Probabilistic Reasoning about Uncertain Relations between Temporal Points

Vladimir Ryabov
Department of Computer Science and Information Systems, University of Jyväskylä, 40351 Jyväskylä, Finland
vlad@jytko.jyu.fi

Seppo Puuronen
Department of Computer Science and Information Systems, University of Jyväskylä, 40351 Jyväskylä, Finland
sepi@jytko.jyu.fi

Abstract

A wide range of AI applications should manage time varying information. Many published research articles in the area of temporal representation and reasoning assume that temporal data is precise and certain, even though in reality this assumption is often false. However, in many real applications temporal information is imperfect and there is a need to find some way of handling it.

An uncertain relation between two temporal points is represented as a vector with three probability values denoting the probabilities of the three basic relations: "<" (before), "=" (at the same time), and ">" (after). The reasoning mechanism includes inversion, composition, addition, and negation operations. We propose formulas to calculate the probability values within the uncertainty vectors representing the resulting relations of the reasoning operations. We also consider an example of using the proposed representation and reasoning mechanism.

1. Introduction

In a wide range of AI research fields there is a need for representation and reasoning about temporal information. Temporal formalisms are applied in natural language understanding, planning, keeping medical records, i.e. in all the areas, where the time course of events plays an important role. Many temporal representation and reasoning approaches assume that precise and certain temporal information is available, and they give little or no support for situations in which we are dealing with imperfect temporal information. However, in many real applications temporal information is imperfect and there is a need to find some way of handling it.

Temporal points and intervals are the main ontological primitives used by temporal formalisms. In this paper we use temporal points as ontological primitives. We propose a probabilistic approach to represent uncertain relations between temporal points and a mechanism for reasoning about these relations. A temporal interval can be represented as a pair of points denoting the start and the end of the interval, and then, the proposed representation can also be applied with intervals, as it is shown in [14]. In many situations there is a need to deal with temporal relations, which are, in reality, not always certain. Uncertainty, which is one kind of imperfect information according to Parsons' classification [11], arises from the lack of information about the state of the world. This lack of information makes it impossible to determine if certain statements about the world are true or false. We are able only to estimate the tendency of the statement to be true or false using, for example, some numerical measure of degree to which one may be sure [11]. Uncertainty can stem from a number of sources, such as many sources of information, granularity mismatch, input errors, or even data can be deliberately made uncertain for reasons of security [8]. Various approaches to the problem of handling imperfect information are mentioned in the bibliography on uncertainty management by Dyreson [5], and in the survey by Parsons and Hunter [12].

The topic of handling uncertainty in temporal knowledge was underlined as a newly emerging and growing subarea of temporal representation and reasoning in the recent survey by Chittaro and Montanari [2]. However, during the past decades a number of approaches to this problem were proposed. The notion of indefinite temporal relation, which is a disjunction of the basic relations, was used by van Beek and Cohen [19]. That representation did not include any numerical measures for uncertainty, mostly concentrating on the reasoning algorithms and the constraint satisfaction problem.

In the research area of integrating time and probability, there are a number of approaches introducing into temporal contexts mathematical formalisms such as Bayesian networks, for example, [13], [7], and Markov processes, for example, [17]. In this paper we use a probabilistic approach to deal with uncertain temporal relations, although, there are several other means of handling uncertainty. For example, Dubois and Prade [3] applied possibility theory to process fuzzy temporal knowledge.

0-7695-1107-4/01 $10.00 © 2001 IEEE

Another growing subarea of temporal representation and reasoning is the area of temporal databases, where there is also a need to handle uncertainty. Dyreson and Snodgrass [6] proposed to support valid-time indeterminacy in temporal databases by a probabilistic approach, concentrating on the temporal query language for uncertain temporal information.

In this paper we propose an approach to represent the uncertain temporal relation between two points as a vector with three probability values denoting the probabilities of the three basic relations ("<" - before, "=" - at the same time, and ">" - after), that can hold between these points. A reasoning mechanism includes inversion, composition, addition, and negation operations. In the definitions of the reasoning operations we provide formulas for calculating the probability values for the uncertainty vectors representing the resulting relation.

The structure of the paper is the following. In the next section we introduce the main concepts used in the paper. In Section 3 we present four operations of the reasoning mechanism. In Section 4 we consider an example of representation and reasoning with uncertain temporal relations, and in Section 5 we discuss about the approach used in this paper. And, finally, in Section 6 we make conclusions and point out some directions for further research.

