Regressão

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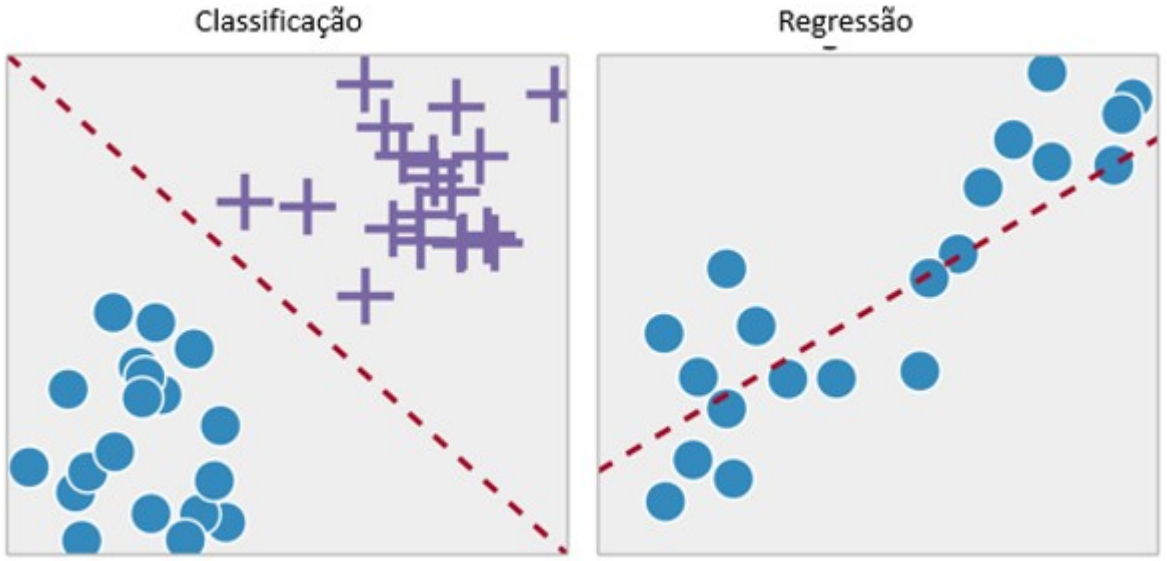
Plano de Aula

- Regressão
- Exercícios

Regressão vs Classificação

- Classificação: Determina uma classe (0,1,2,3)
- Regressão: Determina valores contínuos (preço, clima, vendas, logística, sinais...)



Regressão vs Classificação

Regressão: Compreender a relação entre as características dos dados (variáveis independentes) e a variável dependente (target).

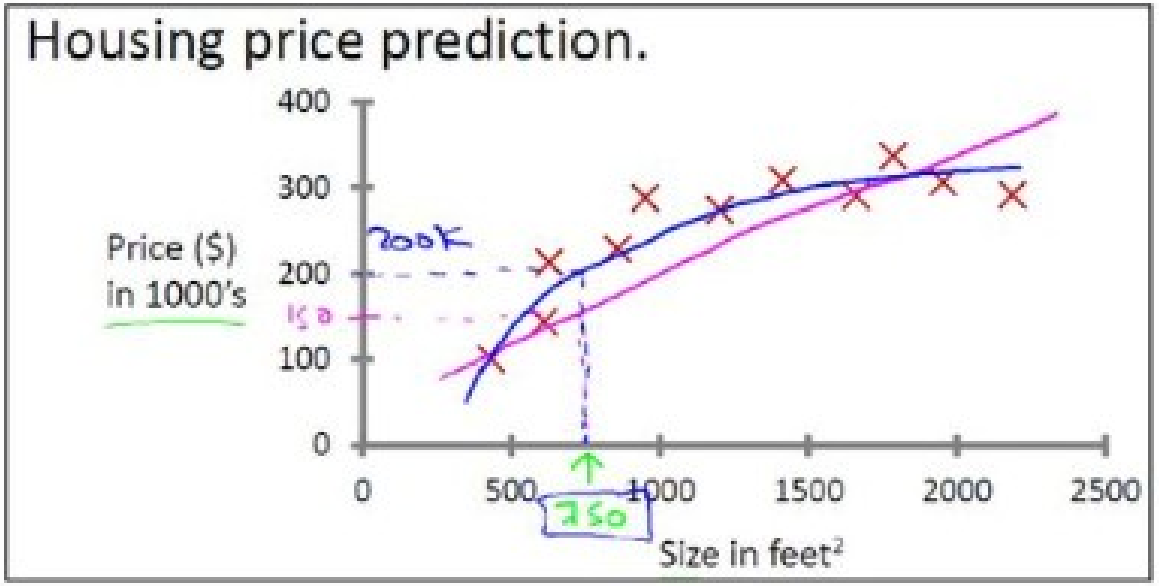
Encontrar a função (f) que aproxima o conjunto {X} em {Y} contínuo

