Posterior predictions, linear predictions, and expectation of posterior predictions cheat sheet

v1.0 (2022-09-26)   Andrew Heiss (https://www.andrewheiss.com; @andrewheiss)

Normal Gaussian models

Normal or Gaussian models are roughly equivalent to standard frequentist ordinary least squares (OLS) models and are generally interpreted the same way.

All the Bayesian prediction functions return values on the original scale of the outcome. In Gaussian models, the results from `posterior_epred()` and `posterior_linpred()` are identical. The draws from `posterior_predict()` will generally have the same mean as the linear predictor, but with more variance since the overall model σ is incorporated into the predictions.

`posterior_epred()` gives the draws from the expected value of the posterior predictive distribution, or the average of each draw from `posterior_predict()`.

In Gaussian regression, this is the **same as the linear predictor μ**.

$$\mathrm{E}(y_i)$$

$$y_i \sim \text{Normal}(\mu_i, \sigma)$$

$$\mu_i = \alpha + \beta x_i$$

`posterior_predict()` gives the draws from a random normal distribution with draws from the posterior distributions of μ and σ.

`posterior_linpred()` gives the posterior draws of the linear model.

Generalized linear models with link transformations

Generalized linear models (e.g., logistic, probit, ordered logistic, exponential, Poisson, negative binomial, etc.) use special link functions (e.g. logit, log, etc.) to transform the outcome variable into a scale that is more amenable to linear regression.

Estimates from these models can be used in their transformed scales (e.g., log odds in logistic regression) or can be back-transformed into their original scale (e.g., probabilities in logistic regression).

When working with links, the various Bayesian prediction functions return values on different scales, each corresponding to different parts of the model.

`posterior_epred()` gives the draws from the expected value of the posterior predictive distribution, or the average of each draw from `posterior_predict()`.

In logistic regression, this is **π on the probability scale** (or inverse logit).

$$\mathrm{E}(y_i)$$

$$y_i \sim \text{Binomial}(1, \pi_i)$$

$$\text{logit}(\pi_i) = \alpha + \beta x_i$$

`posterior_predict()` gives the draws from a random binomial distribution with draws from the posterior distribution of π.

These are 0s and 1s.

`posterior_linpred()` gives the posterior draws of **π on the logit or log odds scale**.

Distributional models with link transformations

Regression models often focus solely on the location parameter of the model (e.g., μ in Normal(μ, σ); π in Binomial(n, π)). However, it is also possible to specify separate predictors for the scale or shape parameters of models (e.g., σ in Normal(μ, σ), φ in Beta(μ, φ)).

More complex models use a collection of distributional parameters (e.g., zero-inflated beta models estimate a mean μ, precision φ, and a zero-inflated parameter `zi`; hurdle lognormal models estimate a mean μ, scale σ, and a hurdle parameter `hu`).

Even plain old Gaussian models become distributional models when a set of predictors is specified for σ (e.g. `brm(y ~ x1 + x2, sigma ~ x2 + x3)`)

When working with extra distributional parameters (`dpar`), the various Bayesian prediction functions return values on different scales for different components of the model.

`posterior_epred()` gives the draws from the expected value of the posterior predictive distribution, or the average of each draw from `posterior_predict()`.

In beta regression, this is **μ on the proportion or probability scale** (or inverse logit).

`posterior_epred(dpar = "phi")` gives the posterior draws of **φ on the unlogged scale**.

$$\mathrm{E}(y_i)$$

$$y_i \sim \text{Beta}(\mu_i, \phi_i)$$

$$\text{logit}(\mu_i) = \alpha_\mu + \beta_\mu x_i$$

$$\log(\phi_i) = \alpha_\phi + \beta_\phi x_i$$

`posterior_predict()` gives the draws from a random beta distribution with draws from the posterior distribution of π.

These are proportions or probabilities between 0–1.

`posterior_linpred()` gives the posterior draws of **μ on the logit or log odds scale**.

