The persistence of statistical information

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Abstract

The frame problem was originally defined in the context of the situation calculus. The problem also manifests itself in more sophisticated temporal logics that can represent interval information. In interval based logics, we call it the persistence problem.

Almost all proposed solutions are only applicable in the context of the situation calculus. A few of the solutions can deal with a restricted class of interval based information (i.e., information that is true throughout an interval).

We consider a version of the persistence problem where we have statistical information about an interval. For example, given the average rainfall over the summer, what is the expected average rainfall during the fall? We describe a logic that can represent statistical interval based information, and then give a solution to the persistence problem for this kind of information.

1 Introduction

In previous work [6], we defined a category of temporal interval-based information called *definite integral information.* For example:

> I spent four hours in the museum. I walked a total of one hour, and paused briefly in front of most of the paintings.

and:

> Over the summer, it rains 20% of the time.

Both examples require representing uncertainty at the subinterval level. In the first example, walking is known to be true for some unknown number of subintervals of a four hour interval, and the total length of all the subintervals is one hour, but the starting points and duration of each subinterval is unknown. In the second example we know how long it rains, but cannot say if it is raining at any particular point.

We are also interested in the kinds of inferences that can be made from definite integral information. For example, from

> Over the summer, it rains 20% of the time.

we may wish to make an inference about rainfall for the month of July or at noon on June 20th. It is impossible to deduce point valued answers to these questions but the information imposes certain constraints on the answer. For example, given a three month summer, the rate of rainfall in July can vary between zero and 60%. As well, we can make certain reasonable inductive inferences. For example, a reasonable inference is that the rate of rainfall in July is 20%, and the probability of rain at noon on June 20 is 0.2. We call the problem of making reasonable inferences about subintervals and interior points for which we have definite integral information the problem of *interpolating definite integral information.* Other reasonable questions to ask are what will be the rainfall during the fall and at 5pm on November 1st? These questions involve making predictions about an interval or point that lies outside the interval for which we have definite integral information. We call this instance of the persistence problem the problem of *extrapolating definite integral information.*

We review previous work that defined definite integral information, presented a solution to the interpolation problem, and a limited solution to the extrapolation problem. We combine and generalize these two solutions to provide a solution to the extrapolation of definite integral information.

2 Definite integral information

The logic we use to represent definite integral information is called RGCH [4]. In RGCH, we represent point based qualitative temporal information with 0–1 valued functions. For example, "the book is on the table at time point 3", is represented as $on(book, table, 3) = 1$ and, "John is not running at time point t_9", is represented as $running(John, t_9) = 0$. In both cases, the last function parameter denotes a time point.

Our representation of point-based qualitative information facilitates the representation of qualitative interval-based information. We derive the duration of truth of temporal information over an interval by integrating the corresponding 0–1 valued function (a similar approach is used in [12]). For example, if the book is on the table between times 3 and 7, then:

$$\forall t \ . \ 3 < t < 7 \quad \rightarrow \quad on(book, table, t) = 1.$$

We integrate:

$$\int_3^7 on(book, table, t)\ dt \quad = \quad 4$$

to get the duration of time (in this case 4 time units) that the book is on the table over the interval (3,7). Another example of qualitative interval-based information is "John ran a while between times t_5 and t_8" which is true if and only if the integral of running over the interval (t_5, t_8) is non-zero:

$$\int_{t_5}^{t_8} running(John, t)\ dt \quad > \quad 0.$$

Recall from Section 1 the following example of definite integral information:

Over the summer, it rains 20% of the time.

This type of information is easily represented in RGCH:

$$\int_{jun21}^{sep20} raining(t)dt = 0.2 \times (sep20 - jun21).$$

In [6], we formally define definite integral information as:

Definition 2.1 (DEFINITE INTEGRAL INFORMATION)

Interval-based information is called *definite integral information* if it can be represented in terms of a definite integral:

$$\int_{t_1}^{t_2} \beta(t)dt = \alpha$$

where $0 \leq \alpha \leq (t_2 - t_1)$ and β is a 0–1 function possibly containing other parameters.

