MATH 392 Problem Set 10

Part I: Changing the changepoint

You're encouraged to start these exercise by copying into your problem set code from the notes that implements the Gibbs Sampler with `set.seed(497)`.

1. Using the `post_samples` matrix (or corresponding dataframe `df`), investigate the shape of the joint posterior distribution by constructing several plots. After contructing these plots, summarize in a few sentences the relationship between the three parameters in the posterior. *Please put all three plots in a single 1 by 3 frame using `library(patchwork)`.*
   - A scatterplot of λ on μ with color mapped to the value of m.
   - The same scatterplot but where you only include samples with values of m that are less than 10 or greater than 50.
   - A hexplot where each hex is filled with either the mean or median value of m.
2. Alter one of the prior distributions of the Poisson rate parameters so that they are much more flat. What is the effect on the joint posterior distribution?

Part II: The Metropolis algorithm

To validate the results from the Gibbs sampler, get a second estimate of the joint posterior (using the original priors) using the Metropolis algorithm. You're encouraged to start these exercise by copying into your problem set code from the notes that implements the Metropolis Algorithm. You will need to put some thought into how to build your symmetric proposal distribution since it combines continuous and discrete parameters.

1. Generate the same 3 x 1 plot as you did for the Gibbs sampler. Do these tell the same story? (these plots should represent your final conclusions after considering questions 2 and 3 below).
2. What is your acceptance rate? Tinker with the variance of the proposal distribution to bring it within the target range.
3. To understand the Markov dependence in this chain (or in fact any form of dependence in index), a helpful tool is an autocorrelation plot. Use the `autoplot()` function in `library(ggfortify)` to get a sense of the correlation in your chain (see example of a correlogram). How heavily do you need to thin the chain to diminish most of this dependence? Go ahead and do so to produce a final set of plots for question 1 above.