Towards a Formal Framework for Spatio-Temporal Granularities

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Abstract

In the spatial context, the research community has not reached yet a widely accepted definition of granularity. There are relatively few papers about "spatial granularities" and many of them give different definitions and often refer to different interpretations of the term "spatial granularity". In this work we propose a formal definition of "spatial granularity" and other related terms involved in the spatial context, as happened for the temporal one. After that, we will merge together our framework for spatial granularity with the usual framework for temporal granularity, giving a definition of spatio-temporal granularity able to manage information about spatial granularities changing over time.

1. Introduction and Motivation

Everyday we use several time granularities (e.g. minutes, days or years) and we also use several spatial granularities, e.g. municipalities, health districts or city quarters. Therefore there is the necessity to manage several spatial granularities as we manage temporal granularities.

A temporal granularity is informally defined as a partition of the time line in groups of instants [1]. This definition is accepted by almost the whole temporal research community and it is used in several papers about temporal granularities.

In the spatial research community a wide acknowledgement on a definition of spatial granularity does not exist. Several papers about spatial granularity give different definitions and, in some cases, they also use "multi-granularity" for modelling "multi-precision" systems [2, 5, 6, 7, 10, 11]. For example, in [11] Stell et al. define a granularity, called "map", as a particular semantic and geometric level of detail. Their basic notion is the "stratified map space", a lattice of granularities partially ordered with respect to their levels of precision. A stratified map space allows one the conversion between two different levels of detail through generalization and refinement operations. In a different way, in [7], Schmidtke et al. use granularities as a multi-resolution representation of space but their approach is based on the notion of "grain-size", i.e. the relevance of the granules. They use a total order over the granules to decide if a grain is important or not and therefore whether it should be represented or not at a given resolution.

In our approach a spatial granularity is, informally, a partition of a space domain in sets of points, called regions, where each region can be seen as an atomic entity. In general, spatial granularities are not directly defined by a spatial metric subdivision of the space. Rather, granularities are defined by different uses, meanings, and interpretations of the space (e.g., provinces, census regions, cities, lakes).

In this paper we discuss a formal framework for spatial granularity. Then we study the possible properties and relations of proposed spatial granularities. After that we merge together our framework with the usual framework for temporal granularity [1] defining spatio-temporal granularities, i.e. a spatial granularity that can change over time. Our spatio-temporal framework allows one to reason about spatial granularities in several time instants by comparing and manipulating spatio-temporal granularities without loss of information. Moreover, the underlying structure for spatial granularity allows one to manipulate, manage, and compare the spatial information.

In order to motivate the need for managing different spatial granularities we focus on applications to land and urban planning. In this context, we deal with several granularities e.g., municipalities, provinces and regions, representing the Italian administrative divisions. Different authorities may need to search for some granules knowing the spatial relations relating them. For example, the province of Verona administration may have to find the municipalities south-eastern to the city of Verona or the quarters of Verona border on a given quarter.

In the same scenario we may also find applications that motivate the use of spatio-temporal granularities. For example, we could aim to analyse the history of the Verona's quarters or to view how a particular peripheral city's quarter

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has grown during the nineties. Moreover we would like to compare the extent of some Verona's municipalities in 1999 and in 2007.

The rest of the paper is organised as follows. In Section 2 we present our framework for spatial granularities. Then, in Section 3 we illustrate our framework for spatio-temporal granularities. Finally, in Section 4, we discuss some literature proposal about spatial and spatio-temporal granularities and sketch some concluding remarks and future work.

2. A Framework for Spatial Granularity

Each spatial granularity has two layers: the spatial domain, over which the spatial regions representing granules are defined, and the index set used to access and manage granules. Even though there are some similarities with the concept of temporal granularity [1], there are some deep differences between temporal granularities and spatial ones.

First, the granules. The most common temporal granularities are "periodical partitions" of the time line and there is some kind of periodicity as for granules extent. Conversely, a spatial granularity is usually not periodical and its granules may have any possible shape.

Second, the relations between the granules. The time line, and also the time granules, are totally ordered by the "before" relation and thus the index set may be any infinite discrete ordered set (e.g. $\mathbb{N}$). Conversely, in a spatial domain we may find many possible relations, thus we must use a more general data structure. For this reason we have chosen to adopt graphs as index sets for spatial granularities, as we will discuss in Section 2.2.

