A Possibility Theory-Based Approach to the Handling of Uncertain Relations between Temporal Points

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Abstract

Uncertain relations between temporal points are represented by means of possibility distributions over the three basic relations "smaller than", "equal to", and "greater than". Operations for computing inverse relations, for composing relations, for combining relations coming from different sources and pertaining to the same temporal points, or for representing negative information, are defined. An illustrative example of representing and reasoning with uncertain temporal relations is given. This paper shows how possibilistic temporal uncertainty can be handled in the setting of point algebra. Moreover, the paper emphasizes the advantages of the possibilistic approach over a probabilistic approach previously proposed. This work does for the temporal point algebra what the authors previously did for the temporal interval algebra.

1. Introduction

Representing and reasoning about time is an essential part of many Artificial Intelligence (AI) tasks (natural language understanding, planning, medical diagnosis and causal explanation, etc). Since the late eighties, temporal reasoning has been attracting the attention of many AI researchers. Several approaches have been proposed in this research area [2][17]: logical formalisms for time, ontological primitives, and algorithms for temporal reasoning and their complexity. Nevertheless, few of these works were concerned with the practical fact that our knowledge about time may be pervaded with vagueness and uncertainty. Dealing with uncertainty in temporal knowledge is considered as one of the major emerging trends in temporal representation and reasoning, as stressed by Chittaro and Montanari [2]. Attempts along this line are not numerous. Dubois and Prade [8] propose an approach for the representation of imprecise or uncertain temporal knowledge in the framework of possibility theory. In this work, fuzzily-known dates, time intervals with ill-known bounds, and uncertain precedence relations between events can be handled. Guesgen *et al.* [14] introduce fuzzy Allen relations as fuzzy sets of ordinary Allen relations agreeing with a neighborhood structure. Dubois *et al*. [5] have proposed a possibilistic temporal logic where each classical logic formula is associated with the fuzzy set of time points where the formula is certainly true to some extent. Let us also mention the work done by Freksa [13] who proposes a generalization of Allen's interval-based approach to temporal reasoning, based on semi-intervals, for processing coarse and incomplete information.

In a more recent paper [4], we have studied different types of problems raised by the fuzziness of the categories used for expressing information in temporal reasoning, when time intervals are used as ontological primitives. Especially, we have provided a fuzzy set-based extension of Allen's approach to interval-based representation of temporal relations. This extension allows for a gradual description of possible relations between time intervals. It is based on a fuzzy partition made of three possible fuzzy relations between dates (*clearly smaller, approximately equal*, and *clearly greater*). Allen's calculus is then extended to the case of fuzzy temporal relations in a convenient and expressive way. Moreover, we have shown that indices for expressing the uncertainty pervading Allen relations between two time intervals (or their fuzzified versions), can be estimated in terms of necessity measures, and used as a basis in deductive reasoning.

Different primitives can be considered for expressing temporal elements. The main candidates are points [18] and intervals [1]. In this paper, we use temporal points as ontological primitives for representing temporal information. We propose an approach for handling uncertainty on temporal relations between points in the framework of possibility theory.

Possibility theory [7] is an uncertainty theory devoted to the handling of incomplete information. It is different from probability theory. From the point of view of

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uncertainty modeling, classical probability theory (assuming a single distribution) is unable to model ignorance (even partial ignorance) in a natural way. Uniform probability distributions on finite sets of elementary events better express randomness than ignorance, namely, the equal chance of occurrence of elementary events. Ignorance rather means that each possible occurrence is viewed as equally plausible by an agent, because there is no available evidence that supports any of them. No probability measure can account for such a state of lacking knowledge (since even a uniform probability distribution leads to assigning non-equal probabilities to at least two possible contingent events in general).

In contrast, possibility theory copes with the situation of complete ignorance in a non-biased way (without making any prior assumption). Another difference between probability theory and possibility theory is that the probabilistic encoding of knowledge is numerical and relies on an additivity assumption while possibilistic encoding can be purely qualitative, hence less demanding in data. Lastly, possibilistic reasoning is computationally less difficult than probabilistic reasoning (but the obtained results may also be less informative).

Two set-functions Π and N, called *possibility and necessity measures*, are used in order to model available information in possibilistic framework. Statements like "A is possible" and "A is certain" are clearly distinguished via the two dual measures Π and N respectively. Only an ordering of elementary events is requested in possibility theory, and it is enough to reconstruct the two orderings of events. Possibility theory can thus be interpreted either as a representation of *ordinal uncertainty* based on linear ordering, or as a numerical theory of uncertainty handling special types of probability bounds.

