Temporal Reasoning with Fuzzy Time-Objects

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Abstract

A novel approach to temporal reasoning is proposed which deals with both uncertain facts and uncertain temporal information in fuzzy logic. Fuzzy time-objects are defined to represent these uncertainties. A point of particular concern when reasoning with both kinds of uncertainty is that temporal uncertainty should not influence factual uncertainty. On the other hand temporal reasoning is exactly about the relation between time and fact. By introducing constrained time-objects we show that a relation between time and fact can be established while avoiding mixing of uncertainties. Then a method to reason with time-objects is introduced. The inference relation between time-objects is thereby decomposed in a fact-fact and a time-time relation. Such a decomposition is only allowed for (semi-) separable time-objects. The decomposition does not prevent time and fact to have mutual influence.

1 Introduction

Temporal reasoning with uncertainty has received growing attention over the past years. Reasoning with both uncertain temporal information and uncertain facts has hardly been dealt with.

In section 2 we investigate how uncertainty is used in temporal reasoning approaches in literature. It will be shown that there are in general two different outlooks on temporal reasoning with uncertainty. It will also be shown that choosing either kind involves a commitment to quite different reasoning strategies.

Given the limitations of (2-dimensional) representations of temporal uncertainty in literature and the aim to represent both uncertainty about time and facts we propose 3-dimensional representations called *time-objects.* These time-objects, with a function similar to formal temporal logic connectives like 'OCCUR' and 'HOLD', are presented in section 3.

Through time-objects uncertain temporal knowledge and uncertain facts are connected. The time-fact relation is defined such that time and fact are independently represented in the time-object. Such time-objects are called *separable.*

Additionally it is thereby sometimes interesting to constrain the domain of a time-object such that a dependence relation between time and fact is established indirectly. Such time-objects are called *semi-separable* if the constraint domain satisfies conditions as discussed in section 4.

Given (constrained) time-objects, these objects become instantiated in the process of reasoning. This involves constraining time, fact and time-object in the light of available evidence, a process which will be explained in section 5.

Finally, reasoning with time-objects is considered in section 6. There we will show how the instantiation of the premises of a temporal rule will lead to the instantiated temporal conclusion. It will also be shown that time and fact do have mutual influence in the reasoning scheme, while mixing of temporal and factual uncertainty is avoided.

2 Use of temporal uncertainty in related work

Many applications require the explicit representation of both uncertain facts and uncertain time. Taking a measurement of some physical quantity, for example, can be both unprecise with respect to the time of measurement as to the measurement itself. Another example is in the medical domain where a patient may give information about symptoms of a disease previously suffered which are both imprecisely dated as well as imprecisely observed.

Factual uncertainty (imprecision of the measurement itself) and temporal uncertainty (imprecision with respect to the time of measurement) are different kinds of uncertainty. Whereas factual uncertainty is about evidence available for some proposition, tem-

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poral uncertainty is about its occurrence in time.

To exemplify this consider that someone may have had symptoms typical for a certain disease, while the time of occurrence of those symptoms does not support the hypothesized disease. This does not make the observed symptoms less true. Temporal and factual uncertainty are not the same! Indeed, the observed symptoms may support the hypothesis of another decease for which the timing is more appropriate.

From the above examples it is clear that it is desirable to know to what extend a proposition is uncertain as a result of factual uncertainty and to what extend temporal uncertainty is involved. In other words, it is desirable to represent factual and temporal uncertainty explicitly.

In our opinion it is thus not sufficient for a temporal reasoning system to add time as another proposition to a system in the way 'color()' or 'size()' might be added. On the contrary, when reasoning with time *all* propositions defined in a system should become time-dependent. Thus a proposition like 'color(house)' becomes something like 'color(house at time t_1)'. All propositions are then placed in time.

Therefore our objective is to represent uncertainty about time and facts independently and to relate these uncertainties by means of a relation similar to formal temporal reasoning connectives like HOLDS and OCCUR. Uncertain temporal information and uncertain facts thus connected will be called *time-objects*. Furthermore this representation should support the definition of uncertain temporal relations (like 'somewhat after/before') and inferencing with uncertain facts.

