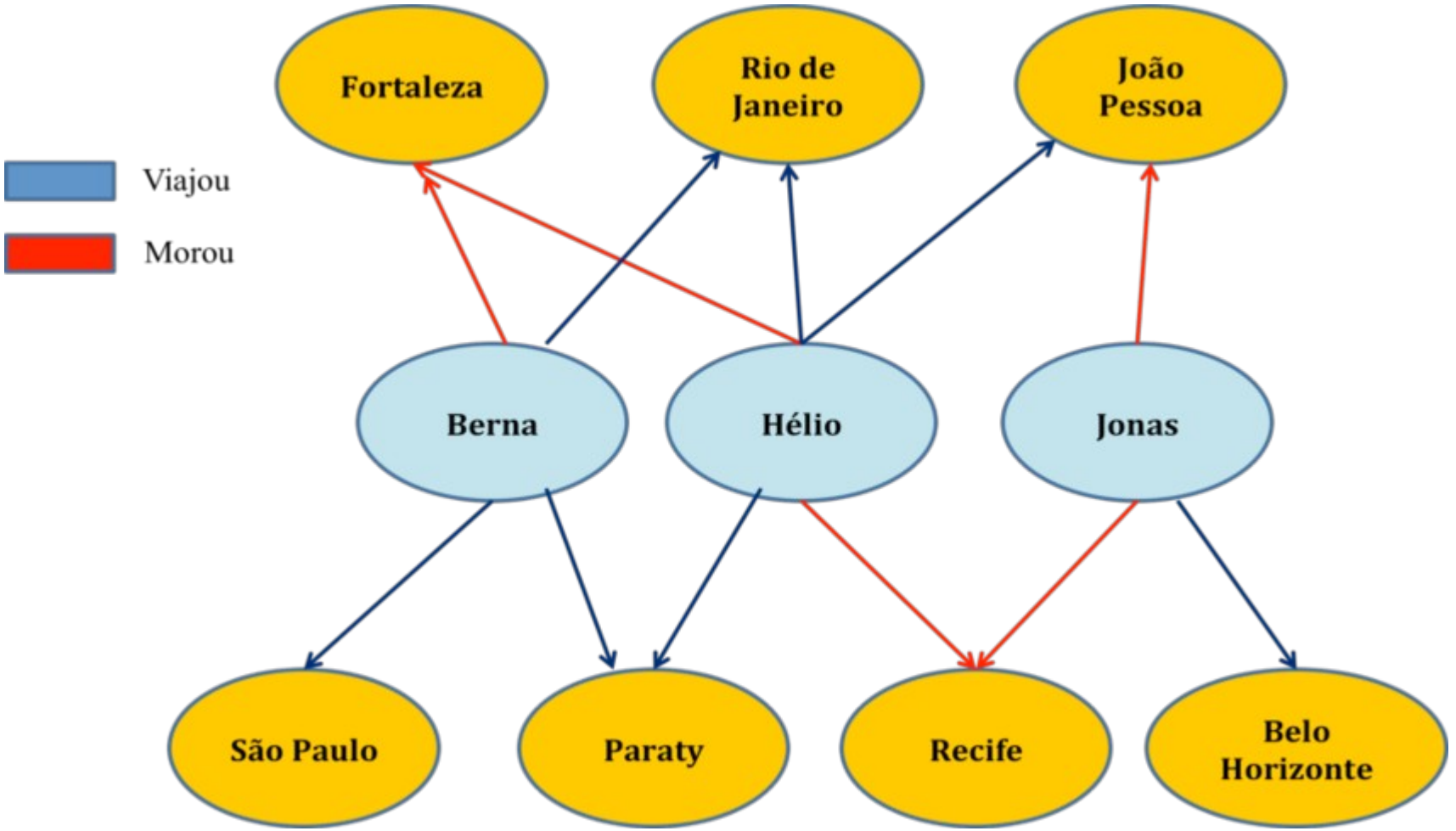


- **Conjunto de Vértices e Arestas**
  - **Direcionado ou Não**
  - **Define graus de relacionamento entre objetos (arestas e vértices)**

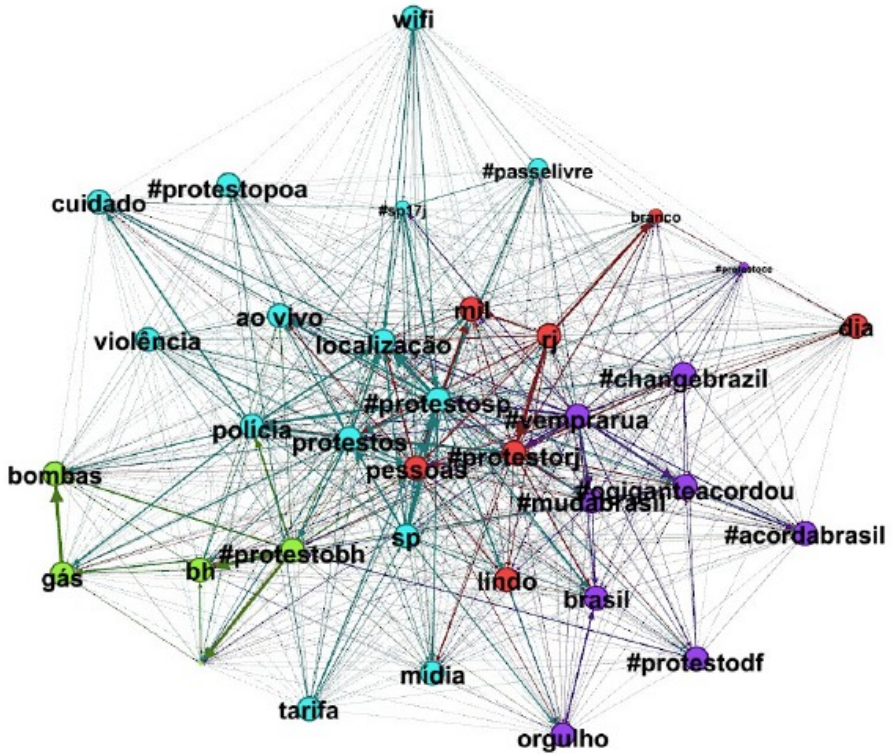
- **Utilizado na modelagem de problemas**
  - **Redes Sociais**
    - **Relacionamento entre Empresas, Pessoas, etc**
  - **Roteamento**
    - **Redes de Computadores**
    - **Rotas Rodoviárias, Aéreas, Malha Elétrica....**
  - **Programação Orientada a Objetos (Classes)**

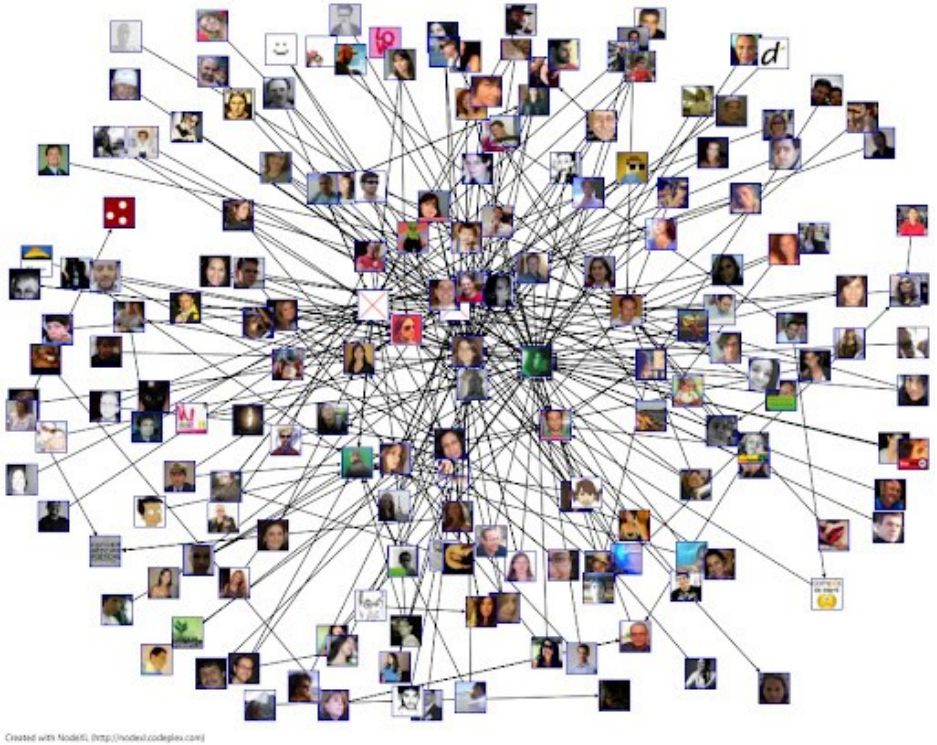


Grafos (Relacionamentos)



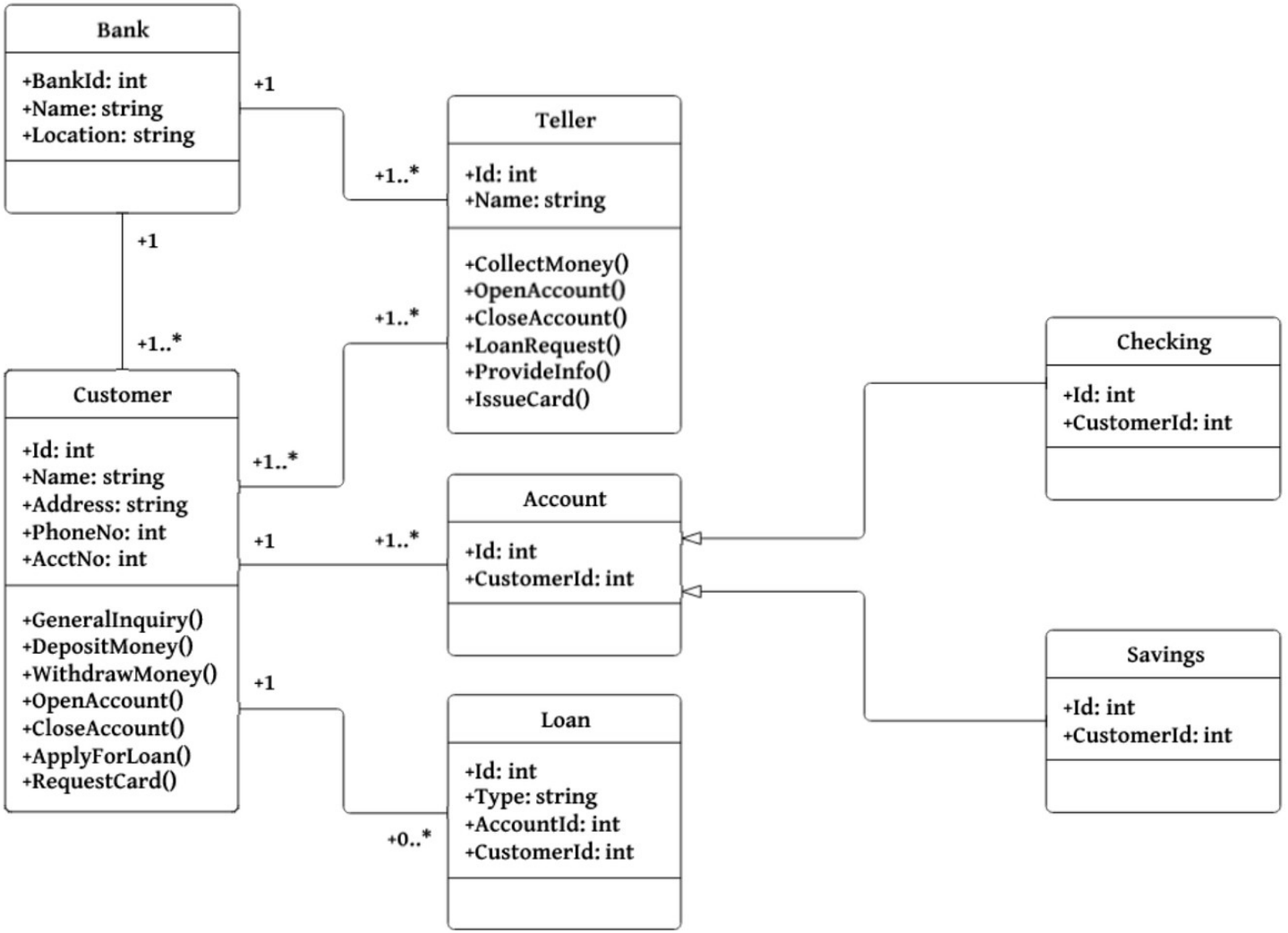
Grafos (Redes Sociais)





Grafos (Roteamento)

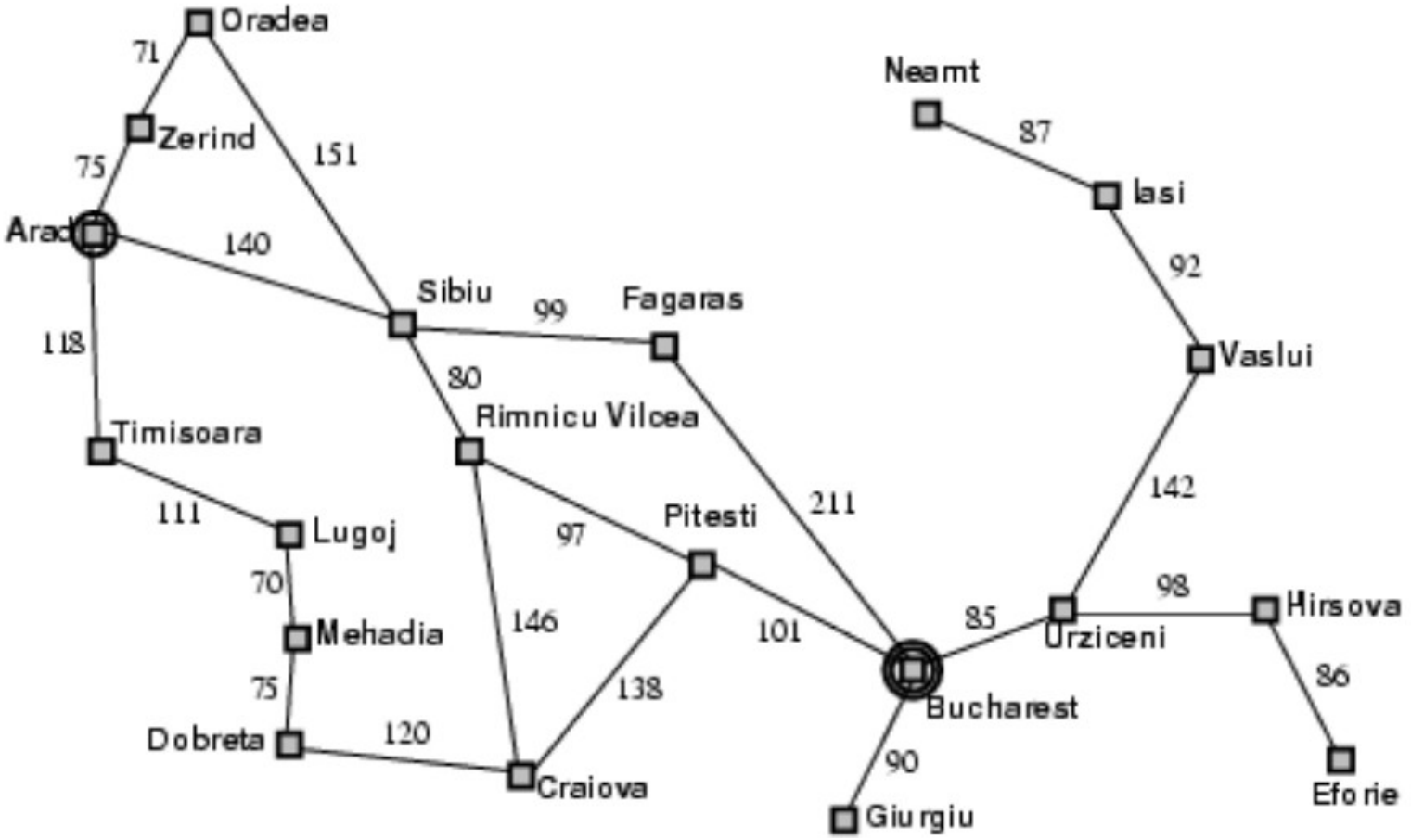
Orientação a Objetos (i.e UML Class Diagram)



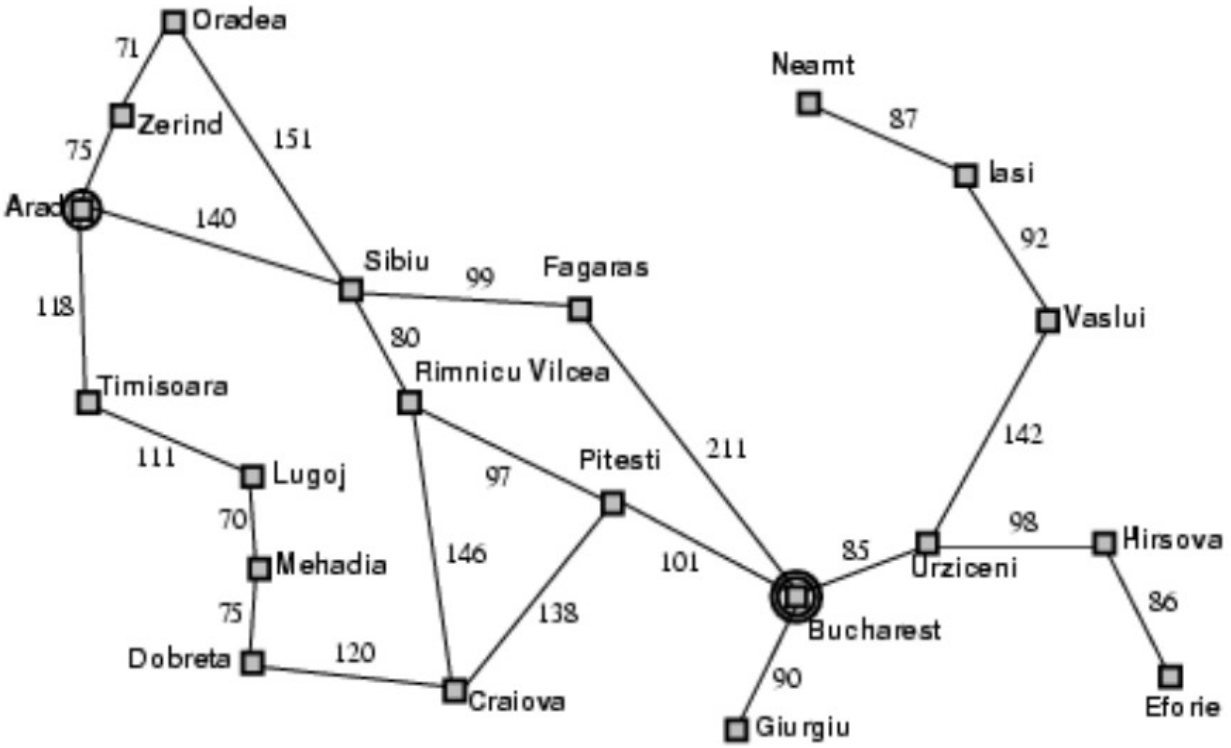
Grafos – Buscas

Busca Cegas (Sem Informação)