Thus, in the spatial context there are more degrees of freedom than in the temporal one and we have to consider this difference in defining notions about spatial granularities. Another consequence of this observation is that some temporal notions have no spatial counterparts.

2.1. Spatial Domain and Multidigraph

In a spatial granularity we may have a continuous or a discrete spatial domain.

Definition 2.1. *A* continuous spatial domain *is a connected subset of $\mathbb{R}^2$ (or $\mathbb{R}^3$) without holes. Thus, it is an infinite set of points.*
A discrete spatial domain *is a regular subdivision (grid of cells) of a chosen space (e.g. $\mathbb{R}^2$ or $\mathbb{R}^3$). Therefore, it is a numerable set of cells, called* chorons.
A relation over points (or chorons) is associated to each spatial domain and it has to be either a partial or total order or preorder relation.

Chorons are the counterpart of the temporal chronons (i.e., an indivisible span of time) and using them we may focus on discrete domains, as it has been done for temporal databases through chronons.

Example 2.1. Given the spatial domain $SD = \mathbb{R}$, we may associate to it the direction relation $North$ such that $North(P, Q)$ is satisfied only if the latitude of P is lower or equal than the one of Q. $North$ is a total preorder: indeed, it is transitive, reflexive and total.

As we will see in the next section, in the definition of spatial granularities we use multidigraphs as index sets, thus we give hereby the definition of multidigraph.

Definition 2.2. *A labelled multidigraph MG is a labelled directed graph with multiple labelled edges defined as $<V, MA, \Sigma_V, \Sigma_A, s, t, l_V, l_A>$ where:*

- *V is the set of nodes;*
- *$MA = (A, m)$ is the multiset of edges. The multiset is composed of the set of edges $A \subseteq V \times V$ and the function $m : A \to \mathbb{N}$ that for each edge in A gives its multiplicity.*
- *Σ_V is the finite alphabet of node labels;*
- *Σ_A is the finite alphabet of edge labels;*
- *$s : A \to V$ is a function indicating the source node of an edge;*
- *$t : A \to V$ is a function indicating the target node of an edge;*
- *$l_V : V \to \Sigma_V$ is the labelling function for the nodes, it is a bijection;*
- *$l_A : A \to \mathcal{P}(\Sigma_A)$ is the labelling function for the edges. l_A must associate to each edge as many labels as its multiplicity, then we impose that, for each edge $e \in A$, $|l_A(e)| = m(e)$.*

2.2. Granularity

Let us start with the definition of granule, assuming the well known concepts of "closed" and "regular" regions [8], before introducing the formal definition of spatial granularity.

Definition 2.3. *Given a spatial domain SD, a granule is any closed and regular subset of SD. We call $SG_{SD} \subseteq \mathcal{P}(SD)$ the set of all possible granules of SD and SR_{SD} the set of all possible spatial relations among granules of SD. Each relation R is represented as a mapping that given a granule $g \in SG_{SD}$ returns the set of granules $R(g)$ related to g in the relation R.*

Note that, with this definition, we allow granules to have holes and be composed of disconnected regions.

Definition 2.4. *A spatial granularity $\mathcal{G}$ is a 4-uple $<SD, MG, D_A, G>$ where:*

