

# Reasoning About the Temporal Aspects of Interacting Agents\*

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## Abstract

*The formalization of temporal reasoning for dealing with actions taking place at different grains of time has been investigated in the recent years. The main concern has been the representation of the real calendar and the mechanisms needed for mapping events “annotated” with any convex or non-convex intervals to their annotation in the calendar. This work presents some results on the representation and reasoning about the actions of agents interacting at different time scales.*

## 1 Introduction

The investigations on the representation and reasoning about the interaction of agents have not paid too much attention to the *temporal aspects of the interaction* itself. We should understand this as *the way models of agents influence each other as time goes by*, despite the fact that such influences may happen at different grains of time. In domains like simulation models of ecological and/or social systems there can be problems when one needs to integrate models (agents) specified at different time granularities.

In this work we shall see how a temporal logic programming approach can be used to formalize the behaviour of an agent when observing the change of other agents’ attribute if such information

is relevant to its behaviour. Three main issues are involved here: a) the representation of information about periods at different levels, b) the exchange of timing information across different models of autonomous agents, and c) the problems we face when combining both issues above in simulation models of autonomous agents. We propose a way to tackle the problem (b) and address a brief discussion on (c).

The rest of this paper is organised as follows. In Section 2 we shall see the main motivation for this work. In Section 3 presents a logic programming approach to representing an agent’s behaviour. Section 4 presents our solution for the problem this work aims to solve. In Section 5 some related work are addressed. Finally, Section 6 presents a brief discussion and the concluding remarks.

## 2 Motivation and Related Work

### 2.1 The Problem to be Solved

When modeling how change takes place at different levels of abstraction on such systems, there can be three different situations of temporal interaction:

1. There are two interacting agents, say  $a_1$  and  $a_2$ , in which their models are expressed at the same level and they update their states at the same tick of the clock. We shall say they are *aligned* in time.

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2. There are two interacting agents, say  $a_1$  and  $a_2$ , but their models are not expressed at the same level of granularity.
3. There are three interacting agents, say  $a_1$ ,  $a_2$  and  $a_3$ . Agents  $a_1$  and  $a_2$  have their models expressed at the same level, but their states change *non-aligned* in time.

The main questions we have to face are: *i*) what is the value of the attribute of each agent at a specific time in between two consecutive time steps at the agent's scale of time? *ii*) what are the values of an agent's attribute during a certain period of time? (for aligned and non-aligned periods in relation to changes on the agent's attributes).

### 3 Representing the Behaviour of Agents

Here, models of agents' behaviour are represented by an extension of Prolog, enhanced with a special unification for temporal labels [1]. The logical connectives for conjunction, disjunction, and implication are represented here by  $\&$ , *or*, and  $\Leftarrow$ , respectively. The only temporal operator used here is the throughout  $@$  operator.

A classical formula is "annotated" (in the sense of [2]) with temporal expressions, and in this case it is called an *atomic temporal formula* if the associated classical formula is atomic. In the case of a classical well formed formula, when each of its elements or the whole formula is annotated, then we call it a *well formed temporal formula* (WTF).

Temporal entities of the hierarchy of time are called *Pure temporal expressions* (PTE). The sorts of PTEs or temporal entities of interest for this work are defined as follows.

- $t(x_1, \dots, x_n)$  to represent one moment of time. For instance,  $t(16, 6, 2001)$  is a moment in a three level hierarchy of time.
- $P$  after  $T$  represents a time in the future obtained from  $T$  after a period of length  $P$ . The form of  $P$  can be either  $p(Value, Scale)$ , or  $p(Value, Scale)$  plus  $P$ , where *plus* is a

constructor of composed lengths of time, and  $P$  is a period. For instance,  $p(10, day)$  plus  $p(3, month)$  after  $t(2, 1, 2001)$ .

Any temporal formula  $A @ T_i$  is written as  $A @ T :: [(T, T_i)]$ . A temporal normal logic program has the form  $\{C_1 :: \theta_1, \dots, C_n :: \theta_n\}$ , where each  $C_i$  is a temporal clause and  $\theta_i$  is its temporal substitution framework (TSF), used to perform a special unification [1]. We now use this language and set notation to describe the model of an agent's behaviour.