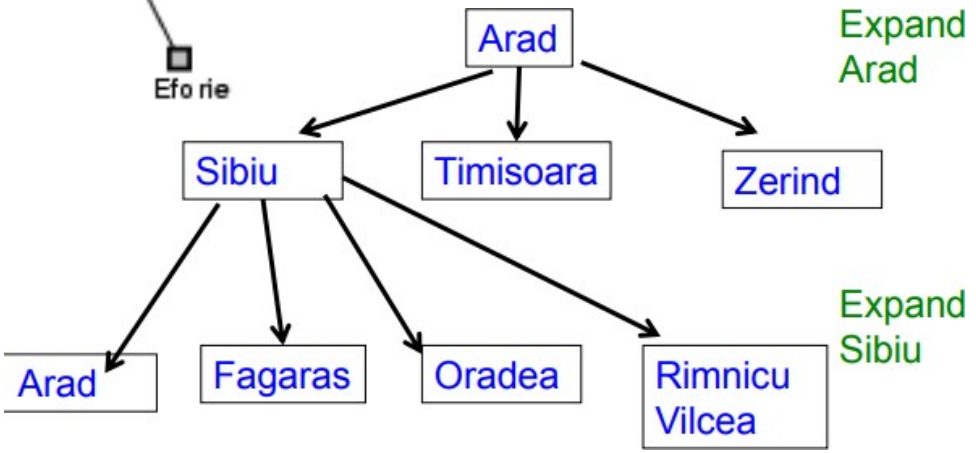
Problema: Arad → Bucharest



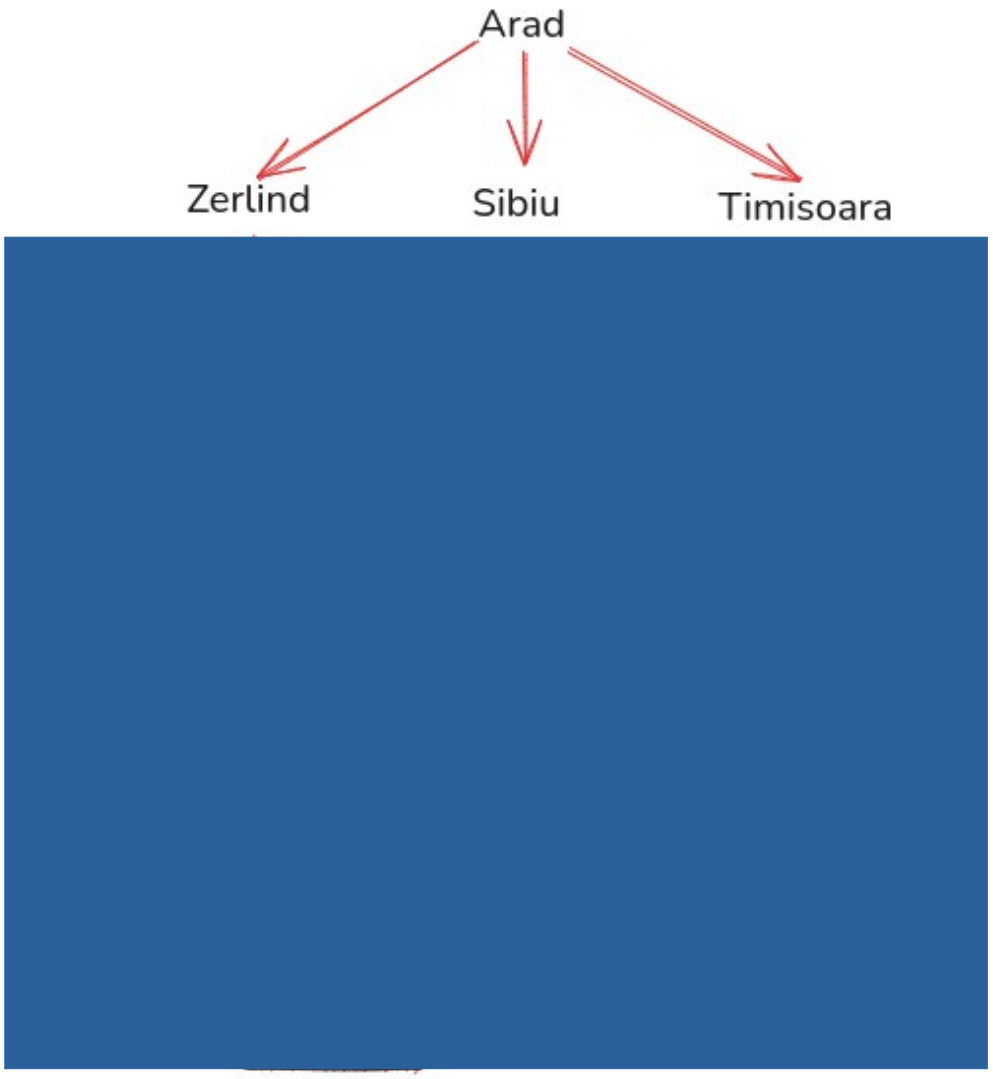
Busca Cegas (Sem Informação)

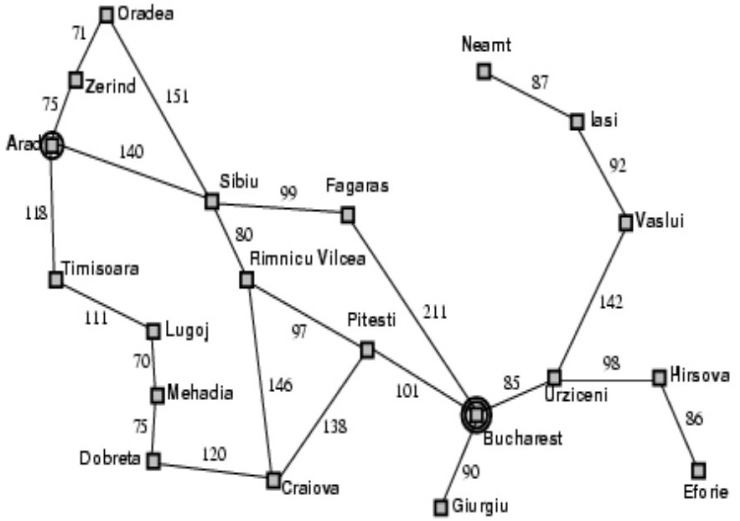


Problema: Qual nodo expandir?

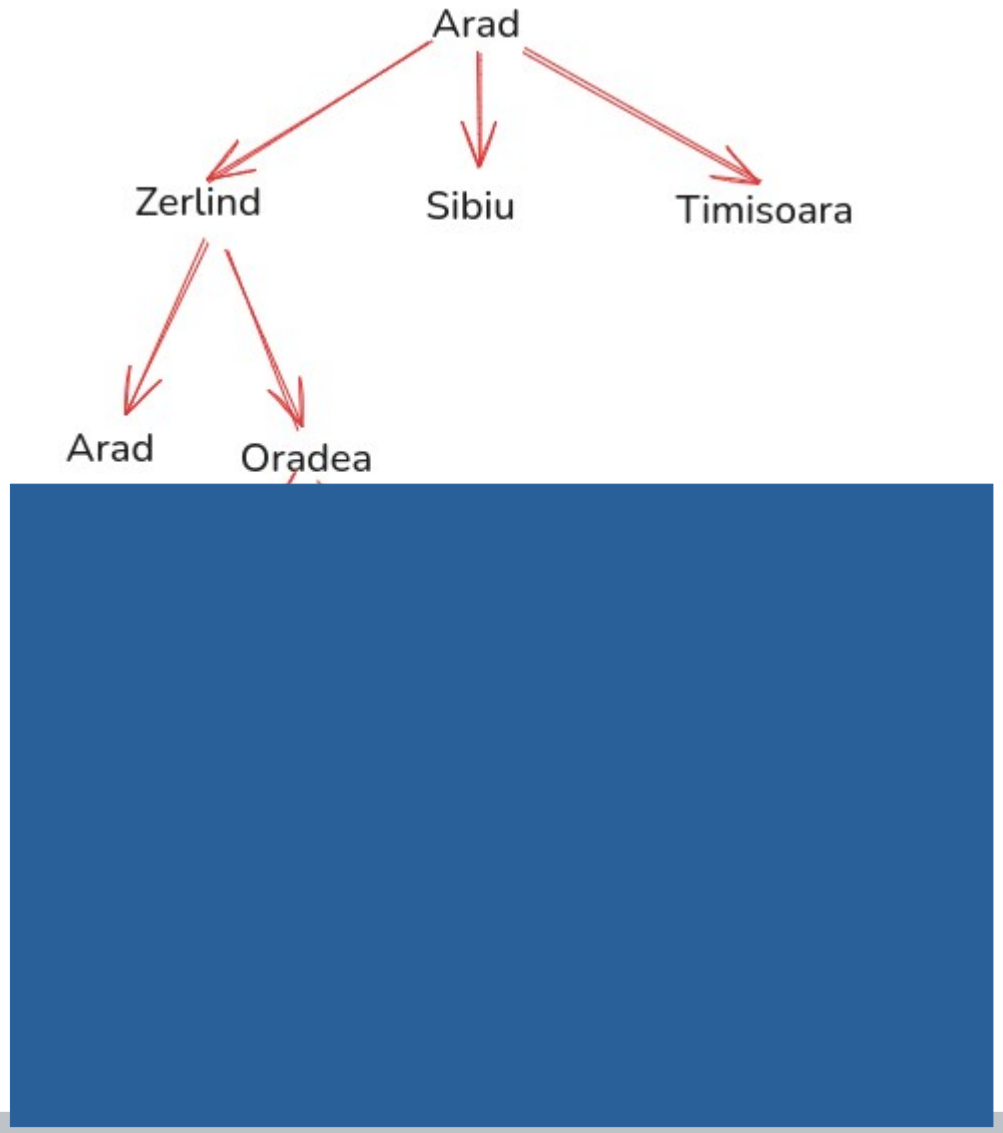


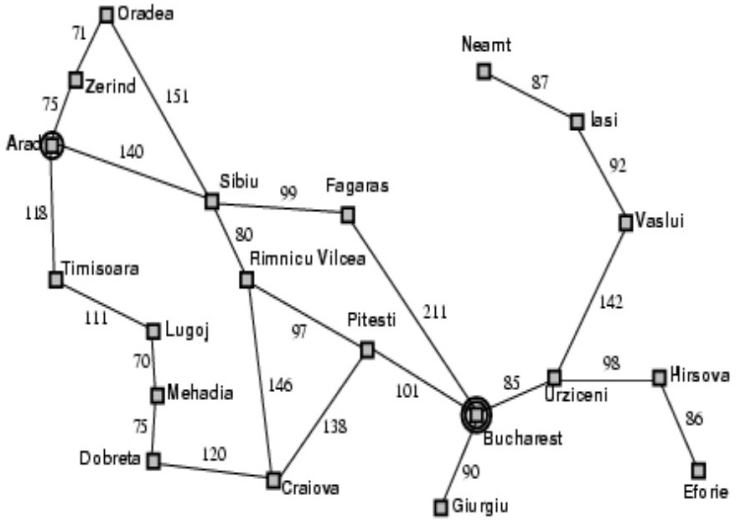
Busca Cega: Profundidade





Busca Cega: Profundidade





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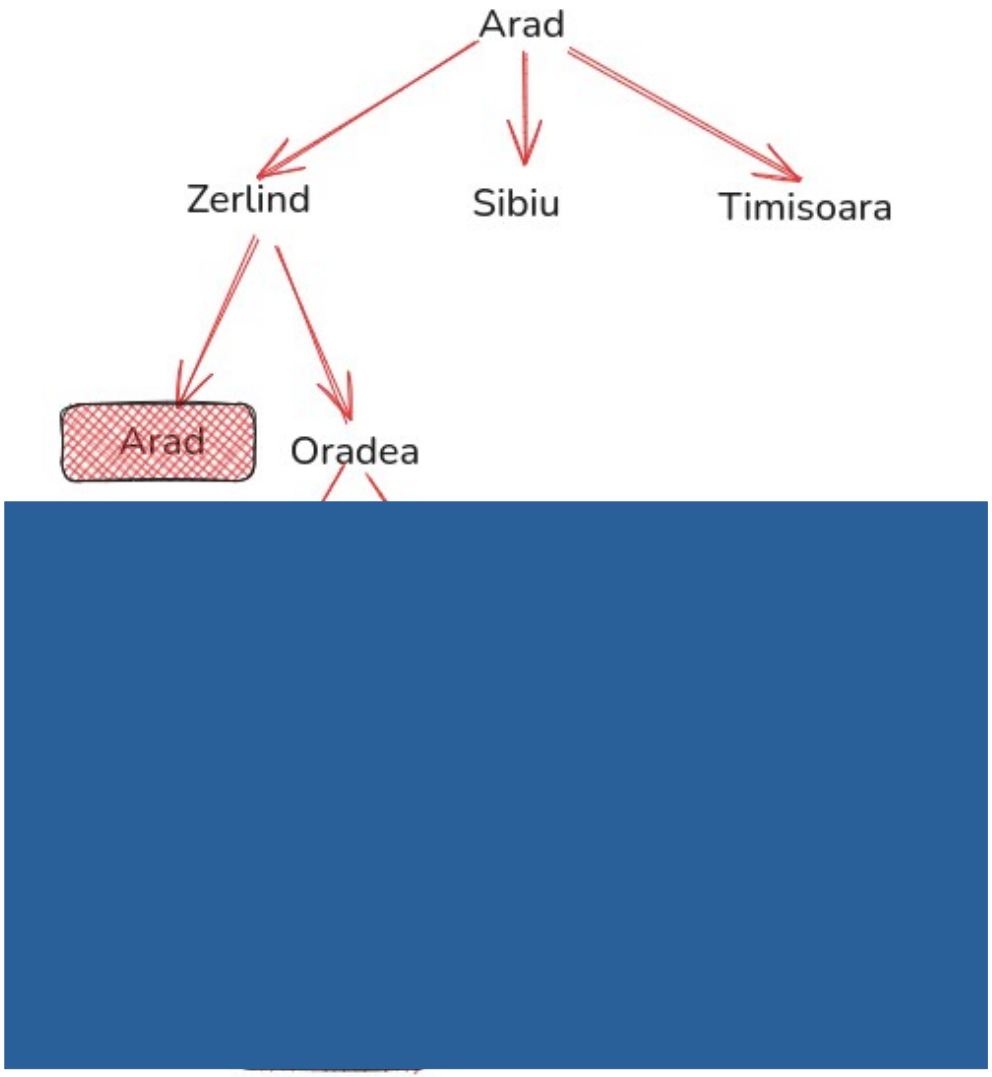

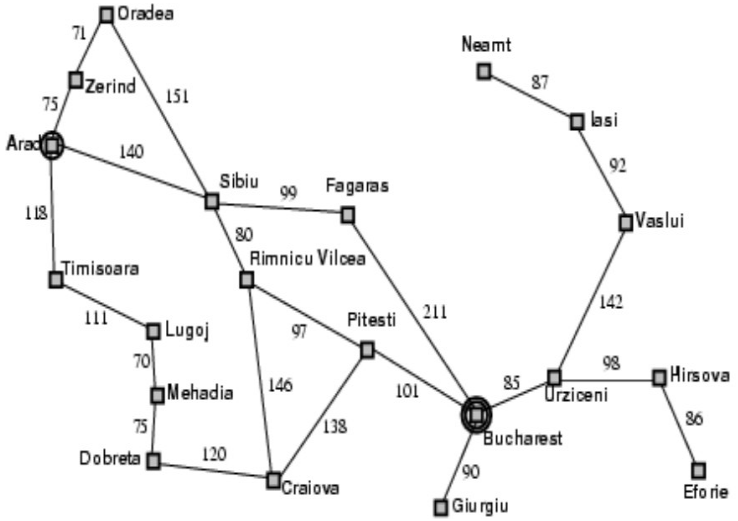

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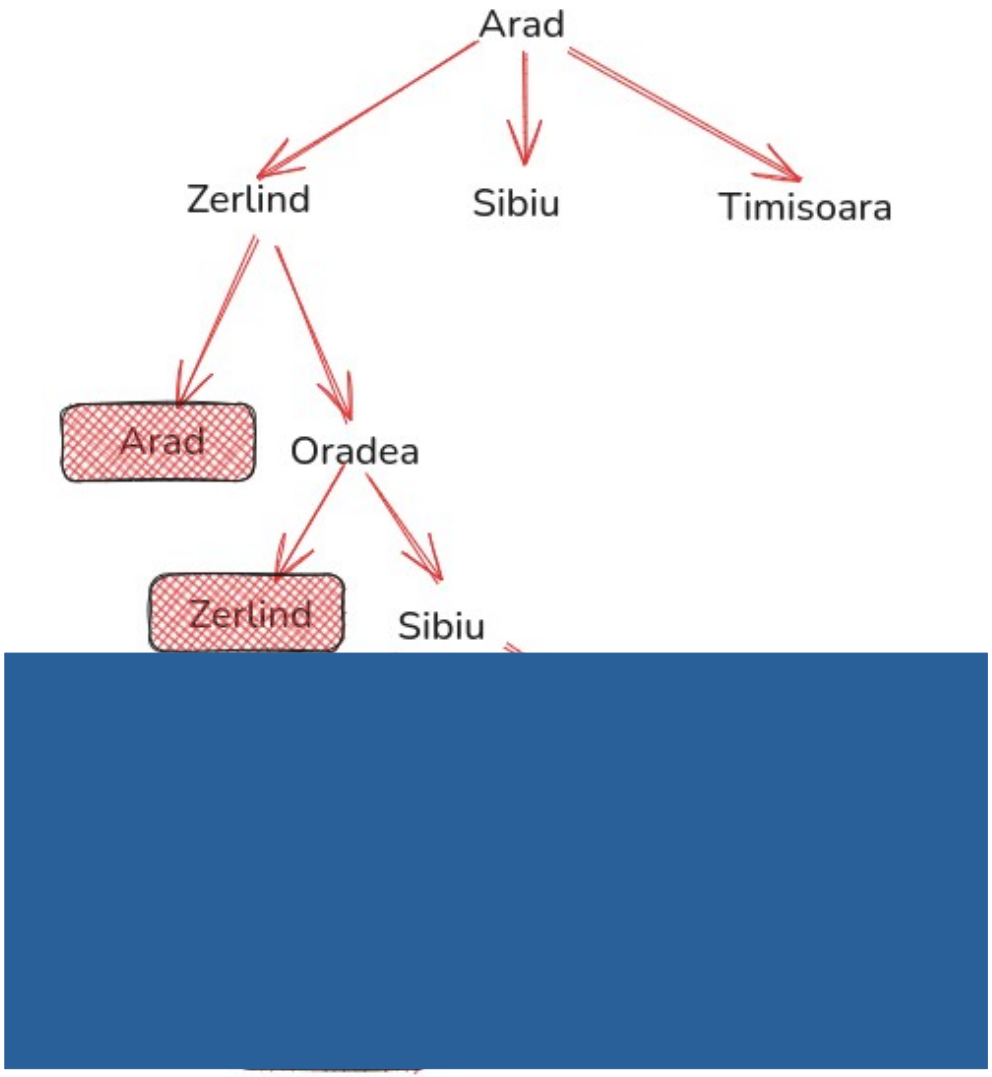

