A Decidable Temporal Logic for Events and States

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Abstract

This paper introduces a new interval temporal logic, TPL. Existing interval temporal logics, we claim, are inadequate to represent the meanings of certain natural language constructions, despite exhibiting high computational complexity. TPL* overcomes these problems, presents the semantics of some natural language constructions, and captures important real-time problems like behaviour of complex systems.*

1. Introduction

In a sentence of natural language temporal information is stored in temporal constructions such as prepositions. In order to understand the semantics of a sentence in English or in any other language it is very important to capture temporal meanings. The formal semantics of temporal constructions in English have been investigated by various researchers [3, 4, 6, 13]. Most of the temporal logics in the literature, particularly interval temporal logics, are not expressive enough to capture the meanings of natural language constructions, and they are not convenient to represent temporal expressions. In addition, these formal systems exhibit high computational complexity. In order to overcome these drawbacks we have introduced a computationally manageable interval temporal logic, called *TPL**, which has affinity with the syntax of temporal constructions in English, and which is convenient in presenting the semantics of natural language constructions. TPL* is an extension of the logic TPL [11]. TPL* covers both *event* and *state* types in order to treat the semantics of different sentence categories.

Modelling real-time requirements is also an important consideration in this paper. That is, we want TPL* to have expressive power of representing the behaviour and specifications of complex systems. Most formal methods of the behaviour of real-time systems are either event-based or state-based. Therefore, TPL* should cover both event-based and state-based views.

TPL* also includes the notion of *duration* since states are characterized by a duration. The duration of a state is the length of the time period during which the system remains in that state. State models and state durations have been found useful for reasoning about time durations for a dynamic system. TPL* can also deal with the duration of an event.

In this paper syntax and semantics of TPL* will be presented. In addition a terminating tableau system will be proposed for the logic to show that its satisfiability problem is decidable, and is in NEXPTIME.

2. Event and State Models

The logic TPL deals only with the semantics of the fragment of English concerning event-type sentences. However, in English there are more sentence categories than event type-sentences. In order to understand the meaning of a sentence in English, the formal system which represents the semantics of the sentence must have sufficient expressive power to capture different sentence categories.

A long-standing tradition in the linguistic and philosophical literature divides simple sentences into categories. In [15] Vendler classified (tense-less) sentences into four different types: *achievements*, *accomplishments*, *activities* and *states*. However, [10] proposes that for the purposes of accounting for the semantics of temporal constructions, the minimal distinction between *event* types and *ongoing process* or *state-types* suffices. Consider the sentences "John wrote the letter", and "John worked on the letter". The former reports a *completed event*, while the latter reports an *ongoing process* or *state-types*. In order to treat the semantics of different sentence categories TPL needs to be extended with the notion of state.

So far, we have only mentioned the natural language semantics considerations. Our aim is to develop a computationally manageable logic which not only has sufficient affinity with the syntax of temporal expressions in English, but also model real-time requirements. Most formal methods of the behaviour of real-time systems are either event-based or state-based. Therefore, a formal method should cover both event-based and state-based views. Although TPL can deal with the semantics of natural language constructions, it lacks the expressive power of representing the behaviour and specifications of complex systems. Hence, we need to extend TPL with the notion of state.

Once we have a temporal logic endowed with state models it is natural to extend it with the notion of *duration* since states are characterized by a duration. The duration of a state is the length of the time period during which the system remains in that state. State models and state durations have been found useful for reasoning about time durations for a dynamic system. TPL* can also deal with the duration of an event.

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3. TPL* : A Temporal Logic for Events and States

TPL* is an interval temporal logic which is an extension of TPL with the notion of state models and durations. Thus, it contains both event-based and state-based views. By extending TPL we aim at developing a decidable logic for specifying properties of finite sequences of states, and presenting the semantics of temporal constructions in English. Since TPL lacks the *chop modality* $\mathcal{C}$, which is necessary to capture important real-time problems like behaviour of complex systems, we make a further extension over TPL, and introduce the chop modality into our new logic.

In this section we present syntax and semantics of TPL*. We also propose a terminating tableau system for the logic, thus showing that its satisfiability problem is decidable. Indeed, this section provides a complexity bound for TPL*-satisfiability, showing that this problem is still in NEXPTIME. In the literature various tableau methods have been developed for linear and branching time point-based temporal logics. However, there exist very few tableau methods for interval temporal logics and duration calculus. The main reason is that operators of interval temporal logics are in many respects more difficult to deal with.

In this section we will also show that addition of chop operator into the logic does not harm the complexity results.

In the sequel, $\mathcal{I}$ denotes the set of intervals, where an interval is closed, bounded, non-empty subset of $\mathbb{R}$. Temporal variables are denoted by the variables $I, J, ...$, which range over $\mathcal{I}$. Given that I represents the interval $[t_1, t_2]$ and J represents the interval $[t_3, t_4]$ where $t_1, t_2, t_3, t_4 \in \mathbb{R}$ and $t_1 \leq t_3 \leq t_4 \leq t_2$, the partial functions $init\,(J, I)$ and $fin\,(J, I)$ denote the intervals $[t_1, t_3]$ and $[t_4, t_2]$, respectively.

