Nonmonotonic Reasoning on a Constructive Time Structure

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Abstract

In this paper, we introduce a temporal logic called Interval Division Logic **IDL** *based on the constructive temporal ontology. In* **IDL**, *time is regarded as a constructive object which is built from an interval by iterating the interval division everytime new temporal fact is recognized.* **IDL** *is a sound and complete logical system which is as expressive as the Bűich tree automata. In order to examine how the persistence problem is treated on this ontology, we extend a nonmonotonic version of* **IDL** *based on the model preference in which any belief is changed as late as possible in the epistemological order rather than the temporal order and Yale Shooting Problem is discussed in this framework.*

1 INTRODUCTION

In this paper, we introduce a temporal logic called Interval Division Logic **IDL** based on the constructive temporal ontology. In **IDL**, *time* is regarded as a constructive object which is built from an interval by iterating the interval division in the process of temporal reasoning. Namely, whenever new (past, present or future) event is recognized in the current epistemological stage, the time structure of the next epistemological stage is formed by dividing the corresponding interval into two intervals before and after that event. Therefore, the time structure of **IDL** has a form of binary tree of which leaves constitute the current sequence of intervals and the depth of tree corresponds to the level of the epistemological stage. In this point, **IDL** is different from the other tree-based temporal logic such as the branching time, in which the depth of tree corresponds to the timeline.[1]

IDL itself is a sound and complete logical system which is as expressive as the *Büich* infinite tree automata. However, it is intrinsically nonmonotonic because of its constructiveness, since it is perfectly reasonable that the beliefs in the current epistemological stage will be changed in the next epistemological stage due to the new information. we extend a nonmonotonic temporal reasoning in **IDL** in order to examine how the persistence problem is treated on this constructive ontology. Since the persistence itself is due to the retention of the belief rather than the inertia of real world, the interpretation for a sequence of events depends not only on the temporal order of the events but also on the epistemological order in which each event is recognized. Therefore, we use the order of the epistemological stage rather than the temporal order for the model minimization.

The following example reveals why the constructive ontology of time is necessary and how the nonmonotonic reasoning on such a time structure works.

Example1. A lamp and a switch problem
Let us assume that we have a lamp with a switch. The pushing the switch results to turn the light on whenever the lamp is off, otherwise it causes to turn the light off.

1) **The Initial Stage:** We have no time structure in the initial stage of thinking.

2) **Event E1:** The lamp is observed to be on at a time $t0$.

3) **Event E2:** The switch is pushed at time t1 ($t0 \leq t1$).

4) **Event E3:** The lamp is observed to be on at time t2 ($t1 \leq t2$).

Assume that these events are recognized in one of the following sequences.

1) **Case1:** A strict sequence: we observe E1 first, then E2 secondly and E3.

2) **Case2:** A sequence in the backward order: we observe E3 first. After that we are informed E2 and informed E1 finally.

3) **Case3:** A sequence in random: we observe E2 first. After that we are informed E1 and informed E3 finally.

The problem is to find out the most preferable interpretation for each case, although we have infinitely many solutions which are logically correct if unknown $SWITCH$ing may occur at anytime.

In the case1, we believe that the lamp holds on after the first observation, so that we infer the swiching

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at the time t1 changes the state of the lamp from on to off. Consequently, we believe the lamp to be off immediately before the event E3 in which we observe the lamp to be on. Then, what time structure do we build in the mind to explain this series of events consistently after we know the event E3? The most reasonable time structure, the solution1, is formed on the assumption that someone pushes the switch between t1 and t2. Another interpretation, the solution2, is an explanation on the assumption that the switch had been already pushed between t0 and t1 so that it was off before the switch was pushed at t2. This time structure is possible but seems to be unnatural in this case.

For the case 2, we prefer the solution2 to the solution1 on the contrary. This difference in the preference of the solutions is based on the simple reason that we usually avoid the annoying update of the already constructed belief in the inferential process.

For the case3, both the solution1 and solution2 are preferable.

