Ockhamistic Logics
and
True Futures of Counterfactual Moments

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Abstract

In this paper various Ockhamistic logics are compared with the aim of making clear the role of true futures of counterfactual moments, that is, true futures of moments outside the true chronicle. First we give an account of Prior's original Ockhamistic semantics where truth of a formula is relative to a moment and a chronicle. We prove that this is equivalent to a semantics put forward by Thomason and Gupta where truth is relative to a moment and a so-called chronicle function which assigns a chronicle to each moment. This is the case because true futures of counterfactual moments do not matter in Thomason and Gupta's semantics. Later we discuss how two options considered by Belnap and Green might be formalised. They come about by assuming either a chronicle or a chronicle function to be given once and for all. *The first of the two options is unable to give an account of certain statements from natural language and the second option invalidates an intuitively valid formula. We propose a new Ockhamistic semantics where the formula in question* is *valid, and furthermore, where true futures of counterfactual moments* are *taken into account. Finally, we discuss possible applications within Artificial Intelligence.*

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1. Introduction

This paper is concerned with certain logics which are Ockhamistic in the sense of involving a notion of true future. The logics in question are compared with the aim of making clear the role of true futures of counterfactual moments, that is, true futures of moments outside the true future.

The paper is structured as follows. In the second section of this paper Prior's original Ockhamistic semantics is introduced. See also [16], p. 126 ff. A notable feature of this semantics is a notion of "temporal routes" or "temporal branches", called chronicles. Here truth of a formula is relative to a moment as well as a chronicle - which is to be understood as truth being relative to a provisionally given true future. In the third section, we give an account of the semantics put forward by Thomason and Gupta in [19]. Here truth of a formula is relative to a moment as well as a so-called chronicle function which assigns a chronicle to each moment - this is to be understood as truth being relative to a provisionally given true future of each moment. We prove that this is equivalent to Prior's original semantics. This is so because true futures of counterfactual moments do not matter in the semantics - despite the fact that truth is relative to a moment *as well as* a chronicle function. In section four we give an account of two options considered by Belnap and Green in [2]. They come about by assuming either a chronicle or a chronicle function to be given *once and for all*. We discuss how such assumptions might give rise to formal semantics. The first of the two mentioned options is unable to give an account of certain statements from natural language in which there are references to true futures of counterfactual moments. The second option invalidates the intuitively valid formula $q \Rightarrow HFq$. In the fifth section we propose a new semantics where the model provides a chronicle function once and for all. Compared to the other Ockhamistic semantics given in this paper, it is a notable feature that the formula $q \Rightarrow HFq$ *is* valid here, and furthermore, true futures of counterfactual moments *are* taken into account. An essential difference between this new semantics and the traditional Ockhamistic semantics is that the involved notion of possibility only refers to what might happen *now* whereas possibility in the traditional sense refers to what might happen *from now on* in general. This new notion of necessity may be called "immediate necessity". In the last section we discuss possible applications of the logics presented in this paper within the area of Artificial Intelligence.

2. Prior's notion of Ockhamistic necessity

In what follows, we shall give an account of the semantics which was put forward by Arthur N. Prior in [16], p. 126 ff., with the aim of formalising some ideas of William of Ockham (ca. 1285 - 1349).

To define Prior's Ockhamistic semantics, we need a set T equipped with a relation $<$ together with a function V which assigns a truth value $V(t,p)$ to each pair consisting of an element t of T and a propositional letter p. The elements of T are to be thought of as moments and $<$ as the before-relation. It is assumed that $<$ is irreflexive and transitive. Furthermore, to account for the branchingness of time it is assumed that $<$ satisfies the condition

$$\forall t, t', t''.\ (t < t' \wedge t'' < t') \Rightarrow (t < t'' \vee t'' < t \vee t = t'').$$

