$$V\Big|_{p_3} \propto \frac{1}{\delta} \exp\left(-\frac{(\mu-\sigma^2-\mu)^2}{2\sigma^2}\right) = \frac{1}{\delta}\exp\left(-\frac{\sigma^2}{2}\right) = \exp(\sigma^2-\mu)\exp\left(-\frac{\sigma^2}{2}\right)$$

$$= \frac{\exp\left(\frac{\sigma^2}{2}\right)}{\exp(\mu)}$$

$$V\Big|_{p_4} \propto \frac{1}{\delta}\exp\left(-\frac{\left(\frac{\sigma}{2}\left(\sqrt{\sigma^2+4}-3\sigma\right)\right)^2}{2\sigma^2}\right) = \frac{1}{\delta}\exp\left(-\frac{1}{8}\left(3\sigma-\sqrt{\sigma^2+4}\right)^2\right)$$

$$= \exp\left(-\mu-\frac{\sigma}{2}\left(\sqrt{\sigma^2+4}-3\sigma\right)\right)\exp\left(-\frac{1}{8}\left(3\sigma-\sqrt{\sigma^2+4}\right)^2\right)$$

$$= \frac{1}{\exp(\mu)}\exp\left(\frac{3\sigma^2}{2}-\frac{\sigma}{2}\sqrt{\sigma^2+4}-\frac{5\sigma^2}{4}+\frac{3\sigma}{4}\sqrt{\sigma^2+4}-\frac{1}{2}\right)$$

$$V\Big|_{p_4} = \frac{1}{\exp(\mu)}\exp\left(\frac{\sigma^2+\sigma\sqrt{\sigma^2+4}-2}{4}\right)$$

$$r_{43} = V\Big|_{p_4} \Big/ V\Big|_{p_3} = \exp\left(\frac{\sigma^2+\sigma\sqrt{\sigma^2+4}-2}{4}-\frac{\sigma^2}{2}\right) = \exp\left(\frac{-\sigma^2+\sigma\sqrt{\sigma^2+4}-2}{4}\right)$$

$$2\ln r_{43} = \frac{-\sigma^2+\sigma\sqrt{\sigma^2+4}-2}{2}$$

From WA:

$$\rightarrow \boxed{\sigma^2 = -2-2\ln r_{43}-\frac{1}{2\ln r_{43}}}$$

$$V\Big|_{p_2} = \frac{1}{\delta}\exp\left(-\frac{1}{8}\left(3\sigma+\sqrt{\sigma^2+4}\right)^2\right) = \exp\left(-\mu-\frac{\sigma}{2}\left(-\sqrt{\sigma^2+4}-3\sigma\right)\right)\exp\left(-\frac{1}{8}\left(3\sigma+\sqrt{\sigma^2+4}\right)^2\right)$$

$$= \frac{1}{\exp(\mu)}\exp\left(\frac{3\sigma^2}{2}+\frac{\sigma}{2}\sqrt{\sigma^2+4}-\frac{5}{4}\sigma^2-\frac{3}{4}\sigma\sqrt{\sigma^2+4}-\frac{1}{2}\right)$$

$$V\Big|_{p_2} = \frac{1}{\exp(\mu)}\exp\left(\frac{\sigma^2-\sigma\sqrt{\sigma^2+4}-2}{4}\right)$$

$$r_{32} = \exp\left(\frac{\sigma^2}{2}-\frac{\sigma^2-\sigma\sqrt{\sigma^2+4}-2}{4}\right) = \exp\left(\frac{\sigma^2+\sigma\sqrt{\sigma^2+4}+2}{4}\right)$$

$$\sigma^2 = \frac{(2\ln r_{32}-1)^2}{2\ln r_{32}} = -2+2\ln r_{32}+\frac{1}{2\ln r_{32}}$$

$$\rightarrow \boxed{\sigma^2 = -2-2\ln r_{23}-\frac{1}{2\ln r_{23}}}$$

$$r_{24} = \exp\left(\frac{\sigma^2 - \sigma^2 - \sigma\sqrt{\sigma^2+4} - \sigma\sqrt{\sigma^2+4 \cdot L+2}}{4}\right) = \exp\left(-\frac{\sigma\sqrt{\sigma^2+4}}{2}\right)$$

$$\rightarrow \boxed{\sigma^2 = 2\left(\sqrt{(\ln r_{24})^2+1} - 1\right) = -2+2\sqrt{(\ln r_{24})^2+1}}$$

Next step is find μ:

$$\delta_3 = t_3 - t_0 = e^{\mu} \cdot \exp(-\sigma^2) = e^{\mu} a_3$$

$$\delta_2 = t_2 - t_0 = e^{\mu} \cdot \exp\left(\frac{\sigma}{2}\left(-\sqrt{\sigma^2+4} - 3\sigma\right)\right) = e^{\mu} a_2$$

$$\delta_4 = t_4 - t_0 = e^{\mu} \cdot \exp\left(\frac{\sigma}{2}\left(\sqrt{\sigma^2+4} - 3\sigma\right)\right) = e^{\mu} a_4$$

$$e^{\mu} = \frac{\delta_3 - \delta_2}{a_3 - a_2} = \frac{t_3 - t_2}{a_3 - a_2}, \quad e^{\mu} = \frac{\delta_2 - \delta_4}{a_2 - a_4}, \quad e^{\mu} = \frac{\delta_3 - \delta_4}{a_3 - a_4}$$

Now find t_0: Pick 2 estimates, average them. Then calculate δ vals.

$$t_0 = t_3 - e^{\mu} a_3$$

$$t_0 = t_2 - e^{\mu} a_2$$

$$t_0 = t_4 - e^{\mu} a_4$$

Find D: again, average the 2 estimates.

$$D = V_n \cdot \sigma\sqrt{2\pi}\, \delta_n \cdot \exp\left(\frac{(\ln \delta_n - \mu)^2}{2\sigma^2}\right).$$

Run with all pairs in $n = 2,3,4$. Pick pair that minimizes MSE from $t_1 \to t_5$.

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calculate ϕ_n for all 5 points. Define $\mathrm{erf}_n = \mathrm{erf}\left(\frac{\ln \delta_n - \mu}{\sigma\sqrt{2}}\right)$

Find θ_s, θ_e:

$$\phi_n = \theta_s + \frac{\theta_e - \theta_s}{2}\left(1 + \mathrm{erf}_n\right)$$

$$\Delta\theta = \theta_e - \theta_s = 2\frac{\phi_a - \phi_b}{\mathrm{erf}_a - \mathrm{erf}_b}$$

$$\theta_s = \phi_n - \frac{\Delta\theta}{2}\left(1 + \mathrm{erf}_n\right)$$

$$\theta_e = \theta_s + \Delta\theta$$

Run θ finder with $n = 1, 3$. Select this θ_s. Run θ finder with $n = 3, 5$. Select this θ_e.