2. Main concepts

In this section we define the basic concepts used throughout the paper.

The three basic relations that can hold between two temporal points are: "<" (before), "=" (at the same time), and ">" (after). These are *certain* temporal relations between points. Possible disjunctions of the certain relations, i.e. "≤" ("<" or "="), "≥" ("=" or ">"), "≠" ("<" or ">"), and "?"("<" or "=" or ">"), are called the *uncertain* relations between temporal points. We propose to extend this representation by providing the probabilities of the basic relations within the uncertain relation.

Definition 1. Let an uncertain relation between two temporal points **a** and **b** be represented by a vector $\left(\mathbf{e}^{<},\mathbf{e}^{=},\mathbf{e}^{>}\right)_{\mathbf{a,b}}$, where the value $\mathbf{e}^{<}_{\mathbf{a,b}}$ is the probability of **a<b**, the value $\mathbf{e}^{=}_{\mathbf{a,b}}$ is the probability of **a=b**, and the value $\mathbf{e}^{>}_{\mathbf{a,b}}$ is the probability of **a>b**. The sum of these probability values is equal to 1, i.e. $\mathbf{e}^{<}_{\mathbf{a,b}}+\mathbf{e}^{=}_{\mathbf{a,b}}+\mathbf{e}^{>}_{\mathbf{a,b}}=1$, since they represent the probabilities of all the basic relations.

For example, in the situation, when the temporal relation between the points **a** and **b** is "<" or "=", and the probability of "<" is 0.7, and the probability of "=" is 0.3, the uncertainty vector is $(0.7, 0.3, 0)_{\mathbf{a,b}}$.

A totally certain relation (TCR) between two temporal points **a** and **b** is a relation represented by the uncertainty vector $\left(\mathbf{e}^{<},\mathbf{e}^{=},\mathbf{e}^{>}\right)_{\mathbf{a,b}}$, where $\mathbf{e}^{<}_{\mathbf{a,b}}=1$, $\mathbf{e}^{=}_{\mathbf{a,b}}=1$, or $\mathbf{e}^{>}_{\mathbf{a,b}}=1$.

Two uncertain temporal relations $\mathbf{r_1}$ represented by the vector $\left(\mathbf{e}^{<}_{1},\mathbf{e}^{=}_{1},\mathbf{e}^{>}_{1}\right)$ and $\mathbf{r_2}$ represented by the vector $\left(\mathbf{e}^{<}_{2},\mathbf{e}^{=}_{2},\mathbf{e}^{>}_{2}\right)$ are *equal* if $\mathbf{e}^{<}_{1}=\mathbf{e}^{<}_{2}$, $\mathbf{e}^{=}_{1}=\mathbf{e}^{=}_{2}$, and $\mathbf{e}^{>}_{1}=\mathbf{e}^{>}_{2}$. Otherwise, the relations $\mathbf{r_1}$ and $\mathbf{r_2}$ are *unequal*. For example, two uncertain relations $(0.7,0.3,0)_{\mathbf{a,b}}$ and $(0.2,0.8,0)_{\mathbf{a,b}}$ between points **a** and **b** are unequal, although they are equal ("≤") at the symbolic level of representation.

In many application domains, even when we know nothing about the relation between any two temporal points, the basic temporal relations between these points are not equally probable. Let us denote the *domain probability values* of the basic relations as $\mathbf{e}^{<}_{\mathbf{D}}$, $\mathbf{e}^{=}_{\mathbf{D}}$, and $\mathbf{e}^{>}_{\mathbf{D}}$. These probability values represent the probabilities of the basic relations between two points in the situation, when we know nothing about the relation between these points in the given domain area. The sum of these probability values is equal to 1. We suggest, that the domain probability values are defined for a particular temporal context or application, otherwise, the basic relations will be considered equally probable, which often introduces some imprecision to the description of the situation. We will use the domain probability values in the definition of the composition and negation operations.

3. Reasoning about uncertain relations

In this section we define four operations for reasoning with uncertain relations between temporal points. Our reasoning mechanism includes inversion, composition, addition, and negation operations, which we consider correspondingly in the following four subsections.

3.1. Inversion

The operation of inversion (~) derives the relation $\mathbf{r_{b,a}}$ between two temporal points **b** and **a**, when the relation $\mathbf{r_{a,b}}$ between **a** and **b** is known, and $\mathbf{r_{b,a}} = \tilde{\mathbf{r}}_{\mathbf{a,b}}$ as presented in Figure 1.

