Characterizing Temporal Repetition

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Abstract

This paper is a preliminary investigation of temporal repetition. We review work in Artificial Intelligence of both formal and practical systems that deal with repetitive temporal objects (i.e. repetitive points and/or intervals). We analyze the essence of repetition, and present an extensive classification of types of repetition.

1 Introduction

This work presents results of our research on representing and reasoning about schedulable, repeated activities, specified using calendars. Examples of such activities include going to a specific class every Tuesday and Thursday during a semester, attending a seminar every first day of a month, and going to fitness classes every other day. This research provides for a valuable framework for scheduling systems, financial systems, process control systems and, in general, date-based systems, and any application where repetition is an important component. Being able to clearly specify the type of repetition encountered in the application can provide for valuable restrictions which can contribute towards a better description and understanding of the domain and more efficient algorithms. Very recently work has been done related to reasoning about repetition, [3, 4, 5, 9, 13, 14, 16, 17, 18, 19, 20]. However, to our knowledge no extensive taxonomy of repetition has been proposed in the literature. We believe that reasoning about repeated activities calls for a study and precise definition of the topological characteristics of the time during which the instances of the activities occur, and the repetition patterns they present. We argue that this kind of study helps comparing existing research in the area and provide insight for possible options not considered before in related research.

Repetition and cycles appear in nature, in everyday life. A myriad of examples can be mentioned: seasons keep recurring, it rains every now and then, the sun rises every day, one year follows another. Many human activities involve some form of repetition in time; waking up every day, eating, going to class every week, the beating of the heart, walking, etc. In all these examples different repetition patterns can be abstracted; some are based on temporal measures, for example a repetition occurring every minute; some repetition patterns are predictable, some not; some repetition may be dependant on the occurrence of other events, and so on. In this paper we analyze the essence of repetition, propose some basic terminology to refer to repetitive temporal objects and present a classification of types of repetition according to parameters determining these various types of repetition. Several notions are considered, some are extremely general, some are very specific. The granularity or precision with which the events in a repetition are expressed is also taken into consideration.

This study focuses on repetition occurring in time. Nonetheless, what we investigate is analogous to or can be extended to other forms of repetition. A straightforward parallel is that of spatial repetition, for example, verbal descriptions (a particular phrase for example) can be repeated in a novel to attain a certain structure in a text, etc. Two dimension or three dimension spatial repetition can also be conceived, for example a figure or pattern can be repeated or combined in many ways to obtain a whole. In this paper we concentrate on one dimensional repetition.

This paper is organized as follows. Next section presents related work. Section 3 discusses the level of ontological similarity between the different instances or *repeats* in a repetition. Section 4 defines the terminology which is used to describe the repetition patterns. Section 5 introduces the parameters according to which repetition is classified, and Section 6 combines these parameters to form some of the classes that belong to a general taxonomy. Section 7 compares proposals that have appeared in the literature under one of the parameters in our classification. We propose that more comparisons of the sort can be done. Finally Section 8 presents a summary of this work and discusses future research venues.

2 Related work

Non-convex intervals (intervals with "gaps") are employed in [12, 13] when referring to recurring periods. This work is an extension of [1], where *(convex) time intervals* are considered as the basic temporal objects, and 13 basic binary relations are defined, as for example "before", "overlaps", "during", etc. between them. [12] presents a taxonomy of binary relations between non-convex intervals. One type of these relations is generated by functors like "mostly", "always", "partially", etc. applied to the basic binary relations between convex intervals. For example, ([12], page 362) "I mostly meets J" indicates that for every component of the non-convex interval J there is a component of the non-convex interval I that meets it. Other binary relations between non-convex intervals include "disjoint from", "strictly intersects" and "bars" (or union) [12].