This example reveals that the temporal structure and its interpretation for a sequence of events depends not only on the temporal order but also the epistemological order in which each event is recognized. Since the temporal reasoning is intrinsically sequential because we can not examine simaltaneously two events which occur at different time, it is an important view that the order of inference plays an essential role in the temporal reasonings. However, this example brings about yet another frame problem because some difficulties arise in attempts to deal with this example in the usual nonmonotonic logics, even in the methods proposed to solve YSP(Yale Shooting Problem) [7].

In the following, we define a formal system of **IDL** that is essentially an intuitionistic logic of time and propose a new method of nonmonotonic temporal reasoning based on the Gabbay's intuitionistic approach.

2 TEMPORAL STRUCTURE

2.1 Time Structure

The time structure of **IDL** is represented with a binary tree of which nodes correspond to intervals and the successor relation is the set inclusion between two intervals.

Definition 1. Time structure

Let U be the set of all finite sequences of 0's and 1's. Namely, $U = \{0,1\}^*$. U can be diagramed as a binary tree. We denote the parent node of x by $\circ x$ and the successor nodes of x by $x0$,$x1$.
We introduce two order relations in U.

1) an epistemological order :
Let x, y denote the elements of U. Define $x \sqsubseteq y$ iff x is an initial segment of y.

2) a temporal order :
Let x, y denote the elements of U. A temporal order $x \preceq y$ is defined by

$$x \preceq y \;\; iff \;\; (\exists z)[x \sqsubseteq z0 \wedge y \sqsubseteq z1]$$

A subset $V : V \subseteq U$ is called a subtree if and only if

$$(\forall x)[x \in V \rightarrow \circ x \in V] \wedge [x0 \in V \; iff \; x1 \in V]$$

A maximal linearly ordered subset of the tree U is called a branch of U. Since a branch is a decreasing sequence of intervals, it has a limitting point in $\{0,1\}^\omega$, which is implicitly corresponding to the time instant.

A tuple $T = (V, \sqsubseteq, \preceq)$ is called a *Time Structure*.

Definition 2. Temporal topology

Let x be an element of U. A subset $[x]$ is given by

$$[x] = \{y \mid x \sqsubseteq y\}$$

As known well, we can introduce a topology into the tree U by the definition that $[x]$ is a neighbor of x. A open set is also defined. Namely, a set Q is open if and only if

$$(\forall x)[x \in Q \rightarrow [x] \subseteq Q]$$

2.2 Temporal Ontology

The elementaly object of **IDL** is an interval ϵ. All intervals are formed from ϵ with an interval division. Therefore, the new interval is added to the current time structure whenever a new event is recognized or an inconsistent interval is generated so that the refinement of time structure is required. Two modal operators $[0]\varphi$ and $[1]\varphi$ are used to represent φ to be true on $x0$ and $x1$ respectively. Time point is implicitly corresponding to the starting or endpoints of intervals. Every predicate is evaluated on intervals rather than time points. The usual modal operator $\Box\varphi$ is used to represent the proposition φ is true in $[x]$ (namely, it is not enough that φ is true in every subinterval of the current time structure but φ must be true in the all subintervals appearing in the future inferential process).

we treat two types of objects used in temporal thinking; namely, beliefs about states and events. A state means a persisting temporal phenomena (namely, propositional fluents in the situation calculus) and an event corresponds to an obsevation of a state or an action, which is usually instantaneous phenomenon so that it is defined at the starting or end points of intervals. These objects are represented using two modal operators $\langle\varphi$ and $\rangle\varphi$.

In general, $\langle\psi$ is true on a interval x if and only if there exists a subinterval y with the same starting point with x such that $\Box\psi$ is true on y. Similaly, $\rangle\psi$ is true on a interval x if and only if there exists a subinterval y with the same endpoint with x, $\Box\psi$ is true on y. Namely, $\rangle\psi$ and $\langle\psi$ means that ψ is true in the some open set of intervals of which starting (end) points is same as the current interval.

Example 2. IDL description

The above example of the lamp and switch is described with the following formulas in **IDL**.

