Consultații EADP, seriile 33 și 34, 02.02.2021, 12^{00}

- examen (quiz): 13 – 15:15, 03.02.2021

  2 ore

- într-un assignment pe MOODLE, separat de quiz, încărcați o copie de CI și legitimație student, în format pdf, jpg, png, între 12:30 – 16:00, 03.02.2021

• Punctaj maxim lucrare = 80 de puncte (10–16 exerciții).

.... : x din 80

• $\text{NOTA} = \dfrac{\text{punctaj lucrare} + \text{punctaj seminar} + 10 \text{ din oficiu}}{10}$.

• minim 45 de puncte pt. nota de trecere.

• seminar 13, 341:

(1) c) sistemul se poate scrie și așa:

$$\begin{pmatrix} x_1' \\ x_2' \\ p_1' \\ p_2' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 0 & 3 & 0 \\ 0 & -2 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ p_1 \\ p_2 \end{pmatrix}$$

$$\begin{pmatrix} x_1' \\ p_1' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ p_1 \end{pmatrix}$$

$$\begin{pmatrix} x_2' \\ p_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x_2 \\ p_2 \end{pmatrix}$$

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(2) a) $(\partial_1 u)^2 - 2(\partial_1 u)(\partial_2 u) + 2(\partial_2 u)^2 - 4u = 0$.

sist. caract.:

$$\begin{cases} \dfrac{dx_1}{dt} = 2p_1 - 2p_2 \\ \dfrac{dx_2}{dt} = -2p_1 + 4p_2 \\ \dfrac{dp_1}{dt} = -0 - p_1(-4) \\ \dfrac{dp_2}{dt} = -0 - p_2(-4) \end{cases}$$

$\dfrac{du}{dt} = p_1 \cdot (2p_1 - 2p_2) + p_2(-2p_1 + 4p_2)$

$\dfrac{du}{dt} = 2p_1^2 - 4p_1p_2 + 4p_2^2$.

$x_1(0) =$

⋮

$u(0) =$

.....

$$! \begin{cases} 1) \ p_1' = 4p_1 \\ 2) \ p_2' = 4p_2 \end{cases} \longrightarrow p_1 = C_1 e^{4t}$$

ec. liniară în p_1: $\boxed{\frac{dp_1}{dt} = a(t) \cdot p_1}$

$a(t) = 4.$

dar $p_1(0) = \dots$ $\Rightarrow$ $p_1(t,1) =$ ...

$\Downarrow$

$$\left. \begin{array}{l} p_2 = C_2 e^{4t} \\ \text{dar } p_2(0) = \dots \end{array} \right\} \Rightarrow p_2(t,1) =$$

3) $x_1' = 2p_1 - 2p_2$ cu p_1, p_2 obtinute mai sus $\Big\} \Rightarrow$

$\Rightarrow x_1' = \underbrace{f_1(t,s)}_{2C_1 e^{4t} - 2C_2 e^{4t}}$ ec. de tip primitivă

4) $x_2' = \dots$

5) $u' =$

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③ Forma canonică a ec:

$$\partial_1^2 u + 2\partial_1\partial_2 u - 3\partial_2^2 + \underbrace{\partial_1 u + \partial_2 u} = 0.$$

I) • metoda Gauss: în $\mathbb{R}^2$

$$g(t_1, t_2) = t_1^2 + 2t_1 t_2 - 3t_2^2$$

$$\| $$

$$(t_1^2 + 2t_1 t_2) - 3t_2^2 =$$

$$= (t_1^2 + 2t_1 t_2 + t_2^2) - t_2^2 - 3t_2^2 =$$

$$= (t_1 + t_2)^2 - 4t_2^2 \Rightarrow g(t_1, t_2) = (t_1 + t_2)^2 - (2t_2)^2$$

$\Rightarrow$ ec. de tip hiperbolic: $r = n = 2$

$0 < m = 1 < 2$

$$\begin{cases} s_1 = t_1 + t_2 \\ s_2 = 2t_2 \end{cases} \Rightarrow \tilde{g}(s_1, s_2) = s_1^2 - s_2^2$$

$$\text{II)} \begin{cases} a = 1 \\ b = \frac{2}{2} = 1 \\ c = -3. \end{cases} \Rightarrow d = b^2 - ac = 1 - 1\cdot(-3) = 4 > 0 \Rightarrow$$

$\Rightarrow$ ec. de tip hiperbolic.

