Contextual Logic of Change and Contextual Proofs

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Abstract

In this paper we discuss CLOC, an alternative approach to reasoning about action and change, which is neither based in the Situation Calculus nor in Circumscription. In our approach, inspired by the Possible Worlds Approach, change is modeled as changing the theory that models the world. We extended First Order Logic, in order to be able to represent elementary and structured changes, and define predicates relating propositions that hold in different situations. We present one rule of inference of this new logic, $\mathcal{EX}_P\mu$, that is used for concluding what propositions hold after the execution of change.

Furthermore, we argue that traditional proof systems, e.g. Fitch's and Lemmon's proof systems, are not suitable for this logic, and propose a proof system where we can distinguish between propositions that hold before and after change has taken place.

1. Introduction

Situation Calculus [15] is a formalism for reasoning about action and change. In this formalism, propositions refer situations, thus creating the need to reformulate the propositions after changing the situation. This problem is strongly related to the frame problem, the problem of deciding which propositions hold after change has occurred.

One of the approaches used to try to solve the frame problem is Circumscription [16, 17, 12], a method for performing common-sense reasoning. The circumscription of predicate relative to a theory minimizes the extent of that predicate. McCarthy [17] proposes the use of simple abnormality theories, whose general facts are described using the predicate Ab, for abnormal. The circumscription of predicate Ab, relative to the theory, with all predicates variable, would presumably be enough to model commonsense reasoning and solve the frame problem. However, as McCarthy realized, this was not the case.

Hanks and McDermott [5] presented an example, the Yale Shooting Problem, which shows that the solutions presented by the use of the circumscription method do not correspond to the intended solutions. Several approaches have been proposed to expand and correct the simple abnormality theories approach, including [7, 10, 1, 13, 6], to name a few. However, the specificity of these approaches still raises some problems. Lifschitz, for example, mentions the difficulties in dealing with the multitude of choices that these methods allow, claiming that those are "the main reason why circumscription is not applied today in knowledge representation as widely as would be expected" [13]. He proposes the use of nested abnormality theories instead, which introduces blocks and the embedding of blocks, about which he writes "...each block can be viewed as a group of axioms that describes a certain collection of predicates and functions, and the embedding of blocks reflects the dependence of these descriptions on each other" [13]. This dependence relation among propositions goes against our understanding of McCarthy's ideas, namely that "the general facts of common sense are described by a collection of sentences that are not oriented in advance to particular problems" [17, Section 11], which we subscribe.

Another approach to the problem of formalizing change is what Sandewall and Shoham [21] named the meta-level approach. In this approach, propositions do not include the identification of situations and change is modeled as changing the set of propositions that holds in situations. Ginsberg and Smith's Possible Worlds Approach [4] is based in these ideas. However, this approach has been criticized by Winslett [24], who proposes the Possible Models Approach, according to which the models, rather than the formulas, are to be changed. We argue, in a forthcoming paper, that change may be modeled using syntactic approaches.

One example of the meta-level approach is Mutation Logic (ML) [19, 20], which is a logic where execution of change is modeled as changing the set of propositions that hold before change is executed into a new set. Change is represented by a proposition describing its pre-conditions and post-conditions. However, this logic has limitations affecting its expressive power, in particular, there is no way to express side-effects of change and the language of the logic does not allow the representation of plans.

The approach we are developing, Contextual Logic of

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Change (CLOC), is an extension of First Order Logic (FOL), augmenting the expressiveness of LM and using some ideas from the Possible World Approach.

In this approach, situations are modeled as belief spaces of contexts (a *context* is a set of hypotheses and a *belief space* of a context is the set of derivable propositions from the context) and are not represented explicitly by any object in the language. A consequence of this representation is the limitation of expressiveness of the language, since we are no longer able to represent propositions about situations.

Another consequence, more important if we want to reason about change, is that propositions are relative to the situation being described and not to other situations. This may raise some problems when modeling reasoning about change, because this involves reasoning about propositions that hold in different situations.

Change, in this approach, is modeled as the change of the set of propositions that describe a situation to another set of propositions (in the framework of predicate calculus, we may think of changing the value of some predicates as we change the situation being represented). When using this representation of situations, however, change can't be represented using FOL without expanding it's expressive power, since change involves propositions that hold in different situations.

