You are provided the marginal distribution of X and the conditional distribution of Y given X. Your task is to find their joint distribution and the marginal distribution of Y.

$$f_X(x) = e^{-x} \quad 0 < x < \infty$$

 $f_{Y|X=x}(y|X=x) = e^{-(y-x)} \quad 0 < x < y < \infty$

- 1. Find the joint distribution of X and Y, $f_{X,Y}(x,y)$.
- 2. Find the marginal distribution of Y, $f_Y(y)$.
- 3. Why doesn't $f_{X,Y}(x,y) = f_X(x)f_Y(y)$?