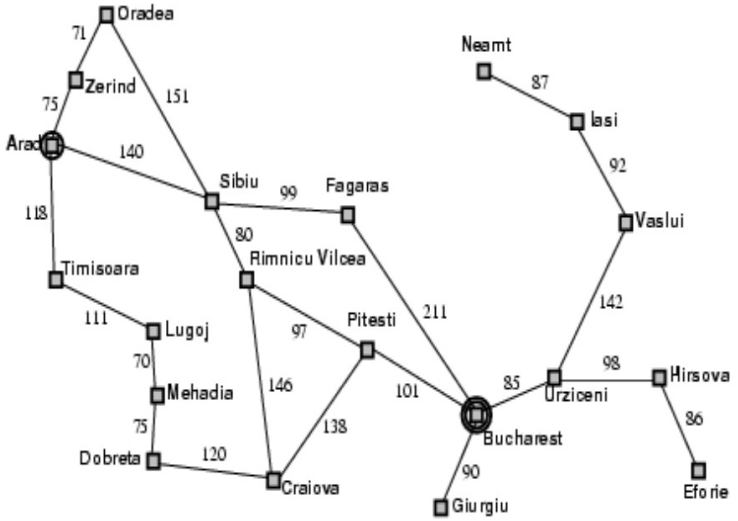

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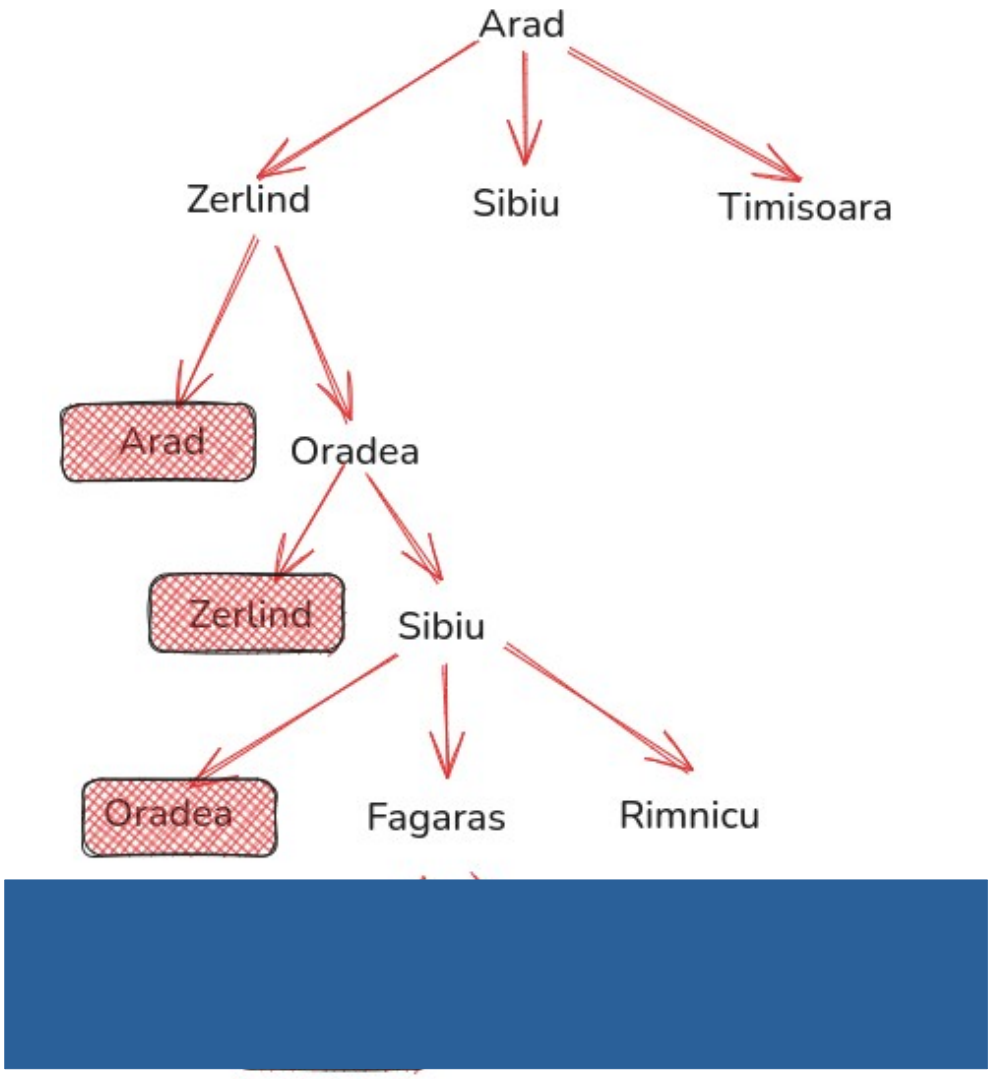

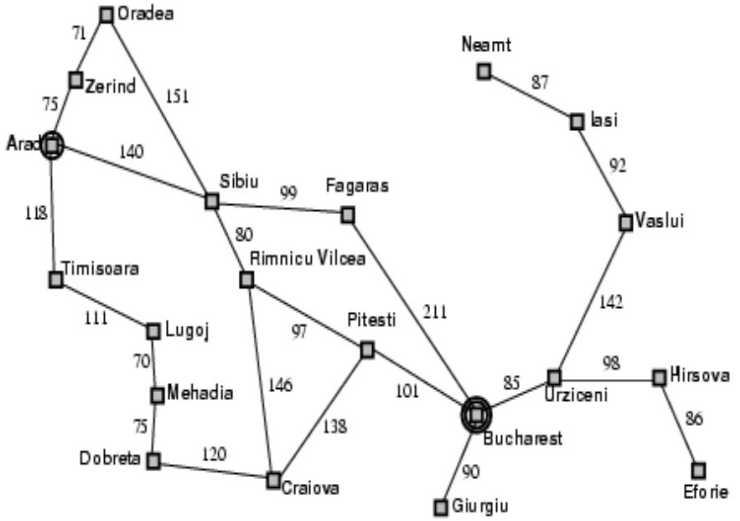

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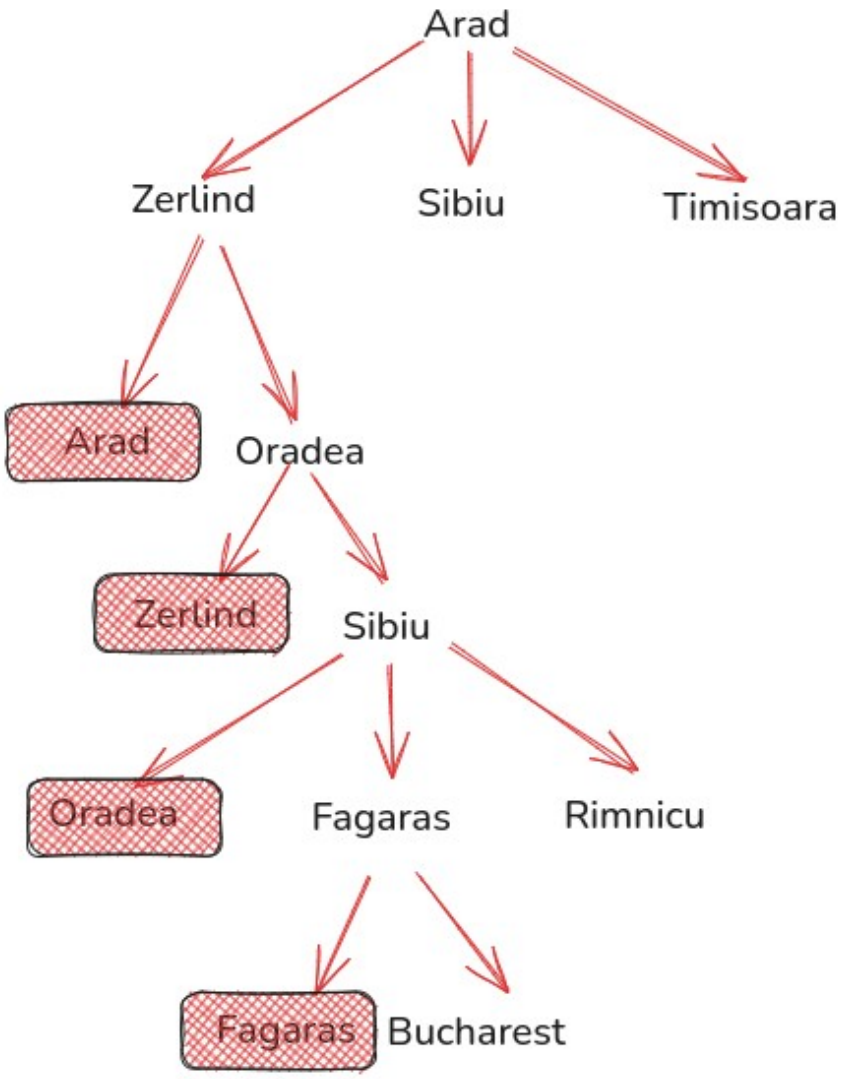
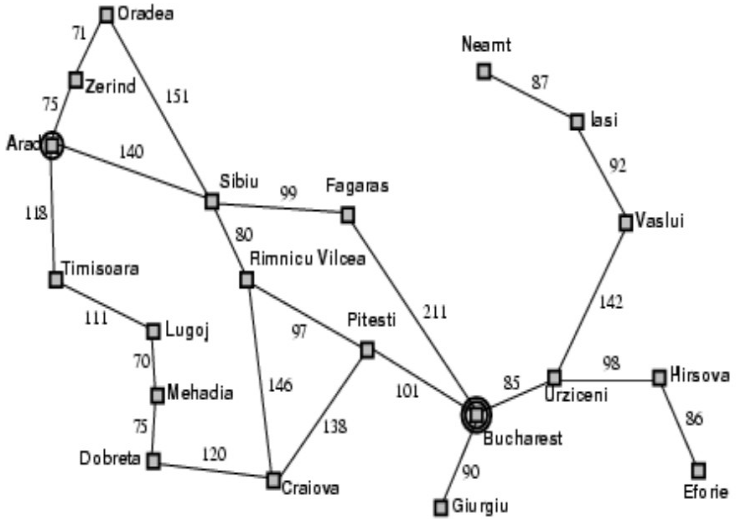
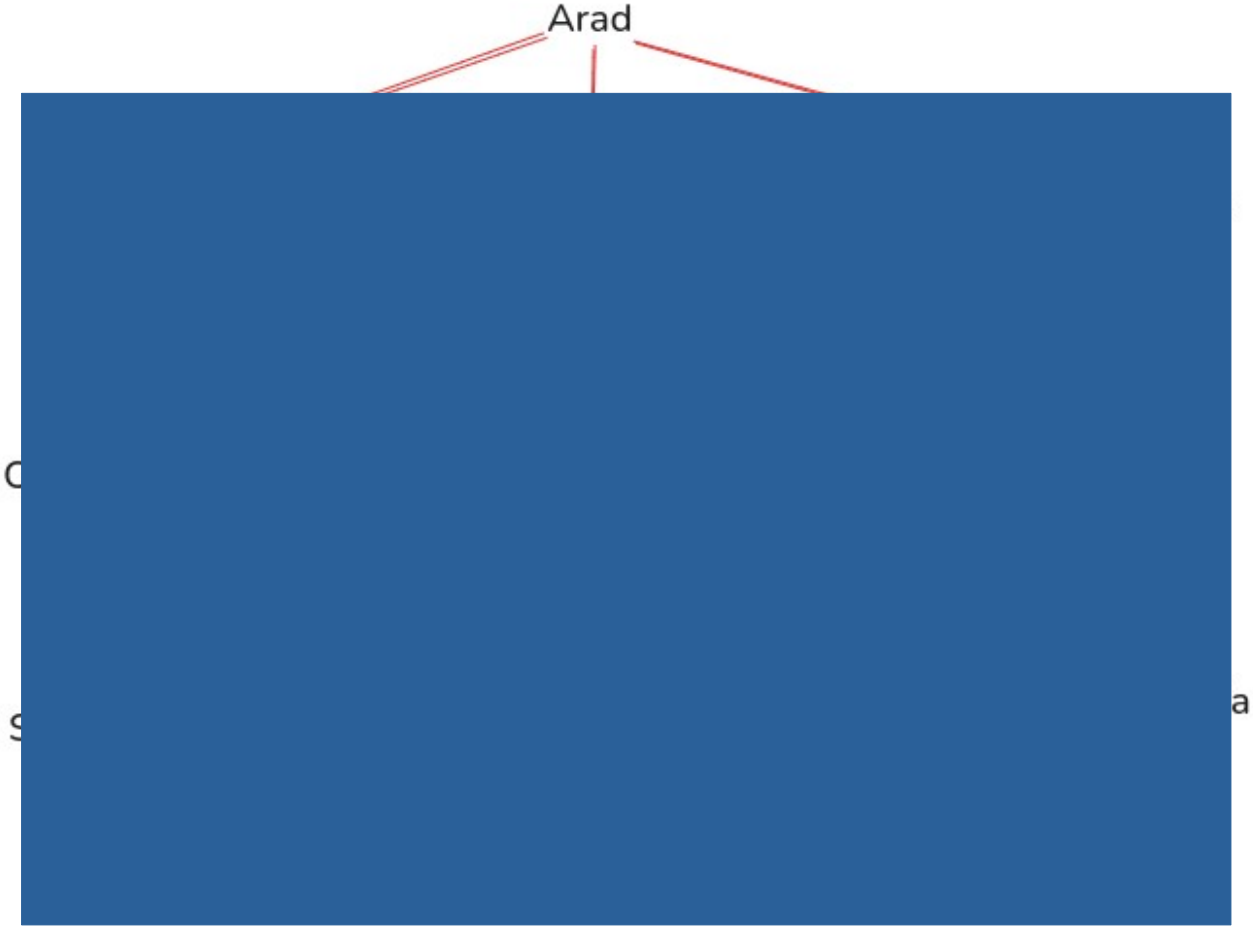


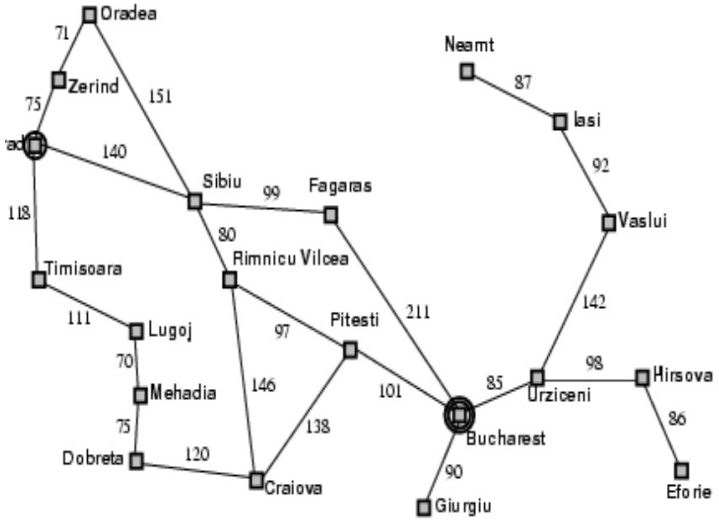


Não Garante a melhor solução!

