Rules for Simple Temporal Reasoning

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Abstract

Simple practical reasoning with propositions whose truth depends on time is a matter of logical engineering. We show that for Boolean logic a reified logic is more appropriate than its non-reified equivalent when time references are interpreted as union-of-convex intervals (UoCI).

1 Introduction

It is an elementary observation that propositions may be true at one time and false at another. Any sort of real-world database must consider such a possibility, and there are various ways of timestamping entries in relation tables (i.e., atomic formulae) to reflect temporal dependencies. Drawing conclusions from data is the domain of logic. How may logic help us in maintaining temporally-dependent information in a current state, and in using this information easily? This is not a new subject, either in AI or CS in general. However, sometimes one can discover important features of reasoning by looking again at simple cases.

We want to find rules capturing a large range of simple but useful inferences for temporally-irreflective propositions, and to implement these rules in a system which attempts to maintain temporally-dependent data. (We were thinking of applications to an ATMS [4] which annotates propositions with their periods of validity.) We find that using union-of-convex intervals is more expressive than other proposals, but that one must use a reified logic rather than a non-reified logic, in contrast to the suggestion of Bacchus *et al.* [3].

1.1 Temporally-Irreflective Propositions

We consider only propositions which are *temporally-irreflective*: intuitively those that in a natural language would be expressed in the present tense with no temporal adverbs or other such explicit reference to time other than the present. Propositions such as '*Fred Smith is employed by Jones Cat-Cleaning Industries, Inc.*' are temporally-irreflective: temporal qualifiers such as '*yesterday*', '*next week*' do not occur. The intuition is that a temporally-irreflective proposition has a truth value which depends on time, but not on temporal indexicals in the proposition itself. This feature validates certain persistence rules: *downward persistence* (DP), that if a proposition is true over a period of time, it's true over any subperiod of that time; and *limited upward persistence* (UP), that if a proposition is true over two periods of time, then it's true over the 'union' period of the two. For purposes of precision, we define a *temporally-irreflective proposition* to be a proposition that satisfies (DP) and (UP).

If p and q are temporally irreflective, it seems intuitively to be the case that Boolean combinations of them are temporally irreflective also. We treat here only Boolean combinations of temporally-irreflective atomic propositions.

1.2 Motivation from Application

We must ensure that the hypotheses constraining the inquiry are plausible. We originally considered enhancing an ATMS with temporal qualification to the truth of its propositions. This application has three features which we adopted as constraints: (1) A focus on syntax. Any information concerning the relations between propositions, intervals and truth must be reduced to syntactic information and syntactic inference from axioms. (2) Inference is quantifier-free. Propositional inference alone is used, and inference follows so-called forward-chaining (i.e., use of Modus Ponens on axioms which are conditionals). (3) Propositions typically are temporally irreflective.

The first feature suggests saying '*proposition p is true on interval i*' within the object-language, rather than as a meta-statement about a logical system. The second suggests searching for axioms of the form $(\bigwedge hypotheses \Rightarrow conclusion)$; the third that we can assume (DP) and (UP). Designing a temporal ATMS is beyond the scope of this paper. We mention it simply to motivate the three constraints.

2 Time

What structure is needed to represent real-world temporal information? Suppose the CEO of your company has two jackets, one red with orange polka dots and the other orange with red polka dots. You naturally want to keep a daily record of his dress, in order to explain how it contributed either to your company's meteoric rise or to its complete misreading of the market, but in either case to make millions with your book on it. The CEO wears his red jacket on Mondays, Wednesdays and Fridays, and his orange jacket on Tuesdays and Thursdays, or the other way round, depending on the week. According to some mathematicians, and some AI researchers, his jacket is red at all points in the set of points from the first point at which he starts work on Monday until the point at which he leaves work on Monday. According to others who simplify, it's red *from* the first point .... *to* the point at which he leaves. However, being ordinary mortals we just want to say it's red for the workday on Monday. But we also want to say that this is true on other days too, with gaps of orange in between.

The first simplification leads to an ontology of convex intervals of time as the temporal reference of truth values of propositions and the second to an ontology of objects which are unions of separated convex intervals, which we call *union-of-convex* intervals (*UoCI*s).

Truth over explicit convex intervals and related structures has been studied in [7, 1, 2, 18]. [8, 9, 16] have studied propositions and reasoning over UoCIs; [13, 14] has further studied the mathematics of representations of convex intervals as sequences of points of varying length.

