Generating Instantiations of Contextual Scenarios of Periodic Events

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Abstract.
In this paper, we consider an expressive formalism to deal with temporal constraints between periodic events which takes into account different components such as frame times, numeric quantification, periods, and qualitative temporal constraints. We define the notions of (contextual) concretization of temporal constraints in such a formalism and of (contextual) scenario of a KB of temporal constraints. We then use these notions in order to introduce an algorithm which generates an instantiation of events which satisfies a scenario.

1 Introduction

The notion of events that repeat more or less regularly in time (henceforth called *periodic events*) is an ubiquitous one in many areas, spanning from scheduling and planning to work flow analysis, protocol management, and temporal databases. Thus, in recent years, a good deal of research both in AI and in temporal databases focused on the treatment of periodic events, following at least three main streams. The first approach focused on the development of formalisms to enable users to introduce user-defined periods (e.g., "the third Tuesday of each month that is a not a vacation"; consider, e.g., [Leban, 86, Soo & Snodgrass, 93, Cukierman & Delgrande, 95]) or dealt with granularity problems in temporal databases [Wang et al., 97]. The second approach introduced first order or temporal logics in order to model different aspects of periodicity (see, e.g., the survey in [Tuzhilin & Clifford, 95]). Finally, the third approach mainly focused on the treatment of *temporal constraints* between periodic events, mainly considering *qualitative* constraints, such as "precede" in Ex.1 (consider, e.g., [Ladkin, 86; Morris et al., 96]).

Ex.1 "breakfast always precedes lunch"

Mny of these papers propose specialised formalisms to represent qualitative temporal constraints and proposes algebraic approaches to temporal reasoning in order to check the consistency of a knowledge base of temporal constraints and to infer new constraints from it. In [Terenziani, 97a, 97b], these types of formalisms and reasoning procedures have been extended in order to deal also with qualitative constraints that are "period-dependent", in the sense that hold in a specific frame of time and on a specific (possibly user-defined) period, and to deal with numeric quantification, such as in Ex.2

Ex.2 "Between 29/9/97 and 19/12/97, <u>twice</u> <u>each Monday</u> there is <u>an</u> hour of History (strictly or not) <u>befor</u>e <u>two</u> hours of Chemistry"

On the other hand, despite their importance in practical applications, the problems of formally *defining* [Morris et al., 96] and of *generating* or *projecting* ([Morris et al., 95; Loganantharaj & Gimbrone, 97; Loganantharaj & Kurkovsky, 97]) a scenario for a KB (knowledge base) of temporal constraints have started to be faced only recently. Roughly speaking, temporal constraints between periodic events may have different instantiations (henceforth called *"concretizations"*, as in [Morris et al., 95]) at different times. For instance, Figure 1 shows a possible concretization of the temporal constraint in Ex.2 in the frame of time between 20/10/97 and 27/10/97. As shown in Figure 1, the concretization of a "period-dependent" temporal constraint in a given frame of time could involve multiple "instantiations" of the constraint (e.g., there are two Mondays between 20/10/97 and 27/10/97). Moreover, concretization have to consider *correlations* between events [Morris et al., 96; Terenziani, 97a,97b]. E.g., Ex.2 does not state that, on Mondays, any hour of History precedes any pair of hours of Chemistry, but that precedence holds between correlated pairs.

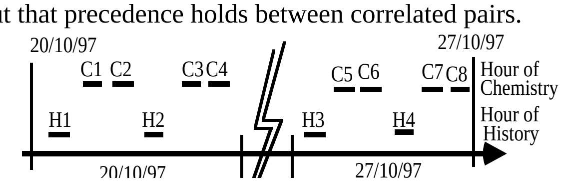

Figure 1. A possible concretization of the temporal constraint in Ex.2: "Between 29/9/97 and 19/12/97, <u>twice</u> <u>each Monday</u> there is <u>an</u> hour of History (strictly or not) <u>befor</u>e <u>two</u> hours of Chemistry"

Roughly speaking, given a KB of temporal constraints about periodic events, a scenario of KB in a given frame of time could be defined as a concretization of all the constraints in KB. Generating an instance of a consistent scenario is very important in many applications, to obtain an instantiation of the activities to be done, and of the temporal constraints between them [Morris et al., 95], or to project the activities in the future [Loganantharaj & Gimbrone, 97; Loganantharaj & Kurkovsky, 97].

In previous papers, we proposed a formalism to deal with different types of temporal constraints between periodic events (such as, e.g., those in Ex.1 and Ex.2 above), and introduced a polynomial-time algorithm to perform temporal reasoning on a KB of constraints in the formalism and to check its consistency [Terenziani, 97a, 97b]. In this paper we extend our previous approach by proposing (i) a formal definition of the notions of concretization and of (contextual) scenario (section 4), and of the notion of satisfaction of a scenario by an instantiation of periodic events (called "*p-instantiation*") (section 5) (ii) a generation mechanism based on the definitions above which generates a p-instantiation which satisfy a scenario (section 6). Section 2 introduces some basic ontological notions underlying the mea[redacted]ing [redacted] temporal constraints between periodic e[redacted]d sect[redacted]n 3 presents our formalism (see also [T[redacted]ia[redacted]a, [redacted]).

