Quantitative Structural Temporal Constraints on Repeating Events

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Abstract

This paper presents a model of the temporal structure of repeating events. This model allows for the representation of quantitative temporal constraints for repeating events, viz., those dealing with their part-whole structure. The temporal structure of repeating events is viewed as being composed of five properties: *number*, *sub-interval duration*, *gap duration*, *period* and *extent*. A set of constraints involving these properties collectively specifies the conditions under which repeating events can be assigned times, and thus partially formalizes the scheduling problem for repeating events. The consistency and tightness of a set of such constraints can be tested by finding a solution to linear equations relating these properties to one another. These equations can also be used to infer constraints about one structural property from the others, when one or more of these constraints is unknown. This paper also introduces a new category of quantitative temporal constraints involving repeating events which specifies that the duration or period of events be distributed randomly over a set of values. Finally, integrating previous work by the same authors, rules are formulated which allow the inferring of structural constraints of one repeating event from those of another, as well as from binary relational constraints between the events.

1 Introduction and background

The scheduling problem is the problem of selecting among alternative sequences of activities and assigning resources and times to them. These assignments must obey a set of constraints that reflect the temporal relationships among activities and the capacity limitations of a set of resources (Zweben and Fox, [17]). A scheduling problem might involve scheduling more than one occurance of the same event. Such an event is called a recurring or *repeating event*. A set of constraints can be defined involving temporal relationships between the different occurrences of the repeating event, and also between one repeating event and others. The former we call *structural constraints*, the latter, *relational constraints*. This paper proposes a model for specifying quantitative structural temporal constraints. Previous work by the same authors (Morris, Shoaff Khatib, [13]) has proposed a basic model of repeating events, and has applied this model to reason with certain kinds of qualitative relational constraints between repeating events. This paper uses the same basic model of repeating events to introduce a mechanism for representing quantitative structural constraints about repeating events. These constraints can be applied to solving the problem of scheduling repeating events.

To motivate the work done in this paper, we introduce a real scheduling problem involving repeating events in the domain of telescope scheduling (Bresina, [3]). In this domain, telescope time for the purpose of observing time-varying phenomena (e.g. eclipsing binary stars) is requested by an astronomer. An astronomer's scientific agenda (e.g., to fill out a light curve for a binary star system), imposes various constraints; for example, on the number of observations and on the number of nights between successive observations (APA, [2]). Thus, an astronomer might request that a given number of repeated observations (specified by an ideal and minimum occurrence count) be executed within a given time window with a given time gap between observations. The ideal gap (in days) is specified either with a fixed gap length or a gap probability distribution (in order to reduce aliasing in the data or determine the period of a recently discovered variable star). An example of a gap probability distribution would be expressed as "gaps should be randomly selected with a uniform probability from the set { 0 days, 1 day, 2 days }". This example illustrates that the properties of a single repeating event can be characterized in terms of a set of *structural properties*, among them the number and duration of each occurrence of the event, the duration of the gap between sucessive occurrences, and the length of time during which all occurrences must complete.

The remaining sections of this paper focus on formalizing the problem of specifying a set of structural constraints for repeating events. In the next section, we review a previously proposed model of repeating events, and define a set of structural properties which, it is claim, comprise necessary and sufficient conditions for characterizing the structure of a single repeating event. A simple language for specifying fixed constraints involving these properties is characterized, followed by a characterization of the conditions required for consistency and tightness of a set of these constraints. Next the issue of representing constraints involving duration (including gap duration) as a probability distribution is addressed. A section follows in which inference rules are formulated for inferring structural constraints of one repeating event in terms of those of another, as well as from relational constraints (again defined in previous work) between distinct repeating events. We conclude with a discussion of related research and possible extensions to the work.