As mentioned above, there are very few works trying to handle uncertainty in temporal reasoning. Recent work done by Ryabov and Puuronen [15] proposed a probabilistic model for dealing with uncertain relations between temporal points. Starting with the three basic relations that can hold between two dates *a* and *b*: "<" (before), "=" (at the same time), and ">" (after), Ryabov and Puuronen define an uncertain relation between *a* and *b* as any possible disjunction of these basic relations, i.e., "≤" ("<" or "="), "≥" ("=" or ">"), "≠" ("<" or ">"), and total ignorance ("<" or "=" or ">"). The uncertainty is then represented by a vector $(e^{<}, e^{=}, e^{>})_{a,b}$, where $e^{<}_{a,b}$ (respectively $e^{=}_{a,b}$, $e^{>}_{a,b}$) is the probability of $a < b$ (respectively $a = b$, $a > b$). Then formulas, which supposedly preserve the probabilistic semantics, are given for propagating uncertainty when composing relations, or when fusing pieces of temporal information about the same dates.

However, the major flaw of this approach is the way to cope with the state of complete ignorance (this is not surprising since it relies on a probabilistic model). When nothing is known about the relation between any two temporal points, Rayabov and Puuronen suggest the use of so-called *domain probability values*, denoted $e^{<}_{D}$, $e^{=}_{D}$ and $e^{>}_{D}$, for representing the probabilities of the basic relations between two temporal points in this situation. This proposal makes sense only if a prior probability distribution is available. However such a prior probability is never made explicit.

In the present paper, a possibilistic approach for the representation and management of uncertainty in temporal relations between two points, is proposed. Uncertainty is represented as a vector involving three *possibility* values expressing the plausibility of the three basic relations ("<", "=", and ">") that can hold between these points. Reasoning about these uncertain temporal relations is considered through a set of operations including inversion, composition, combination, and negation, like in [15]. These different operations govern the uncertainty propagating in the inference process. We show that the whole reasoning process can be handled in possibilistic logic [6].

The paper is organized as follows. Section 2 presents the possibilistic representation of uncertain relational knowledge about dates and compares it to the probabilistic approach. Section 3 discusses the inference rules that form the basis of the reasoning method. The illustrative example of [15] for reasoning with uncertain temporal relations is considered in section 4. To conclude, the main interesting points of the approach are briefly recalled and some future working directions are outlined.

2. Representation issue

Three basic relations can hold between two temporal points: *"<" (before), "=" (at the same time),* and *">" (after)*. These relations are fully certain temporal relations between points. When knowledge about temporal relations is lacking, the number of alternative options is finite. For instance, we may only know that date *a* does not take place after date *b*, that is, either date *a* takes place before date *b*, or *a* is at the same time than *b*, but these options exclude the remaining alternative. More generally, an uncertain relation between temporal points is any possible disjunction of basic relations, i.e., "≤" ("<" or "="), "≥" (">" or "="), "≠" ("<" or ">"), and "?" ("<", "=", or ">"). The last case represents total ignorance, i.e., any of the three relations is possible.

In the following, we extend this representation by assuming that each of the three basic relations are more or less plausible.

2.1. Background on possibility theory

In the last decade, there has been a major trend in uncertainty modeling (especially regarding partial belief) emphasizing the idea that the degree of confidence in an event is not totally determined by the confidence in the opposite event, as assumed in probability theory. Possibility theory belongs to this trend. It was coined by L.A. Zadeh in the late seventies [20] as an approach where uncertainty is induced by pieces of vague linguistic information, described by means of fuzzy sets [19]. Possibility theory offers a simple, non-additive modeling of partial belief, which contrasts with probability theory. It provides a potentially more qualitative treatment of partial belief since the operations "max" and "min" play a role in possibility theory somewhat analogous to the sum and the product in probability calculus.

In the possibilistic framework, the estimation of an agent's confidence about the occurrence of related events, is based on the two set-functions Π and N, called *possibility* and *necessity* measures respectively [7]. Let U be a reference set and A and B subsets of U.

- A possibility measure is a mapping Π from $P(U) = 2^U$ to [0, 1] which satisfies the following axioms:

i) $\Pi(\varnothing) = 0$,

ii) Normalization: $\Pi(U) = 1$,

iii) Maxitivity, which in the finite case reads $\Pi(A \cup B) = \max(\Pi(A), \Pi(B))$.