We expanded FOL: i) by introducing new terms that represent change; ii) by defining new functions on these terms that are used to represent structured changes; iii) by introducing new predicates relating propositions in different situations, for example $\mathcal{X}_P$, which is applied to a term representing change, and means that it has been executed; and iv) adding inference rules to allow the derivation of the effects of change.

The application of these rules of inference, that mention proposition in different situations, raises some problems if traditional proof styles, discussed in Section 2, are used. The problem is that traditional proof styles are appropriate for reasoning within the same situation, but are not for reasoning when change occurs and situations are not reifyed. When dealing with change, even if a proof was made for a proposition in the situation before change was executed, the proposition might not hold in the context that results from change and, therefore, should not be used in any further derivation. Contextual proofs are different from traditional proofs because they enable to cross lines out (we call this *strokes*), to distinguish between proof lines that may and may not be used for deriving new propositions after a change was executed.

We present, in Section 2, two classical proof styles, argue that they are not appropriate to represent proofs for CLOC, and then propose the contextual proof, a generalization of Lemmon's proof system [9]. In Section 3, we present one of CLOC's inference rule that allows the derivation of the consequences of change, and, in Section 4, we give two examples of this new inference rule using the new proof style.

1	$a \to b$			Hyp	$\{a \to b\}$
2		$b \to c$		Hyp	$\{a \to b, b \to c\}$
3			a	Hyp	$\{a \to b, b \to c, a\}$
4			$a \to b$	$1, Reit$	$\{a \to b, b \to c, a\}$
5			b	$3, 4, MP$	$\{a \to b, b \to c, c\}$
6			$b \to c$	$2, Reit$	$\{a \to b, b \to c, a\}$
7			c	$5, 6, MP$	$\{a \to b, b \to c, a\}$
8		$a \to c$		$3-7, \to I$	$\{a \to b, b \to c\}$
9	$(b \to c) \to (a \to c)$			$2-8, \to I$	$\{a \to b\}$

Figure 1: Example of Fitch's proof.

2. Proof systems

In this section we discuss two traditional proof styles, Fitch's proof and Lemmon's proof, and argue that they are not appropriate for dealing with change, when situations are not reifyed. A new proof system is then proposed.

2.1. Fitch's proof

According to Fitch ([3], §5.5.), a proof is a sequence of items satisfying at least one of the following conditions: (1) it is an axiom of the system; (2) it is a direct consequence of preceding items of the sequence; (3) it is a hypothesis of the sequence; or (4) it is a subordinate proof.

A proof is said to be *subordinate* to another if it is an item of the other proof. In a subordinate proof, we can reiterate items from the subordinating proof into a subordinate proof, but we cannot reiterate the other way round, from the subordinate to it's subordinating proof.

In Figure 1, we present a proof that illustrates Fitch's proof representation. Each line has: a unique line number; a wff; the indication of the rule of inference used in this line (we changed the names of the inference rules for the sake of uniformization); and the context, the set of hypotheses that were raised and were not dropped in the proof up to this line (this is different from Fitch's presentation, since he doesn't present the context in his proofs).

In this proof, we introduce hypothesis $a \to b$ (Line 1), use a subordinate proof to derive $a \to c$ based on $b \to c$ (Lines 2–8), and use Implication Introduction to deduce $(b \to c) \to (a \to c)$ (Line 9). The subordinate proof in Lines 2–8 uses another subordinate proof (Lines 3–7).

The main point in this example is to illustrate the change of the contexts when entering and leaving subor-

{1}	1	$a \to b$	Hyp
{2}	2	$b \to c$	Hyp
{3}	3	a	Hyp
{1,3}	4	b	$3, 1, MP$
{1,2,3}	5	c	$4, 2, MP$
{1,2}	6	$a \to c$	$3, 5, \to I$
{1}	7	$(b \to c) \to (a \to c)$	$2, 6, \to I$

Figure 2: Example of Lemmon's proof.

dinate proofs: in Line 2, the context is augmented with the hypothesis raised in the subordinate proof and becomes $\{a \to b, b \to c\}$; in Line 3, the context is changed to $\{a \to b, b \to c, a\}$. When leaving the subordinate proof, the context is reduced and the hypothesis raised in the subordinate proof can no longer be used. In Line 8, the hypothesis a raised in the subordinate proof in Lines 3–7 is dropped, and the context is now $\{a \to b, b \to c\}$. In Line 9, the same happens to the hypothesis raised in subordinate proof in Lines 2–8 and the context is, therefore, $\{a \to b\}$.

