Let  $X_1, X_2, \dots X_k \sim \chi^2_{n_i}$ ,  $i = 1, \dots, k$ , independently. Show that  $X_1 + X_2 + \dots + X_k = \sum_{i=1}^k X_i \sim \chi^2_{n_1 + n_2 + \dots + n_k}$ . (Hint: use moment generating functions.)