2.1 Choosing a Representation

UoCIs correspond to no fixed number of points - one UoCI may have four components (maximal convex subintervals) with thus 8 points, and another six components (12 points). However, if we represent UoCIs directly, rather than via points, we utilise one temporal argument only in an assertion of truth (the UoCI), as in the convex case. This move allows us to use a standard logical language to write temporally-qualified assertions. Concerning the UoCI data structure, Ligozat (*op. cit.*) represents a UoCI as a sequences of real numbers (representing component endpoints). We choose the TUS. This allows us to use logical reasoning within a theory of UoCIs, as well as numerical calculation.

2.2 Can We Use an Existing Truth-Over-Intervals Theory?

We show in [5] the inadequacy of Humberstone's formulation [7].

Shoham's reified logic [17] adds axioms simulating propositional reasoning in the arguments to the predicate *TRUE*, to enable the usual propositional inferences to be performed, e.g.

$$TRUE(i, p) \& TRUE(i, q) \Rightarrow TRUE(i, (p \& q))$$

It is somewhat inelegant to have to add all these rules.

A Bacchus-style propositional logic, which subsumes Shoham's [3], treats an atomic proposition as having an extra argument which is a UoCI, and in contrast to Shoham's, doesn't allow Boolean combinations as arguments in atomic formulae. Thus we write $p(I)$ instead of $TRUE(i, p)$, and the formula corresponding to Shoham's atomic $TRUE(i, (p\&q))$ is the conjunction $p(I)$ & $q(I)$, which also corresponds to $TRUE(i, p)$ & $TRUE(i, q)$, showing how the reified rule is absorbed by the underlying logic in the non-reified formulation. We need to add a temporal theory (Bacchus *et al.* don't propose any) – we pick UoCIs over the rationals, and varying temporal references in some of the axioms, so one can infer the truth of propositions over different intervals from those they came with (else adding temporal references wouldn't give us anything over propositional logic!). In addition, a reified theory needs to add *simulation rules* as above for propositional reasoning.

Using (the theory of) UoCIs over the rationals allows us to employ a single temporal argument in either reified or Bacchus-type predicate symbols. The atomic formula *models*(I, p) (which we write $I \models p$) asserts that proposition p is true over interval I.

2.3 Applicable Reasoning With Intervals

The logic of interval reasoning is considered in [18], in particular the first-order theory of convex intervals over a dense unbounded linear order is proved countably categorical. Special reasoning techniques, using methods described in [11], have been developed for some quantifier-free formulas [1, 12]. McKenzie has noted that the theory of finite-UoCIs (fUoCI, a UoCI with finitely many components) on the rationals is decidable via the decision procedure for S2S (which is super-exponential!) [15]; Ligozat has shown how to perform some quantifier-free reasoning with them [13, 14] and Morris, Shoaff and Khatib have adapted the methods of convex-interval reasoning to some special cases [16].

These works all treat the intervals as objects of a mathematical structure. They show that we can fulfil some desiderata by choosing fUoCIs as our temporal reference. But what about the data? One can represent real convex intervals of time as clock-like sequences

[year, month, day, hour, ...]

of all possible finite lengths and [10] showed that, with the appropriate interval relations between them, these form a notation for the convex rational interval structure. This notation is called the BTU (*Basic Time Units*) (called TU in [10]) and is the foundation of the TUS (*Time Unit System*), which includes non-convex intervals formed from BTUs by application of the operators *periodify* [8], and *conglom* and *intersect* (see Section 2.4).

2.4 Conglomeration and Intersection

In [5], we define the *conglomeration* operation of two intervals I and J, denoted *conglom*(I, J) in the code and $I + J$ in math notation, to be the 'union' of I and J, i.e. that interval which is the set of components of I and J, except that those different components which have some common subinterval or those which *meet* are merged into one component [8]. Conglomeration is an associative and commutative operation, and so generalises to an arbitrary set of interval arguments. A formal definition of conglomeration is a straightforward formalisation of the intuitive definition. Similarly, the *intersection* of two intervals, $intersect(I, J)$ or $I \cdot J$, is that interval which consists precisely of those subintervals which are common to both I and J (i.e. the overlapping parts of their components). Algorithms for computing both these operations in time linear in the number of components are given in [5].

It's easy to show that the fUoC rational intervals form a distributive lattice [6] under + and '$\cdot$'. Relaxing the finiteness requirement, a general UoCI may have infinitely many components. The general UoC rational intervals form a complete lattice (arbitrary sums and products exist) under the generalisation of + and $\cdot$ to arbitrary sets of intervals.