$$f : X \rightarrow Y, \quad Y \in \mathbb{R}$$

Regressão Linear

Regressão Não Linear

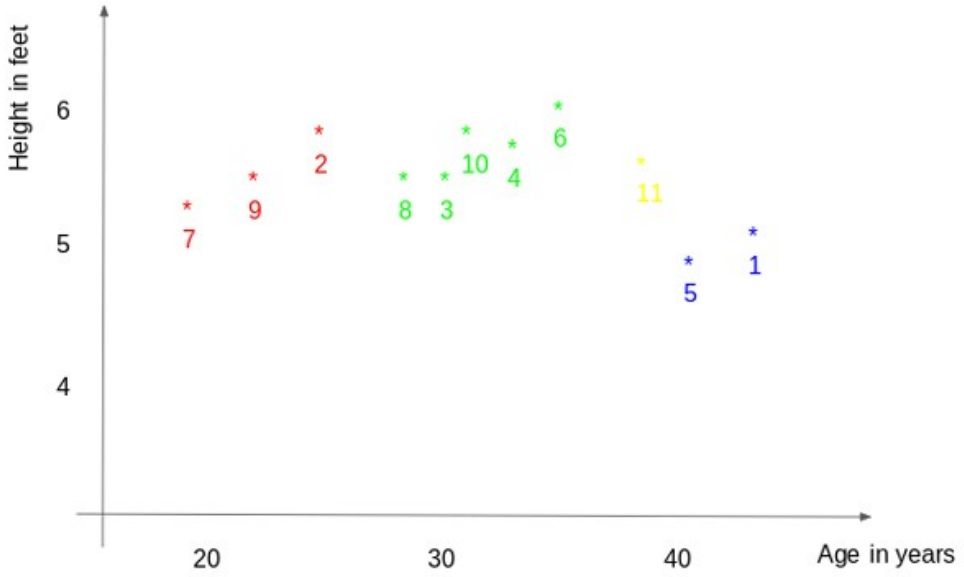
- Polinomial
- Exponencial
- .....



Regressão - KNN

Considere o dataset para determinar o peso de uma pessoa:

ID	Height	Age	Weight
1	5	45	77
2	5.11	26	47
3	5.6	30	55
4	5.9	34	59
5	4.8	40	72
6	5.8	36	60
7	5.3	19	40
8	5.8	28	60
9	5.5	23	45
10	5.6	32	58
11	5.5	38	?