2 The structure of repeating events

The temporal substratum of a repeating event can be depicted as a (for our purposes finite) set or sequence of intervals $I = \{I_1, I_2, \ldots I_n\}$, following the account in (Morris-Shoaff-Khatib, [13]). Each I_j is called a *sub-interval* of I. We assume here that each interval in I is non-overlapping and non-meeting. Thus each "gap" between sucessive events is itself an interval meeting the following and met by the preceeding sub-interval. An interval is represented in the customary way, viz., as a pair of natural numbers $I_i = \langle I_i^1, I_i^2 \rangle$, where $I_i^1 < I_i^2$. [1]

Drawing from the motivating example of the telescope observation request, the structural properties of a set of intervals are viewed as describing the number and relative durations of the sub-intervals and their gaps. We distinguish five such properties of I, characterizing the

- *number* of subintervals in I;
- sub-interval *duration*;
- *gap duration* (the duration between consecutive sub-intervals);
- *period* (the duration between consecutive sub-interval start times); and

[1] The account presented here could be extended to allow overlapping sub-intervals by allowing gaps of "negative duration", expressing degree of overlap. This possibility is not considered for reasons of simplicity in the presentation.

- *extent* (the duration from the start of the earliest subinterval to the end of the latest).

Each property defines a type of quantitative structural constraint for repeating events. Each constraint will be expressed here as a *range* of values, and in all cases except number those values are durations, expressed as differences between times. Corresponding to each property are *atomic* constraints of the following form:

- number n: $L_n \leq |I| \leq U_n$ (i.e., number is a constraint on the cardinality of I);
- sub-interval duration d: $L_d \leq I_i^2 - I_i^1 \leq U_d, i = 1 \ldots |I|$
- gap duration g: $L_g \leq I_{i+1}^1 - I_i^2 \leq U_g, i = 1 \ldots |I| - 1$
- period p: $L_p \leq I_{i+1}^1 - I_i^1 \leq U_p, i = 1 \ldots |I| - 1$
- extent e: $L_e \leq I_{|I|}^2 - I_1^1 \leq U_e$

Henceforth, we use the following abbreviations for the sake of uniformity: n_I for $|I|$, d_{I_j} for $I_i^2 - I_i^1$, g_{I_j} for $I_{i+1}^1 - I_i^2$, and so on. Where the context is clear, we will further abbreviate g_{I_j} to g_j, etc. The atomic constraints can be combined logically to express constraints of arbitrary complexity, e.g., to express a constraint on a repeating event E such as *the first, second, and last occurrence of E should have a duration between 3 and 5 time units; all the others should have between 4 and 6 time units*. In the examples that follow, we restrict our attention to durations of *constant* length (i.e., each subinterval duration is in the same range of values).

It is clear from this characterization that a number of dependencies exist among the five structural properties. First, sub-interval duration and gap duration collectively determine period. Similarly, duration and period together determine gap duration. Further, number, duration and gap (equivalently, number and period) collectively determine extent. Consequently, assuming these properties comprise a complete set of structural properties, it follows that a specification of the number and period of a repeating event (or number, sub-interval duration and gap duration) completely characterizes the temporal structural properties of the event. We express this more formally as follows:

Proposition 1 For each set I of subintervals of a repeating event,

$$\Sigma_{i=1}^{n_I}(d_i) + \Sigma_{i=1}^{n_I - 1}(g_i) = e_I; \text{ equivalently}$$

$$\Sigma_{i=1}^{n_I}(p_i) = e_I.$$

I.e., the extent of a repeating event is the sum of all its sub-interval durations plus the sum of its gaps durations (equivalently, the sum of its periods).

Because of these relationships, one problem that can be solved using this model is determining whether a set of structural constraints on a single repeating event is consistent.

Example 1 The following is a set of structural constraints for a repeating event I:

- number: $3 \leq n_I \leq 5$
- sub-interval duration: $4 \leq d_i \leq 7, i = 1 \ldots |I|$
- gap duration: $2 \leq g_i \leq 3, i = 1 \ldots |I| - 1$
- extent: $30 \leq e_I \leq 50$

Clearly this example formalizes the constraint on I that the number of subintervals be between 3 and 5, the duration of each interval be between 4 and 7 time units, the gap between subintervals be between 2 and 3 time units, and the extent be between 30 and 50 time units.