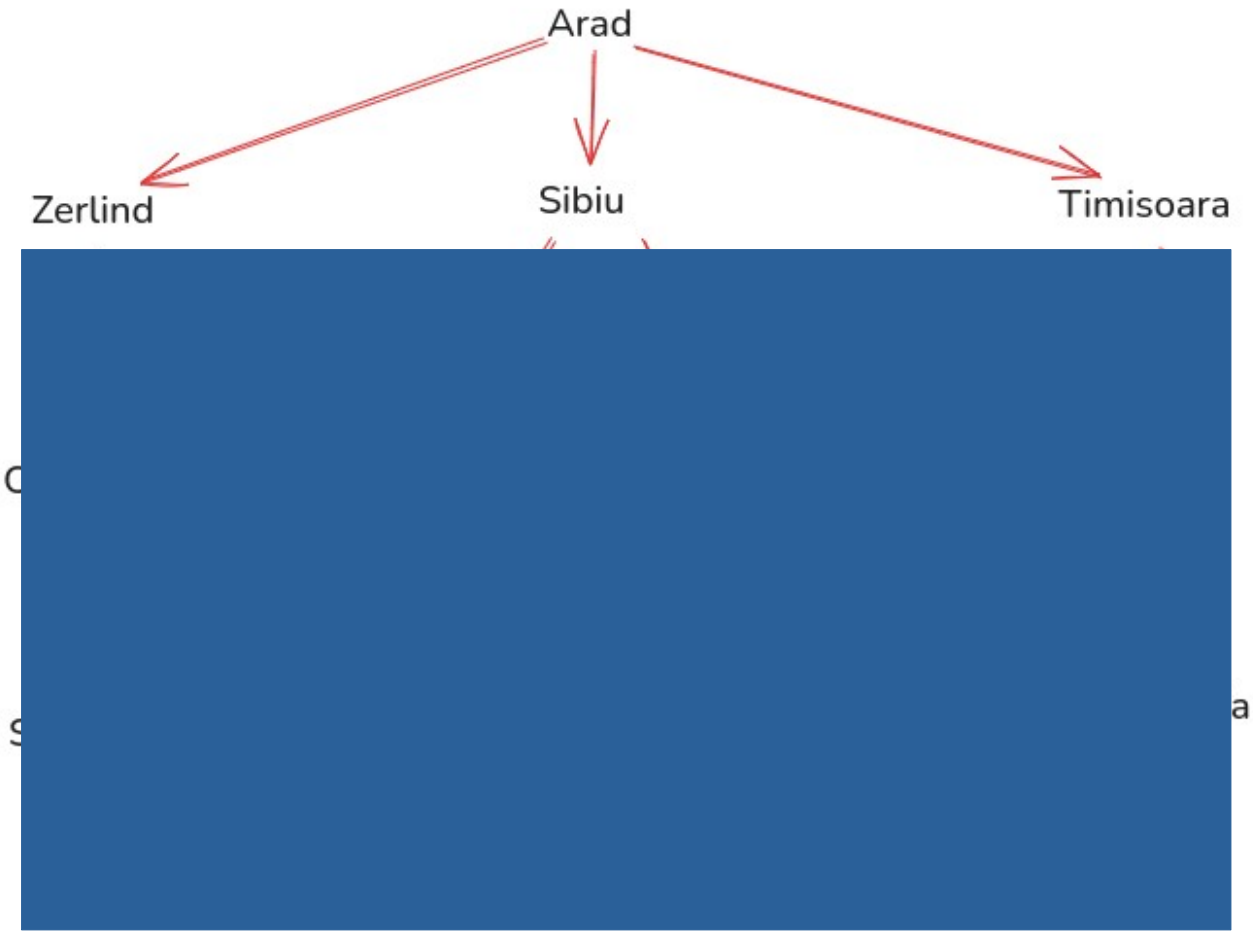
Ciclos devem ser tratados!

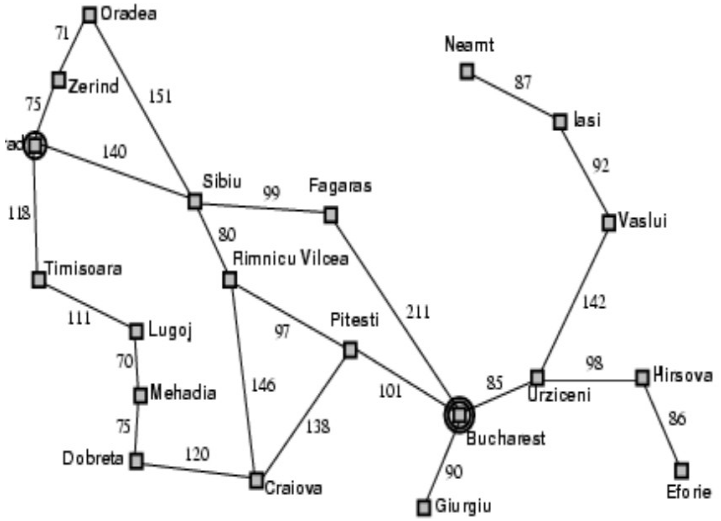
Busca Cega: Largura





Busca Cega: Largura





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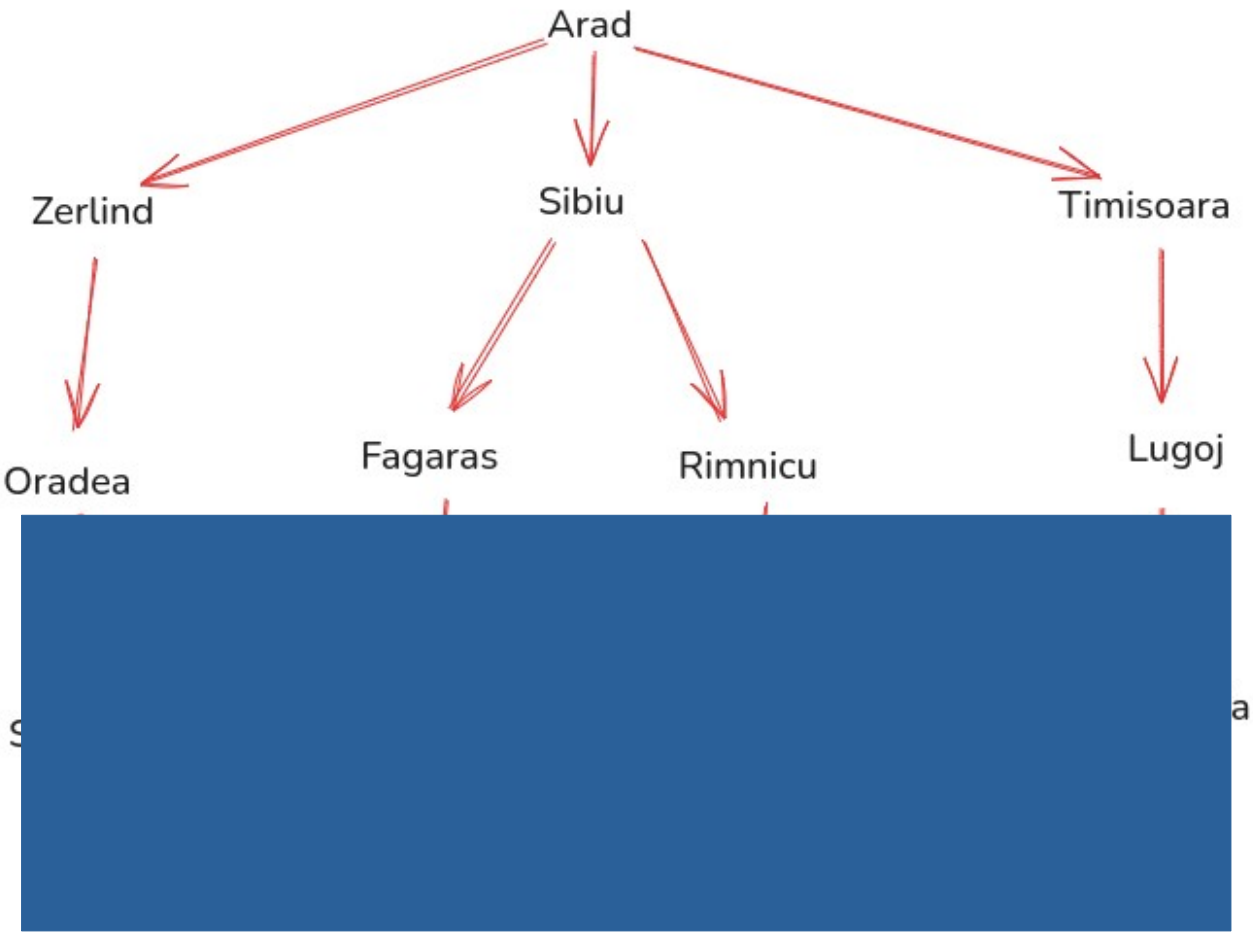

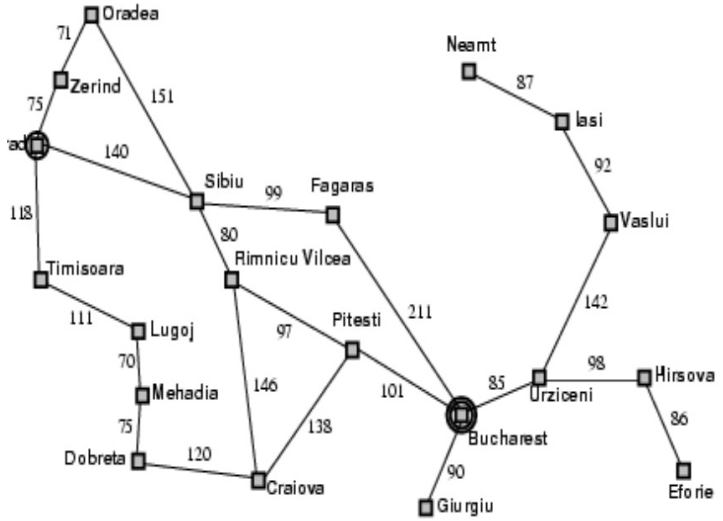

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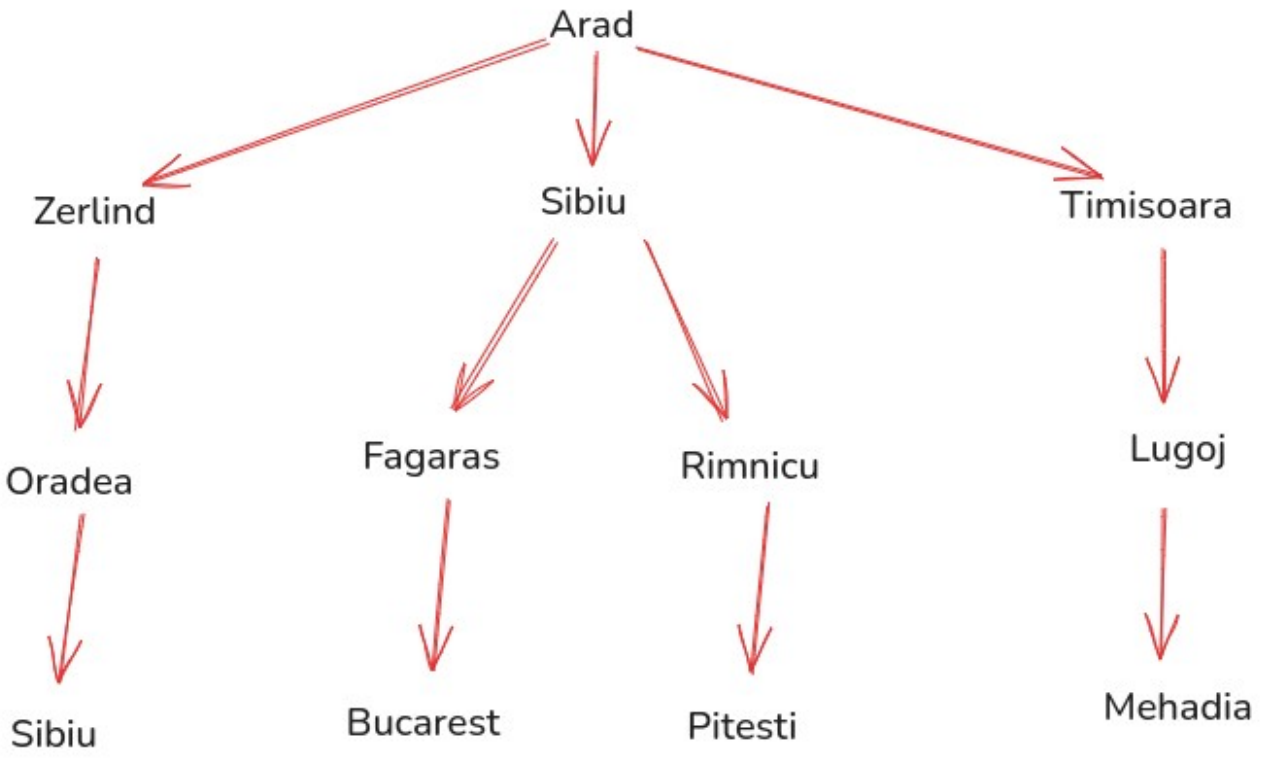
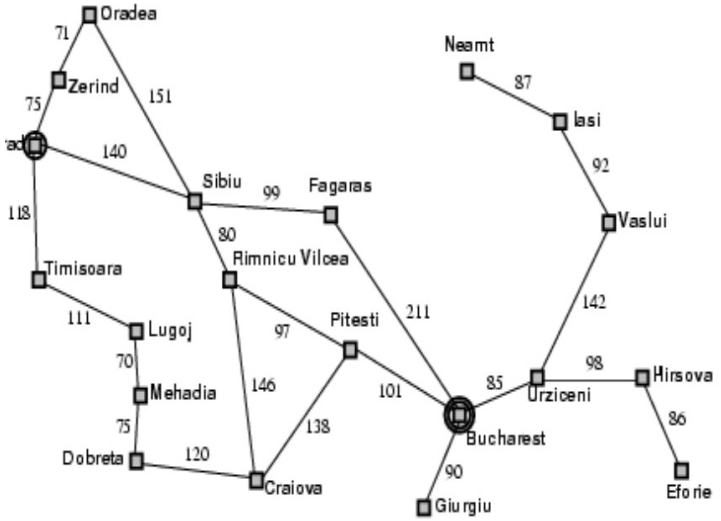




Garante a melhor solução!

Ciclos devem ser tratados!

Busca Cega: Largura

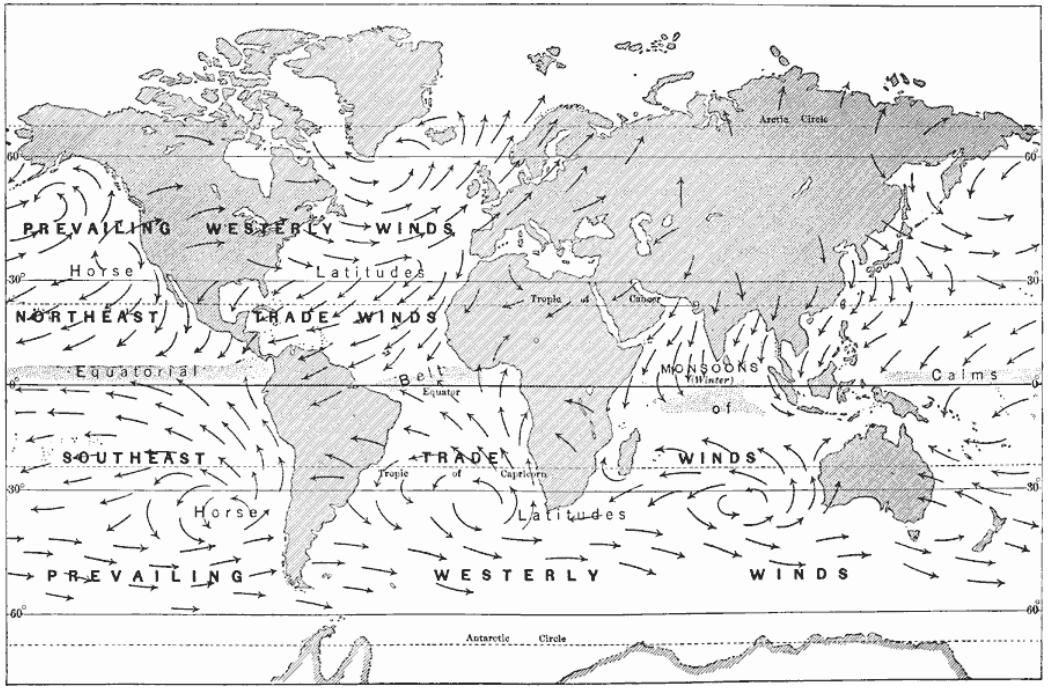
- **Bons resultados para arvores com pouca profundidade**
- **Caso contrário, o custo computacional é alto**
- **Exemplo**
  - **Ramificação b=10, 1 M nodos/seg , 1 KB por node**

$1+b^1+b^2+(b^3-b) = 1+10+100+(1000-10) = 1101$

profundidade	nós	tempo	memória
2	1100	0,11 seg	1 MB
4	111.100	11 seg	106 MB
6	10^7	19 min	10 GB
8	10^9	31 horas	1 TeraB
10	10^{11}	129 dias	101 TeraB
12	10^{13}	35 anos	10 PentaB
14	10^{15}	3.523 anos	1 exaB

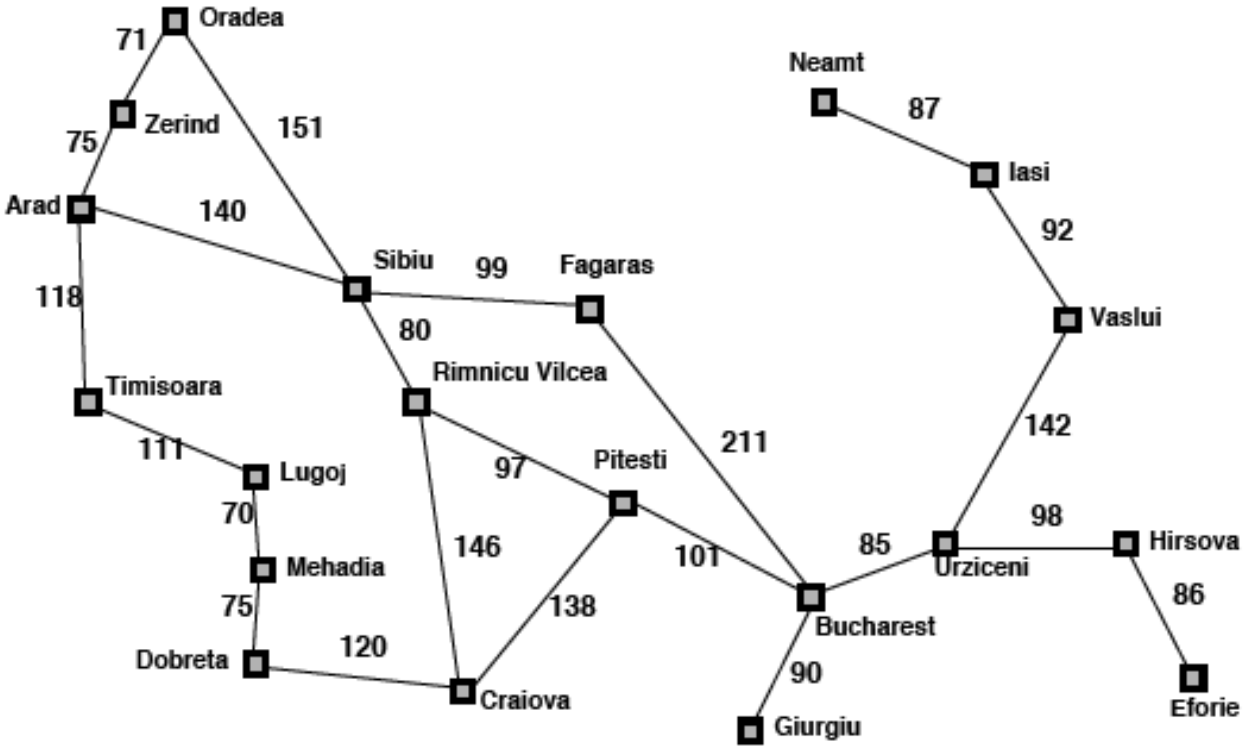
Busca Heurística (Informada)