Regressão - KNN

Dado K vizinhos, computa-se a média

K=3

```
ID11 = (77+72+60)/3

ID11 = 69.66 kg
```

K=5

```
ID 11 =  (77+59+72+60+58)/5

ID 11 = 65.2 kg
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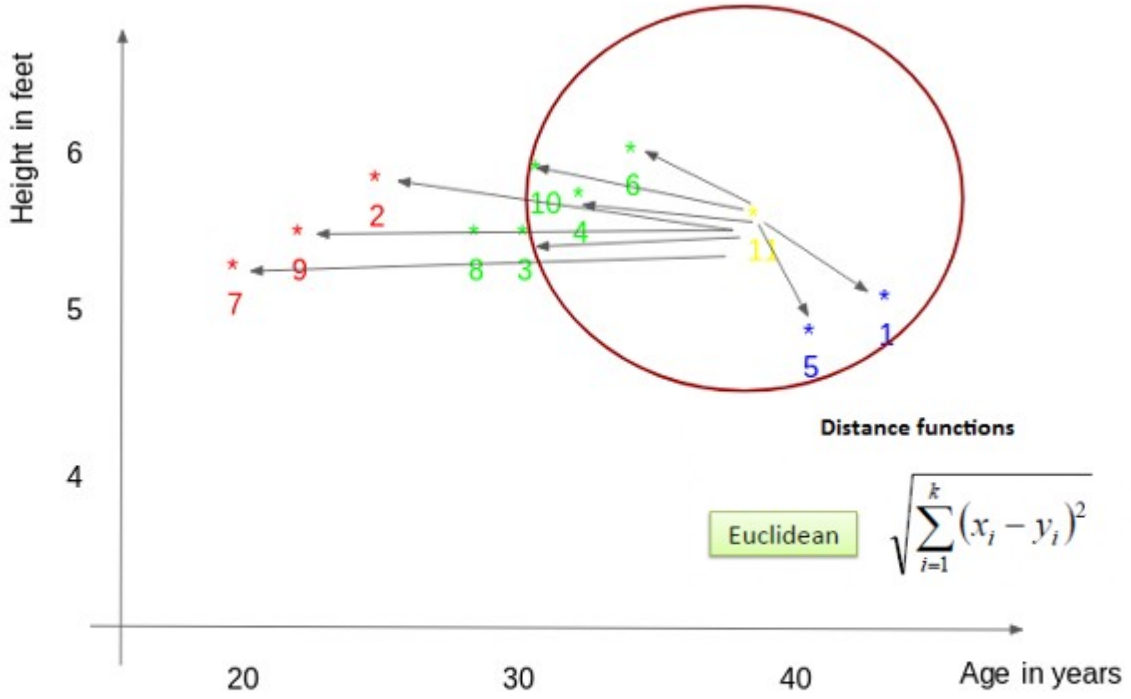


ID	Height	Age	Weight
1	5	45	77
4	5.9	34	59
5	4.8	40	72
6	5.8	36	60
10	5.6	32	58

Regressão

Como avaliar o erro ?

Mean-Square-Error (MSE)

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

Root Mean-Square-Error (RMSE)

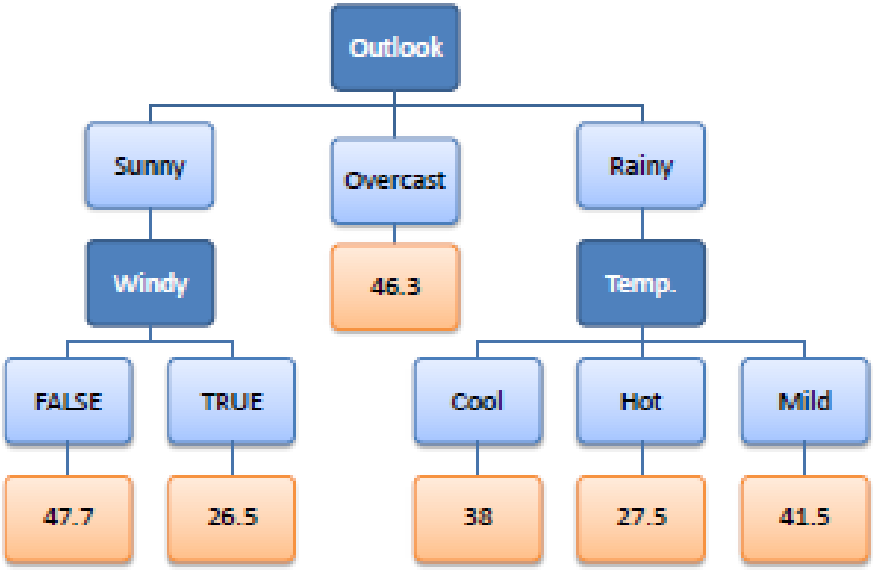
$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - p_i)^2}$$

Height	Age	Weight	Predicted	Diff
5	40	71	69	-2
5.9	23	86	80	-6
4.5	51	65	77	12
5.3	36	89	82	-7
5.1	48	75	67	-8
			MSE	59.4
			RMSE	7.71

Regressão - Árvores

Predictors | Target

Outlook	Temp.	Humidity	Windy	Hours Played
Rainy	Hot	High	False	25
Rainy	Hot	High	True	30
Overcast	Hot	High	False	48