2 Basic ontological notions

A *periodic event* can be seen as a *collection*[redacted]*s of events* of the same type which do not ov[redacted]in[redacted]. E.g., "in March 1997, John went to the worki[redacted]ffic[redacted] each Monday" can be seen as the colle[redacted] instances of the event of John going to [redacted]ffic[redacted] which took place on Mondays in Ma[redacted]. Let [redacted]e function from periodic event[redacted]o ins[redacted]ces o[redacted]uch that, given any [redacted]t ev* [redacted](ev*)= {ev[redacted], ev_n} is th[redacted] i[redacted]e) [redacted]f t[redacted]stances of ev*. Every ins[redacted]ce o[redacted]ev[redacted]k[redacted]ace[redacted] *time interval*, which is a conve[redacted] of[redacted]ts[redacted]vee[redacted]tar[redacted]g an[redacted]n ending point. Thu[redacted]ver[redacted] i[redacted]t ev_i [redacted]**xt**(ev_i)=i_{ev_j} where i_{ev}[redacted]he[redacted]im[redacted]onv[redacted]e i[redacted]al in which ev_j happe[redacted]Th[redacted]co[redacted]ion[redacted]nts[redacted]s place on a set of time[redacted]rv[redacted]wh[redacted]re [redacted]por[redacted]tents of the instances [redacted]ve[redacted]co[redacted] it. [redacted]*iods* (e.g., "Mo[redacted]e[redacted]ses o[redacted]ic [redacted]s, since the ins[redacted]cur [redacted]in[redacted]a[redacted]me[redacted] particular, th[redacted]functi[redacted]o appl[redacted]o [redacted]ds[redacted] example, (Mondays*[redacted]es as result the [redacted]f a[redacted]ondays. For the sake of brevity we also [redacted]funct[redacted]**xt** to set of instances, intending that it [redacted]plies[redacted]nstance in the set. For example, **Ext**((ev*)) den[redacted]the set of time intervals when the instances of ev* took place.

Let us now take into account the ontological notions needed in order to deal with the semantics of the ***temporal constraints*** between periodic events. Constraints such as that in Ex.2 locate in time periodic events by pairing each instance (or set of instances) of a periodic event with the instance of period when it happened. Following Morris [Morris et al., 96], we call this pairing relation "*association*", defined as follows:

Definition. Given a period C*, and given two instances of events e_1 and e_1', the relation $ASSOC_{C*}(e_1,e_1')$ holds if and only if e_1,e_1' happen in the same instance C_i of C*.

For example, given the period Mondays*, and given an instance $Monday_k$ of Mondays* (e.g., September 29, 1997), $ASSOC_{Monday_k}(e_1,e_1')$ holds if and only if both e_1,e_1' happened on $Monday_k$.

$ASSOC_{C*}$ is an equivalence relation (reflexive, symmetric and transitive).

Definition. We indicate as $[s_i]_{C*}$ the equivalence classes introduced by equivalence relation $ASSOC_{C*}$ (i.e., $[s_i]_{C*} = [s_j]_{C*}$ if and only if $ASSOC_{C*}(s_i,s_j)$ holds)

The treatment of qualitative temporal constraints between (numerically quantified) periodic events is quite complex. First of all, qualitative constraints hold between sets of inst[redacted]ces o[redacted]vents. E.g., in Ex.2, a precedence relation is [redacted]d bet[redacted]een the temporal extent of an instance of an [redacted]r o[redacted]tory and the extent of two instances of an hour [redacted]Che[redacted]ry. We thus introduce the function CV-CB

[redacted]**ini**[redacted] Given a set S={i_1, ..., i_k} of time intervals, i = [redacted]-C[redacted] is the minimal convex time interval which [redacted]ers[redacted]he intervals in S.

[redacted]rec[redacted]in the usual interpretation, a qualitative relation [redacted]wee[redacted]o periodic events ev1* and ev2* involves an [redacted]erl[redacted] relation which pairs instances (or collections [redacted]inst[redacted]s) of the two events. As in [Morris et al., 96], [redacted]is relation *correlation*. Correlation is a relation b[redacted]en instances of periodic events, which holds as a result of some contingent relation in the world between them (see [Morris et al., 96; Terenziani, 97a] for more details). For instance, Ex.2 does not state that, on Mondays, any hour of History precedes any collection of two hours of Chemistry, but that precedence holds between correlated pairs. In our approach, the relation COR represents correlation:

Definition. Given two sets of instances of events s1={e_1, ... ,e_h} and s2={e'_1, ... ,e'_k}, the relation COR(s1,s2) holds if and only if s1 and s2 are correlated.

COR is an equivalence relation (reflexive, symmetric and transitive).

Definition. We indicate as $[s_i]$ the equivalence classes introduced by equivalence relation COR (i.e., $[s_i] = [s_j]$ if and only if COR(si,sj) holds)

E.g., in the example in Figure 1, we could have COR({H1},{C1,C2}), COR({H2},{C3,C4}), COR({H3},{C5,C6}), COR({H4},{C7,C8}).

3 Temporal formalism

Our "high-level" temporal formalism has been defined in such a way that temporal constraints such as that in Ex.2 can be expressed, but path-consistency on a knowledge base of temporal specifications in our formalism can be computed in polynomial time. The syntax of temporal constraints in our formalism is the following:

(SYN) <Frame_Time> <Num_Quant> EACH
<Period_Name> <Num_Quant> <Periodic_Event>
<Qual_Rel> <Num_Quant> <Periodic_Event>

<Frame_Time> indicates a frame of time (time interval) ranging from a starting point to an ending point, and is represented by a pair of dates (e.g., [29/9/97-19/12/97]). The frame time ($-\infty,+\infty$) indicates the whole time-line. **<Num_Quant>** is a numeric quantifier expressed as "n-times", where n is a natural number. We assume a "strong" interpretation of numeric quantifiers: for example, in our approach, (S1) below involves the cardinality constraint that there are exactly 4 instances of H_Chem* and exactly 2 instances of H_Hist* contained (strictly or not) into each instance of Monday in the frame time [29/9/97 - 19/12/97]. **<Periodic_Event>** is the representation of an event repeated in time. In this paper, for the sake of simplicity, we use a representation of the form "event_type(e)" (e.g., the predicate H_Chem*(e) is true for each event e of attending an hour of lesson in Chemistry). However, the approach we propose is independent of the type of representation used to describe periodic events. **<Qual_Rel>** is a qualitative relation between the temporal extent of two periodic events. In our approach, it can be expressed using any relation (ambiguous or not) in Allen's Interval Algebra [Allen, 83]. **<Period_Name>** is a user-defined identifier of a period (calendric definition; e.g., "1st-Tuesdays-of-Januarys*"). In our approach, the definitions of the periods must be provided by the user using a slight adaptation of Leban's language [Leban et al., 87], which we discussed in [Terenziani, 97a]. Thus, our approach also deals with user-defined periods (calendar-dates). E.g., given the definition of Mondays*, Ex.2 can be represented by (S1) in our formalism:

(S1) [29/9/97-19/12/97] 2-times EACH Mondays* 1-times H_Hist*(e) (BEFORE,MEETS) 2-times H_Chem*(e')

While an intuition of the meaning of (S1) is shown in Figure 1, a more accurate description will be provided in the next section, where we will define the concretizations of (S1). For the sake of brevity, in this paper we limit the description of our formalism to the essential. However, notice that we also deal with "period-independent" qualitative constraints such as those in Ex.1, and with constraints on the number of repetition of events in (user-defined) periods. A complete description of our high-level formalism can be found in [Terenziani, 97b]. Moreover, in [Terenziani, 97a] we proposed a logical semantics (in F.O.L) for our formalism, we widely discussed the motivations underlying our choice of the high-level formalism, and we proposed the comparisons (e.g., in the expressive power) with others specialised and/or logical approaches to periodic events (see, e.g., the survey in [Tuzhilin & Clifford, 95]).

KB1 below is the example of KB of temporal constraints we carry on throughout this paper. Predicates H_TOPIC*(e) represent the fact that e is an event of an hour of lesson of TOPIC; we regard each hour of lesson as an independent event. We suppose that the user provided the definitions of the periods s/he used in the temporal constraints (Mondays*, Tuesdays* and Mon+Tue* -representing the period Monday plus Tuesday- in KB1), using our adaptation of Leban's language [Terenziani, 97a].

(KB1)
(S1) [29/9/97-19/12/97] 2-times EACH Mondays* 1-times H_Hist*(e) (BEFORE,EQUAL) 2-times H_Chem*(e')
(S2) [29/9/97-19/12/97]1-times EACH Tuesdays* 2-times H_Chem*(e)(BEFORE,EQUAL) 1-times H_Phys*(e')
(S3) [29/9/97-19/12/97]1-times EACH Mon+Tue* 1-times H_Math*(e) (BEFORE,EQUAL) 1-times H_Phys*(e')

4 Concretizations and scenarios

Intuitively, a concretization of a temporal constraint TC between periodic events is a *temporal constraint TC'* obtained by "instantiating" TC. However, in our high-level formalism different related components (e.g., period, qualitative temporal constraint) have to be instantiated. Thus, we define concretization in an incremental way. First (subsection 4.1), we define the concretization [illegible] the components of a temporal constraint (i.e. [illegible] period and of the qualitative relation). Then, in [illegible] we [illegible] concretization of a whole temporal constraint pe[illegible] and 4.3 we specialize such a definition to [illegible] concretization of a temporal constraint in a gi[illegible] of time. Finally, 4.4 proposes an example, and [illegible] the notion of a scenario of a KB in a given [illegible]

4.1 Periods and qualitative relation[illegible]

Definition . A concretization Conc$_P$(C[illegible] of a [illegible] is an instantiatio[illegible] of C* (i.e., C_i [illegible] (C*)). For example, the [illegible]st week of September 97 is a concretizat[illegible] week. In the following, for the sake of brevity, we [illegible]enote each instance of a period with the time interval in wh[illegible]h it t[illegible]es place. Thus, we have [1/9/97 - 7/9/97] [illegible] Conc$_P$([illegible]eeks*). Notice also that the same instance may be the concretization of different periods: e.g., October 1st. 1997 is a concretization of day, of Wednesday, of first-Wednesday-of-each-month, etc.

A concretization of a (possibly ambiguous) qualitative temporal constraint <Qual-Rel> component of our high-level temporal constraints is a selection of one of the atomic unambiguous qualitative constraint, and an n-concretization is a selection of n (not necessarily distinct) atomic qualitative constraint (e.g., BEFORE

$Conc_Q$(BEFO[redacted],ME[redacted]S): {BEFORE, MEETS, BEFORE} 3-Co[redacted]EFO[redacted], MEETS, EQUAL)

Definition . A conc[redacted] $Conc_Q$(R) of a qualitative constraint R={R1, [redacted] Rk[redacted]here each Ri 1≤i≤k is one of the 13 atomic relatio[redacted]f A[redacted]s interval algebra) is an atomic relation Rj {[redacted]k}.

A n-concretization [redacted]-$Conc_Q$(R) of a qualitative constraint R={R1, ..., [redacted]k} is [redacted]set of[redacted]omic[redacted]lations $\{R_{j1}, ..., R_{jn}\}$, R_{ji} {R1,[redacted]1≤ x [redacted].

4.2 Concretization of a temporal c[redacted]rai[redacted]

In order to define concretizations o[redacted]mpo[redacted]onstraints (of type SYN), we use the following[redacted]tio[redacted]e indicate as |s| the cardinality of a set s. [redacted]et [redacted]stances of events s={e_1, ..., e_k} and [redacted] peri[redacted]*, [redacted]use $[s]_{C*}$ where s={e_1, ..., e_k} as [redacted]shortha[redacted]r [redacted]* = ... = $[e_k]_{C*}$ (i.e., to state that [redacted] s [redacted]during the very same insta[redacted]e of C*). [redacted]urthe[redacted]en periodic event ev*, we wri[redacted]s_i (ev[redacted] to [redacted] that s_i is a set of n distin[redacted]instan[redacted]s of ev*.