The weak relationship between the possibility of an event A and that of its complement A^c ("not A") can be expressed by $\max(\Pi(A), \Pi(A^c)) = 1$ due to $A \cup A^c = U$ and $\Pi(U) = 1$. In case of total ignorance, both A and A^c are fully possible $\Pi(A) = \Pi(A^c) = 1$. Note that this leads to a representation of ignorance ($\forall$ A, $\Pi(A) = 1$) which presupposes nothing about the number of elements in the reference set U (elementary events), while the latter aspect plays a crucial role in probabilistic modeling. Indeed the normalization constraint in probability theory enforces the constraint $\Sigma_{u \in U}\, p(u) = 1$. In case of no information, there is no reason to assign a probability to an elementary event higher than to another. Hence $p(u) = 1/\mathrm{card}(U)$, which depends on the way elementary events are defined. Worse, except if $\mathrm{card}(U) = 2$, there will be two contingent events A, B with $P(A) \neq P(B)$, which questions the idea that this uniform probability represents ignorance (see Dubois, Prade and Smets [11]). The case when $0 < \min(\Pi(A), \Pi(A^c)) < 1$ corresponds to partial belief about A or its complement.

The interpretation of the endpoints of the [0, 1] scale for a possibility measure are clearly different from the probabilistic use: $\Pi(A) = 0$ means that A is impossible (i.e., A is certainly false), while $\Pi(A) = 1$ only expresses that A is completely possible, which leaves $\Pi(A^c)$ completely unconstrained. The weak relationship between $\Pi(A)$ and $\Pi(A^c)$ forces us to use both quantities for the description of uncertainty about the occurrence of A. $\Pi(A^c)$ estimates the possibility of "not A", which is related to the certainty (or necessity) of occurrence of A, since when "not A" is impossible then A is certain. It is thus natural to use this duality when defining the degree of necessity of A.

- A necessity measure is the dual of the possibility measure and is defined as follows

$$N(A) = 1 - \Pi(A^c).$$

It estimates the certainty of event "A" as the degree of impossibility of the event "not A". Note that $N(A) = 1$ means that A is certainly true, while $N(A) = 0$ only says that A is not certain at all (however A might still be possible). It is easy to verify that $N(A) > 0$ implies that $\Pi(A) = 1$ (an event is completely possible before being somewhat certain). Necessity measures satisfy an axiom dual of the one of possibility measures, namely: $N(A \cap B) = \min(N(A), N(B))$. For more details about possibility theory, see also [9] and [10].

For convenience, we have presented possibility and necessity measures valued on [0, 1] scale. However, it can be straightforwardly generalized to any linearly ordered scale. Then, $1 - (\cdot)$ is replaced by the order-reversing map on the scale. This remark emphasizes the qualitative nature of the possibilistic setting.

2.2. Modeling temporal uncertainty

Let us consider the basic granule of uncertain temporal information that can be naturally expressed by an agent, in the form of uncertain statements concerning pairs of dates:

Definition 1. Let *a* et *b* be two temporal points. An uncertain relation $r_{a,b}$ between *a* et *b* is represented by a vector $\Pi_{ab} = (\Pi^<, \Pi^=, \Pi^>)_{a,b}$, where $\Pi^<_{a,b}$ (respectively $\Pi^=_{a,b}$, $\Pi^>_{a,b}$) is the possibility of $a < b$ (respectively $a = b$, $a > b$).

Obviously, the normalization property of the measure of possibility holds, i.e., $\max(\Pi^<_{a,b}, \Pi^=_{a,b}, \Pi^>_{a,b}) = 1$. This property means that at least one of the three basic relations holds with a possibility equals to 1. For instance, if we know that the temporal relations between points *a* and *b* is ">" with possibility 0.8, "=" with possibility 0.4, hence ">" with a possibility 1 then, the uncertainty vector is $(0.8, 0.4, 1)_{a,b}$. It is worth noticing that from the uncertainty vector ($\Pi^{<}_{a,b}$, $\Pi^{=}_{a,b}$, $\Pi^{>}_{a,b}$) we can derive the necessity that a ≥ b (respectively a ≤ b, a ≠ b) using the usual duality between possibility and necessity. Namely,

$N^{\geq}_{a,b} = N(a \geq b) = 1 - \Pi^{<}_{a,b}$,

$N^{\leq}_{a,b} = N(a \leq b) = 1 - \Pi^{>}_{a,b}$,

$N^{\neq}_{a,b} = N(a \neq b) = 1 - \Pi^{=}_{a,b}$.

In a similar way, we can also obtain

$N(a < b) = 1 - \max(\Pi^{=}_{a,b}, \Pi^{>}_{a,b})$,

$N(a > b) = 1 - \max(\Pi^{<}_{a,b}, \Pi^{=}_{a,b})$,

$N(a = b) = 1 - \max(\Pi^{<}_{a,b}, \Pi^{>}_{a,b})$.

