MANOLACHE ANDREI 29. 01. 2020

344

MR17

EXAMEN CALCUL NUMERIC

I.

$$\begin{cases} 3x_1 + 3x_2 + 4x_3 = 21 \\ 3x_1 + 11x_2 + 11x_3 = 58 \\ 15x_1 + 19x_2 + 22x_3 = \cancel{8}\ 119 \end{cases}$$

$$A = \begin{pmatrix} 3 & 3 & 4 \\ 3 & 11 & 11 \\ 15 & 19 & 22 \end{pmatrix} \quad b = \begin{pmatrix} 21 \\ 58 \\ 118 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$w = (1,2,3)$ – vectorul de permutări pt ~~...~~ soluțiilor

$p = (1,2,3)$ – vectorul de permutări ordinea (pt coloane)

$\underline{k=1}$

$$|a_{pm}| = \max_{i,j=\overline{1,3}} |a_{ij}| = a_{33} \Rightarrow \begin{cases} p = 3 \\ m = 3 \end{cases}$$

$L_1 \leftrightarrow L_3 \rightarrow w_1 \leftrightarrow w_3 \rightarrow w = (3,2,1)$

$C_1 \leftrightarrow C_3 \rightarrow p_1 \leftrightarrow p_3 \Rightarrow p = (3,2,1)$

$$A \overset{L_1 \leftrightarrow L_3}{\sim} \begin{pmatrix} 15 & 19 & 22 \\ 3 & 11 & 11 \\ 3 & 3 & 4 \end{pmatrix} \overset{C_1 \leftrightarrow C_3}{\sim} \begin{pmatrix} 22 & 19 & 15 \\ 11 & 11 & 3 \\ 4 & 3 & 3 \end{pmatrix}$$

$m_{21} = \frac{11}{22} = \frac{1}{2}$ $\quad L_2 \leftarrow L_2 - \frac{1}{2} L_1$

$m_{31} = \frac{4}{22} = \frac{2}{11}$ $\quad L_3 \leftarrow L_3 - \frac{2}{11} L_2$

$$A \sim \begin{pmatrix} 22 & 19 & 15 \\ 0 & 3/2 & -\frac{9}{2} \\ 0 & -\frac{5}{11} & \frac{3}{11} \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{2}{11} & 0 & 1 \end{pmatrix}$$

$\underline{\underline{k=2}}$ $\quad |a_{pm}| = \max\limits_{i,j=\overline{2,3}} |a_{ij}| = a_{23} \Rightarrow \begin{cases} p = 2 \\ m = 3 \end{cases}$

$m \neq k \Rightarrow C_2 \leftrightarrow C_3 \Rightarrow p = (3, 1, 2)$

$$A \overset{C_2 \leftrightarrow C_3}{\sim} \begin{pmatrix} 22 & 15 & 19 \\ 0 & -\frac{9}{2} & 3/2 \\ 0 & \frac{3}{11} & -5/11 \end{pmatrix}$$

$$m_{32} = \cancel{\frac{3/10}{-\frac{9}{2}} = -\frac{1}{3}} \quad \frac{\frac{3}{11}}{-\frac{9}{2}} = \frac{3}{11} \cdot \frac{-2}{9} = \cancel{-\frac{1}{15}}\ \frac{-2}{33}$$

$$L_3 \leftarrow L_3 + \cancel{\tfrac{1}{15}}\,\tfrac{2}{33} L_2$$

$$A \overset{L_3 \leftarrow L_3 + \frac{2}{33} L_2}{\sim} \begin{pmatrix} 22 & 15 & 19 \\ 0 & -9/2 & 3/2 \\ 0 & 0 & -\frac{5}{11} \ldots \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ \frac{2}{11} & \cancel{\frac{1}{15}}\ \frac{-2}{33} & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 22 & 15 & 19 \\ 0 & -9/2 & 3/2 \\ 0 & 0 & -\frac{5}{11} \end{pmatrix}$$