- **Utiliza uma função heurística para determinar a próxima expansão**
- **Problema: Procurar um barco no oceano**
  - **Cega: M2 por M2**
  - **Heurística: Corrente Marítima, Ventos, ...**



Busca Heurística

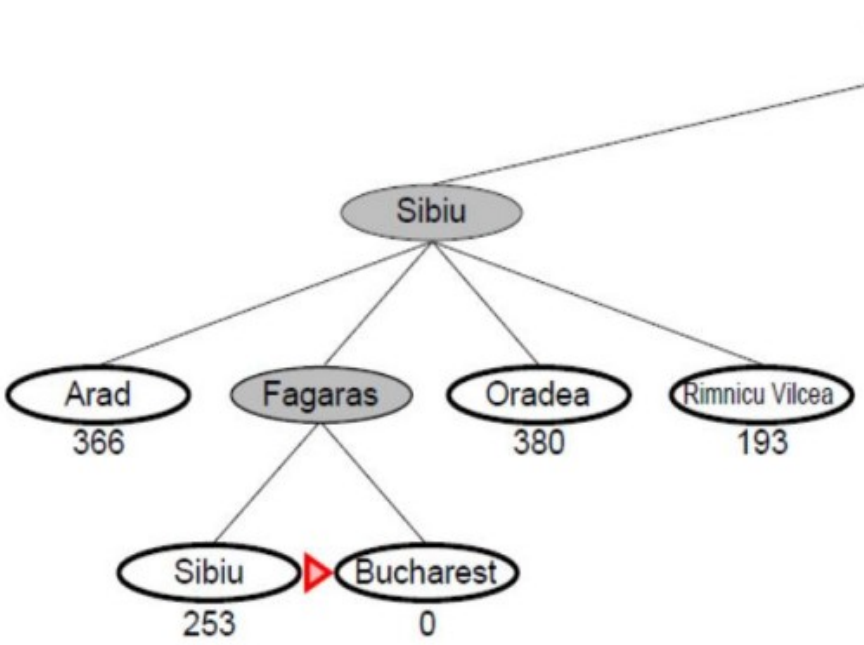
Romania with step costs in km



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Busca Heurística: “Ambiciosa” (Greedy Search)

- **f(n) = h(n)**
  - **h(n): Custo de n até folha (n→destino)**



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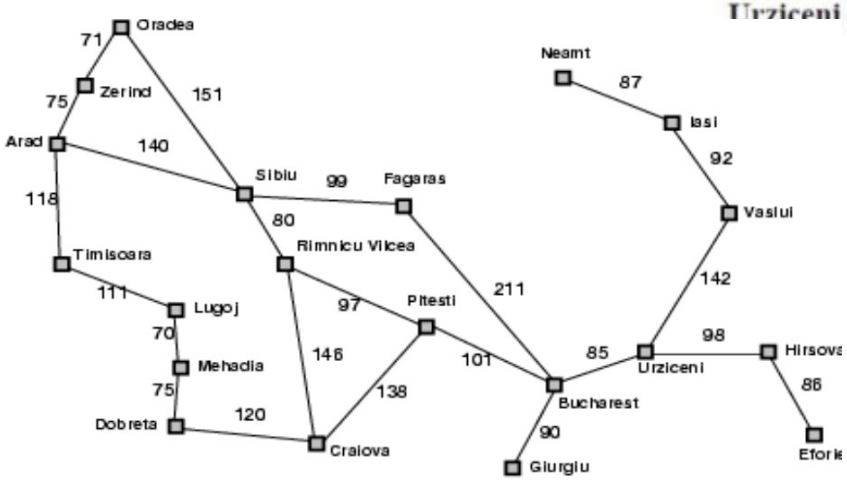
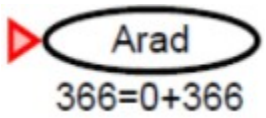


Busca Heurística: A*

- **f(n) = g(n) + h(n)**



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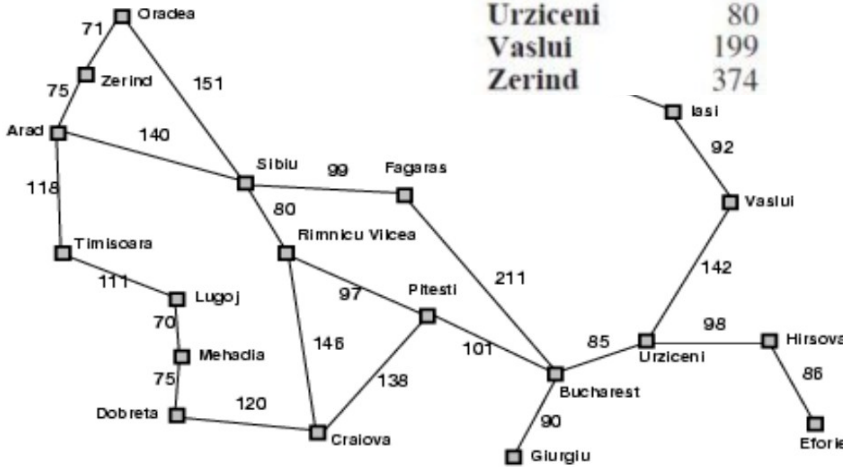

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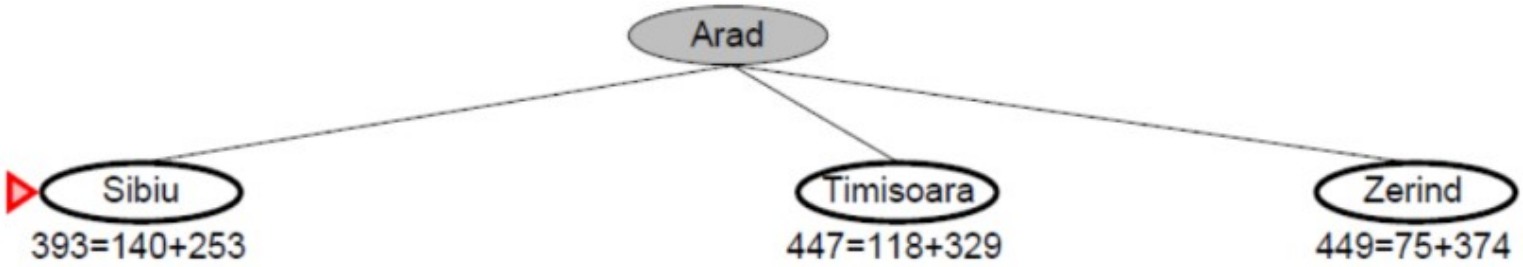

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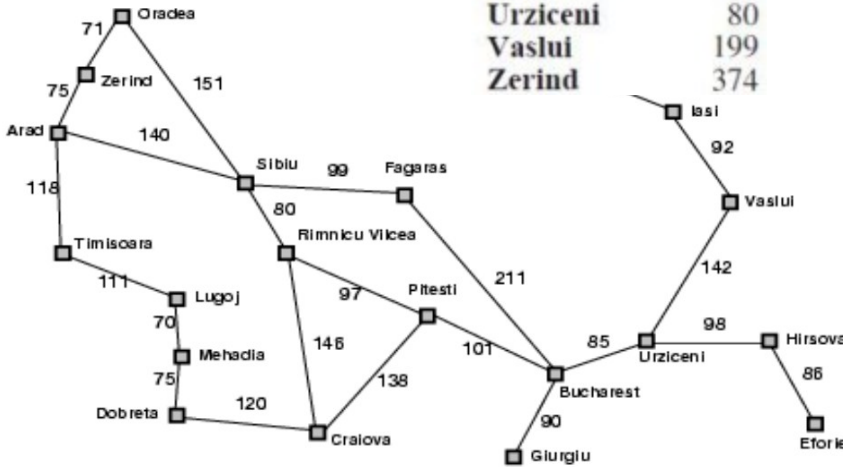
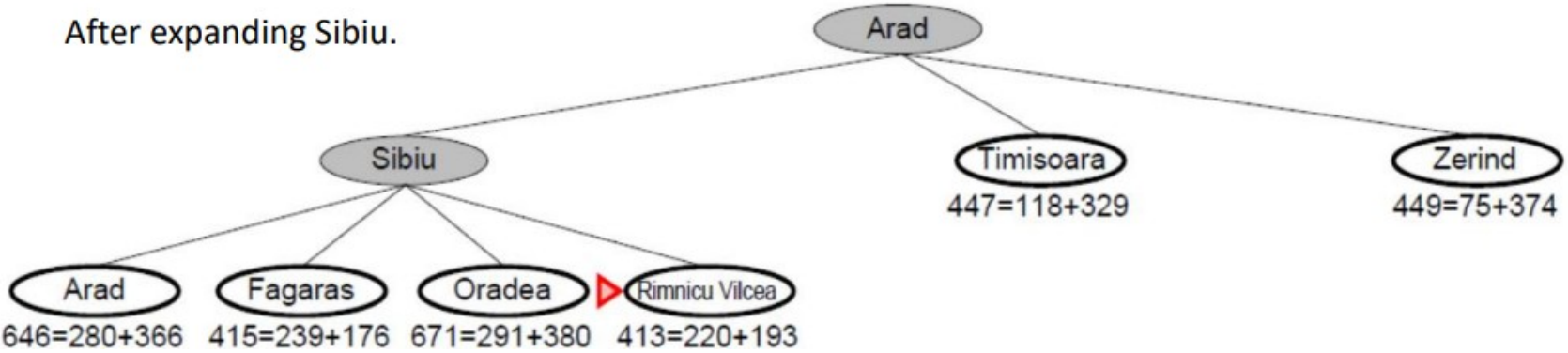


Busca Heurística: A*

After expanding Sibiu.



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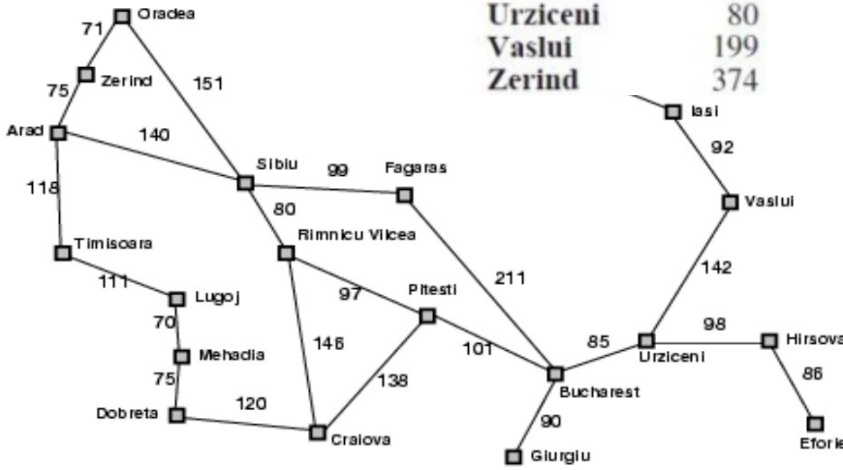

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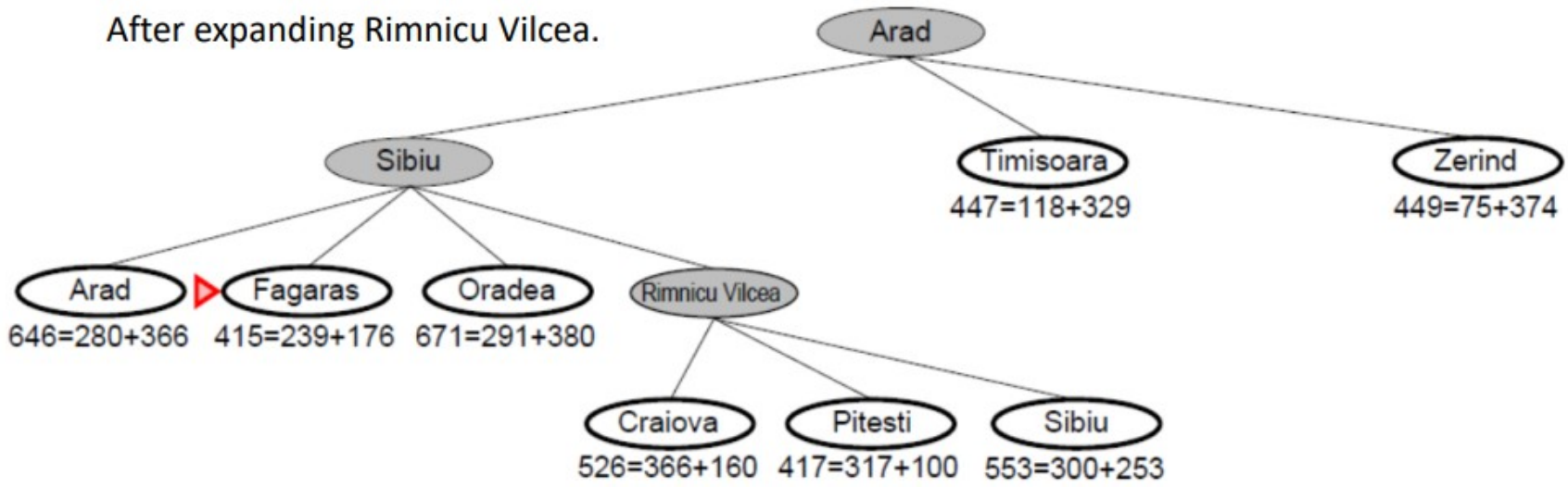

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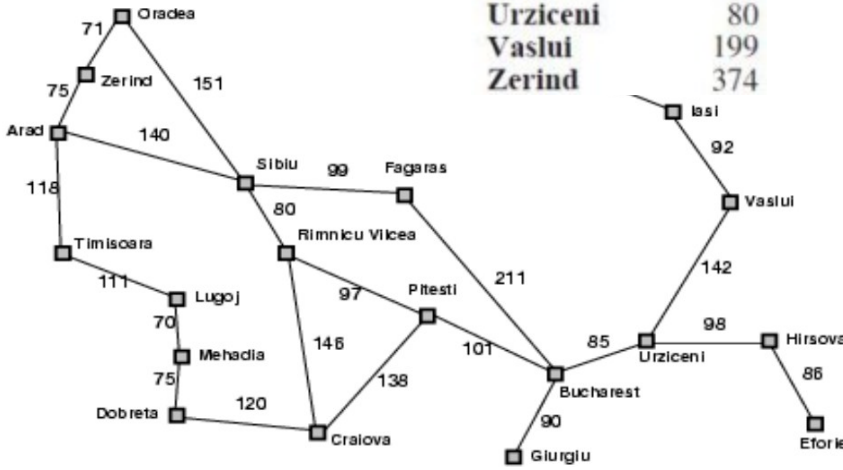
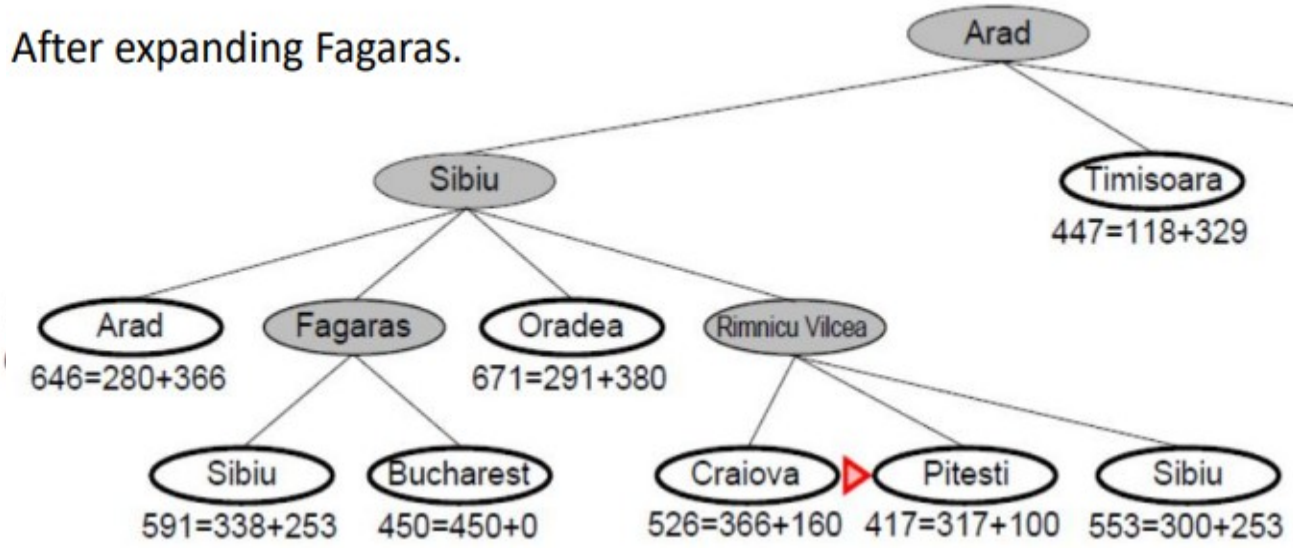


Busca Heurística: A*

After expanding Fagaras.