2.2. Lemmon's proof

Lemmon's definition of a proof is slightly different from Fitch's definition: he doesn't include neither subordinate proofs nor axioms.

According to Lemmon [9], a proof is a sequence of lines, where each line may represent: (1) a hypothesis; or (2) a conclusion drawn from the application of inference rules.

In Figure 2, we show the proof corresponding to the example of last section, but now using Lemmon's proof representation. In each line, we represent: the set of hypotheses that supports the wff in the line (notice that it is not the context of the proof); a unique line number; a wff; and an indication of the rule of inference used to generate the line.

In this proof, and beyond the line content differences between Fitch's and Lemmon's proofs, such as supports (omitted in Fitch's proof) and contexts (omitted in Lemmon's proof), the main difference is that Lemmon does not use subordinate proofs. Therefore, it makes no sense to reiterate proof lines and, thus, there are no lines corresponding to Lines 4 and 6 of the previous example.

When using Lemmon's proof style, the structuring capabilities introduced by subordinate proofs in Fitch's proofs are lost, since this subordination scheme is not used by Lemmon, and it makes no sense to describe the evolution of context in Lemmon's proofs.

2.3. Discussion

When using traditional proofs styles, there is no way to express that some propositions should no longer be derivable after performing change.

Consider Fitch's proof style. Given a proof, we can augment it raising the hypothesis of executing a given change and conclude its consequences (the representation of this proposition and the rule of inference that describes this derivation are discussed in Section 3). One possible consequence is to remove a set of propositions from the belief space, but, unfortunately, there is no general way to represent it. Even though subordinate proofs allow hypotheses to be raised and dropped, it is not general enough, because it imposes an order according to which hypotheses are dropped (we cannot drop the hypotheses in subordinating proofs without dropping the hypotheses in its subordinate proofs).

Lemmon's proof isn't better, because there is no way to differentiate propositions in different situations, using the information available in the proof. There is no way to conclude that a true proposition in a situation is no longer true after change has occurred, like our next example shows.

Suppose, for example, that the proposition that w is a green wall is represented as hypothesis $Green(w)$ and that there is a proposition to represent that w was painted white (we will discuss the representation of this proposition in Section 3). From these two propositions, we want to derive, using appropriate inference rules, that the wall is white, $White(w)$. However, using Lemmon's proof style, the conjunction $Green(w) \wedge White(w)$ can be derived, meaning that the wall is both green and white, a proposition that makes no sense, because there is no way to say that the hypotheses that underlies $Green(w)$ don't hold any longer.

The problem is that lines, in Lemmon's proof, are never reduced and the wff of a line may always be used by an inference rule. A proof is "monotonic" in the sense that the set of conclusions always grows as the proof is augmented.

We now propose the contextual proof, which distinguishes derivations based on hypotheses in a given description of a situations from the other derivations.

2.4. Contextual proof

The reason why we have developed the new proof system is that using Lemmon's proof system with CLOC would led to confusions, namely because there is no way to distinguish between propositions that hold in different situations.

The idea of contextual proof is to generalize Lemmon's proof, allowing strokes to be made in some lines, meaning that the wff in that line should not be used in any future derivation, at least until all line strokes are removed. We

{1}	1	$a \to b$	Hyp	
{2}	2	$b \to c$	Hyp	
~~{3}~~	~~3~~	~~a~~	~~Hyp~~	6
~~{1,3}~~	~~4~~	~~b~~	~~$3,1,MP$~~	6
~~{1,2,3}~~	~~5~~	~~c~~	~~$4,2,MP$~~	6
{1,2}	6	$a \to c$	$3,5,\to I$	

Figure 3: Deduction of $a \to c$.