[18] also elaborate on the notions of non-convex interval relations defined in [12, 13]. They deal with qualitative relations between periodic events, considering for example a quantifier "always before". These relations are defined so that *correlated* subintervals relate with a basic interval binary relation. They define an algebra of relations between what they call *n-intervals*, a subclass of Ladkin's non-convex intervals. New qualitative relations between repetitive events can be derived. [15] proposes a generalization of non-convex intervals as an ordered, finite sequence of points in a linear order. Poessio and Brachman [19] are mainly concerned with the implementation of algorithms to detect overlapping repeated activities. This work relies on temporal constraint satisfaction results and algorithms [6]. Repeated activities have non-convex intervals as their temporal counterpart. Leban et al deal with repetition and time units [14]. This work relies on sequences of consecutive intervals combined into "collections". The collection representation makes use of "primitive collections" (essentially circular lists of integers), and two basic operators, *slicing* and *dicing*, which subdivide an interval and select a subinterval from another collection respectively. They propose to build new collections with these operators, and so represent calendar based repetitive intervals. [10] presents a first-order axiomatization of recurrence, based on Allen's interval calculus which allows atemporal entities to be associated "incidentally or repeatedly" with temporal intervals. To refer to repetition associated to calendar based dates, and also allowing for repetition to include gaps, this proposal relies on Leban et al's collections. [9] proposes a notation based on first order logic to represent recurrence of points and intervals. [20] presents a temporal formalism combining research done in several streams of temporal reasoning involving calendar based repetition. [2, 7, 8] include the formalization of time granularity. A brief classification of repeated events appears in [17]. "Repetitive events" are partitioned into "periodic" and "aperiodic", and aperiodic is partitioned as "random" and "stochastic". [16] also introduces the term of "near-periodic event".

In [3, 4, 5] we define an abstract hierarchical unit structure (a *calendar structure*) that expresses specific relations and properties among the units that compose it, for example *year*, *week*. Furthermore, we define a *decomposition* relation between time units and categorize it with two properties, *constancy* and *alignment*. Decomposition thus takes into account repetitive containment between time units. Based in this formalism, we represent specific intervals or time moments in the time line with *calendar expressions*. Examples include: $([(year, 1995), (month, M)]$ `where` $(M = 1$ `or` $M = 4))_{CE}$, i.e. January or April 1995; $\exists_3 D\ [(year, 1995), (week, W), (day, D)]_{CE}$, i.e. three days per week in 1995.

These expressions subsume the language of [14]. Their slicing can be paralleled with the use of composed time units and the intersection operator in our calendar expressions, and their dicing is similar to assigning values to the different time units in our calendar expressions. Uncertainty is covered by both formalisms (e.g. three days per week). Our calendar expressions additionally allow universal quantification, the "union" and "difference" operations, and displacement of the whole repetitive series by a duration.

3 Ontological similarity among the occurrences in a series

Each occurrence in a series of events or activities should have certain characteristics in common to be considered as a repeated series. In this respect, some philosophers distinguish "naked" repetition from "clothed" repetition [11]. The first kind involves the "unvarying repetition of the same". This case of repetition can be found for example in a cyclic numerical series, where identical numbers are repeated, as in 123123.... Naked repetition can also appear in a literary work, for example an identical phrase being repeated to stress a certain aspect of the novel. On the other hand, clothed repetition introduces difference at each of the occurrences. Arguably, repetition of events or activities in time correspond to "clothed" repetition, given that no event or activity ever gets repeated exactly; it is a matter highly dependant on the *level of abstraction*. At some level of abstraction repetition is naked.

Having said this, for the present study, we ignore these differences. Thus, for example, having lunch every day is considered a repeated activity even though quite likely it could include different meals, etc. Furthermore, we are concerned about repetition in terms of temporal objects, and thus in the previous example we look at the intervals during which the lunches occur and not at the quality of the lunches per se. Hence, when giving examples, we assume instances of repetition as "similar to other instances" when they have "certain" essence in common, without entering into the philosophy of the similarity aspect. Another aspect that we abstract in this classification is the nature of the objects that are repeated. We won't distinguish a repeated series of, for example, "activities" from a repeated series of "properties", i.e. we don't distinguish the different ontological nature of the occurrences or repeats. Hence "going to a meeting every Friday" and "we had sunny weather every day last week" are both valid examples. Other kind of examples simply consist of a graphical picture.