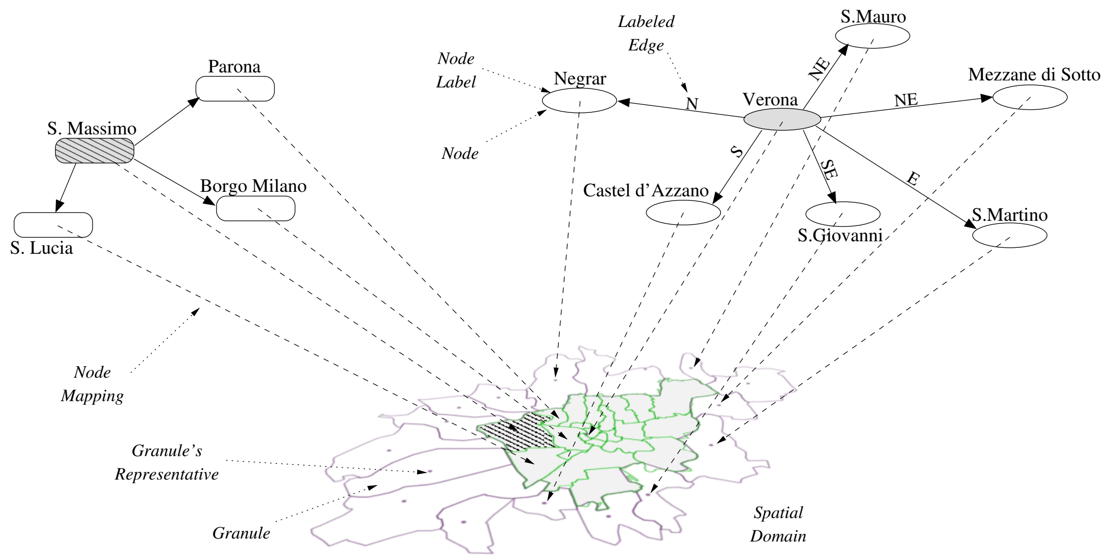

Figure 1. Two partial spatial granularities: Verona's quarters and southeast municipalities

1. *SD is a spatial domain;*
2. *MG is a multidigraph;*
3. *$D_A : MG.\Sigma_A \rightarrow SR_{SD}$ is a mapping from the edge labels to the relations between the granules that are represented in the granularity;*
4. *$G : MG.V \rightarrow SG_{SD}$ is a mapping from the nodes of the multidigraph to the spatial granules such that for each node $v \in MG.V$, $G(v) \neq \emptyset$ and any pair of granules must have disjoint interior.*

For any pair (v_1, v_2) of nodes of $MG.V$ and any label $l \in MG.\Sigma_A$, given $g_1 = G(v_1)$, $g_2 = G(v_2)$, and $R = D_A(l)$, in MG there must be an edge labeled with l between v_1 and v_2 iff the two granules g_1 and g_2 are related by the relation R, i.e. $g_2 \in R(g_1)$.

The multidigraph must reflect the spatial information. For this reason, each edge label corresponds to a relation in SR_{SD} through the total function D_A and further we impose the constraint at the end of the definition.

The multidigraph MG, hereinafter "graph", is a high level representation of a set of spatial granules and their relations, and it allows one to manage and query spatial information more efficiently than the direct use of granules geometries. Each node represents a different granule and each edge a relation between two granules. A granularity is then made up of the granules which are identified by the mapping G, that associates to each node its spatial extent.

For simplicity, we now introduce some shorthands. With G we indicate, if there are not ambiguity problems, both the mapping from the nodes to the granules and the whole granularity. The components (e.g. graph, spatial domain) of a granularity G are pointed using their name with the subscript G. Finally, note that, since the mapping $MG.l_V$ from the nodes to the node labels is a bijection, we can indicate without ambiguity a node or a granule with its label. Moreover, when we talk about a spatial relation over the granules we use its correspondent edge label.

Figure 1 depicts an example of two spatial granularities. These granularities represent the Verona's quarters (rectangular nodes) with the *Touch* relation and the Italian municipalities (round nodes) with the usual eight direction relations, respectively. In the figure dashed lines represent the mapping from nodes to granules, while continuous lines represent edges between nodes.

2.3. Relations Between Granularities

Considering the spatial definition of the granules, we are able to define several relations between granularities, such $GroupsInto$, $FinerThan$, $Subgranularity$ and $Partition$. Each of these relations has a weak definition and a strong one. In the weak version we consider only how the granules of the two granularities are related, while in the strong version also the spatial relations between the granules are considered. Let us consider, for example, the relation $GroupsInto$.

Groups Into: We say that a granularity G groups weakly into a granularity H ($GroupsInto(G, H)$) if for each node $w \in V_H$ there exists a subset $S_w \subseteq V_G$ such that $H(w) = \bigcup_{v \in S_w} G(v)$.

Moreover we say that a granularity G groups strongly into a granularity H with respect to a spatial relation R, written $GroupsInto_R(G, H)$, if G groups weakly into H

and the edges in G between granules belonging to different granules of H and representing R are preserved also in H.

3. A Framework for Spatio-Temporal Granularities

In this section we merge together our framework for spatial granularity with that of temporal granularity [1], obtaining a framework for spatio-temporal granularities.

In our proposal a spatio-temporal granularity represents and traces the changes over time of a particular spatial granularity. Then, our framework maintains for every temporal instant the spatial granularity that is valid at that time. Every time something happens, changing the spatial granularity we are representing, we register a new "version" of the spatial granularity. Thus, a spatio-temporal granularity represents several spatial granularities representing just one (evolving) world aspect, represented by the node label set. This choice is based on the idea that any possible change in a spatial granularity does not change completely it, i.e. two consecutive versions of a granularity must have some common granules.