2.5 Some Reasoning Principles

If we add the two points at infinity to the rationals, we can add the empty interval $\langle\rangle$ and the full line $\mathbf{1} = \langle -\infty, \infty\rangle$. They satisfy the following laws: for any interval I, $I + \langle\rangle = I, I \cdot \langle\rangle = \langle\rangle, I + \mathbf{1} = \mathbf{1}, I \cdot \mathbf{1} = I$. It would also make sense under this supposition to allow intervals to have components which are half-infinite convex intervals in the set:
$Half = \{\langle -\infty, a\rangle \mid a \in \mathcal{Q}\} \cup \{\langle b, \infty\rangle \mid b \in \mathcal{Q}\}$
where $\mathcal{Q}$ is the rational numbers. Call these intervals the extended-rational UoCIs. It's easy to show that every extended-rational UoCI I has a complement: an interval $\sim I$ such that $I + \sim I = \mathbf{1}, I \cdot \sim I = \langle\rangle$. A structure on which the binary operations $+, \cdot$ form a distributive lattice, with constants $\langle\rangle$, $\mathbf{1}$ and unary operation $\sim$, all satisfying the stated laws, is a Boolean algebra. Thus the extended-rational UoCIs form a Boolean algebra under these operations. How may we use this observation to explicate the relation between intervals and formulas?

We have taken the following rule of *downward-persistence* [17] as part of the definition of temporally-irreflective propositions:

$$(\mathrm{DP}) : (I \models p) \,\&\, (J \subseteq I) \Rightarrow (J \models p)$$

where "$\subseteq$" denotes the *interval-containment* relation. Interval containment is defined for convex intervals as $S \cup F \cup D$: a UoC I is contained in J just in case each component of I is contained in some component of J (it is straightforward to formalise this informal definition). For the interval structure we use, interval-containment is definable from '+' using the *composition principle* below.

Similarly, we have taken the following rule of limited upward-persistence to be part of the definition of temporal irreflectivity:

$$(\mathrm{UP}) : (I \models p) \,\&\, (J \models p) \Rightarrow (I + J \models p)$$

With a proposition p we may associate the interval

$$I_p \;=\; conglom(\{J \mid J \models p\})$$

when it exists. In the *complete* lattice of extended-rational UoC intervals, which includes arbitrary sums and products and thus some 'infinite-UoC' intervals, I_p always exists, whereas in the extended-rational fUoCIs it may sometimes not. In the complete lattice, we may generalise upward-persistence to the rule of complete upward persistence

$$(\mathrm{CUP}) : I_p \models p$$

so I_p is thus the maximal interval on which p is satisfied, in this lattice.

The *composition principle* (CompP) that

$$\forall J \subseteq I : \exists K \; disjoint\ from J : J + K = I$$

although false for convex intervals over the rationals (for example, take $I = \langle 1, 4\rangle$ and $J = \langle 2, 3\rangle$), is easily provable for UoC rational intervals (take $K = \{\langle 1,2\rangle, \langle 3,4\rangle\}$). One may prefer to define $\subseteq$ in terms of $+$, namely that

$$J \subseteq I \triangleq \exists K : J + K = I$$

In the presence of this definition, the composition principle is equivalent to the existence of a complement for every interval (complements require also $\langle\rangle$ and $\mathbf{1}$).

In the lattice of fUoCIs over the extended rationals, it follows from (DP) and (UP) along with (CompP) that the *persistence* condition is satisfied, namely that

$$I \models p \Leftrightarrow \forall J \subseteq I : J \models p$$

The rule $I_p \models p$ conjoined with downward persistence is equivalent to this persistence condition over the complete lattice. We can take *persistence* in the complete lattice to mean either of these equivalent formulations. The following rules are intuitively plausible for temporal irreflectivity:
$(I \models p \,\&\, I \models q) \Leftrightarrow I \models (p \,\&\, q)$
$(I \models p \;\vee\; I \models q) \Rightarrow I \models (p \vee q)$
The following two more general rules follow directly from these and (DP).

$$(\&\text{-I})\ (I \models p \& J \models q) \Rightarrow (I \cdot J) \models (p \,\&\, q)$$

$$(\vee\text{-I})\ (I \models p \;\vee\; J \models q) \Rightarrow (I \cdot J) \models (p \vee q)$$

It easily follows from (&-I) and ($\vee$-I) that in the complete lattice:

$$I_{p\&q} = I_p \cdot I_q \text{ and } I_{p\vee q} = I_p + I_q$$

In order to obtain propositional reasoning within the reified form, we add the principle that tautologies are true over any interval:

$$(\text{Taut})\ I \models p,\ \textit{for any tautology } p$$

which may be expressed in the complete lattice in the presence of downward-persistence as: $\mathbf{1} \models p$, *for any tautology* p.

