Classical and Fuzzy Neighborhood Relations of the Temporal Qualitative Algebra

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Abstract

In this paper we study the problem of representing different forms of imperfect temporal knowledge. Imperfection in knowledge can be present in the forms of coarse knowledge about temporal events or even in the form of incomplete, imprecise, vague or uncertain temporal information. The two orthogonal notions of coarseness and fuzziness seem to be quite adequate to manage these two kinds of temporal ignorance and they can be combined to develop a more general model. The first part of the present study is dedicated to the definition of a new Neighborhood Temporal Qualitative Algebra nQA starting from Freksa's work about Allen's Algebra neighborhood relations. Freksa's Algebra is merged with the Convex Point Algebra and all the neighborhood relations needed to close the nQA Algebra are computed automatically. The Algebra nQA is tractable. In the second part of the paper, the fuzziness notion is considered and combined with that of conceptual neighborhood. The algebra nQA^{fuz} *is defined as a fuzzy extension of the algebra nQA, and the conditions of its tractability are studied.*

1 Introduction

According to Smets [22, 23] imperfection in knowledge, and hence in temporal knowledge, may assume many aspects. It may be in the form of incomplete information when the value of a variable is missing; it may be also due to vagueness and/or imprecision when the value of a variable is given but not with the precision required or the value is given in a coarse way. Another aspect can be related to the presence of uncertainty: in this case it depends on the state of knowledge of an agent about the world. All these aspects can be present and can be combined. The distinctions between these categories are vague, as well.

Coarse knowledge is a special form of incomplete knowledge. The missing knowledge corresponds to fine distinctions which are not made. When knowledge about temporal events is coarse, it does not appear adequate to represent it in terms of disjunctions of finely grained alternative propositions: the alternatives can lie in the same ballpark of a conceptualization, they are "conceptual neighbors" introduced by Freksa in 1992 [8]. They allow processing coarse knowledge directly with a good advantage from a cognitive point of view.

To model imprecise and vague temporal knowledge the theory of Fuzzy Sets seems to be a more natural method: it characterizes the concepts in terms of fuzzy possibility distributions [27]. It allows to relate the qualitative linguistic terms to a quantitative interpretation, providing in this way a sort of interface between the qualitative and the quantitative levels of descriptions [10]. Moreover, it has been proven [6] that fuzzy methodologies are also adequate to represent the uncertain aspect that can affect knowledge.

In this paper, we study how to deal with these two orthogonal notions of coarseness and fuzziness in order to develop a more general way to represent imperfect temporal knowledge. To this aim, the previous points of view have been merged together: the neighborhoods represent the "horizontal" dimension [9] connecting cooperating and competing concepts, while the fuzzy membership values are the "vertical" dimension which connects the concepts with their definitions.

After having introduced classical algebras used in Temporal Reasoning (Section 2), namely Allen's Interval Algebra and Convex Point Algebra, in Section 3 the neighborhood relations of the Temporal Qualitative Algebra (i.e. their union) are identified starting from Freksa's Interval Algebra neighborhood relations. A new Neighborhoods Qualitative Algebra nQA is defined. In the same section we discuss about tractability of nQA. Then, following the vertical dimension, we extend the nQA algebra using the notion of fuzziness: membership degrees are added to the nQA

relations defining a new nQA^{fuz} algebra. Applying the metholodology of α-cut, already exploited in [3], the conditions of its tractability are studied.

2 Qualitative Temporal Algebras

In the literature, there are three choices regarding the primitive for the ontology of time: (1) instantaneous points, (2) durative intervals and (3) both points and intervals [16, 15]. Temporal Qualitative Reasoning is typically realized in form of calculi over sets of relations, for example over the 13 Allen's relations [1] for reasoning about intervals (that constitute the Allen's Interval Algebra IA) or over the set $\{<, >, =\}$ to reason about points (Point Algebra PA, [25]). Indeterminacy in these calculi is usually expressed by means of disjunctions of atomic relations, and so a general relation can be written as

$$\{rel_1, \ldots, rel_m\}$$

where each rel_i is an atomic relation. It is worth noting here that expressing indeterminacy by means of disjunctions is little plausible from a cognitive point of view, since as indeterminacy grows, complexity of the relations increases as well, while in common life vague definitions are indeed used to avoid complex definitions of ill-known entities.