A relation satisfying this condition is said to be *backwards linear*. An important feature of the semantics is a notion of "temporal routes" or "temporal branches" which are to be thought of as possible courses of events. We shall call such branches *chronicles*[1]. Formally, chronicles are maximal linear subsets in $(T,<)$. The set of chronicles induced by $(T,<)$ will be denoted $C(T,<)$, and moreover, for any moment t we let $\Delta(t)$ denote the set of chronicles to which t belongs. Note that the conditions of transitivity and backwards linearity make each chronicle c backwards-closed, that is, if $t \in c$ and $t' < t$ then $t' \in c$. Truth is relative to a moment as well as to a chronicle to which the moment belongs - which is to be understood as truth being relative to a provisionally given true future of the moment in question. In [16], p. 126, Prior calls it a "prima facie" assignment. By induction, we define the valuation operator $Prior$ as follows:

$$\begin{array}{rcl} Prior(t,c,p) & \Leftrightarrow & V(t,p), \text{ where } p \text{ is a prop. letter} \\ Prior(t,c,p \wedge q) & \Leftrightarrow & Prior(t,c,p) \wedge Prior(t,c,q) \\ Prior(t,c,\neg p) & \Leftrightarrow & \neg Prior(t,c,p) \\ Prior(t,c,Pp) & \Leftrightarrow & \exists t' < t.\ Prior(t',c,p) \\ Prior(t,c,Fp) & \Leftrightarrow & \exists t' > t.\ t' \in c \wedge Prior(t',c,p) \\ Prior(t,c,\Box p) & \Leftrightarrow & \forall c' \in \Delta(t).\ Prior(t,c',p) \end{array}$$

So $Prior(t,c,p)$ amounts to p being true at the moment t in the chronicle c. A formula p is said to be Prior-valid if and only if p is Prior-true in any structure $(T,<,V)$, that is, we have $Prior(t,c,p)$ for any moment t and chronicle c such that $t \in c$.

For instance, let us consider the formula $q \Rightarrow HFq$ (where Hp is an abbreviation for $\neg P \neg p$). From the above definitions it is obvious that $Prior(t,c,q \Rightarrow HFq)$ for any t and any c with $t \in c$. Therefore $q \Rightarrow HFq$ is valid in this system. Likewise, the formula $q \Rightarrow \Box q$, where the tense operator F does not occur in q, is valid. This formula is in good accordance with the medieval dictum "unumquodque, quando est, oportet esse" ("anything, when it is, is necessary"), see [17], p. 191.

It may be doubted whether Prior's Ockhamistic system is in fact an accurate representation of the temporal logical ideas propagated by William of Ockham. According to Ockham, God knows the contingent future, so it seems that he would accept an idea of absolute truth, also when regarding a statement Fq about the contingent future - and not only prima facie assignments like $Prior(t,c,Fq)$. That is, such a proposition can be made true "by fiat" simply by constructing a concrete structure which satisfies it. But Ockham would say that Fq can be true at t without being relativised to any chronicle. It is possible to establish a system which seems a bit closer to Ockham's ideas by taking chronicles to be disjoint and having a relation between chronicles corresponding to identity up to a certain moment. A system along these lines is in [12] called "Leibnizian".

3. Thomason and Gupta's notion of Ockhamistic necessity

In this section, we shall compare the traditional Ockhamistic semantics given by Prior to the semantics given by Richmond Thomason and Anil Gupta in [19]. An essential difference is that in the traditional semantics truth is relative to a moment as well as a chronicle whereas in Thomason and Gupta's semantics, truth is relative to a moment as well as a so-called chronicle function which to each moment assigns a chronicle. We shall prove that the two semantics are equivalent. This is the case because true futures

[1] Some authors call them *histories*.

of counterfactual moments do not matter in the semantics - despite the fact that truth is relative to a moment *as well as* a chronicle function. It should be mentioned that besides the usual Ockhamistic connectives the semantics given in Thomason and Gupta's paper also interprets counterfactual implication - in such a context true futures of counterfactual moments *do* make a difference.

A formal account of Thomason and Gupta's semantics is based on the same notion of a structure as the one used previously in this paper. So we need a set T equipped with an irreflexive, transitive and backwards linear relation $<$ together with a function V which assigns a truth value to each pair consisting of a moment and a propositional letter. Furthermore, it involves a relation on T relating "copresent" (or "pseudo-simultanous") moments and also certain additional machinery to interpret counterfactual implication. Here we do not consider counterfactual implication so we shall disregard such machinery, and moreover, we adapt Thomason and Gupta's semantics to the previous setting of this paper where no notion of copresentness is taken into account.