1) Causal rules of the switch and obsevation:
   $\Box\{SW \rightarrow [0]\rangle on \equiv [1]\langle off \vee [0]\rangle off \equiv [1]\langle on\}$
   $\Box\{OBL(\varphi) \rightarrow [1]\langle\varphi\}$
   $\Box\{OBR(\varphi) \rightarrow [1]\rangle\varphi\}$

2) Event
   Event E1: $E1 \equiv OBL(on)$
   Event E2: $E2 \equiv SW$
   Event E3: $E3 \equiv OBR(on)$

3) the sequence of inference:
   The Case1 (the strict sequence):
   $E1 \wedge [1]E2 \wedge [1][1]E3$
   The Case2 (the backward order):
   $E3 \wedge [0]E2 \wedge [0][0]E1$
   The Case3 (the randam order):
   $E2 \wedge [0]E1 \wedge [1]E3$

3 FORMAL SYSTEM

3.1 Syntax

A formula of **IDL** is an usual well formed formula of a standard modal logic with the additional modal operator [0],[1] and $\lceil, \rceil$.

We use the following abbreviation,

1) $\Diamond\varphi \equiv \neg\Box\neg\varphi$
2) $\lfloor\varphi \equiv \neg\lceil\neg\varphi$
3) $\rfloor\varphi \equiv \neg\rceil\neg\varphi$

3.2 Semantics

Let $T = (V, \sqsubseteq, \preceq)$ be a time structure. A valuation ϑ on T is a function mapping from propositional letter P to a subset of V, i.e. $\vartheta(P) \subset V$. $\vartheta(P)$ means the elements of V in which P holds.

$M =< T, \vartheta >$ is called a model.

Definition 3.
For given wff's φ,ψ and $x \in V$, $M \models \varphi(x)$ is defined through the following conditions,

1) $M \models P(x)$ iff $x \in \vartheta(P)$.
2) $M \models \neg\varphi(x)$ iff not $M \models \varphi(x)$.
3) $M \models (\varphi \wedge \psi)(x)$ iff $M \models \varphi(x)$ and $M \models \psi(x)$.
4) $M \models \Box\varphi(x)$ iff for all y such that $x \sqsubseteq y$, $M \models \varphi(y)$.
5) $M \models \lceil\varphi(x)$ iff for all $n > 0$ $M \models \varphi(x0^n)$.
6) $M \models \rceil\varphi(x)$ iff for all $n > 0$ $M \models \varphi(x1^n)$.
7) $M \models [0]\varphi(x)$ iff $M \models \varphi(x0)$.
8) $M \models [1]\varphi(x)$ iff $M \models \varphi(x1)$.

A wff φ is said to be satisfiable iff there exists a model M such that $M \models \varphi$, where
$M \models \varphi$ iff $M \models \varphi(\epsilon)$

3.3 Axiom system

A formal system of **IDL** is described by the following axioms.

Axioms

1) Distrubution of operator:
   $\vdash \Box(\varphi \supset \psi) \supset \Box\varphi \supset \Box\psi$
   $\vdash \lceil(\varphi \supset \psi) \supset \lceil\varphi \supset \lceil\psi$
   $\vdash \rceil(\varphi \supset \psi) \supset \rceil\varphi \supset \rceil\psi$
   $\vdash [0](\varphi \supset \psi) \supset [0]\varphi \supset [0]\psi$
   $\vdash [1](\varphi \supset \psi) \supset [1]\varphi \supset [1]\psi$

2) Reflection:
   $\vdash \Box\varphi \supset \varphi$
   $\vdash \lceil\varphi \supset \varphi$
   $\vdash \rceil\varphi \supset \varphi$

3) Decomposition:
   $\vdash \Box\varphi \supset < 0 > \Box\varphi \wedge < 1 > \Box\varphi$
   $\vdash \lceil\varphi \supset [0]\lceil\varphi$
   $\vdash \rceil\varphi \supset [1]\rceil\varphi$

4) Induction:
   $\vdash \Box(\varphi \supset [0]\varphi \wedge [1]\varphi) \supset (\varphi \supset \Box\varphi)$
   $\vdash \lceil(\varphi \supset [0]\varphi) \supset (\varphi \supset \lceil\varphi)$
   $\vdash \rceil(\varphi \supset [1]\varphi) \supset (\varphi \supset \rceil\varphi)$

5) Mixing of negation:
   $\vdash \neg[0]\varphi \equiv [0]\neg\varphi$
   $\vdash \neg[1]\varphi \equiv [1]\neg\varphi$