When the entities involved are both points and intervals, a set PI defined by Meiri [16] can be used. It is formed by 5 atomic point-interval relations b, s, d, f, a which stand for *before*, *starts*, *during*, *finishes* and *after* respectively. The union of IA algebra with PI and PA algebras is called the temporal Qualitative Algebra (QA).

2.1 Neighborhood relations

In many cases qualitative descriptions with indeterminate reference value work because potential reference candidates provide a neighborhood of similar values or, in terms of the terminology of Qualitative Reasoning, the values form a conceptual neighborhood [8]. To this aim Freksa proposes a generalization of Allen's algebra based on semi-intervals (beginning or ending of an event). This allows restricting Allen's algebra in a more plausible way, from a cognitive point of view: the less it is known, the simpler representation can be used, in contrast with disjunctions, that complicate the representation in case of uncertain knowledge.

From the same paper the following definitions are reported:

Definition 1. Two relations between pairs of events are *(conceptual) neighbors*, if they can be directly transformed into one another by continuously deforming (i.e. shortening, lengthening, moving) the events in a topological sense.

Table 1. Freksa's neighborhood relations.

Relation set	Name	Inverse
$\{b, m, o, fi, di, si, eq, s, d, f, oi, mi, a\}$	?	?
$\{b, m, o, fi, di, si, eq, s, d, f, oi\}$	*bd*	*db*
$\{o, fi, di, si, eq, s, d, f, oi, mi, a\}$	*db*	*bd*
$\{o, fi, di, si, eq, s, d, f, oi\}$	*ct*	*ct*
$\{b, m, o, fi, di\}$	*ol*	*yo*
$\{b, m, o, s, d\}$	*sb*	*sv*
$\{di, si, oi, mi, a\}$	*sv*	*sb*
$\{d, f, a, oi, mi\}$	*yo*	*ol*
$\{b, m, o\}$	*ob*	*ob*
$\{o, fi, di\}$	*oc*	*yc*
$\{o, s, d\}$	*bc*	*sc*
$\{fi, eq, f\}$	*tt*	*tt*
$\{si, eq, s\}$	*hh*	*hh*
$\{di, si, oi\}$	*sc*	*bc*
$\{d, f, oi\}$	*yc*	*oc*
$\{oi, mi, a\}$	*ys*	*ob*
$\{b, m\}$	*pr*	*sd*
$\{mi, a\}$	*sd*	*pr*

Definition 2. A set of relations between pairs of events forms a *(conceptual) neighborhood* if its elements are path-connected through "conceptual neighbor" relations.

Definition 3. Incomplete knowledge about relations is called *coarse knowledge* if the corresponding disjunction of at least two relations forms a conceptual neighborhood.

Freksa has identified 18 neighborhoods relations, reported in Table 1; two of these, $\{b, m\} \Leftrightarrow pr$ and $\{a, mi\} \Leftrightarrow sd$, do not appear in Allen's composition table, therefore just 29 entries (13 atomic relations + (18 - 2) Freksa's relations) are sufficient to form an algebra, in the following also referred as Freksa's Algebra (FA).