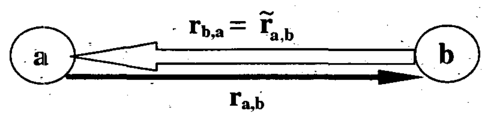

Figure 1. Operation of inversion

The relations "<" and ">" are mutually inverted, and an inversion of "=" is the relation "=". To obtain the uncertainty vector for an inverted uncertain temporal relation between two points, we only need to exchange the values of the probabilities of the relations "<" and ">", as in the following definition.

Definition 2. Let the relations $\mathbf{r}_{a,b}$ and $\mathbf{r}_{b,a}$ be defined by the vectors $\left(e^{<},e^{=},e^{>}\right)_{a,b}$ and $\left(e^{<},e^{=},e^{>}\right)_{b,a}$ correspondingly, and $\mathbf{r}_{b,a}=\tilde{\mathbf{r}}_{a,b}$. Then, $\mathbf{e}^{<}_{b,a}=\mathbf{e}^{>}_{a,b}$, $\mathbf{e}^{=}_{b,a}=\mathbf{e}^{=}_{a,b}$, and $\mathbf{e}^{>}_{b,a}=\mathbf{e}^{<}_{a,b}$.

From the above definition it is easy to derive the property of double inversion, according to which an uncertain temporal relation $\mathbf{r}_{a,b}$ is equal to the double inversion of this relation, i.e. $\mathbf{r}_{a,b}=\tilde{\tilde{\mathbf{r}}}_{a,b}$.

3.2. Composition

The operation of composition ($\otimes$) derives the relation $\mathbf{r}_{a,c}$ between the temporal points **a** and **c**, when there exist the relation $\mathbf{r}_{a,b}$ between the points **a** and **b** and the relation $\mathbf{r}_{b,c}$ between the points **b** and **c**, as presented in Figure 2.

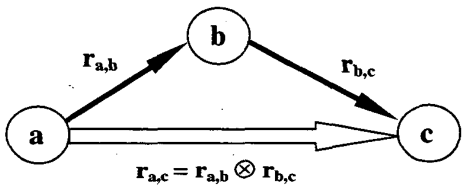

Figure 2. Operation of composition

The goal of the composition operation is to find out the probabilities of the three basic relations $\mathbf{e}^{<}_{a,c}$, $\mathbf{e}^{=}_{a,c}$, and $\mathbf{e}^{>}_{a,c}$ between the temporal points **a** and **c**, with the known uncertainty vectors for the relations $\mathbf{r}_{a,b}$ and $\mathbf{r}_{b,c}$. There exist 9 combinations of the values of the relations $\mathbf{r}_{a,b}$ and $\mathbf{r}_{b,c}$ (in each combination the first value is the probability of the relation between the points **a** and **b**, and the second value between the points **b** and **c**): 1) "<" and "<"; 2) "<" and "="; 3) "<" and ">"; 4) "=" and "<"; 5) "=" and "="; 6) "=" and ">"; 7) ">" and "<"; 8) ">" and "="; 9) ">" and ">". In the combinations 1, 2, and 4 the basic relation "<" is supported. In the combination 5 the basic relation "=" is supported. In the combinations 6, 8, and 9 the basic relation ">" is supported. And, finally, in the combinations 3 and 7 all the three basic relations are supported. In each combination the probability of the particular value of the relation is derived as a multiplication of the correspondent probability values from the uncertainty vectors. For example, the probability obtained in the combination 1 is a multiplication of $\mathbf{e}^{<}_{a,b}$ and $\mathbf{e}^{<}_{b,c}$ *supporting* $\mathbf{e}^{<}_{a,c}$. In the combinations 3 and 7, the probability needs to be divided between the probabilities of all three basic relations, according to the domain probability values $\mathbf{e}^{<}_{D}$, $\mathbf{e}^{=}_{D}$, and $\mathbf{e}^{>}_{D}$.