`posterior_linpred(dpar = "phi")` gives the posterior draws of **φ on the log scale**.

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Gaussian regression example

Normal or Gaussian models are roughly equivalent to standard frequentist ordinary least squares (OLS) models and are generally interpreted the same way.

All the Bayesian prediction functions return values on the original scale of the outcome. In Gaussian models, the results from `posterior_epred()` and `posterior_linpred()` are identical. The draws from `posterior_predict()` will generally have the same mean as the linear predictor, but with more variance since the overall model σ is incorporated into the predictions.

```
model_normal <- brm(
  bf(body_mass_g ~ flipper_length_mm),
  family = gaussian(),
  data = penguins
)
```

$$\mathrm{E}(y_i)$$

$$y_i \sim \mathrm{Normal}(\mu_i, \sigma)$$

$$\mu_i = \alpha + \beta x_i$$

`posterior_epred()` gives the draws from the expected value of the posterior predictive distribution, or the average of each draw from `posterior_predict()`.

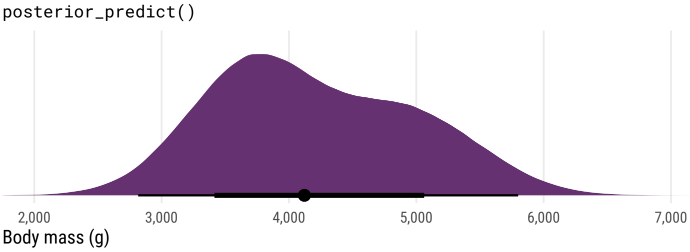
In Gaussian regression, this is the **same as the linear predictor μ**.

`posterior_linpred()` gives the posterior draws of the linear model.

`posterior_predict()` gives the draws from a random normal distribution with draws from the posterior distributions of μ and σ.

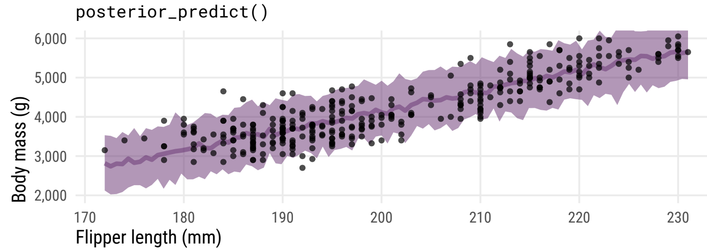
Posterior predictions Random draws from posterior Normal(μ, σ)

`posterior_predict()`



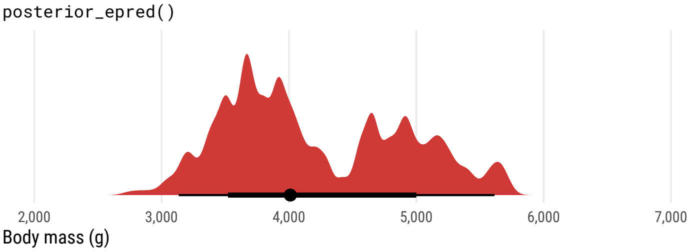
Posterior predictions Random draws from posterior Normal(μ, σ)

`posterior_predict()`



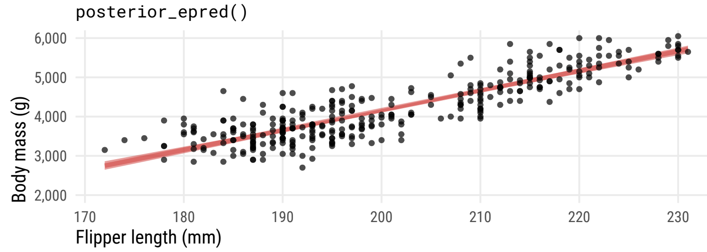
Expectation of the posterior E[y] and μ in the model