Consistency can be defined and computed directly from the proposition above.

Definition 1 A set of structural constraints on a repeating event I is *consistent* if

$$\exists m \in [L_n, U_n] \exists d_1, \ldots d_n \in [L_d, U_d], \exists g_1, \ldots g_{n-1} \in [L_g, U_g], \exists e \in [L_e, U_e] (d_1 + \ldots d_n) + (g_1 + \ldots g_{n-1}) = e$$

The $[L_x, U_x], x \in \{n, d, g\}$ are the ranges for number, sub-interval duration, and gap duration, respectively.

Example 2 The set of constraints in Example 1 is consistent; among others, the assignment

$$n = 5; d_1 = 4; d_2 = 7; d_3 = 6; d_4 = 5; d_5 = 7; g_1 = 2; g_2 = 3; g_3 = 2; g_4 = 3; e = 39$$

is a solution.

Another useful property concerns whether the ranges of values for the constraints are as small as possible; we refer to this condition as constraint *tightness*. One collection of tightness conditions consists of n-tightness, d-tightness, g-tightness and e-tightness. These are defined as follows:

Definition 2 A set of constraints is n-tight if

$$\forall n \in [L_n, U_n] \exists d_1, \ldots d_n \in [L_d, U_d], \exists g_1, \ldots g_{n-1} \in [L_g, U_g], \exists e \in [L_e, U_e] (d_1 + \ldots d_n) + (g_1 + \ldots g_{n-1}) = e.$$

A set of constraints is e-tight if

$$\forall e \in [L_e, U_e], \exists n \in [L_n, U_n] \exists d_1, \ldots d_n \in [L_d, U_d], \exists g_1, \ldots g_{n-1} \in [L_g, U_g] (d_1 + \ldots d_n) + (p_1 + \ldots p_{n-1}) = e.$$

A similar characterization of d-tightnesss and g-tightness can be constructed; details are omitted here.

Example 3 The constraints in Example 1 are d-tight and g-tight, since the solution in Example 2 assigns all the values in the range to the solution. On the other hand, the constraints are neither n-tight nor e-tight, since neither $n = 3$ nor $e = 50$ can be part of any consistent solution.

Other varieties of tightness can be defined in terms of pairs, or triples of values from different ranges. For example, the constraints in Example 1 are not (g, d)-tight because the assignment $d = 4; g = 2$ can't be extended to a consistent solution, i.e., one within the range of the extent.

Determining the consistency of a set of constraints with finite ranges can clearly be performed in time that is bounded by the number of possible assignments of values in each of the ranges of values, which is exponential in n, p, d, g and e. A simple algorithm for determining consistency of a set of structural constraints (expressed here as a combination of number, sub-interval, gap, and extent) is found in Figure 1. In the algorithm, where R is an interval of values viewed as a set, $\Pi_m(R)$ refers to the cartesian product of m copies of R. This algorithm can also be used to generate an assignment of number and sub-interval and gap duration, viz., by returning the consistent vector of value assignments of the d_i, p_i, and n.

The proposed model of structural constraints can be applied to inferring constraints from others, given the dependencies identified. For example, if the extent constraint were omitted from the specification in Example 1, the remaining constraints could be used, in conjunction with the consistency condition, to infer an extent in the range $[16, 47]$. Furthermore, intersecting this with the given range yields $[30, 47]$, which makes the specification e-tight. Also, the period of I can be inferred to be in the range $[6, 10]$, the result of adding the sub-interval duration and gap ranges.

```
input: [L_n, U_n], number range;
       [L_d, U_d], sub-interval duration range ;
       [L_g, U_g], gap range;
       [L_e, U_e], extent range;
output: true if a set of structural constraints is consistent; else false.
begin
consistent = false;
while not consistent do
    for each m ∈ [L_n, U_n]
        for each ⟨d_1, ...d_m⟩ ∈ Π_m[L_d, U_d]
            for each ⟨g_1, ...g_{m-1}⟩ ∈ Π_{m-1}[L_g, U_g]
                if Σ_i d_i + Σ_i g_i ∈ [L_e, U_e]
                then consistent = true;
return consistent
end
```

Figure 1: Determining consistency of a set of structural constraints involving number, and gap and sub-interval duration

3 Extensions to the model of repeating event structure

This section expands the representation of structural constraints to address issues of duration distribution using a probability density function, and optimizing constraints, with the telescope observation domain being the motivating force. Consequences of these extensions for determining consistency and tightness are briefly discussed.