Sunny	Mild	High	False	46
Sunny	Cool	Normal	False	62
Sunny	Cool	Normal	True	23
Overcast	Cool	Normal	True	43
Rainy	Mild	High	False	35
Rainy	Cool	Normal	False	38
Sunny	Mild	Normal	False	48
Rainy	Mild	Normal	True	48
Overcast	Mild	High	True	52
Overcast	Hot	Normal	False	44
Sunny	Mild	High	True	30



Regressão - Árvores

Determina a homogeneidade pelo do desvio padrão, média e coeficiente de variação

Hours Played
25
30
46
45
52
23
43
35
38
46
48
52
44
30

$$\boldsymbol{Count} = n = 14$$

$$\boldsymbol{Average} = \bar{x} = \frac{\sum x}{n} = 39.8$$

$$\boldsymbol{Standard\ Deviation} = S = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} = 9.32$$

$$\boldsymbol{Coeffeicient\ of\ Variation} = CV = \frac{S}{\bar{x}} * 100\% = 23\%$$

Regressão - Árvores

Desvio padrão para dois atributos:

$$S(T,X) = \sum_{c \in X} P(c)S(c)$$

		Hours Played (StDev)	Count
Outlook	Overcast	3.49	4
	Rainy	7.78	5
	Sunny	10.87	5
			14

S(Hours, Outlook) = **P**(Sunny)*__S__(Sunny) + **P**(Overcast)*__S__(Overcast) + **P**(Rainy)*__S__(Rainy)

= (4/14)*3.49 + (5/14)*7.78 + (5/14)*10.87

= 7.66

Regressão

Redução do Desvio Padrão

$$SDR(T,X) = S(T) - S(T,X)$$

SDR(Hours , Outlook) = **S**(Hours) – **S**(Hours, Outlook)

= 9.32 – 7.66 = 1.66

O maior SDR é escolhido como raiz

		Hours Played (StDev)
Outlook	Overcast	3.49
	Rainy	7.78
	Sunny	10.87
SDR=1.66		

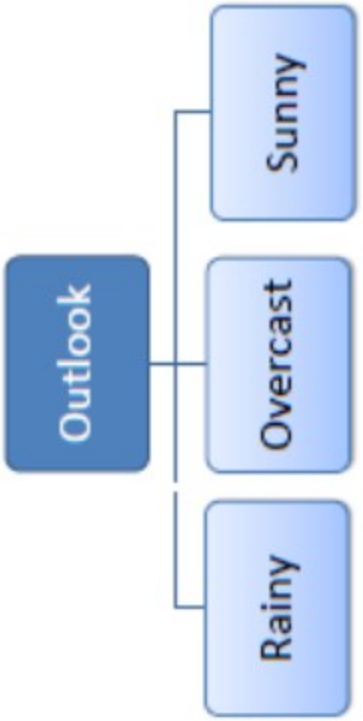
		Hours Played (StDev)
Temp.	Cool	10.51
	Hot	8.95
	Mild	7.65
SDR= 0.48		

		Hours Played (StDev)
Humidity	High	9.36
	Normal	8.37
SDR=0.28		

		Hours Played (StDev)
Windy	False	7.87
	True	10.59
SDR=0.29		

Regressão

Como escolher o critério de parada?



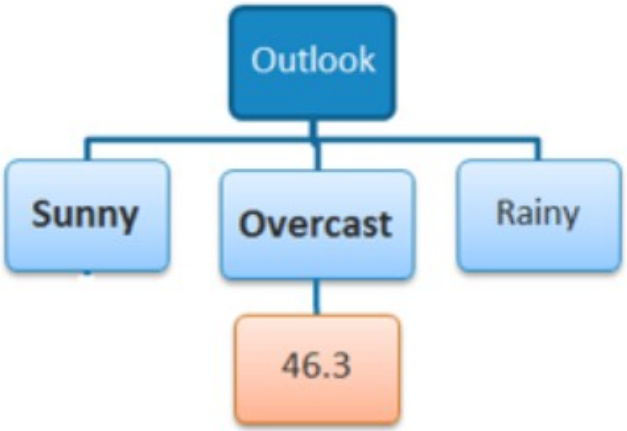
Outlook	Temp	Humidity	Windy	Hours Played