However, any change to a granularity can modify not only the granules shape or extent but also the granule number. Any granule can be splitted in two or more granules or, conversely, several granules can be merged in a single granule. We handle these cases managing the split and merge operations and then tracing the granules history with respect to the previous granularity versions.

Unlike other approaches [2, 9], in our framework we do not associate a single spatial granularity to a temporal granule, but we propose to associate to each temporal granule a sequence of spatial granularities. This approach has several advantages.

Maintaining the spatial granularity associated to each temporal instant, allows us to compare and to reason about different spatio-temporal granularities based on uncomparable temporal granularities. In fact, using the main literature approaches [2, 9], the reasoning operations are limited only to temporal granularities related by $FinerThan$ or $GroupsInto$ relation. Our framework overcomes this constraint. In other words, we can manage, build, and reason about any spatio-temporal granularity without loss of information just partitioning the spatial information associated to instants accordingly to the temporal granularity we are interested in.

Moreover, our approach, allowing the spatial information to change during a single temporal granule, can represent homogeneously both discontinuous (e.g., administrative divisions changes) and continuous (e.g., pollution areas evolution) changes of spatial granularities.

Figure 2 shows an example of a spatio-temporal granularity. For clarity, in the figure we focus our attention only

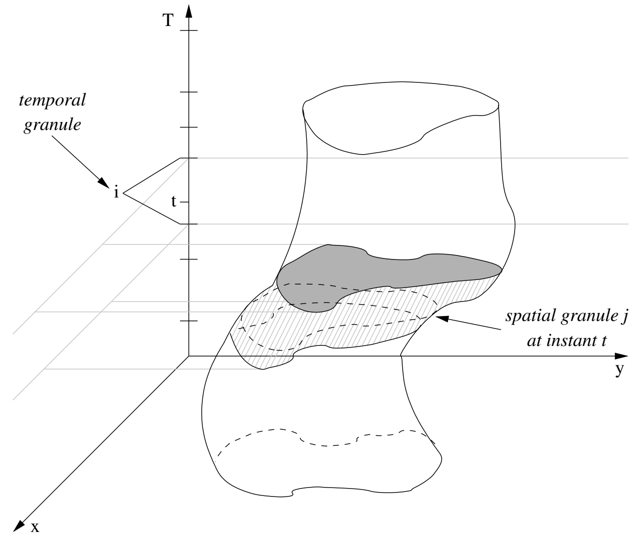

Figure 2. A spatio-temporal granularity example

on a single spatial granule changing over time. The x and y axes represent the spatial dimensions, while the T axis represents the time. On the T axis we can identify the instants associated to the temporal granules. The grey slice is composed by the spatial granularities (one for each time point) associated to the temporal granule i.

Definition 3.1. *Let T be a temporal domain and $GF = \{G_S^k\}$ a family of spatial granularities with the same node and edge labels sets, representing the same spatial relations. A* spatial evolution *E is a mapping from T to GF such that:*

$$\forall t \in T : \exists G_S^k \in GF : E(t) = G_S^k$$

i.e., given a temporal instant, E provides the spatial granularity valid at that instant.

Given a spatial node label j, $E(t)(j)$ represents the spatial granule j valid at instant t.

Definition 3.2. *Let G_T be a temporal granularity and E a spatial evolution both over domain T. A* spatio-temporal granularity *G_{ST} is a pair $<G_T, E>$. Moreover, given a temporal granule index i and a spatial node label j, $G_{ST}^j(i) = \{E(t)(j)\}_{t \in G_T(i)}$ is the* spatio-temporal granule *representing the spatial granule j during the temporal granule i.*

Our framework can be extended to aggregate and split the spatial granularities through the temporal granularities. In other words, the aggregation and splitting operations permit to create a spatio-temporal granularity G_{ST_2} based on a temporal granularity T_2 from a spatio-temporal granularity G_{ST_1} based on a finer or coarser temporal granularity T_1. The spatial granularities associated to the granules of T_1 are splitted and/or aggregated in the granules of T_2. Moreover, in our approach, the aggregations do not loss information: from G_{ST_2} we can build G_{ST_3} based on a finer temporal granularity T_3.

Usually proposals in literature allow aggregation and splitting only using temporal granularities related by $FinerThan$ or $GroupsInto$ relations. Instead, our framework, associating a spatial granularity to each time point of temporal granules, allows us to aggregate and split using any pair of temporal granularities because every temporal granule, and its associated spatial granularities, can be splitted at any time point.