Calc: $\begin{cases} \lambda_1 = \dfrac{b-\sqrt{d}}{a} = \dfrac{1-2}{1} = -1 \\ \lambda_2 = \dfrac{b+\sqrt{d}}{a} = \dfrac{1+2}{1} = 3. \end{cases}$

Integram ec: $\left[\begin{array}{l} \dfrac{dx_2}{dx_1} = -1 \Rightarrow dx_2 = -dx_1 \Rightarrow x_2 = -x_1 + C_1 \\ \qquad \text{(variabile separate)} \\ \dfrac{dx_2}{dx_1} = 3 \Rightarrow dx_2 = 3dx_1 \Rightarrow x_2 = 3x_1 + C_2 \end{array} \right.$

$\Rightarrow \begin{cases} \underbrace{x_1 + x_2}_{\varphi_1} = C_1 \\ \underbrace{-3x_1 + x_2}_{\varphi_2} = C_2 \end{cases} \Rightarrow \underbrace{\begin{cases} y_1 = x_1 + x_2 \\ y_2 = -3x_1 + x_2 \end{cases}}$

transf. de coordonate prin care functia u devine $\tilde{u}(y_1, y_2)$,
$u(x_1, x_2)$

adica: $u(x_1, x_2) = \tilde{u}(y_1(x), y_2(x))$

Calc: $\partial_1 u = \dfrac{\partial u}{\partial x_1} = \dfrac{\partial}{\partial x_1}\left(\tilde{u}(y_1(x_1,x_2), y_2(x_1,x_2))\right) =$

$= \dfrac{\partial \tilde{u}}{\partial y_1} \cdot \underbrace{\dfrac{\partial y_1}{\partial x_1}}_{1} + \dfrac{\partial \tilde{u}}{\partial y_2} \cdot \underbrace{\dfrac{\partial y_2}{\partial x_1}}_{-3} \Rightarrow$

$\Rightarrow \boxed{\dfrac{\partial u}{\partial x_1} = \dfrac{\partial \tilde{u}}{\partial y_1} - 3\dfrac{\partial \tilde{u}}{\partial y_2}}$

$\partial_2 u = \dfrac{\partial u}{\partial x_2} = \dfrac{\partial \tilde{u}}{\partial y_1} \cdot \underbrace{\dfrac{\partial y_1}{\partial x_2}}_{1} + \dfrac{\partial \tilde{u}}{\partial y_2} \cdot \underbrace{\dfrac{\partial y_2}{\partial x_2}}_{1} \Rightarrow$

$\Rightarrow \boxed{\dfrac{\partial u}{\partial x_2} = \dfrac{\partial \tilde{u}}{\partial y_1} + \dfrac{\partial \tilde{u}}{\partial y_2}}$

$\partial_1^2 u = \dfrac{\partial}{\partial x_1}\left(\dfrac{\partial u}{\partial x_1}\right) = \dfrac{\partial}{\partial x_1}\left(\underbrace{\dfrac{\partial \tilde{u}}{\partial y_1} - 3\dfrac{\partial \tilde{u}}{\partial y_2}}_{v}\right) = \dfrac{\partial v}{\partial x_1} =$

$= \dfrac{\partial v}{\partial y_1} \cdot \underbrace{\dfrac{\partial y_1}{\partial x_1}}_{=1} + \dfrac{\partial v}{\partial y_2} \cdot \underbrace{\dfrac{\partial y_2}{\partial x_1}}_{=-3} \Rightarrow$

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$$\Rightarrow \partial_1^2 u = \frac{\partial}{\partial y_1}\left(\frac{\partial \tilde u}{\partial y_1} - 3\frac{\partial \tilde u}{\partial y_2}\right)\cdot 1 + \frac{\partial}{\partial y_2}\left(\frac{\partial \tilde u}{\partial y_1} - 3\frac{\partial \tilde u}{\partial y_2}\right)\cdot(-3) \Rightarrow$$

$$\Rightarrow \boxed{\partial_1^2 u = \frac{\partial^2 \tilde u}{\partial y_1^2} - 6\frac{\partial^2 \tilde u}{\partial y_1 \partial y_2} + 9\frac{\partial^2 u}{\partial y_2^2}}$$

$$\partial_1\partial_2 u = \frac{\partial}{\partial x_1}\left(\frac{\partial u}{\partial x_2}\right) = \frac{\partial}{\partial x_1}\left(\frac{\partial \tilde u}{\partial y_1} + \frac{\partial \tilde u}{\partial y_2}\right) =$$