Inferential rule

1) Propositional tautology:
   If $\vdash_{PL} \varphi$ then $\vdash \varphi$
2) Modus ponens:
   IF $\vdash \varphi \supset \psi$ and $\vdash \varphi$ then $\vdash \psi$
3) Necessity:
   If $\vdash \varphi$ then $\vdash \Box\varphi$

Definition 4.
The modal operators $\langle\varphi$ and $\rangle\varphi$ are defined by the following equations:

$$\langle\varphi \equiv \lfloor\Box\varphi$$
$$\rangle\varphi \equiv \rfloor\Box\varphi$$

Namely,

$$\langle\varphi \equiv \Box\varphi \vee [0]\langle\varphi (\equiv [0]^*\Box\varphi)$$
$$\rangle\varphi \equiv \Box\varphi \vee [1]\rangle\varphi (\equiv [1]^*\Box\varphi)$$

Definition 5.

A wff φ said to be a theorem iff $\vdash \varphi$ is deduced from the axioms by the successive application of the inferential rules.

We introduce the deductive relation $\varphi \vdash \psi$ by:

$\varphi \vdash \psi$ if and only if $\vdash \Box\varphi \supset \Box\psi$

Note that φ is informally interpreted to be true at an interval x when $\Box\varphi$ holds in x. Also φ is true at a time point (the starting point or the end point) in interval x when $\langle\varphi$ (or $\rangle\varphi$) holds in x. Therefore, the deductive relation $\varphi \vdash \psi$ means that if φ is true at any interval x then ψ is true at the interval x.

Soundness, completeness and decidability

The above axiom system is a sound and complete characterization for **IDL**. Also $\varphi \vdash \psi$ is decidable relation.

This is proved from a theorem that **IDL**'s formula is as expressive as the Büich infinite tree automata[4][9].

4 NON-MONOTONICITY

4.1 general default rules

In order to deal with the temporal reasoning about the persistence, we extend the **IDL**-based nonmonotonic logic in the framework of Gabbey's intuitioninstic theory of nonmonotonicity [5].

We introduces a default rule:

$A \rightsquigarrow C(\Box\Diamond B)$ if and only if $A \wedge \Diamond B \vdash C$

which means we can infer C nonmonotocally from A by using a default $\Box\Diamond B$.

This default rule means that we can conclude tentatively C from the knowledge A and a belief B at the current epistemological stage. From the view of the model minimization, this method corresponds to prefer the model in which any default is changed as late as possible in the epistemological order only if it is necesarry.

4.2 persistence of beliefs

In order to deal with the temporal reasoning about the persistence of beliefs, we employ transform $\langle\psi$, $\rangle\psi$ into the following default rules

L1: $\langle\psi \rightsquigarrow \psi\ (\Box\Diamond\psi)$

L2: $\langle\psi \rightsquigarrow [0]\psi \wedge [1]\langle\psi\ (\Box\Diamond[0]\psi)$

L3: $\langle\psi \rightsquigarrow [0]\psi\ ([0]\Box\Diamond\psi)$

R1: $\rangle\psi \rightsquigarrow \psi\ (\Box\Diamond\psi)$

R2: $\langle\psi \rightsquigarrow [1]\psi \wedge [0]\rangle\psi\ (\Box\Diamond[1]\psi)$

R3: $\rangle\psi \rightsquigarrow [1]\psi\ ([1]\Box\Diamond\psi)$

$\langle\psi$ originally means that ψ is true at the starting point of the cerrent interval, namely ψ holds in some neighbore of that point. In the other words, the $\langle\psi$ (or $\rangle\psi$) is true in the current interval iff $\Box\psi$ is true in some subinterval x with the same starting point (the same endpoint) as the interval. In this case, we say $\langle\psi$ (or $\rangle\psi$) is realized at x.

The above default rules requires that if $\langle\psi$ then ψ must be realized in the largest consistent interval of which starting point is same as the current interval.

EXAMPLE3: The lamp with switch nonmonotonic reasoning

For each case of the example2, the inferential process is given in the following.