Consider now the law of non-contradiction,

$$(\text{NonCon})\ I \neq \langle\rangle \;\Rightarrow\; I \not\models (p \,\&\, \neg p)$$

In the complete lattice, it follows from the fact that $p \vee \neg p$ is a tautology, from (Taut), persistence, and (NonCon) that $I_{\neg p} = {}^{\sim}(I_p)$. Thus the mapping $p \mapsto I_p$ is an embedding (a one-to-one homomorphism) of the Boolean algebra of propositional logic (the free Boolean algebra on countably many generators) into the complete extended-rational UoC intervals. This makes temporal inference very easy! But we don't yet have all required rules. Fix interval I, and suppose $I \models p$. How may we infer that all the propositional consequences of p also hold on I? That is, suppose $I \models (p \rightarrow q)$. The intuition behind temporal irreflectivity along with persistence would lead us to infer $I \models q$. Thus, we need the rule:

$$(\text{MP})\ I \models (p \rightarrow q) \,\&\, I \models p \;\Rightarrow\; I \models q$$

However, using I_p comes at a high price. Adding $\langle\rangle$ and the points at infinity destroy the *uniformity* property that for any Allen relation R and convex interval i, there exists a j such that iRj: consider *meets* and the interval $i = \langle 2, \infty\rangle$. There is no interval j such that $i\,M\,j$. Similarly there is no j such that $\langle\rangle\,M\,j$. This in turn destroys the validity of the Allen composition table and renders the relation algebra **IA** of intervals much more complex (indeed possibly infinite). If the algebra is infinite, path-consistency computations may no longer terminate (example in [11]), and so on.

Thus we choose to stay with inference rules over the basic lattice or the basic complete lattice, and not include $\langle\rangle$, $\mathbf{1}$, or *Half*. This affects the rules concerning negation, and prevents us from simplifying (Taut). However, we retain as axioms all the rules mentioned, modifying (NonCon) as follows:

$$(\text{NC})\ I \not\models (p \,\&\, \neg p)$$

Name	Rule	Justification
DP	$I \models p \mathbin{\&} J \subseteq I \Rightarrow J \models p$	Axiom
UP	$I \models p \mathbin{\&} J \models p \Rightarrow (I + J) \models p$	Axiom
	$I \models p \vee J \models p \Rightarrow (I \cdot J) \models p$	DP, PrLogic
	$I \models \neg p \Rightarrow \neg(I \models p)$	NC, &-I, PrLogic
NC	$\neg(I \models (p \& \neg p))$	Axiom
Taut	$I \models p$ for every tautology p	Axiom
	$I \models (\neg p) \mathbin{\&} J \subseteq I \Rightarrow \neg(J \models p)$	NC, DP
	$I \models p \mathbin{\&} I \models q \Leftrightarrow I \models (p \& q)$	Taut, MP (&-I)
	$I \models p \vee I \models q \Rightarrow I \models (p \vee q)$	MP, PrLogic
MP	$I \models (p \rightarrow q) \mathbin{\&} I \models p \Rightarrow I \models q$	Axiom
&-I	$I \models p \mathbin{\&} J \models q \Rightarrow (I \cdot J) \models (p \& q)$	Axiom
∨-P	$I \models p \mathbin{\&} J \models q \Rightarrow (I + J) \models (p \vee q)$	Taut, MP, UP
∨-I	$I \models p \vee J \models q \Rightarrow (I \cdot J) \models (p \vee q)$	Axiom
EMP	$I \models (p \rightarrow q) \mathbin{\&} J \models p \Rightarrow (I \cdot J) \models q$	MP, DP

