Interval temporal logics with chop-like operators

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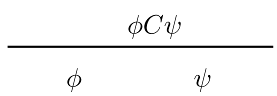
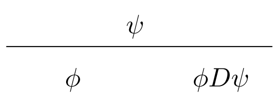
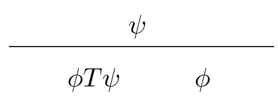
Much of the research on formal temporal logic since Arthur Prior has used semantics based on points of time. Nonetheless, there is a fair body of work on logics with semantics based on intervals of time. Interval temporal logics are thought appropriate when dealing with events that take place over extended periods, and they have found uses in artificial planning and intelligent agents, for example.

Like point-based systems, interval logics allow a wide choice of temporal operators. Although most of the attention has been on unary operators with semantics given by the so-called Allen relations (such as 'I is wholly before J' and 'I overlaps J'), binary operators have also been considered. In 1991, Venema [4] introduced an interval logic based on binary operators called C ('chop'), D, and T. For example, $\phi C\psi$ holds on an interval if it can be decomposed into two subintervals, the first satisfying ϕ and the second satisfying ψ:



D and T (read as 'done' and 'to come'?) are similar, but the 'decomposition' lies before (D) or after (T) the current interval:





These operators are reminiscent of Kamp's Until and Since, and are related to composition of binary relations.

The logic CDT has a number of potential applications and is also of theoretical interest. It is rather powerful: Venema proved that over intervals of linear time, it is exactly as expressive as the three-variable fragment of first-order logic with binary relation symbols. However, it can be axiomatised: Venema gave finite axiomatisations that are sound and complete for intervals over several common classes of linear flows of time. Note that these axiomatisations use a non-orthodox 'Burgess–Gabbay-style' inference rule.

Still, many questions about CDT and its fragments remained open, including decidability, and finite axiomatisability using only orthodox inference rules. Recently, in joint work with Angelo Montanari and Guido Sciavicco [2], it was shown that almost all fragments of CDT containing at least one operator are

1. undecidable for intervals over various classes of finite or infinite linear time flows,
2. for intervals over any class of linear flows of time containing $(\mathbb{Q}, <)$, not finitely axiomatisable with standard rules.

Undecidability can be shown by encoding known undecidable tiling problems and (for finite time flows) the Post correspondence problem or the problem of whether two given context-free grammars can produce a word in common. The non-finite axiomatisability proofs use ideas from the theory of Tarski's relation algebras — in particular, algebras related to ones devised by Monk [3] to show that for finite $n \geq 3$, the class of representable n-dimensional cylindric algebras is not finitely axiomatisable.

I will discuss this work in my talk. See [1] for a 'road map' of related systems.

References

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[3] J. D. Monk. Nonfinitizability of classes of representable cylindric algebras. *J. Symbolic Logic*, 34:331–343, 1969.

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