In literature most approaches which address uncertainty in temporal reasoning start either from the perspective of formal temporal theories or from fuzzy logic. Examples of the former are the first-order probability logics by Goodwin[9], Haddawy[10], Hanks and McDermott[11], and Kanazawa[12]. Examples of fuzzy logic approaches are by Dubois and Prade[6, 7], Dutta[8] and Qian[14].

Although uncertainty plays a different role in each of these examples two common approaches can be distinguished. The first approach considers propositions which may be uncertain, while the time related to those propositions is not. That is, time serves only as an index to the probability of propositions. Hanks and McDermott, for example, represent the probability of a proposition ϕ at a time t as $P(\phi \text{ at } t)$ or $P(\phi_t)$. Work by Haddawy[10] is also of this nature.

The other approach is to represent (crisp) propositions ϕ which may occur at an uncertain time t_ϕ. We might say that in these approaches the probability $P(t_\phi \text{ at } t)$ is represented. This representation is for example used by [7], [8] and [15].

Somewhat at odds with these approaches is the work by Dean and Kanazawa[5] who represent events as being certain but occurring at an uncertain time, while the truth of a fact is represented as being itself uncertain at a certain time t. It is thus a mix of the aforementioned approaches.

Obviously, choosing either representation $P(\phi \text{ at } t)$ or $P(t_\phi \text{ at } t)$ introduces some limitations with respect to both the reasoning process and to the kind of knowledge that can be represented. When no uncertainty in time is allowed typical imprecise temporal relations like 'somewhat before' or 'somewhat after' cannot be represented . Similar arguments apply when facts are represented to occur with certainty at some not precisely known time.

Extending either representation to allow uncertainty of the other kind is also not easily done, since this will usually cause mixing of temporal and factual uncertainty. This is highly undesirable, because it becomes unclear to what extend the uncertainty about a proposition derives from temporal or factual uncertainty.

Limiting ourselves to either representation also has a strong impact on the reasoning process. The main effort when reasoning about uncertain time is usually in constraining temporal relations between facts and events. The aim is then to arrive at a consistent set of temporal relations. Most approaches dealing with uncertain time are thus based on temporal networks or graphs. Work by Console et al.[3, 4], Chen[2], and Kirillov[13] is of this nature.

On the other hand reasoning about uncertain propositions can usually not be embedded in temporal networks due to its open-ended future. In that case reasoning often involves branching futures and possible world scenarios. Haddawy[10] and Hanks and McDermott[11] provide examples of this type of reasoning.

Summarizing, temporal representations in literature do not explicitly represent both temporal and factual uncertainty. Additionally simultaneously reasoning with both uncertain facts and uncertain time has also not been dealt with. Further, present temporal representations are not easily extended such that uncertainty about time and fact may still be distinguished. In addition, the temporal representation chosen also restricts reasoning to certain modes of inference.

In this paper we represent uncertain time and uncertain facts with fuzzy time-objects to overcome the above problems and limitations. In time-objects uncertainty about time and fact are independently represented. All propositions in our system are time-objects and thus every proposition has both a factual and a temporal component. Further reasoning with time-objects is not restricted by the representation and uncertainty about fact and time can be inferred independently as we will show in the next sections.

3 Representing time-objects

From the previous section it follows that current (2-dimensional) representations do not allow for the representation of both uncertain facts and uncertain times without also allowing the mixing of uncertainty about time and fact. Instead we propose a 3-dimensional representation called a *time-object* to evade such problems.

Time-objects are represented as objects in a space spanned by the time axis, the uncertainty axis and the axis along which facts are represented. The time-object is thus a 2-dimensional fuzzy set in which both time and fact are connected (the third dimension is the possibility attached). In figure 1 an example is given of such a time-object.