Assume that $\mathcal{E}$ and $\mathcal{S}$ denotes a fixed infinite set of event atoms and state variables, respectively, and $e \in \mathcal{E}$, $s \in \mathcal{S}$. The set of event expressions α is defined by $\alpha ::= e \mid e^f \mid e^l$, where the letter f and l stand for the adjectives *first* and *last,* respectively. That is, e^f denotes the first of finitely many events of type e, and similarly e^l denotes the last of finitely many events of type e within a temporal context I (see [11] for more details).

The set of *duration expressions*, denoted by θ, is defined as follows:

$$\theta ::= \neg\theta \mid \int \langle e \rangle \tau k \mid \int [e] \tau k \mid \int \Diamond s \tau k \mid \int \Box s \tau k$$

where the symbol $\int$ denotes the time length of a given state or event, k is a constant, and $\tau \in \{=, \neq, <, >, \leq, \geq\}$.

The terms, denoted by σ, have the following syntax:

$$\sigma ::= \top \mid s \mid \theta \mid \neg\sigma \mid \langle e \rangle \sigma \mid [e] \sigma \mid \{\alpha\} \sigma \mid \{\alpha\}_> \sigma \mid \{\alpha\}_< \sigma.$$

Then the formulas of TPL* can be recursively defined as follows:

$$\phi ::= \sigma \mid \phi \wedge \psi \mid \phi \vee \psi \mid \phi \mathcal{C} \psi.$$

where ϕ and ψ are TPL* formulas. The connectives $\rightarrow$ and $\leftrightarrow$ can be defined in usual way.

Definition 1. Let $\mathcal{I}$ be the set of all bounded, closed, non-empty intervals of real numbers $\{[t_1, t_2] : t_1 \leq t_2 \wedge t_1, t_2 \in \mathbb{R}\}$, $\mathcal{E}$ be an infinite set of event atoms, and $\mathcal{S}$ be an infinite set of states. A TPL* model $\mathcal{M}$ is a finite subset of $(\mathcal{I} \times \mathcal{E}) \cup (\mathcal{I} \times \mathcal{S})$ such that for all $J, J' \in \mathcal{I}$ if $\langle J, s \rangle, \langle J', s \rangle \in \mathcal{M}$, then $J \cap J' = \varnothing$. For any $J \in \mathcal{I}$, $e \in \mathcal{E}$, and $s \in \mathcal{S}$, $\mathcal{M}(J)$, $\mathcal{M}(e)$ and $\mathcal{M}(s)$ are defined as follows:

$$\begin{aligned}
&\mathcal{M}(e) \equiv \{J \in \mathcal{I} \mid \langle J, e \rangle \in \mathcal{M}\} \\
&\mathcal{M}(s) \equiv \{J \in \mathcal{I} \mid \langle J, s \rangle \in \mathcal{M}\} \\
&\mathcal{M}_e(J) \equiv \{e \in \mathcal{E} \mid \langle J, e \rangle \in \mathcal{M}\} \\
&\mathcal{M}_s(J) \equiv \{s \in \mathcal{S} \mid \langle J, s \rangle \in \mathcal{M}\} \\
&\mathcal{M}(J) \equiv \mathcal{M}_e(J) \cup \mathcal{M}_s(J)
\end{aligned}$$

The definition implies that if $\langle J, s \rangle \in \mathcal{M}$, then $\langle J, s \rangle$ is a *maximal pair* meaning that the state s holds over an interval J and any subinterval J' of J. That is, $\langle J, s \rangle \in \mathcal{M}$ intuitively implies that the state s holds throughout $J' \subset J$; thus $\langle J', s \rangle \notin \mathcal{M}$.

Definition 2. Let $\mathcal{M}$ be a model for TPL* and $I \in \mathcal{I}$. The formal semantics is then defined as follows:

$\mathcal{M} \models_I s$ iff $\exists I' \supseteq I$ such that $\langle I', s \rangle \in \mathcal{M}$
$\mathcal{M} \models_I \int \langle e \rangle \tau k$ iff $\exists J \subset I$ such that $\langle J, e \rangle \in \mathcal{M}$ and $|J| \tau k$
$\mathcal{M} \models_I \int [e] \tau k$ iff $\forall J \subset I$ $\langle J, e \rangle \in \mathcal{M}$ implies $|J| \tau k$
$\mathcal{M} \models_I \int \Diamond s \tau k$ iff $\exists J \subset I$ such that $\langle J, s \rangle \in \mathcal{M}$ and $|J| \tau k$
$\mathcal{M} \models_I \int \Box s \tau k$ iff $\forall J \subset I$ $\langle J, s \rangle \in \mathcal{M}$ implies $|J| \tau k$
$\mathcal{M} \models_I \phi \mathcal{C} \psi$ iff $\exists J \subset I$ such that $\mathcal{M} \models_J \phi$ and $\mathcal{M} \models_{fin(J,I)} \psi$ and $|init\,(J, I)| = 0$

where $|J|$ denotes the length of the interval J. The rest of the semantics is defined as in TPL (see [11]).