Note that the cases where $\alpha = 0$ and $\alpha = t_2 - t_1$ coincide with Shoham's [11] point-point-liquid proposition types, Allen's [1] properties, and McDermott's [9] facts. For example, the block is red over the interval (0,10) is written as:

$$\int_0^{10} colour(block, red, t)dt \ = \ 10$$

and the block is not green:

$$\int_0^{10} colour(block, green, t)dt \ = \ 0.$$

3 Interpolation

In this section, we review a solution to the interpolation problem that appears in [5, 6].

For interpolation, we wish to make plausible inferences that go beyond those deductively entailed by the definite integral information. These inferences hinge on an assumption of epistemological randomness (or, roughly, a principle of indifference); that is, taking all we know into account, each possible interpretation is assumed to be interchangeable (i.e., equally expected). From this we can infer an expected value for the function β at a particular point. We can compute this value from the given definite integral information by first determining an interval of points, all of which have the same expected value for β, and using definite integral information about the interval to determine the expected value at the particular point. This interval of points having the same expected value for β is a *maximally specific reference interval.*

Definition 3.1 (MAX. SPECIFIC REF. INTERVAL) The (possibly non-convex) interval R is a *maximally specific reference interval* for $\beta(t_0)$ if

1. $t_0 \in R$, and
2. β has the same expected value at every point in R.

We do not need to know the expected value of β at every point in an interval to determine if it is maximally specific—we only need to know the expected values are the same. If the information we have concerning two points is identical (with respect to β), then the expected value of β at the points is the same. So all we need do is verify that we have the same information for every point in the interval.

Definition 3.2 (INTERCHANGEABLE POINTS)

Suppose the only definite integral information our knowledge base contains concerning β is for the intervals: $R_1, R_2, \ldots, R_n$ (we refer to these as *explicit reference intervals* for β). We say two points t_1 and t_2

are *interchangeable* with respect to β, by which we mean that β has the same expected value at t_1 and t_2, if for every explicit reference interval R_i for β, we have $t_1 \in R_i$ iff $t_2 \in R_i$.

This says if the points fall within the same explicit reference intervals, then the expected values are (defined to be) the same.

Proposition 3.3 (Finding Max. Spec. Ref. Int.) If I is the intersection of the explicit reference intervals for β that contain the point t_0, and U is the union of the explicit reference intervals for β that *do not* contain the point t_0, then a maximally specific reference interval for $\beta(t_0)$ is $R = I - U$.

Once we have identified a maximally specific reference interval for $\beta(t_0)$, we can relate the expected value of $\beta(t_0)$ to the definite integral information concerning the reference interval. To do this, we use the following property of mathematical expectation:

Proposition 3.4 (Expectation of Sums)

For any interval I

$$E\left(\int_I \beta(t)dt\right) = \int_I E(\beta(t))dt.$$

Interval I need not be convex. For example, if $I = (x,y) \cup (u,v)$ and (x,y) and (u,v) are disjoint, then $\int_I = \int_x^y + \int_u^v$.

We use Proposition 3.4 and the fact that the points of the reference interval have the same expected value for β to derive the following direct inference rule:

Proposition 3.5 (Direct Inference Rule 1)

If R is a maximally specific reference interval for $\beta(t_0)$ then

$$E(\beta(t_0)) = E\left(\frac{\int_R \beta(t)dt}{|R|}\right).$$

The expected value of β at t_0 is equal to the average value of β over the interval R. Note that this follows trivially from the fact that β has the same value at every point in R since R is a maximally specific reference interval.

The following property relates subintervals to intervals:

Proposition 3.6 (Direct Inference Rule 2)

If S is a (possibly non-convex) subinterval of a maximally specific reference interval R for β then

$$E\left(\frac{\int_S \beta(t)dt}{|S|}\right) = E\left(\frac{\int_R \beta(t)dt}{|R|}\right).$$

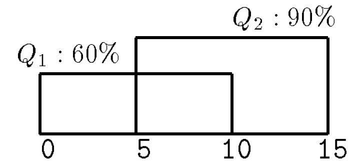

Figure 1: Overlapping reference intervals.