Case 1

The first event $E1$, the observation that the lamp is on at time t1, divides the empty interval ϕ into 0, 1 where $\langle on$ is true in 1 . This is immediately realized in this interval by the default rule L1 under the assumption $\Box\Diamond on$.

The next event $E2$, namely someone pushs the switch at time t2, urges the interval division of 1 into 10 and 11. Note that $\Box on$ is believed to be true in 1. Therefore, $\rangle on$ is true in 10. From the causal rule of switch, we can infer $\langle off$ in 11, which falsifies the default $\Box\Diamond on$ in 1. Since application of the rule2 for $\langle on$ also falsifies its assumption, By the rule L3, $\langle on$ in 1 is realized in 10 by the rule L3. $\langle off$ in 11 itself is realized by the rule L1 so that $\Box off$ holds in 11 under the assumption $\Box\Diamond off$.

The third event $E3$, the obsevation that the lamp is on at time t3 again forces the decomposition of the interval 11 into 110 and 111 where the $\rangle on$ to be true in 110. From this information, the default $\Box\Diamond off$ in 11 is falsified and the rule L2 is applied to $\langle off$ in 11. But this is failed because $\Box\Diamond off$ does not hold even in 110. Therefore, by the rule L3, $[110]\langle off$ is examined. Finally, this is realized in 1100 by the rule L2 under the assumption $\Box\Diamond off$ in 1100. $\rangle on$ in 110 is also realized in 1101 under the assumption $\Box\Diamond on$ in 1101.

Case 2

The first event $E3$ gives $\Box on(\Box\Diamond on)$ in 0 by the rule R1 and the next switching $E2$ divides this interval into 00, 01 where $\Box off(\Box\Diamond off)$ in 0 and $\Box on(\Box\Diamond on)$ in 1.

The third event $E1$ falsifies the default $\Box\Diamond off$ in 0. By applying the rule R3,R1,R2, we have a time line in which $\Box on$ in 0010,$\Box off$ in 0011 and $\Box on$ in 11.

Consequently, we select the solution1 for the case1 and the solution2 for the case2, respectively but there is no difference between two solutions for the case3.

4.3 Yale Shooting Problem

The Yale Shooting Problem **YSP** is the famous example which exhibits the persistence problem can not be treated completely by the non-monotonic version of the simple temporal logic [7]. We show how this problem is dealt with in the **IDL**.

The problem itself is very simple. Let assume the following event sequence.

1. In the initial situation, it is observed that a gun is unloaded ($\neg loaded$) and Teddy is alive ($alive$).
2. Then, the gun is loaded($LOAD$).
3. Some time after that ($WAIT$),
4. Teddy is shooted($SHOOT$)

5. Is Teddy alive or not?

In this example, the initial condition is represented by an axiom :
$OBL(\neg loaded \wedge alive)$, namely,
$[1]\langle\neg loaded \wedge [1]\langle alive$

The sequence of actions is informed in the following order:
$[1]LOAD \wedge [1][1]WAIT \wedge [1][1][1]SHOOT$

The causal laws of the actions $LOAD$ and $SHOOT$ are given by
$\Box\{LOAD \rightarrow [1]\langle loaded\}$
$\Box\{Shoot \rightarrow [[0]\langle\rangle loaded \wedge\rangle alive) \rightarrow [1]\langle\neg alive]\}$

The action $WAIT$ has no causal law because it does not change any situation.

At first, we observe the gun is unloaded and Teddey is alive in the initial situation. This information divides the initial interval ϕ into 0 and 1 where $\langle\neg loaded \wedge \langle alive$ is true at the interval 1. These two $\langle$-formula are immediately realized so that $\Box\neg loaded \wedge \Box alive$ is assumed to be true in 1. The action $LOAD$ forces the division 1 into two subintervals 10 and 11. From the causal rule for $LOAD$, $\langle loaded$ is beleaved to be true in 11. Therefore, $\Box\neg loaded$ in 11 is not consistent with the action $LOAD$ so that $\langle\neg loaded$ should be realized again in the interval 10, while $\Box alive$ is assumed to be true in the both intervals 10 and 11. The action $WAIT$ urges the division 11 into 110 and 111 but it does not change any situation. The action $SHOOT$ divides the interval 111 into 1110 and 1111. We can assume $loaded$ is true in 1110 because $\langle loaded$ is realized in 11. By the causal rule for $SHOOT$, $\langle\neg alive$ become to be true in the interval 1111. This is immediately realized in this interval, so that $\neg alive$ is true in the interval 1111.