Sunny	Mild	High	FALSE	45
Sunny	Cool	Normal	FALSE	52
Sunny	Cool	Normal	TRUE	23
Sunny	Mild	Normal	FALSE	46
Sunny	Mild	High	TRUE	30
Overcast	Hot	High	FALSE	46
Overcast	Cool	Normal	TRUE	43
Overcast	Mild	High	TRUE	52
Overcast	Hot	Normal	FALSE	44
Rainy	Hot	High	FALSE	25
Rainy	Hot	High	TRUE	30
Rainy	Mild	High	FALSE	35
Rainy	Cool	Normal	FALSE	38
Rainy	Mild	Normal	TRUE	48

Regressão

Considere um limiar de CV < 10% ou count <= 3

Overcast => OK
Rainy e Sunny ainda precisam de ‘divisões’

		Hours Played (StDev)	Hours Played (AVG)	Hours Played (CV)	Count
Outlook	Overcast	3.49	46.3	8%	4
	Rainy	7.78	35.2	22%	5
	Sunny	10.87	39.2	28%	5



Regressão

Sunny:

'Windy' é determinante

Maior SDR

CV < 10% ou count <= 3

Outlook - Sunny

Temp	Humidity	Windy	Hours Played
Mild	High	FALSE	45
Cool	Normal	FALSE	52
Cool	Normal	TRUE	23
Mild	Normal	FALSE	46
Mild	High	TRUE	30
			S = 10.87
			AVG = 39.2