Verificam $LU = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ \frac{2}{11} & -\frac{2}{33} & 1 \end{pmatrix} \begin{pmatrix} 22 & 15 & 19 \\ 0 & -9/2 & 3/2 \\ 0 & 0 & -\frac{5}{11} \end{pmatrix}$

$$= \begin{pmatrix} 22 & 15 & 19 \\ 11 & 3 & 11 \\ 4 & 3 & 3 \end{pmatrix} = A'$$

$$\left.\begin{array}{l} b = \begin{pmatrix} 21 \\ 58 \\ 119 \end{pmatrix} \\ w = (3,2,1) \end{array}\right\} \Rightarrow b' = \begin{pmatrix} 119 \\ 58 \\ 21 \end{pmatrix}$$

$$Ly^t = b' \iff \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ \frac{2}{11} & -\frac{2}{33} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 119 \\ 58 \\ 21 \end{pmatrix}$$

$$\Rightarrow \boxed{y_1 = 119}$$

$$\underline{y_2} = 58 - \frac{1}{2} y_1 = 58 - \frac{119}{2} = -\frac{3}{2}$$

$$\underline{y_3} = 21 - \frac{2}{11} \cdot 119 + \frac{2}{33} \cdot \frac{3}{2}$$

$$= 21 - \frac{238}{11} + \frac{1}{11} = \underline{-\frac{9}{11}}$$

$Ux' = y$

$$\begin{pmatrix} 22 & 15 & 19 \\ 0 & -9/2 & 3/2 \\ 0 & 0 & -4/11 \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 119 \\ -3/2 \\ -\frac{9}{11} \end{pmatrix}$$

$$-\frac{4}{11} \cdot x_3' = -\frac{9}{11} \rightarrow \cancel{x_3' = \frac{6}{4} = \frac{3}{2}} \quad x_3' = \frac{9}{4}$$

$$x_2' = \frac{-\frac{3}{2} - \frac{3}{2} \cdot \frac{9}{4}}{-\frac{9}{2}} = \frac{5}{9} - \frac{39}{36}$$

$$x_1' = \frac{119 - 15x_2' - 19x_3'}{22} = 8{,}0909$$

$$p = (3,1,2) \Rightarrow x = \left(\frac{9}{4}, 8{,}0909, -\frac{39}{36}\right) = (2.25, 8.09, -1{,}0833)$$

MANCA CHE YIT

II. $x = (x_1, x_2, x_3, x_4)$ $\quad y = f(x)$

$P_3(x) = ?$

$f(x) = \sin(2x) \qquad x \in \left(0, \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{4}\right)$

$P_3(x) = L_{3,1}(x) y_1 + L_{3,2}(x) y_2 + L_{3,3}(x) y_3 + L_{3,4}(x) y_4$

$y_1 = \sin(0) = 0$

$y_2 = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$y_3 = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$

$y_4 = \sin\left(\frac{\pi}{2}\right) = 1$

$$L_{3,1}(x) = \frac{\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{6}\right)\left(x-\frac{\pi}{4}\right)}{\left(0-\frac{\pi}{12}\right)\left(0-\frac{\pi}{6}\right)\left(0-\frac{\pi}{4}\right)}$$

$$L_{3,2}(x) = \frac{(x-0)\left(x-\frac{\pi}{6}\right)\left(x-\frac{\pi}{4}\right)}{\left(\frac{\pi}{12}-0\right)\left(\frac{\pi}{12}-\frac{\pi}{6}\right)\left(\frac{\pi}{12}-\frac{\pi}{4}\right)}$$

$$L_{3,3}(x) = \frac{(x-0)\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{4}\right)}{\left(\frac{\pi}{6}-0\right)\left(\frac{\pi}{6}-\frac{\pi}{12}\right)\left(\frac{\pi}{6}-\frac{\pi}{4}\right)}$$

$$L_{3,4}(x) = \frac{(x-0)\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{6}\right)}{\left(\frac{\pi}{4}-0\right)\left(\frac{\pi}{4}-\frac{\pi}{12}\right)\left(\frac{\pi}{4}-\frac{\pi}{6}\right)}$$