Definition 3. Let the relations $\mathbf{r}_{a,b}$, $\mathbf{r}_{b,c}$, and $\mathbf{r}_{a,c}$ between the temporal points **a**, **b**, and **c** be represented by the uncertainty vectors $\left(e^{<},e^{=},e^{>}\right)_{a,b}$, $\left(e^{<},e^{=},e^{>}\right)_{b,c}$, and $\left(e^{<},e^{=},e^{>}\right)_{a,c}$ correspondingly. Then $\mathbf{r}_{a,c}=\mathbf{r}_{a,b}\otimes\mathbf{r}_{b,c}$ is:

$$\mathbf{e}^{<}_{a,c}=\mathbf{e}^{<}_{a,b}\mathbf{e}^{<}_{b,c}+\mathbf{e}^{<}_{a,b}\mathbf{e}^{=}_{b,c}+\mathbf{e}^{=}_{a,b}\mathbf{e}^{<}_{b,c}+\mathbf{e}^{<}_{a,b}\mathbf{e}^{>}_{b,c}\mathbf{e}^{<}_{D}+\mathbf{e}^{>}_{a,b}\mathbf{e}^{<}_{b,c}\mathbf{e}^{<}_{D},$$

$$\mathbf{e}^{=}_{a,c}=\mathbf{e}^{=}_{a,b}\mathbf{e}^{=}_{b,c}+\mathbf{e}^{<}_{a,b}\mathbf{e}^{>}_{b,c}\mathbf{e}^{=}_{D}+\mathbf{e}^{>}_{a,b}\mathbf{e}^{<}_{b,c}\mathbf{e}^{=}_{D}, \text{ and}$$

$$\mathbf{e}^{>}_{a,c}=\mathbf{e}^{>}_{a,b}\mathbf{e}^{>}_{b,c}+\mathbf{e}^{>}_{a,b}\mathbf{e}^{=}_{b,c}+\mathbf{e}^{=}_{a,b}\mathbf{e}^{>}_{b,c}+\mathbf{e}^{<}_{a,b}\mathbf{e}^{>}_{b,c}\mathbf{e}^{>}_{D}+\mathbf{e}^{>}_{a,b}\mathbf{e}^{<}_{b,c}\mathbf{e}^{>}_{D}.$$

The sum of the probability values $\mathbf{e}^{<}_{a,c}$, $\mathbf{e}^{=}_{a,c}$, and $\mathbf{e}^{>}_{a,c}$, defined above, is equal to 1, which can be easily proved by transforming and simplifying their sum.

The operation of composition is obviously non-commutative (i.e., $\mathbf{r}_{a,b}\otimes\mathbf{r}_{b,c}\neq\mathbf{r}_{b,c}\otimes\mathbf{r}_{a,b}$), associative (i.e., $\mathbf{r}_{a,b}\otimes(\mathbf{r}_{b,c}\otimes\mathbf{r}_{c,d})=(\mathbf{r}_{a,b}\otimes\mathbf{r}_{b,c})\otimes\mathbf{r}_{c,d}$) for symbolic representation of relations, non-associative for particular probability values within the uncertainty vectors, and TCR "=" is the identity for composition (i.e., "="$\otimes\mathbf{r}_{b,c}=\mathbf{r}_{b,c}$ and $\mathbf{r}_{a,b}\otimes$"=" $=\mathbf{r}_{a,b}$). Moreover, an inversion of the composition of two uncertain temporal relations $\mathbf{r}_{a,b}$ and $\mathbf{r}_{b,c}$ is equal to the composition of their inversions (i.e. $\sim\!\left(\mathbf{r}_{a,b}\otimes\mathbf{r}_{b,c}\right)=\tilde{\mathbf{r}}_{b,c}\otimes\tilde{\mathbf{r}}_{a,b}$). The latter property can be easily proved using the formulas in Definitions 2 and 3.

3.3. Addition

In many situations, we need to deal with more than one possible uncertain relation between two temporal points. This happens when the information about a relation, for instance, is collected from a number of information sources or experts. For example, according to the first expert we might know that the relation between points **a** and **b** is "≤" and is represented by the uncertainty vector $(0.6,0.4,0)_{a,b}$. At the same time, the second expert suggests that the relation between these points is "≥"with the vector $(0,0.8,0.2)_{a,b}$. In this situation, it can be helpful to combine these two uncertain temporal relations into a single uncertain relation $\mathbf{r}_{a,b}$ between the points **a** and **b**. To be able to do this, we propose the binary operation of addition ($\oplus$), illustrated in Figure 3.

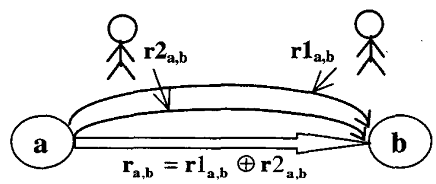

Figure 3. Operation of addition

Operation of addition is similar to the operation of intersection defined, for example, in the interval algebra of Allen [1], in the point algebra of Vilain and Kautz [20], and further in the extensions of that algebras by van Beek [18]. We extend the operation of intersection, which uses the symbolic representation of temporal relations, with probabilistic measures of uncertainty, and provide the formulas to calculate the probability values within the resulting vector.