`posterior_epred()`



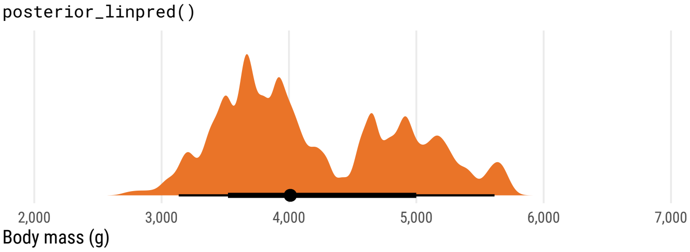
Expectation of the posterior E[y] and μ in the model

`posterior_epred()`



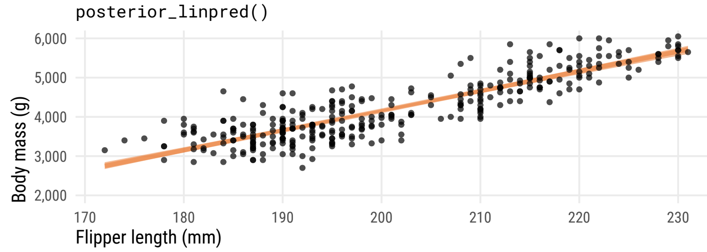
Linear predictor μ in the model

`posterior_linpred()`



Linear predictor μ in the model

`posterior_linpred()`



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Logistic regression example (generalized linear model with link transformation)

Generalized linear models (e.g., logistic, probit, ordered logistic, exponential, Poisson, negative binomial, etc.) use special link functions (e.g. logit, log, etc.) to transform the outcome variable into a scale that is more amenable to linear regression.

Estimates from these models can be used in their transformed scales (e.g., log odds in logistic regression) or can be back-transformed into their original scale (e.g., probabilities in logistic regression).

When working with links, the various Bayesian prediction functions return values on different scales, each corresponding to different parts of the model.

```r
penguins <- penguins |>
  mutate(is_gentoo = species == "Gentoo")

model_logit <- brm(
  bf(is_gentoo ~ bill_length_mm),
  family = bernoulli(link = "logit"),
  data = penguins
)
```

$$\mathrm{E}(y_i)$$

$$y_i \sim \text{Binomial}(1, \pi_i)$$

$$\text{logit}(\pi_i) = \alpha + \beta x_i$$

`posterior_epred()` gives the draws from the expected value of the posterior predictive distribution, or the average of each draw from `posterior_predict()`.

In logistic regression, this is **π on the probability scale** (or inverse logit).

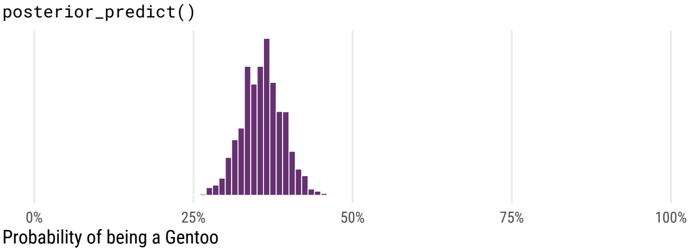
`posterior_linpred()` gives the posterior draws of **π on the logit or log odds scale**.

`posterior_predict()` gives the draws from a random binomial distribution with draws from the posterior distribution of π.

These are 0s and 1s.

Posterior predictions Random draws from posterior Binomial(1, *π*)

`posterior_predict()`

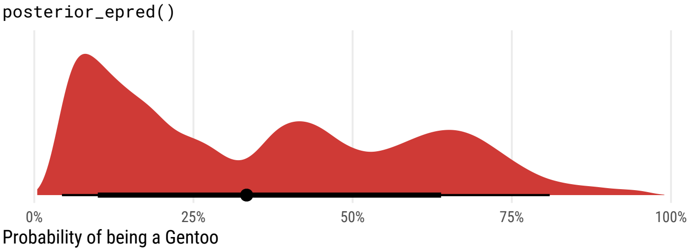


Posterior predictions Random draws from posterior Binomial(1, *π*)

