# Representing Temporal Relationships Between Events and Their Effects

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## Abstract

*Temporal relationships between events and their effects are complex. As the effects of a given event, a proposition may change its truth value immediately after the occurrence of the event and remain true until some other events occur, while another proposition may only become true/false from some time after the causal event has occurred. Expressing delayed effects of events has been a problematic question in most existing theories of action and change. This paper presents a new formalism for representing general temporal causal relationships between events and their effects. It allows expressions of both immediate and delayed effects of events, and supports common-sense assertions such as "effects cannot precede their causes".*

## 1. Introduction

The notion of state is central in most conceptualisations of the physical world. The states of the world, which may be expressed as a set of propositions, are changed by certain relevant patterns which are termed in the literature as *actions/events*. The concepts of change and time are deeply related since changes are caused by events, while events can be expressed as actions performing over certain time intervals/points. Over the last decade, several extensions to McCarthy and Hayes' framework of situation calculus [8,9], and to Kowalski and Sergot's event calculus [6], have been proposed (including that of Schubert [15], of Lin and Shoham [7], of Gelfond *et al* [4, 5], of Pinto and Reiter [12, 13, 14], and of Miller and Shanahan [11]) in order to enrich the temporal ontology in reasoning about action and change. These formalisms usually associate entities such as *situations/states*, *fluents*, and *actions* with some special *time*, where time elements are characterised as points and intervals are constructed out of points. However, these approaches have not gone as far as one would like for dealing with temporal issues in representing and reasoning about actions and their effects, and there are still some problematic issues which have not been satisfactorily solved.

Generally speaking, the world persists in a given state until some action is carried out to change it into another state. While some actions may be instantaneous, most of them perform over some interval of time. Hence, both points and intervals are needed for expressing the time spans of situations and actions. However, temporal relationships between events and their effects are complex. For instance, on the one hand, as the effect of a given event, a proposition may change its truth value immediately after the occurrence of the event, i.e., there is no delay time between the occurrence of the event and the change of the truth value; on the other hand, a proposition may only become true/false from some time after the event that is supposed to make it to change its true value has occurred.

Expressing the delayed effects of events has been a problematic question in reasoning about action and change. In fact, in most existing temporal theories, including that of Schubert [15], of Lin and Shoham [7], of Pinto and Reiter [13, 14], and of Miller and Shanahan [11], the effects of actions have been represented as the immediate results just after the execution of the corresponding actions. Actions with delayed effects have not been addressed in these systems. In [5], Gelfond et. al. propose an approach in terms of *duration of actions* to describe an action with delayed effects. They simply count the delay time in the duration of actions. In this way, actions have been considered to continue until the results of actions appear. However, this does not really match the common-sense idea of an action with delayed effect, which in fact intuitively means that there is a delay time between the performance of the action and its effect. E.g., *25 seconds after a pedestrian starts pressing the button at the crosswalk, the pedestrian crossing light* *turns to yellow from red, and after another 5 seconds it, turns to green.*

The objective of this paper is to propose a new formalism for representing general temporal causal relationships between events and their effects. In section 2, we present the logical preliminaries of the new formalism. Section 3 illustrates the expressive power of the new formalism. Section 4 concludes the paper.

## 2. The Logical Preliminaries

The formalism consists of three disjoint sorts of objects **T**, **F** and **A**, called *times*, *fluents* and *actions*, respectively (By fluents in this paper, we mean propositions whose truth values are dependent on time). We shall denote the elements of **T**, **F** and **A** by letters (possibly indexed) $t$, $f$, and $a$.

### 2.1 The Underlying Time Structure

In [10], a general time theory is proposed which addressed both points and intervals as primitive time elements. It is in fact an extension to Allen and Hayes' interval based temporal logic [1, 2]. In what follows, we briefly describe such a time structure which will be used as the temporal basis of the formalism.

We use *Duration* to denote a duration assignment function which assigns a non-negative real number for each time-element in **T**. A time-element $t$ is called a (time) interval if $Duration(t) > 0$, otherwise $t$ is called a (time) point. The primitive relation over time elements, *Meets*, is axiomatized by:

(A1) $\forall t_1,t_2,t_3,t_4(Meets(t_1,t_2) \wedge Meets(t_1,t_3) \wedge Meets(t_4,t_2) \Rightarrow Meets(t_4,t_3))$

i.e., the "place" where two time elements meet is unique and closely associated with the time elements;

(A2) $\forall t \exists t',t''(Meets(t',t) \wedge Meets(t,t''))$

i.e., every time element has at least one neighbouring time element proceeding it, and another succeeding;

(A3) $\forall t_1,t_2(\exists t',t''(Meets(t',t_1) \wedge Meets(t_1,t'') \wedge Meets(t',t_2) \wedge Meets(t_2,t'')) \Rightarrow t_1 = t_2)$

i.e., the time element standing between any two meeting places is unique;

