

Temporal Reasoning: A Three Way Analysis

Vineet Padmanabhan¹, Abdul Sattar¹, Arun K. Pujari², and Chinmoy Goswamy³

¹ Knowledge Representation & Reasoning Unit(KRRU)
School of Comp. & Info. Technology
Griffith University, Nathan, Queensland, Australia.
[vineet, sattar]@cit.gu.edu.au

² Dept. of Computing & Info. Sciences
University of Hyderabad, Hyderabad, India
akpcs@uohyd.ernet.in

³ Department of Philosophy
University of Hyderabad, Hyderabad, India
cgsh@uohyd.ernet.in

Abstract

*The study of Temporal Representation and Reasoning is a core area of research in Linguistics, Philosophy and Artificial Intelligence. However, very little work has been done in analysing the principles common to these three areas. This paper attempts to investigate major issues and approaches studied in these disciplines. In this attempt, we discovered the Philosophical origin of Allen's Interval Algebra. We present an analysis of major issues and approaches proposed in these areas to handle temporal representations. Further, a variant of Neighbourhood Logic, Neighbourhood End-Point Model (**NEPM**) is proposed. It is shown that the Disjunctive Normal Neighborhood Logic Form (**DNNLF**) of NEPM provides an elegant interpretation for the tautologyhood of Allen's Interval Relations. We expect NEPM to be further developed for unifying temporal reasoning approaches proposed in Linguistics, Philosophy and Artificial Intelligence.*

1 Introduction

Reasoning about time has been a core area of research in Linguistics [8, 18], Philosophy [3, 5, 13] and Artificial Intelligence [1, 15–17]. The Linguists are concerned with extracting and capturing how temporal and tense information are provided in sentences. For the Philosophers the logical theory of time related propositions is of substantial interest because explicit temporal considerations arise in a wide variety of philosophically relevant contexts. In Artificial Intelligence temporal reasoning is a vital task in many areas such as planning, scheduling and natural language processing and hence an explicit understanding of the underlying temporal concepts need to be developed. Though immense work has been done by each of these disciplines in the field of temporal reasoning, not much effort has been taken to analyse the ideas that are common in them. Such an effort

would definitely pave the way for a better understanding of the concept itself.

This paper adopts an interdisciplinary approach linking the fields of Philosophy, Linguistics and Artificial Intelligence. The major outcome of such an approach was the discovery of the Philosophical origin of Allen's Interval Algebra (IA) [10] in one of the early Philosophical works of C.D Broad [3]. It is surprising because in none of the works upto date in AI has such a reference been made about Broad's work, to the best of our knowledge.

Further, an analysis of modeling time (the way in which the concept of time is visualised) in each of the three disciplines is dealt with. The temporal representation with which we are concerned take temporal intervals as primitives and thereby explain the relations between them. It is shown that some of the issues related to the nature of temporal relations as visualised by the Philosophy and Linguistic community can be adequately described using AI models. For instance, Philosopher's attribute a static, (*B-series*), as well as dynamic, (*A-series*), nature to temporal relations and always finds it difficult to account for this contradictory notion of time. The AI models of Interval Algebra (IA) [1] and Neighbourhood Logic(NL) [14] offers an explanation to this issue. It can be seen that the notion of A-series and B-series of time arises due to the limitation of interval relations to the basic three (i.e before, after and simultaneous). In a similar manner the Linguistic notion of *Proximity* (how to account for the neighborhoods of an interval) as well as *Origo* (the concept of "present moment"), is given an explanation with the help of IA and NL models.

A new model called **NEPM** (Neighborhood Endpoint Model) is introduced which has got the characteristics of both Interval Algebra and Neighborhood Logic. This new model is a slightly modified version of the one presented in [11]. The thirteen relations of Allen's Interval Algebra is shown in a new light using the model. A normal form, **DNNLF** (*Disjunctive Normal NeighborhoodLogic Form*)

for the new model is proposed on the basis of end-point relations. Further it is shown how the normal form differs from the work of [11] where the **NNLF** is shown in terms of interval length instead of end-point relations. It can be seen that the new model accounts for the tautologyhood of Allen's Interval Relations. One of the advantages of doing such an interdisciplinary work is to provide uniform framework for reasoning about time and thereby provide a better understanding of the concept.