Note that an agent positively asserting a relation between *a* and *b* should use necessity degrees rather than possibility degrees. For instance, declaring a ≥ b is more faithfully modeled by N(a ≥ b) = 1 than by Π(a ≥ b) = 1, the latter being very weak and uninformative. Possibility degrees can only express information negatively ("it is more or less impossible that…"). Hence although the uncertainty vector Π_{ab}= ($\Pi^{<}_{a,b}$, $\Pi^{=}_{a,b}$, $\Pi^{>}_{a,b}$) is the primitive representation in terms of a possibility distribution on elementary mutually exclusive situations involving *a* and *b*, it is more convenient in practice to use a vector of necessities N_{ab}= ($N^{\geq}_{a,b}$, $N^{\neq}_{a,b}$, $N^{\leq}_{a,b}$) on the complements of elementary events.

An important principle in possibility theory is the principle of minimal specificity which assumes that any elementary event is possible unless ruled out by an available piece of information. In view of this principle any piece of information can be expressed by means of a single possibility distribution that is the least committed one in agreement with this piece of information, obtained by maximizing possibility degrees. This choice is tentative and can be questioned upon the arrival of a new piece of information that will lead to a more specific possibility distribution. A piece of information like "a ≥ b is sure to level α" expressed as N(a ≥ b) ≥ α, corresponds to $\Pi^{<}_{a,b} \leq 1 - \alpha$, hence ($\Pi^{<}_{a,b}$, $\Pi^{=}_{a,b}$, $\Pi^{>}_{a,b}$) = (1− α, 1, 1), or equivalently ($N^{\geq}_{a,b}$, $N^{\neq}_{a,b}$, $N^{\leq}_{a,b}$) = (α, 0, 0). This possibility distribution is the least upper bound of all such distribution that respect the constraint N(a ≥ b) ≥ α.. If another piece of information of the form N(a ≠ b) ≥ β, is obtained, it corresponds to revising (actually expanding) ($\Pi^{<}_{a,b}$, $\Pi^{=}_{a,b}$, $\Pi^{>}_{a,b}$) into Π_{ab}= (1− α, 1− β, 1) and so on. Note that it is obtained as the (fuzzy set) intersection of (1− α, 1, 1) and (1, 1− β, 1). Information of the form N(a > b) > γ is expressed likewise as Π_{ab}= (1− γ, 1− γ, 1). Added to the two other items yields Π_{ab} = (min(1− α,1− γ) min(1− β,1− γ), 1). In case of complete ignorance, namely, when nothing is known about the relations between two temporal points, *each of the three alternatives receives possibility value 1*. Π_{ab}= (1, 1, 1). Hence *any* information pertaining to the relative position of *a* and *b* can be expressed by a unique Π_{ab} triplet.

The above setting is very similar to the one in [15] where the basic information granule is of the form of probability vectors P_{ab} = ($e^{<}_{a,b}$, $e^{=}_{a,b}$, $e^{>}_{a,b}$) whose components are summing to 1. However these authors assume that at least two such probabilities are available, while this is not requested in possibility theory. The standard probabilistic framework cannot express the knowledge of one piece of information such as $e^{<}_{a,b}$. In case of plain ignorance, resorting to P_{ab} = (1/3, 1/3, 1/3) is hardly convincing since it entails P(a ≥ b) > P(a < b) for instance, which is a non trivial piece of knowledge.

3. Reasoning about uncertain temporal relations

To reason on the basis of uncertain temporal relations, we revisit the set of inference rules considered in [15]. They enable us to infer new temporal information and to propagate uncertainty in a possibilistic way. This reasoning tool relies on four operations expressing: inversion, composition, combination, and negation, which are presented in the following subsections. We show that the probabilistic rules used in [15] are sometimes debatable, and that their possibilistic counterparts are more sound (if sometimes providing less information).

3.1. Inversion operation

Let $r_{a,b}$ be a known relation between two temporal points *a* and *b*, the inversion operation (~) enables to produce the relation $r_{b,a}$ between the points *b* and *a* as pictured in Figure 1.

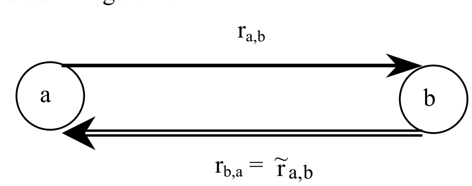

Figure 1. Operation of inversion

The basic relations "<" and ">" are mutually exchanged, and the inversion of relation "=" is the relation "=" itself. To derive the uncertainty vector of an inverted temporal relation, we only need to exchange the possibility values of the basic relations "<" and ">". Now, if the uncertainty vector of $r_{a,b}$ is represented by Π_{ab} and $r_{b,a} = \tilde{r}_{a,b}$ then, the uncertainty vector Π_{ba} of the relation $r_{b,a}$ is such that $\Pi^{<}_{b,a} = \Pi^{>}_{a,b}$, $\Pi^{=}_{b,a} = \Pi^{=}_{a,b}$ and $\Pi^{>}_{b,a} = \Pi^{<}_{a,b}$. It is obvious that the normalization property for the derived uncertain relation $r_{b,a}$ holds (i.e., $\max(\Pi^{<}_{b,a}, \Pi^{=}_{b,a}, \Pi^{>}_{b,a}) = 1$), knowing that it holds for the initial uncertain relation $r_{a,b}$. The same inversion process is valid for probability triplets [15] or necessity triplets. Basically, it comes down to exchanging *a* and *b*.