Atomic constraints on duration have been interpreted to mean that each sun-interval (or gap or period) is constrained to have a duration in a given range. A special case of this interpretation is that each duration must be in the range and all durations must have the same value in the range. This would require another kind of atomic constraint to be added, of the form $d_i = d_j$, where d_i and d_j are durations. Still another interpretation, the one to be explored in this section, is that each duration is to be assigned according to a probability density function. Typically, as in the telescope scheduling example, it is the duration of the gaps between event occurrences that will be assigned in this manner. We focus on this case, although the representation can be extended to handle sub-interval durations as well.

For example, a uniform distribution will be expressed here (combining the terminology introduced here with that adopted in (Bresina, [3])), as $U(L_g \leq I_g \leq U_g)$ where I_g is a gap duration. This notation is to be interpreted to mean that a constant probability density function is to be associated with the assignment of ranges. Other possible distributions are normal, negative-exponential, or gamma.

Example 4 Consider the following set of constraints:

- number: $2 \leq n_I \leq 6$
- sub-interval duration: $4d_i \leq 7, i = 1 \ldots n_I$
- gap duration: $U(4 \leq g_i \leq 9), i = 1 \ldots n_I$
- extent: $30 \leq e_I \leq 100$

This example formalizes the constraint on the gaps between occurrences of I that their duration should be assigned uniformly from the interval $[4, 9]$.

When structural constraints have been assigned a probability density function in this manner, the notion of consistency, as we have defined it, must be generalized. One way is to define a notion of ρ-consistency for a set of constraints. A formal treatment of this notion is beyond the scope of this paper; a simple motivating example will suffice here:

Example 5 Consider the following set of constraints:

- number: $3 \leq n_I \leq 3$
- sub-interval duration: $4 \leq d_i \leq 7, i = 1 \ldots n_I$
- gap duration: $U(4 \leq g_i \leq 9), i = 1 \ldots n_I - 1$
- extent: $39 \leq e_I \leq 40$.

I must occur exactly 3 times, with a minimal extent of 39 time units. This is only possible when all the gap durations are assigned to be 9 time units. This assignment is, however, very unlikely, given the uniform probability function f assigned to it; this assigns a probability of $f(X) = \frac{1}{6}$ to each element in the extent range. Hence, although this set of constraints is consistent, it is not ρ-consistent for values of ρ much greater than 0.

Another modification of the basic structural model is to allow for the representation of optimizing constraints. To accomplish this, we introduce into each of the number, and sub-interval and gap duration constraint terminology a distinction between required and *preferred* constraints. Further, we require that the range associated with a preferred constraint be a sub-interval of the corresponding range of the required constraint. Thus, consider the following modification of the previous example:

- required: $2 \leq n_I \leq 6$; preferred: $5 \leq n_I \leq 6$
- required: $4 \leq d_i \leq 7, i = 1 \ldots n_I$; preferred: $6 \leq d_i \leq 7, i = 1 \ldots n_I$;
- $U(4 \leq g_i \leq 9), i = 1 \ldots n_I - 1$
- $30 \leq e_I \leq 100$

This set assigned preferences to the number and sub-interval duration constraints. In addition to determining consistency of this set, it is useful to determine whether a set of assignments is optimal with respect to a set of preferences. A straight-forward way to do this is to modify the consistency-checking algorithm introduced earlier in this section so that it first examines values in the preferred ranges. If it finds a consistent assignment, it also finds an optimal one. Otherwise, an optimal assignment with respect to all the preferences does not exist. Subsets of preferred ranges can then be examined to determine if partial optimality can be achieved.