{1}	1	$a \to b$	Hyp	
~~{2}~~	~~2~~	~~$b \to c$~~	~~Hyp~~	7
~~{3}~~	~~3~~	~~a~~	~~Hyp~~	6,7
~~{1,3}~~	~~4~~	~~b~~	~~$3,1,MP$~~	6,7
~~{1,2,3}~~	~~5~~	~~c~~	~~$4,2,MP$~~	6,7
~~{1,2}~~	~~6~~	~~$a \to c$~~	~~$3,5,\to I$~~	7
{1}	7	$(b \to c) \to (a \to c)$	$2,6,\to I$	

Figure 4: Deduction of $(b \to c) \to (a \to c)$.

also allow axioms to be present in a proof, as in Fitch's proof.

We base our proof definition in Lemmon's proof definition. A proof is a sequence of lines, where each line represents: (1) a hypothesis; (2) a conclusion drawn from the application of inference rules; or (3) an axiom. Each line may or may not be struck out.

Based on the representation of Lemmon's proofs, we add to the representation of a line a fifth item, containing a sequence of pairs, where the first item of each pair is the *justification of the stroke* and corresponds the number of the line that caused the stroke, and the second item of the pair, if it exists, is the *justification of stroke erasing*, represented between brackets, and corresponds to the line that caused the removal of the stroke (stroke erasing will not be illustrated in this paper).

As an example of contextual proof, we present an example using an extension of Lemmon's rule of Implication Introduction. According to this rule, lines corresponding to the subordinate proof are struck through and cannot to be further used, at least until the strokes are removed. As a matter of fact, using contextual proof, presumes changing all inference rules so that they consider only the lines that are not struck through, instead of considering all lines.

In Figure 3, we derive the implication $a \to c$ based on hypotheses $a \to b$ and $b \to c$. Lines 3–5 are struck through and the justification of the strokes is Line 6. We use indentation to emphasize the structure of the proof. This proof is continued, in Figure 4, with the derivation of $(b \to c) \to (a \to c)$ in Line 7, based in $a \to b$, and Lines 2–6 are struck through by this line (the lines that were struck and are being struck again are represented with double strokes). Again, none of these lines may be used in any further derivations.

Even though we don't present the context analysis like the one when presenting Fitch's proof example, it is interesting to notice that the context evolution would be the same as Fitch's proof example evolution, if we ignore the lines in Fitch's proof example where the reiteration inference rule was used.

3. Contextual logic of change

In this section, we discuss the contextual logic of change, the logic we propose to handle change, and present one of its rules of inference.

CLOC expands First Order Logic by: i) introducing terms that represent change, the *mutations*; ii) introducing operations for building structured changes (also called plans); iii) introducing new predicates relating propositions in different situations; and iv) defining inference rules that handle change.

A *mutation* corresponds to an atomic change. A mutation μ_i is described by two sets of propositions, the pre-conditions, $\Psi(\mu_i)$, and post-conditions, $\Phi(\mu_i)$, and is written as $\mu(\Psi(\mu_i); \Phi(\mu_i))$. The set $\Psi(\mu_i) - \Phi(\mu_i)$ contains propositions that are no longer known to be true after the execution of change. For example, painting white a green wall w may be represented by the mutation $\mu(\{Green(w)\}; \{White(w)\})$.

We define *changes* as generalizations of mutations. There are two kinds of changes: *elementary* that corresponds to mutations, and *structured changes*, that corresponds to the composition of mutations and plans. We consider two types of structured changes: (1) sequential execution plans and (2) unordered execution plans. Given changes Π_1 and Π_2, then $\Pi_1 - \Pi_2$ represent the sequential execution of Π_1 and Π_2 and $\prec \Pi_1, \Pi_2 \succ$ represent the unordered execution of Π_1 and Π_2.

Since mutations are terms in CLOC, propositions about change may now be written, using, for example, the predicate $\mathcal{X}_P$, which means that the term it applies to has been executed in the past. When considering only one change, the proposition "the mutation μ has been executed" is written as $\mathcal{X}_P(\mu)$.