A temporal c[redacted]strain[redacted]n our form[redacted]lism may have different concretizations[redacted]ep[redacted]ding on the [redacted]osen concretizations for the peri[redacted] the [redacted]ualitative [redacted]onstraint, an[redacted]for the instances [redacted]e[redacted]ts. On[redacted]atural way [redacted] capturing th[redacted] fact is to ado[redacted]-[redacted]rm[redacted]lae of [redacted] forms (F[redacted] and (F2) belo[redacted]:

(F1) [redacted] xn [whe[redacted]Cond(x, x1[redacted] xn)[redacted]C'(x, x[redacted]..., [redacted])

(F2) **AND {** set-definition **}** -[redacted]rmula

The first part of (F1) is a declaration of -[redacted]uantified varia[redacted]es. The second part (in square brackets; called "where[redacted]art in the following) is a logical formula which defines[redacted] set of conditions that the bindings of the variables [redacted]ve to satisfy. Finally, the third part (called "TC" [redacted]te[redacted]oral Constraint- part) is the logical formula whic[redacted] repr[redacted]ents the concretization of the temporal co[redacted]traint. T[redacted]e intended meaning of the application of a -[redacted]rmula of t[redacted] form (F1) to a set of elements a1, ... ,an is the foll[redacted]ing: the formula can only be applied to elements th[redacted] satisfy the conditions in the "where" part (otherw[redacted] th[redacted]esult of the application is undefined), and gives [redacted] the formula obtained binding the - variable[redacted] t[redacted]elements a1, [redacted] an and substituting the bindin[redacted] T[redacted]part. The r[redacted]ult[redacted]f a partial application is a [redacted]in w[redacted]ich on[redacted] so[redacted]e [redacted] the variables have bee[redacted] example,

([redacted]whe[redacted]d(x) [redacted]en(y)] Even([redacted]y)) 3, 4 = [redacted]ven[redacted]2) [redacted]e bindings [redacted]e x=3, y[redacted])

([redacted]y [where O[redacted](x) [redacted]ven(y)] Even([redacted]*y)) 4 = [redacted][where Od[redacted]x)] Even(x*4) (the bindings are y=4)

(F2) represents the AND-ing of the application of a -[redacted] formula like (F1) to each one of the elements in set-definition.

We can now define the ("gener[redacted]") co[redacted]retization of a temporal constraint in [redacted]ism.

Definition . The c[redacted]ncret[redacted]on[redacted]nc$_{TC}$(T[redacted] of a temp[redacted]ral constraint TC o[redacted] fo[redacted] SYN), i.e., of the form "[redacted]2-times EACH C[redacted]-ti[redacted]ev1* R n3-times ev2*", [redacted]ere I is a frame tim[redacted] a[redacted]-defined) period ev1* and [redacted]2* two[redacted]iodi[redacted]nts[redacted] R a qualitative tempo[redacted]nstra[redacted] is d[redacted]ol[redacted]

Conc[redacted](I n[redacted]es E[redacted]CH C[redacted]1* R n3-times

[redacted]

[whe[redacted]

[redacted]

$[s_1]_{c_i}$ = ... = $[s_{n2}]_{c_i}$ = [s[redacted]

N_S_DUR$^=$(CV-[redacted]t(s_1)),I[redacted]

CB(**Ext**(s'_{n2})),I[redacted]

R_{i_1}(CV-C[redacted]

$R_{i_{n2}}$(CV-[redacted]**Ext**(s[redacted]

|{e_j \ e_j [redacted]

N_S_DUR[redacted]**xt**(e_j)[redacted]

|{e_j \ e_j $[e_j]_{c_i}$ [redacted] (ev2[redacted]

N_S_DUR$^=$(**Ext**(e_j),I)}}| = n3*n2

This formula reflects the fact that constraints in our formalism may have [redacted]rent concretizations, depending on the concret[redacted]tions[redacted] c_i of the[redacted]period, on the concretization R[redacted]of the qualitati[redacted]e [redacted]nt and on the concretization (i[redacted]ever[redacted]. In[redacted]rticul[redacted] c_i must be a concreti[redacted] th[redacted]ram[redacted] time (i.e., c_i Co[redacted]c$_i$),I) [redacted]here N_S_DUR$^=$ st[redacted]nds [redacted]g or e[redacted]al, and N_S_D[redacted]R[redacted]2) [redacted] for [redacted]ARTS(i1,i2) END[redacted]) [redacted]2) E[redacted]UAL(i1,i2) of Allen[redacted] al[redacted]$_1$, ..., $R_{i_{n2}}$ [redacted] must be a[redacted]n2-conc[redacted]at[redacted]ative constraint R (i.e., [redacted]R_i[redacted]C[redacted](R)); there must be n2 se[redacted]s_1, ...[redacted]ce[redacted]ev1* (i.e., s_1, ... ,s_{n2} (ev[redacted])[redacted]n2 se[redacted]sta[redacted]es of ev2* (i.e., s'_1, ... ,s'_{n2} (ev[redacted]hat th[redacted] temporal extent of each set is during th[redacted]frame[redacted]e I (i.e[redacted] N_S_DUR$^=$(CV-CB(**Ext**((s_1)),I)

N_S_DUR$^=$(CV-CB(**Ext**(s'_{n2})),I)), and such that all the instances in the sets are associated with c_i (i.e., $[s_1]_{c_i}$ = ... = $[s_n]_{c_i}$ = $[s'_1]_{c_i}$ = ... = $[s'_{n2}]_{c_i}$; in other words, each instance oc[redacted] du[redacted] c_i) a[redacted] airwise correlated (i[redacted] [s[redacted] In such a c[redacted] stra[redacted] th[redacted] n are that the [redacted] actly n3*n[redacted] nstan[redacted] d[redacted] cu[redacted] ng [redacted] (i.[redacted] (ev1[redacted]

N_S[redacted] UR [redacted] x[redacted] }}| [redacted] n1*n2 [redacted] {e_j \ e_j [e_j]$_{c_i}$ [redacted] j

(ev2[redacted] _DUR=(**E**[redacted](e$_j$),I)}}| [redacted] n3[redacted] 2) and the atomic qualitative relations R_{i_1}, ..., R[redacted] n2 [redacted] d [redacted] ween correlated pair of set of instances of [redacted] ent[redacted] i. [redacted] R_{i_1}(CV-CB(**Ext**(s_1)),CV-CB(**Ext**(s'_1))) [redacted] $R_{i_{n2}}$(CV-CB(**Ext**(s_{n2})),CV-CB(**Ext**(s'_{n2})))).