3.2. Composition operation

Knowing the uncertain relation $r_{a,b}$ between two temporal points *a* and *b*, and the uncertain relation $r_{b,c}$ between the points *b* and *c*, the operation of composition ($\otimes$) enables us to derive the temporal relation $r_{a,c}$ that may hold between the points *a* and *c*. Figure 2 illustrates this operation.

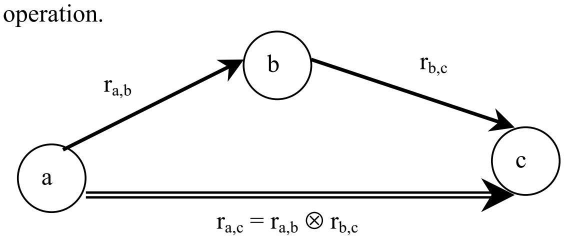

Figure 2. Operation of composition

This operation expresses somewhat the transitive closure of the three basic relations that can hold between two time points. Instead of assuming that the uncertain relations $r_{a,b}$ and $r_{b,c}$ are defined by the vectors Π_{ab} and Π_{bc} respectively, we shall use necessity triplets N_{ab} and N_{bc}. The reason is that we can straightforwardly use possibilistic logic [6] to compute the uncertainty vector for the derived relation $r_{a,c}$, i.e. N_{ac}. Indeed possibilistic logic is sound and complete with respect to the computation of possibility distributions exploiting the principle of minimal specificity. The basic inference rule we shall need here is possibilistic resolution, in the form $N(a\ \theta_1\ c) \geq \min(N(a\ \theta_2\ b), N(b\ \theta_3\ c))$ provided that $a\ \theta_2\ b$ and $b\ \theta_3\ c$ imply $a\ \theta_1\ c$, where θ_1, θ_2 and θ_3 are any relations among dates.

- *Computing $N(a \geq c)$.* The only existing sufficient condition for deriving $a \geq c$ with certainty from information concerning $a\ \theta_2\ b$ and $b\ \theta_3\ c$ where θ_2 and θ_3 are of the form $\geq$, $\leq$ or $\neq$ is when both $a \geq b$ and $b \geq c$ hold. Hence it follows: $N(a \geq c) \geq \min(N(a \geq b), N(b \geq c))$, where the latter two values are obtained from Π_{ab} and Π_{bc}. Of course, $N(c \geq a)$ is computed likewise.

- *Computing $N(a \neq c)$.* The only existing sufficient conditions for deriving $a \neq c$ with certainty from information concerning $a\ \theta_2\ b$ and $b\ \theta_3\ c$ where θ_2 and θ_3 are of the form $\geq$, $\leq$ or $\neq$ are when at least one of $a \neq b$ or $b \neq c$ hold together with $a \geq b \geq c$ or $a \leq b \leq c$. Indeed, suppose not, then either $a = b = c$, or $a \geq b$, for instance, and $c > b$ which is not enough to conclude $a \neq c$ (and likewise for any other similar conditions). Hence

$N(a \neq c) \geq N((a \neq b \text{ or } b \neq c) \text{ and } (a \geq b \geq c \text{ or } a \leq b \leq c))$
$= \min(N(a \neq b \text{ or } b \neq c), N(a \geq b \geq c \text{ or } a \leq b \leq c)$
$\geq \min(\max(N(a \neq b), N(b \neq c), \max(N(a \geq b \geq c), N(a \leq b \leq c))$, since $N(p \text{ or } q) \geq \max(N(p), N(q))$
$= \min(\max(N(a \neq b), N(b \neq c), \max(\min(N(a \geq b), N(b \geq c)), \min(N(a \leq b), N(b \leq c)))$.