The organisation of the paper is as follows. We begin with an analysis of the Linguistic, Philosophical and AI approach to time. The issues related to Linguistic and Philosophical approaches is explained in the next section with the help of the AI models. A new approach linking NL and endpoint relations is shown, whereby all the 13 relations in Allen's Interval Algebra is deduced. A normal form based on our new approach is given for Allen's Interval relations in the final section.

2 The Linguistic and Philosophical Approaches to Time

There is no single approach comprehensive enough to reveal the entire concept of time. Different approaches bring different aspects of time into prominence and each view needs to be supplemented on occasion by others. A brief survey of the different approaches to Temporal Representation and Reasoning in the area of Philosophy and Linguistics and the related issues is discussed in this section.

2.1 Linguistic Approach to Time

According to the standard tense theory [18, ?] temporality appears in Language in three different ways. For example, in a sentence like *The Light was on* the three ways in which temporality appears are:

- Time at which the light was on - Time of Situation SIT_t ;
- Time at which such a claim is made - Topic Time T_t ; and
- Time at which the Utterance is made - Time of Utterance U_t .

The relation between T_t and U_t is called *Tense* and that between T_t and SIT_t is known as *aspect* (the way in which some situation is hooked upto some T_t). *Tense* refers to the grammatical expression of the time of the situation described in the proposition relative to some other time. Aspect is not relational like tense, rather designates the internal temporal organisation of the situation described by the verb. From the above example it can be seen that the simple past, present and future can be analysed in terms of the

standard tense theory but when it comes to other interval relations it fails to provide a proper theory of representation. The following example may illustrate the point. *They found John in the bath tub. He was dead.* If such sentences are to be adequately stated in terms of standard tense theory, then, *Was* can be put in the form of SIT_t before U_t . But by stating that *he was dead*, the speaker does not want to assert that the time of his being dead precedes the time of utterance, since John will be dead at the time of utterance. Hence the fact of his being dead cannot precede the U_t . The U_t cannot be after John ceases to be dead. It includes it. What the speaker does by uttering the sentence is to make an assertion about some time in the past, when the event of John's being dead began and it is asserted that the event of John's being dead includes the time at which the utterance is made. But if one could contrast this with the sentence *John was in the garden*, it may not be the case that he is still in the garden. The standard tense theory fails in its endeavour to provide an adequate representation for such sentences as it is restricted to the basic three relations of *before*, *after*, *simultaneous*.

The failure of standard tense theory motivated some Linguists to adopt a slightly deviated method in which the basic time concept is viewed as a structure [IR, ti, Ri, Before, IN, O] where

- **IR** is a set of real numbers with the usual order relation $<$. This is called segmentability by the Linguists, i.e. time can be divided into smaller segments.
- **ti** is the set of closed intervals of IR, the time spans. For the linguists time spans are not qualitative whereas lexical contents have qualitative nature.
- **RI** is a subset of ti, such that for each ti, there is exactly one RI which properly includes ti (R_i is the region of ti). This is called *Proximity* in Linguistic terms. If 'a' and 'b' are time spans then 'a' may be near to or far from 'b'. For example expressions like *soon*. It also shows up in time distinctions like *near future* and *far future*. It is not easy to construct this intuition of proximity by metrical distance. *soon* can mean *in five minutes*, as in *the coffee will be ready soon*. It can also mean *five months* as in *they soon got divorced*. The issue of proximity is dealt with when we compare it with the AI notion of NL.
- **Before** is a partial order on ti, such that if $S=[r_i, r_j]$ and $T=[r_k, r_l]$ are in ti, then S before T iff ($r_j < r_k$).
- **IN** is a relation on ti such that S IN T iff they have at least one element in common. This is called "*inclusion*". If 'a' and 'b' are time spans then 'a' may be included in 'b'. This inclusion may be full or partial.
- **O** is a distinguished element of ti, the *Origo*. The Linguists takes this as a distinguished moment of present experience and considers it to be crucial to the way in which Language encode temporal relations.

The basic time concept mentioned above allows one to define more complex relations like *Immediately before*, *Long before* etc. Refer [18] for a more detailed discussion.