In the traditional semantics, truth is relative to a moment and a chronicle to which the moment belongs. In Thomason and Gupta's semantics, truth is relative to a moment and a chronicle function. A *chronicle function* is a function C which assigns to each moment a chronicle such that the following two conditions are satisfied:

(C1) $\forall t.\ t \in C(t)$.

(C2) $\forall t, t'.\ (t < t' \wedge t' \in C(t)).\Rightarrow C(t) = C(t')$

The first condition says that the chronicle assigned to a moment has as element the moment itself. The second condition says that chronicles assigned to later moments of a chronicle coincide with chronicles assigned to earlier ones. The definition of truth of a formula as relative to a moment and a chronicle function is to be understood as truth being relative to a moment and a provisionally given true future of each moment. The chronicle function with respect to which truth is relative, is assumed to be normal at the moment at hand. Given a moment t, a chronicle function C is said to be *normal at* t if and only if

$$\forall t' < t.\ C(t') = C(t).$$

We let $N(t)$ denote the set of chronicle functions which are normal at the moment t. Normality at a moment means that the moment is in the true future of any earlier moment. Without restricting attention to chronicle functions normal at a given moment, the formula $q \Rightarrow HFq$ would not be valid.

How should $\Box p$ be interpreted? As Thomason and Gupta explains in [19], p. 311, one should not simply say that $\Box p$ is true at t with respect to C normal at t if and only if p is true at t with respect to *all* chronicle functions C' normal at t. This is because $\Box p$ is supposed to say that p holds no matter how things *will* be. Hence, C' should differ from C at most at t and moments in the future of t. It follows from normality at t that C' also has to be allowed to differ from C at moments in the past of t. This leads to the following definition: The chronicle functions C and C' are said to *differ at most at* t *and its past and future* if and only if

$$\forall t'.\ (t' \neq t \wedge t' \not< t \wedge t \not< t') \Rightarrow C(t') = C'(t').$$

We let $\Delta_{P,F}(t, C)$ denote the set of chronicle functions which differ from C at most at t and moments in the past and future of t. By induction, we define the valuation operator V as follows:

$$\begin{array}{rcl} V(t,C,p) & \Leftrightarrow & V(t,p), \text{ where } p \text{ is a prop. letter} \\ V(t,C,p \wedge q) & \Leftrightarrow & V(t,C,p) \wedge V(t,C,q) \\ V(t,C,\neg p) & \Leftrightarrow & \neg V(t,C,p) \\ V(t,C,Pp) & \Leftrightarrow & \exists t' < t.\ V(t',C,p) \\ V(t,C,Fp) & \Leftrightarrow & \exists t' > t.\ t' \in C(t) \wedge V(t',C,p) \\ V(t,C,\Box p) & \Leftrightarrow & \forall C' \in N(t) \cap \Delta_{P,F}(t,C).\ V(t,C',p) \end{array}$$

A formula p is said to be valid if and only if p is true in any structure $(T, <, V)$, that is, we have $V(t, C, p)$ for any moment t and chronicle function C normal at t.

We shall now show that this notion of validity is equivalent to the traditional notion of Ockhamistic validity. As alluded to above, this is so because true futures of counterfactual moments do not matter. In the semantics above, $\Box q$ says that q holds no matter how things will be, where "things" do not only refer to the true future of the present moment but also to true futures of future moments. Thus, even if an Ockhamistic model does provide true futures of counterfactual moments, these futures do not matter if things will be different (and obviously not if things will be the same either). With the aim of proving the two notions of validity equivalent, we shall first prove a theorem which essentially says that chronicles can be "mimicked" by chronicle functions.

Theorem 3.1 *Given a chronicle* c *there exists a chronicle function* C *such that* $\forall t \in c.\ C(t) = c$.