We can exploit this idea to permit many interesting inferences. For example, suppose we know (see Figure 1):

$$\int_0^{10} \beta(t)dt = 6 \ \wedge \ \int_5^{15} \beta(t)dt = 9.$$

Notice the explicit reference intervals $Q_1 = [0,10]$ and $Q_2 = [5,15]$ overlap. For $\beta(7)$, we have $Q_3 = Q_1 \cap Q_2 = [5,10]$ is a maximally specific reference interval. We derive from the definite integral information that

$$\int_5^{10} \beta(t)dt \in [4,5]$$

and then by Direct Inference Rule 1, we have that

$$E(\beta(7)) = E\left(\frac{\int_5^{10} \beta(t)dt}{5}\right) \in [\tfrac{4}{5}, 1].$$

In the limit, it can be used to obtain the probability that temporal information is true at a point. In [5], we show that given additional reasonable constraints, for example, that the temporal information was continuous, we can establish bounds for interior point and subinterval inferences.

4 Limited extrapolation

Almost all previous work on persistence hinges on McCarthy's common sense law of inertia (CSLI): things tend not to change. The obvious consequence of adopting this view is that it becomes reasonable to infer that the duration of non-change is arbitrarily long. For instance, a typical inference in systems that appeal to CSLI is that if a person is alive now, the person will remain alive (arbitrarily long) until something happens that results in the person's death.

CSLI seems like a reasonable assumption in microworlds where the duration of events is relatively short and the number of measurable forces acting upon any object is small. In the real world, it seems that a more natural assumption is that *nothing* persists forever, but instead persists according to a probability distribution. Inferences, such as a wallet dropped on a busy street tends to remain where it fell for a shorter duration than a wallet lost on a hunting trip, can be drawn in this framework. Unlike the CSLI approach, this inference is possible without knowing what happened to change the wallet's location.

Another issue is the direction of persistence: does information persist into the past as well as the future? For example, if we notice a building on the way home from work, then is it not just as reasonable to assume that the building was there the previous day as it is to assume it will be there the following day. Most approaches (e.g., [8, 10, 7, 3] to name only a few) restrict persistence to the forward direction only.

"Arbitrary-duration" persistence is too crude an approximation. Instead, temporal information persists for some finite period of time into the future and/or past. How long does information actually persist? In most cases, we cannot give a definitive answer to this question. For example, if John is currently alive, we cannot determine with certainty the time of his death (assuming we don't murder John). But, from actuarial tables, we can estimate his life expectancy. This is true of most temporal reasoning: although we don't know exactly how long something should persist, we can make reasonable statistical estimates.

In this section, we review a bi-directional limited-duration solution to the extrapolation problem that appears in [5]. Note that this is a solution to the case where the temporal information is true throughout the interval (i.e., only applicable to a subset of definite integral information).

We approximate the truth values of a piece of information over time with *regions of persistence*. For example, let running be true at time 100. Assume that a person usually runs for 30 minutes and may sometimes run for up to 50 minutes. We expect running to be true for some 30 minute interval that contains time 100. For simplicity, we assume 100 is located in the center of the interval. We then expect running to persist over the interval (100-15,100+15) and we expect running not to persist outside (100-25,100+25). Over the intervals $(100-25, 100-15)$ and $(100+15, 100+25)$ we are unwilling to predict whether running persists. The regions of persistence for running are shown in Figure 2.

The regions of persistence *(rop)* are represented by the relation:

$$rop(running, -25, -15, 15, 25).$$

The general form is:

$$rop(\beta,\ -t_1,\ -t_2,\ t_3,\ t_4)$$

where β is a point-based item of information. If β is true at time X and nothing is known to affect β, then β is expected to persist throughout the interval $(X-t_2, X+t_3)$, β may or may not persist over $(X-t_1, X-t_2)$ and $(X+t_3, X+t_4)$, and β is expected to be false before $X-t_1$ and after $X+t_4$. So, we predict β is true over $(X-t_2, X+t_3)$, we predict β is false before $(X-t_1)$ and after $(X+t_4)$ and otherwise we make no prediction. The general regions of persistence are shown in Figure 3. Note the regions are not necessarily symmetric around X. It may be that $t_2 \neq t_3$ and/or $t_1 \neq t_4$.