4 Basic terminology and definitions

In this section we formally define terminology which is used throughout this paper. As we discussed in the previous section, the objects we consider to repeat are "temporal objects":

Definition 1 (Temporal objects) *Temporal objects include time intervals and time points during* *which the repeated event, activity or property of interest occurs. Gaps are also conceived as temporal objects, representing the (temporal) separation between successive repeats in a series.*

Definition 2 (Temporal Series) *We define* temporal series *occurring within a* reference frame *as a sequence of temporal objects as follows:*

A temporal series $\mathcal{T}$ *of* $n \in N$ repeats *is a sequence of 2n elements:* $\mathcal{T} =< r_1, g_1, r_2, g_2, \ldots, r_n, g_n >$, *where* r_i *and* g_i *are temporal objects, the* i^{th} repeat *and the* i^{th} gap *respectively;* $i \in N, 1 \leq i \leq n$. *The* reference frame *is the interval that starts when the first repeat starts and finishes when the* n^{th} *gap finishes*[1].

Even though we stick to temporal series, these definitions are directly applicable to one dimension spatial series, and in fact, we omit the "temporal" adjective whenever this does not lead to ambiguities.

Definition 3 (Interval series) *An* interval series *is a temporal series whose repeats are intervals. The beginning point of a subsequent repeat has to be equal or after the beginning point of the previous one. Therefore two contiguous intervals relate with the relations in the set {before, meets, overlaps, finished-by, contains, starts, started by, equals} or a disjunction of the previous. These relations are part of the basic 13 interval relations in the Interval Algebra of [1].*

To illustrate interval series graphically, a repeat will be indicated with equal signs, `===` and a gap with hyphens `---`. Lengths will be proportional to the duration, and a vertical bar `|` will separate each couple repeat-gap. So, for example, in the series "last week it rained every day", contiguous (one day) repeats meet. `|==|==|==` .... In the series "they go to swim Mondays and Wednesdays", contiguous repeats are one before the next one. `|==--|==---------|==--` .... If a series contains intervals during which a device is manufactured, and if this device is manufactured in an assembly line, then these intervals overlap.

```
|=====---------|
    |=====---------|
        |=====---------|  ...
```

A series can be conceived of singers singing the same melody (over and over again) in a choir; hence (the time during which) each person sings one instance of the melody can be considered a repeat. If they all sing together, then all repeats are equal, ie., all singers start and finish at the same time each instance of the melody. Now consider a choir singing a canon; a canon is a contrapuntal musical composition in which various instruments or voices take up the same melody successively, before the previous one has finished [21]. Canon singing is thus an example where contiguous repeats overlap. As well, this can be seen as a number of combined or superimposed repetitive series (one for each singer) with the same repeat duration but "out of phase".

```
|=====|=====|=====|=====|
    |=====|=====|=====|=====|
        |=====|=====|=====|=====|  ...
```

Other variations of a choir singing the same melody in some counterpoint style can be conceived to illustrate other relations between repeats which intersect. Parallel processing and pipelining architectures provide for yet other (computer related) examples. In the series "reading and understanding a scientific paper" there can be several papers being "processed" at the same time, thus repeats may relate with any of the relations that the definition above includes.

Definition 4 (Point series) *A* point series *is a temporal series where the repeats are points.*

We consider *point* series to stress the fact that each repeat corresponds to a (durationless) time point, or to an event in time where the duration concept does not apply. An example of a point series is that of the "daily stock market values". Also, a point series could be *representing* an interval series, via the extreme points of each subinterval. Such is the case of "P-(generalized) intervals", where "P" is the set of even integers [15]. Furthermore, a point series can be extracted or derived from an interval series. For example we may refer to the series of "beginning times of an (interval) series of lecture classes".

Definition 5 (Duration of a temporal object) *A* point *has duration 0. A* convex interval *duration is the difference between the beginning and ending point of the interval. If the ending point of an interval is conventionally defined to be before the beginning point, the duration is conventionally defined negative. Durations are expressed with* time units. *The* precision *of a duration is the finest time unit the duration is expressed with.*

We denote the beginning and ending points of an interval i as $beg(i)$ and $end(i)$ respectively. We measure durations with time units. See [3, 4] for a formal definition of time units. Examples of time units include *year*, *week*, etc. A duration is for example "2 years, 10 days and 10 hours". In this case the precision is *hours.* But the same duration could be expressed in a *coarser* time unit. We consider durations rounded up to integers, for example "2 years, 10 days and 10 hours" becomes "2 years and 11 days". More precisely, we define a duration round up function as follows:

Definition 6 (Duration function) *The function* $\mathcal{D}_\pi$: *Temporal_Object* $\longrightarrow Z$ *associates a temporal object with an integer, which denotes its duration in the time unit or precision* π. *This function* $\mathcal{D}_\pi$ *maps an interval to a number whose absolute value is the lowest integer greater or equal than the "exact" (possibly fractional) duration absolute value. Duration values are negative when they correspond to temporal intervals where conventionally* $end(i)$ *is before* $beg(i)$.