Proof: In this proof we shall use Zermelo's Theorem and Zorn's Lemma which are both equivalent to the Axiom of Choice. Zermelo's Theorem says that any set can be well-ordered, that is, it can be equipped with a linear partial order such that any non-empty subset has a least element. By Zermelo's Theorem, we can assume that T is well-ordered by a relation $\sqsubseteq$. For any non-empty subset T' of T we let $\sqcap T'$ denote its $\sqsubseteq$-least element. Zorn's Lemma says that if each linear subset of a non-empty partially ordered set A has an upper bound, then A has a maximal element. By Zorn's Lemma, we can assume that for any moment t there exists a chronicle c' such that $t \in c'$, and hence, by the Axiom of

Choice, we can assume that there exists a function f which to any moment t assigns a chronicle c' such that $t \in c'$. By transfinite induction using $\sqsubseteq$, we define a function

$$C : T \rightarrow C(T, <)$$

such that

$$C(t) = \begin{cases} c & \text{if } t \in c \\ C(\sqcap\{t' \sqsubset t | t \in C(t')\}) & \text{if } \exists t' \sqsubset t.\, t \in C(t') \\ f(t) & \text{otherwise} \end{cases}$$

for any moment t. We only have to check that C satisfies the conditions (C1) and (C2). Clearly, (C1) is ok. Assume that $t < t'$ and $t' \in C(t)$. We then have to prove that $C(t) = C(t')$. It is straightforward to check that $t \in c \Leftrightarrow t' \in c$, so without loss of generality we can assume that $t \notin c$ and $t' \notin c$. Let $t_0 = \sqcap\{u | t \in C(u)\}$. Thus, t_0 is the $\sqsubseteq$-least moment where t belongs to the assigned chronicle. Hence, $t \in C(t_0)$ and $t_0 \sqsubseteq t$, and furthermore, $C(t_0) = C(t)$. Now, $t' \in C(t_0)$ as $t' \in C(t)$ so $t'_0 \sqsubseteq t_0$ where $t'_0 = \sqcap\{v | t' \in C(v)\}$. Thus, $t'_0 \sqsubseteq t'$ and $C(t'_0) = C(t')$. But $t < t'$ so $t \in C(t'_0)$ and hence $t_0 \sqsubseteq t'_0$ by definition of t_0. We conclude that $t_0 = t'_0$ so $C(t) = C(t')$. QED.

The theorem above gives rise to an important corollary.

Corollary 3.2 *Let a chronicle function C be given. For any moment t and chronicle c such that $t \in c$ there exists a chronicle function C' normal at t such that $C'(t) = c$ and $C' \in \Delta_{P,F}(t, C)$.*

Proof: Let $K = \{t' | t' \neq t \wedge t' \not< t \wedge t \not< t'\}$ and note that $T \setminus K$ is itself a structure, that is, irreflexive, transitive and backwards linear. We have $c \cap K = \emptyset$ because $t \in c$, so the preceeding theorem can be applied to c considered as a chronicle in $T \setminus K$ to obtain a chronicle function C' as appropriate. Note that any chronicle in $T \setminus K$ is also a chronicle in T. Furthermore, note that $T \setminus K$ is forwards closed. QED.

The lemma we shall prove now essentially says that given a chronicle function, the semantics does not take into account true futures of counterfactual moments.

Lemma 3.3 *Let a formula q be given. For any chronicle c and chronicle functions C and C' normal at a moment t such that $C(t) = C'(t)$ it is the case that $V(t, C, q) \Leftrightarrow V(t, C', q)$.*

Proof: Induction on the structure of q. We only check the $\Box$ case; the other cases are straightforward. Assume that $V(t, C, \Box p)$. By definition of the semantics we have $V(t, C', \Box p)$ if

$$\forall C'' \in N(t) \cap \Delta_{P,F}(t, C').\, V(t, C'', p).$$

It follows from Corollary 3.2 that for any

$$C'' \in N(t) \cap \Delta_{P,F}(t, C')$$

there is a

$$C''' \in N(t) \cap \Delta_{P,F}(t, C)$$

such that $C'''(t) = C''(t)$. But $V(t, C''', p) \Leftrightarrow V(t, C'', p)$ according to the induction hypothesis. By definition of the semantics we have $V(t, C, \Box p)$ only if

$$\forall C''' \in N(t) \cap \Delta_{P,F}(t, C).\, V(t, C''', p).$$

Thus, it is the case that $V(t, C'', p)$. QED.

An important consequence of the preceeding results is the following lemma.

Lemma 3.4 *Let a formula q be given. For any chronicle c and moment t such that $t \in c$ the existence of a chronicle function C normal at t such that $C(t) = c$ and $V(t, C, q)$ is equivalent to $V(t, C', q)$ being the case for any chronicle function C' normal at t such that $C'(t) = c$.*

Proof: Follows from Theorem 3.1 and Lemma 3.3. QED.