Hence we can reconstruct N_{ac} from necessity triplets N_{ab} and N_{bc}. The uncertainty vector Π_{ac} can be obtained from the necessity triplet N_{ac} as follows: $\Pi^{<}_{a,c} = 1 - N^{\geq}_{a,c}$, $\Pi^{=}_{a,c} = 1 - N^{\neq}_{a,c}$ and $\Pi^{>}_{a,c} = 1 - N^{\leq}_{a,c}$.
The above derivations make no assumption such as e.g. independence and the like. It contrasts with the work in [15]. These authors compute for instance probability $e^{<}_{a,c}$ as

$$e^{<}_{a,c} = e^{<}_{a,b}\, e^{<}_{b,c} + e^{<}_{a,b}\, e^{=}_{b,c} + e^{=}_{a,b}\, e^{<}_{b,c} + e^{<}_{a,b}\, e^{>}_{b,c}\, e^{<}_{U} + e^{>}_{a,b}\, e^{<}_{b,c}\, e^{<}_{U},$$

the three first terms stem from sufficient conditions for $a < c$. Moreover, identities such as $P(a < b < c) = P(a < b) \cdot P(b < c)$ are taken for granted. Lastly, the two last terms in the right hand side of the equality involve so-called domain probabilities, and account for cases where $a < c$ is not forbidden by relations on *(a, b)* and *(b, c)*. They are added with the hope to derive $P(a < c)$ exactly. Unfortunately, this derivation looks debatable in several respects. The precise meaning of domain probabilities remains obscure, let alone their practical assessment. The authors never clarify what is the probability space underlying their calculations. Insofar as several (at least 3) dates are involved, probabilities like $P(a < b)$ and $P(b < c)$ are marginal probabilities stemming from an unknown underlying joint distribution. Hence it is not likely that the triplet P_{ac} can be exactly derived from marginal probabilities such as triplets P_{ab} and P_{bc}.
Maximal sufficient conditions for deriving $a < c$ in terms of $a\ \theta_2\ b$ and $b\ \theta_3\ c$ where θ_2 and θ_3 are of the form $>$, $<$ or $=$ are $a < b < c$, $a = b < c$, and $a < b = c$. Hence, only the inequality

$$P(a < c) \geq P(a < b < c) + P(a = b < c) + P(a < b = c)$$

holds without any additional assumptions.
It seems that $P(a < c)$ can be exactly computed only if one can define a unique joint probability function on the proper space S. Suppose only variables *a*, *b*, *c* are involved. Then the probability space S is made of

sequences x θ_2 y θ_3 z where (x, y, z) is a permutation of the dates *a*, *b*, *c* and $\theta_2, \theta_3 \in \{>, =\}$. There are 13 such sequences corresponding to elementary events whose probability must be specified. Then for instance

$$P(a < c) = P(a < b < c) + P(a = b < c) + P(a < b = c) + P(b < a < c) + P(a < c < b).$$

But these terms are not derivable from the only knowledge of P_{ab} and P_{bc}. In particular this is especially true for $P(b < a < c)$ and $P(a < c < b)$. Besides $P(a < b < c)$ will generally differ from $P(a < b)\cdot P(b < c)$. Only $\mathrm{Max}(0, P(a < b) + P(b < c) - 1) \le P(a < b < c) \le \min(P(a < b), P(b < c))$ is valid. So a valid local inference on $P(a < c)$ in terms of a θ_2 b and b θ_3 c where θ_2 and θ_3 are of the form >, < or = without any assumption is

$$e^{<}_{a,c} \ge \mathrm{Max}(0,\ e^{<}_{a,b} + e^{<}_{b,c} - 1) + \mathrm{Max}(0,\ e^{<}_{a,b} + e^{=}_{b,c} - 1) + \mathrm{Max}(0,\ e^{=}_{a,b} + e^{<}_{b,c} - 1),$$

but this inference is not clearly complete. So the probabilistic scheme proposed in [15] looks debatable, and making it sound leaves us with only (weak) probability bounds. Possibilistic inference rules look simpler and stronger, even if more qualitative. The possibility distribution on S is defined by applying the principle of minimal specificity to all information items considered as projections.

3.3. Operation of combination

This operation deals with the situation where the information about a relation is coming from two or more distinct sources or experts. For instance, assume that the uncertain relation that holds between two temporal points *a* and *b* is provided by two experts E_1 and E_2. The former (respectively the latter) suggests the relation $r1_{a,b}$ (respectively $r2_{a,b}$) defined by the uncertainty vector $\left(\Pi_1^{<}, \Pi_1^{=}, \Pi_1^{>}\right)_{a,b}$ (respectively $\left(\Pi_2^{<}, \Pi_2^{=}, \Pi_2^{>}\right)_{a,b}$). Our goal is how to combine these two uncertain temporal relations into a single uncertain relation $r_{a,b}$. Let us denote the combination operation by the symbol $\oplus$. Then, we write $r_{a,b} = r1_{a,b} \oplus r2_{a,b}$. The idea here is that all raw information pertaining to the pair of dates (a, b) should be merged so as to produce a unique possibility triplet Π_{ab}.