$$= \frac{\partial}{\partial y_1}\left(\frac{\partial \tilde u}{\partial y_1} + \frac{\partial \tilde u}{\partial y_2}\right)\underbrace{\frac{\partial y_1}{\partial x_1}}_{1} + \frac{\partial}{\partial y_2}\left(\frac{\partial \tilde u}{\partial y_1} + \frac{\partial \tilde u}{\partial y_2}\right)\cdot\underbrace{\frac{\partial y_2}{\partial x_1}}_{-3}$$

$$\Rightarrow \boxed{\partial_1\partial_2 u = \frac{\partial^2 \tilde u}{\partial y_1^2} - 2\frac{\partial^2 \tilde u}{\partial y_1 \partial y_2} - 3\frac{\partial^2 \tilde u}{\partial y_2^2}}$$

$$\partial_2^2 u = \frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_2}\right) = \frac{\partial}{\partial x_2}\left(\frac{\partial \tilde u}{\partial y_1} + \frac{\partial \tilde u}{\partial y_2}\right) =$$

$$= \frac{\partial}{\partial y_1}\left(\frac{\partial \tilde u}{\partial y_1} + \frac{\partial \tilde u}{\partial y_2}\right)\cdot\underbrace{\frac{\partial y_1}{\partial x_2}}_{1} + \frac{\partial}{\partial y_2}\left(\frac{\partial \tilde u}{\partial y_1} + \frac{\partial \tilde u}{\partial y_2}\right)\cdot\underbrace{\frac{\partial y_2}{\partial x_2}}_{1} \Rightarrow$$

$$\Rightarrow \boxed{\partial_2^2 u = \frac{\partial^2 \tilde u}{\partial y_1^2} + 2\frac{\partial^2 \tilde u}{\partial y_1 \partial y_2} + \frac{\partial^2 \tilde u}{\partial y_2^2}}$$

Ec. derive (ec. $\underset{y_1, y_2}{\text{în}}$ $\tilde u$):

$$\underbrace{\cancel{\frac{\partial^2 \tilde u}{\partial y_1^2}} - 6\frac{\partial^2 \tilde u}{\partial y_1 \partial y_2} + 9\cancel{\frac{\partial^2 u}{\partial y_2^2}}} + \underbrace{2\cancel{\frac{\partial^2 \tilde u}{\partial y_1^2}} - 4\frac{\partial^2 \tilde u}{\partial y_1 \partial y_2} - 6\cancel{\frac{\partial^2 \tilde u}{\partial y_2^2}}} -$$

$$\underbrace{-3\cancel{\frac{\partial^2 \tilde u}{\partial y_1^2}} - 6\frac{\partial^2 \tilde u}{\partial y_1 \partial y_2} - 3\cancel{\frac{\partial^2 \tilde u}{\partial y_2^2}}} + \frac{\partial \tilde u}{\partial y_1} - 3\frac{\partial \tilde u}{\partial y_2} +$$

$$+ \frac{\partial \tilde u}{\partial y_1} + \frac{\partial \tilde u}{\partial y_2} = 0. \Rightarrow$$

$$\Rightarrow -16\frac{\partial^2 \tilde u}{\partial y_1 \partial y_2} + 2\frac{\partial \tilde u}{\partial y_1} - 2\frac{\partial \tilde u}{\partial y_2} = 0 \ \Big|\cdot(-16) \Rightarrow$$

$$\Rightarrow \left|\ \frac{\partial^2 \tilde u}{\partial y_1 \partial y_2} - \frac{1}{8}\frac{\partial \tilde u}{\partial y_1} + \frac{1}{8}\frac{\partial \tilde u}{\partial y_2} = 0\ \right|$$

...

Obs: dacă s-ar fi redus derivatele de ordin 1, atunci ec. de tip hiperbolic se poate scrie:

$$\frac{\partial^2 \tilde{u}}{\partial y_1 \partial y_2} = 0,$$

iar în forma asta se poate integra:

$$\frac{\partial}{\partial y_1}\left(\frac{\partial \tilde{u}}{\partial y_2}\right) = 0 \Rightarrow \frac{\partial \tilde{u}}{\partial y_2} = f(y_2)$$

$$\Rightarrow \tilde{u}(y_1, y_2) = F(y_2) + C(y_1) \Rightarrow$$

$$\Rightarrow \boxed{u(x_1, x_2) = F(-3x_1 + x_2) + C(x_1 + x_2)}$$

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④ $\begin{cases} 2x_2\, \partial_1 u + (x_1 + x_2)\, \partial_2 u = x_1^2 \\ u(x_1, x_2) = \underbrace{\frac{7 x_1 x_2}{4}}_{u_0(x_1, x_2)} \quad \text{pe } S = \{x \in \mathbb{R}^2 \mid x_1 = 4x_2\} \end{cases}$