The negation operator ($\neg$) in a duration expression can be moved inwards as follows:

$$\neg \int \langle e \rangle \tau k \equiv \int [e] \tau' k$$

$$\neg \int \Diamond s \tau k \equiv \int \Box s \tau' k$$

where τ' the corresponding inverted operator of τ (For example, $\neq$ is the inverted operator of $=$, $<$ is that of $\geq$, etc.) $\neg \int [e] \tau k$ and $\neg \int \Box s \tau k$ can be defined similarly.

3.1. Semantics of Some Sentences

In order to show how TPL* represents the English sentences including the temporal constructions we will consider the semantics of some sentences in a fragment of English. Consider the following sentences:

(3.1) `A warning is received during every control period until the water level becomes normal.`

(3.2) `After a drop in the water level, a warning is received during every control period until the water level becomes normal.`

The meaning of (3.1) is that, within the given temporal context I, there is a definite interval J over which the water level is normal and over every interval J', which is subsumed by the initial segment of I up to the beginning of I, a control period occurs, and J' subsumes some interval over which a warning is received. The sentence (3.1) is represented by the TPL* formula

(3.3) $\{normal\}_< [control] \langle warning \rangle \top$.

The sentence (3.2) can be represented by the TPL* formula

(3.4) $\{drop\}_> \{normal\}_< [control] \langle warning \rangle \top$.

Let us look at how TPL* represents the event-typed and state-typed English sentences including duration. Consider the following sentences:

(3.5) `John solved a problem in less than ten minutes during every lunch break.`

(3.6) `John worked on the exercises for a period of less than ten minutes during every lunch break.`

These sentences can be represented in TPL*, respectively, as follows:

(3.7) $[break] \int \langle solve \rangle \leq 10$

(3.8) $[break] \int \Diamond work \leq 10$

As we can see, TPL* differentiates between the state-typed and event-typed sentences. If a sentence includes a duration, a duration expression is associated with it.

3.2. Decidability Results

In this part we will show that TPL* is a decidable logic. Indeed, the satisfiability problem for TPL* is in NEXPTIME. This is proved by building models whose sizes are exponentially bounded. Certain parts of the following proof have been taken verbatim from [11].

Lemma 1. In a TPL* formula $\neg$ operator can be moved inwards such that $\neg$ exists only in $\neg \{e\} \top, \neg s$ and $\bot$*(*which is equivalent to $\neg\top$*)*.

Proof. The proof is trivial for $\neg s$ and $\bot$. In a TPL* formula the $\neg$ operator can be moved inwards as follows:

$$\begin{aligned}
&\neg \langle e \rangle \phi \equiv [e] \neg\phi \\
&\neg [e] \phi \equiv \langle e \rangle \neg\phi \\
&\neg \{e\} \phi \equiv \neg \{e\} \top \vee \{e\} \neg\phi \\
&\neg \{e^\lambda\} \phi \equiv [e] \bot \vee \{e^\lambda\} \neg\phi \\
&\neg \int \langle e \rangle \tau k \equiv \int [e] \tau' k \\
&\neg \int \Diamond s \tau k \equiv \int \Box s \tau' k
\end{aligned}$$

where $e \in \mathcal{E}$, $s \in \mathcal{S}$, $\lambda \in \{f, l\}$, $\phi \in$ TPL* and τ' the corresponding inverted operator of τ. $\neg \{e\}_\rho \phi, \neg \{e^\lambda\}_\rho \phi$, $\neg \int [e] \tau k$, $\neg \int \Box s \tau k$ can be treated similarly ($\rho \in \{<, >\}$). By means of this lemma we can normalize the forms of TPL* formulas.

Definition 3. Given a non-empty model $\mathcal{M}$, the depth of $\mathcal{M}$ is the greatest m for which there exist $J_1 \subset J_2 ... \subset J_m$ such that for all i and for some $e \in \mathcal{E}$, $1 \leq i \leq m$, $\langle J_i, e \rangle \in \mathcal{M}$. The *depth* of an empty model is defined to be 0.

When we determine the depth of a model, we only consider event types. Since states are maximal, and true over all subintervals, they are not involved in the definition of depth.

Now we will show that we can reduce the size of satisfying models in such a way that the depth of resulting models are polynomially bounded on the length of the formula. More formally, for a given satisfying model $\mathcal{M}$ we will find a reduced satisfying model $\mathcal{M}^* \subseteq \mathcal{M}$, whose depth is bounded by $|\phi|^2$, such that $\mathcal{M} \models_I \phi$ implies $\mathcal{M}^* \models_I \phi$. Before starting the formal proof, we will give some definitions.

