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344

Examen

1. a) $P = (l_0, l_1, l_2, l_3)$

$$l_0 = (8,5)$$
$$l_1 = (6,1)$$
$$l_2 = (a,b) = (2,5)$$
$$l_3 = (c,d) = (4,7)$$

Alegem $\begin{cases} a=2 \\ b=5 \end{cases}$, $\begin{cases} c=4 \\ d=7 \end{cases}$

P.t schema de Casteljau aplicăm recurența:

$$l_i^m(t) = (1-t)\, l_i^{m-1}(t) + t\, l_{i+1}^{m-1}(t)$$

$l_0^0 = l_0$

$l_1^0 = l_1 \rightarrow l_0^1 = (7,3)$

$l_2^0 = l_2 \rightarrow l_1^1 = (4,3) \rightarrow l_0^2\left(\frac{11}{2}, 3\right)$

$l_3^0 = l_3 \rightarrow l_2^1 = (6,6) \rightarrow l_1^2\left(5, \frac{9}{2}\right) \rightarrow l_0^3\left(\frac{21}{4}, \frac{15}{4}\right)$

b) Forma Bernstein:

$$l_0^2(t) = \sum_{k=0}^{2} B_k^2(t) \cdot l_k \qquad ; B_k^n(t) = C_n^k (1-t)^{n-k}\cdot t^k$$

$$B_0^2(t) = C_2^0 (1-t)^2 t^0 = (1-t)^2$$
$$B_1^2(t) = C_2^1 (1-t)t = 2(1-t)t$$
$$B_2^2(t) = C_2^2 t^2 = t^2$$

$$\Rightarrow l_{0(t)}^2 = B_0^2(t)\cdot l_0 + B_1^2(t)l_1 + B_2^2(t)l_2$$

$$l_0^2\left(\tfrac{1}{2}\right) = \tfrac{1}{4}\cdot(8,5) + 2\left(1-\tfrac{1}{2}\right)\tfrac{1}{2}\cdot(6,1) + \tfrac{1}{2^2}\cdot(2,5)$$

$$= \left(2,\tfrac{5}{4}\right) + \left(3,\tfrac{1}{2}\right) + \left(\tfrac{1}{2},\tfrac{5}{4}\right)$$

$$= \left(\tfrac{11}{2},3\right)$$

2) a) $r = 2$

$\alpha \in (3,5)$

$(a * f)(\alpha) = ?$

Fie $\alpha = 3,2$.

$$(a * f)(\alpha) = \sum_{j \in \mathbb{Z}} a[j]\, f(\alpha - j)$$

Cum a semnal discret, va avea valori doar în punctele $j \in \mathbb{Z}$. Cum f filtru continuu de rază $r = 2$, doar pe intervalul $(-2, 2)$ va avea valori nenule. Deci, termenii din sumă au valori nenule doar pt $j \in \mathbb{Z}$ și $\alpha - j \in (-2, 2)$

$$\alpha = 3,2 \Rightarrow 3,2 - j \in (-2,2) \Rightarrow j \in \{2; 3; 4; 5\}$$

~~Cu alte cuvinte, valoarea obținută depinde și are sens doar în punctele $x \in \mathbb{Z}$ date de mărginirea de rază $r = 2$~~

$$(a * f)(3,2) = a[2]\, f(1,2) + a[3]\, f(0,2) + a[4]\, f(-0,8) + \text{~~a[5]f(-1,8)~~}$$
$$+ a[5]\, f(-1,8)$$

Obținem o mediere a valorilor semnalului a din jurul lui $\alpha = 3,2$ folosind filtrul f.

b) Alegem $\begin{cases} \alpha = 20 \\ \beta = 35 \\ \gamma = 40 \end{cases}$

$$(A * f)[i,j] = \sum_{i',j'} A[i',j'] \, f[i-i', j-j']$$

$\Rightarrow \quad 0 \cdot 50 + 1 \cdot 60 + (-1) \cdot 80 + (-1) \cdot 30 + 2 \cdot 80 + 1 \cdot 20 + 0 \cdot 20 + (-2) \cdot 35$
$$+ \; 3 \cdot 40$$

$= \quad 60 - 80 - 30 + 160 + 20 - 70 + 120 = 180$

Valoarea pixelului central în urma aplicării filtrului pe matricea A este 180.

3) $f(x) = (2\cos x + \sin x)^2 = 4\cos^2 x + 4\cos x \cdot \sin x + \sin^2 x$

$= \; 3\cos^2 x + 4 \cdot \cos x \cdot \sin x + (\underset{1}{\underbrace{\cos^2 x + \sin^2 x}})$

$= \; 3\cos^2 x + 2 \cdot 2 \cdot \cos x \cdot \sin x + 1$

$= \; 3\left(\dfrac{1}{2} + \dfrac{1}{2}\cos 2x\right) + 2 \cdot \sin(2x) + 1$

$= \; \dfrac{3}{2} + \dfrac{3}{2}\cos 2x + 2\sin(2x) + 1$

$= \; \dfrac{5}{2} + 3\cos(2x) + 2\sin(2x)$

$\boxed{\begin{array}{l} \cos^2 x = \dfrac{1 + \cos 2x}{2} \\[2mm] \sin 2x = 2 \cdot \sin x \cdot \cos x \end{array}}$

coeficienții $\to$ nenuli $\begin{cases} a_0 = \dfrac{5}{2} \\[2mm] a_2 = 3/2 \\[2mm] b_2 = 2 \end{cases}$