We now present a sketch of one rule of inference of CLOC, *Elimination of Execution of a Mutation in the Past*, $E\mathcal{X}_P\mu$, that governs changes in contexts due to the execution of an elementary change. We will be interested, in the rest of this paper, in the theories that Lifschitz [11] called "theories of a single action", in which only one elementary change is considered. The rule that handles the execution of a plan, instead of a mutation, will be discussed elsewhere. In order to present the rule $E\mathcal{X}_P\mu$, we define the *support* of a derivation as the set of hypotheses that are used in the derivation.

{1}	1	$Green(w)$	Hyp
{2}	2	$\mathcal{X}_P(\mu(\{Green(w)\};\{White(w)\}))$	Hyp

Figure 5: Proof in the painting example before the application of $E\mathcal{X}_P\mu$.

~~{1}~~	~~1~~	~~$Green(w)$~~	~~Hyp~~	3
{2}	2	$\mathcal{X}_P(\mu(\{Green(w)\};\{White(w)\}))$	Hyp	
{1,2}	3	$White(w)$	$1, 2, E\mathcal{X}_P\mu$	

Figure 6: Proof in the painting example after the application of $E\mathcal{X}_P\mu$.

Consider a context, that represents the initial situation, and a mutation μ, that represents a change from the initial situation into another situation. If there are no hypotheses referring the predicate $\mathcal{X}_P$ in the context, we may introduce hypothesis $\mathcal{X}_P(\mu)$, and if from the hypotheses in the context there is a derivation of the conjunction of pre-conditions of μ, $\bigwedge(\Psi(\mu))$, then the execution of rule $E\mathcal{X}_P\mu$ has the following effects: i) it removes a set of hypotheses from the context such that both the pre-conditions in $\Psi(\mu)$ that are not post-conditions in $\Phi(\mu)$, and negations of the post-conditions, are no longer derivable, and the set of removed hypotheses is minimal relative to set inclusion; and ii) it adds the conjunction of the post-conditions, $\bigwedge(\Phi(\mu))$, to the belief space, with support $\{\mathcal{X}_P(\mu)\} \cup Sup(\bigwedge(\Psi(\mu)))$, where $Sup(\bigwedge(\Psi(\mu)))$ is the support of the derivation of $\bigwedge(\Psi(\mu))$ (for clarity, in step i, we are ignoring propositions such as "the mutation μ *will be* executed", which should also be removed from the belief space after executing change μ).

The contextual proofs are appropriate for being used with this logic since some rules of inference in this logic presume the change of contexts, with removing and adding hypotheses to the original context.

4. Examples

In this section, we present two examples of CLOC using contextual proof. In the first example, we formalize the example presented earlier of painting walls, and in the second example we discuss the ramification problem.

4.1. The wall painting example

We now present an example of a deduction in CLOC, using contextual proof.

We now present the solution of the problem raised in Section 2.3, using CLOC. In Figure 5, we raise hypotheses $Green(w)$ and $\mathcal{X}_P(\mu(\{Green(w)\};\ \{White(w)\}))$.

{1}	1	$\forall(r)(Saucer(r) \leftrightarrow Cup(r))$	Hyp
{2}	2	$Saucer(r_1)$	Hyp
{1}	3	$Saucer(r_1) \leftrightarrow Cup(r_1)$	$1, \forall E$
{1,2}	4	$Cup(r_1)$	$2, 3, \leftrightarrow E$
{5}	5	$\mathcal{X}_P(\mu(\{Saucer(r_1)\};\ \{Saucer(r_2)\}))$	Hyp

Figure 7: Proof in the saucer/cup example before the application of $E\mathcal{X}_P\mu$.

{1}	1	$\forall(r)(Saucer(r) \leftrightarrow Cup(r))$	Hyp	
~~{2}~~	~~2~~	~~$Saucer(r_1)$~~	~~Hyp~~	6
{1}	3	$Saucer(r_1) \leftrightarrow Cup(r_1)$	$1, \forall E$	
~~{1,2}~~	~~4~~	~~$Cup(r_1)$~~	~~$2, 3, \leftrightarrow E$~~	6
{5}	5	$\mathcal{X}_P(\mu(\{Saucer(r_1)\};\ \{Saucer(r_2)\}))$	Hyp	
{2,5}	6	$Saucer(r_2)$	$2, 5, E\mathcal{X}_P\mu$	

Figure 8: Proof in the saucer/cup example before the application of $E\mathcal{X}_P\mu$.