4 Integration with relational model of repeating events

In this section, we sketch the framework within which structural constraints on repeating events can be inferred from structural and relational constraints involving other repeating events. Space limitations, as well as the fact that this is on-going research, prevents a complete treatment to be offered here. We restrict our attention to temporal reasoning for *single resource scheduling*, such

Propositional Form	Interpretation
$I\ \forall\forall(R)\ J$	Every sub-interval of I has R to every sub-interval of J.
$I\ \exists\forall(R)\ J$	Some sub-interval of I has R to every sub-interval of J.
$I\ \forall\exists(R)\ J$	Every sub-interval of I has R to some sub-interval of J.
$I\ \exists\exists(R)\ J$	Some sub-interval of I has R to some sub-interval of J.

Table 1: Classification of relations between pairs of collections using quantifiers.

as the telescope observation domain, in which no event overlaps any others. This simplifies the account by limiting the number of distinct types of temporal relations between any two intervals, while retaining a sense of the expressive power of the model. The general structure of inference we will be examining in this section is one of two forms:

- From a set of structural constraints $S(I)$ on repeating event I, and a unary relational constraint $R(I)$ defined on I, infer a new set of structural constraints $S'(I)$ for I.
- From $S(I)$, and a relational constraint $R(I, J)$ between I and another repeating event J, infer a set of structural constraints $S'(J)$.

First, we introduce a non-structural unary quantitative constraint on repeating events, called a *time window*. A time window specifies the interval of time within which an entire set of repeating events can begin and end. A time window is not a structural constraint because it does not describe the relationship between a repeating event and its parts. For example, in the structural constraint *twice a week for 5 weeks, between January and August, 1996*, the time window is between January and August, 1996. Notice the difference between the time window and extent (in the example, the latter is 5 weeks). Clearly, an extent of a repeating event must fit completely within the time window; it is inconsistent, for example to say *twice a week for 6 weeks during June*. Thus, a time window $[tw_l, tw_u]$ specifies the earliest start time and latest completion times for the extent of a repeating event. This leads to the first of the set of constraints between structural and other constraints:

Constraint 1 **(TW-E)**If $e_l \leq I_e \leq e_u$ is a constraint on the extent of I, and the time window of I is the interval $[tw_l, tw_u]$ then the extent of I is constrained to be $e_l \leq I_e \leq min(e_u, (tw_u - tw_l))$.

This rule constrains extents to be no longer than time windows. For example, if I has the extent constraint $10 \leq I_e \leq 50$ and the time window is determined to be in the range $[0, 45]$, then the extent constraint can be updated to $10 \leq I_e \leq 45$.

Next, we apply the model for relational constraints introduced in (Morris, Shoaff & Khatib,[13]) to infer structural constraints from binary qualitative relational constraints between pairs of repeating events. Four categories of qualitative constraint were identified based on different combinations of the use of *all* and *some* used to specify the constraint. These categories are summarized in Table 1. Here, I and J are any repeating events and R is an interval temporal relation. In the remainder of this section we focus upon the structural constraints about a repeating event J that can be inferred from knowledge about the structural constraints of I and a relational constraint of either the form: *Every sub-interval of I has R to every sub-interval of J* (henceforth referred to as $\forall\forall$ constraints, following the earlier terminology) or *Every sub-interval of I has R to some sub-interval of J* ($\forall\exists$ constraints). Similar constraints can be inferred about the other two classes of constraints, and therefore will be considered here only briefly.

First, we consider the $\forall\forall$ constraint type. These define constraints between pairs of repeating events considered as complete units. With single-resource scheduling, the possible qualitative relationships are *all before, all after, disjoint overlaps, disjoint overlapped-by, disjoint contains, disjoint contained in.* Figure 2 displays three of these relations; the others in the set are inverses.