2.2 Tractability

A frequent issue in Qualitative Reasoning is to identify subsets of relations that lead to model *tractable* problems, that is problems that can be solved in polynomial time and therefore can be of practical use. Allen's polynomial time algorithm for Temporal Reasoning never infers invalid consequences from a set of assertions, but it does not guarantee that all the inferences that follow from the assertions are generated; thus the algorithm is incomplete. Vilain and Kautz [25] have in fact shown that computing the closure in the full interval algebra is an $\mathcal{NP}$-complete problem (which only can be solved in exponential time). For this reason, sub-algebras have been developed to restrict the complexity

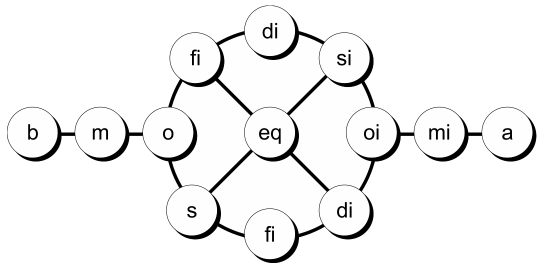

Figure 1. the neighborhood structure of type "A" [8].

of the full Interval Algebra. Nöekel [18] discusses a significant subset of Allen's algebra which has a tractable closure algorithm. This subset is defined by a convexity property and means that for any two interval end points belonging to a common semi-interval relation, intermediate end points belong to the relation as well. By this method, the continuous uncertainty property [24] generates the set of "convex interval relations" on the structure defined by the neighborhood of the relations (Figure 1 shows the type "A" structure). 82 of these neighborhoods are convex relations in this structure and form a tractable algebra, called SA_c. Recall that PA is tractable.

When defining such fragments, it is important to ensure that they are actually (sub-)algebras, namely closed under the operations of inversion, intersection and composition, i.e. that by applying these operations on relations belonging to a fragment relations in the same fragment are obtained. This ensures the applicability of constraint propagation algorithms to problems modeled by relations based on these sub-algebras. The maximal tractable sub-algebra of IA is the ORD-Horn Algebra $\mathcal{H}$ identified by Nebel [17].

3 Neighborhoods of the Qualitative Algebra

In this Section the neighborhoods of the Qualitative Algebra QA will be studied. To do this, the neighborhoods of all the algebras belonging to QA, that will be called nPA, nPI and nIA have to be investigated. In Figure 2 a general schema with all these algebras is shown.

In analogy with Freksa's Algebra FA, coarse relations based on the Point Algebra PA can be defined. Here just the convex PA, that is $PA_c = PA \backslash \{\neq\}$, will be considered. This novel algebra, called nPA is defined as follows:

Definition 4. nPA is the algebra formed by the set $\{<, >, =, pr, sd, ?\}$ where $pr = \{<, =\}$ and $sd = \{>, =\}$.

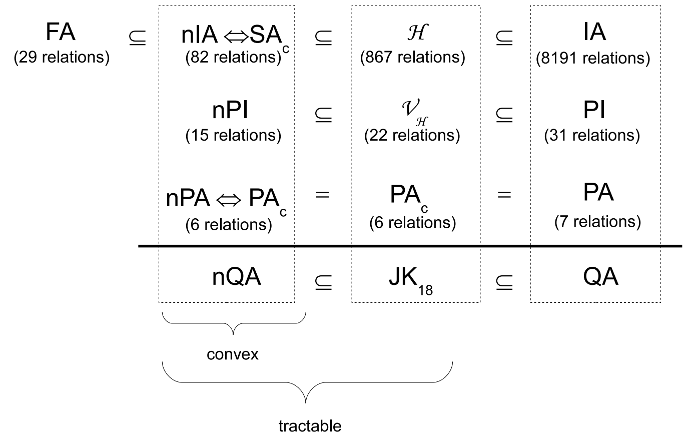

Figure 2. relations between the algebras cited in the paper (the empty set is not considered a relation to be counted).

Notice that nPA is equivalent to PA_c, in the sense that both contain relations that can be mapped into one another by semantic equivalences. Coarse relations of PA are intuitive, in fact they are also more commonly named as "less or equal to" ($\leq$) and "greater or equal to" ($\geq$).

Definition 5. The inverse of pr relation is sd relation, and vice versa.