Definition 4. Assume that $\neg$ operators in a given TPL* formula ϕ are moved inwards, as shown in Lemma 1, and $\mathcal{M}$ contains only event-atoms and state variables involved in ϕ. Let Φ be the set of subformulas of ϕ event atom e in ϕ, and every interval $J \in \mathcal{I}$, we define $L_e(J)$, as follows:

$$\begin{aligned}
&L(J) = \{\psi \in \Phi \mid \mathcal{M} \models_J \psi\} \\
&L_e(J) = L(J) \setminus \bigcup \{L(J) \mid K \subset J, K \in \mathcal{M}(e)\}
\end{aligned}$$

$\langle J, e \rangle \in \mathcal{M}$ is *redundant* if $L_e(J) = \varnothing$ and there exist $K, K' \in \mathcal{M}(e)$ such that $K \subset K' \subset J$.

If we look at the definition, we can see that $L_e(J)$ records the subformulas of ϕ which are true at an interval J, except the subformulas which are true at some subinterval K of J with $\langle K, e \rangle \in \mathcal{M}$. Any $\langle J, s \rangle \in \mathcal{M}$ is a maximal pair, and it is not redundant.

Lemma 2. Let the number of symbols in a given TPL* formula be denoted by $|\phi|$. For a given model $\mathcal{M}$ and interval I, if $\mathcal{M} \models_I \phi$, then there exists a model $\mathcal{M}^* \subseteq \mathcal{M}$, with depth at most $O\left(|\phi|^2\right)$, such that $\mathcal{M}^* \models_I \phi$.

Proof. Now we will reduce the model $\mathcal{M}$ to $\mathcal{M}^*$ by removing redundant pairs:

$$\mathcal{M}^* = \mathcal{M} \setminus \{\langle J, e \rangle \mid \langle J, e \rangle \ is\ redundant\}$$

Let m be the number of event atoms occurring in ϕ, m' be the number of event atoms occurring in ϕ, and n be the number of subformulas of ϕ. If $J \subset J'$ such that $\langle J, e \rangle \in \mathcal{M}$ and $\langle J', e \rangle \in \mathcal{M}$, then $L_e(J)$ and $L_e(J')$ are disjoint. That is, the depth of the chain of the intervals at which e occurs is bounded by the number of the subformulas of ϕ in which e is mentioned. The depth of the chain of the intervals at which a state s holds is 1 since we only consider maximal states. Therefore, $\mathcal{M}^*$ is bounded by $m(n+2) + m'$. Since we know that $m < |\phi|$, $m' < |\phi|$ and $n < |\phi|$, it easily follows that the depth of $\mathcal{M}^*$ is bounded by $|\phi|^2$.

Now we must show that, for all I and all $\psi \in \Phi$, $\mathcal{M} \models_I \psi$ implies $\mathcal{M}^* \models_I \psi$. We will use structural induction on the complexity of ϕ.

Base Case : Suppose $\mathcal{M} \models_I \psi$

$\psi = \top$ or $\psi = \bot$: Trivial

$\psi = s$: By the semantics definition we know that there is $I' \supseteq I$ such that $\langle I', s\rangle \in \mathcal{M}$. Since $\langle I', s\rangle$ is a maximal pair, it cannot be redundant; therefore $\langle I', s\rangle \in \mathcal{M}^*$. Thus, $\mathcal{M}^* \models_I s$.

$\psi = \neg s$: By the semantics definition we know that there is no $I' \supseteq I$ such that $\langle I', s\rangle \in \mathcal{M}$. Since $\mathcal{M}^* \subseteq \mathcal{M}$, it is clear that $\langle I', s\rangle \notin \mathcal{M}^*$. Thus, $\mathcal{M}^* \models_I \neg s$.

$\psi = \neg\{e\}\top$: There are two cases that we need to consider. In the first case, there is no $J \subset I$ with $\langle J, e\rangle \in \mathcal{M}$. Since $\mathcal{M}^* \subseteq \mathcal{M}$, it is clear that $\langle J, e\rangle \notin \mathcal{M}^*$, and thus $\mathcal{M}^* \models_I \neg\{e\}\top$. In the other case, there exist two different intervals $J \subset I$ and $J' \subset I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $\langle J', e\rangle \in \mathcal{M}$. If neither of $\langle J, e\rangle$ and $\langle J', e\rangle$ is redundant , then $\langle J, e\rangle \in \mathcal{M}^*$ and $\langle J', e\rangle \in \mathcal{M}^*$. Thus, $\mathcal{M}^* \models_I \neg\{e\}\top$. Otherwise, if $\langle J, e\rangle$ is redundant, there must exist $K \subset K' \subset J$ such that $\langle K, e\rangle \in \mathcal{M}$ and $\langle K', e\rangle \in \mathcal{M}$. Since these pairs are non-redundant, they must occur in $\mathcal{M}^*$, i.e. $\langle K, e\rangle \in \mathcal{M}^*$ and $\langle K', e\rangle \in \mathcal{M}^*$. Thus, $\mathcal{M}^* \models_I \neg\{e\}\top$. The situation in which $\langle J, e\rangle$ is redundant can be handled similarly.