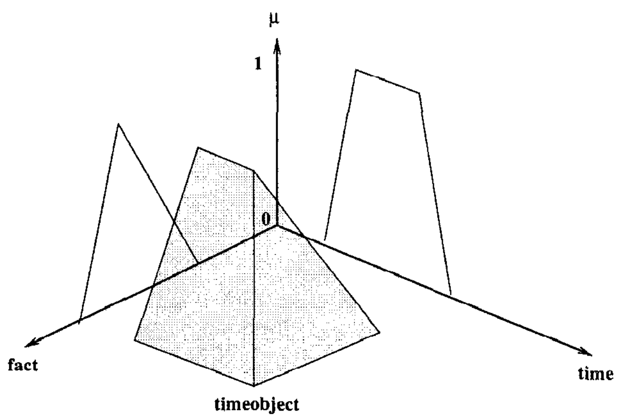

Figure 1: An example of a separable time-object represented as a 3-dimensional object combining uncertain time and uncertain fact.

A time-object should be interpreted as a fuzzy valuation serving the same purpose as connectives like 'OCCURS' and 'HOLDS' in formal temporal logics, with the exception that time-objects have a truth *range* between 0 and 1, while in formal logics a time-object is either true (1) or false (0). Fuzzy logic thereby allows for a large number of possible definitions of the fuzzy valuation function.

It is important in this place to reflect on the nature of the relation between the time-object and 2-dimensional representations of uncertainty about time and fact. Can these 2-dimensional representations be derived from the time-object, or is the time-object itself constructed from uncertain time and fact, or is their possibly some other relation?

In our representation the relation between time, fact and time-object is defined with the notion of *separability*. A time-object is called separable if uncertainty about time and fact are mutually independent. In that case time and fact can be derived from the time-object and vice versa. If a time-object is separable either the time-object or the independent time and fact representations are sufficient to represent all knowledge. This notion is formalized below in eqs. (1), (3) and (4).

Only (semi-) separable time-objects are considered in the remainder of this paper. This choice allows decomposition of reasoning with time-objects in a fact-fact and time-time relation as will be explained in section 6.

Obviously there are many time-objects thinkable which are not separable. A subset of those, which will be considered in the next section, are *semi-separable*. Semi-separability means that the correct 2-dimensional representations can be derived from the time-object, but not the other way around.

Under the assumption of separability the fuzzy valuation μ_{tobj_f} of the time-object $tobj_f$ can be constructed from the fuzzy valuations of a fact f_x and the related corresponding fuzzy valuation of its temporal occurrence f_t. Separability is partially defined by eq. (1):

$$\mu_{tobj_f}(x,t) = TOV(\mu_{f_x}(x), \mu_{f_t}(t)) \quad (1)$$

where TOV denotes the *time-object valuation* which needs to be defined. We define TOV as:

$$TOV(\mu_{f_x}(x), \mu_{f_t}(t)) = \mu_{f_x}(x) \times \mu_{f_t}(t) \quad (2)$$

Motivation for this definition of TOV is that it reflects the mutual independence, and thus the separability of the time-object. Eq. (2) may be interpreted as a fuzzy analogy to the probability $P(f_x, f_t)$.

Because of the assumed separability of the time-object the sets of valuations of f_x and f_t can be constructed in case $tobj_f$ is given. That is, under the condition that $\max(\mu_{tobj_f}(x,t)) = 1$ or, stated otherwise, that the *core* of μ_{tobj_f} is not empty. Time-objects which satisfy this condition are called *normal* time-objects. As a consequence the derived fuzzy sets μ_{f_x} and μ_{f_t} will also be normal.

In case of normal separable time-objects the valuations of f_x and f_t are retrieved from $tobj_f$ in the following way:

$$\mu_{f_x}(x) = \max_t \mu_{tobj_f}(x,t) \quad (3)$$

$$\mu_{f_t}(t) = \max_x \mu_{tobj_f}(x,t) \quad (4)$$

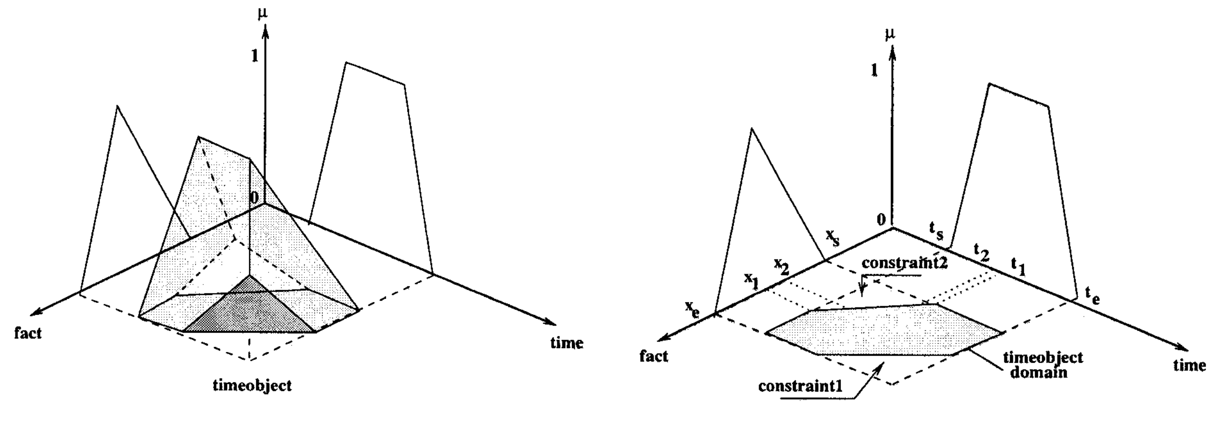

Figure 2: Constraining the domain of a separable time-object

These equations represent the inverse relations of eq. (1). A time-object is now called separable when eq. (1), (3) and (4) are valid which formalizes the definition of separability.

The above conditions for separability are not satisfied when the time-object is not normal. To retain separability for non-normal time-objects eqs. (3) and (4) need to be generalized. Given the choice for TOV in eq. (2) this can be done in the following way:

$$\mu_{f_x}(x) = \max_t(\mu_{tobj_f}(x,t)) / \max(\mu_{f_t}(t)) \tag{5}$$

$$\mu_{f_t}(t) = \max_x(\mu_{tobj_f}(x,t)) / \max(\mu_{f_x}(x)) \tag{6}$$

Note that eqs. (5) and (6) involve a circular argument. To break the circularity prior knowledge of $\max(\mu_{f_t}(t))$ and $\max(\mu_{f_x}(x))$ is needed.

In addition to separability time-objects should also be *convex*. Convexity is defined as:

$$\begin{aligned} &\forall x_1, x_2, x, t_1, t_2, t \\ &\quad \text{if } ((x_1 \leq x \leq x_2) \text{ and } (t_1 \leq t \leq t_2)) \text{ then} \\ &\quad \mu_{tobj}(x,t) \geq \min(\mu_{tobj}(x_1,t_1), \mu_{tobj}(x_2,t_2)) \end{aligned} \tag{7}$$

Convexity guarantees that the more or less possible values for $tobj(x,t)$ are clustered together. Obviously, if the time-object is convex, the projections to the time and fact axis are also convex.

When reasoning with time-objects in the next sections it is assumed that all time-objects are convex. Further, if time-objects are not normal, it is assumed that additional knowledge is available such that separability may still be claimed. The notion of separability itself is allowed to be relaxed somewhat as we will see in the next section.

4 Constraining time-objects

There is an undeniable relation between the temporal and factual parts of a time-object. However, temporal uncertainty should not become inextricably mixed with factual uncertainty at any other place than in the time-object. On the other hand we *do want* to include some kind of influence between time and fact to do justice at least to our intuition. This section deals with a solution which allows for such an influence without the mixing of uncertainties.

The solution we present is to restrict the influence between time and fact to their respective *domains* by constraining the domain of the time-object itself. Figure 2 shows a rather simple way to constrain the domain of a time-object. In this case the range of possible time points which correspond to the smallest possible instantiation x_s of the fact f_x is constrained as well as the range of time corresponding to the largest instantiation x_e of f_x. Similarly the range of fact instantiations corresponding to the earliest time point instantiation t_s of f_t is constrained, as also those fact instantiations corresponding to t_e. These constraints are summarized in eq. (8) and shown graphically in figure 2.

$$\begin{array}{ll} x_s \longrightarrow [t_s, t_1] & x_e \longrightarrow [t_2, t_e] \\ t_s \longrightarrow [x_s, x_1] & t_e \longrightarrow [x_2, x_e] \end{array} \tag{8}$$

In eq. (8) t_1, t_2, x_1 and x_2 are the constraints applied to the domain of the time-object.