Proposition 6. *nPA is an algebra closed under the operations of inversion, intersection and composition.*

Now it is possible to merge nPA point algebra with Freksa's interval algebra through a set of point-interval (PI) relations and discover which additional relations are needed for defining a set of coarse relations for QA; this new set will be named nPI. The new PI coarse relations have been computed automatically by imposing the closure of the canonical operations (*i.e.*: intersection and composition) using two algorithms implemented in Prolog, one for each operation. The pseudo-code of the algorithm for composition, the most complex among the two, has been reported in the listing named Algorithm 1; it uses a knowledge base composed by atomic and known coarse relations expressed by the predicate $relations(N)$, which identifies the set of relations starting from a name (*e.g.*: $relations(pr) = \{b, m\}$). From the composition table of QA atomic relations [16], all combinations can be checked by exploiting the following property of relation algebras [5]:

Proposition 7. *Given two relations $R = \{r_1, \ldots, r_n\}$ and $S = \{s_1, \ldots, s_m\}$*, the composition between R and S is $R \circ S = \{r_1 \circ s_1, \ldots, r_n \circ s_m\}$.

Algorithm 1 defining new coarse relations by composition

DCompose(N_1, N_2)
input: the names of the relations to be composed
output: the name of an existing relation or the atomic relations of a new one

```
/* this function computes the composition between an
atomic relation r1 and a general relation S, and adds the
result to the set T */
function f2(r1, S, T)
    if S = ∅ then
        return T
    else
        {s1, s2, ..., sm} = S
        t1 ← r1 ∘ s1 // lookup in the composition table
        T' ← T ∪ t1
        return c2(r1, {s2, ..., sm}, T')

/* this function computes the composition between two gen-
eral relations R and S, and adds the result to the set T */
function f1(R, S, T)
    if R = ∅ then
        return T
    else
        {r1, r2, ..., rn} = R
        t1 ← f2(r1, S, T)
        T' ← T ∪ t1
        return f1({r2, ..., rn}, S, T')

/* this function looks up the name of a coarse (or atomic)
relation starting from a set of relations R or emits a warning
and stops computation */
function identify(R)
    if ∃r(R) = N then
        return N
    else
        print('New relation found')
        stop

/* this function returns the set of atomic relations R corre-
sponding to the name N */
function relations(N)
    return R : r(R) = N

// main
R1 ← relations(N1)
R2 ← relations(N2)
Res1 ← f1(R1, R2, ∅)
Res2 ← sort(Res1)
return identify(Res2)
```

Table 2. new coarse relations for nPI.

Sets found (compos. and inters.)	Name	Inverse
$\{b, d, f, s\}$	na	nb
$\{a, d, f, s\}$	nb	na
$\{b, s, d\}$	bd	db
$\{a, d, f\}$	db	bd
$\{b, s\}$	pr	sd
$\{a, f\}$	sd	pr
$\{d, f, s\}$	ct	ct
$\{d, f\}$	yc	bc
$\{d, s\}$	bc	yc

The algorithms applied to nPA do not find additional relations. When applied to the set $\{nPA \cup PI_{atomic} \cup FA\}$, where $PI_{atomic} = \{b, s, d, f, a\}$, Algorithm 1 finds 6 new point-interval relations, while the algorithm for intersection finds 3 additional relations; these relations have been reported in Table 2 with their names; for example $na = \{b, d, f, s\}$ stands for "not after".

Definition 8. nPI is the set of qualitative temporal relations given by union of the 5 classical atomic PI relations with the set $\{nb, na, bd, db, pr, sd, ct, bc, yc\}$ (relations reported in Table 2).

Definition 9. The inverse relations of nb, na, bd, db, pr, sd, ct, bc, yc are the relations na, nb, db, bd, sd, pr, ct, yc, bc respectively.

Once nPA and nPI have been defined, FA has been considered. Six additional relations for intervals have been characterized by composition (Table 3); pr and sd relations, which were not needed to close FA, are now required too. The algorithm for intersection applied to the interval-interval relations finds 45 additional coarse relations (Table 4) that complete the algebra of relations between intervals. It has to be noticed that the final set of relations obtained in this way is formed by 82 relations, which can be mapped to SA_c, by semantic equivalences.