$\psi = \int\langle e\rangle\tau k$: By the semantics there exits $J \subset I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $|J|\tau k$. If $\langle J, e\rangle$ is not redundant, $\langle J, e\rangle \in \mathcal{M}^*$. Thus, $\mathcal{M}^* \models_I \int\langle e\rangle\tau k$. Otherwise, there must exist $K \subset K' \subset J$ such that $\langle K, e\rangle$ and $\langle K', e\rangle$ are not redundant; therefore $\langle K, e\rangle \in \mathcal{M}^*$ and $\langle K', e\rangle \in \mathcal{M}^*$. Since $L_e(J) = \varnothing$, we have either *(i)* $L(J) = \varnothing$ and $L(K) = \varnothing$ or *(ii)* $L(J) = \{\psi\}$ and $L(K) = \{\psi\}$. For the first case we have $\mathcal{M} \not\models_I \psi$. So ψ must be true at both J and K, implying $|K|\tau k$ and $|K'|\tau k$. Therefore, we have $\mathcal{M}^* \models_I \int\langle e\rangle\tau k$.

$\psi = \int[e]\tau k$: By the semantics we know that for all $J \subset I$ $\langle J, e\rangle \in \mathcal{M}$ implies $|J|\tau k$. If there is no redundant $\langle J, e\rangle$, then $\mathcal{M}^* \models_I \int[e]\tau k$. If there are some redundant pairs of $\mathcal{M}$, they will not be included in $\mathcal{M}^*$. For all other non-redundant $\langle J, e\rangle \in \mathcal{M}$, $\langle J, e\rangle \in \mathcal{M}^*$ implies $|J|\tau k$. Then, $\mathcal{M}^* \models_I \int[e]\tau k$.

$\psi = \int\Diamond s\tau k$: By the semantics there is $J \subset I$ such that $\langle J, s\rangle \in \mathcal{M}$ and $|J|\tau k$. Since $\langle J, s\rangle$ is a maximal pair, it cannot be redundant. So, we have $\mathcal{M}^* \models_I \int\Diamond s\tau k$.

$\psi = \int\Box s\tau k$: By the semantics, for all $J \subset I$ $\langle J, s\rangle \in \mathcal{M}$ implies $|J|\tau k$. Since we know that $\langle J, s\rangle$ is a maximal pair, it cannot be redundant. Thus, $\mathcal{M}^* \models_I \int\Box s\tau k$.

Inductive Case:

Suppose $\mathcal{M} \models_I \psi$

$\psi = \langle e\rangle\theta$: There is $J \subset I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $\mathcal{M} \models_J \theta$. If $\langle J, e\rangle$ is not redundant $\langle J, e\rangle \in \mathcal{M}^*$. By the inductive hypothesis $\mathcal{M} \models_J \theta$ implies $\mathcal{M}^* \models_J \theta$. Otherwise, there must exist $K \subset K' \subset J$ such that $\langle K, e\rangle$ and $\langle K', e\rangle$ are not redundant; therefore $\langle K, e\rangle \in \mathcal{M}^*$ and $\langle K', e\rangle \in \mathcal{M}^*$. Since $L_e(J) = \varnothing$, it is clear that $\mathcal{M} \models_{K'} \theta$. By the inductive hypothesis $\mathcal{M} \models_{K'} \theta$ implies $\mathcal{M}^* \models_{K'} \theta$. Then, $\mathcal{M}^* \models_I \langle e\rangle\theta$.

$\psi = [e]\theta$: By the semantics, for all $J \subset I$ $\langle J, e\rangle \in \mathcal{M}$ implies $\mathcal{M} \models_J \theta$. We know that non-redundant elements $\langle J, e\rangle$ of $\mathcal{M}$ do not occur in $\mathcal{M}^*$. By the inductive hypothesis $\mathcal{M} \models_J \theta$ implies $\mathcal{M}^* \models_J \theta$ for each $J \subset I$ such that $\langle J, e\rangle$ is non-redundant . Thus, $\mathcal{M}^* \models_I \langle e\rangle\theta$.

$\psi = \{e\}\theta$: There exists a unique $J \subset I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $\mathcal{M} \models_J \theta$. Since there is no $K \subset J$ with $\langle K, e\rangle \in \mathcal{M}$, $\langle J, e\rangle$ is not redundant; therefore $\langle J, e\rangle \in \mathcal{M}^*$. By the inductive hypothesis $\mathcal{M} \models_J \theta$ implies $\mathcal{M}^* \models_J \theta$. So, we have $\mathcal{M}^* \models_I \langle e\rangle\theta$.