[1] The term "reference frame" appears in [18], is referred to as "R-interval" in [17], "frame-time" in [20] and "reference-date" in [19].

We refer to this "round up to the next integer duration" as duration, unless otherwise specified. For example: $\mathcal{D}_{day}$("Meeting held on Tuesday from 10:00 to 12:30") = 1 *day*. (Notice that the "exact" duration in days would be $2.5/24 \cong 0.1$ days). The duration of this same example, with a different precision is $\mathcal{D}_{hour}$("Meeting held on Tuesday from 10:00 to 12:30") = 3 *hours*. The finest precision in the preceding example is *minutes*, or *half hours*, if such time unit existed in the time unit hierarchy; time units can be added to a time unit hierarchy to contemplate the required precision for the application [3]. Duration values can be negative; for example, $\mathcal{D}_{day}$("a gap whose exact duration is $-2.5/24 \cong -0.1$ days") = -1 day. This is further addressed in the following lemma:

Lemma 1 (Repeats duration and gaps duration) *Let $\mathcal{T} = < r_1, g_1, r_2, g_2, \ldots, r_n, g_n >$, $n, i \in N, 1 \leq i \leq n$, be a temporal series. If $\mathcal{T}$ is a* point *series, then, for any precision π, $1 \leq i \leq n$, $\mathcal{D}_\pi(r_i) = 0$ and $\mathcal{D}_\pi(g_i) > 0$. If $\mathcal{T}$ is an* interval *series, then, for any precision π, $1 \leq i \leq n$, $\mathcal{D}_\pi(r_i) > 0$ and*

$$\mathcal{D}_\pi(g_i) \begin{cases} > 0 & \text{if } r_i \text{ (before) } r_{i+1}, \\ = 0 & \text{if } r_i \text{ (meets) } r_{i+1}, \\ < 0 & \text{if } r_i \text{ (overlaps or finished-by or} \\ & \text{contains or equals or} \\ & \text{starts or started-by)) } r_{i+1}. \end{cases}$$

Specifically, $\mathcal{D}_\pi(g_i) = -|\mathcal{D}_\pi(r_{i+1})|$ if r_i (equals or starts or started-by) r_{i+1}.

Definition 7 (Repeats-precision, gaps-precision) *The* repeats-precision *is the precision used to measure* all *the repeats in a series. The* gaps-precision *(possibly different than the repeats-precision) is used to measure all gaps.*

For example, the series "someone's birthdays" can be measured with *day* as the repeats-precision and *year* as the gaps-precision. The intuitive idea of having a precision, is that the finest possible is the one that the natural language expression of the series provides information about. Hence, there is no information to express durations in *seconds* in the previous example. But this is not imposed as a restriction, and therefore the series "someone's birthdays" could be expressed in seconds, if the application so requires. Similarly, a precision of *millennium* wouldn't be very natural in the previous example, but similarly it could be so required. Some repeats may have an undefined duration, start and/or end (and therefore an undefined precision), as in "they go rafting every Spring". In this example a point series would be an adequate representation, where gaps would be of one *year* of duration. Since points are conventionally durationless, the distance between two successive points in a point series is equal to the duration of the gap between them, i.e. the difference between the two points. For example, "the daily stock market value" series has a distance of one day between each pair of successive points.

Definition 8 (Distances between two intervals) *We define 4 different distances between two intervals i_1 and i_2:*

1. Strict-distance *or* gap, $beg(i_2) - end(i_1)$.
2. B-distance, *i.e.* $beg(i_2) - beg(i_1)$
3. E-distance, $end(i_2) - end(i_1)$
4. Convex distance, *the period minimally covered by the two intervals, i.e.* $end(i_2) - beg(i_1)$.

For example consider the series "Monday of every week". The b-distance and e-distance between each successive subinterval are *1 week* (or *7 days*). The strict distance or gap is *6 days*. The convex distance is *8 days*. For a graphical picture of the different distances between two disjoint intervals see Figure 1. Both, the b-distance and e-distance include the duration of one of the two intervals involved. Furthermore, the strict distance is negative when the two intervals intersect. Our convex distance definition is inspired in the convexification operation defined in [13].