In the possibilistic framework, the general ideas that govern the fusion of information issued from several distinct sources are, first, that there is no unique combination mode, and, the choice of the combination mode depends on an assumption about the reliability of sources [12]. Then, the uncertainty vector Π_{ab} for the relation $r_{a,b}$ can be computed as follows.

i) if all the sources agree and are considered as equally and fully reliable, then Π_{ab} is such that

$$\Pi^{<}_{a,b} = \min(\Pi_1^{<},\ \Pi_2^{<}),$$

$$\Pi^{=}_{a,b} = \min(\Pi_1^{=},\ \Pi_2^{=}),$$

$$\Pi^{>}_{a,b} = \min(\Pi_1^{>},\ \Pi_2^{>}).$$

This means that the source that assigns the least possibility degree to a given relation is considered as the best-informed with respect to this relation. There is no reinforcement with the operator "min". The idempotence of this operator copes with the problem of possible redundancy of sources and duplicated information. However, the obtained result may be a subnormalized possibility distribution if the sources are partially conflicting, i.e. $\max(\Pi^{<}_{a,b}, \Pi^{=}_{a,b}, \Pi^{>}_{a,b}) < 1$. In that case, a quantitative renormalization exists which consists in dividing all the possibility degrees by $\max(\Pi^{<}_{a,b}, \Pi^{=}_{a,b}, \Pi^{>}_{a,b})$. This is a combination rule commonly used in possibility theory [12]. A qualitative renormalization exists also, which is defined by setting to one the highest possibility degree(s), other degree(s) remaining unchanged.

ii) if the sources disagree and at least one of them is wrong, then $(\Pi^{<}, \Pi^{=}, \Pi^{>})_{a,b}$ is such that

$$\Pi^{<}_{a,b} = \max(\Pi_1^{<},\ \Pi_2^{<}),$$

$$\Pi^{=}_{a,b} = \max(\Pi_1^{=},\ \Pi_2^{=}),$$

$$\Pi^{>}_{a,b} = \max(\Pi_1^{>},\ \Pi_2^{>}).$$

In this case, the source that assigns the greatest possibility degree to a given relation is considered as the best-informed with respect to this relation. But then there is no need for renormalization.

Note that in the absence of conflict the combination rule is just an application of the principle of minimal specificity. In contrast, when the local pieces of information are modeled by probability distributions, Ryabov and Puuronen [15] proposed an ad hoc combination rule similar to Dempster rule of combination [3], but invariant with respect to negation. It is basically a renormalized harmonic mean of the components of the probability vectors. It clearly presupposes independence between sources. arious other options are available as for instance a weighted average of the uncertainty triplets, or a geometric mean (see Cooke, [3]).

3.4. Operation of negation

It may happen that we have no information about the relations which are possible between two temporal points, but information about relations that cannot hold between these points. The aim of the operation of negation is to infer information about the possible uncertain temporal relation $r_{a,b}$ between two points *a* and *b* when it is known that an uncertain temporal relation between *a* and *b* is not possible. For instance, it might be known that the relation

$r_{a,b}$ is definitely not "=". In other terms, it is certain with a degree equals to 1 that $r_{a,b}$ is not "=". In this case, $r_{a,b}$ can be still "<" or ">".

The pieces of information, like it is certain at level α that the relation $r_{a,b}$ is "not θ" where $\theta \in \{<, =, >\}$, are represented by assigning possibility degree $1 - \alpha$ to θ, while the two other relations have possibility degrees equal to 1. This is in agreement with the fact that $N(a\ \theta\ b) = 1 - \Pi(a\ r\ b)$ where r is the set of relations representing "not θ". For instance, if it is certain at level 0.6 that $r_{a,b}$ is not "=" then, the uncertainty vector $(\Pi^{<}, \Pi^{=}, \Pi^{>})_{a,b} = (1, 0.4, 1)$. Moreover, if it is α-certain that the relation $r_{a,b}$ is not ">" and β-certain is not "=" then, this is represented as the min-combination of the two possibility distributions. Namely, $(\Pi^{<}, \Pi^{=}, \Pi^{>})_{a,b} = \min[(1, 1, 1 - \alpha), (1, 1 - \beta, 1)] = (1, 1 - \beta, 1 - \alpha)$, applying the combination rule.