Let  $Att_a$  be an attribute of an agent  $A$  which changes every  $P$  units of time, and  $Att_b$  be an attribute of another agent  $B$  which affects  $A$ . The *simulation clause scheme* (SCS) of  $A$ 's behaviour which changes  $Att$  is given by

$$\begin{aligned}
 &value(Att, A, V_i) @ T_i :: [(T_i, X_i)]. \\
 &value(Att, A, V_f) @ T \Leftarrow \\
 &\quad value(Att, A, V_p) @ T_p \& \\
 &\quad V_L = \{V \mid value(Att_b, B, V) @ T_j \text{ and } \\
 &\quad \quad T_j \text{ is included in } T_p \dots T\} \& \\
 &\quad \mathcal{R}(V_p, V_L, V_f) :: [(T_p, X_p), (T, P \text{ after } T_p)].
 \end{aligned}$$

where  $value(Att, A, V_i) @ T_i$  is the initial state of  $Att$  and  $\mathcal{R}(V_p, V_L, V_f)$  represents a sequence of sub-goals which relates  $V_p$ ,  $V_L$  and  $V_f$ .

This clause scheme is used as an abstraction and it could be plugged within any other standard logic of action and change. The actual difference to standard computational logic is the mechanism of inference for dealing with non-aligned processes which we shall see next.

## 4 Reasoning about Interacting Agents

### 4.1 Agents Aligned in Time

For this case the deduction process tries to satisfy a given query relative to an attribute's value at an specific time  $T$ . If  $T_f$  is the final time point of the search interval with length  $P$ , then we start from the first value of the attribute and repeatedly get new instances of simulation clause for the changes of the attribute. We stop when either  $T = T_f$  or  $T_f$  precedes  $T$ , where  $T_f = P$  after  $T_p$ .

We extend the meta-interpreter of [1] as follows, where  $var(X)$  is true if  $X$  is a variable and

$body\_of\_scs(A)$  is true if  $A$  can be matched with the body of a SCS.

$$\begin{aligned} solve(value(Att, Ag, V) @ T, \theta_s, \theta_f) \Leftarrow & \\ (T, T_1) \in \theta_s, \& \ var(T_1) \& & \\ body\_of\_scs(value(Att, Ag, V) @ T) \& & \\ clause(value(Att, Ag, V_i) @ T_i \Leftarrow \top :: \theta_n) \& & \\ free\_search(value(Att, Ag, V_i) @ T_i, \theta_n, & \\ \quad value(Att, Ag, V) @ T, \theta_s, \theta_f). & \end{aligned}$$

The declarative meaning of it is that the value of the attribute  $Att$  of the agent  $Ag$  is  $V$  throughout  $T$  if  $T_1$  is the temporal substitution of  $T$  in  $U_s$ , and  $T_1$  is a variable, and  $value(Att, Ag, V) @ T$  can be matched with the body of a simulation clause, and there is a clause  $value(Att, Ag, V_i) @ T_i \Leftarrow \top$  with TSF  $\theta_n$ , and the search free of fixed time yields a solution for  $value(Att, Ag, V) @ T$  at time  $T$ , with  $T$  in  $\theta_s$ , and  $\theta_f$  is its TSF.

In the  $free\_search/5$  the use of new instances of SCS involves temporal combination to generate them. This shall be represented by the predicate  $inst\_simul\_cls/5$  which associates  $X_p @ T_p$  of the body of a SCS and its TSF  $\theta$  to the head of this new clause instance, the constraints after  $X_p @ T_p$ , as well as in the new TSF resulting from this new instance. This is formalised as follows.

$$\begin{aligned} free\_search(X @ T_i, \theta_i, X @ T, \theta, \theta_f) \Leftarrow & \\ temp\_comb(\theta_i, T_i, \theta, T, \theta_f). & \\ free\_search(X_p @ T_p, \theta_p, X_r @ T_r, \theta_s, \theta_f) \Leftarrow & \\ inst\_simul\_cls(X_p @ T_p, \theta_p, X_f @ T, C, \theta_s) \& & \\ \theta_{s'} = \theta_s \setminus \{(T_p, V_p)\} \& & \\ temp\_comb((T_f, V), \theta_f, (T_s, V_s), \theta_{s'}, \theta_n) \& & \\ solve(C, \theta_n, \theta_e) \& & \\ free\_search(X_f @ T_f, \theta_e, X_r @ T_r, \theta_s, \theta_f). & \end{aligned}$$