4.3 Contextual concretizations

In practice, in most applications where temporal constraints between periodic events are involved, the interest is focused on a specific "temporal context" (interval of time). Thus, the following notion of "contextual concretization" is a crucial one for the practical applicability of our approach.

Definition . **A contextual concretization** of a temporal constraint (in the fo[redacted] sm presented in Section 3) **in an instance C'** of a [redacted] iod ca[redacted] be defin[redacted] (using application) as follows[redacted]

Conc$^{\{C}$[redacted]$_{TC}$([redacted]-times EACH C* n1-time[redacted] ev1* R n3-t[redacted] es ev[redacted]*) =

AND[redacted] i \ c_i [redacted] nc[redacted] ([redacted]) [redacted]_S_DUR=(**E**[redacted](c_i),**Ext**(C')) [redacted]_S_DUR=(**E**[redacted]t(c_i),I)**}** (ConcTC(I n2-times EACH C* n1-times ev1* R n3-times ev2*)) c_i

The contextual concretization of a constraint TC in our formalism is a concretization where t[redacted] eriod has been fixed (bounded). However, in our a[redacted] proach, we deal with multiple (user-defined) periods [redacted] ranul[redacted] ities). Thus, it may be the case that more t[redacted] n [redacted] ncretization (instance) of the period C* is cont[redacted] ed in [redacted] the context C'. Let S = {x \ x [redacted] P([redacted]*) [redacted] N_S_DUR=(**Ext**(x),**Ext**(C')) [redacted]_S_DUR=(**E**[redacted](x),I)} the set of all such concretizations. Conc$^{\{C'\}}_{TC}$(TC) is thus obtained by and-ing the application of the concretization Conc$_{TC}$(TC) to all the instances in S. We can achieve a further step in the process of defining a specific concretization for a temporal constraint by binding also the qualitative constraint, thus obtaining contextual q-concretizations, in which only the instances of events remain to be bound.

Definition. A **contextual q-concretization** of a temporal constraint **in an instance C'** of a period can be defined (using application) as follows:

QConc$^{\{C'\}}_{TC}$(I n2-times EACH C* n1-times ev1* R n3-times ev2*) =

AND[redacted] i \ c_i [redacted] nc[redacted]) [redacted]_S_DUR=(**E**[redacted](c_i),**Ext**(C')) [redacted]_S_DUR[redacted](c_i),I)**}** ConcT[redacted] (I n2-[redacted] es EACH C* [redacted]-times ev1* R[redacted] 3-tim[redacted] ci Ri (w[redacted] re Ri n2-C[redacted]

4.4 A[redacted] example

For exa[redacted] ple, given R=[redacted] F[redacted] B[redacted] RE} 2-Conc$_Q$([redacted] EFORE,M[redacted]) [redacted] si[redacted] 29/9/97-29/9/97] is the o[redacted] oncretiz[redacted] y [redacted] 2[redacted] 9/97-30/9/97], QCon[redacted]{[2[redacted]/9/97-30[redacted] c[redacted] is[redacted] l t[redacted] (C[redacted] c$_{TC}$(S1[redacted] [29/[redacted] B[redacted] ,BEFORE} = [redacted]

s_2 , s'_1[redacted]

[where s[redacted] ([redacted]

s'_1, s'_2 [redacted] m*)[redacted]

N_S_DUR$^=$(CV-CB(**E**[redacted]**xt**(s_1)),[29/9/9719/[redacted] 97])[redacted]

N_S_[redacted] UR$^=$(C[redacted] C[redacted](**Ex**[redacted] s'_2)),[29/9/97-19/12/97][redacted]

[redacted] 1][29/9/97[redacted] 9/97[redacted] = [s_2][29/9/97-29/9[redacted]

[s'_1][29/9/97-2[redacted] 97] [redacted] [s[redacted] [29/9/97-29/9/9[redacted]

[s_1] = [s'_1] [redacted] 2] = [s'_2] **]**

BEFORE([redacted] CB(**E**[redacted](s_1)),CV-CB(**Ext**[redacted]

BEFORE([redacted] CB(**E**[redacted](s_2)),CV-CB(**E**[redacted]

|{e_j \ e_j [redacted] 29/9/97-29/9/97] [redacted]

N_S_DUR[redacted] xt(e_j)[redacted] 29/9/97-19/12[redacted] 7][redacted] = 2[redacted]

|{e_j \ [redacted] [e_j][2[redacted] 9/97-29/9/97] [redacted] j (H_[redacted])

[redacted]_S_DUR$^=$(**E**[redacted](e_j),[29/9/97-19/12/97])}| = 4

Roughly speaking, this formula states that, in the context [29/9/97-30/9/97], and choosing the qualitative relations {BEFORE,BEFORE} between the events, the possible concretizations of (S1) are given by any two sets of one concretization of H_Hist* each, and any two sets of two c[redacted] ncretizations of H_Chem* each, such that the temporal [redacted] xte[redacted] of each set is during the whole frame time [29/9/97-19/12/[redacted] 7], all sets are in the same class [29/9/97-29/9/97] for the[redacted] ssociation relations (in other words, the temporal extent o[redacted] ach set is during [29/9/97-29/9/97]), and sets of concretizations of H_Hist* and H_Chem* are pairwise correlated. In such a case, the concretization of the constraint in (S1) imposes the qualitative constraint BEFORE between correlated pairs, and the cardinality constraints that there are no other instances of H_Hist* and H_Chem* in the context [29/9/97-29/9/97].

4.5 Scenarios

Given a KB of temporal constraints, a scenario is a contextual q-concretization of all the temporal constraints in KB at a given instance of period C' (independent concretizations of different temporal constraint at different instances of periods provide scenarios that are quite useless from the practical point of view).

Definition. Given a KB of tem[redacted] constraints, and a period C', a **scenario in C'** is defin[redacted] follo[redacted]s:

Scenario$^{\{C'\}}$(KB) = **AND {**TC KB**}** [redacted]nc$^{\{C'\}}_{TC}$(TC)

5 P-instantiations and satisfaction

We can now introduce the notion of "instantiation" of a set of periodic events (called p-instantiation), and we can define when a p-instantiation satisfies a scenario for a KB of temporal constraints.