In general constraints applied to the domain of the time-object may take any form. Some restrictions apply, however. For one, if all possible instantiations of time and fact are to be covered by the time-object (complete specification).

In addition, in order to retain perfect reconstruction of the set of possible temporal and factual instantiations from a given *normal* time-object using eqs. (3) and (4) the following restrictions apply:

$$\begin{array}{lll} t_2 < t_1 & \wedge & \{\exists t \in [t_2, t_1] \mid \mu_{tobj}(x,t) = 1\} \\ x_2 < x_1 & \wedge & \{\exists x \in [x_2, x_1] \mid \mu_{tobj}(x,t) = 1\} \end{array} \quad (9)$$

A time-object is called *semi-separable* if it satisfies the above conditions provided the unconstrained time-object is separable as well. Temporal and factual parts can be retrieved from the constrained time-object, but not the other way around. Although the above equations apply to normal time-objects, definitions can easily be generalized to include also non-normal time-objects.

The effect of constraining time-objects is shown in the next section where the process of instantiating a time-objects is considered.

5 Instantiating time-objects

Suppose evidence for f_x and f_t is obtained, denoted by respectively f'_x and f'_t. The instantiated semi-separable time-object, called $tobj'_f$, is then defined to be the fuzzy intersection of $tobj_f$ and $tobj_i$:

$$tobj'_f = tobj_f \cap tobj_i \quad (10)$$

where $tobj_i$ is an unconstrained separable time-object calculated from f'_x and f'_t according to eq. (2). This is, by the way, an instant where f'_x and f'_t become available in the process of reasoning and not $tobj'_f$ directly. We noted earlier that this is the common thing to expect.

There are many functions possible to define fuzzy intersection (so called T-norms). These should, however, relate to the way time-objects are defined. Given our definition of TOV in eq. (2) the appropriate T-norm to choose is the maximal T-norm which is defined as:

$$\mu_{tobj_f \cap tobj_i}(x,t) = \min_{x,t}(\mu_{tobj_f}(x,t), \mu_{tobj_i}(x,t)) \quad (11)$$

where $\mu_{tobj}(x,t)$ represents the fuzzy valuation of the time-object $tobj$ at the point (x,t) in the fact-time space.

Given (semi-) separability of $tobj_f$ and $tobj_i$, the factual and temporal instantiations f'''_t and f'''_x are calculated below under the condition of normality:

$$\mu_{f'''_t}(t) = \min_{t \in tobj'_f}(\max_x(\mu_{tobj_f}(x,t)), \max_x(\mu_{tobj_i}(x,t))) \quad (12)$$

$$\mu_{f'''_x}(x) = \min_{x \in tobj'_f}(\max_t(\mu_{tobj_f}(x,t)), \max_t(\mu_{tobj_i}(x,t))) \quad (13)$$

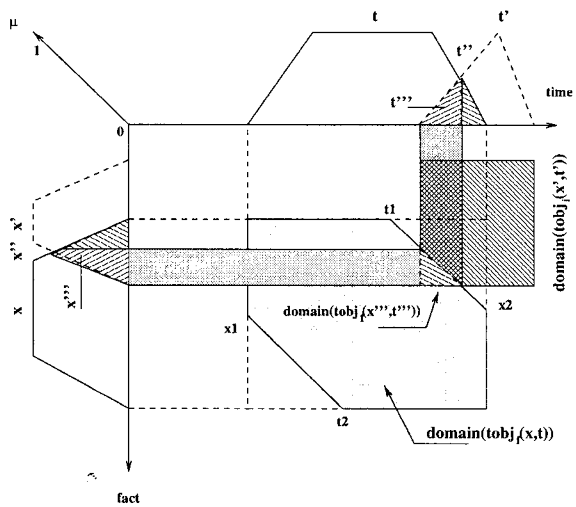

Figure 3: Instantiation and factual and temporal projections of a semi-separable time-object.