#### 4.2 Agents Aligned and Non-Aligned in Time

The idea now is to align the search so that the last time stamp found, or  $V_f$  will always be at the given  $T$  of the query, i.e.  $T = T_f$  will always hold. This policy forces the search to take any new influence into account. Formally we have the following  $solve/3$  definition for this case.

$$\begin{aligned} solve(value(Att, Ag, V) @ T, \theta_s, \theta_f) \Leftarrow & \\ (T, T_1) \in \theta_s \& & \end{aligned}$$

$$\begin{aligned} \neg \varphi(T_1) \& & \\ body\_of\_scs(value(Att, Ag, V) @ T) \& & \\ clause(value(Att, Ag, V_i) @ T_i \Leftarrow \top :: \theta_i) \& & \\ (T_i, T_2) \in \theta_i \& \ T_1 \text{ does not precede } T_2 \& & \\ T_1 = P_a \text{ after } T_2 & & \\ aligned\_search(P_a, value(Att, Ag, V_i) @ T_i, \theta_i, & \\ \quad value(Att, Ag, V) @ T, \theta_s, \theta_f). & \end{aligned}$$

The declarative meaning of this clause is similar to the previous one. In what follows we show the possible cases and their specification, where  $change(Att, Proc)$  is true if  $Att$  is changed by process  $Proc$ ,  $scale(Agent, Proc, S)$  is true if  $Agent$  has a process  $Proc$  which runs at scale of time  $S$ , and  $smaller\_period(P_1, P_2)$  is true if period  $P_1$  is a length of time smaller than  $P_2$ .

$$\begin{aligned} aligned\_search(0, X @ T_n, \theta_n, X @ T_s, \theta_s, \theta_f) & \\ \Leftarrow & \\ \theta_{s'} = \theta_s \setminus \{(T_s, V_s)\} \& \ \theta_{n'} = \theta_n \setminus \{(T_n, V_n)\} \& & \\ temp\_comb((T_s, V_s), \theta_{s'}, (T_n, V_n), \theta_{n'}, \theta_f). & \\ aligned\_search(P_a, X_i @ T_i, \theta_i, X @ T, \theta_s, \theta_f) & \\ \Leftarrow & \\ P_a > 0 \& & \\ X = value(Att, Agent, -) \& & \\ change(Att, Proc) \& & \\ scale(Agent, Proc, S) \& & \\ (equivalents(P_a, p(1, S)) \& & \\ search(X_i @ T_i, \theta_i, X @ T, \theta_s, \theta_f)) \text{ or } & \\ (\neg equivalents(P_a, p(1, S)) \& & \\ (smaller\_period(P_a, p(1, S)) \& & \\ \theta_f = \theta_s) \text{ or } & \\ (smaller\_period(p(1, S), P_a) \& & \\ mod\_decomposed(P_a, S, P_1) \& & \\ (P_1 = p(-, S) \& & \\ search(X_i @ T_i, \theta_i, X @ T, \theta_s, \theta_f)) \text{ or } & \\ (P_1 = p(D_1, S_1) \text{ plus } p(-, S) \& & \\ temp\_comb(T_j, [(T_j, V_j)], T, & \\ [(T, p(D_1, S_1) \text{ after } T_i) | \theta_i], \theta_j) \& & \\ search(X_i @ T_j, \theta_j, X @ T_s, \theta_s, \theta_f)))))) & \end{aligned}$$

The  $search/5$  stops when the the final time is reached, otherwise a new instance of simulation clause is matched with the value of the attribute at the previous time, the constraints are solved and the search re-starts with this new value. Formally we have.