$\forall\forall$-constraints can sometimes be used to imply structural constraints dealing with extent or sub-interval or gap duration. In particular, disjoint containment restricts the extent of one interval to be contained in that of the other. Specifically:

Constraint 2 ($\forall\forall$-**ST**) Let $L_e \leq I_e \leq U_e$ be an extent constraint on I, and let $L_d \leq I_d \leq U_d$ and $L_g \leq I_g \leq U_g$ be sub-interval and gap duration constraints on I. Let *All* I R *All* J be a $\forall\forall$ constraint between I and J. Then there exists a constraint $L'_e \leq J_e \leq U'_e$ on the extent of J, where

- If $R = disjoint\ contains$ then $L'_e = L_e - 2(U_d + 1) \wedge U'_e = U_e - 2(L_d + 1)$;
- If $R = disjoint\ contained\ in$ then $L'_e = L_e + 2(L'_d + 1) \wedge U'_e = \infty$.

Furthermore,

- if $R = disjoint\ contains$ then there exists a sub-interval duration constraint on J of the form $(L_g - 2) \leq J_d \leq (U_g - 2)$; and
- if $R = disjoint\ contained\ in$ then there exists a gap interval duration constraint on J of the form $(L_d - 1) \leq J_g \leq (U_d - 1)$;

Intuitively, this set of constraints bounds the extent and durations of repeating events that are either completely contained in, or completely contain, others. The soundness of this constraint will not be proven formally here; the following example illustrates the constraint.

Example 6 From *All* I *contains all* J and the structural constraints on I:

- extent: $30 \leq I_e \leq 50$;
- gap duration: $5 \leq I_d \leq 10$

The following structural constraints can be inferred for J:

- extent: $8 \leq J_e \leq 39$;
- sub-interval duration: $3 \leq J_g \leq 8$

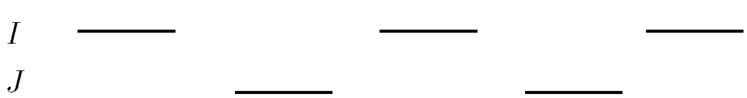
The second example of interactions between structural and qualitative relational constraints involves $\forall\exists$ relations. This class was discussed extensively in (Morris-Shoaff-Khatib, [13]). Briefly, this type of constraint implies a repeating *pairing* of some, or each sub-interval of one repeating event with sub-intervals of another. For example, the expression I *is always before* J says that each sub-interval of J follows some sub-interval of I. The result is a mapping between sub-intervals of each repeating event; the sub-intervals so mapped are said to be "correlated". Examples of different types of mapping are found in Figure 3. Here we restrict the discussion to "single-event" correlation, i.e., where the mapping is one to one. This type of constraint allows inferences to be made about number, as well as possibly subinterval and gap duration. For example, if a repeating event I is constrained to have a number of sub-intervals, and there is a one to one mapping between I and J, then clearly J is constrained to have the same number of sub-intervals. More generally:

All I before all J

All I overlap all J

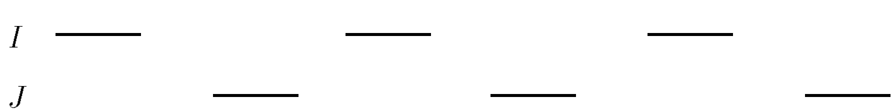
All I contains all J

Figure 2: $\forall\forall$ constraints on non-overlapping repeating events



I always before J

I only before J



I always and only before J

Figure 3: $\forall\exists$ constraints on non-overlapping repeating events

Always before: constraint on sub-interval duration of J in terms of gap duration of I

Always after: constraint on gap duration of J in terms of sub-interval duration of I

Figure 4: Interactions between structural constraints and relational constraints involving "always'

Constraint 3 (**∀∃-ST**) Let $L_n \leq I_n \leq U_n$ be a number constraint on I and let $I\ QR\ R\ J$ be a ∀∃ constraint between I and J, where $QR \in \{always, only, always - and - only\}$, and $R \in \{before,\ after\}$[2]. Then there exists a constraint $L'_n \leq J_n \leq U'_n$ on the extent of J, where

- If $QR = always$ then $U'_n \leq U_n$;
- If $QR = only$ then $L'_n \geq L_n$;
- If $QR = always - and - only$ then $L'_e = L_e \wedge U'_e = U_e$;

In addition, we infer the additional constraints involving gap and sub-interval durations:

- If $QR = always$ and $R = before$ then $U'_d \leq U_g$;
- If $QR = always$ and $R = after$ then $U'_g \leq U_d$;
- If $QR = only$ and $R = before$ then $U'_d \leq U_g$;
- If $QR = only$ and $R = after$ then $U'_g \leq U_d$;
- If $QR = always\ and\ only$ then $U'_d \leq U_g \wedge U'_g \leq U_d$.