We have introduced notations f''_t, f'''_t, f''_x and f'''_x to show the reader the difference between separable and semi-separable time-objects. In case $tobj_f$ is separable temporal and factual projections of instantiations of $tobj_f$ equal f''_x and f''_t. In case of semi-separable time-objects, these instantiations can be further constrained to f'''_x and f'''_t as will be shown below. The difference between (f''_x, f''_t) and (f'''_x, f'''_t) expresses the mutual influence time and fact can have on their respective domains.

Suppose we start with a time-object which is separable. Then instantiations of the time-object can directly be calculated from the temporal and factual projections in the following way:

First, calculate the instantiations f''_x and f''_t as those fuzzy subsets which are maximal compatible with f_x and f_t given the evidence f'_x and f'_t. As for the time-objects, also for these fuzzy-sets the fuzzy intersection is taken to represent this notion (see also figure 3):

$$f''_x = f_x \cap f'_x \quad (14)$$

$$f''_t = f_t \cap f'_t \quad (15)$$

The instantiated time-object can now be calculated directly as:

$$\mu_{tobj'_f}(x,t) = TOV(f''_x(x), f''_t(t)) = f''_x(x) \times f''_t(t) \quad (16)$$

If the instantiated time-object is normal and separable no further constraints apply and $f'''_x = f''_x$ and $f'''_t = f''_t$. If the instantiated time-object is not normal it is needed to keep the maximum values of f_x''' and f_t''' in memory to retain separability.

If the time-object $tobj_f$ is only semi-separable, the situation becomes slightly more difficult as can be seen from figure 3. In that case f_x'' and f_t'' can be further constrained on there *domains* to f_x''' and f_t''' respectively. The constrained domains of f_x'' and and f_t'', denoted $[x_s, x_e]$ and $[t_s, t_e]$, are calculated as follows:

$$t_s = \{\min(t) \mid \mu_{tobj_f'}(x,t) > 0\} \tag{17}$$

$$t_e = \{\max(t) \mid \mu_{tobj_f'}(x,t) > 0\} \tag{18}$$

$$x_s = \{\min(x) \mid \mu_{tobj_f'}(x,t) > 0\} \tag{19}$$

$$x_e = \{\max(x) \mid \mu_{tobj_f'}(x,t) > 0\} \tag{20}$$

Now the fuzzy valuation of f_x''' and f_t''' are calculated as:

$$\mu_{f_x'''}(x) = \begin{cases} \mu_{f_x''}(x) & \text{, if } x_s \le x \le x_e \\ 0 & \text{, otherwise} \end{cases} \tag{21}$$

$$\mu_{f_t'''}(t) = \begin{cases} \mu_{f_t''}(t) & \text{, if } t_s \le t \le t_e \\ 0 & \text{, otherwise} \end{cases} \tag{22}$$

The instantiated time-object $tobj'$ and its compatible instantiations x''' and t''' have now been calculated the other way around. Note that these instantiations are equal to those already directly inferred with eqs. (12) and (13) above.

Whether f_t' and f_x' are inferred from $tobj_i$ or the other way around depends on how these instantiations become available to the reasoning process, but is of no consequence as long as $tobj_i$ is separable and normal. Otherwise, if $tobj_i$ is not normal, the maximum possibilities of the projections of $tobj_i$ along the time and fact axis need to be known in addition.

In this section we have shown how time and fact can have mutual influence on their respective domains without inextricable mixing of uncertainties. In the next section it will be shown that this influence is also carried over to the reasoning process.

