

MATH 392 Problem Set 1

Exercises from the Book

7.1: 1 - 4

Practice with LaTeX

Please transcribe the mathematics found below into LaTeX using the snippets of LaTeX available in the Slack thread. If you see errors or inconsistencies in my hand-written notes, please correct them. We'll start class on Friday by discussing the link between the code shown in the slides and the analytical approach below.

Ex 3 | Stratified Random Sampling

Divide population into $J=2$ strata (S_1, S_2)

$$N_1 + N_2 = N$$

$$\sigma_1^2 = \frac{1}{N_1} \sum_{i \in S_1} (x_i - \mu_1)^2$$

$$\sigma_2^2 = \frac{1}{N_2} \sum_{i \in S_2} (x_i - \mu_2)^2$$

$$\mu_1 = \frac{1}{N_1} \sum_{i \in S_1} x_i$$

$$\mu_2 = \frac{1}{N_2} \sum_{i \in S_2} x_i$$

rewrite:

$$\mu = \frac{1}{N} \sum_{i \in S_1} x_i + \frac{1}{N} \sum_{i \in S_2} x_i$$

$$= \frac{N_1}{N} \mu_1 + \frac{N_2}{N} \mu_2$$

$$\sigma^2 = \frac{1}{N} \sum_{i \in S_1} (x_i - \mu)^2 + \frac{1}{N} \sum_{i \in S_2} (x_i - \mu)^2$$

$$= \frac{N_1}{N} (\sigma_1^2 + (\mu_1 - \mu)^2) + \frac{N_2}{N} (\sigma_2^2 + (\mu_2 - \mu)^2)$$

Draw $n \cdot \frac{N_1}{N} = n_1$ from S_1 and $n \cdot \frac{N_2}{N} = n_2$ from S_2 .

Estimate μ w/ $\bar{x} = \frac{1}{n} \sum_{j=1}^2 \sum_{i=1}^{n_j} x_{ji}$.

$$E(\bar{x}) = \frac{1}{n} E\left(\sum \sum x_{ji}\right)$$

$$= \frac{1}{n} n \mu = \mu$$

$$V(\bar{x}) = \frac{1}{n^2} V\left(\sum \sum x_{ji}\right)$$

$$= \frac{1}{n^2} \sum_{j=1}^2 \sum_{i=1}^{n_j} V(x_{ji})$$

$$= \frac{1}{n^2} \sum_{j=1}^2 n_j \sigma_j^2$$

$$= \frac{1}{n} \left(\frac{n_1}{n} \sigma_1^2 + \frac{n_2}{n} \sigma_2^2 \right)$$

Practice with R

Adapt the code from the slides to produce a single ggplot with two density curves on it: one with the density of 5000 \bar{x} s estimated through SRS and 5000 \bar{x} s estimated through stratified sampling. All of the code to make this plot can be copied from the slides but I'd like you to make one important change: create a scenario that has *three* strata. It's up to you to pick the parameter values that define that population. Include in your pdf both the single plot and all of the code necessary to create it.