			CV = 28%

		Hours Played (StDev)	Count
Temp	Cool	14.50	2
	Mild	7.32	3

SDR = 10.87-((2/5)*14.5 + (3/5)*7.32) = **0.678**

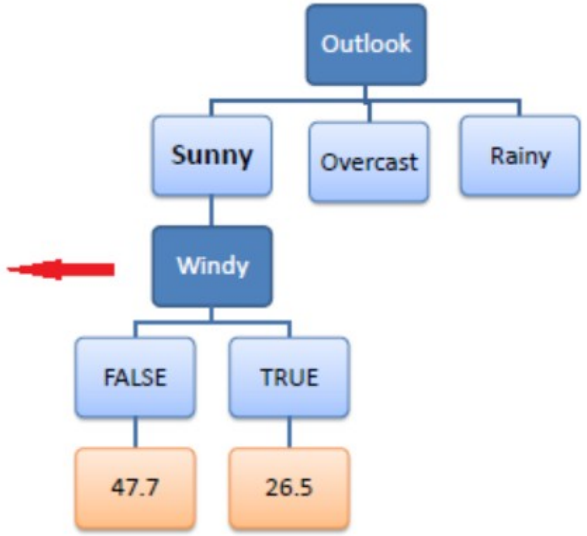
		Hours Played (StDev)	Count
Humidity	High	7.50	2
	Normal	12.50	3

SDR = 10.87-((2/5)*7.5 + (3/5)*12.5) = **0.370**

		Hours Played (StDev)	Count
Windy	False	3.09	3
	True	3.50	2

SDR = 10.87-((3/5)*3.09 + (2/5)*3.5) = **7.62**

Temp	Humidity	Windy	Hours Played
Mild	High	FALSE	45
Cool	Normal	FALSE	52
Mild	Normal	FALSE	46
Cool	Normal	TRUE	23
Mild	High	TRUE	30



Regressão

Rainy:

'Temp' é determinante

Maior SDR

CV < 10% ou count <= 3

Outlook - Rainy

Temp	Humidity	Windy	Hours Played
Hot	High	FALSE	25
Hot	High	TRUE	30
Mild	High	FALSE	35
Cool	Normal	FALSE	38
Mild	Normal	TRUE	48
			S = 7.78
			AVG = 35.2
			CV = 22%

		Hours Played (StDev)	Count
Temp	Cool	0	1
	Hot	2.5	2
	Mild	6.5	2