Definition. A **p-instantiation** is a triple **<E,Ext,COR>** where E is a [redacted]f instances of events, Ext is the **Ext** function, a[redacted]gning [redacted]he exact temporal location to the instances [redacted] in E (e.g., expressed as pair of dates), and COR [redacted] the correlation between set of instances in E. A **p-instan[redacted]tion [redacted] C'** is a p-instantiation such that for each [redacted]E, N_[redacted]UR$^{=}$(**Ext**(e),**Ext**(C')) holds.

Defini[redacted]on. Given a contextual q-concretization QConc$^{\{C}$[redacted]$_{TC}$(TC) of a temporal constraint TC in the instan[redacted]period C', and given **a p-instantiation PI = <E,E[redacted],COR[redacted] in C'**, **PI satisfies QConc$^{\{C'\}}_{TC}$(TC)** if an[redacted]nly if t[redacted] set E of instances of events bounds all the [redacted]riables in[redacted]QConc$^{\{C'\}}_{TC}$(TC) satisfying both the conditions in the "where" part of QConc$^{\{C'\}}_{TC}$(TC) an[redacted] the constraints in the "TC" part of QConc$^{\{C'\}}_{TC}$(TC):

- A cardinality constraint |{e\Condition(e)}| = n[redacted]is satisfied if the number of instances of events e[redacted]E such that Condition (e) is true is exactly n
- An association constraint [e]$_{C'}$ (where e E) is satisfied if **Ext** is such that N_S_DUR$^{=}$(**Ext**(e),**Ext**(C') holds.
- A correlation constraint [e1] = [e2] is satisfied if COR(e1,e2) holds in PI (COR is symmetric and transitive)
- A qualitative temporal constraint of the form R_i(CV-CB(**Ext**(Si)),CV-CB(**Ext**(Sj))) where Si and Sj are two correlated sets of instances of events in E is satisfied if Ext is such that R_i((CV-CB(**Ext**(Si)),CV-CB(**Ext**(Sj))) is true. A set of constraints is satisfied if all the constraints in the set are satisfied.

Finally, we can define the notion of satisfaction of a contextual scenario.

Definition. A p-instantiation PI=<E,Ext,COR> satisfies a contextual scenario Scenario$^{\{C'\}}$(KB) for a KB of temporal constraints if, for each temporal constraint TC in KB, PI satisfies QConc$^{\{C'\}}_{TC}$(TC).

For example, the p-instantiation <E,EXT,COR> in [29/9/97-30/9/97] (29/9/97 is a Monday) satisfies Scenario$^{\{[29/9/97-30/9/97]\}}$(KB1), in case Conc$_Q$ is always {BEFORE} for all qualitative constraints.

E= {H_Hist*(H1), H_Hist*(H2), H_Chem*(C1), H_Chem*(C2), H_Chem*(C3), H_Chem*(C4), H_Chem*(C5), H_Chem*(C6), H_Math*(M1), H_Math*(M2), H_Phys*(P1), H_Phys*(P2)};
Ext(H1)=[29/9/97 at 8:00 - 29/9/97 at 9:00],
Ext(H2)=[29/9/97 at 12:30 - 29/9/97 at 13:30],
Ext(C1)=[29/9/97 at 9:10 - 29/9/97 at 10:00],
Ext(C2)=[29/9/97 at 10:30 - 29/9/97 at 11:30],
Ext(C3)=[29/9/97 at 15:00 - 29/9/97 at 16:00],
Ext(C4)=[29/9/97 at 16:00 - 29/9/97 at 17:00],
Ext(C5)=[30/9/97 at 9:10 - 30/9/97 at 10:00],
Ext(C6)=[30/9/97 at 10:30 - 30/9/97 at 11:30],
Ext(M1)=[30/9/97 at 12:30 - 30/9/97 at 13:30],
Ext(P1)=[30/9/97 at 15:00 - 30/9/97 at 16:00];
COR={<{H1},{C1,C2}>,<{H2},{C3,C4}>,
<{M1},{P1}>, <{C5,C6},{M2}>, <{C5,C6},{P2}>}

6 Generating p-instantiations that satisfy scenarios

In this section, we describe the algorithm GEN which, given a KB, generates a p-instantiation that satisfies a contextual scenario for KB. GEN is based on the definitions in sections 4 and 5. For the sake of brevity, the [redacted]resentation of GEN is only sketched, and we propose on[redacted]a very simple application example.

GEN(KB, KB_period, C')

[redacted] check the consistency of KB and infer [redacted]w temporal constraints from KB. Let K[redacted]e the new kn[redacted]wledge base re[redacted]ting from this step

2) % For each temp[redacted]l constraint TC_i KB', [redacted]pute its contextual concret[redacted]onc$^{\{C'\}}_{TC}$(TC_i)and the "fragments" in C' it cove[redacted] %

- FOR EA[redacted] TC_i KB' [redacted]pute CSet(TC_i)={C_{i_1}, .., C[redacted] the set of the temporal extents of all co[redacted]tizations of the pe[redacted]C* in TC_i in context C'
- Le[redacted]Set(KB')={I_1 [redacted] I_k} [redacted]e set of all intervals obtai[redacted] usi[redacted] the i[redacted]rvals in CSet(TC_i), for each TC_i KB', [redacted]der to p[redacted]tion the period C' into the finest subintervals (called "f[redacted]ment[redacted])
- FOR EACH TC_i KB', [redacted]en ISet(KB')={I_1, ..., I_k} and given CSet(TC_i)={C_{i_1}, .., C_{i_n}} compute ISet(TC_i)= {{I_{i_1}, .., I_{i_r}}, ..., {I_{i_h} .., I_{i_k}}} where, e.g., {I_{i_1}, .., I_{i_r}}is the set [redacted]agments that covers C_{i_1} and{I_{i_h} .., I_{i_k}}is th[redacted]set o[redacted]ra[redacted]nts that covers C_{i_n}

3) % For each [redacted]gment I[redacted]and f[redacted] each event ev_k* , compute the num[redacted]nces of ev_k* that have to be instantiated into the [redacted]terva[redacted] %

FOR EACH I_j ISet([redacted]

- FOR EACH TC_i [redacted]nsider its contextual concretization to ch[redacted]k wh[redacted]er ISet(TC_i) contains I_j .
- Let EvSet(I_j) = [redacted]ev_1*, ..., ev_j*} the set of all events ev* considered th[redacted]conc[redacted]tization of a temporal constraint TC_i KB' [redacted]that ISet(TC_i) contains I_j .