$\psi = \{e^f\}\theta$: There exists a unique *minimal-first* interval $J \subset I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $\mathcal{M} \models_J \theta$. Since there is no *minimal-first* interval $K \subset J$ with $\langle K, e\rangle \in \mathcal{M}$, $\langle J, e\rangle$ is not redundant; therefore $\langle J, e\rangle \in \mathcal{M}^*$. By the inductive hypothesis $\mathcal{M} \models_J \theta$ implies $\mathcal{M}^* \models_J \theta$. Then, $\mathcal{M}^* \models_I \langle e\rangle\theta$.

$\psi = \{e\}_<\theta$: There exists a unique $J \subset I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $\mathcal{M} \models_{init(J,I)} \theta$. Since there is no $K \subset J$ with $\langle K, e\rangle \in \mathcal{M}$, $\langle J, e\rangle$ is not redundant; therefore $\langle J, e\rangle \in \mathcal{M}^*$. By the inductive hypothesis $\mathcal{M} \models_{init(J,I)} \theta$ implies $\mathcal{M}^* \models_{init(J,I)} \theta$. Thus, $\mathcal{M}^* \models_I \langle e\rangle\theta$.

$\psi = \{e^l\}\theta$, $\psi = \{e\}_>\theta$, $\psi = \{e^f\}_<\theta$, $\psi = \{e^l\}_<\theta$, $\psi = \{e^f\}_>\theta$, $\psi = \{e^l\}_>\theta$: These cases can be dealt with similarly.

$\psi = \theta_1\mathcal{C}\theta_2$: By the semantics, there exists $J \subset I$ such that $\mathcal{M} \models_J \theta_1$ and $\mathcal{M} \models_{fin(J,I)} \theta_2$ and $|init(J, I)| = 0$. By the inductive hypothesis $\mathcal{M}^* \models_J \theta_1$ and $\mathcal{M}^* \models_{fin(J,I)} \theta_2$. We then easily have $\mathcal{M}^* \models_I \theta_1\mathcal{C}\theta_2$.

$\psi = \theta_1 \vee \theta_1$, $\psi = \theta_1 \wedge \theta_1$: These cases are treated in a similar way to $\psi = \theta_1\mathcal{C}\theta_2$.

Before constructing a sub-interpretation $\mathcal{M}^*$ of $\mathcal{M}$, we define:

Definition 5. Given $\mathcal{M} \models_I \phi$, assume that $\neg$ operators in ϕ are moved inwards, as shown in Lemma 1, and the depth of $\mathcal{M}$ is at most $|\phi|^2$ by Lemma 2. Let Φ be the set of subformulas of ϕ. A formula φ is *basic* if the major connective of φ is neither $\vee$ nor $\wedge$. For any interval J and any $\psi \in \Phi$, the *set of all maximal basic subformulas* φ of ψ with $\mathcal{M} \models_J \varphi$ is denoted by $S(\psi, J)$ which entails ψ.

Lemma 3. If a TPL* formula ϕ is satisfiable , then there is a terminating tableau procedure for ϕ which constructs a tree, from which $\mathcal{M}^*$ is extracted, of size bounded by $2^{p(|\phi|)}$ for some fixed polynomial p.

Proof. Given the assumptions in the definition, suppose that $\mathcal{M} \models_{I_0} \phi$. The procedure $\texttt{tree}(\phi, I_0)$ grows a labelled tree with nodes V, edges E, and two labelling functions $\lambda : V \to \mathcal{I}$ and $L : V \to \mathcal{P}(\Phi)$, where $\mathcal{P}(\Phi)$ denotes the power set of Φ. For $v \in V$, $\lambda(v)$ denotes the interval represented by the node v, and $L(v)$ denotes some collection of formulas which must be true at this interval. The procedure $\texttt{tree}(\phi, I_0)$ is shown in Figure 1. In the figure, $\alpha \in \{e, e^f, e^l\}$, and Q

denotes the queue of nodes in V awaiting processing. The steps in $\texttt{tree}(\phi, I_0)$ ensure that existential formulas in Φ have witnesses as required, and the embedded calls to $\texttt{univ}(u)$ ensure that universal formulas in Φ are not falsified by these witnesses.

We claim that the procedure $\texttt{tree}(\phi, I_0)$ terminates after finitely many iterations, and upon termination the tree (V, E) satisfies the size bound of $2^{p(|\phi|)}$. The steps in the procedure $\texttt{tree}(\phi, I_0)$ ensures that if the tree (V, E) contains a path $v_0 \rightarrow ... \rightarrow v_m$, then $\lambda(v_0) \supset ... \supset \lambda(v_m)$. On the other hand, if we investigate the procedure $\texttt{univ}(v_i)$ we can see that $\texttt{univ}(v_i)$ adds at most $|\phi|^2$ symbols to $L(v_i)$ at most d different values, where d is the depth of $\mathcal{M}$, and $1 \leq i \leq m$. Since we know that d must be at most of order $|\phi|^2$, the length of the path $v_0 \rightarrow ... \rightarrow v_m$ is at most of order $|\phi|^4$. Thus, the eventual size of the tree is bounded by $2^{|\phi|^4}$.