$$\Rightarrow P_3(x) = \frac{\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{6}\right)\left(x-\frac{\pi}{4}\right)}{\frac{-\pi^3}{288}} \cdot 0 + \frac{(x-0)\left(x-\frac{\pi}{6}\right)\left(x-\frac{\pi}{4}\right)}{\frac{2\pi^3}{12^2}} \cdot \frac{1}{2}$$

$$+ \frac{(x-0)\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{6}\right)}{\frac{-\cancel{1}\pi^3}{\cancel{6}\cdot 12^2}} \cdot \frac{\sqrt{2}}{2} + \frac{(x-\cancel{0})\left(x-\frac{\pi}{12}\right)\left(x-\frac{\pi}{6}\right)}{\frac{\pi^3}{2\cdot 12^2}}$$

MANOLACHE ANDREI VII

III. $f(x) = \sin(2x)$ $\quad x \in \left(0, \frac{\pi}{12}, \frac{\pi}{6}\right)$

$$S(x) = \begin{cases} S_1(x) = a_1 \cancel{x}\ b_1 x + c_1 x^2 + d_1 x^3 & x \in \left[0, \frac{\pi}{12}\right) \\ S_2(x) = a_2 + b_2\left(x-\frac{\pi}{12}\right) + c_2\left(x-\frac{\pi}{12}\right)^2 + d_2\left(x-\frac{\pi}{12}\right)^3 & x \in \left[\frac{\pi}{12}, \frac{\pi}{6}\right] \end{cases}$$

1) S interpolată în nodurile x_1, x_2, x_3

$S_1(x_1) = S(x_1) = f(x_1) = f(0) = \boxed{a_1 = \sin 0 = 0}$

$S_2(x_2) = S(x_2) = f(x_2) = f\left(\frac{\pi}{12}\right) = \boxed{\frac{1}{2} = a_2}$

$$S(x_3) = S_2(x_3) = f(x_3) = f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \quad a_2 + b_2 \frac{\pi}{12} + c_2 \cdot \left(\frac{\pi}{12}\right)^2 + d_2 \left(\frac{\pi}{12}\right)^3 = \frac{\sqrt{3}}{2}$$

$$\boxed{b_2 \frac{\pi}{12} + c_2 \left(\frac{\pi}{12}\right)^2 + d_2 \left(\frac{\pi}{12}\right)^3 = \frac{\sqrt{3}}{2} - \frac{1}{2}}$$

2) S cont în nodurile interioare $\left(x_2 = \frac{\pi}{12}\right)$

$$S_1(x_2) = S_2(x_2) \Rightarrow S_1\left(\frac{\pi}{12}\right) = S_2\left(\frac{\pi}{12}\right)$$

$$\boxed{a_1 + b_1 \cdot \frac{\pi}{12} + c_1 \cdot \left(\frac{\pi}{12}\right)^2 + d_1 \left(\frac{\pi}{12}\right)^3 = \frac{1}{2}}$$

3) S' cont. în nodurile interioare $\left(x_2 = \frac{\pi}{12}\right)$

$$S'(x) = \begin{cases} b_1 + 2c_1 x + 3d_1 x^2 & x \in \left[0, \frac{\pi}{12}\right) \\ b_2 + 2c_2 \left(x - \frac{\pi}{12}\right) + 3d_2 \left(x - \frac{\pi}{12}\right)^2 & x \in \left[\frac{\pi}{12}, \frac{\pi}{6}\right] \end{cases}$$

$$S_1'(x_2) = S_2'(x_2) \Leftrightarrow S_1'\left(\frac{\pi}{12}\right) = S_2'\left(\frac{\pi}{12}\right)$$

$$\Leftrightarrow \boxed{b_1 + 2c_1 \cdot \frac{\pi}{12} + 3d_1 \cdot \left(\frac{\pi}{12}\right)^2 = b_2}$$