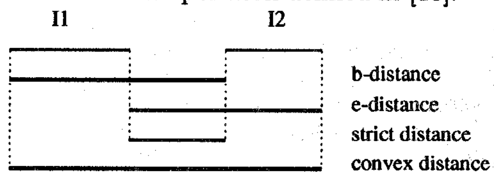

Figure 1: Graphical representation of four different distances between two disjoint intervals

Definition 9 (Convex closure) *The convex closure of successive repeats in an interval series is the minimal convex interval covering all those repeats.*

This definition generalizes the convex distance between two intervals just defined. A new series can be obtained by convexifying subseries in it. For example 'classes are held on Mondays, Wednesdays and Sundays every week" can be convexified to "Classes are held every week"; each subseries composed of three (day) intervals and the gaps between them was convexified into one interval.

5 Parameters of a temporal series: Towards a classification.

In this section we analyze *parameters* or classification axes that characterize temporal series. Series are organized in a lattice according to possible combinations of "values" each parameter takes. We distinguish five parameters to classify the repetition pattern:

1. Interval series qualitative structure.
2. Duration of repeats pattern.
3. Distance between repeats pattern.
4. Frequency of repeats per period of time.

Additionally, to present a broader taxonomy, we analyze repetition situations which stem from the application domain and not the temporal domain:

5. Temporally vs. event driven repetition.

Finally, we analyze how the reference frame (time during which the series occurs) can be specified:

6. Reference frame specification.

Arguably, these parameters analyze the patterns of the essential elements in a repetitive series. Hence, we propose that cyclic, probabilistically cyclic, random, and any other type of series results from the combination of possibilities of these parameters. Further sections define the possible values for the parameters, along with numerous examples.

Parameter - Interval series qualitative structure: Let $\mathcal{T} =< r_1, g_1, r_2, g_2, \ldots, r_n, g_n >$, $n, i \in N, 1 \leq i \leq n$, be a temporal series. There are $2^8 - 1 = 255$ possible values for this parameter, which reflects how r_i relates to r_{i+1} for all i: by definition, the possible relations are 8 basic *meets*, *before*, *overlaps*, *finished-by*, *contains*, *equals*, *starts*, *started-by* and any disjunction of them. One possible value (of the 255) is *all-before*, as in the series "artificial intelligence seminar meetings take place on Fridays". Another possible structure is *all-(meet or before)*, as in "sunny days in Vancouver", where the repeats-precision is expressed in days. Another possible structure is *all-overlaps* as in the canon singing example above. Notice that a new series can be obtained by convexifying subseries in it, and the qualitative structure (as well as other parameters) of the convexified series can vary from the original one. For example, `|=====|=----|=====|=----|` ..., with an all-(before or meets) structure can be convexified to `|===========|===========|` ..., with an all-meets structure. Similarly, series with intersecting consecutive repeats can be convexified to non-intersecting repeats. Clearly, the structure should reflect the detail according to the precision level desired for the application. We believe that it useful and arguably needed to specify precisely this parameter when defining a repetitive structure. This parameter qualitatively characterizes a repetitive series and thus provides information which importantly can restrict the usage and algorithms needed to deal with it.

Parameter - Repeats duration repetition pattern: We define 9 possible values for this parameter. Table 1 summarizes these values and how they are structured. Given this structure of possible values, it appears that they cover all possibilities within the parameter; the value is either completely, partially or not specified, etc.

The definition of these 9 values with examples follow. Let $\mathcal{T} =< r_1, g_1, r_2, g_2, \ldots, r_n, g_n >$, $n, i \in N, 1 \leq i \leq n$, be a temporal series.

1. **Constant durations:** For all $i, j, 1 \leq i, j \leq n$, $\mathcal{D}_\pi(r_i) = \mathcal{D}_\pi(r_j)$, for any precision π. Example:"series of experiments taking 10 min each".