Theorem 1. The satisfiability problem for TPL* is in NEXPTIME.

Proof. If a TPL* formula ϕ is satisfiable , then ϕ is satisfied in a model $\mathcal{M}^*$ of size bounded by $2^{p(|\phi|)}$ for some fixed polynomial p. $\mathcal{M}^*$ can be extracted from the tree which is constructed by $\texttt{tree}(\phi, I_0)$. Let $\mathcal{M}^* = \{\langle J, e\rangle \in \mathcal{M} \mid$ for some $v \in V,\ J = \lambda(v)\ \}$. By Lemma 3 we know that $\mathcal{M}^*$ is of size bounded by $2^{p(|\phi|)}$, for some fixed polynomial p. In [8] it was proved that $\mathcal{M}^* \models_{I_0} \phi$.

We make a last remark that we have assumed TPL* is interpreted over a linear time flow and only finitely many events and states can occur over a bounded-time interval.

4. Conclusion

In this paper we introduced an interval temporal logic TPL* to represent meanings of sentences in English, and capture important real-time problems like behaviour of complex systems. TPL* is an extended version of TPL with the notion of state models, durations and the chop modality. Thus, it contains both event-based and state-based views.

We mentioned some drawbacks concerning the problems of interval temporal logics in natural language discourse, and real-time systems domain. In particular, in order to overcome the drawbacks with TPL we extended the syntax of TPL while keeping the logic computationally decidable. By this extended notation we aimed at presenting the semantics of natural language constructions, and modelling behaviour of real-time systems.

In order to preserve decidability we also extended the tableau system for the logic TPL. By showing that the tableau method terminates even for this extended logic we proved the satisfiability problem is decidable. Indeed, we showed that this problem is in NEXPTIME.

The future research directions include using other standard proof techniques to investigate the decidability of the satisfiability problem of TPL*, giving an axiomatization of TPL* complementing the semantic view, providing a framework for processing multiple temporal constraints and extending the

begin $\texttt{tree}(\phi, I_0)$

Select object v_0, and set

$Q := \{v_0\}$; $V := \{v_0\}$; $\lambda(v_0) := I_0$; $L(v_0) := S(\phi, I_0)$; $E := \varnothing$

until $Q = \varnothing$ **do**

Select $v \in$Q, and set $I := \lambda(v)$; $Q := Q \setminus \{v\}$

for *every* $\psi \in L(v)$ **do**

1) If $\psi = s$, there exits $J \supseteq I$ such that $\langle J, s\rangle \in \mathcal{M}$. Select $w \notin V$ and set $\lambda(w) := J$; $L(w) := \varnothing$; $Q := Q \cup \{w\}$; $V := V \cup \{w\}$; $E := E \cup \{(v, w)\}$.
2) If $\psi = \neg s$, there is no $J \supseteq I$ with $\langle J, s\rangle \in \mathcal{M}$. Since $\mathcal{M}^* \subseteq \mathcal{M}$, $\langle J, s\rangle \notin \mathcal{M}^*$.
3) If $\psi = \neg\{e\}\top$, there exit $J \subset I$, $J' \subset I$ such that $J \neq J'$, $\langle J, e\rangle \in \mathcal{M}$ and $\langle J', e\rangle \in \mathcal{M}$. Choose either J or J'. Select $w, w' \notin V$ and set $\lambda(w) := J$; $\lambda(w') := J'$; $L(w) := \varnothing$; $L(w') := \varnothing$; $Q := Q \cup \{w, w'\}$; $V := V \cup \{w, w'\}$; $E := E \cup \{(v, w), (v, w')\}$. Execute $\texttt{univ}(w)$ and $\texttt{univ}(w')$.
4) If $\psi = \int\langle e\rangle \tau k$, there exits $J \subset I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $|J|\,\tau k$. Select $w \notin V$ and set $\lambda(w) := J$; $L(w) := \varnothing$; $Q := Q \cup \{w\}$; $V := V \cup \{w\}$; $E := E \cup \{(v, w)\}$. Execute $\texttt{univ}(w)$.
5) If $\psi = \int\Diamond s \tau k$, there exits $J \subset I$ such that $\langle J, s\rangle \in \mathcal{M}$ and $|J|\,\tau k$. Select $w \notin V$ and set $\lambda(w) := J$; $L(w) := \varnothing$; $Q := Q \cup \{w\}$; $V := V \cup \{w\}$; $E := E \cup \{(v, w)\}$.
6) If $\psi = \langle e\rangle\,\theta$, there exits $J \subset I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $\mathcal{M} \models_J \theta$. Select $w \notin V$ and set $\lambda(w) := J$; $L(w) := S(\theta, J)$; $Q := Q \cup \{w\}$; $V := V \cup \{w\}$; $E := E \cup \{(v, w)\}$. Execute $\texttt{univ}(w)$.
7) If $\psi = \{\alpha\}\,\theta$, there exits a unique $J \subset I$ such that $\mathcal{M}, J \models_I \alpha$ and $\mathcal{M} \models_J \theta$. Select $w \notin V$ and set $\lambda(w) := J$; $L(w) := S(\theta, J)$; $Q := Q \cup \{w\}$; $V := V \cup \{w\}$; $E := E \cup \{(v, w)\}$. Execute $\texttt{univ}(w)$.
8) If $\psi = \{\alpha\}_<\,\theta$, there exits a unique $J \subset I$ such that $\mathcal{M}, J \models_I \alpha$ and $\mathcal{M} \models_{init(J,I)} \theta$. Let $J' = init(J, I)$. Select $w, w' \notin V$ and set $\lambda(w) := J$; $\lambda(w') := J'$; $L(w) := \varnothing$; $L(w') := S(\theta, J')$; $Q := Q \cup \{w, w'\}$; $V := V \cup \{w, w'\}$; $E := E \cup \{(v, w), (v, w')\}$. Execute $\texttt{univ}(w)$ and $\texttt{univ}(w')$.
9) If $\psi = \{\alpha\}_>\,\theta$, proceed similarly.
10) $\mathcal{M} \models_I \theta_1 \mathcal{C} \theta_2$, there exists $J \subset I$ such that $\mathcal{M} \models_J \theta_1$ and $\mathcal{M} \models_{fin(J,I)} \theta_2$ and $|init(J, I)| = 0$. Let $J' = fin(J, I)$. Select $w, w' \notin V$ and set $\lambda(w) := J$; $\lambda(w') := J'$; $L(w) := S(\theta_1, J)$; $L(w') := S(\theta_2, J')$; $Q := Q \cup \{w, w'\}$; $V := V \cup \{w, w'\}$; $E := E \cup \{(v, w), (v, w')\}$. Execute $\texttt{univ}(w)$ and $\texttt{univ}(w')$.