6 Reasoning with time-objects

The question addressed in this section is how to infer one time-object from another. As already noted before there are technical reasons for considering only semi-separable time-objects. Below we will discuss these reasons starting from the following single inference step:

IF $tojb_f(x, t_1)$ **THEN** $tojb_g(y, t_2)$

The relation between two time-objects in a fuzzy inference scheme is a 5-dimensional relation between the 3-dimensional time-object $tobj_f$ and the 3-dimensional time-object $tobj_g$. The main difference between the subspaces in which $tobj_f$ and $tobj_g$ reside is in the dimensions where the factual parts of the time-objects involved are represented (f_x and g_y). We perceive the relation between $tobj_f$ and $tobj_g$ as represented graphically in figure 4.

Figure 4 shows the decomposition of the inference relation between $tobj_f$ and $tobj_g$ into two separate relations $R(f_x, g_y)$ and $R_t(f_t, g_t)$. In this figure the relation is mapped to three dimensions by omitting the uncertainty axis and mapping the temporal content of $tobj_f$ and $tobj_g$ onto the same axis. The relation $R(f_x, g_y)$ denotes the fact-fact relation between the factual parts and $R_t(f_t, g_t)$ denotes the time-time relation between the temporal parts of $tobj_f$ and $tobj_g$.

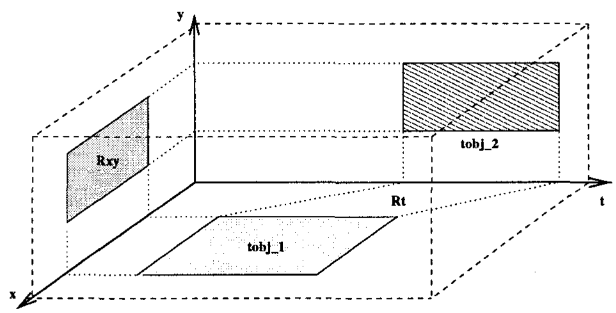

Figure 4: The decomposed fuzzy inference relation between two separable time-objects.

Such a decomposition of the inference relation is only allowed in case the time-objects involved are semi-separable. This is exactly the reason why we consider only semi-separable time-objects. But why would we like to decompose the inference relation at all?

Suppose the inference relation is *not* decomposed and a semi-separable time-object $tobj_g'$ is directly inferred from a semi-separable time-object $tobj_f'$ with an ordinary fuzzy inference relation R:

$$\mu_{tobj_g'}(y, t_2) = \max_{x,t_1} \min_{x,t_1,y,t_2}(\mu_{tobj_f'}(x, t_1), R(x, t_1, y, t_2)) \tag{23}$$

where R is a standard fuzzy implication function like Mamdani's or Larsen's.

In that case any instantiations f_x''' and f_t''' derived in the manner explained in the previous sections would leave $tobj_g$ unaltered in case $\max(tobj_f'(x,t)) = 1$. This means that a variety of temporal instantiations f_t''' would result in the same conclusion $tobj_g$ as long as f_t''' would be a normal fuzzy set. In addition, instantiations f_x''' would directly influence the valuation of g_t', which would result in an undesired mixing of uncertainties in g_t'.

Another (pragmatic) reason for the inference decomposition is that 5-dimensional relations are generally unintelligible to humans and thus hard to obtain from knowledge acquisition. Further such direct relations do not reveal the temporal relations between two time-objects, nor the fact-fact implications.

Except for evading problems with knowledge acquisition and uncertainty mixing an additional technical advantage of the proposed decomposition is that typical time relations like 'before', 'after' and 'persist' can now be performed in their proper domain. Similarly advantage can be taken of the large variety of fuzzy inferencing techniques to relate the factual parts of time-objects.

6.1 The fact-fact relation

The fact-fact relation $R(f_x, g_y)$ is a general modus-ponens inference rule:

$$\begin{array}{llll} \textbf{if} & x \text{ is } f_x & \textbf{then} & y \text{ is } g_y \\ & x \text{ is } f_x' & & \\ \hline & & & y \text{ is } g_y' \end{array}$$

where the if-then statements are represented as fuzzy implications.

Suppose f_x is defined on X and f_y is defined on Y. The fuzzy implication $R(f_x, g_y)$ is then a fuzzy relation defined on $X \times Y$ of the following general form:

$$\mu_{R(f_x,g_y)}(x,y) = I(\mu_{f_x}(x), \mu_{g_y}(y)) \qquad (24)$$

where $I(\mu_{f_x}(x), \mu_{g_y}(y))$ is an implication function, like, for example, Mamdani's or Larsen's implication function.