$\begin{cases} x_1 = \varphi_1(s) = 4s \\ x_2 = \varphi_2(s) = s. \end{cases}$

$$\begin{cases} \frac{dx_1}{dt} = 2x_2 \\ \frac{dx_2}{dt} = x_1 + x_2 \\ \frac{du}{dt} = x_1^2 \\ x_1(0) = 4s \\ x_2(0) = s. \\ u(0) = \frac{7}{4} \cdot 4s \cdot s = 7s^2 \end{cases}$$

Dacă: $x_1' = 2x_1$
$\Downarrow$
$x_1 = C_1 e^{2t}$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow \underline{x_1'' = \overset{tr(A)}{1} \cdot x_1' - \overset{\det A}{(-2)}\, x_1}$$

$$r^2 = r + 2 \Rightarrow \begin{vmatrix} r_1 = -1 \\ r_2 = 2 \end{vmatrix} \Rightarrow \boxed{x_1(t) = C_1 e^{-t} + C_2 e^{2t}}$$

$\Downarrow$

$\varphi_1(t) = e^{-t}$

$\varphi_2(t) = e^{2t}$

Pt. x_2 : îl aflam din prima ec. (adică, ec. în care avem x_1')

$$x_1' = 2x_2 \Rightarrow x_2 = \frac{1}{2}x_1' = \frac{1}{2}\left(c_1e^{-t} + c_2e^{2t}\right)' \Rightarrow$$

$$\Rightarrow \boxed{x_2(t) = -\frac{1}{2}c_1e^{-t} + c_2e^{2t}}$$

c_1 și c_2 se determină din $\left.\begin{array}{l} x_1(0) = 4s \\ x_2(0) = s \end{array}\right| \Rightarrow \begin{cases} c_1 + c_2 = 4s \\ -\frac{1}{2}c_1 + c_2 = s \end{cases}$

$$\frac{3}{2}c_1 \;/\; = 3s \Rightarrow \boxed{c_1 = 2s} \Rightarrow \boxed{c_2 = 2s}$$

$$\Rightarrow \boxed{\begin{array}{l} x_1(t,s) = 2s\left(e^{-t} + e^{2t}\right) \\ x_2(t,s) = s\left(-e^{-t} + 2e^{2t}\right) \end{array}}$$

mai departe se află u $\Rightarrow$ soluția parametrică.

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(5) $(x')^2 - 2xx'' + 1 = 0$

$$x^2 + (x')^2 - 2xx'' = 0.$$

$$\begin{cases} x(1) = 0 \\ x'(1) = 0 \end{cases}$$

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(6) $\varphi_0(t) = \alpha$, $\alpha \in \mathbb{R}$

$\left.\begin{array}{l} x' = -\frac{1}{t}x^2 + \frac{4}{t}x - \frac{3}{t} \\ \varphi_0'(t) = 0 \qquad t > 0 \end{array}\right| \Rightarrow 0 = -\frac{1}{t}\alpha^2 + \frac{4}{t}\alpha - \frac{3}{t} \quad | \cdot t$

$$\Rightarrow -\alpha^2 + 4\alpha - 3 = 0$$

$$\alpha_1 = 1 \qquad \alpha_2 = 3. \Rightarrow$$

$$\Rightarrow \varphi_0(t) = 1 \text{ sau } \varphi_0(t) = 3.$$

$x' = \frac{1}{t}\left(-x^2 + 4x - 3\right)$ ec. cu var. sep.

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(7) $\begin{cases} x_1' = x_2 \\ x_2' = -x_1 - 2x_1^3 \end{cases}$

a) $F(t, x_1, x_2) = x_1^4 + x_1^2 + x_2^2$.
e integrală primă pt. sistem.

b) $\underbrace{C_1 = x_1^4 + x_1^2 + x_2^2}$ $\Rightarrow$ $x_2^2 = C_1 - x_1^4 - x_1^2$ $\Rightarrow$

$\Rightarrow x_2 = \pm\sqrt{C_1 - x_1^4 - x_1^2}$ $\Rightarrow$

$\Rightarrow$ $\underbrace{\frac{dx_1}{dt} = \sqrt{C_1 - x_1^4 - x_1^2}}$ $\Rightarrow$ $\underline{x_1(t)}$

sau

$\underbrace{\frac{dx_1}{dt} = -\sqrt{C_1 - x_1^4 - x_1^2}}$