Problems: One of the fundamental problems that arises out of the above model is its inability to account for change. In order to describe the changing world we need to reason about both the static and dynamic aspects of the world. The static aspects are captured by properties (Eg. *vineet owns a car*) that hold over a period of time while the dynamic aspects are captured by occurrences (Eg. *vineet running a race*) which describe a change over a period of time. For instance, if one considers the earlier example of *The light was on*, in order to describe the changing world, we need to have an interval of time during which the light was off followed by an interval during which it was on. This process consists of a closed time-interval when the light is *off*, followed by an open interval during which the light's intensity is increasing, and ending with a closed interval when the light is *on*. Since this process is hidden from the human eye the series of events is perceived as being collapsed into an instant, and as occurring simultaneously. The price that is paid for this abstraction is that the process is no longer viewed as continuous. The above given Linguistic model cannot account for the dynamic aspect of such temporal sentences. Allen's Interval Algebra (IA) provides a solution, whereby the temporal intervals in the representation are considered to be neither closed nor open but are described as *meeting*.

Though the linguistic community has put forward the notion of *Proximity* and *Origo* they don't give an adequate representation for these notions. These two notions can be captured by Allen's Interval Algebra and Neighborhood Logic which will be explained later.

2.2 Philosophical Approach to Time

The problem of temporal reasoning in Philosophy goes back to the days of Aristotle. It is impossible to give an analysis of the whole literature in this paper. Two views which we feel are of importance to this work is considered here. *A-series* and *B-series* of time as proposed by McTaggart [9] and C.D. Broad's concept of *Interval Relations* [3].

McTaggart assigns a static and dynamic nature to time. He holds the view that events are represented as being in the past, present or future and also as continually changing with respect to the tensed determinations. Events of the past become more remote and events of the future are drawn to the present. This he labels as the dynamic nature of time and calls it by the name **A-series**. The very same events which are continually changing in respect to their pastness, presentness or futurity can be laid out in a permanent order

whose generating relation is *earlier than*. In such a conception, the temporal relations between events like that of precedence and subsequence can be described in timelessly true or false statements. This he calls the static nature of time and gives the name of **B-series**. McTaggart claims that the events in the B-series never change their position, for if some event is earlier than some other event by certain time units, then it is always the case that the one is so many time units earlier than the other. The only change an event can undergo is in the A-series. For example if the term 'P' is taken as earlier than the term 'Q', then the relation between these two terms can be shown in two different ways i.e. *P is always in the past while Q is in the present* or *P is always in the present while Q is in the future*.

The other view which we consider from the Philosophical Literature is that of C.D. Broad. This particular analysis of Broad's work is extremely important because it throws light into the Philosophical origin of Allen's Interval Algebra. Broad states that *If A and B are two experiences of the same person and no assumption is made about the relative duration's of A and B, there are in fact 13 alternative possible relations in which A may stand to B*¹ (Allen's Interval Algebra maintains the same view of thirteen relations). Of the thirteen relations, Broad maintains that six are independent of the relative duration of A and B. One can hold only if A and B are of equal duration, three can hold from A to B only if A is shorter than B and the remaining three can hold from A to B only if A is longer than B. At present we are not concerned with the other views put forward by Broad as that will be discussed at a different platform.

Inspired by Broad's view, an extended Interval Algebra called Interval Duration (INDU) Algebra is proposed in [12]. The INDU has 25 basic relations between two intervals. Each of these relations represents not only a relation between two interval events, but also includes qualitative information about durations of the two intervals.

Problems: The major drawback with the A-series of time is its inability to visualise the Past, Present and Future concept of temporality. McTaggart is of the view that since every event in the A-series possesses all the three mutually incompatible A-determinations of-*past, present, future*- it involves a contradiction. Neighborhood Logic provides an elegant interpretation to the notion of A-series of time. Similarly B-series faces the problem of being restricted to the three standard relations of *earlier, later, simultaneous*. Allen's Interval Algebra gives an adequate explanation for the notion of B-series of time.

¹ From C.D. Broad, An examination of McTaggart's Philosophy, Volume 2, Part 2, Cambridge University Press, Cambridge 1938.

3 AI Approaches to Time

AI approach to problem solving requires sophisticated world models that can capture change. Temporal reasoning seeks to provide the linguistic and inferential apparatus for exact discourse and rigorous reasoning in this sphere. Two important models for representing and reasoning about time is considered here.