Definition 4. Let the uncertain relations $\mathbf{r1_{a,b}}$, $\mathbf{r2_{a,b}}$, $\mathbf{r_{a,b}}$ between the temporal points **a** and **b** be represented by the uncertainty vectors $\left(\mathbf{e}_1^<,\mathbf{e}_1^=,\mathbf{e}_1^>\right)_{\mathbf{a,b}}$, $\left(\mathbf{e}_2^<,\mathbf{e}_2^=,\mathbf{e}_2^>\right)_{\mathbf{a,b}}$, and $\left(\mathbf{e}^<,\mathbf{e}^=,\mathbf{e}^>\right)_{\mathbf{a,b}}$ respectively. Then $\mathbf{r_{a,b}} = \mathbf{r1_{a,b}} \oplus \mathbf{r2_{a,b}}$ is:

$$\mathbf{e}_{\mathbf{a,b}}^< = \frac{\mathbf{s}^<}{\mathbf{S}},\ \mathbf{e}_{\mathbf{a,b}}^= = \frac{\mathbf{s}^=}{\mathbf{S}},\ \text{and } \mathbf{e}_{\mathbf{a,b}}^> = \frac{\mathbf{s}^>}{\mathbf{S}},\ \text{where } \mathrm{S=s^<+s^=+s^>},$$

$$\mathbf{s}^< = \frac{\mathbf{e}_1^< \cdot \mathbf{e}_2^<}{\mathbf{e}_1^< + \mathbf{e}_2^<},\ \mathbf{s}^= = \frac{\mathbf{e}_1^= \cdot \mathbf{e}_2^=}{\mathbf{e}_1^= + \mathbf{e}_2^=},\ \text{and } \mathbf{s}^> = \frac{\mathbf{e}_1^> \cdot \mathbf{e}_2^>}{\mathbf{e}_1^> + \mathbf{e}_2^>}.$$

The operation of addition is commutative (i.e., $\mathbf{r1_{a,b}}\oplus\mathbf{r2_{a,b}}=\mathbf{r2_{a,b}}\oplus\mathbf{r1_{a,b}}$), idempotent (i.e., $\mathbf{r1_{a,b}}\oplus\mathbf{r1_{a,b}}=\mathbf{r1_{a,b}}$), and TCR is an annihilator for addition (i.e., $\mathbf{r1_{a,b}}\oplus\mathrm{TCR}=\mathrm{TCR}\oplus\mathbf{r1_{a,b}}=\mathrm{TCR}$). Addition is associative (i.e., $(\mathbf{r1_{a,b}}\oplus\mathbf{r2_{a,b}})\oplus\mathbf{r3_{a,b}}=\mathbf{r1_{a,b}}\oplus(\mathbf{r2_{a,b}}\oplus\mathbf{r3_{a,b}})$) at the symbolic level of representation, and non-associative for particular probability values within the uncertainty vectors. Moreover, an inversion of the addition of two uncertain temporal relations $\mathbf{r1_{a,b}}$ and $\mathbf{r2_{a,b}}$ is equal to the addition of their inversions (i.e., $\sim\!\left(\mathbf{r1_{a,b}} \oplus \mathbf{r2_{a,b}}\right) = \left(\tilde{\mathbf{r}}\mathbf{1_{a,b}} \oplus \tilde{\mathbf{r}}\mathbf{2_{a,b}}\right)$). The latter property can be proved using formulas in Definitions 2 and 4.

Combining a number of relations into one, we irrevocably lose the information about the added relations. This means, that after deriving the uncertainty vector as a result of the addition it is in most cases impossible to know which relations were added. This can be crucial in the systems where it is important to know the ancestors of the derived relation. One possible solution to this problem is to keep the history of added relations, although, in many complex reasoning systems this will appear to be too expensive and, hence, unreasonable.

3.4. Negation

There are situations, when we do not have the information about the relations which are possible between two temporal points, but we might know the relations that are impossible. The operation of negation (—) derives the possible uncertain temporal relation $\mathbf{r_{a,b}}$ between points **a** and **b**, when it is known that the uncertain temporal relation $\mathbf{r1_{a,b}}$ between the points **a** and **b** is impossible. For example, it might be known that the relation $\mathbf{r_{a,b}}$ is definitely not "<". In this case, $\mathbf{r_{a,b}}$ can still be ">" or "=".