$$\begin{aligned}
& \text{search}(X @ T_n, \theta_n, X @ T_s, \theta_s, \theta) \Leftarrow \\
& \quad \theta_{n'} = \theta_n \setminus \{(T_n, V_n)\} \ \& \ \theta_{s'} = \theta_s \setminus \{(T_s, V_s)\} \ \& \\
& \quad \text{temp\_comb}((T_s, V_s), \theta_{s'}, (T_n, V_n), \theta_{n'}, \theta_f). \\
& \text{search}(X_i @ T_i, \theta_i, X_r @ T_r, \theta_r, \theta_f) \Leftarrow \\
& \quad (T_i, V_i) \in \theta_i \ \& \ (T_r, V_r) \in \theta_r \ \& \\
& \quad V_i \text{ precedes } V_r \ \& \\
& \quad \text{inst\_simu\_cls}(X_i @ T_i, \theta_i, X_f @ T, C, \theta) \ \& \\
& \quad \theta' = \theta \setminus \{(T, V)\} \ \& \\
& \quad \text{temp\_comb}((T_j, V_j), [], (T, V), \theta', \theta_j) \ \& \\
& \quad ((\text{precedes}(V_r, V_j) \ \& \ X_r = X_i \ \& \ \theta_f = \theta_i) \ \text{or} \\
& \quad (V_r \text{ does not precede } V_j \ \& \ \text{solve}(C, \theta_j, \theta_{j'}) \ \& \\
& \quad \text{search}(X_j @ T_j, \theta_{j'}, X_r @ T_r, \theta_s, \theta_f))).
\end{aligned}$$

#### 4.3 Representing Interaction Between Agents

Two agents  $A_i$  and  $A_j$  interact at different grains of time, and an attribute  $Att_j$  of  $A_j$  affects an attribute  $Att_i$  of  $A_i$ . Agent  $A_i$  has to “observe” the changes on the values of  $Att_j$ . The *observer process* (OP) will get the progression of the values of  $Att_j$ , in a list  $L$ , during a specific period of time of the length of  $k$ . This idea is depicted in Figure 1.

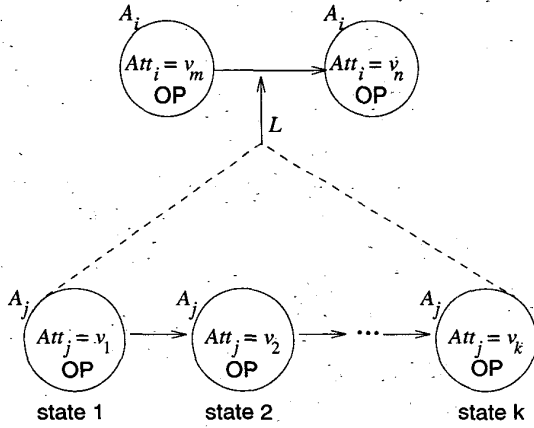


Figure 1: Interaction between agents  $A_i$  and  $A_j$ , where the OP of  $A_i$  get the list  $L$  of values of  $Att_j$  during a period  $k$  units of time.

The extension of the meta-interpreter for dealing with it is simply to call for the predicate  $\text{progress\_observed}(\text{value}(\text{Att}, \text{Agent}) @ T, P, L, \theta_s, \theta_f)$ . As the value of the observed attribute at the right ending point is unimportant to compute the state of the agent for that time,

then the period observation is open on the right. We use  $\text{length\_of\_time}(I_L, P)$  to associate any linear interval to its length (i.e. its ending points inclusive). From this point there are two possible computations as we show as follows. First the case where agents are aligned.

$$\begin{aligned}
& \text{progress\_observed}(\text{value}(\text{Att}, \text{Agent}) @ T, \\
& \quad P, [V|R], \theta_s, \theta_f) \Leftarrow \\
& \quad \text{change}(\text{Att}, \text{Proc}) \ \& \ \text{scale}(\text{Agent}, \text{Proc}, S) \ \& \\
& \quad (T, T_p) \in \theta_s \ \& \ \text{length\_of\_time}(T_p \dots T_f, P) \ \& \\
& \quad \text{solve}(\text{value}(\text{Att}, \text{Agent}, V) @ T, \theta_s, \theta_n) \ \& \\
& \quad \text{compute\_rest}(\text{value}(\text{Att}, \text{Agent}, V) @ T, \\
& \quad \quad T_f, P, S, \theta_n, \theta_f, R).
\end{aligned}$$

Second, for the case where the initial state of the attribute is either included within the given period or the initial state occurs at some time after the last time of observation, we have.