In this instance, we can give the *rop* relation a simple statistical semantics. Assume the duration of β is normally distributed with typical duration *(mean)* μ and typical variation *(variance)* σ^2 about that mean. Suppose we are satisfied to predict β remains true if the probability of β remaining true is greater than 50% and we wish to predict β is *false* if the probability is less than 5% (approximately) and otherwise we make no prediction. In this case, the relation $rop(\beta,\ -t_1,\ -t_2,\ t_3,\ t_4)$ holds if and only if $t_2+t_3=\mu$, and $t_1+t_4=\mu+2\sigma$. This statistical semantics subtly changes the meaning of persistence; rather than stating that we can be reasonably sure β persists over $(X-t_1, X+t_4)$ it states that we can be reasonably sure it does not persist *beyond* the interval. This is consistent with the usual interpretation of a continuous probability distribution function. For example, if *running* truly has a normal distribution, the duration of a run is less than the mean 50% of the time. Thus at time X we expect the run to end within t_3 minutes with probability 0.5.

The semantics of the *rop* relation may vary with the problem domain. For example, if we must be 95% sure that running is true to predict that it is true, we let $t_2+t_3=\mu-2\sigma$ and t_1+t_4 is unchanged.

As with the case of interpolation, this formalism can be extended to reason in other interesting settings. For example, suppose running is known to be true over the interval (80,120). In [5, 6] we show how to estimate the time at which running will end. Alternately, suppose instead the same parameters for *running*, but John has stopped running; we can compute an estimate for when John began his run. Thus the *rop* formalism, unlike others, is bidirectional.

5 General extrapolation

Given statistical information (i.e., definite integral information) about a time interval, we wish to make reasonable inferences about past or future intervals. For example, assume it rains 20% of the time during the summer. What is the rainfall during the months that immediately precede and follow the summer? For how many months following the summer should we make predictions about?

The rop solution from the previous section is not applicable in this case because the rainfall is not continuous throughout the summer. The temporal projection technique of Hanks and McDermott [7] is also inappropriate. We cannot determine from the statistical information if it was raining during the last day of summer. Even if we knew it was raining at that time, it does not make sense to allow raining to persist indefinitely. We have no information about

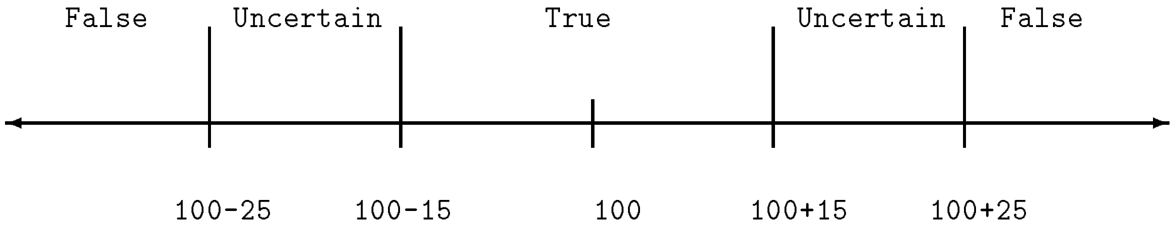

Figure 2: The regions of persistence for running

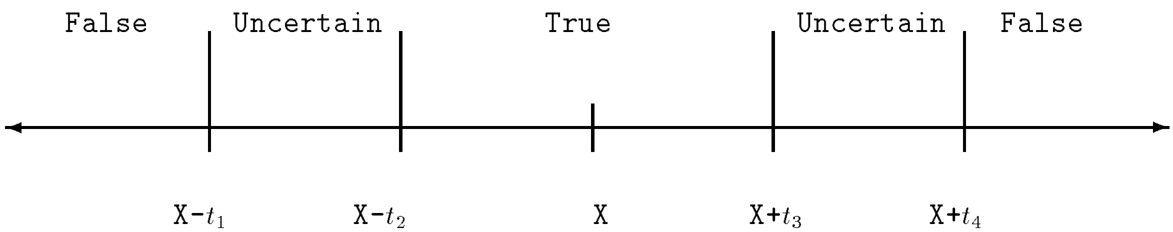

Figure 3: The general regions of persistence

actions or events that may affect raining. Finally, Dean and Kanazawa's probabilistic temporal projection [2] cannot be used as it requires the construction of a survivor function for *raining* based on many observations of raining changing from true to false. In our example, we have no observations of raining at particular points. No solution to the extrapolation problem appears in the temporal litterature.