The implication of g_y', given f_x''', is now calculated according to the following general scheme:

$$\mu_{g_y'}(y) = \max_x \min_{x,y}(\mu_{f_x'''}(x), \mu_{R(f_x,g_y)}(x,y)) \qquad (25)$$

6.2 The time-time relation

The time-time relation R_t between f_t and g_t actually stands for a number of relations which are a special class of the general fact-fact relations discussed above. Where fact-fact relations are relating a domain X to a domain Y ($R : X \to Y$), time-time relations are defined in the temporal domain T only and of the form $R_t : T \to T$. The temporal domain T is thereby independent from X and Y.

Temporal reasoning is different from ordinary reasoning in the sense that special relations apply to the common temporal part of all time-objects. Relations in the temporal domain are closely related to intuitions about time, which, when condensed in relations, constitute a *structure of time.* In such a time-structure usually the ontology, ordering, flow, and density of time are defined.

We discussed the ordering of *fuzzy* primitive time-elements in [1]. Other possible relations in the temporal domain are temporal statements like "approximately at the same time", or "t_x minutes after" and the notion of *persistency.* Dubois and Prade suggest in [7] also relations like "much after", "before or slightly after", "much before". For lack of space we will below only describe the relation "t_x minutes after" to provide the reader with the general idea.

The relation "t_x minutes after" is defined as an algebraic operation with fuzzy numbers. Fuzzy numbers denote fuzzy sets whose membership functions are continuous and convex and have a non-empty core.

Suppose g_t' is the fuzzy set which is "t_x minutes after" f_t'''. Then g_t' is calculated with following relation, based on the extension principle:

$$\mu_{g_t'}(t) = \sup_{t=t_i+t_j} \min\{\mu_{t_f}(t_i), \mu_{t_x}(t_j)\} \qquad (26)$$

where μ_{t_x} represents the fuzzy valuation of the notion "t_x minutes". Eq. (26) is a standard fuzzy algebraic operation.

6.3 Inference results

Given that g_y' is derived with eq. (25) and g_t' with, for example, eq. (26), the conclusion of the inference process can be calculated. The conclusion $tobj_g'$ will be the instantiation of $tobj_g$ with g_y' and g_t' which will themselves be constrained to g_y''' and g_t''' in the manner explained in section 4. This concludes the inference process.

Note that if time-objects are constrained on their domains, f_x has an indirect influence on g_t through f_t, but also through f_y and $tobj_g$. Similarly f_t has indirect influence on g_y. The influence does not affect the valuations and is restricted to the domains. This is what we meant earlier: the mutual influence between fact and time in semi-separable time-objects carries over to the inference process.

Note also that another consequence of the inference process is that the maximum possibility of the inferred time-object $tobj_g'$ is bounded by the maximum possibilities of f_x''' and f_t''' which in turn are bound by $tobj_f'$:

$$\max_{y,t}(\mu_{tobj_g'}(y,t)) \leq \max_{x,t}(\mu_{tobj_f'}(x,t)) \qquad (27)$$

In practice this means that the inferred time-object is never more possible than its causing time-object, which is according to intuition.

7 Conclusions

We have presented a new approach to temporal reasoning with uncertainty which incorporates reasoning with both uncertain time and uncertain facts. It is shown that approaches in literature dealing with either uncertain time or uncertain facts can not be extended to include uncertainty of the other kind without also mixing temporal and factual uncertainty. Therefore we propose to use 3-dimensional time-objects to represent uncertainty about time and facts. The time-objects considered in this paper are semi-separable such that decomposition in factual and temporal parts is possible. A new method is proposed which constrains the domain of a time-object. Constraining the domain of a time-object introduces a relation between the time and fact components of a time-object, but avoids mixing of uncertainty about time and fact. A method for reasoning with time-objects is presented and it is shown how one time-object may be inferred from another time-object by decomposition of the inference relation under the assumption of (semi-) separability.

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