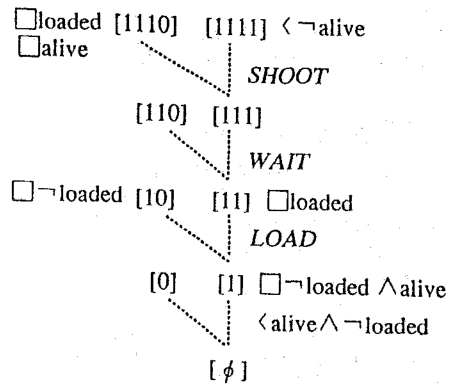
As a result, we construct the preference model shown in the Figure 3-a, which gives the prediction that teddy is dead after shooting, so that the **IDL** gives the natural answer for this problem as is similar to the other logic such as Shoham's chronological ignorance [8].

IDL also gives the intuitively natural answer for the problem which requires the reasoning backward in time. Let assume that we are informed that Teddy is alive for some time after shooting in the above example. How the temporal structure is modified and the sequences of events is explained?

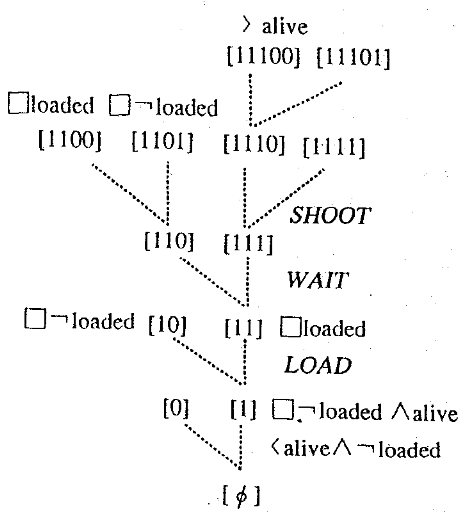
The answer is simple if we can believe that Teddy can raise from the dead. In this case, we get a modified model with new time-line in which Teddy is dead in the result of shooting but he is recovered again afterwards. However, we have a knowledge that he can't be revived usually. Namely,

$\Box(\neg alive \rightarrow \Box\neg alive)$

Assume that the interval 1111 is devided into 11110 and 11111 where $\rangle alive$ is observed at 11110. This means that $\Box alive$ is true at the interval 11110 from the above axiom. Namely, $\langle alive$ holds in 1111. From the causal rule of $SHOOT$, $\rangle\neg loaded$ should be true in the interval 1110. Therefore, $\langle loaded$ can not be realized in 11 so that it should be realized again in 110. The result (Figure 3-b) shows that something happens to cause the gun unloaded while $WAIT$ing. (Here we use the action WAIT following the original formulation of **YSP** but the same result is derived without WAIT, because the time structure in **IDL** is dense so that actions may occur at any time. In the case without $WAIT$, the interval before $SHOOT$ is devided and the gun is unloaded at that time.)



1-a. Temporal prediction



1-b. Reasoning in backword

Figure1. Time structure for the YSP

5 Concluding Remarks

In this paper, we introduce a temporal logic which is based on an idea that *time* is constructed step-by-step along with the thinking process for temporal phenomena. Although we omit the discussion here, the concept of tense is also introduced via the interval division. For instance, $\mathbf{F}\psi$, which means ψ is true in every future interval, is equivalent to the statement that ψ is true for every right interval of all previous interval divisions. The tense of past can be similarly defined.

The **IDL** is essentialy intuitionistic, although it is formulated in the cotegory of S4 logic, so that Gabbey' intuitionistic approach to nonmonotonic reasoning is naturally incorporated.

Temporal logic is extended to the intuitonistic one for the point ontology in [3] and for the interval ontology in [2]. However, these logics are the bi-modal logics constituted from $S4 \times S4$, $S4$ for epistemological order and another $S4$ for the temporal order, so that it is radically different from **IDL**.

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