2. **Time unit based constant durations:** For all i, j $1 \leq i, j \leq n$, $\mathcal{D}_\pi(r_i) = \mathcal{D}_\pi(r_j)$, but this constancy is apparent because of the time unit. Example: "series of observations of 1 month each". If the precision is *month*, then all the durations are the same, however, because months have different lengths in days, durations clearly are not "really" constant.

a. Fully specified
   - i. With a function
     1. constant durations
     2. time unit based constant durations
     3. cyclical equal durations
     4. cyclical functionally related durations
   - ii. By extension
     5. known a-priori durations

b. Partially specified
   - i. Approximate information
     6. bounded durations
     7. probabilistic durations
   - ii. Incomplete information
     8. incomplete duration information

c. Not specified
     9. not known duration repetition pattern

Table 1: Possible values for the *duration repetition pattern* parameters.

3. **Cyclical equal durations:** There exist $k > 0$ different values v_j, and these values repeat cyclically, i.e: For all $j \in N, 1 \leq j \leq k$, for all $t \in N, (j + tk) \leq n$, $\mathcal{D}_\pi(r_{j+tk}) = \mathcal{D}_\pi(r_j) = v_j$. k is the *repeats duration cycle*. Example: "series of experiments such that the even numbered experiments take 10 min., and the odd numbered experiments take 15 min." ($k = 2$).

4. **Cyclical functionally related durations:** There exist $k > 0$ different values v_j, and these values repeat cyclically modulo a function (not the identity), i.e: For all $j \in N, 1 \leq j \leq k$, for all $t \in N, (j + (t+1)k) \leq n$, $\mathcal{D}_\pi(r_j) = v_j$, $\mathcal{D}_\pi(r_{j+(t+1)k}) = f_j(\mathcal{D}_\pi(r_{j+tk}))$. Example: consider the series of intervals during which a pendulum moves between each (almost instantaneous) stop when it reaches one of the two highest positions. Since there is friction, the intervals during which the pendulum moves decrease monotonously (according to some function f), until the pendulum stops. So, $\mathcal{D}_\pi(r_{i+1}) = f(\mathcal{D}_\pi(r_i))$.

5. **Known a-priori durations:** Durations are known a priori but with no specific repetition pattern, i.e, there exist $n > 0$ different values v_i, and $\mathcal{D}_\pi(r_i) = v_i$, $1 \leq i \leq n$. Example: "in a seminar, attending a talk of 1 hour, then another during half hour and then another for 1 hour.

6. **Bounded durations:** For all $i, 1 \leq i \leq n$, lowest_value $\leq \mathcal{D}_\pi(r_i) \leq$ highest_value. Example: "series of songs which take from 2 min. to 5 min.".

7. **Probabilistic durations:** There exist $w > 1$ different values v_j, and the probability of a repeat having one of these values as a duration is known: For all $i, 1 \leq i \leq n$, $P(\mathcal{D}_\pi(r_i) = v_j) = p_j$, $\sum_{j=1}^{w} p_j = 1$. Example: "series of experiments

which take 10 min 40% of the repetitions and 15 min 60% of the repetitions".

8. **Incomplete duration information:** The duration information is not known for all n repeats, but there exists a proper *subseries* about which information regarding repeats duration is known with some of the previous criteria.

9. **Not known duration repetition pattern:** Each repeat has no known a priori duration, i.e., there is no specific pattern. Example: "reading and understanding scientific papers" (in principle, it is not possible to bound the time it takes to read different papers and understand them).

Parameter - Distance between repeats pattern: The formulation of the possibilities for a parameter analyzing the strict distance between repeats is analogous to the one for durations of repeats. In both cases we analyze durations of intervals, in one case we analyze repeat durations, in the other, distances between repeats durations. Therefore there are 9 possible values just as in the previous case, see Table 5. Definitions are analogous; for example, constant *repeats duration* was defined as $r_i = r_j$, for all i, j, $1 \leq i, j \leq n$, whereas a constant *strict distances* (or constant *gaps*) occurs when $g_i = g_j$, for all i, j, $1 \leq i, j \leq n$. Furthermore, in Section 4 we defined four kinds of distances between intervals. Possible values of a parameter analyzing these distances are also the same for the four different distances. Thus the same 9 possible values are considered for each distance. Obviously the resulting series may present very different patterns for the same kind of value. For example the following series has constant b-distance but not constant strict distance: `===--|===--|=----|==---` .... Notice that for those distances which include at least one of the repeats, there is an interconnection between the parameter analyzing the durations of the repeats and the parameter about distances between them. For example, if the durations are constant and the strict distances are cyclic, the b-distances are cyclic: `===---|===-|===---|===-` ....