(A4) $\forall t_1,t_2(Meets(t_1,t_2) \Rightarrow \exists t \forall t',t''(Meets(t',t_1) \wedge Meets(t_2,t'') \Rightarrow Meets(t',t) \wedge Meets(t,t''))$

i.e., if two meeting places are separated by a sequence of time elements, then there is a time element which connects these two meeting places. Hence, by recalling axiom (A3), for any two adjacent time elements, $t_1$ and $t_2$, we may denote the *adjacent union* of $t_1$ and $t_2$ as a time interval, $t = t_1 \oplus t_2$. N.B., $t_1 \oplus t_2$ always implies that $Meets(t_1, t_2)$.

(A5) $\forall t_1,t_2(Meets(t_1,t_2) \Rightarrow Duration(t_1) > 0 \vee Duration(t_2) > 0)$

i.e., points can not meet other points;

(A6) $\forall t_1,t_2(Meets(t_1,t_2) \Rightarrow Duration(t_1 \oplus t_2) = Duration(t_1) + Duration(t_2))$

i.e., the duration assignment is consistent with the adjacent union operator "$\oplus$" and the conventional addition "+";

(A7) $\forall t_1,t_2,t_3,t_4(Meets(t_1,t_2) \wedge Meets(t_3,t_4) \Rightarrow Meets(t_1,t_4) \nabla \exists t'(Meets(t_1,t') \wedge Meets(t',t_4)) \nabla \exists t''(Meets(t_3,t'') \wedge Meets(t'',t_2)))$

i.e., meeting places of time elements are totally ordered, which ensures the linearity of time structure (here, "$\nabla$" represents "exclusive-or").

### 2.2. Fluents

The truth values of fluents are dependent on times. We use $Holds(f, t)$ to denote that fluent $f$ is true over time t.

We impose the following axioms on predicate *Holds*:

(A8) $\forall f \forall t(Holds(f, t) \Leftrightarrow \forall t'(Sub(t', t) \Rightarrow Holds(f, t')))$

N.B. Here, $Sub(t_1,t_2)$ intuitively denotes that time $t_1$ is a part of time $t_2$:

$$\forall t_1,t_2(Sub(t_1,t_2) \Leftrightarrow Equals(t_1,t_2) \vee Starts(t_1,t_2) \vee Finishes(t_1,t_2) \vee During(t_1,t_2))$$

where,

$$Equals(t_1,t_2) \Leftrightarrow t_1 = t_2,$$
$$Starts(t_1,t_2) \Leftrightarrow \exists t'(t_1 \oplus t' = t_2)$$
$$Finishes(t_1,t_2) \Leftrightarrow \exists t'(t' \oplus t_1 = t_2),$$
$$During(t_1,t_2) \Leftrightarrow \exists t',t''(t' \oplus t_1 \oplus t'' = t_2)$$

(A9) $\forall t_1,t_2 \forall f(Holds(f, t_1) \wedge Holds(f, t_2) \wedge Meets(t_1, t_2) \Rightarrow Holds(f, t_1 \oplus t_2))$

Following Galton's notation [3], we refer to the operator "not" as fluent-negation, so that not($f$) represents the negation of fluent $f$, to be kept distinct from ordinary sentence-negation, symbolised by "$\neg$". Properties about the negation of fluents can be given as below:

$$\forall t(Holds(\text{not}(f), t) \Rightarrow \forall t'(Sub(t', t) \Rightarrow \neg Holds(f, t')))$$

$$\forall t(Holds(f, t) \Rightarrow \forall t'(Sub(t', t) \Rightarrow \neg Holds(\text{not}(f), t')))$$

Hence, any fluent and its negation can not be both true over the same time $t$. In other words, they are in conflict with each other. However, it is important to note that, for a given fluent, say $f$, it may be the case that its negation not($f$) is not the only fluent which conflicts with it. That is, there may be some fluents other than not($f$) that can not be true together with fluent $f$. Consider the example of traffic signals: let *GreenOn*, *YellowOn* and *RedOn* denote fluents "the green light is on", "the yellow light is on" and "the red light is on", respectively. On the one hand, we know that at any time $t$, if fluent *GreenOn* is true, then its negation not(*GreenOn*) must be false. On the other hand, while fluent *GreenOn* is true, fluent *RedOn* must be false as well, although fluent *YellowOn* may be true.

We use Conf($f_1$, $f_2$) to denote that fluent $f_1$ is in conflict with fluent $f_2$:

$$\text{(A10)}\ \forall f_1,f_2(Conf(f_1, f_2) \Leftrightarrow \forall t(\ Holds(f_1, t) \Rightarrow Holds(\text{not}(f_2), t)) \wedge Holds(f_2, t) \Rightarrow Holds(\text{not}(f_1), t)))$$

Specially, the negation of fluent $f$ is in conflict with $f$.