We now prove a theorem making clear how truth in the traditional Ockhamistic semantics is related to truth in the new semantics.

Theorem 3.5 *Let a formula q be given. Also, let a moment t and a chronicle c such that $t \in c$ be given. Then $Prior(t, c, q)$ if and only if for any chronicle function C normal at t such that $C(t) = c$ it is the case that $V(t, C, q)$.*

Proof: Induction on the structure of q. We proceed on a case by case basis. The case where q is a propositional letter follows from Theorem 3.1. The $\wedge$ case is straightforward. The $\neg$ case follows immediately from Lemma 3.4. The P and F cases follow from Lemma 3.4. The $\Box$ case goes as follows: By definition $Prior(t, c, \Box p)$ is equivalent to $Prior(t, c', p)$ being the case for any c' such that $t \in c'$. By the induction hypothesis this is equivalent to $V(t, C', p)$ being the case for any C' normal at t such that $C'(t) = c'$ where c' is any chronicle such that $t \in c'$. This is equivalent to $V(t, C', p)$ being the case for any C' normal at t. But cf. Corollary 3.2 this is equivalent to $V(t, C, \Box p)$ being the case for any C normal at t such that $C(t) = c$. QED.

The theorem above enables us to conclude that the traditional semantics is equivalent to Thomason and Gupta's semantics.

Corollary 3.6 *A formula is Prior-valid if and only if it is valid in Thomason and Gupta's semantics.*

Proof: Straightforward by the preceeding theorem. QED.

4. Belnap and Green's argument

In the interesting paper [2], Nuel Belnap and Mitchell Green consider a model where a chronicle is given once and for all - the true chronicle. They denote such a chronicle TRL (*T*hin *R*ed *L*ine). Belnap and Green argue that this kind of model is unable to give an account of certain statements from ordinary language in which there are references to true futures of counterfactual moments. This is remedied by assuming that instead of just one true chronicle the model provides one true chronicle for each moment. But also in this case problems apparently crop up. In this section we shall discuss how Belnap and Green's arguments might be formalised.

We will first consider the case where the model provides a chronicle once and for all. Belnap and Green have based their arguments on the following example of a statement from ordinary language:

> The coin will come up heads. It is possible, though, that it will come up tails, and then *later* (*) it will come up tails again (though at that moment it could come up heads), and then, inevitably, still *later* it will come up tails yet again. ([2], p. 379)

As Belnap and Green explain in [2], p. 379, the trouble is that at (*) the example says that tails *will* happen. The point is here that (*) is future relative to a counterfactual moment, that is, a moment outside the true chronicle. Now, how should tenses and possibility in the sentence above be interpreted? Belnap and Green give the following informal account of the semantics they have in mind:

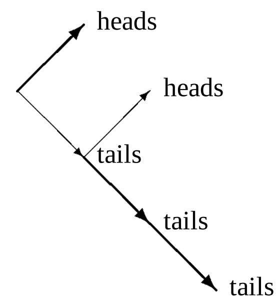
> In the semantic theory of branching+TRL the *future tense* moves you forward along TRL and the *past tense* moves you backward along it. Any talk of *possibility* or *necessity* or *inevitability* refers to some histories other than TRL. ([2], p. 379)

So any future tensed sentence has to be interpreted with respect to the true chronicle. But this does obviously not make sense outside the true chronicle. We agree with Belnap and Green that this is problematic.

Belnap and Green remedies this deficiency by assuming that instead of just one true chronicle the model provides a function which to each moment assigns one true chronicle. They discuss which conditions such a function C has to satisfy, but rather than condition (C2) they assume the condition

$$\forall t, t'.\, t < t' \Rightarrow C(t) = C(t')$$

to be satisfied - which amounts to the function being normal at every moment of the model. This is problematic because it forces the before-relation to be forwards linear, as they do indeed point out[2]. In what follows, we shall therefore restrict our attention to chronicle functions in the previous sense of this paper unless otherwise is indicated. Using a chronicle function, truth of the statement above amounts to truth at the left-most moment of the structure



where we have used a thick line to represent the future part of a chronicle (the past part needs no representation as it is uniquely determined).