An example of the interaction between gap constraints of one repeating event and duration constraints of another is illustrated in Figure 4. It should be noted again that many of the constraints noted in this section assume the single resource restriction. If overlapping among sub-intervals between different repeating events is allowed, then many of these constraints are no longer sound.

5 Extended example

6 Related research

There are two approaches to formalizing knowledge about time. The first stems from the modal logic tradition, and is based on a temporal interpretation of the possible worlds framework (Goldblatt, 1987). The other approach stems from the literature on constraint-based reasoning, involving a temporal interpretation of the variables, domains, and constraints being manipulated by constraint solvers. The work here, as with most of the related research, stems from the second tradition. The work reported here overlaps with the time literature in at least four locations:

- In offering a formalization of intervals representing recurring events;

[2]Again, this restriction on R is due to the single resource constraint, which prohibits and overlapping of sub-intervals

- In reasoning with collections of intervals;
- In applying to problems of scheduling recurring events;
- In integrating qualitative and quantitative constraints.

In the remainder of this section, we briefly review the related literature at each of these points.

The idea of explicitly reasoning with set of intervals was first presented, at least in the AI literature, with Allen's notion of *reference interval* (Allen 1983); Davis (Davis 1991) and (Kooman 1989) expand this idea. Ladkin (Ladkin, 1987), first proposed a constraint-based approach to representing collections (unions) of convex intervals. This proposal was extended by Ligozat (1991), who investigated the calculus of *generalized intervals* in an algebraic setting. Formal systems for reasoning about collections of intervals are often in the context of reasoning about calendar information in a database (Niezette and Stevenne 1992), and (Leban et. al 1986); more recently, (Terenziani, 1997). In addition to the authors, recurrence has been studied by (Cukierman and Delgrande, [5]) and, in the temporal database literature, by (Tuzhilin and Clifford, [15]), in the context of finite representations of queries about infinite temporal data. Koomen (Kooman, [8]) described a system incorporating recurrence information into a plan generation application. The idea of incorporating quantitative and qualitative temporal constraints into a temporal reasoning system has been pursued by Kautz and Ladkin (Kautz and Ladkin, [7]) and by Meiri (Meiri, [10]). Finally, in addition to the telescope observation problem motivating the examples in the paper, other scheduling problems require the efficient manipulation of recurring events. The earliest system to address these concerns in detail seem to be MAESTRO (Britt, Geoffrey and Gorey, [4]), which schedules spacecraft activities. More recently, GPSS (Ground Processing Scheduling System), for scheduling space shuttle maintenance activities, also deals with repeating events.

Much work remains in making the proposed system fully general. We conclude this section by identifying six areas of potential future work. First, there is a natural generalization of structural constraints to identify classes of quantitative (sometimes called metric) relational constraints between pairs of repeating events. For example, a constraint such as $3 \leq I_i^1 - J_i^1 \leq 5$ is a quantitative correlate to $\forall\exists$-constraints, and specifies that each I sub-interval must start between 3 and 5 time units after their correlated J sub-interval. Second, more work is required in formalizing the notion of ρ-consistency introduced earlier. Third, the effect of generalizing the proposed system to handle overlapping repeating events (either within a single repeating event, or between distinct repeating events, or both) should be studied. Fourth, a more complete characterization of interactions between class of constraints should be investigated. For example, the interactions between $\forall\forall$ and $\forall\exists$ qualitative constraints have yet to be investigated rigorously. Finally, a less restricted constraint language shoud be constructed to allow for complex constraints like *every other occurence of I should have a duration of 1 time unit* to be specified. Finally, the system proposed here should be mapped to representations of calender systems, such as those referred to earlier. This would allow for more natural specifications of structural constraints, such as *twice a day, for one hour each, once in the morning and once at night, every other week, for 6 months, between 1997 and 1999.*