### 2.3 States/Situations

A *state*, $s$, is defined as a subset of $\mathbf{F} \cup \mathbf{F}^{\sim}$, where $\mathbf{F}^{\sim} = \{\text{not}(f) \mid f \in \mathbf{F}\}$.

A state, $s$, is *complete* if and only if:

$$\forall f(f \in s \vee \text{not}(f) \in s)$$

otherwise, $s$ is called an *incomplete* state.

A state is called *consistent* if and only if

$$\forall f_1, f_2(f_1 \in s \wedge f_2 \in s \Rightarrow \neg Conf(f_1, f_2))$$

otherwise, $s$ is called an *inconsistent* state.

In what follows, we shall use **S** to denote the set of all the consistent complete states.

A situation, $st$, is a state $s$, associated with a particular time (point or interval) $t$, over which the world persists in that state. We shall write $st$ as $st = \langle s, t \rangle$, and denote by **Sit** the set of all situations.

$$\text{(A11)}\ \forall st \exists s \exists t(st = \langle s, t \rangle)$$

$$\text{(A12)}\ \forall s \forall t(\langle s, t \rangle \in \mathbf{Sit} \Leftrightarrow \forall f(\ f \in s \Leftrightarrow Holds(f, t) \wedge f \notin s \Leftrightarrow Holds(\text{not}(f), t)))$$

$$\text{(A13)}\ \forall st(\exists s_1,s_2 \exists t_1,t_2(st = \langle s_1, t_1 \rangle \wedge st = \langle s_2, t_2 \rangle) \Rightarrow s_1 = s_2 \wedge t_1 = t_2)$$

Axiom (A13) states that the representation for any situation is unique. Hence, for any situation $st$, if $st = \langle s, t \rangle$, we shall call $s$ and $t$ the *reference state* and the *reference time* of situation $st$, denoted as *State*($st$) and *Time*($st$), respectively.

By (A12) and (A8), we can directly infer that:

$$\forall s \forall t,t'(\langle s, t \rangle \in \mathbf{Sit} \wedge Sub(t', t) \Rightarrow \langle s, t' \rangle \in \mathbf{Sit})$$

and by (A12) and (A9):

$$\forall s \forall t,t'(\langle s, t_1 \rangle \in \mathbf{Sit} \wedge \langle s, t_2 \rangle \in \mathbf{Sit} \wedge Meets(t_1, t_2) \Rightarrow \langle s, t_1 \oplus t_2 \rangle \in \mathbf{Sit})$$

For reasons of simplicity, we may sometimes use *Holds*($f$, $st$) to denote that fluent $f$ is observed as true in situation $st$, provided that:

$$\forall st \forall f(Holds(f, st) \Leftrightarrow Holds(f, Time(st)))$$

### 2.4 Actions/Events

Actions, given in terms of certain relevant patterns, may perform over some specified time points or intervals. We use *Performs*($a$, $t$) to denote that action $a$ performs over time $t$.

$$\text{(A14)}\ \forall a \forall t_1,t_2(\ Performs(a, t_1) \wedge Performs(a, t_2) \wedge Meets(t_1, t_2) \Rightarrow Performs(a, t_1 \oplus t_2))$$

i.e., if an action performs over two adjacent times respectively, then it performs over the ordered union of these two times.

The world holds in one state until an action is performed over some special time to change it into another state (may be still the same one). We shall call such a phenomenon an event. Hence, analogously to the form of a situation which are defined as a pair of a state and a time element, an event is given in the form of a pair of an action and a time, such that:

(A15) $\forall a \forall t(<a, t> \in \mathbf{E} \Leftrightarrow Performs(a, t))$

where **E** denotes the set of all events.

Again, analogously to the definitions of the reference state and the reference time of a given situation, for an event $e$ with the form of $<a, t>$, we shall call $a$ and $t$ the *reference action* and the *reference time* of event $e$, and, without confusion, denote them as $a = Action(e)$ and $t = Time(e)$, respectively.