3.1 Allen's Interval Algebra (IA)

Allen proposed that there are 13 basic relations among intervals and showed that these thirteen relations among intervals represent all possible relationships between any two time intervals [1, 2]. This view of Allen is similar to that put forward by Broad [3] (refer previous section). In IA time interval 'X' is an ordered pair X^-, X^+ such that $X^- < X^+$, where X^- and X^+ are interpreted as points on the real line. Hence interval Interpretation means mapping of intervals into pairs of distinct real numbers in such a manner that the beginning of an interval is strictly before the end of an interval. Therefore given two interpreted time intervals, their relative positions can be described exactly by one of the elements of the set of thirteen basic relations where each basic relation can be defined in terms of its endpoint relations. For a detailed discussion refer [1]. The basic seven relations along with their endpoint relations is given in the table below.

Basic Interval Relations	Endpoint Relations
X before Y	$X^- < Y^- \wedge X^+ < Y^-$
X meets Y	$X^- < Y^- \wedge X^+ = Y^-$
X overlaps Y	$X^- < Y^- \wedge X^+ > Y^-$
X during Y	$X^- > Y^- \wedge X^+ > Y^-$
X starts Y	$X^- = Y^- \wedge X^+ > Y^-$
X finishes Y	$X^- > Y^- \wedge X^+ > Y^-$
X equals Y	$X^- = Y^- \wedge X^+ = Y^-$

3.2 Neighborhood Logic

A complete First Order Logic for intervals called Neighborhood Logic (NL) was first proposed by Hansen and Zhou [4]. This Logic has two expanding modalities, namely, \Diamond_l and \Diamond_r . In [4] the adequacy of the neighborhood modalities is proved by deriving the other unary and binary modalities in a first order logic of the neighborhood modalities and the interval length. It also defines various notions of real analysis in terms of \Diamond_l and \Diamond_r and hence the possibility of constructing a formal theory to specify and reason about hybrid systems. We follow the definitions and notations of NL as given in [11, 14]. Since we are interested in showing how the concept of NL accounts for the problem of A-series, we

are not describing technical details. However some of the basic definitions are mentioned here.

The formulas ϕ, ψ of NL are generated by the grammar

$$\phi := A \mid \neg\phi \mid \phi \vee \psi \mid \Diamond_r \phi \mid \exists x. \phi \mid l \bowtie x \mid x \bowtie y,$$

where A,B,C are the atomic formulas, x, y are global variables and \bowtie is $<, >, \leq, \geq, \neq, =$.

Definition 1. A formula is a mapping from set of intervals I to true, false

$A: I \rightarrow \text{true, false}.$

Definition 2. l is a mapping from set of intervals to non-negative real numbers such that $l[b, e] = (e - b)$.

Definition 3. An interpretation I and an interval $[b, e]$ satisfy $\Diamond_l A$ (or $\Diamond_r A$), if there exists $\delta \geq 0$ such that I and $[b-\delta, b]$ (or I and $[e, e+\delta]$) satisfy A .

$$\Diamond_l A \equiv (\exists \delta) (\delta > 0 \wedge \mid b-\delta, b)$$

$$\Diamond_r A \equiv (\exists \delta) (\delta > 0 \wedge \mid e, e+\delta)$$

Definition 4. The combination of modalities viz. $\Diamond_r \Diamond_l$ (or $\Diamond_l \Diamond_r$) can be used to reach left (or right) neighborhoods of the ending point e (or the beginning point b) of an interval. For a model M , valuation V and interval $[b, e]$

$$M, V, [b, e] \models \Diamond_r \Diamond_l A$$

iff there exists $\delta' \geq 0$ such that

$$M, V, [e, e+\delta'] \models \Diamond_r A$$

iff there exists $\delta \geq 0$ such that

$$M, V, [e-\delta, e] \models A. \text{ In a similar manner it can be shown}$$

that

$$M, V [b, e] \models \Diamond_l \Diamond_r A$$

iff there exists $\delta' \geq 0$ such that

$$M, V, [b-\delta', b] \models \Diamond_l A$$

iff there exists $\delta \geq 0$ such that

$$M, V, [b, b+\delta] \models A$$

$(\Diamond_r \Diamond_l), (\Diamond_l \Diamond_r)$ can be seen as the converse of $\Diamond_l (\Diamond_r)$.

This can be shown as

$${}^c \Diamond_l \equiv \Diamond_r \Diamond_l$$

$${}^c \Diamond_r \equiv \Diamond_l \Diamond_r$$

4 Comparative Analysis

An interdisciplinary analysis of a particular concept always leads to a better understanding of the concept itself. Temporal reasoning from the view point of three different disciplines has been considered. The positive outcome of such an analysis is being dealt in this section.