Our proposed solution to the extrapolation problem is based on the interpolation formula:

$$E\left(\frac{\int_S \beta(t)dt}{|S|}\right) = E\left(\frac{\int_R \beta(t)dt}{|R|}\right)$$

that states we may infer that the proportion of points in R for which $\beta(t)$ is true is equal to the proportion of points in S for which $\beta(t)$ is true, provided that $S \subseteq R$ (and R is a maximally specific reference interval). To use this for extrapolation, we change the restrictions on regions R and S so that S is a region adjacent to R rather than a subinterval. We then use rop information to estimate the "amount" of persistence while the modified interpolation formula determines the "spread" of persistence.

Definition 5.1 (Extrapolation Assumption)

Let R be a convex region for which we have definite integral information about β, and S be a convex region adjacent to R for which we have no information about β. The *extrapolation assumption* is that:

$$E\left(\frac{\int_S \beta(t)dt}{|S|}\right) = E\left(\frac{\int_R \beta(t)dt}{|R|}\right) \quad (1)$$

where $\int_S \beta(t)dt$ is estimated using rop information.

This assumption is an implementation of bi-directional limited-duration persistence. The justification for it is that it captures the intuition that the extrapolated region S is like the adjacent region R.

For example, suppose running is known to be true over the interval $R = (80, 120)$. How long should running persist beyond time 120 (i.e., $S = (120, X)$)? Using formula (1), we have:

$$E\left(\frac{\int_{120}^{X} running(t)dt}{X - 120}\right) = E\left(\frac{\int_{80}^{120} running(t)dt}{40}\right).$$

Substituting for $\int_{80}^{120} running(t)dt = 40$, gives:

$$E\left(\frac{\int_{120}^{X} running(t)dt}{X - 120}\right) = 1.$$

Solving for $E(X)$ gives:

$$E(X) = E\left(\int_{120}^{X} running(t)dt\right) + 120 \quad (2)$$

Using rop information from the previous section, we can estimate the value of the integral on the right hand side. Assume that running is normally distributed with a mean of 30. Suppose we wish to predict running will persist if we are 50% sure it will continue, and we wish to predict it won't if we are 97.5% sure it won't and we make no prediction otherwise. We consider the expected remaining time for those runs longer than 40 minutes. By conventional methods, we find that about 50% of runs longer than 40 minutes last about another 4 minutes and 97.5% are completed within about 16 minutes. Thus, $E(\int_{120}^{X} running(t)dt) = 4$, and formula (2) reduces to:

$$E(X) = 124.$$

This holds only in the case where $\int_R \beta(t) = |R|$; i.e., continuous running. We now show that the interpolation rule can be combined with the *rop* rule to allow the extrapolation of general definite integral information.

Consider the case where *running* is not continuous, for example $\int_{60}^{120} running(t) = 40$ ($R = (60, 120)$, $S = (120, X)$). Using formula (1), we have:

$$E\left(\frac{\int_{120}^{X} running(t)dt}{X - 120}\right) = E\left(\frac{\int_{60}^{120} running(t)dt}{60}\right). \tag{3}$$

Using the rop information (taking the amount of running, rather than running interval length, to be normally distributed with a mean of 30) we have that $E\left(\int_{120}^{X} running(t)dt\right) = 4$, and formula (3) is reduced to:

$$E\left(\frac{4}{X - 120}\right) = E\left(\frac{40}{60}\right).$$

So $E(X) = 126$. Thus we expect that $\int_{120}^{126} running(t)dt = 4$. We can now also compute the expected value of running at a point in the interval (120,126) by doing interpolation.

6 Future work and Conclusion

The extrapolation assumption is only one of many possible assumptions for extrapolating our knowledge. One weakness of the assumption is that it does not take trends into account. Points near R may be expected to behave more like points inside R than points further away. In future work, we will study variations of the extrapolation assumption which incorporate such effects. Another weakeness is the assumption's restriction to convex intervals. We are currently investigating ways to extend the assumption to handle non-convex intervals as well.

The problem of extrapolation appears in the AI literature as the frame problem and in the statistical literature (as well as the uncertainty in AI literature) as the problem of forecasting. Previous work on *regions of persistence* and extrapolation from definite integral information considered the special case where the temporal information persisted throughout the entire interval for which integral information was available. This work consolidates previous work on interpolation and regions of persistence into a unified framework that allows bidirectional reasoning about persistence about events.

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