Parameter - Frequency: The frequency of repetition over a certain period is related to the previous parameters in many cases and is independent in others. For example "Mondays and Tuesdays every week" has a frequency of two repeats per week. This corresponds to a *cyclical equal gaps* series, with cycle length equal to 2. The series "going to fitness classes three times a week" has uncertainty with respect to the duration of the repeats and the distance between them, thus the previous two parameters could be assigned the 9^{th} value, i.e. *not known duration repetition pattern.* However, clearly this series has a known *frequency* repetition pattern. An example of a bounded frequency series is "my heart beats between 150 and 174 times per minute after a series of cardio exercises", as is "at least three times a week". This parameter then can have the 9 possible values like in the previous parameters, adopted to frequencies.

Parameter - Temporally or event driven repetition: We analyze here event-driven repetition as opposed to temporally driven repetition. This parameter is orthogonal to the previous. Consider the example "change the oil in the car every 6000 km". It can be noticed that there is no temporal information that can predict when the change of oil will take place. However, there is a certain repetitive pattern, the event that the car covers 6000 km. In fact in this case we are comparing two series. The series which has an unknown temporal repetition pattern: "the car covers 6000 km" and the series "the oil in the car is changed", which is related to and inherits the randomness from the former. The two series are related in this example with the operator *always after* [12].

Parameter - Reference frame: The reference frame of a series is the interval minimally covering it, Definition 2. One way of looking at the reference frame is thus as one repeat series, the result of convexifying any whole series. Therefore the values considered for the previous parameters also suggest possibilities for classifying the reference frame. Since it particularly constitutes the interval during which there is a series of repeats, other values can specify it, such as total number of repeats. Table 2 summarizes the structure used to classify this parameter and following we provide the definition and examples.

Temporally determined reference frame
1. Fully specified
2. Partially specified
   i. Approximate information
   ii. Incomplete information
3. Not specified

Application determined reference frame
4. Repeat since
5. Repeat while
6. Repeat until

Table 2: Possible values for the *reference frame* parameter.

1. **Fully specified reference frame.** Reference frames are convex intervals, and as such can be fully specified by two values: the two extreme points or an extreme point and a duration. Since they also constitute the interval during which there is a series of repeats, they can also be defined as an extreme point and total number of repeats. Examples include a series taking place "from January 1^{st} 1996 until December 31^{st} 1996", or a series of "10 classes, with a certain repetition pattern, starting March 11^{th} 1996". Infinite intervals also are considered fully specified intervals, for example AD.

2. **Partially specified reference frames** include bounded or probabilistically known values. For example the starting point could be defined by

"one day in March 1^{st} - March 3^{rd} or "March 1^{st} 1996 with 10% probability, March 2^{nd} with 70% probability" and March 3^{rd} with 20% probability.

3. **Incomplete information** includes cases where only one value specifying the reference frame is known. For example only the duration is known or only the number of repeats, but no extreme point.

4-6. **Application determined.** Finally, the reference frame of a series, just as the repetition pattern, can be determined by the application domain. Hence the series can be defined **since**, **while** or **until** a condition holds. The options of specifying the reference frame by the *number of repeats*, *while* or *until* a condition holds correspond to the basic iterative constructs in an imperative programming language.

6 Combining the values of patterns to form a taxonomy

The combination of all the possible values of the parameters defined is currently under study. Maybe not all the parameters generate a realistic repetition pattern, and several combinations may result in the same class. We preview a machine case analysis as well, to obtain a complete taxonomy and further investigate repetition characteristics. However, we believe the characterization we present in this work already provides for a simple and complete classification of repetitive series, with respect to the suggested parameters. We present here only some classes that should belong to such taxonomy to exemplify the coverage of our classification.

An important class is that of **periodic** series, which we define as the series having constant b-distances (constant distances from beginning point to beginning point of subsequent repeats). However, given the parameters and values proposed we distinguish several variations of periodic series, a matter which is not distinguished in the literature surveyed.