On spatio-temporal granularities we can also select information using spatial and temporal projections (respectively Π_S and Π_T). The temporal projection permits us to retrieve the spatial granularities valid at one or more time points or temporal granules, while the spatial projection allows us to retrieve the temporal evolution only for some spatial granules.

4. Discussion and Conclusions

In this section we present and discuss the main proposals in the literature related to the formalisation of spatial and spatio-temporal granularities.

In [4] Erwig et al. define a partition as a function from the space point ($\mathbb{R}^2$) to the set of labels and define regions as the sets of points with the same labels. Besides, they do not use an "index" or a symbolic representation for the partitions. This lack is partially solved in [6], where McKenney and Schneider, using the definitions presented in [4], show how to represent a partition using the graph induced by the intersection points (the nodes) and the boundaries of the regions (the edges). However, they use graphs as a synthetic geometric representation of a partition and do not represent the relations between the regions.

As we do in this paper, in [3] Camossi et al. define spatial granularities following the standard temporal definition, i.e. as a mapping from an index set to a space domain. They do not explore and analyse completely the notion of spatial granularities and all the possible relations among them. Moreover, they do not organise the index set using a particular data structure. They focus their attention on the representation of the same information at different levels of detail and they define data conversions between different granularities using generalization and refinement operators.

In [2] and [9] two approaches to the spatio-temporal granularities are presented with some similarities to our proposal. However, both proposals cannot represent spatial granularities changing continuously. Moreover, these proposals define a spatio-temporal granularity attaching the valid time to a single spatial granularity and derive the evolution of a spatial granularity indirectly comparing several spatio-temporal granularities.

As for future work, we want to study the improvement in the spatial and spatio-temporal reasoning resulting from the use of the graphs as high-level representation of a spatial granularity. Moreover we want to formally define spatial and spatio-temporal operations, exploring their computational complexity and semantics.

References

[1] C. Bettini, X. S. Wang, and S. Jajodia. A general framework for time granularity and its application to temporal reasoning. *Ann. Math. Artif. Intell*, 22(1-2):29–58, 1998.

[2] E. Camossi, M. Bertolotto, and E. Bertino. A multigranular object-oriented framework supporting spatio-temporal granularity conversions. *International Journal of Geographical Information Science*, 20(5):511–534, 2006.

[3] E. B. E. Camossi, M. Bertolotto and G. Guerrini. Issues on modeling spatial granularities. In *COSIT 03 Workshop: Fundamental Issues in Spatial and Geographic Ontology*, 2003.

[4] M. Erwig and M. Schneider. Partition and conquer. In S. C. Hirtle and A. U. Frank, editors, *Spatial Information Theory: A Theoretical Basis for GIS, International Conference COSIT '97*, volume 1329 of *LNCS*, pages 389–407. Springer, 1997.

[5] F. Fonseca, M. Egenhofer, C. Davis, and G. Câmara. Semantic granularity in ontology-driven geographic information systems. *Ann. Math. Artif. Intell.*, 36(1–2):121–151, 2002.

[6] M. McKenney and M. Schneider. Spatial partition graphs: A graph theoretic model of maps. In D. Papadias, D. Zhang, and G. Kollios, editors, *SSTD 2007*, volume 4605 of *LNCS*, pages 167–184. Springer.

[7] H. R. Schmidtke and W. Woo. A size-based qualitative approach to the representation of spatial granularity. In M. M. Veloso, editor, *IJCAI 2007*, pages 563–568, 2007.

[8] M. Schneider and T. Behr. Topological relationships between complex spatial objects. *ACM Trans. Database Syst*, 31(1):39–81, 2006.

[9] S. sheng Wang and D. you Liu. Spatio-temporal database with multi-granularities. In Q. Li, G. Wang, and L. Feng, editors, *WAIM 2004*, volume 3129 of *LNCS*, pages 137–146. Springer, 2004.

[10] B. Smith and B. Brogaard. Quantum mereotopology. *Ann. Math. Artif. Intell*, 36(1-2):153–175, 2002.

[11] J. Stell and M. Worboys. Stratified map spaces: a formal basis for multi-resolution spatial databases. In T. Poiker and N. Chrisman, editors, *Proceedings of Eighth International Symposium on Spatial Data Handling*, pages 180–189, 1998.