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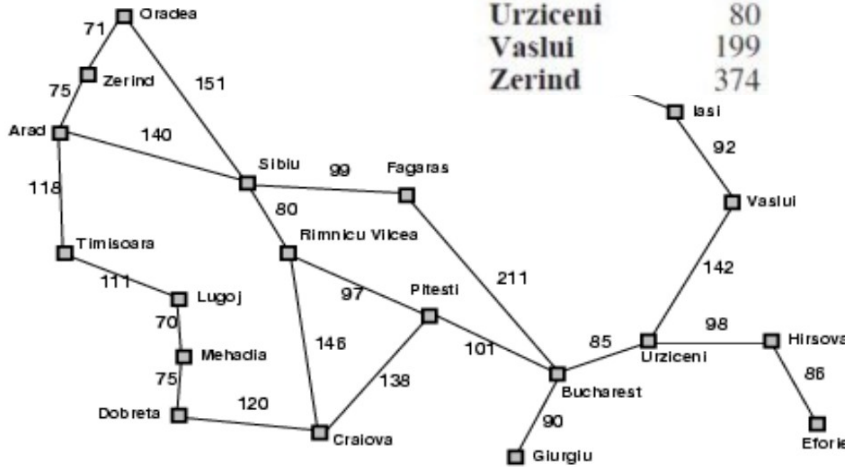
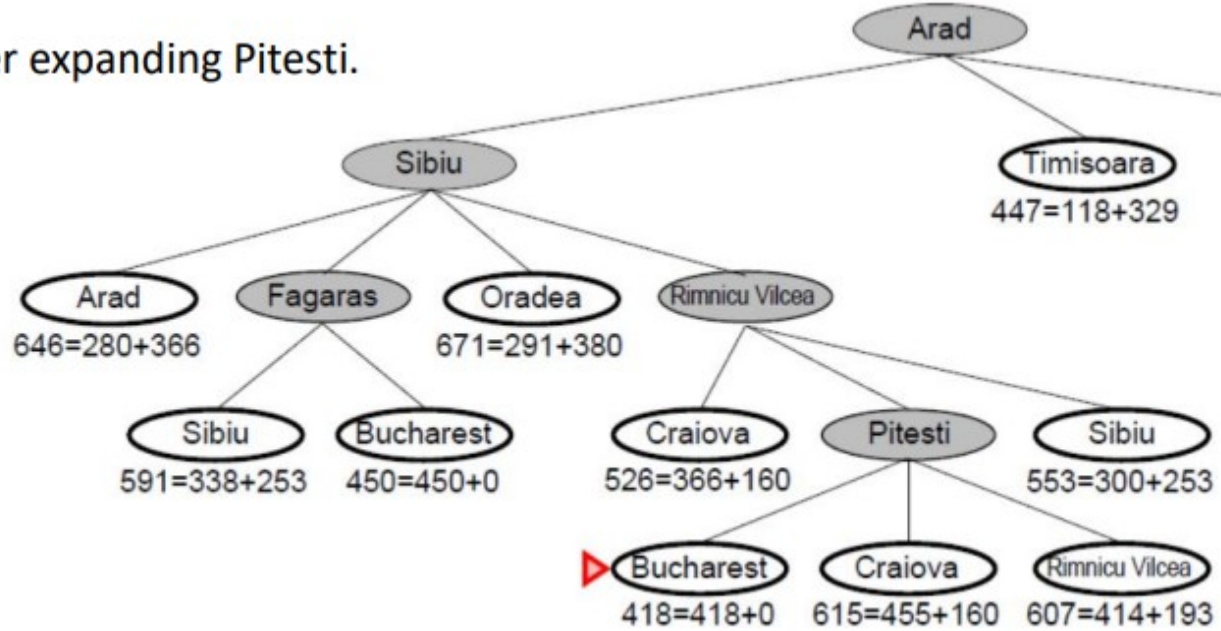


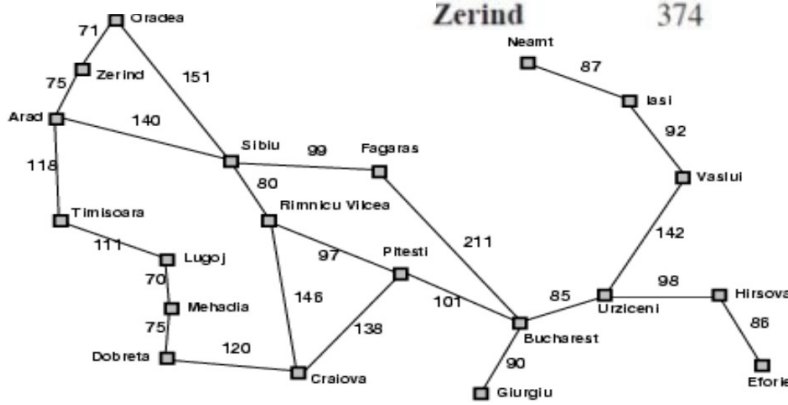
Busca Heurística: A*

After expanding Pitesti.



straight-line distances to Bucharest

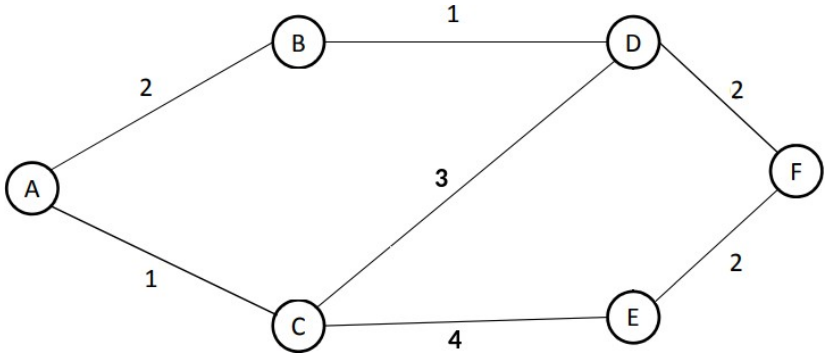
Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

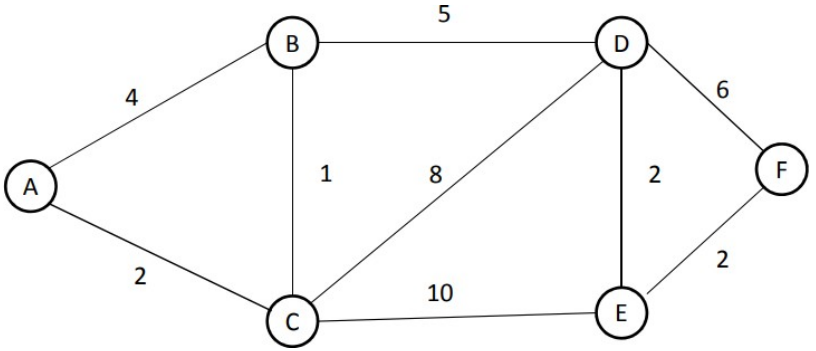


Grafos – Caminho Mínimo

Algoritmo de Dijkstra

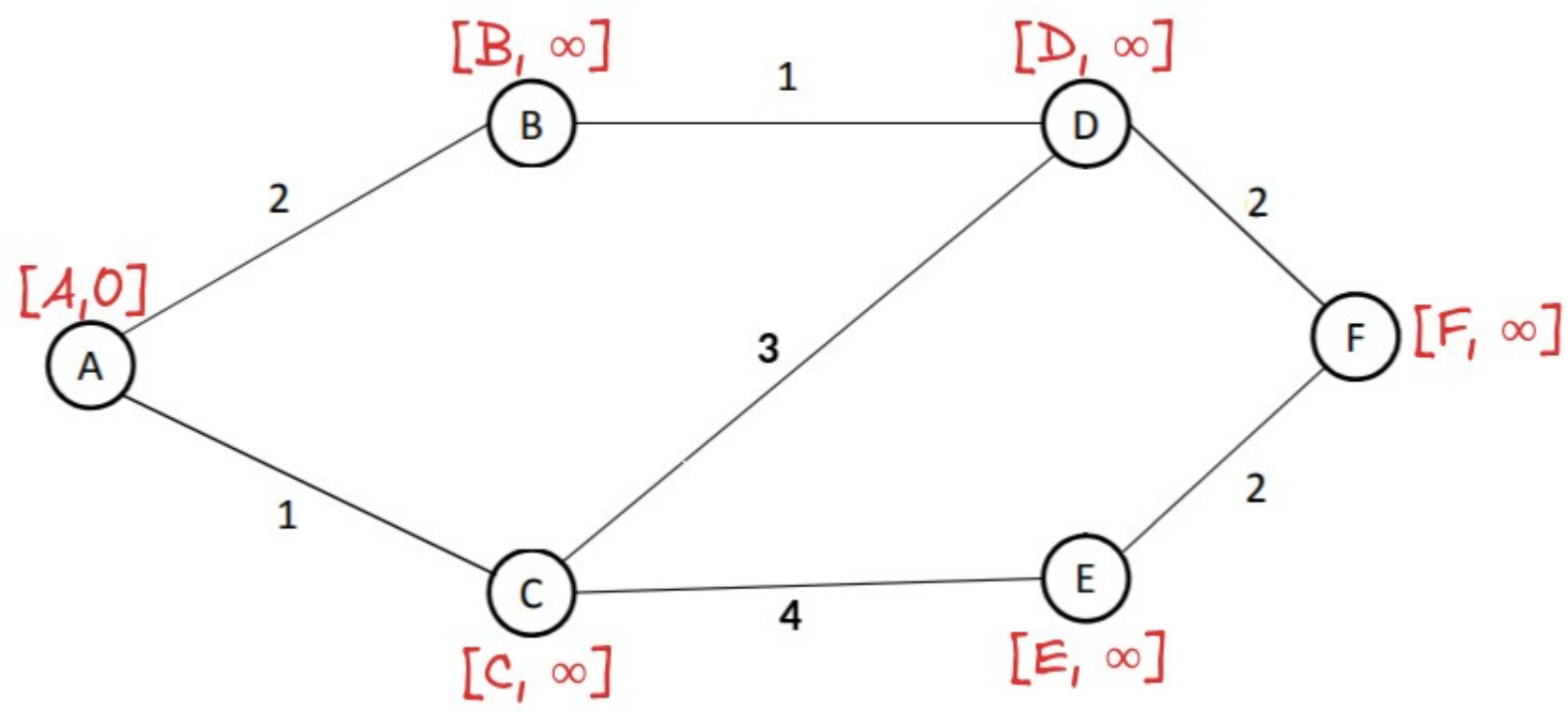
- **Edsger Dijskstra – 1959**
- **Caminho mais curto em grafo**
- **De A para F**





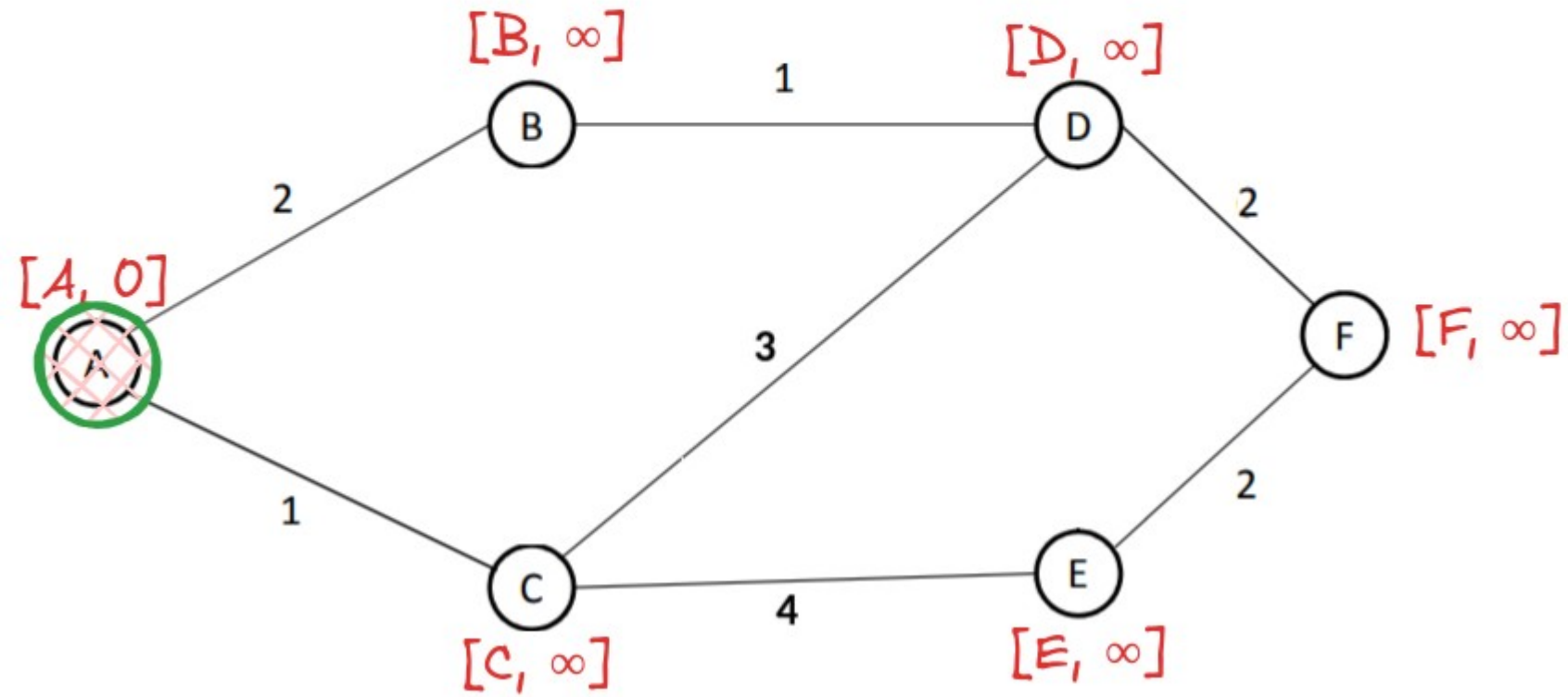
Algoritmo de Dijkstra

Inicialização:
Distância do nó de origem = 0;
Demais nós = ∞;



Algoritmo de Dijkstra

X 1. Selecione o nó não visitado com menor distância.
2. Para cada vizinho, calcule a nova distância via o nó atual.
Atualize a distância do vizinho se a nova for menor.
3. Repita até todos os nós serem visitados



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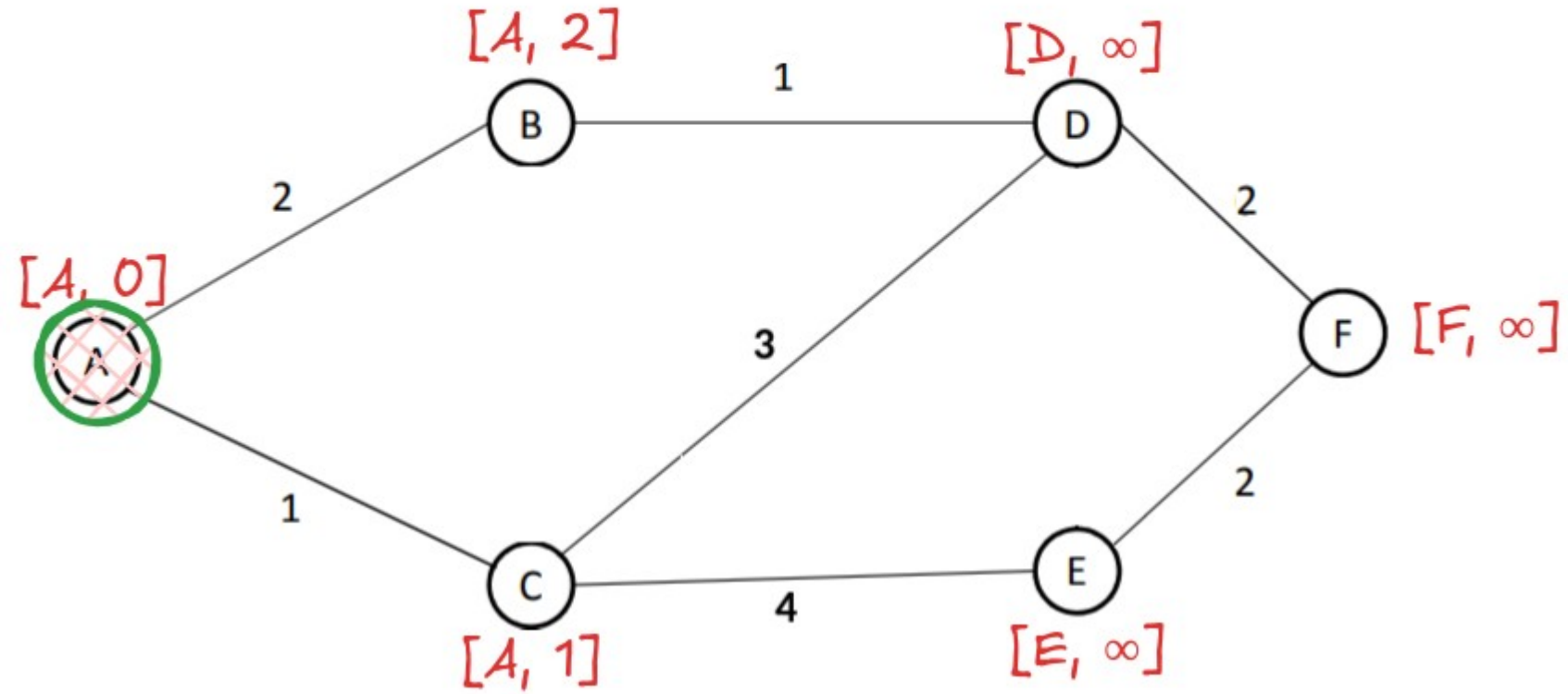