Table 1: 'Inference Rules' and Their Justification

Name	Rule	Justification
BDP	$p(I) \mathbin{\&} J \subseteq I \Rightarrow p(J)$	Axiom
BUP	$p(I) \mathbin{\&} p(J) \Rightarrow p(I + J)$	Axiom
	$p(I) \vee p(J) \Rightarrow p(I \cdot J)$	BDP, PrLogic
	$\neg p(I) \Rightarrow \neg p(I)$	Tautology
	$\neg(p(I) \& \neg p(I))$	Tautology
	(Tautologies)	Tautologies
	$\neg p(I) \mathbin{\&} J \subseteq I \Rightarrow \neg p(J)$	False!!
	$p(I) \mathbin{\&} q(I) \Rightarrow p(I) \mathbin{\&} q(I)$	Tautology
	$p(I) \vee q(I) \Rightarrow p(I) \vee q(I)$	Tautology
	$p(I) \rightarrow q(I) \mathbin{\&} p(I) \Rightarrow q(I)$	Tautology
&-I-L	$p(I) \mathbin{\&} q(J) \Rightarrow p(I \cdot J)$	Axiom
&-I-R	$p(I) \mathbin{\&} q(J) \Rightarrow q(I \cdot J)$	Axiom
∨-U-I	$p(I) \mathbin{\&} q(J) \Rightarrow p(I + J) \vee q(I + J)$	False!!
∨-I	$p(I) \vee q(J) \Rightarrow p(I \cdot J) \vee q(I \cdot J)$	Axiom
BEMP	$p(I) \rightarrow q(I) \mathbin{\&} p(J) \Rightarrow q(I \cdot J)$	New Axiom !!

Table 2: The Rules in Bacchus Form

An argument that these rules suffice (in the sense that no more can be reasonably added) may be found in [5].

2.6 Converting to Bacchus Form

For Bacchus-form, we rewrite $I \models p$ as $p(I)$ and we have noted that one writes $I \models (p \& q)$ as $p(I, ..) \mathbin{\&} q(I, ..)$.

3 Reified Logic Wins

Table 2 shows the axioms converted into Bacchus-form. (NC), (Taut) and (MP) become tautologies. (&-I) and (∨-I) become three formulas in Bacchus-form, all of which follow by propositional logic from (BDP). We're left with the persistence rules (BDP) and (BUP). How elegant! Since we regarded these rules as defining the temporally-irreflective propositions, it may seem that the extra rules we introduced for the reified case are there only because of reification, and that Bacchus-form is preferable. But hold on – in fact, Bacchus-form turns out to be simply less expressive. Table 2 shows the Bacchus translation of the rules of Table 1, which are either axioms or derived from axioms (the *Justification* column). The Bacchus-form of the derived rules yields some anomalies.

1. The rule (NC) turns into a false statement in Bacchus-form. The reason is straightforward – the reified logic distinguishes between a negation of a proposition being true over an interval I, and it not being the case that the proposition is true over I. This distinction may not be made in Bacchus-form.

2. The rule (∨-P), which is derivable in the reified logic, corresponds to (∨-U-I) in Bacchus-form. (∨-U-I) is false, and we don't see a way to translate (∨-P) well into Bacchus-form.

3. The Bacchus-form of (EMP) is a new rule (BEMP) which isn't derivable from (BDP) or (BUP), even though (EMP) is derivable from (MP) (whose Bacchus-form is a tautology) and (DP). (BEMP) may look as though it should be derivable from propositional modus ponens

and (BDP), but in fact it's not, since although one can conclude $p(I \cdot J)$ from $p(I)$ using (BDP), from $p(I) \to q(I)$ there's no rule which would enable one to conclude $p(K) \to q(K)$ for $K \subseteq I$ (and then we would use $K = I \cdot J$). The hypothesis $p(I)$ has been weakened – of course one could infer $p(I) \to q(K)$ but this doesn't help. The implication is a compound formula composed from two formulas evaluated on intervals, whereas in the reified case, it's one compound formula evaluated on a single interval, and thus (DP) is applicable.

So Bacchus-form needs three rules, (BDP), (BUP) and (BEMP), not just two as we'd originally thought. Crucially, it cannot easily effect the important distinction between a negation being true on an interval and it not being the case that a proposition is true on the interval; neither does it seem that the upward persistence of disjunction from a conjunction, ($\vee$-U-I), can be expressed in the simple form of an implication.

A reified logic yields discriminations that appear not to be obtainable easily with Bacchus-form for UoCIs. Even supposing an equivalent form of the rules could be found, they may not have the form $\bigwedge$ *hypotheses* $\Rightarrow$ *atomic-formula* suitable for forward-chaining, and they may involve formulas other than Boolean combinations of qualified temporally-irreflective atomic predicates. We conclude that a reified logic is more suitable for evaluating propositions on UoC intervals.

4 Conclusions

We considered the simple propositional logic of temporally-irreflective propositions whose truth varies with time. Under the supposition of linear time, we proposed a temporal ontology of UoCIs, and proposed to represent these directly, using the TUS with *conglomeration* and *intersection*, as in [5]. Given this structure, we considered the issue of reified logic versus a Bacchus-style non-reified logic for evaluating propositions over UoCIs, and concluded in favor of the reified logic.

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