In Figure 6, we continue this proof eliminating the execution of change $\mu(\{Green(w)\};\{White(w)\})$, and we change the proof's context by removing the pre-condition of the mutations, $Green(w)$, since it does not belong to the set of the post-conditions, justifying the strokes with Line 3, and deriving $White(w)$ with support $\{Green(w), \mathcal{X}_P(\mu(\{Green(w)\};\ \{White(w)\}))\}$, that is, the wall w is white because in the initial situation it was green and it has been painted white.

4.2. The saucer and cup example

In this example, similar to examples presented in [18, 1], we consider two objects, a saucer and a cup, and the cup is on the saucer. In the initial situation, both objects are in room r_1. Changing the saucer to room r_2 results in changing the position of both the saucer and the cup.

In Figure 7, we present the proof before rule $E\mathcal{X}_P\mu$ was applied, and in Figure 8 we present the proof after the rule was applied.

It is interesting to notice that the results from this approach depends strongly on the set of hypotheses that was chosen to model initial situation. According to CLOC, there is only one description of the world resulting from the execution of change. If we considered the hypotheses $Cup(r_1)$ instead of $Saucer(r_1)$, the set of hypotheses raised would describe the same belief space, but there would be two possible situations resulting from the execution of change, corresponding to moving both the saucer and the cup, or only moving the cup. This happens because, if we considered hypothesis $Cup(r_1)$ instead of

$Saucer(r_1)$, then the change would have not been completely specified.

It is quite common to have different contexts resulting from the execution of change. This happens when change is not completely specified for the given set of hypotheses and different choices of hypotheses to be removed from a context must be considered.

Baker [1] discusses a similar scenario and concludes that his approach doesn't solve the ramification problem because both situations are acceptable by his method and he claims that the two alternatives should be considered. However, CLOC only considers one of the alternatives, because the initial situation is modeled as the belief space generated from a set of propositions that were considered relevant, and those propositions should also be mentioned in the definition of change if they are to be removed from the belief space.

Our example shows that the ramification problem, when understood as deriving the representation of world resulting from applying a certain change to a situation, cannot be solved when there are alternative derivations of the propositions that are to be removed from the belief space.

5. Concluding remarks

We present an alternative approach to Situation Calculus [15] and Circumscription [16, 17, 12] for modeling change, the CLOC approach, which is a logic based on the Possible Worlds Approach [4].

We argue that the lack of expressiveness resulting from not reifying situations may result in a solution to the frame problem (as presented by McCarthy and Hayes, in [15]). Therefore, the resulting logic is expected to model appropriately reasoning about action and change.

In this paper, we presented part of the proof theory of this logic. The semantic theory has not yet been developed, and, therefore, the semantics is still informal.

According to the proof theory, execution is modeled by the change of contexts. After executing change, some propositions should no longer be derived. We discuss Fitch's and Lemmon's proof systems [3, 9], and argued that they are not adequate to be used when representing some proofs in CLOC. We proposed, as an alternative, the contextual proof system, in which lines in a proof may by struck out, allowing the distinction between propositions that hold before and after the execution of change.

CLOC is based in the Possible Worlds Approach, according to which situations are represented by sets of propositions. We defined an extension of First Order Logic, introducing new terms representing change, operations to build structured representations of change, and new predicates relating propositions that hold in different situations. We presented $E\mathcal{X}_P\mu$, one inference rule that describes the derivation of the consequences of the execution of actions. We presented two examples of derivations using this rule.

CLOC is a result of project we are developing to extend the SNePS system [23, 22]. In this project, we are going to integrate three descendants of logic SWM [14], the logic underlying the SNePS, which are: the Mutation Logic [19, 20], a logic for dealing with change, as described above; the logic SWMC [2], a logic for default reasoning underlying SNePSwD, a version of the SNePS system using defaults; and the OK BDI formalism of [8], the formalism underlying the SNePS actuator. The logic CLOC results from the integration of Mutation Logic with OK BDI formalism. The resulting system will be used to perform reasoning about action and change using incomplete information.

Other papers relative to this approach will be available in `http://www.gia.ist.utl.pt/~pedro`.

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