7 Concluding remarks

The purpose of this paper has been to apply a model of repeating events to the formalization of part of the complex problem of scheduling repeating events, and to devise a way of solving this part of the problem. This simple framework allows for constraints to be defined which specify the part-whole temporal structure of the repeating event.

References

[1] Allen, J. 1983. Maintaining Knowledge About Temporal Intervals. In *Readings in Knowledge Representation*, Brachman, R., and Levesque, H., (editors). (SanMateo:Morgan Kaufman), 510-521.

[2] Long Term Telescope Loading in the Associate Principal Astronomer.

`http://ic-www.arc.nasa.gov/ic/projects/xfr/loader/loader.html`

[3] Bresina, J.L. 1994. Telescope Loading: A Problem Reduction Approach. In *Proceedings of the Third International Symposium on Artificial Intelligence, Robotics, and Automation for Space.* Pasadena, CA, Jet Propulsion Labs.

[4] Britt, D., Geoffrey, A., and Gohring, J. The Impact of the Utility Power System Concept on Spacecraft Activity Scheduling. *Proceedings of the 23rd Intersociety Energy Conversion Engineering Conference*, vol. III. (1988).

[5] Cukierman, D. and Delgrande, J. Characterizing Temporal Repitition. In *Proceedings of TIME-96*, Chittaro, Goodwin, Hamilton, and Montanari, eds. IEEE Press, 80-87.

[6] Dechter, R., Mehri, I., and Pearl, J. Temporal Constraint Networks. In Brachman, R., Levesque, H., and Reiter, R., eds. *Proceedings of the First International Conference on Principles of Knowledge Representation and Reasoning.* Morgan Kaufmann, 1989, 83-93.

[7] Kautz, H., and Ladkin, P., Integrating Qualitative and Metric Temporal Reasoning. *AAAI Spring Symposium on Constraint-based Reasoning, Working Notes*, 1991, 66-78.

[8] Koomen, J., Reasoning about Recurrence. *International Journal of Intelligent Systems* Vol 6, (1991), 461-496.

[9] Ladkin, P.B. 1986. Primitives and Units for Time Specification. In *Proceedings of AAAI-86*, (San Mateo:Morgan Kaufman), 354-359.

[10] Meiri, I., 1991. Combining Qualitative and Quantitative Constraints in Temporal Reasoning. *AAAI Spring Symposium Workshop Notes*, Stanford University.

[11] Morris, R., and Al-Khatib, L. 1991. An Interval-Based Temporal Relational Calculus For Events With Gaps. *The Journal of Experimental and Theoretical Artificial Intelligence*, 3:87-107.

[12] Morris, R., Shoaff, W., and Khatib, L. 1993. Path Consistency in a Network of Non-Convex Intervals. *Proceedings of the 13th International Joint Conference on Artificial Intelligence (IJCAI-93)*, Chambery, France. 650-655.

[13] Morris, R., Shoaff, W., and Khatib, L., (1996) Domain Independent Reasoning About Recurring Events. *Computational Intelligence*, Volume 12, Number 3, 450-477.

[14] Rohrer, C. 1980. *Time, Tense, and Quantification: Proceedings of the Stuttgart Conference on Logic of Tense and Quantification*, Max Niemeyer Verlag.

[15] Tuzhilin, A., and Clifford, J., 1995. On Periodicity in Temporal Databases. *Information Systems* **30**(5), 619-639.

[16] Van Beek, P. 1992. Reasoning about Qualitative Temporal Information. *Artificial Intelligence* **58**:297-326.

[17] Zweben, M. and Fox, M., *Intelligent Scheduling.* San Francisco: Morgan-Kaufmann Pub. Inc., 1994.