The unary operation of negation was also included in the algebra by Ladkin and Maddux [9], [10] (called operation of complement there). We extend their definition with our representation of uncertain relations using uncertainty vectors and provide the formulas for calculating the probability values within the resulting vector.

We suppose, that the probability of a particular basic temporal relation which is impossible between two points needs to be divided between the probabilities of the other two basic temporal relations in the resulting uncertainty vector. This division between these two contributed relations should be made according to the domain probability values.

Definition 5. Let the temporal relations $\mathbf{r_{a,b}}$ and $\mathbf{r1_{a,b}}$ be represented by the uncertainty vectors $\left(\mathbf{e}^<,\mathbf{e}^=,\mathbf{e}^>\right)_{\mathbf{a,b}}$ and $\left(\mathbf{e}_1^<,\mathbf{e}_1^=,\mathbf{e}_1^>\right)_{\mathbf{a,b}}$ correspondingly. Then $\mathbf{r_{a,b}}=\overline{\mathbf{r1_{a,b}}}$ is:

$$\mathbf{e}_{\mathbf{a,b}}^< = \frac{\mathbf{e}_\mathbf{D}^<\mathbf{e}_1^>}{\mathbf{e}_\mathbf{D}^< + \mathbf{e}_\mathbf{D}^=} + \frac{\mathbf{e}_\mathbf{D}^<\mathbf{e}_1^=}{\mathbf{e}_\mathbf{D}^< + \mathbf{e}_\mathbf{D}^>},\ \mathbf{e}_{\mathbf{a,b}}^= = \frac{\mathbf{e}_\mathbf{D}^=\mathbf{e}_1^<}{\mathbf{e}_\mathbf{D}^= + \mathbf{e}_\mathbf{D}^>} + \frac{\mathbf{e}_\mathbf{D}^=\mathbf{e}_1^>}{\mathbf{e}_\mathbf{D}^< + \mathbf{e}_\mathbf{D}^=},$$

$$\text{and } \mathbf{e}_{\mathbf{a,b}}^> = \frac{\mathbf{e}_\mathbf{D}^>\mathbf{e}_1^<}{\mathbf{e}_\mathbf{D}^= + \mathbf{e}_\mathbf{D}^>} + \frac{\mathbf{e}_\mathbf{D}^>\mathbf{e}_1^=}{\mathbf{e}_\mathbf{D}^< + \mathbf{e}_\mathbf{D}^>}.$$

For example, the probability value $\mathbf{e}_1^<$ is decomposed into the probabilities $\frac{\mathbf{e}_\mathbf{D}^=\mathbf{e}_1^<}{\mathbf{e}_\mathbf{D}^= + \mathbf{e}_\mathbf{D}^>}$ and $\frac{\mathbf{e}_\mathbf{D}^>\mathbf{e}_1^<}{\mathbf{e}_\mathbf{D}^= + \mathbf{e}_\mathbf{D}^>}$ which contribute to the values $\mathbf{e}_1^=$ and $\mathbf{e}_1^>$ correspondingly. In a similar way we divide the probabilities $\mathbf{e}_1^=$ and $\mathbf{e}_1^>$.

A negation of an inversion of an uncertain temporal relation is equal to the inversion of the negation of this relation (i.e., $\overline{\tilde{\mathbf{r}}_{\mathbf{a,b}}} = \sim\!\left(\overline{\mathbf{r_{a,b}}}\right)$), which can be easily proved.

In the next section we consider an example of representation of uncertain temporal relations and the use of different operations to reason with them.

4. Example

Let us consider three temporal points **a**, **b**, and **c**, and uncertain relations between them. Let the domain probabilities of the basic relations be: $\mathbf{e}_\mathbf{D}^<$=0.3, $\mathbf{e}_\mathbf{D}^=$=0.1, and $\mathbf{e}_\mathbf{D}^>$=0.6. Let the information about the relation between the points **a** and **b** be obtained from two information sources. According to the first information source, the probabilities of the basic relations "<", "=", and ">" between **a** and **b** are 0.7, 0.1, and 0.2 correspondingly. The second information source suggests that the probabilities of the basic relations between these points are: 0.45, 0.4, and 0.15. And, finally, it is known that the relation between the points **b** and **c** is definitely not ">". Let us find the probabilities of the basic relations between the points **c** and **a**. The given information is: $\mathbf{r1}_{a,b}$=(0.7,0.1,0.2), $\mathbf{r2}_{a,b}$=(0.45,0.4,0.15), and $\mathbf{r1}_{b,c}$= (0,0,1), as it is illustrated at Figure 4a.