If we followed the above reasoning in the probabilistic case, the negation of P_{ab} would consist in assigning probability mass $e^{<}_{a,b}$ to the event $a \geq b$ (the disjunction of > and =), $e^{>}_{a,b}$ to the event $a \leq b$, and $e^{=}_{a,b}$ to the event $a \neq b$. Clearly, what is obtained is no longer a probability measure, but a random set describing body of uncertain evidence in the sense of belief functions [16]. However Ryabov and Puuronen [15] never envisage this possibility because they insist on ever getting a unique probability measure as the result of any inference operation. They propose to systematically share the probability carried from $\theta \in \{>, =, <\}$ over to the complement of θ between the two elements of the disjunction (e.g. $e^{<}_{a,b}$ is shared between > and =). This process is debatable since the negation of a probability triplet corresponds to a weaker form of information.

4. An illustrative example

Consider a slightly modified version of the example already discussed in [15]. Let *a*, *b* and *c* be three temporal points where the relations that could hold between them are uncertain. We know that the relation between the points *a* and *b* is provided by two information sources (supposedly reliable). According to the first source, the possibility degrees of the basic relations "<", "=", and ">" between *a* and *b* are 1, 0.2, and 0.3 respectively. The second source suggests that the possibility values of the basic relations between these two points are 1, 0.4, and 0.25. We know also that it is certain at level 0.6 that the relation between *b* and *c* is not ">". The problem is to estimate the uncertainty vector $(\Pi^{<}, \Pi^{=}, \Pi^{>})_{a,c}$ of the temporal relation that could hold between the points *a* and *c*.

The available pieces of information can be summarized as follows:

$r1_{a,b} = (1, 0.2, 0.3)$, $r2_{a,b} = (1, 0.4, 0.25)$,
$r_{b,c} = (1, 1, 0.4)$, using the negation operation.

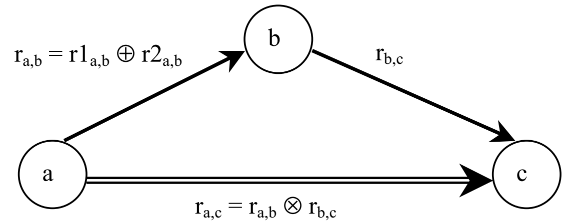

Figure 3. The structure of the example

The combination operation produces $r_{a,b} = r1_{a,b} \oplus r2_{a,b} = (1, 0.2, 0.25)$. In terms of possibilistic logic, this information reads $N(a \neq b) \geq 0.8$, $N(a \leq b) \geq 0.75$. The possibility distribution $r_{b,c}$ expresses that $N(b \leq c) \geq 0.6$. Then we get

$N(a \leq c) \geq \min(N(a \leq b), N(b \leq c)) = 0.6.$

Similarly, $N(a \neq c) \geq \min(N(a \neq b), N(a \leq b \leq c)) \geq \min(0.8, 0.6) = 0.6$. Hence $(\Pi^{<}, \Pi^{=}, \Pi^{>})_{a,c} = (1, 0.4, 0.4)$ which means that the basic relation "<" is completely possible between the temporal points *a* and *c*, and both the basic relations "=" and ">" are possible between these points with a degree 0.4.

5. Conclusion

Possibility theory provides an expressive tool for the representation and the treatment of uncertainty pervading pieces of local information in temporal reasoning. There are two interesting features of this theory in uncertainty modeling that make it different from standard probabilistic modeling. First, it can be purely qualitative, thus avoiding the necessity of quantifying uncertainty if information is poor. Second, it is capable of modeling ignorance in a non-biased way. In [4], we had discussed how to use possibility and necessity measures to estimate to what extent it is possible, or certain, that some Allen relations (or their fuzzified versions) hold between two time intervals, when knowledge is uncertain. In this work, we have shown, for the case of point algebra, how possibility theory can be directly used for handling uncertainty about the relative position of time points. The difficulty of proposing a sound probabilistic approach lies in the fact that relational temporal information is sparse, granular so that it is not possible to define a unique probability distribution over the sequences of time points without making debatable assumptions or using ad hoc inference rules. For instance it is not clear how to define a Bayesian net from the available probabilistic information. An unbiased probabilistic approach inevitably leads to handle a family of probability, and cannot rule out the case where the available information is inconsistent. The use of possibility theory naturally copes with such incomplete information.

Several developments of this preliminary work can be envisaged. First it seems that a direct encoding of possibilistic relational time information into possibilistic logic would provide a sound and complete reasoning method, attaching degrees of necessity to relational statements of the formal a θ b. There is no need to resort to ad hoc inference rules. It would provide for an automated reasoning tool. Since the strict equality rarely holds between two temporal points in practice, the proposed approach could also be extended in order to allow for handling a fuzzy partition made by the three basic relations "smaller than (<)", "approximately equal (≈)", and "greater than (>)". Lastly, it may be possible to reconsider the probabilistic approach in the light of evidence theory of Shafer, so as to more simply account for the incompleteness of information that seems to pervade the declarative approach to temporal reasoning.

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