$$\begin{aligned}
& \text{progress\_observed}(\text{value}(\text{Att}, \text{Agent}) @ T, P, \\
& \quad \text{Prog}, \theta_s, \theta_f) \Leftarrow \\
& \quad \text{change}(\text{Att}, \text{Proc}) \ \& \ \text{scale}(\text{Agent}, \text{Proc}, C) \ \& \\
& \quad (T, B_p) \in \theta_s \ \& \ \text{length\_of\_time}(B_p \dots B_f, P) \ \& \\
& \quad \neg \text{solve}(\text{value}(\text{Att}, \text{Agent}, -) @ T, \theta_s, -) \ \& \\
& \quad \text{solve}(\text{value}(\text{Att}, \text{Agent}, V) @ T_i, \theta_s, \theta_n) \ \& \\
& \quad (T_i, B_i) \in \theta_n \ \& \\
& \quad (\neg \text{precedes}(B_f, B_i) \ \& \\
& \quad \text{Prog} = [V|R] \ \& \\
& \quad \text{compute\_rest}(\text{value}(\text{Att}, \text{Agent}, V) @ T_i, \\
& \quad \quad B_f, P, C, \theta_n, \theta_f, R) \ \text{or} \\
& \quad \text{precedes}(B_f, B_i) \ \& \\
& \quad \text{Prog} = []).
\end{aligned}$$

In the rest of the progress, represented by  $\text{compute\_rest}/5$ , the important thing to describe is how the period of observation relates to the agent's scale of change. For the simplest case where  $P$  is smaller than the length of  $C$ , we have four different cases as depicted in Figure 2. First,  $P$  starts at a time aligned to an updating time step of the agent's attribute (a). Second,  $P$  finishes at an updating time (b). Third, it is in between two consecutive updating time steps, (c), and finally it includes a single updating time step (d). This gives us the following specification.

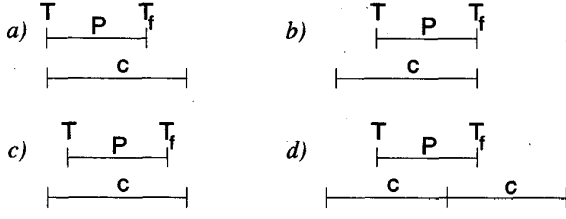


Figure 2: Case 1 - a)  $P$  starts aligned; b) ends aligned; c)  $P$  is included in between two consecutive time steps; d)  $P$  includes a single time step.

$compute\_rest(value(Att, Agent, -) @ -, T_f,$   
 $p(D, C_1), C_2, \theta_s, \theta_f, [V_f]) \Leftarrow$   
 $\neg (C_1 = C_2) \ \&$   
 $smaller\_period(p(D, C_1), p(1, C_2)) \ \&$   
 $solve(value(Att, Agent, V_f) @ T, [(T, T_f)] | \theta_s, \theta_f).$

The other case is the one in which the length of  $c$  is smaller than or equal to the length of  $P$ . This can lead us to other possibilities which we classify in two ways. In the first case, depicted in Figure 3,  $P$  can be decomposed into  $C$  and either it matches the actually updating time steps of the agent's attribute (a), or it may be non-aligned (b). It may also not be decomposable and then it matches at the beginning (c) but not at the end, or vice-versa (d).

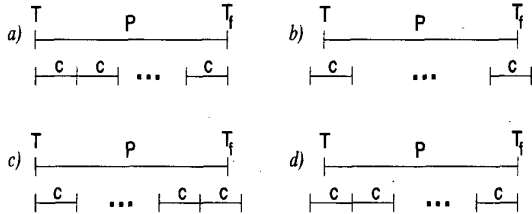


Figure 3: Case 2 - The Period of observation is modularly decomposable into  $C$ , (a) and (b), and  $P$  is not modularly decomposable, (c) and (d).

The final case is where  $P$  and unitary length at  $c$  are equivalent and either (a) they are totally aligned (starts and ends at the same time step), or (b) they are non-aligned. This is depicted in Figure 4.

What has to be done, in the context of  $compute\_rest/5$ , is simply to check if  $P$  has a

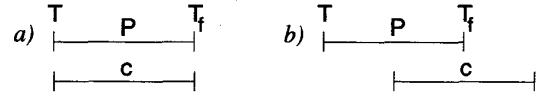


Figure 4: Case 3 - a)  $P$  is aligned to  $c$ ; b)  $P$  and  $c$  are non-aligned.

length greater than or equal to the length of  $c$ , then it should compute the progression of the value of the attribute until time  $T_f$ , taking these cases above into account. This is formally specified as follows.