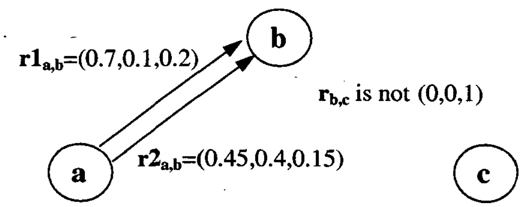

Figure 4. Initial relations between a, b, and c

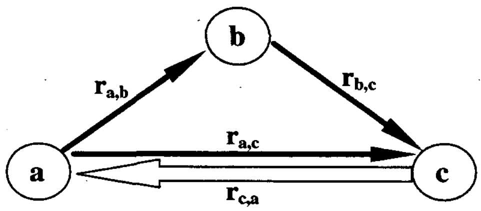

Figure 5. Derived temporal relations

The relation $\mathbf{r}_{c,a}$ can be derived as an inversion of the relation $\mathbf{r}_{a,c}$, which is a composition of the relations $\mathbf{r}_{a,b}$ and $\mathbf{r}_{b,c}$ as it is illustrated at Figure 5. The relation $\mathbf{r}_{a,b}$ can be found as an addition of the relations $\mathbf{r1}_{a,b}$ and $\mathbf{r2}_{a,b}$, and the relation $\mathbf{r}_{b,c}$ is a negation of the relation $\mathbf{r1}_{b,c}$:

$$\mathbf{r}_{c,a} = \tilde{}\left(\mathbf{r}_{a,b} \otimes \mathbf{r}_{b,c}\right) = \tilde{}\left(\left(\mathbf{r1}_{a,b} \oplus \mathbf{r2}_{a,b}\right) \otimes \overline{\mathbf{r1}_{b,c}}\right).$$

Applying the formulas of Definitions 3, 4, 5, and 6:

$$\mathbf{r}_{c,a} = \tilde{}\left(\left((0.7,0.1,0.2) \oplus (0.45,0.4,0.15)\right) \otimes \overline{(0,0,1)}\right) =$$

$$\tilde{}\left((0.623,0.182,0.195) \otimes (0.75,0.25,0)\right) =$$

$$= \tilde{}(0.803,0.06,0.137) = (0.137,0.06,0.803).$$

The derived uncertainty vector includes the probabilities of the basic relations between the temporal points **c** and **a**: $\mathbf{e}^{<}_{c,a} = 0.137$, $\mathbf{e}^{=}_{c,a} = 0.06$, and $\mathbf{e}^{>}_{c,a} = 0.803$.

5. Discussion

Probabilistic approach, which was applied in this paper to handling uncertain temporal relations, is actually one of the numerical formalisms for dealing with uncertainty. The other widely used techniques are possibility theory [4], Dempster-Shafer's theory of evidence [15], certainty factors [16], etc. The selection of the probabilistic approach was made based on the criteria for the evaluation of uncertainty management techniques proposed in [21]. These include: "interpretation", "imprecision", "calculus", "consistency", "assessment", and "computation". Moreover, to be able to model accurately uncertain temporal relations, a formalism needs to model "alternative relations" (we suppose that only one of the basic relations actually holds between two points), "dependent values" (for example, the values of the endpoints of two temporal intervals are dependent; to model the relation between these intervals we need to model these dependencies, as shown in [14]), and, finally, we need to have clear and explicit rules for combining the uncertainty measures.

Based on criteria above, we suggest that the probabilistic approach is better suited to model the uncertain temporal relations. At the same time, fuzzy set theory can be successfully used to model another aspects of temporal knowledge, for example, generalized events [3]. We also suppose that Dempster-Shafer theory of evidence can also be applied for dealing with uncertain temporal relations, since it uses the notion of probability. It is different from our approach in a way that we do not consider probabilities of all possible subsets of the basic relations. Although, in some particular applications it can be useful, and we suppose that our approach can be extended to do this.

6. Conclusions

In this paper we proposed a probabilistic approach to represent uncertain relations between temporal points and a mechanism for reasoning with these relations. The uncertain relation between two points is represented by the uncertainty vector with three probabilities of the basic relations ("<", "=", and ">"). The reasoning mechanism includes inversion, composition, addition, and negation operations. Further research is needed to analyze the applicability of the proposed mechanism in different applications of temporal formalisms. As another direction for further research we consider the development of the means to measure the degree of certainty of a particular temporal relation and study the change of this degree while performing different reasoning operations.