4) S'' cont. în nodurile interioare $\left(x_2 = \frac{\pi}{12}\right)$

$$S''(x) = \begin{cases} S_1''(x) = 2c_1 + 6d_1 x & x \in \left[0, \frac{\pi}{12}\right) \\ S_2''(x) = 2c_2 + 6d_2 \left(x - \frac{\pi}{12}\right) & x \in \left[\frac{\pi}{12}, \frac{\pi}{6}\right] \end{cases}$$

$$S_1''\left(\frac{\pi}{12}\right) = S_2''\left(\frac{\pi}{12}\right)$$

$$\boxed{2c_1 + 6d_1 \cdot \frac{\pi}{12} = 2c_2}$$

5) Considerăm

$$\begin{cases} S''(x_1) = f''(x_1) \\ S''(x_3) = f''(x_3) \end{cases}$$

$f(x) = \sin 2x$

$f'(x) = 2 \cdot \cos 2x$

$f''(x) = -4 \sin 2x$

$S''(0) = S_1''(0) = f''(0)$

$\Leftrightarrow \quad 2c_1 = 0 \quad \Rightarrow \boxed{c_1 = 0}$

$S''\left(\frac{\pi}{6}\right) = S_2''\left(\frac{\pi}{6}\right) = f''\left(\frac{\pi}{6}\right)$

$2c_2 + 6d_2 \frac{\pi}{12} = -4 \sin \frac{\pi}{3}$

$$\boxed{2c_2 + 6d_2 \cdot \frac{\pi}{12} = -4 \cdot \frac{\sqrt{3}}{2}}$$

$$\begin{cases} a_1 + b_1 \cdot \frac{\pi}{12} + c_1 \left(\frac{\pi}{12}\right)^2 + d_1 \cdot \left(\frac{\pi}{12}\right)^3 = \frac{1}{2} \\ b_2 \cdot \frac{\pi}{12} + c_2 \cdot \left(\frac{\pi}{12}\right)^2 + d_2 \left(\frac{\pi}{12}\right)^3 = \frac{\sqrt{3}-1}{2} \\ b_1 + 2c_1 \frac{\pi}{12} + 3d_1 \left(\frac{\pi}{12}\right)^2 = b_2 \\ 2c_2 + 6d_2 \cdot \frac{\pi}{12} = -2\sqrt{3} \end{cases}$$

Rezolvăm sistemul

$$\begin{cases} b_1 \cdot \frac{\pi}{12} + d_1 \left(\frac{\pi}{12}\right)^3 = \frac{1}{2} \\ b_2 \cdot \frac{\pi}{12} + c_2 \cdot \left(\frac{\pi}{12}\right)^2 + d_2 \left(\frac{\pi}{12}\right)^3 = \frac{\sqrt{3}-1}{2} \\ b_1 + 3d_1 \cdot \left(\frac{\pi}{12}\right)^2 = b_2 \end{cases}$$

$$2c_2 + 6d_2 \cdot \frac{\pi}{12} = -2\sqrt{3} \iff c_2 + 3d_2 \cdot \frac{\pi}{12} = -\sqrt{3}$$

$$2\overset{0}{c_1} + 6d_1 \cdot \frac{\pi}{12} = 2c_2 \iff 3d_1 \frac{\pi}{12} = c_2$$

Rezolvăm sistemul

$$\Rightarrow \left[b_1 + 3d_1 \left(\frac{\pi}{12}\right)^2\right] \frac{\pi}{12} + c_2 \cdot \left(\frac{\pi}{12}\right)^2 + d_2 \left(\frac{\pi}{12}\right)^3 = \frac{\sqrt{3}-1}{2}$$

$$\begin{cases} b_1 + 3d_1 \left(\frac{\pi}{12}\right)^2 = b_2 \quad / \cdot \frac{\pi}{12} \\ b_1 \frac{\pi}{12} + d_1 \left(\frac{\pi}{12}\right)^3 = \frac{1}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} b_1 \frac{\pi}{12} + 3d_1 \left(\frac{\pi}{12}\right)^3 = b_2 \cdot \frac{\pi}{12} \\ b_1 \frac{\pi}{12} + d_1 \left(\frac{\pi}{12}\right)^3 = \frac{1}{2} \end{cases}$$