- FOR EACH ev_k* Ev[redacted](I_j), co[redacted]pute Card(I_j(ev_k*)), the number of instances of [redacted]* that have to be instantiated into the interval I_j .

4) % Given the qualitative constraints between [redacted]ents imposed by $Conc^{\{C'\}}{}_{TC}$(TC[redacted]), for each TC_i [redacted]B', given Card(I_j(ev_k*)), f[redacted] each [redacted]vent in KB' and [redacted]or each fragment, instantiate [redacted]ent[redacted]in C' %

FOR EACH $Conc^{\{C}$[redacted]TC[redacted]) such that TC_i KB' ([redacted]
="I n2-times EACH C*[redacted] ev_1* R n3-times ev_2*")

FOR EACH C_i CSet([redacted])

FOR EACH I_j ISet([redacted]) which is (strictly or not) during C_i

- Instantiate Card(I_j(ev_1*)) non-overlapping instances of ev_1* and Card(I_j(ev_2*)) non-overlapping instances of ev_2* in I_j , respecting the qualitative temporal constraint R

The inputs of GEN are a KB of temporal constraints in our formalism, a separate knowledge base KB_period containing the definitions of the user-defined periods in KB (using our adaptation of Leban's language [Leban et al., 87; Terenziani, 97a]), a period of concretization C' (e.g., [29/9/97 - 30/9/97]). GEN consists of four main steps. The first step is mainly an optimisation, since consistency is already detected when generating p-instantiations (there is no p-instantiation satisfying a contextual scenario for a KB which is inconsistent in C'). However, generation is computationally more expensive than consistency checking using the (incomplete) polynomial algorithm we proposed in [Terenziani, 97a]. In our example, (KB1) is checked to be consistent, and is not changed by the temporal reasoning process. The second step [redacted]sponds to computing the contextual concretizati[redacted]s of a[redacted] the tempo[redacted] constraints (t.c.) in KB. The cruc[redacted]point is to genera[redacted], fo[redacted] each period C* in any t.c., all th[redacted]concretizations [redacted] C* wh[redacted] are (starkly or not) during C' a[redacted] duri[redacted] the [redacted]me tim[redacted] of t.c. (i.e., the set {x \ x Conc[redacted]) [redacted]_S_DUR=([redacted]**t**(x),**Ext**(C')) N_S_DUR=(**Ext**(x), I)}). This computation is performed on the basis of the definition of periods in KB_period. The algorithm we used is similar to the one in [Chandra et al., 93]. Given KB1, and the concretization period C'= [29/9/97 - 30/9/97], we have just one concretization of Mondays* in C' (i.e., the day [29/9/97-29/9/97]), of Tuesdays* (i.e., [30/9/97 - 30/9/97]), and of Mon+Tue* ([29/9/97-30/9/97]). Thus, we have ISet(KB1)={[29/9/97-29/9/97] and [30/9/97 - 30/9/97]}, Iset(S1)={{[29/9/97-29/9/97]}}, Iset(S2) = {{[30/9/97-30/9/97]}}, Iset(S3) = {{[29/9/97-30/9/97]}}. The computation of the number of instances of each type of event in each fragment at step three require solving a system of equations. For each constraint TC_i ="I n2-times EACH C* n1-times ev_1* R n3-times ev_2*", for each concretization C_i of C* in C', given the set {I1, ...,Ik} of fragments in ISet(KB') which are contained into C_i, the system contains the equations ev_1*(I1)+...+ev_1*(Ik)=n1*n2 and ev_2*(I1)+ ...+ ev_2*(Ik) =n3*n2 where ev*(Ii) is a variable representing the number of concretizations of ev* in the interval Ii. For example, from (S2) we have H_Phys*([30/9/97-30/9/97]) = 1 and H_Chem*([30/9/97-30/9/97])=2, while from (S3) we have H_Math*([29/9/97-29/9/97]) + H_Math*([30/9/9730/9/97]) =1 and H_Phys*([29/9/97-29/9/97]) + H_Phys*([30/9/9730/9/97]) =1.

Thus, the overall system of equations admits two solutions, one where the concretization of H_Math* takes place on [29/9/97-29/9/97], and the other with H_Math* on [30/9/97-30/9/97].

Given one of the solutions to the system of equations, in the fourth step we instantiate the events and assert the COR relations between them, as fixed by the contextual concretizations. Notice that the fourth step can be seen a non-deterministic one which (although implicitly) computes the q-concretization of the constraints. In fact, during the instantiation process, whenever a qualitative temporal constraint R={R1,...,Rk} in the contextual concretizations has to be considered, one of the atomic relations in {R1,...,Rk} is selected, provided that it is consistent with the temporal locations of the already instantiated events. The algorithm implementing the fourth step thus operates by cases, depending on the selected atomic qualitative relation.

Finally, let us briefly consider the computational complexity of the GEN algorithm. First, it is important to notice that the overall algorithm contains two backtracking points. The first point correspond to the fact that there may be multiple solutions (i.e., multiple combinations of values for Card(I_j(ev_i*)), for each fragment I_j and each event ev_i*) satisfying the equations in [redacted]ep 3 (however, some of them may be inconsistent [redacted]th[redacted]e qualitative temporal constraints in the contextual [redacted]concret[redacted]ations). The second point concerns the choice of the ato[redacted]c relation in step 4. Thus, in the worst case, the generatio[redacted] of p-instantiation is exponential. We are currently studying the possibility of introducing optimisations in the GEN algorithm, besides cutting inconsistent qualitative constraints in the pre-computation in step 1.