4.1 B-series vs Interval Algebra

It can be seen that the problem of B-series of time arises due to the limitation of the relations to three. Allen's thirteen relations provide a solution to this problem. In IA each atomic relation is characterised by a conjunctive condition involving the endpoints of the two intervals considered. For instance, for two intervals $I = (I^-, I^+)$ and $J = (J^-, J^+)$, I overlaps J iff $I^+ < J^+$. In order to express indefinite information unions of the atomic interval relations are used. Since there are 13 atomic relations, there are 2^{13} possible unions of atomic relations which form the set of binary interval relations. This helps one to forego the static nature of time as attributed by the B-series in terms of interval relations.

4.2 Neighborhood Logic vs A-series

The problem with the A-series of time was its inability to visualise the Past, Present and Future concept of temporality. How to account for these three notions when dealing with the concept of time. When one visualises the problem in the background of NL it can be seen that the Logic has two expanding modalities \Diamond_r and \Diamond_l with the help of which one can reach *left (right)* Neighbourhoods of the beginning (ending) point of an interval. For instance if an interval A is taken in which the condition *The light was on* holds, then according to the NL interpretation of \Diamond_l one can show that the condition was true for some interval of time in the left neighborhood of A (i.e.past) and by \Diamond_r it can be shown that it remains true for some interval of time in the right neighborhood (i.e.future). Similarly by using the combination of modalities one can reach the inner intervals within a given interval. The problem with A-series was that it was trying to model a continuous process in a discontinuous manner. NL on the other hand provides an adequate representation for both the dynamic as well as the static nature of time.

4.3 Linguistic Model vs NL

The linguistic model was inappropriate to capture the notions of *Proximity* and *Origo*. It can be seen that the NL and IA models provide a suitable explanation for these two notions. As stated earlier the problem of proximity arises from expressions like *soon* or *just*. For example in a sentence like *The coffee will be ready soon*, if "A" is the interval in which the coffee gets ready, then according to the Linguistic model there is an interval *before* or *Just before* to A where the process of *coffee getting ready* is true. But this information is not enough to reason about the above given sentence as there could be an interval which *meets* A and A becomes true from that point onwards. The advantage with the NL model is that it can not only account for the left and right neighbourhoods of an interval with the help of \Diamond_l and

\Diamond_r but when the interval is a point interval i.e. $b = e$, can become the modalities for the conventional left and right neighbourhoods of the point. Such a model can give an adequate representation for the concept of change implicit in sentences of the above type.

The notion of *Origo* can also be seen in the same light. The linguists always had a problem with the time of utterance U_t , which was discussed earlier while dealing with the Linguistic approach to time. They claim that there is some time point called *now* or *the present moment of time* and find it difficult to account for this particular notion. In the model put forward by Allen [1], the notion of *Reference Interval* is introduced to account for the *present moment of time*. A reference interval is simply another interval but is used to group together clusters of intervals. For example, in modeling facts about the history of a person, the key events might be his birth, his first going to school etc. The intervals between such key events are called the reference intervals. Thus a hierarchy of reference intervals could be held, each containing the present moment. When one of the reference intervals in the hierarchy ceases to contain the present moment a new reference interval is selected. For example in English we can refer to the exact moment of utterance (eg, at a race the starter may say *Go now*) as well as to larger intervals as *This morning* and also longer intervals as *Today*, *This Year* etc. If such type of intervals which have well defined starting and termination points are maintained in the hierarchy representing the *present* the Linguistic problem of *Origo* and the Philosophical notion of *now* can be dealt with.

The interesting point to be noted here is that one can visualise Allen's Interval Algebra as representing the static nature of time (B-series) and Neighborhood Logic, the dynamic aspect (A-series). It has been shown in [11] that all the thirteen relations of IA can be formulated in terms of NL. One such representation of *Before* relation in terms of the NL modalities is given below. In this work the interval length is taken into consideration.

$$A \wedge (l = x) \wedge \Diamond_l(l = y_1 \wedge \Diamond_l(l = y_2 \wedge B))$$

This can be interpreted as: If the interval A holds at $[b, e]$ then by

$$\Diamond_l(l = y_1) \wedge \Diamond_l(l = y_2 \wedge B)$$

we mean that B holds at a neighborhood Interval

$$\Diamond_l(l = y_2) \text{ of the interval } [b - y_1, b].$$

This means that B holds at $[b - y_1 - y_2, b - y_1]$.