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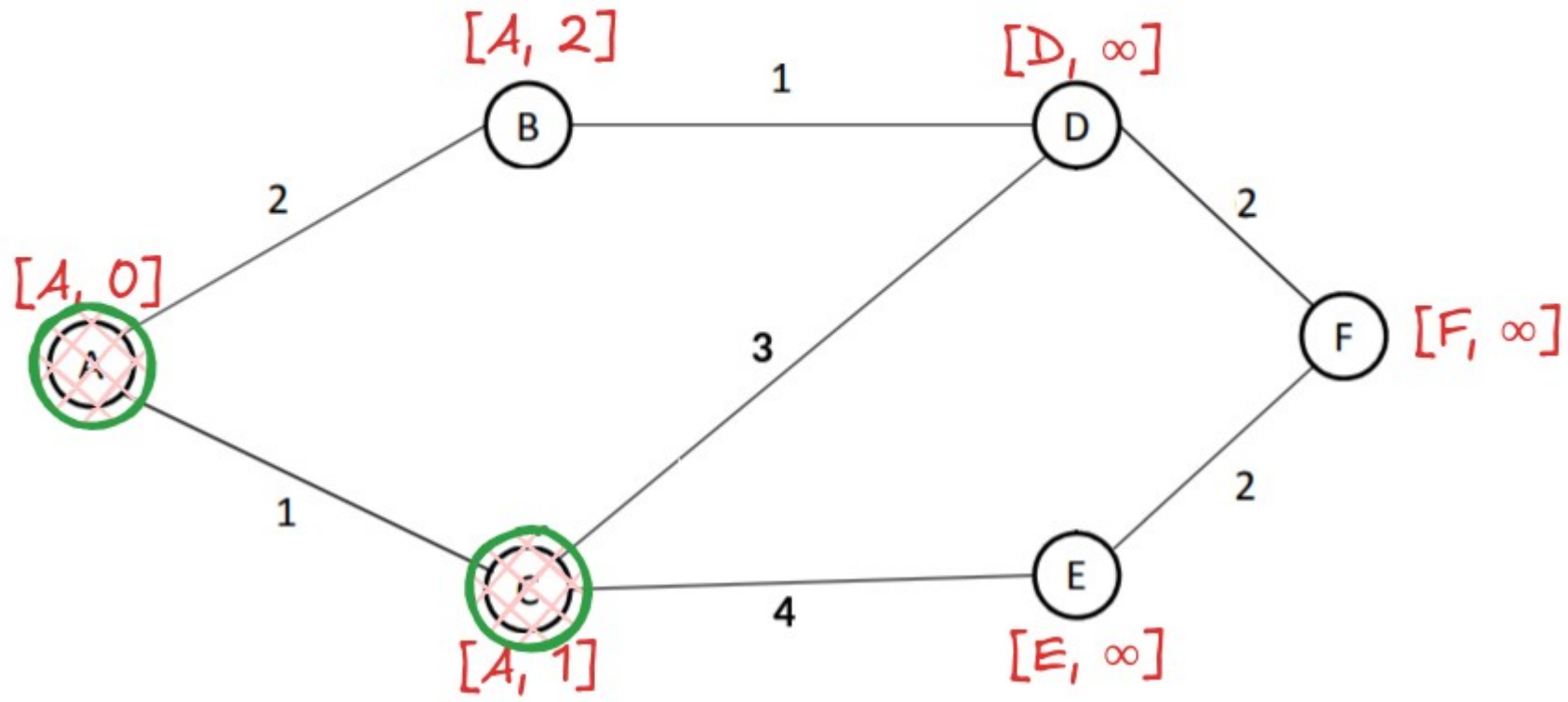

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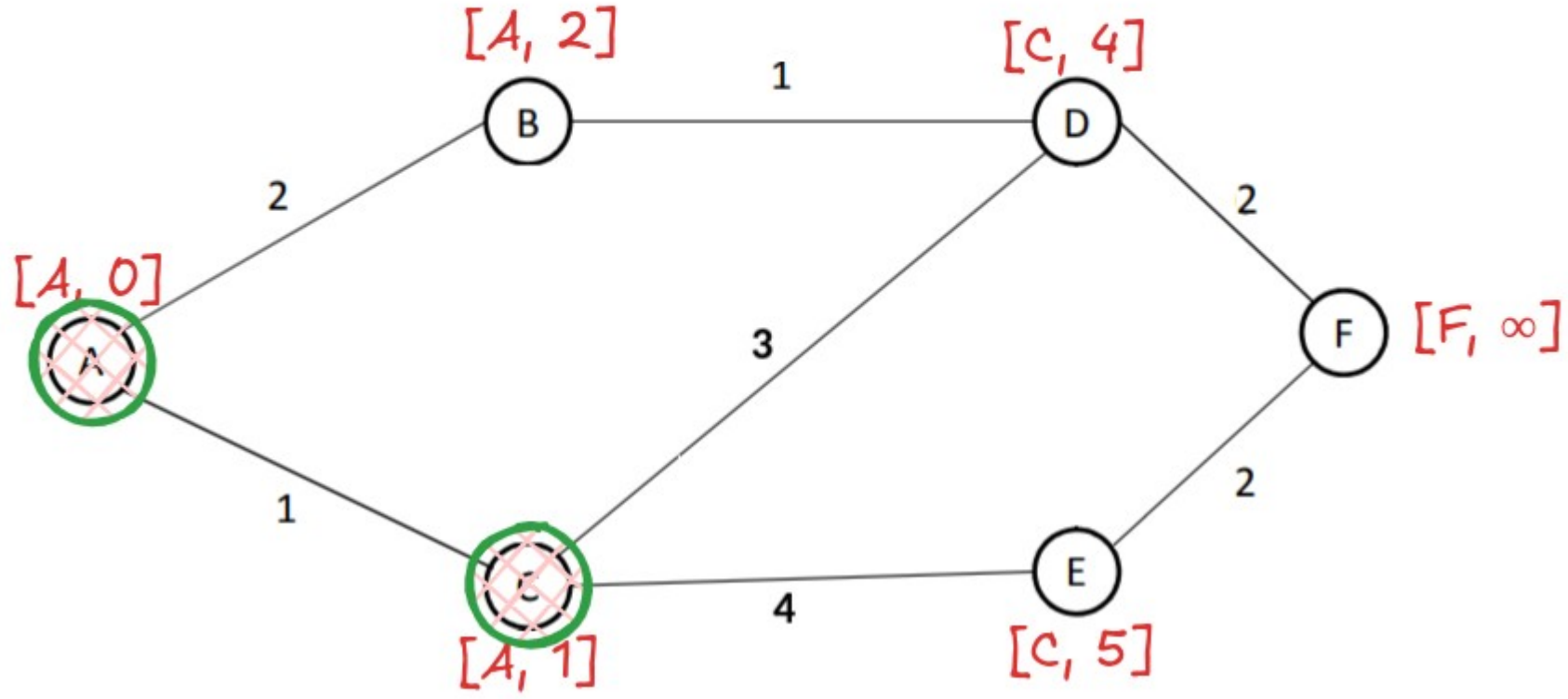

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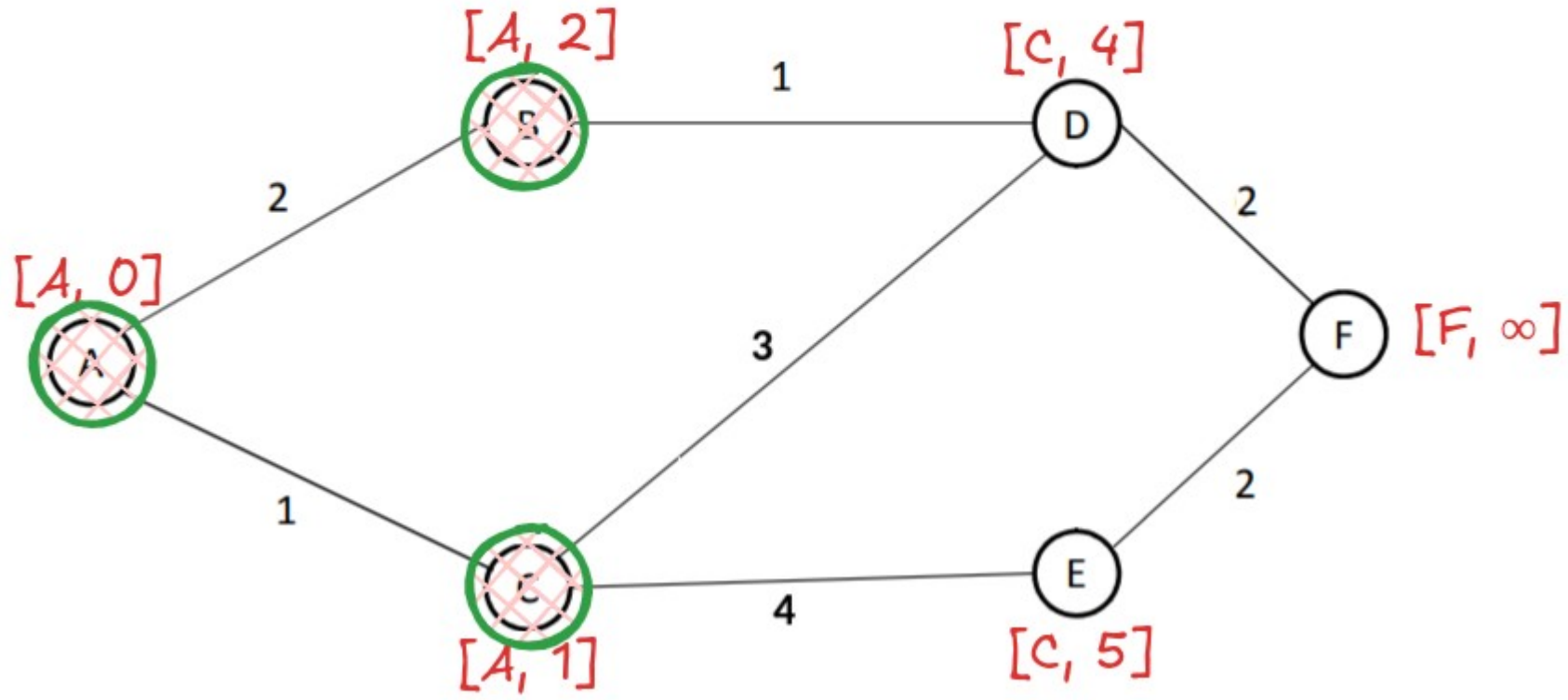

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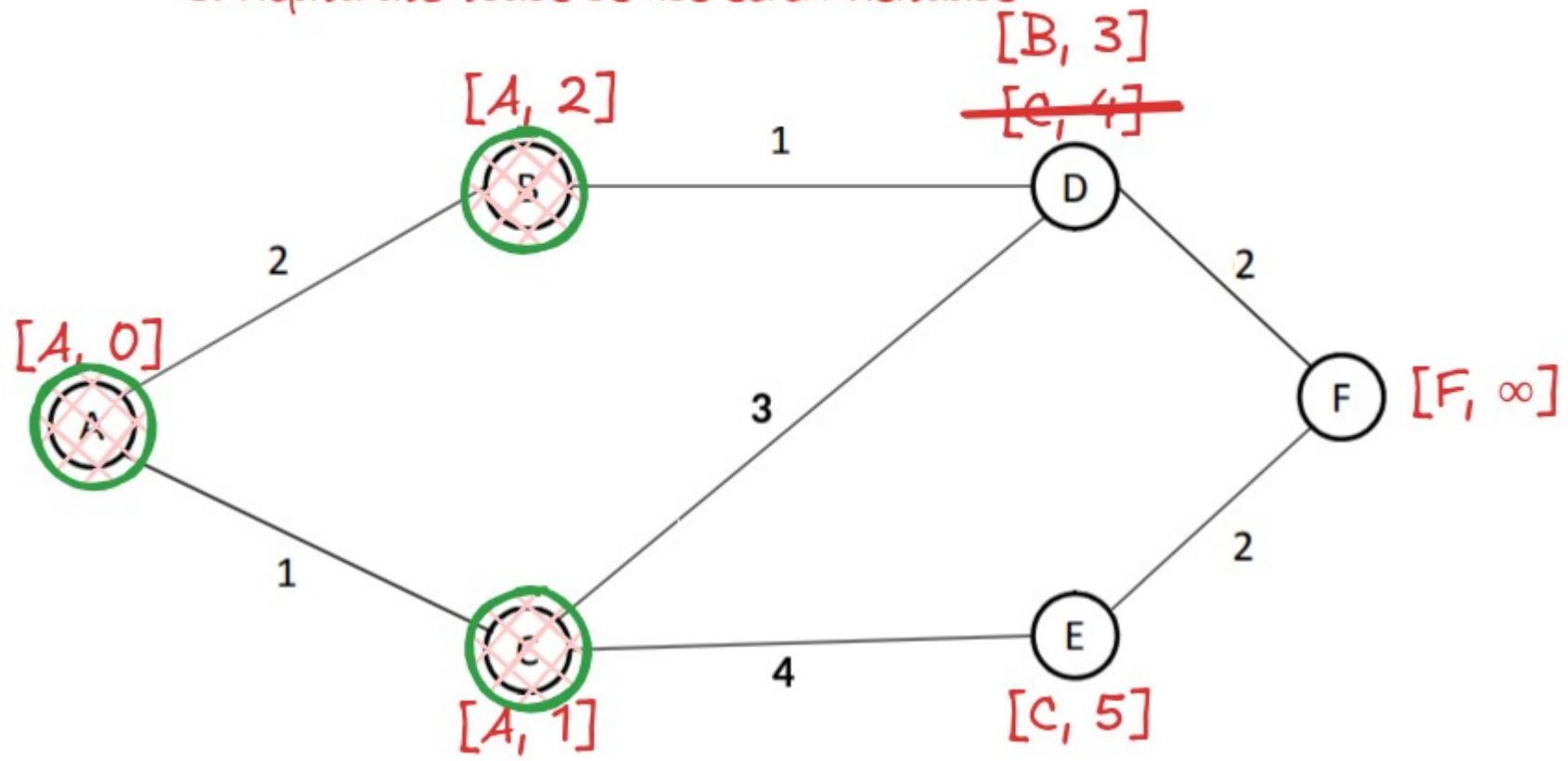

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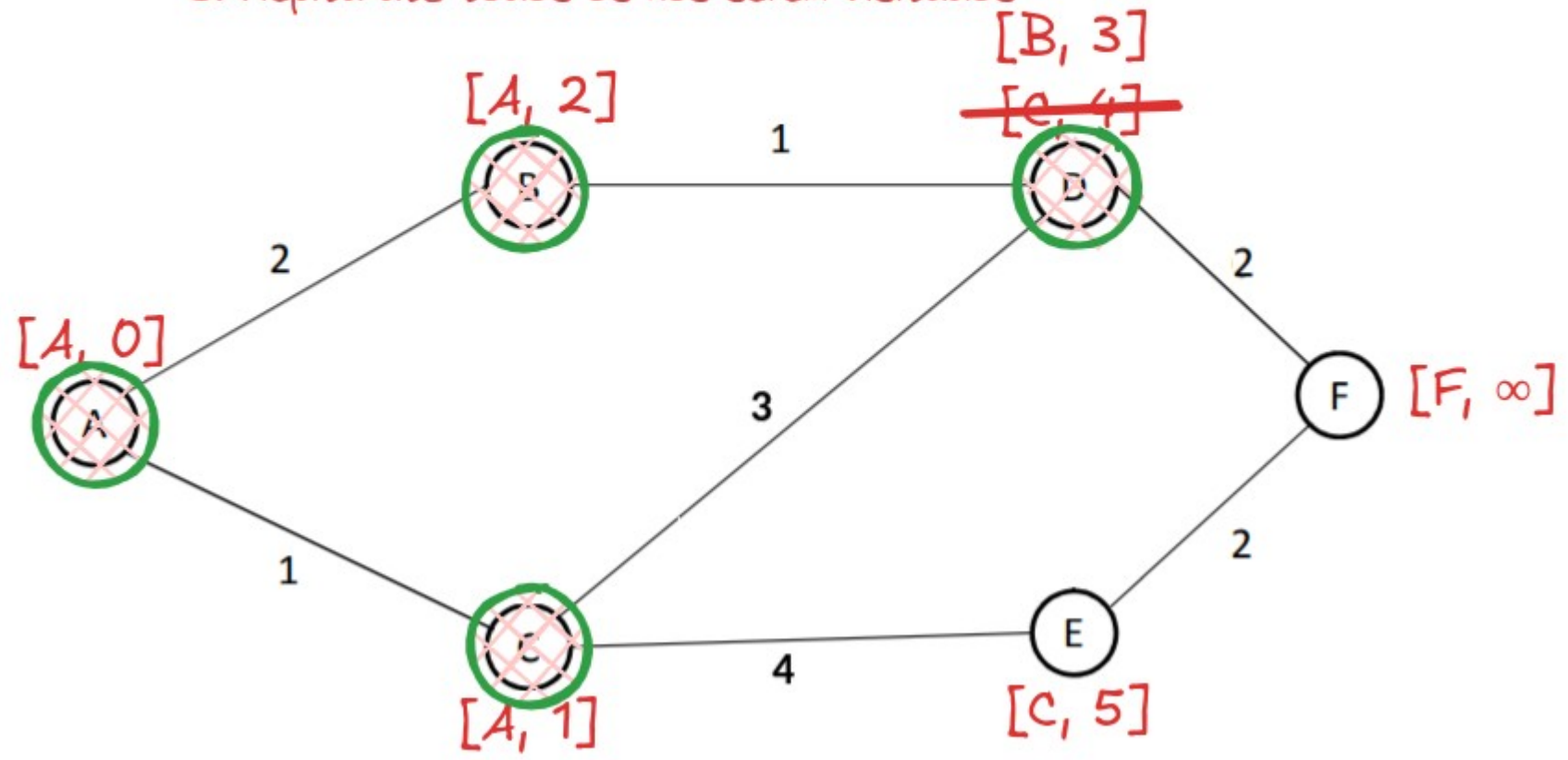