$compute\_rest(value(Att, Agent, V) @ T_i, T_f,$   
 $p(D, C_1), C, \theta_s, \theta_f, R) \Leftarrow$   
 $smaller\_period(p(1, C), p(D, C_1)) \ \&$   
 $evolve(value(Att, Agent, V) @ T_i, T_f, C, \theta_s, \theta_f, R).$

Now, there are three possibilities when  $evolve/4$  is called:

1.  $T_i$  is equal to  $T_f$ , then  $R$  should be associated with an empty list.
2.  $T_i$  is not equal to  $T_f$ , then apply resolution between  $value(Att, Agent, V) @ T_i$  and the body of a simulation clause  
 $value(Att, Agent, V_j) @ p(1, C) \text{ after } T_i \Leftarrow$   
 $value(Att, Agent, V_j) @ p(1, C) \text{ after } T_i \ \&$   
 $Constraints.$

Either:

- (a)  $T_f$  precedes  $T_j$ , i.e.  $T_i$  after  $p(1, C)$  or the next value to be computed, then find  $V_f$  at  $T_f$  associate  $R$  to  $[V_f]$
- (b)  $T_f$  does not precedes  $T_j$ , then solve the  $Constraints$ , and associate  $R$  to  $[V_j | R_1]$  and compute the rest of progression in  $R_1$  from  $T_j$  until  $T_f$ .

## 5 Related Work

This work identified four important cases when the period of an agent's observation is shorter than the time scale of the other agent. We can related them to the Allen's interval calculus [3]. Here we described ending interval, beginning interval, wholly within the interval, and overlapping.

Other approaches on temporal granularity did not concern with this identification, [4, 5]. A logic programming approach similar to this work is Chronolog(MC) [6] which is flexible for multiple granularities, but no exploration was made on representing interacting agents working at different clocks.

Kraus et. al. [7] proposed a strategic model of interaction that takes the passage of time during the negotiation process itself into account. They investigated agents that lose over time and need to share resources. They base their investigation in cooperation of agents while ours does not need to assume cooperation. This means that our approach is not suitable for strategic models.

## 6 Discussions and Final Remarks

Consider a scenario of interaction as depicted in Figure 5 where all agents work at grain 7. The goal is for the time stamp  $t(20, 1, 1)$ , and agents  $a_3$  and  $a_4$  both affect  $a_1$  and  $a_2$  but are introduced some time later. According to policy we used the initial state for the search is  $t(6, 1, 1)$ . However, this is not fully accurate because  $a_3$  would have affected  $a_1$  since  $t(4, 1, 1)$ .

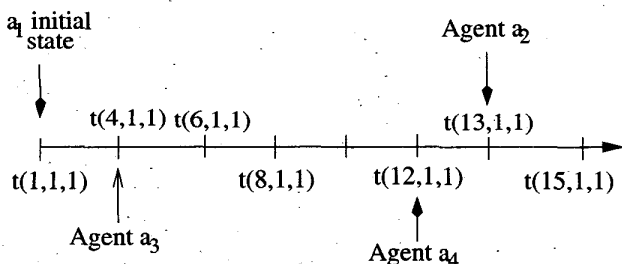


Figure 5: Another situation in which two other non-aligned agents are present.

Alternatively we could take the earliest initial state of all agents affecting (in this case)  $a_1$  and start the search from it. Note that in this situation the last change in  $a_1$  before  $t(13, 1, 1)$  is  $t(11, 1, 1)$ . Thus, the value assumed for  $a_1$  state at  $t(13, 1, 1)$  would be the one found at  $t(11, 1, 1)$ . However, this would be no better than the one proposed here because it would not take the influence of  $a_4$  upon  $a_1$  into account. This other agent has been introduced at  $t(12, 1, 1)$  which is

in between  $t(11, 1, 1)$  and  $t(13, 1, 1)$ . As a matter of fact this approach is symmetric to what is proposed here.

This problem only shows how complicated can be the use of models of interacting autonomous agents to predict the future within a temporal structure. Our main contribution is to raise the problems of communication between interacting agents at different grains of time, and to address the possible initial direction to cope with it.

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