In a similar manner all the other relations can be constructed. The advantage of such a modelling is that it helps us to visualise the static nature (B-series) of time as well as the dynamic nature (A-series) in a single perspective (i.e in terms of Neighborhood Logic).

5 Neighbourhood End point Model (NEPM)

An explanation of the issues related to temporal representation and reasoning as viewed by the Philosophy and Linguistic community has been dealt with in the previous sections with the help of IA and NL. We propose a new model which we believe would account for unifying temporal reasoning approaches in Linguistics, Philosophy and AI in the future. In this model Allen's 13 relations is addressed in terms of NL and Endpoint relations and it differs from that of [11] where interval length is taken into consideration instead of end-point relations. Some of the nested formulas formed using the new model can be written as:

A is before B:

$$A = \diamond_l B \wedge (A^+ < B^-) \wedge \diamond_l \wedge (A^- < B^-) \wedge A \\ B = \diamond_r A \wedge (B^+ > A^+) \wedge \diamond_r \wedge (B^+ > A^+) \wedge B$$

A meets B:

$$A = \diamond_l B \wedge (A^- < B^-) \wedge \diamond_r \wedge (A^+ = B^-) \wedge A \\ B = \diamond_r A \wedge (B^+ > A^+) \wedge \diamond_l \wedge (B^- = A^+) \wedge B$$

A overlaps B:

$$A = \diamond_l B \wedge (A^- < B^-) \wedge \diamond_r \wedge (A^+ > B^- \wedge A^+ < B^+) \wedge A \\ B = \diamond_r A \wedge (B^+ > A^+) \wedge \diamond_l \wedge (B^- > A^- \wedge B^- < A^+) \wedge B$$

A starts B:

$$A = \diamond_l B \wedge (A^+ < B^+ \wedge A^+ > B^-) \wedge \diamond_l \wedge (A^- = B^-) \wedge A$$

$$B = \diamond_r A \wedge (B^+ > A^+) \wedge \diamond_l \wedge (B^- = A^-) \wedge B$$

A during B:

$$A = \diamond_l B \wedge (A^- > B^- \wedge A^+ < B^+) \wedge \diamond_r \wedge (A^+ < B^+) \wedge A$$

$$B = \diamond_r A \wedge (B^+ > A^+) \wedge \diamond_l \wedge (B^- < A^-) \wedge B$$

A finishes B:

$$A = \diamond_r B \wedge (A^- > B^- \wedge A^- < B^+) \wedge \diamond_r \wedge (A^+ = B^+) \wedge A$$

$$B = \diamond_l A \wedge (B^- < A^-) \wedge \diamond_r \wedge (B^+ = A^+) \wedge B$$

A equals B:

$$A = \diamond_l B \wedge (A^- = B^-) \wedge \diamond_r \wedge (A^+ = B^+) \wedge A$$

$$B = \diamond_r A \wedge (B^+ = A^+) \wedge \diamond_l \wedge (B^- = A^-) \wedge B$$

In a similar manner other relations can be worked out. The above formulas can be called *Nested Formulas*. The significance of the given interpretation is that we can take either A as the reference interval and B as the target interval or vice versa and show a particular relation. More *Nested Formulas* can be formed by composing the modal operators in different ways. But that doesn't seem to solve our problem. Can we give a common structure to these formulas? We define all these relations by taking into consideration only the \diamond_l and \diamond_r operators. Elsewhere [11] such a formulation has been termed as *Normal Neighborhood Logic form*. We adopt the same term. We go on to describe the *Disjunctive Normal Neighborhood Logic form (DNNLF)* of NEPM.