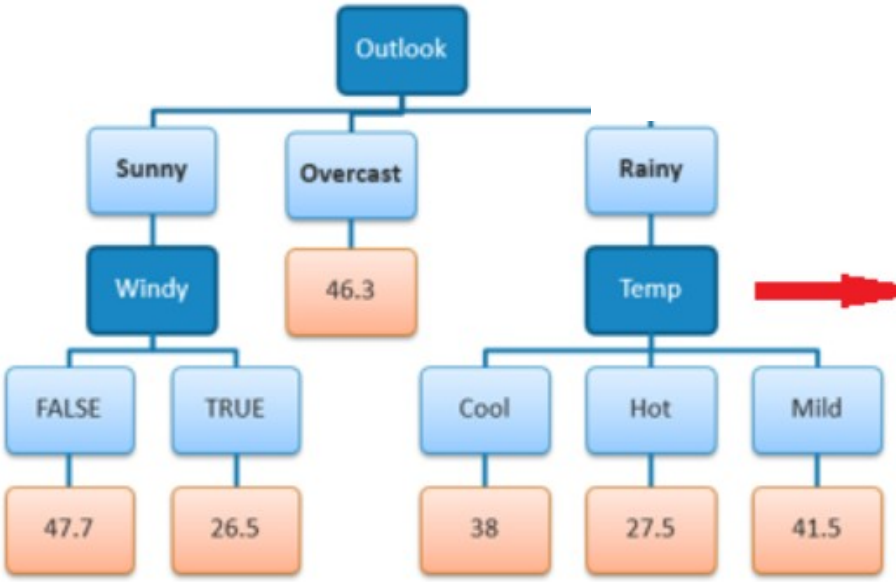
SDR = 7.78 - ((1/5)*0+(2/5)*2.5 + (2/5)*6.5) = **4.18**

		Hours Played (StDev)	Count
Humidity	High	4.1	3
	Normal	5.0	2

SDR = 7.78 - ((3/5)*4.1 + (2/5)*5.0) = **3.32**

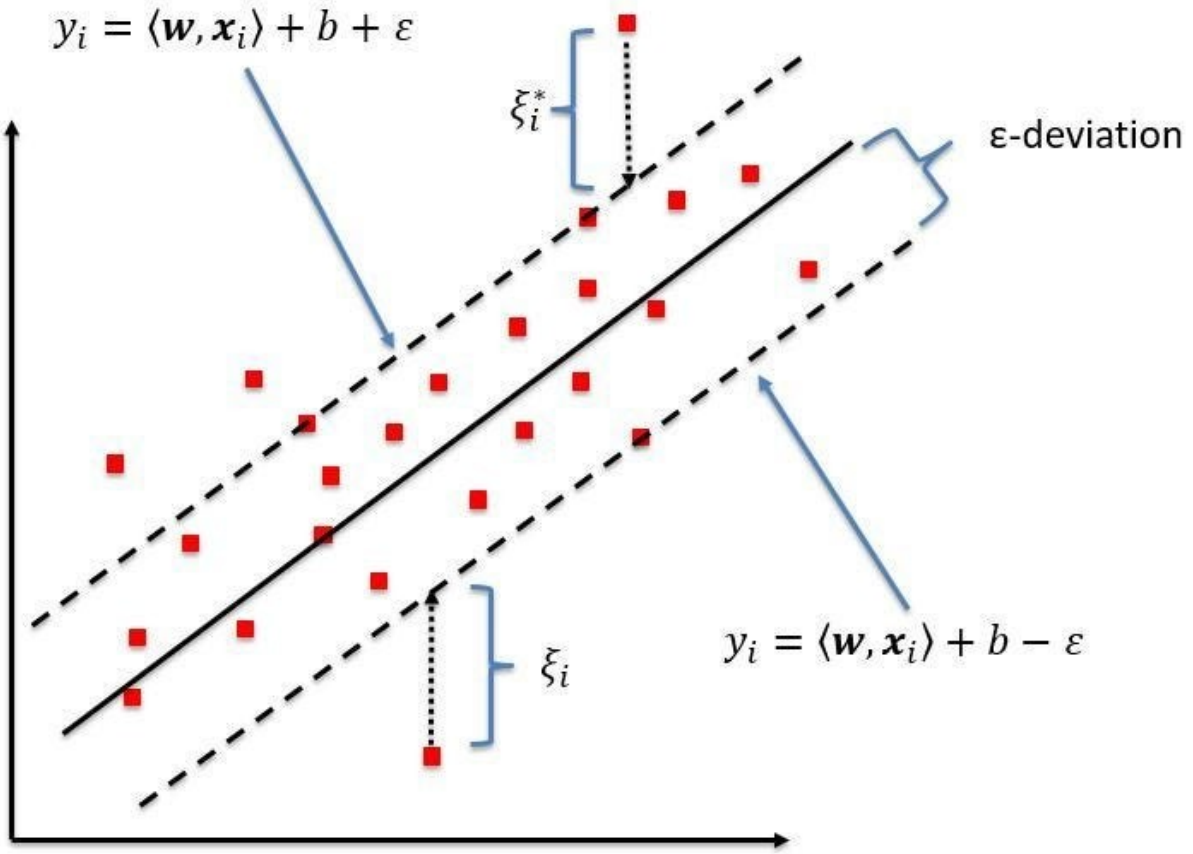
		Hours Played (StDev)	Count
Windy	False	5.6	3
	True	9.0	2

SDR = 7.78 - ((3/5)*5.6 + (2/5)*9.0) = **0.82**



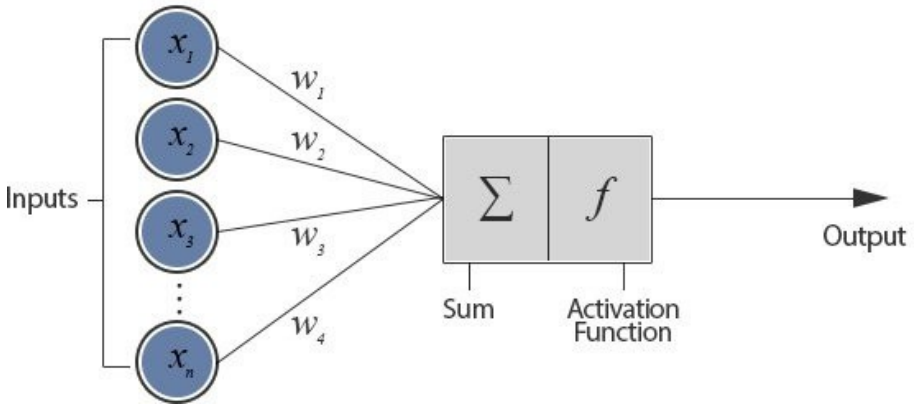
Temp	Hours Played
Cool	38
Hot	25
Hot	30
Mild	35
Mild	48

Regressão – SVM

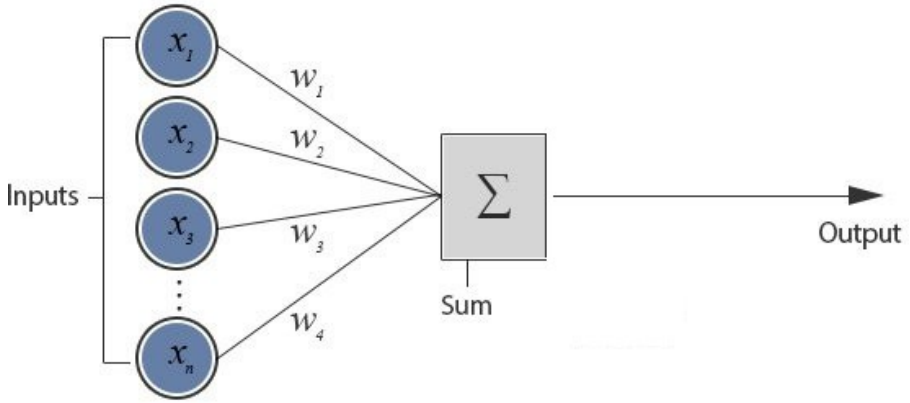


Regressão – MLP

Perceptron – Classificação

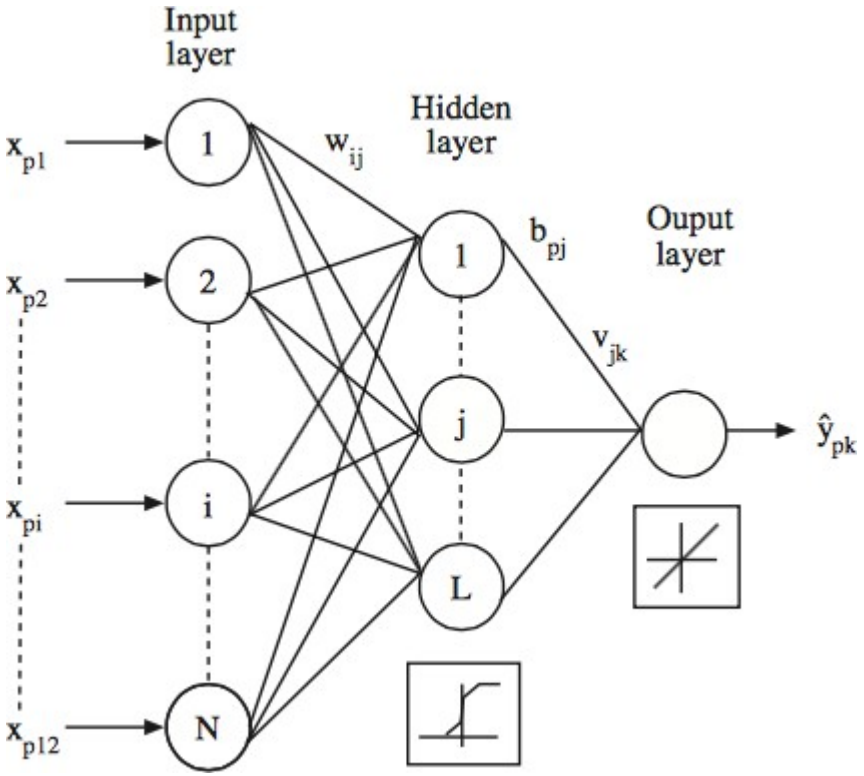


Perceptron - Regressão



Regressão – MLP

MLP – Classificação



Lets Code

Abordaremos a construção dos modelos de regressão utilizando um dataset para prever preços de casas.

Acompanhe e crie sua implementação em conjunto com o professor.

Código base em: Tópico 02 - Regressão.ipynb