Belnap and Green have argued that the formulae $FFq \Rightarrow Fq$ and $q \Rightarrow PFq$ should be valid. They do not give a formal semantics but it is clear that they interpret tenses by moving in the appropriate direction along the true chronicle of the moment at hand. It is not clear how the interpretation of possibility goes in their semantic theory, but it seems unavoidable to take into account chronicle functions which are different from the given chronicle function at the moment at hand (if that was not the case then the interpretation of possibility could not refer to chronicles other than the true chronicle of the moment at hand). Hence, truth has to be relative to a moment as well as a chronicle function. This suggests a semantics like the one put forward by Thomason and Gupta *except* that a chronicle function is given once and for all and $\Box p$ is true at t with respect to C if and only if p is true at t with respect to all chronicle functions C' such that C' differ from C at most at the moment t. Formally, the chronicle functions C and C' are said to *differ at most at* t if and only if

$$\forall t'.\, t' \neq t \Rightarrow C(t') = C'(t').$$

We let $\Delta(t, C)$ denote the set of chronicle functions which differ from C at most at t. Then the valuation operator V is defined as follows:

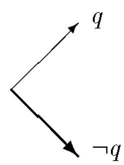
$$\begin{array}{rcl}
V(t,C,p) & \Leftrightarrow & V(t,p), \text{ where } p \text{ is a prop. letter} \\
V(t,C,p \wedge q) & \Leftrightarrow & V(t,C,p) \wedge V(t,C,q) \\
V(t,C,\neg p) & \Leftrightarrow & \neg V(t,C,p) \\
V(t,C,Pp) & \Leftrightarrow & \exists t' < t.\, V(t',C,p) \\
V(t,C,Fp) & \Leftrightarrow & \exists t' > t.\, t' \in C(t) \wedge V(t',C,p) \\
V(t,C,\Box p) & \Leftrightarrow & \forall C' \in \Delta(t,C).\, V(t,C',p)
\end{array}$$

A formula p is then said to be valid if and only if p is true in any structure $(T, <, V, C)$, that is, we have $V(t, C, p)$ for any moment t.

Given such a semantics, it is straightforward to check that $FFq \Rightarrow Fq$ is valid whereas it is not valid if (C2) is

[2] In [3] we remedied this by suggesting the more appropriate condition (C2). Later, we realized that this condition was introduced in [19]. In fact, it occurs already in the paper [11] by Vaughn McKim and Charles Davis where, however, it is formulated in a different way.

left out. But $q \Rightarrow PFq$ is invalid. We agree that this is troublesome. Furthermore, the formula $q \Rightarrow HFq$ is invalid; it is not true at the upper-most moment of the structure



Faced with the mentioned problems, Belnap and Green simply abandon the idea of assuming a chronicle function to be given once and for all.

5. The new semantics

Also in this section we assume that the model provides a chronicle function once and for all - the true chronicle function. What we are after here is a semantics where the formula $q \Rightarrow HFq$ *is* valid, and furthermore, where true futures of counterfactual moments *are* taken into account. With this goal in mind we shall propose a new semantics.

A notable difference between this new semantics and the traditional Ockhamistic semantics is that in the new semantics the involved notion of possibility just refers to what might happen *now* whereas possibility in the traditional sense refers to what might happen *from now on* in general. In other words, in the traditional semantics $\Box q$ means that q is true no matter what happens now and in the future whereas in the new semantics $\Box q$ means that q is true no matter what happens now. This new notion of necessity may be called ”immediate necessity”. The new semantics corresponds to considering the first occurrence of the word ”possible” in Belnap and Green's example as referring to the first tossing of the coin rather than the whole sequence of tossings. We find this very natural.

In what follows, we shall give an account of the new semantics. What we do is we take the semantics of the last section where a chronicle function is given once and for all and where truth is relative to a moment and a chronicle function (note that conditions (C1) and (C2) are satisfied here). We then add the condition that the chronicle function has to be normal at the moment at hand. This makes the resulting semantics validate the formula $q \Rightarrow HFq$. The restriction to normal chronicle functions forces us to change the semantics of necessity such that it takes into account not only chronicle functions differing from the given chronicle function at the moment at hand but also chronicle functions differing at moments in the past of the moment at hand. Thus, the key feature of the semantics is that $\Box p$ is true at t with respect to C normal at t if and only if p is true at t with respect to all chronicle functions C' normal at t such that C' differ from C at most at the moment t and its past. This leads to the following definition: C and C' are said to *differ at most at t and its past* if and only if it is the case that