We use $Changes(sit_1, e, t, sit_2)$ to denote the proposition that, immediately after time $t$, event $e$ changes situation $sit_1$ into situation $sit_2$ (see Fig.1):

$$\begin{aligned}(A16)\ \forall e{\in}\mathbf{E}\forall sit_1,sit_2{\in}\mathbf{Sit}\forall t{\in}\mathbf{T}(&Changes(sit_1,e,t,sit_2) \Leftrightarrow \\ &Meets(Time(sit_1), Time(e)) \\ &\wedge (Equals(Time(e), t) \vee Starts(Time(e), t) \\ &\wedge Meets(t, Time(sit_2)))\end{aligned}$$

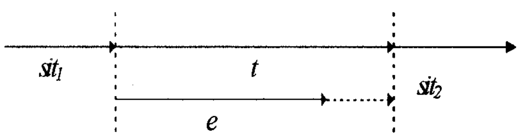


Figure 1

## 3. The Expressive Power

Axiom (A16) ensures that, the reference time of any event "*Meets*" or is "*Before*" the reference time of the corresponding situation caused by the event. Hence, one can directly infer the common-sense assertion that "effects cannot precede their causes". In fact, axioms (A16) allows a flexible temporal relationship between effects and their causal events. On the one hand, some fluents may change their truth values immediately after the reference time of the causal event. In other words, there is no delay time between the effect and its causal event. This case corresponds to the form of axiom (A16) where the temporal relation between $Time(e)$ and $t$ in (A16) is $Equals(Time(e), t)$, rather than $Starts(Time(e), t)$. On the other hand, it may be the case that some fluents change their truth values from some time after the reference time of the causal event, i.e., there may be some delay times that stand between the effect and its causal event. Again, this case can be successfully expressed by taking the temporal relation between $Time(e)$ and $t$ in (A16) as $Starts(Time(e), t)$, rather than $Equals(Time(e), t)$. For instance, consider the example of the pedestrian crossing lights mentioned in the introduction. We employ the following three fluents:

*RedOn*: the red light at the crosswalk is on;
*YellowOn*: the yellow light at the crosswalk is on;
*GreenOn*: the green at the crosswalk is on;

Let *PressButton* denote the action of pressing the button, and $Sit_{Red}$ denote a situation in which the red light is on:

$Holds(RedOn, Sit_{Red})$

Assuming in situation $Sit_{Red}$ a pedestrian presses the button, e.g., for 1 second, then we have event $E = <PressButton, T_E>$, and can express the effects of event $E$ as below:

$Changes(Sit_{Red}, E, Time(E), Sit_{Effect}')$,
$Changes(Sit_{Red}, E, T_1, Sit_{Effect}'')$,
$Changes(Sit_{Red}, E, T_2, Sit_{Effect}''')$

where

$Dur(Time(E)) = 1$,
$Dur(T_1) = 25$,
$Dur(T_2) = 30$,
$Holds(RedOn, Sit_{Effect}')$,
$Holds(YellowOn, Sit_{Effect}'')$,
$Holds(GreenOn, Sit_{Effect}''')$.

Here, we have directly expressed the fact that there is no time delay between the reference time of event $E$ and the reference time of the effect situation $Sit_{Effect}'$.

Also, we can successfully express the fact there is a delay time, say $T_{D1}$, standing between the reference time of event $E$ and the reference time of the effect situation $Sit_{Effect}''$, that is:

$Meets(Time(E), T_{D1})$,
$Meets(T_{D1}, Time(Sit_{Effect}''))$

Similarly, we can express that there is a delay time, say $T_{D2}$, standing between the reference time of event $E$ and the reference time of the effect situation $Sit_{Effect}'''$, that is:

$Meets(Time(E), T_{D2})$,
$Meets(T_{D2}, Time(Sit_{Effect}'''))$

Additionally, we can easily infer that:

$T_{D2} = T_{D1} \oplus Time(Sit_{Effect}'')$
where

$Dur(Time(Sit_{Effect}')) = Dur(T_{D1}) = 24,$
$Dur(Time(Sit_{Effect}'')) = 5,$

The above knowledge can be graphically presented as Figure 2:

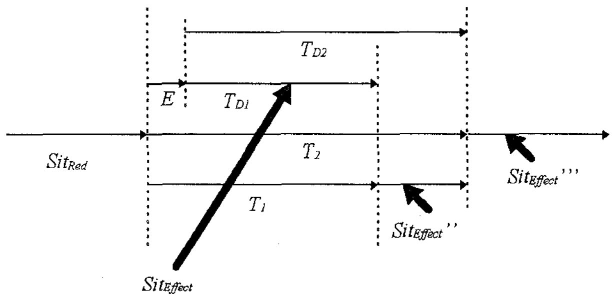


**Figure 2**

## 4. Conclusions

In this paper, we have presented a new formalism for representing general temporal causal relationships between events and their effects. This formalism is in fact achieved by means of synthesising the quintessence of some representative theories, including that of McCarthy and Hayes [8, 9], Allen [2], Kowalski and Sergot [6], Lin and Shoham [7], Gelfond et al [4, 5], Pinto and Reiter [13, 14], and Miller and Shanahan [11], etc. The main technical contribution of the revised formalism in its power of expressing flexible temporal relationships between effects and their causes, including both immediate and delayed effects of events. Also, it is shown that the new formalism supports common-sense assertions such as "effects cannot precede their causes".

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