A before B :

$$\{ \diamond_l B \wedge (A^- < B^-) \wedge \diamond_r \wedge (A^+ < B^-) \wedge A \} \\ \vee \\ \{ \diamond_r A \wedge (B^+ > A^+) \wedge \diamond_l \wedge (B^- > A^+) \wedge B \}$$

It can be seen that the formula A before B is modeled with the help of only two (\diamond_l and \diamond_r) operators. The advantage of using these two operators for modeling the relations stems from the fact that the model itself can account for all the different endpoint relations needed for making a particular relation true. For instance from the model given above one can find that the conditions needed for A before B is ($A^- < B^- \wedge A^+ < B^-$) or ($B^+ > A^+ \wedge B^- > A^+$). The whole relation can be seen as a disjunction of conjunctive clauses and this we term as *Disjunctive Normal form* [6, 7] of NEPM. In other words the formula A before B receives the truth value **T** iff both ($A^- < B^-$, $A^+ < B^-$) receive **T** or both ($B^+ > A^+$, $B^- > A^+$) receive **T**. This simple fact can be expressed by a general form such as ($P \wedge \neg Q$) \vee ($\neg P \wedge Q$). In the above given formula P stands for ($A^- < B^-$), $\neg Q$ for ($A^+ < B^-$), $\neg P$ for ($B^+ > A^+$), and Q for ($B^- > A^+$). It can be said that such a form informs us about the tautologyhood of the formula. We see that the models for making the formula A before B true can be directly found out from the shape itself. The rest of the relations are shown in a similar manner.

A meets B:

$$\{ \diamond_l B \wedge (A^- < B^-) \wedge \diamond_r \wedge (A^+ = B^-) \wedge A \} \\ \vee$$

$$\{ \diamond_r A \wedge (B^- = A^+) \wedge \diamond_l \wedge (B^+ > A^+) \wedge B \}$$

A overlaps B :

$$\{ \diamond_l B \wedge (A^- < B^- \wedge \diamond_r \wedge (A^+ > B^-) \wedge (A^+ < B^+ \wedge A) \} \vee$$

$$\{ \diamond_r A \wedge (B^+ > A^+) \wedge \diamond_l \wedge (B^- < A^- \wedge (B^- > A^-) \wedge B \}$$

A starts B :

$$\{ \diamond_l B \wedge (A^- = B^- \wedge \diamond_r \wedge (A^+ > B^-) \wedge (A^+ < B^+) \wedge A) \} \vee$$

$$\{ \diamond_r A \wedge (B^+ > A^+) \wedge \diamond_l \wedge (B^- = A^-) \wedge B \}$$

A during B:

$$\{ \diamond_l B \wedge (A^- > B^-) \wedge (A^- < B^+ \wedge \diamond_r \wedge (A^+ < B^+) \} \vee$$

$$\{ \diamond_r A \wedge (B^+ > A^+) \wedge \diamond_l \wedge (B^- < A^-) \wedge B \}$$

A finishes B :

$$\{ \diamond_l B \wedge (A^- > B^-) \wedge (A^- < B^+ \wedge \diamond_r \wedge (A^+ = B^+) \} \vee$$

$$\{ \diamond_r A \wedge (B^+ = A^+) \wedge \diamond_l \wedge (B^- < A^-) \wedge B \}$$

A equals B:

$$\{ \diamond_l B \wedge (A^- = B^-) \wedge \diamond_r \wedge (A^+ = B^+) \wedge A \} \\ \vee$$

$$\{ \diamond_r A \wedge (B^+ = A^+) \wedge \diamond_l \wedge (B^- = A^-) \wedge B \}$$

For a complete analysis of the different views on time and the way they are seen in the background of Linguistics, Philosophy and Artificial Intelligence one can refer [10].

6 Conclusion

This paper makes an attempt to analyse the work in Linguistics, Philosophy and Artificial Intelligence concerning Temporal Reasoning by studying the principles common in them. In this study we discovered the Philosophical origin of one of the most influential Temporal Representation models in Artificial Intelligence, (Allen's Interval Algebra). We identified major issues concerning temporal reasoning in Linguistics and Philosophy. These issues were discussed in the light of formal approaches for temporal representations being used in Artificial Intelligence and Computer Science, such as, Interval Algebra and Neighborhood Logic respectively. Two variants of Neighborhood Logic are proposed: *Neighborhood End Point Model (NEPM)* and *Disjunctive Normal Neighborhood Logic Formula (DNNLF)*. We believe that these models can be further developed to provide a uniform treatment of temporal representations in Linguistics, Philosophy and Artificial Intelligence.

Acknowledgements

The authors would like to thank members of the *Knowledge Representation and Reasoning Unit (K.R.R.U)* for valuable suggestions and criticism. The first two authors would like to gratefully acknowledge Australian Research Council for their financial support.

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