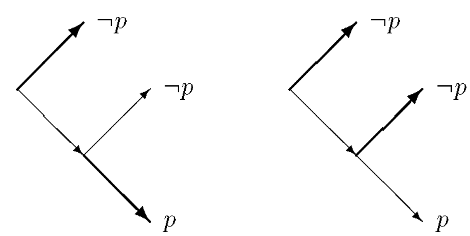
$$\forall t'.\ (t' \neq t \wedge t' \not< t) \Rightarrow C(t') = C'(t').$$

We let $\Delta_P(t, C)$ denote the set of chronicle functions which differ from C at most at t and moments in the past of t. We now define the valuation operator V as follows:

$$\begin{array}{rcl}
V(t,C,p) & \Leftrightarrow & V(t,p), \text{ where } p \text{ is a prop. letter} \\
V(t,C,p \wedge q) & \Leftrightarrow & V(t,C,p) \wedge V(t,C,q) \\
V(t,C,\neg p) & \Leftrightarrow & \neg V(t,C,p) \\
V(t,C,Pp) & \Leftrightarrow & \exists t' < t.\ V(t',C,p) \\
V(t,C,Fp) & \Leftrightarrow & \exists t' > t.\ t' \in C(t) \wedge V(t',C,p) \\
V(t,C,\Box p) & \Leftrightarrow & \forall C' \in N(t) \cap \Delta_P(t,C).\ V(t,C',p)
\end{array}$$

A formula p is said to be valid if and only if p is true in any structure $(T, <, V, C)$, that is, we have $V(t, C, p)$ for any moment t at which C is normal.

It is straightforward to check that both of the formulae $FFq \Rightarrow Fq$ and $q \Rightarrow HFq$ are valid. The fact that the last mentioned formula is validated makes us believe that the formal semantics given here is not the one Belnap and Green had in mind when writing the paper [2]. Now, consider the structures

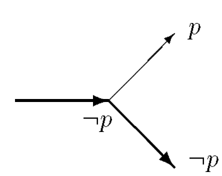


Clearly, the formula $\Diamond Fp$ (where $\Diamond q$ is an abbreviation for $\neg\Box\neg q$) is true in the left-most moment of the first structure whereas it is false in the corresponding moment of the second structure. But the structures are identical except that the involved chronicle functions differ at a counterfactual moment. This shows that true futures of counterfactual moments *do* make a difference in this new semantics.

The new semantics is not equivalent to Prior's Ockhamistic semantics (and thus neither to Thomason and Gupta's semantics). The new semantics invalidates the formula

$$F\Diamond Fp \Rightarrow \Diamond FFp$$

which is valid in Prior's semantics (note that in the context of dense time, this formula is equivalent with $F\Diamond Fp \Rightarrow \Diamond Fp$). The formula is not true in the left-most moment of the structure



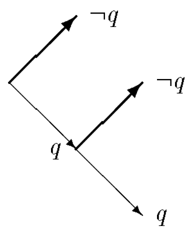
So Prior-validity does not imply validity in the sense proposed above. However, it should be mentioned that all axioms of the modal logic S5 are validated (for an introduction to S5, see [7]).

The invalidity of the above mentioned formula actually reveals a notable feature of this new semantics: It allows for the emergence of possibility in time, that is, it allows for a situation in which the truth of a proposition is possible in the future whereas the future truth of the proposition is impossible.

In [3] we suggested another semantics which also has the property that it allows for the emergence of possibility in time. The two semantics are not equivalent. The semantics of the mentioned paper invalidates the formula

$$H(Gq \Rightarrow \Diamond Gq)$$

(where Gp is an abbreviation for $\neg F \neg p$). It is not true in the lower-most moment of the structure



On the other hand, the formula is valid with respect to the new semantics given above. We find the formula intuitively valid wherefore we consider the semantics of the present paper as an improvement compared to the one of [3].

6. The relevance to Artificial Intelligence

The main concern of this paper has been to work out the formalities of a revised Ockham-semantics as a basis for AI-research and -applications, rather than actually investigating its use for various purposes. That remains to be done, but we shall briefly mention some points and areas of application, where an Ockhamistic logic enhanced as suggested is likely to be useful.