In a **simple periodic** series, repeats durations are constant, gaps are constant (and, in case of an interval series it follows that the other distances are constant as well). Frequency is constant too. Furthermore, the structure of the series can be all-meet or all-before. For example "a TV show that takes place every day, only in one TV channel, at the same time, with a one hour duration" constitutes such series.

An **intersecting periodic** series would be a variation of the previous where repeats may intersect. The same example applies, but the same (pre-recorded) show would be forecasted in other TV channels as well, at intersecting time intervals.

A **time unit based periodic** series is "apparently" periodic, because of the time unit chosen to measure distances between repeats, such as in a repetition once a month.

A **periodic beginnings only** series is such that the duration of the repeat and the strict distances are not constant, though the b-distances are, for example: `==---|=----|=====|==---|===--| ...`

A **doubly cyclic** series includes the class whose durations are cyclic and distances are cyclic as well. The general cycle is the minimum common multiple of the two cycles, the repeats cycle and the gaps cycle. For example $\mathcal{D}_\pi(r_1) = 2$, $\mathcal{D}_\pi(r_2) = 3$, and then it repeats (i.e repeat cycle is 2). Likewise, $\mathcal{D}_\pi(g_1) = 3$, $\mathcal{D}_\pi(g_2) = 1$, $\mathcal{D}_\pi(g_3) = 2$ and then it repeats (gaps cycle is 3). The general cycle is 6. Graphically: `==---|===-|==--|===---|==-|===--|==--- ....`

A **cyclic series** (not "doubly") could be cyclic with respect to distances between repeats, but have some other pattern of repetition with respect to repeats duration. For example "One hour classes taking place Monday and Tuesday every week".

A **triple stochastic** series is the class with probabilistic repeats duration, gaps duration and frequency.

7 Classification of different proposals under the present parameters

It is possible to classify and compare the different approaches that have appeared in the literature according to the parameters suggested here, and therefore be able to compare them. This kind of classification should also help as a guide to define or review languages and algorithms dealing with repetition. We next analyze several approaches that have recently been reported according to the qualitative structure parameter. A complete comparison considering all parameters is out of the scope of this paper.

In the literature surveyed, when the subsequent intervals have a gap between them, the series is considered as *one* "non-convex interval" [12, 13]. Thus it corresponds to interval series where subsequent repeats are one before the next one. A series with either disjoint or meeting contiguous subintervals is referred to as a "general non-convex interval" in [12], and can be paralleled with an "order 1 collection of intervals" [14] and with the temporal counterpart of "repeated activities" [19]. Collections of greater order also constitute interval series of either disjoint or meeting repeats. A "partition of intervals" [18] is a series of meeting subintervals. Similarly, meeting or disjoint repeats are contemplated in "sequences of occurrences within an R-interval" [17]. Expressions representing "periodic events" [20] have a calendar-based repetition subexpression which rely on Leban et al's collections. Repetition as axiomatized by Koomen [10] allows for meeting contiguous intervals only, and to express calendar based repetition and allow gaps between repeats this axiomatization relies on Leban's collections. Gabbay's language to represent temporal repetition includes interval series and point series, where the distance between subsequent beginning points of the repeats is always constant [9]. Our calendar expressions represent repetitive series where successive repeats are disjoint or meet [4]. Additionally, these expressions can be combined with "set operations": union, intersection and difference or displaced by a duration. It is therefore possible to express, for example, the union of series which would result in overlapping repeats. We are not aware of repetitive temporal series being defined with overlapping nor any kind of intersecting repeats in the AI and temporal reasoning literature.

8 Summary

In this paper we have analyzed the essence of temporal repetition, and presented a classification according to parameters determining the various types of repetition. We believe that reasoning about repeated activities calls for such a study. Several notions have been considered, some are extremely general, some are very specific. The analysis of all possible combinations of the parameter values is under study. We are also investigating how operations on repetitive series interact with the values the parameters take.

We believe that the set of parameters proposed involve the essential components in a repetitive series: the repeats, the distances between them and the reference frame. Different applications could lead to the consideration of additional parameters, or a more detailed partitioning in the values the ones proposed here take, however we believe this characterization provides for a simple and complete classification or scheme of classification of repetitive series, with respect to the proposed parameters. Finally, we find that this kind of classification can help comparing different proposals addressing problems where the domain involves repetitive structures, and guide new developments of both declarative languages and implementations.

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