7 Comparisons and conclusions

Our overall approach is an (algebraic) approach of the third type discussed in the introduction. However, unlike other approaches in the AI and database literature, our formalism allows one to deal also with composite types of temporal constraints such as that in Ex.2, where frame time, numeric quantifiers, periods and qualitative relations are considered at the same time (see also footnote 1). For example, Morris et al.'s algebraic approach [Morris, 95, 96] is very close to ours, but it considers only "period-independent” constraints between not numerically quantifiers events (see, e.g., Ex.1; however, different forms of quantifications are taken into account by Morris et al.). On the other hand, Loganantharaj mainly faced the problem of associating possibilistic distributions to qualitative temporal constraints between periodic events [Loganantharaj & Gimbrone, 95] and to metric constraints concerning the durations of events, which are also expressed using transition rules [Loganantharaj & Gimbrone, 97; Loganantharaj & Kurkovsky, 97]. However, user-defined periods and "period-dependent" qualitative constraints were not considered by Loganantharaj et. al..

The notions of "concretization" and "n-concretization" of a temporal constraint between periodic events and of scenario have been studied by Morris et al. in [Morris et al., 95] considering the language they proposed [Morris et al.., 93]. Moreover, Morris et al. also defined the notion of consistent scenario [Morris et al., 95,96]. While our definition of n-concretization of a qualitative temporal constraint R (see the definition. of n-$Conc_Q(R)$ in section 4) is basically taken from their approach, our definition of the concretization of temporal constraints is more elaborate. In fact, our formalism is more expressive, since we took into account also numeric quantifiers, frame times and (user-defined) periods. Moreover, since our temporal constraints may be period-dependent, we had to introduce the notion of "contextual" concretization, and of "contextual" scenario. On the other hand, Loganantharaj et al., considered possibilistic distributions on the duration of events [Loganantharaj & Gimbrone, 97; Logantharaj & Kurkovsky, 97] and faced the problem of projecting the constraints on the durations in the future using the current domain information.

Finally, it is important to notice that the notions of (contextual) concretizations and scenario we introduced in this paper not only are basic notions in generative approaches, but also they may be used in a “recognition” way: given a KB of temporal constraints, and given a set of observations (i.e., a set of instantiations of activities observed at specific intervals of time), one could check whether these observations respect the temporal constraints in the KB or, in other words, if they are consistent with a scenario for the KB.

References

[Allen, 83] J.F. Allen, "Maintaining Knowledge about Temporal Intervals", *Comm. ACM*,. vol. 26, no. 11, pp. 832-843, Nov. 1983.

[Chandra et al., 93] R. Chandra, A. Segev, and M. Stonebraker, "Implementing Calendars and Temporal Rules in Next Generation Databases", *Proc. Int'l Conf. on Data Engineering*, Huston, pp. 264-273, 1994. (see also LBL Tech. Report 34229, University of California, Berkeley, 1993).

[Cukierman, & Delgrande, 95] D. Cukierman, and J. Delgrande, "A Language to Express Time Intervals and Repetition", *Proc. TIME-95,* pp. 41-48, 1995.

[Ladkin, 86] P. Ladkin, "Primitive and Units for Time Specification", *Proc. Fifth National Conf. on Artificial Intelligence*, pp. 354-359, Philadelphia, PA, 1986.

[Leban et al., 86] B. Leban, D.D. McDonald, and D.R. Forster, "A representation for collections of temporal intervals", *Proc. Fifth National Conf. on Artificial Intelligence*, pp. 367-371, Philadelphia, PA, 1986.

[Loganantharaj & Gimbrone, 95] R. Loganantharaj, and S. Gimbrone, "Probabilistic Approach for Representing and Reasoning with Repetitive Events", *Proc.FLAIRS-95.*, pp. 26-30, Melbourne, FL, April 1995.

[Loganantharaj & Kurkovsky, 97] R. Loganantharaj, and S. Kurkovsky, "A new model for projecting temporal distance using fuzzy temporal constraints", In Proc. IEA/AIE'97, 1997.

[Loganatharaj & Gimbrone, 97] R. Loganantharaj, and S. Gimbrone, "Issues on Synchronizing when Propagating Temporal Constraints", AAAI97 Workshop on Spatial and Temporal Reasoning, 1997.

[Morris et al., 95] R.A. Morris, L. Khatib, and G. Ligozat, "Generating Scenarios from specifications of Repeating Events", *Proc. of the TIME-95* , pp. 41-48, 1995.

[Morris et al., 96] R.A. Morris, W.D. Shoaff, and L. Khatib, "Domain Independent Temporal Reasoning with Recurring Events", *Computational Intelligence 12(3),* 450-477, 1996.

[Soo & Snodgrass, 93] M. Soo, and R. Snodgrass, "Multiple Calendar Support for Conventional Database Management Systems", *Proc. Int'l Workshop on an Infrastructure for Temporal Databases*, Arlington, Texas, June 1993.

[Terenziani, 97a] P. Terenziani, "Integrating calendar-dates and qualitative temporal constraints in the treatment of periodic events", *IEEE Transactions on Knowledge and Data Engineering 9(5),* 1997.

[Terenziani, 97b] P. Terenziani, "Qualitative and Quantitative Temporal Constraints about Numerically Quantified Periodic Events", In Proc. *TIME-97*, IEEE Computer Society Press, pp. 94-101, 1997.

[Tuzhilin & Clifford, 95] A. Tuzhilin and J. Clifford, "On Periodicity in Temporal Databases", *Information Systems,* vol. 20, no.8, pp. 619-639, December 1995.

[Wang et al., 97] X. Wang, C. Bettini, A. Brodsky, S. Jajodia, "Logical Design for Temporal Databases with Multiple Granularities", TODS 22(2), pp. 115-170, 1997.