6.1. Natural language understanding

Our version of Ockhamistic semantics can at the same time provide two advantages in this field: Firstly, an Ockhamistic logic can, as pointed out in for example [12], make a genuine distinction between the following three statements:

1. Possibly, Mr. Smith will commit suicide.

2. Necessarily, Mr. Smith will commit suicide.

3. Mr. Smith will commit suicide.

This is a distinction arguably required for natural language understanding, regardless of any different metaphysical assumptions one might have about time and the contingent future. Secondly, within the same frame, linguistic examples as the one given by Belnap and Green can be dealt with. This also makes the system promising for evaluating counterfactual statements (see also [6]).

6.2. Planning

Branching time temporal logics are used for formalising temporal aspects of planning problems. Roughly, planning amounts to choosing appropriate actions with the aim of achieving a desirable outcome. Notable in this respect are the papers [10] and [14]. See also [5] which besides temporal notions also involves chance. The model of the last mentioned paper differs from the models of the present paper by having disjoint chronicles - which corresponds to the Leibniz system of [12]. The above mentioned applications do not explicitly involve true futures of counterfactual moments, but we conjecture that this notion - and also the naturally associated notion of immediate necessity put forward in the previous section - does have a place in such contexts.

6.3. Partial information reasoning

Branching time as well as chronicle functions occur naturally within the area of partial information reasoning, which recently has been pointed out in the paper [1]. In this context the notion of branching time comes about when considering the future courses of events which are compatible with given (partial) knowledge. In such a situation one particular future course of events can be singled out by using various criteria such as minimal change principles, probability and typicality. As explained in [1], such criteria simply correspond to chronicle functions on the underlying branching time structure. Our notion of immediate necessity is therefore readily available in such contexts.

6.4. Causal and counterfactual reasoning

The close relation between causal and counterfactual reasoning has been pointed out by a number of authors. Moreover, both of these areas have a clearly temporal component. Thus, David Lewis' possible world semantics for counterfactuals [8] was related to branching time in [9]. In [4] it was shown how Lewis' possible world semantics for counterfactuals related to non-monotonic reasoning, and how this could be utilized in AI-contexts (among other things fault diagnosis). In [18] Shoham made further progress in this direction (subsequently refined in several papers), and showed its relevance in non-monotonic reasoning - a crucial area in current AI-research. In [12] Øhrstrøm and Hasle presented a system which integrates counterfactual, causal and temporal reasoning, based on an Ockhamistic logic. See also [13]. The idea of true futures at conterfactual moments plays an important role in these systems, but only for finite models and not explicitly worked out as here. However, in our view the line of development in causal and counterfactual reasoning since [8] makes it credible that much can be gained from the study of how to apply the logic presented here to current problems in AI, especially those related to causal, counterfactual and temporal non-monotonic reasoning.

6.5. Knowledge representation

From the area of knowledge representation it is relevant to mention Javier Pinto and Raymond Reiter's work on the situation calculus, which is a formalism for representation and reasoning about actions and their effects. Compared to the formalisms of the present paper, this formalism is based on predicate logic rather than modalities. In the situation calculus, a branching time structure is implicitly present where the branches are possible sequences of situations starting from an initial situation. Such a branching time structure is in [15] endowed with a preferred branch describing the world's true evolution. This is obtained by extending the situation calculus with a predicate singling out situations belonging to the preferred branch. Given this, it is possible to express and answer certain hypothetical queries like:

> At time T_p in the past, when you put A on B, could A have been put on C instead? ([15], p. 7)

However, it is not possible to express counterfactual queries such as:

> If I had not paid the bookie, would I have lived to bet again? ([15], p. 12)

This is so because the situation calculus of [15] do not take into account true futures of counterfactual moments which the authors call "hypothetical actual" lines of situations. In fact, they write:

> The present formalism allows us to express knowledge about what happens in the world .... Unfortunately, we do not have hypothetical actual lines that would allow us to express knowledge about what would occur if certain non-actual actions were to be performed, for example, counterfactuals in which alternative action occurrences can be postulated. ([15], p. 7)

It seems that formalisation of hypothetical actual lines would (explicitly or implicitly) have to involve the notion of a chronicle function. This would make the considerations of the present paper relevant.

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