b) $\quad h = a\phi_0^2 + b\phi_1^2 + c\phi_2^2 + d\phi_3^2$

$$\text{Alegem} \begin{cases} a = 2 \\ b = 1 \\ c = 3 \\ d = 2 \end{cases}$$

$$\Rightarrow h = 2\phi_0^2 + \phi_1^2 + 3\phi_2^2 + 2\phi_3^2$$

Reprezentarea grafică :
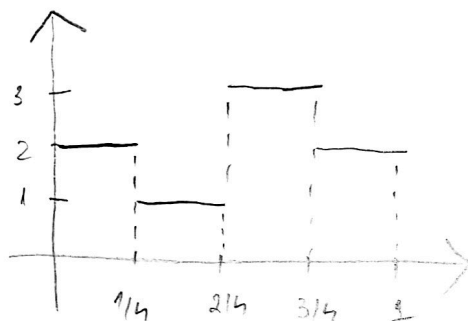
$$h\left(\tfrac{1}{6}\right) + h\left(\tfrac{2}{6}\right) + h\left(\tfrac{4}{6}\right) + h\left(\tfrac{5}{6}\right) = ?$$

$$h\left(\tfrac{1}{6}\right) = 2\phi_0^2\left(\tfrac{1}{6}\right) + \phi_1^2\left(\tfrac{1}{6}\right) + \phi_2^2\left(\tfrac{1}{6}\right) + \phi_3^2\left(\tfrac{1}{6}\right)$$

Cum $\quad 0 \le \tfrac{1}{6} < \tfrac{1}{4}$ și $\phi_1^2, \phi_2^2, \phi_3^2$ nule pe intervale
$\left[0, \tfrac{1}{4}\right) \Rightarrow h\left(\tfrac{1}{6}\right) = 2\phi_0^2\left(\tfrac{1}{6}\right) = 2$

Analog procedăm cu $h\left(\tfrac{2}{6}\right) + h\left(\tfrac{4}{6}\right) + h\left(\tfrac{5}{6}\right)$

$$\tfrac{2}{6} \in \left[\tfrac{1}{4} ; \tfrac{1}{2}\right) \Rightarrow h\left(\tfrac{2}{6}\right) = \phi_1^2\left(\tfrac{2}{6}\right) = 1$$

$$\tfrac{4}{6} \in \left[\tfrac{1}{2} ; \tfrac{3}{4}\right) \Rightarrow h\left(\tfrac{4}{6}\right) = 3\phi_2^2\left(\tfrac{4}{6}\right) = 3$$

$$\tfrac{5}{6} \in \left[\tfrac{3}{4} ; 1\right] \Rightarrow h\left(\tfrac{5}{6}\right) = 2\phi_3^2\left(\tfrac{5}{6}\right) = 2.$$

$$\Rightarrow h\left(\tfrac{1}{6}\right) + h\left(\tfrac{2}{6}\right) + h\left(\tfrac{4}{6}\right) + h\left(\tfrac{5}{6}\right) = 2 + 1 + 3 + 2 = 8.$$

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4)

26	27	26	29	28
27	28	41	42	29
28	42	33	38	28
26	41	42	45	29
28	42	35	37	27
24	27	28	27	26

Aleg $u = 30$. Parcurgem matricea și comparăm fiecare valoare cu cea a pragului (Marcăm cu 1 dacă $>= 30$, 0 altfel)

$\Rightarrow$

0	0	0	0	0
0	0	1	1	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Marcăm cu ● pt 0
○ pt 1

$\Rightarrow$

●	●	●	●	●
●	●	○	○	●
●	○	○	○	●
●	○	○	○	●
●	○	○	○	●
●	●	●	●	●

Luăm o fereastră de 2×2 care glisează și analizează valorile

Obținem diverse cazuri: etc. 5/7

Conturul final obținut este:

b)
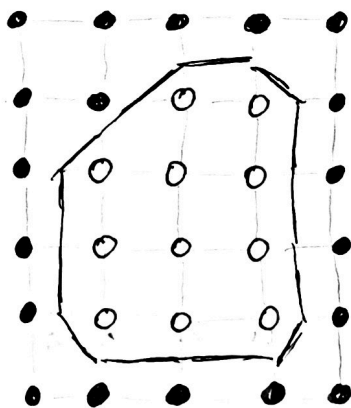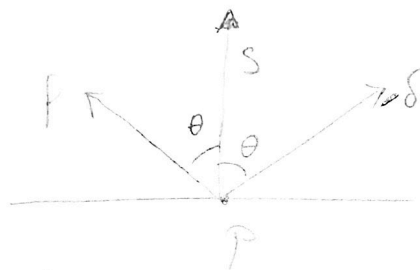
δ - raza incidentă

P - raza reflectată

Fie $\begin{cases} a = 2 \\ b = 1 \\ c = 3 \end{cases}$

Știm că $P - \delta = \omega = 2(-\delta \cdot S)S$

$\underbrace{}_{\text{proiecția ortogonală}}$

Direcția de reflexie:

$$P = \delta - 2(\delta \cdot S)S$$

$S = (0, 0, 1)$

$\delta = (2, 1, 3)$

$\Rightarrow P = (2, 1, 3) - 2 \cdot (2, 1, 3) \cdot (0, 0, 1) \cdot (0, 0, 1)$

$\quad = (2, 1, 3) - 2 \cdot (2 \cdot 0 + 1 \cdot 0 + 3 \cdot 1) \cdot (0, 0, 1)$

$$P = (2,1,3) - 6(0,0,1) = (\underline{\underline{2, 1, -3}})$$