`posterior_predict()`

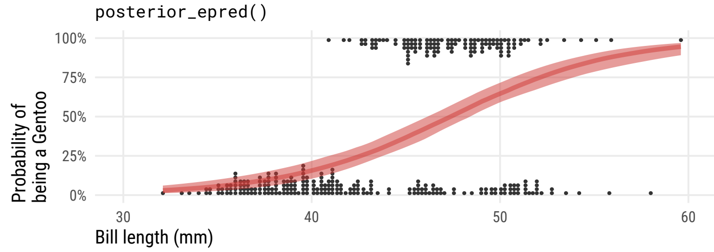
Expectation of the posterior E[*y*] and *π* in the model

`posterior_epred()`



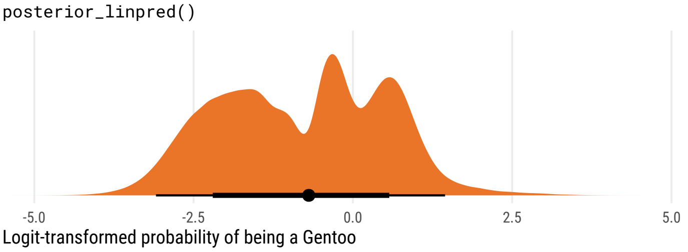
Expectation of the posterior E[*y*] and *π* in the model

`posterior_epred()`



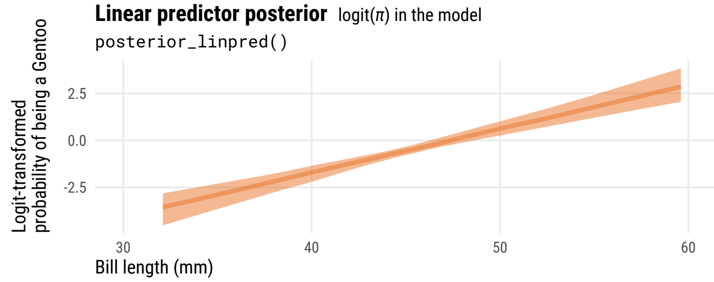
Linear predictor logit(*π*) in the model

`posterior_linpred()`



Linear predictor posterior logit(*π*) in the model

`posterior_linpred()`



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Beta regression example (distributional model with multiple link transformations)

Regression models often focus solely on the location parameter of the model (e.g., μ in Normal(μ, σ); π in Binomial(n, π)). However, it is also possible to specify separate predictors for the scale or shape parameters of models (e.g., σ in Normal(μ, σ), φ in Beta(μ, φ)).

More complex models use a collection of distributional parameters (e.g., zero-inflated beta models estimate a mean μ, precision φ, and a zero-inflated parameter `zi`; hurdle lognormal models estimate a mean μ, scale σ, and a hurdle parameter `hu`).

Even plain old Gaussian models become distributional models when a set of predictors is specified for σ (e.g. `brm(y ~ x1 + x2, sigma ~ x2 + x3))`

When working with extra distributional parameters (`dpar`), the various Bayesian prediction functions return values on different scales for different components of the model.

```r
penguins <- penguins |>
  mutate(bill_ratio = bill_depth_mm /
                      bill_length_mm)

model_beta <- brm(
  bf(bill_ratio ~ flipper_length_mm,
     phi ~ flipper_length_mm),
  family = Beta(),
  data = penguins
)
```

$$\mathrm{E}(y_i)$$

$$y_i \sim \mathrm{Beta}(\mu_i, \phi_i)$$

$$\mathrm{logit}(\mu_i) = \alpha_\mu + \beta_\mu x_i$$

$$\log(\phi_i) = \alpha_\phi + \beta_\phi x_i$$

`posterior_epred()` gives the draws from the expected value of the posterior predictive distribution, or the average of each draw from `posterior_predict()`.

In beta regression, this is **μ on the proportion or probability scale** (or inverse logit).