$$\ominus \quad 2d_1 \left(\frac{\pi}{12}\right)^3 = b_2 \cdot \frac{\pi}{12} - \frac{1}{2} \quad / \cdot 12$$

$$\frac{1}{6} d_1 \left(\frac{\pi}{12}\right)^3 = b_2 - 6$$

$$\begin{cases} c_2 + 3d_2 \cdot \frac{\pi}{12} = -\sqrt{3} \\ 3d_1 \cdot \frac{\pi}{12} = c_2 \end{cases} \Leftrightarrow \begin{cases} 3d_1 \frac{\pi}{12} + 3d_2 \cdot \frac{\pi}{12} = -\sqrt{3} \quad / \cdot \left(\frac{\pi}{12}\right)^{-1} \\ 3d_1 + 3d_2 = \frac{-\sqrt{3} \cdot 12}{\pi} \end{cases}$$

$$d_1 = \frac{\frac{-12\sqrt{3}}{\pi} - 3d_2}{3}$$

De rezolvat sistemul în continuare

MANOLACHE ANDREI VIT

IV. $\int_0^2 f(x)\,dx = 5$ (cu formula trapezului)

$\int_0^2 f(x)\,dx = 4$ (cu formula dreptunghiului)

Formula trapezului

$\int_0^2 f(x)\,dx = 5$

$x_1 = a = 0$
$x_2 = b = 2$
$h = 2-0 = 2$

$I_1(f) = w_1 f(x_1) + w_2 f(x_2) = w_1 f(a) + w_2 f(b)$

$w_1 = h\int_1^2 \frac{t-2}{-1}\,dt = \frac{h}{2} = 1$

$w_2 = h\int_1^2 (t-1)\,dt = \frac{h}{2} = 1$

$I_1(f) = f(a) + f(b) = 5 \iff \boxed{f(0) + f(2) = 5}$

Formula dreptunghiului

$\int_0^2 f(x)\,dx = 4$

$x_0 := a = 0$
$x_2 := b = 2$
$x_1 := \frac{a+b}{2} = 1$
$h = \frac{b-a}{2} = 1$

$I_0(f) = w_1 f(x_1) = w_1 f\left(\frac{a+b}{2}\right) = w_1 f(1) = 4$

$w_1 = \int_0^2 L_{0,1}(x)\,dx = b-a = 2$ $\Big\} \Rightarrow$

10/

$\Rightarrow I_0(f) = 2f(1) = 4 \rightarrow \boxed{f(1)=2}$

<u>Formula Simpson</u>

$x_1 = a = 0$

$x_2 = \frac{a+b}{2} = 1$

$x_3 = b = 2$

$h = \frac{b-a}{2} = 1$

$$I_2(f) = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)$$
$$= w_1 f(0) + w_2 f(1) + w_3 f(2)$$

$w_1 = h\int_1^3 \frac{1}{2}(t-2)(t-3)\,dt = \frac{h}{3} = \frac{1}{3}$

$w_2 = h\int_1^3 -(t-1)(t-3)\,dt = \frac{4h}{3} = \frac{4}{3}$

$w_3 = h\int_1^3 \frac{1}{2}(t-1)(t-2)\,dt = \frac{h}{3} = \frac{1}{3}$

$$I_2(f) = \frac{1}{3}\cdot f(0) + \frac{4}{3}\cdot 2 + \frac{1}{3} f(2)$$
$$= \frac{1}{3}(\underbrace{f(0)+f(2)}_{5}) + \frac{8}{3} = \frac{5}{3} + \frac{8}{3} = \frac{13}{3}$$