7. Acknowledgements

We are grateful to anonymous reviewers for useful comments and constructive criticism. This work was partly funded by COMAS graduate school at the University of Jyväskylä.

8. References

[1] J.F. Allen, "Maintaining Knowledge about Temporal Intervals", *Communications of the ACM*, Vol.26, 1983, pp.832-843.

[2] L. Chittaro and A. Montanari, "Temporal Representation and Reasoning in Artificial Intelligence: Issues and Approaches", *Annals of Mathematics and Artificial Intelligence*, Vol.28, 2000, pp.47-106.

[3] D. Dubois and H. Prade, "Processing Fuzzy Temporal Knowledge", *IEEE Transactions on Systems, Man, and Cybernetics*, Vol.19(4), 1989, pp.729-744.

[4] D. Dubois and H. Prade, *Possibility Theory: an Approach to the Computerized Processing of Uncertainty*, Plenum Press, New York, 1998.

[5] C.E. Dyreson, "A Bibliography on Uncertainty Management in Information Systems", in *Uncertainty Management in Information Systems: From Needs to Solutions*, Kluwer Academic Publishers, 1997, pp.415-458.

[6] C.E. Dyreson and R.T. Snodgrass, "Supporting Valid-Time Indeterminacy", *ACM Transactions on Database Systems*, Vol.23(1), 1998, pp.1-57.

[7] P. Haddawy, "Representing Plans under Uncertainty: a Logic of Time, Chance, and Action", *Lecture Notes in Computer Science*, Vol.770, Springer, 1994.

[8] S. Kwan, F. Olken, and D. Roten, "Uncertain, Incomplete, and Inconsistent Data in Scientific and Statistical Databases", in *Second Workshop on Uncertainty Management and Information Systems: from Needs to Solutions*, Catalina, USA, 1993.

[9] P.B. Ladkin and R. Maddux, "On Binary Constraint Networks", *Technical Report*, KES.U.88.8, Kestrel Institute, Palo Alto, California, 1988.

[10] P.B. Ladkin and R. Maddux, "The Algebra of Binary Constraint Networks", *Technical Report*, KES.U.88.9, Kestrel Institute, Palo Alto, California, 1988.

[11] S. Parsons, "Current Approaches to Handling Imperfect Information in Data and Knowledge Bases", *IEEE Transactions on Knowledge and Data Engineering*, Vol.8(3), 1996, pp.353-372.

[12] S. Parsons and A. Hunter, "A Review of Uncertainty Handling Formalisms", in *Applications of Uncertainty Formalisms*, A. Hunter and S. Parsons (eds.), Lecture Notes in Artificial Intelligence, Vol.1455, Springer, 1998, pp.8-37.

[13] J. Pearl, "Probabilistic Reasoning in Intelligent Systems", 2-d Edition, Morgan Kaufmann, 1992.

[14] V. Ryabov, "Uncertain Relations between Indeterminate Temporal Intervals", *Proceedings of the 10-th International Conference on Management of Data*, R. Agrawal, K. Ramamritham, and T. Vijayaraman (eds.), Tata McGraw Hill Publishing Company Limited, New Delhi, India, 2000, pp.87-95.

[15] G. Shafer, *A Mathematical Theory of Evidence*, Princeton University Press, 1976.

[16] E. Shortliffe, *Computer-Based Medical Consultations: MYCIN*, Elsevier, New York, 1976.

[17] A. Tawfik and E. Neufeld, "Irrelevance in Uncertain Temporal Reasoning", *Proceedings of the 3-d International Workshop on Temporal Representation and Reasoning (TIME'96)*, IEEE Computer Society, 1996, pp.182-187.

[18] P. van Beek, "Exact and Approximate Reasoning about Qualitative Temporal Relations", Ph.D. thesis, Department of Computer Science, University of Waterloo, Canada, 1990.

[19] P. van Beek and R. Cohen, "Exact and Approximate Reasoning about Temporal Relations", *Computational Intelligence*, Vol.6, 1990, pp.132-144.

[20] N. Vilain and H. Kautz, "Constraint Propagation Algorithms for Temporal Reasoning", *Proceedings of the 5-th National Conference of the American Association for Artificial Intelligence*, Morgan Kaufmann, 1986, pp.377-382.

[21] P. Waley, "Measures of Uncertainty in Expert Systems", *Artificial Intelligence*, Vol.83, 1996, pp.1-58.