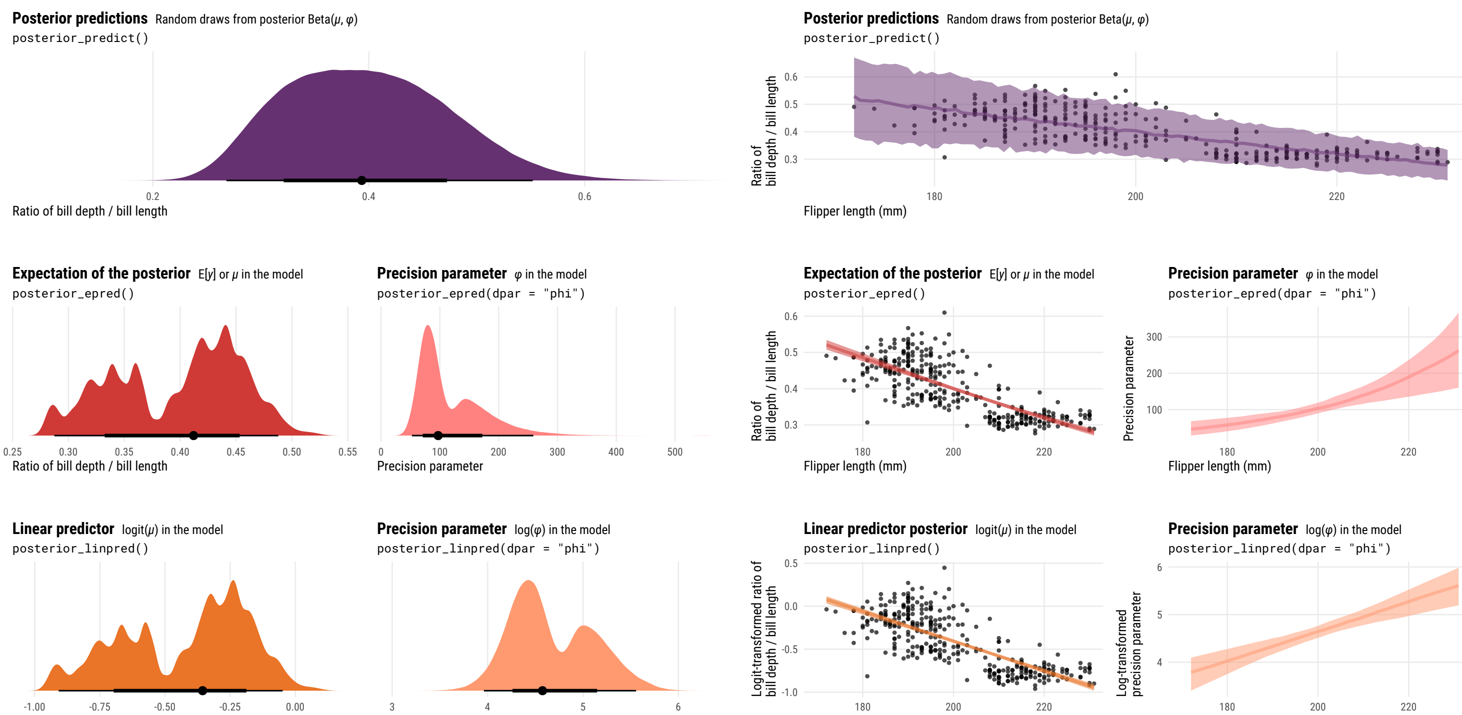
`posterior_epred(dpar = "phi")` gives the posterior draws of **φ on the unlogged scale**.

`posterior_predict()` gives the draws from a random beta distribution with draws from the posterior distribution of π.

These are proportions or probabilities between 0–1.

`posterior_linpred()` gives the posterior draws of **μ on the logit or log odds scale**.

`posterior_linpred(dpar = "phi")` gives the posterior draws of **φ on the log scale**.



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