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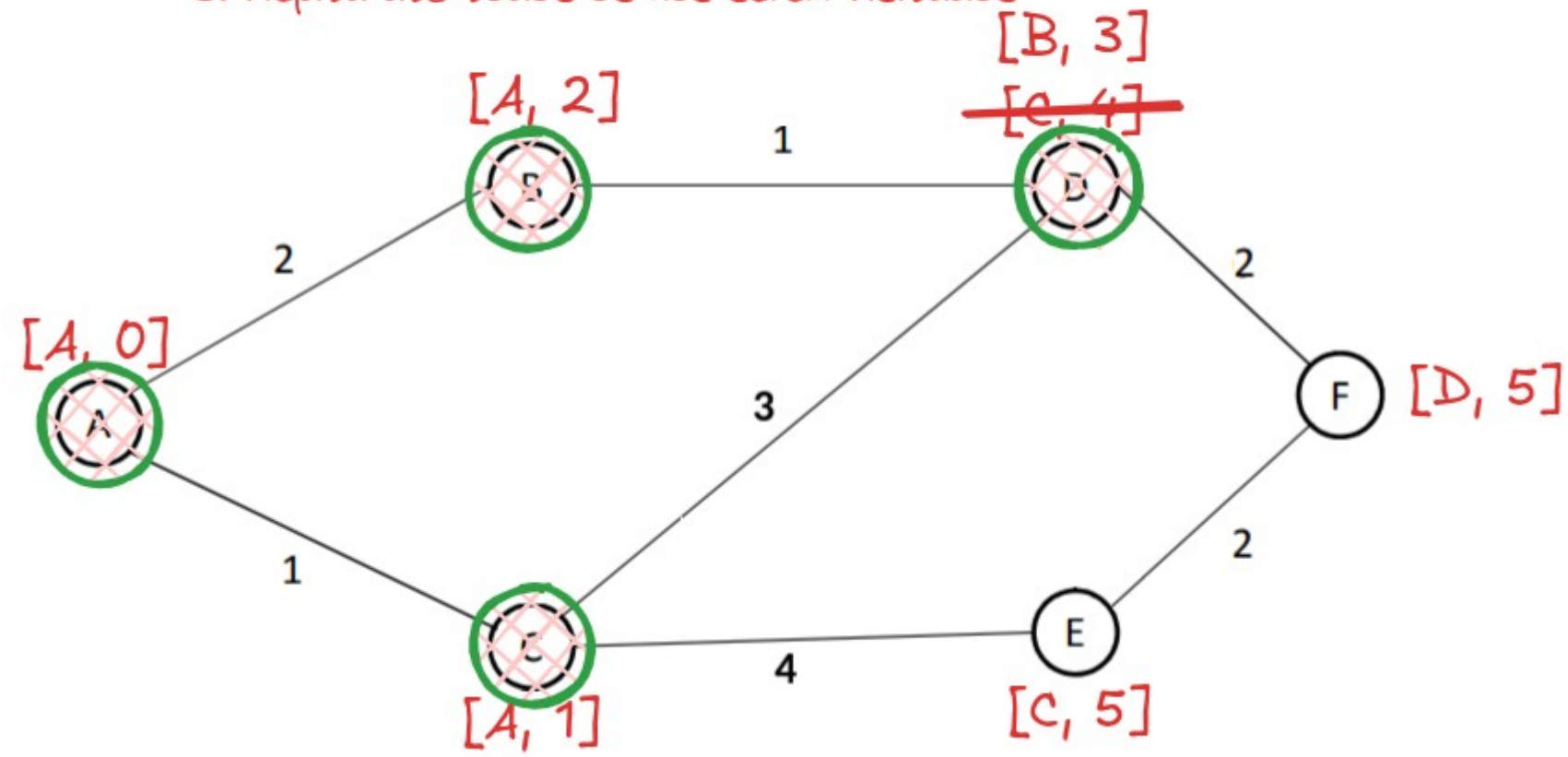

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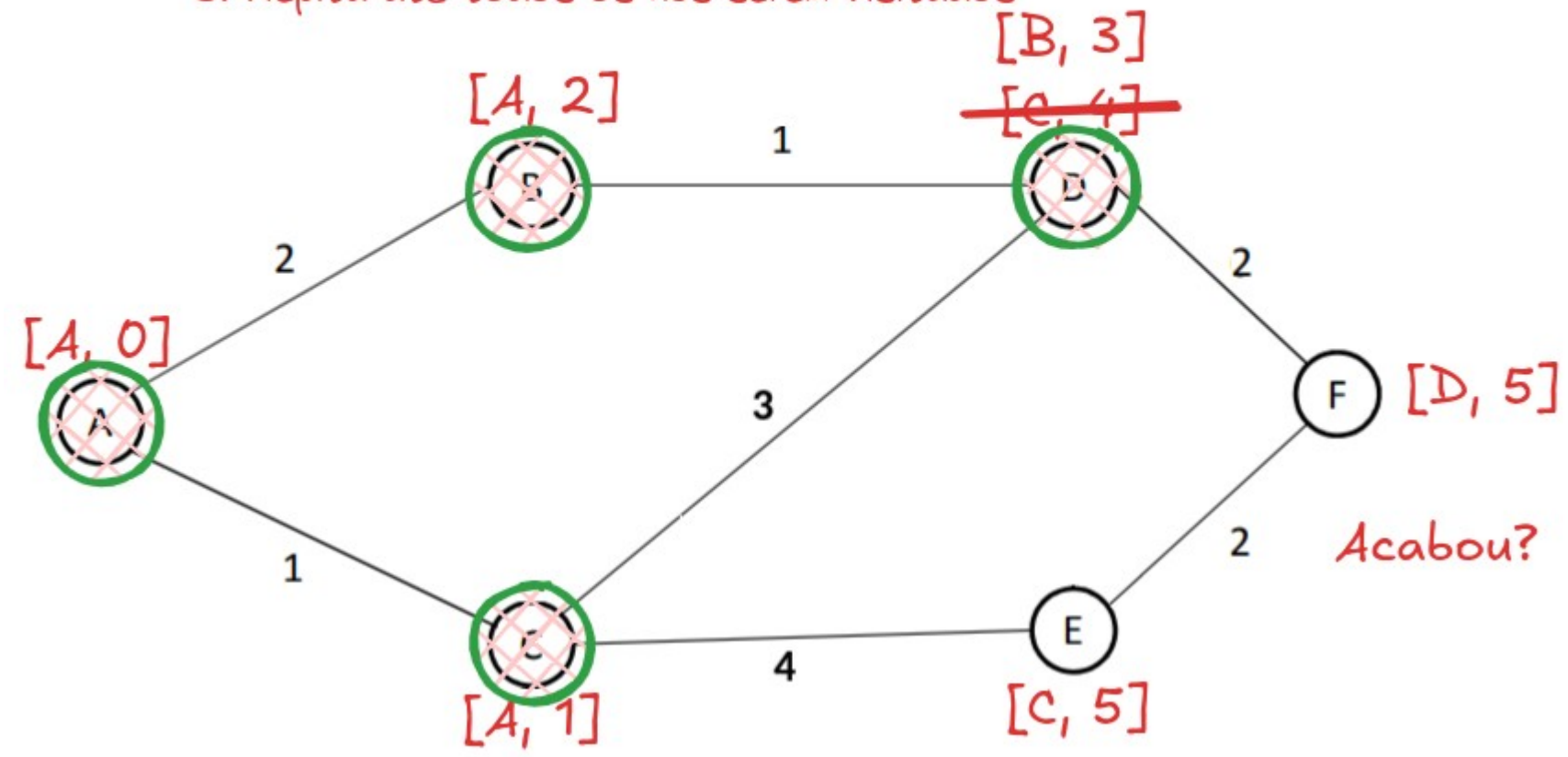

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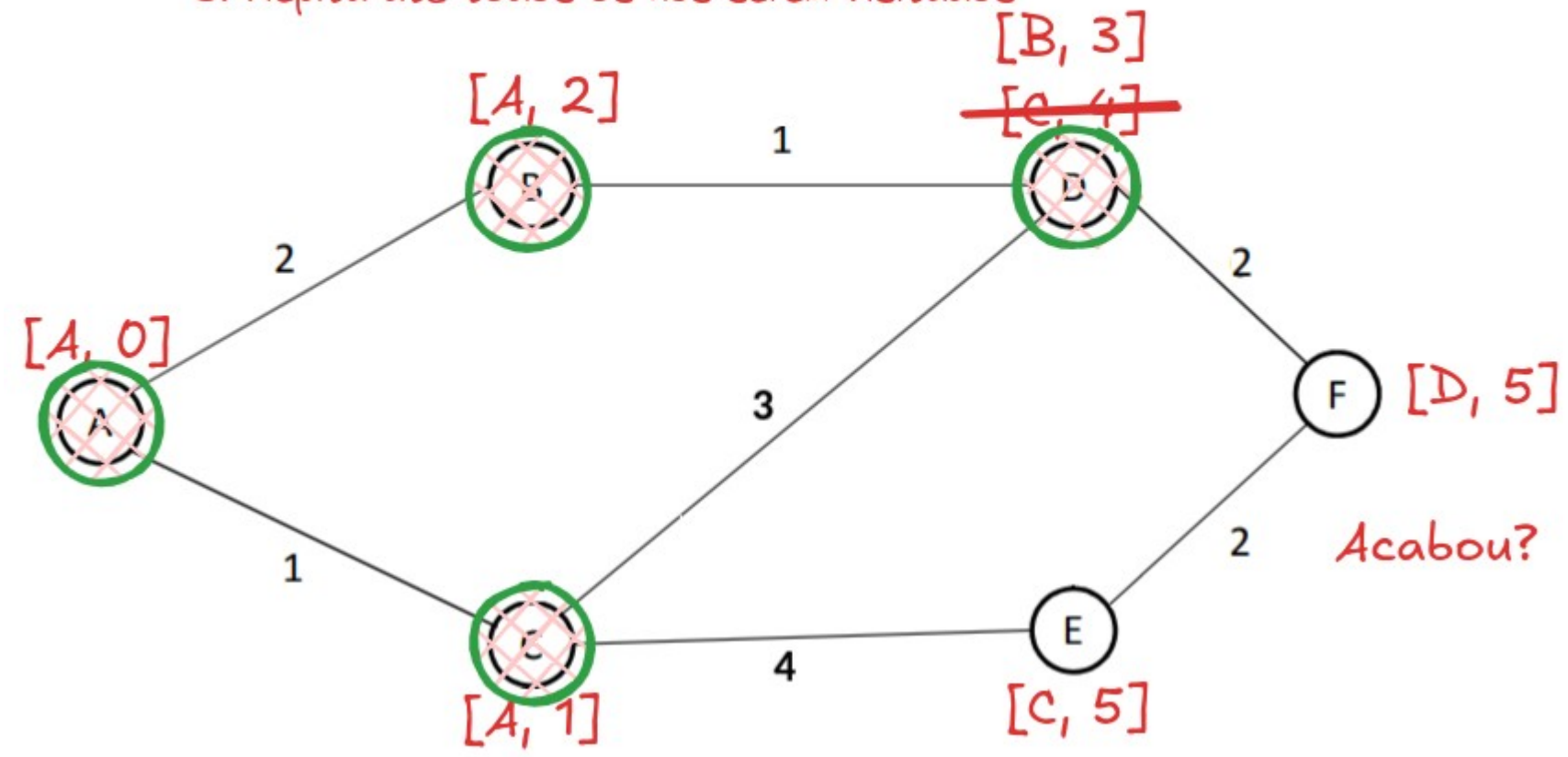

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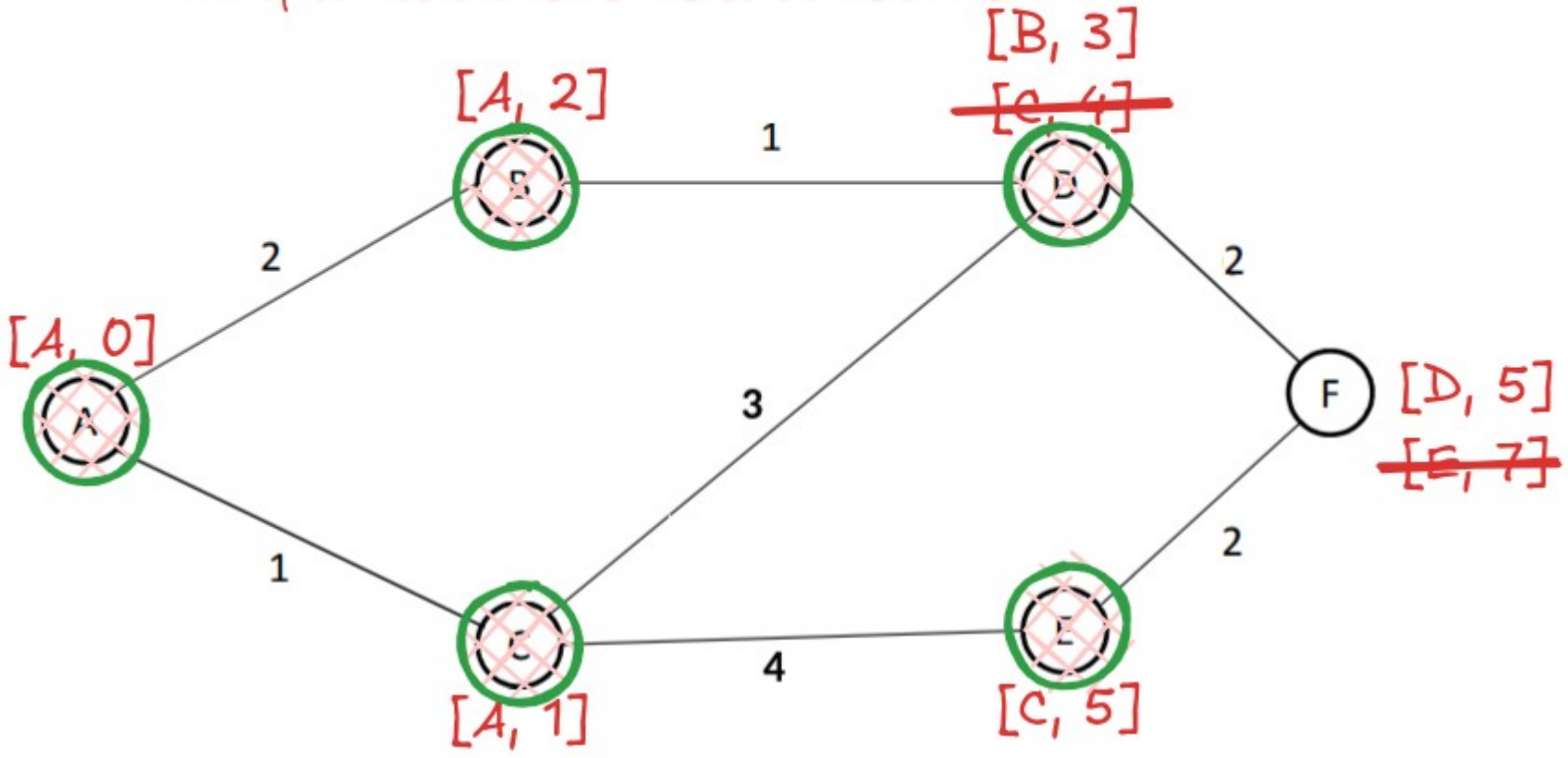
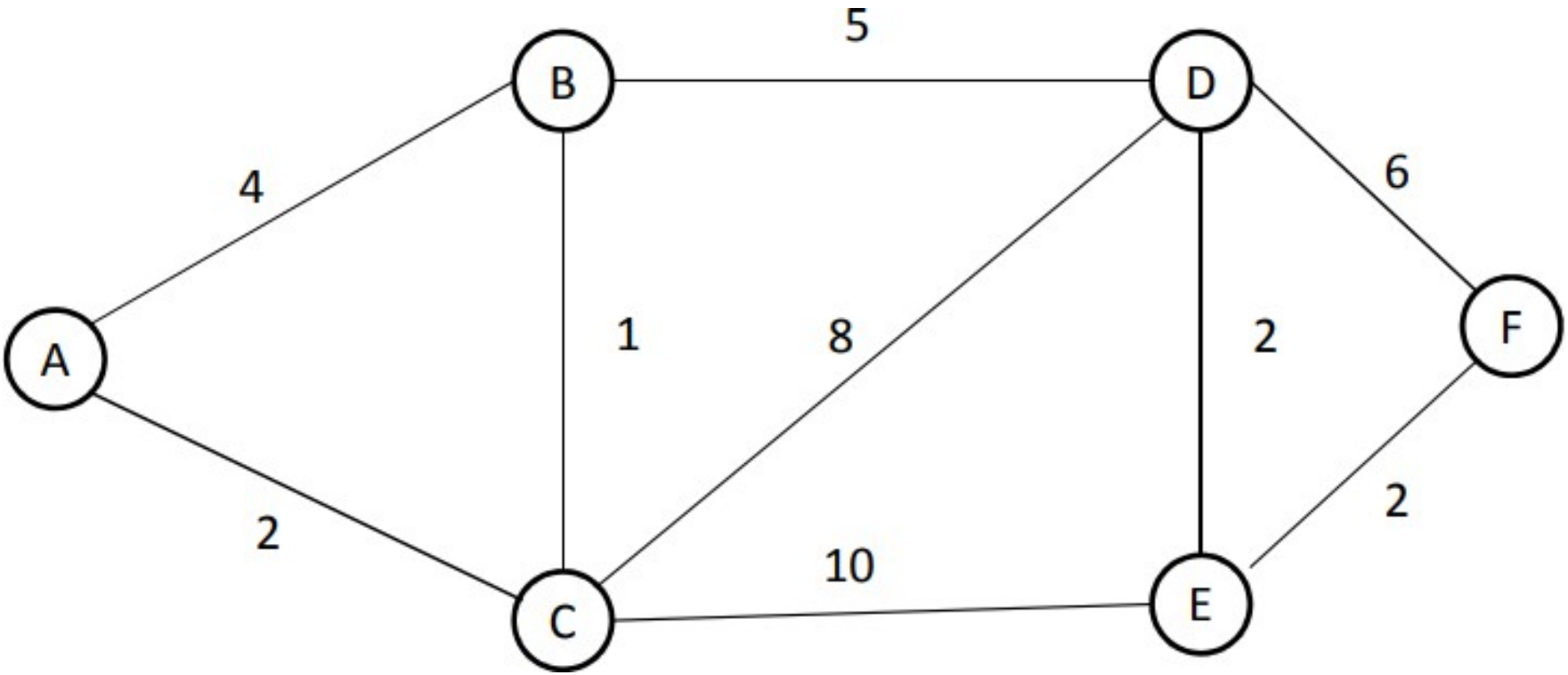


Algoritmo de Dijkstra - Exercício



Trabalho

- **Represente um problema real**
- **Analise de buscas**
- **Utilize a biblioteca *pyvis* para ilustrar o grafo**
- **Entrega via AVA**

- **VERIFIQUE OS DETALHES E REQUISITOS NO AVA!!**