Valoarea integralei, folosind formula Simpson este $\frac{13}{3}$

MANOLACHE ANDREI VI7

V. $\varphi(h)$ $\quad f'(x_0) = \varphi(h) + a_2 h^2 + a_3 h^3 + a_4 h^4 + \dots \qquad (1)$

Ec(1) are loc pt $\forall\, h>0 \quad h \to h/4$

$$4^2 \cdot /\ f'(x_0) = \varphi(h/4) + a_2\left(\frac{h}{4}\right)^2 + a_3\left(\frac{h}{4}\right)^3 + a_4\left(\frac{h}{4}\right)^4 + \dots$$

$$4^2 f'(x_0) = 4^2 \varphi\left(\frac{h}{4}\right) + a_2 h^2 + a_3 \frac{h^3}{4} + a_4 \frac{h^4}{4^2} + \dots$$

$$f'(x_0) = \varphi(h) + a_2 h^2 + a_3 h^3 + a_4 h^4 + \dots$$

$(-)$

$$\frac{1}{4^2-1} : /\ (4^2-1) f'(x_0) = 4^2 \varphi\left(\frac{h}{4}\right) - \varphi(h) + a_3 h^3\left(\frac{1}{4}-1\right) + a_4 h^4\left(\frac{1}{4^2}-1\right) + \dots$$

$$\varphi_2(h) := \frac{1}{4^2-1}\left[4^2 \varphi\left(\frac{h}{4}\right) - \varphi(h)\right]$$

$$(2) \quad f'(x_0) = \varphi_2(h) + b_3 h^3 + \cancel{b_3}\, b_4 h^4 + \dots$$

EC (2) valabilă pt $\forall\, h>0$

$h \to h/4$

$$4^3 /\ f'(x_0) = \varphi_2\left(\frac{h}{4}\right) + b_3\left(\frac{h}{4}\right)^3 + b_4\left(\frac{h}{4}\right)^4 + \dots$$

$$4^3 f'(x_0) = 4^3 \varphi_2\left(\frac{h}{4}\right) + b_3 h^3 + b_4 \frac{h^4}{4} + \dots$$

$$f'(x_0) = \varphi_2(h) + b_3 h^3 + b_4 h^4 + \dots$$

$(-)$

$$\frac{1}{4^3-1} : /\ (4^3-1) f'(x_0) = 4^3 \varphi_2\left(\frac{h}{4}\right) - \varphi_2(h) + b_4\left(\frac{1}{4}-1\right) h^4 + \dots$$

$$\varphi_3(h) := \frac{1}{4^3-1}\left[\varphi_2\left(\frac{h}{4}\right) - \varphi_2(h)\right]$$

$$f'(x_0) = \varphi_3(h) + c_4 h^4 + c_5 h^5 + \dots$$
$$= \varphi_3(h) + O(h^4)$$

$$\varphi(x_0) = \varphi_3(h) + O(h^4)$$

$$= \frac{1}{4^3-1}\left[\varphi_2\left(\frac{h}{4}\right) - \varphi_2(h)\right]$$

$$= \frac{1}{4^3-1}\left[\frac{1}{4^2-1}\left[4^2\varphi\left(\frac{h}{16}\right) - \varphi\left(\frac{h}{4}\right)\right] - \frac{1}{4^2-1}\left[4^2\varphi\left(\frac{h}{4}\right) - \varphi(h)\right]\right]$$

$$= \frac{1}{63}\left[\frac{1}{15}\left[16\varphi\left(\frac{h}{16}\right) - \varphi\left(\frac{h}{4}\right)\right] - \frac{1}{15}\left[16\varphi\left(\frac{h}{4}\right) - \varphi(h)\right]\right]$$

$$= \frac{1}{63}\left[\frac{16}{15}\varphi\left(\frac{h}{16}\right) - \frac{\varphi\left(\frac{h}{4}\right)}{15} - \frac{16}{15}\varphi\left(\frac{h}{4}\right) + \frac{1}{15}\varphi(h)\right]$$

$$= \frac{1}{63}\left[\frac{16}{15}\varphi\left(\frac{h}{16}\right) - \frac{17}{15}\varphi\left(\frac{h}{4}\right) + \frac{1}{15}\varphi(h)\right]$$

$$= \frac{16}{63\cdot 15}\varphi\left(\frac{h}{16}\right) - \frac{17}{63\cdot 15}\varphi\left(\frac{h}{4}\right) + \frac{1}{63\cdot 15}\varphi(h)$$