end for *every*

end until

end $\texttt{tree}$

begin $\texttt{univ}(u)$

for *every* formula $[e]\,\theta \in \Phi$ such that $\langle\lambda(u), e\rangle \in \mathcal{M}$ and there exists $L \supset \lambda(u)$ with $\mathcal{M} \models_L [e]\,\theta$ **do**

Set $L(u) := L(u) \cup S(\theta, \lambda(u))$

end for *every*

end $\texttt{univ}$

Fig. 1. tree

decision method with a temporal constraint network resolution algorithm, investigating the expressive power of TPL* in natural language discourse and specification of real-time systems, and investigating feasible model checking methods for TPL*.

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References

[1] van Benthem J. F. The Logic of Time. Dordrecht, 1983.

[2] Chaochen Z., Hansen M. R. Duration Calculus. A Formal Approach to Real-Time Systems. EATCS Series of Monographs in Theoretical Computer Science, Springer, 2004.

[3] Crouch R., Pulman S. G. Time and Modality in a Natural Language Interface to a Planning System. Artificial Intelligence, vol. 63, num. 1-2, 1993, pp. 265-304.

[4] Dowty D. Word Meaning and Montague Grammar. Dordrecht: D. Reidel, 1979.

[5] Goranko V., Montanari A., Sciavicco G. Road Map of Interval Temporal and Duration Calculi. Journal of Applied Non-Classical Logics, vol. 14, 2004.

[6] Hwang C. H., Schubert L. K. Interpreting tense, aspect and time adverbials. Proceedings of the First International Conference on Temporal Logic, Lecture Notes in Computer Science, Springer, vol. 827, 1994, pp. 238-274.

[7] Kamp H. Events, Instants and temporal reference. Semantics from Different Points of view, Springer, 1979, pp. 376-417.

[8] Konur S. A Decidable Interval Temporal Logic. First Year Progress Report, University of Manchester, 2005.

[9] Moszkowski B. Reasoning about Digital Circuits. PhD Thesis, Stanford University, Computer Science Department, 1983. .

[10] Pratt-Hartmann I., Bree D. The Expressive Power of the English Temporal Preposition System. University of Manchester, Department of Computer Science Technical Report, UMCS-93-1-7.

[11] Pratt-Hartman I. Temporal Prepositions and their logic. Artificial Intelligence, vol. 166(1-2), 2005, pp. 1-36.

[12] Pratt-Hartman I., Francez N. Temporal Prepositions and Temporal Generalized Quantifiers. Linguistics and Philosophy, 24(2), 2001, pp. 187-222.

[13] Roper P. Intervals and tenses. Journal of Philosophical Logic, 1980, pp. 451-469.

[14] Ter Meulen A. G. Representing Time in Natural Language. MIT Press, Cambridge, 1996.

[15] Venema Y. A Modal Logic for Choppping Intervals. Journal of Logic and Computation, vol. 1, 1991, pp. 453-476.

[16] Vendler Z. Linguistics and Philosophy. Cornell University Press, 1967.