Definition 10. nIA is the set of qualitative temporal relations given by union of the 13 Allen's atomic relations, the 18 Freksa's coarse relations, the set of 6 coarse relations listed in Table 3 and the set of 45 coarse relations listed in Table 4.

Definition 11. The inverse relations of pr, sd, nyo, nol, nsv, nsb, na, nb are the relations sd, pr, nol, nyo, nsb, nsv, nb, na respectively.

Table 3. new coarse relations for FA found by composition.

Sets found	Name	Inverse
$\{a, d, di, eq, f, fi, m, mi, o, oi, s, si\}$	nb	na
$\{b, d, di, eq, f, fi, m, mi, o, oi, s, si\}$	na	nb
$\{b, di, eq, fi, m, o, s, si\}$	nyo	nol
$\{b, d, eq, f, fi, m, o, s\}$	nsv	nsb
$\{a, di, eq, f, fi, mi, oi, si\}$	nsb	nsv
$\{a, d, eq, f, mi, oi, s, si\}$	nol	nyo

Table 4. 45 new coarse relations for FA found by intersection.

ol_nsb	yo_na	sv_nol
ol_nsv	sb_nyo	sv_na
ol_nb	sb_nol	bd_nol
hh_nsb	sb_nb	bd_nsb
hh_nsv	tt_nyo	bd_nb
yo_nsb	tt_nol	ct_nyo
yo_nsv	sv_nyo	ct_nsv
db_na	nyo_nsb	nsv_na
ob_nb	nyo_nsv	nb_na
oc_nsv	nyo_nb	ol_nsv_na
sc_nol	nol_nsb	yo_nsb_na
bc_nyo	nol_nsv	sb_nyo_nb
yc_nsb	nol_na	sv_nol_na
ys_na	nsb_na	bd_nol_nsb
ct_nyo_nsv	nyo_nsv_na	nol_nsb_na

The inverse relations of the 45 relations reported in Table 4 can be obtained by inverting the relations involved.

Example 12. The inverse of the relation sb_nyo_nb is the relation sv_nol_na.

The union of nPA with the new sets of coarse relations allows combining Freksa's coarse interval-interval relations with points: the new algebra will be called nQA.

Definition 13. nQA is the set of qualitative temporal relations given by union of nIA, nPI and nPA.

Proposition 14. *The set nQA is closed under inversion, intersection and composition, i.e. it is an algebra.*
The algebra nQA is tractable.

Proof. The Algebra nQA is, by definition, the union of nIA, nPI and nPA, therefore any relation belonging to nQA belongs to one of that three sets. nIA is tractable, because it is equivalent to SA_c, nPA is tractable because it is equivalent to the Convex Point Algebra PA_c, nPI is a subset of the tractable set $\mathcal{V}_{\mathcal{H}}$ defined by Jonsson and Krokhin [13] proposed to integrate punctual events in the Nebel's ORD-Horn Algebra [17]. □

The previous proposition can be also proven by noting that nQA is a subset of the 18th tractable algebra found by Jonsson and Krokhin in [13].

4 Fuzzy Qualitative Algebras

Zadeh has introduced the notion of "fuzzy set" to represent concepts whose boundaries are ill-defined. Given a set F defined over a referential set U, its characteristic function $\chi_F : U \rightarrow \{0, 1\}$ has been generalized into a so-called membership function $\mu_F : U \rightarrow [0, 1]$. The membership degree of $x \in F$ is represented by $\mu_F(x)$. It is possible to assign preference degrees by means of membership functions to the classical qualitative algebras.

4.1 Fuzzy Interval Algebra and Fuzzy Qualitative Algebra

Allen's Interval Algebra has been extended to the framework of Possibility Theory by several authors ([7, 12, 19, 4, 21]). Here, the approach proposed in [4] is considered: a degree $\alpha_i \in [0, 1]$ has been attached to every atomic relation rel_i. It indicates the *preference degree* of the corresponding assignment among the others

$$I_1 \; R \; I_2 \text{ with } R = (rel_1[\alpha_1], \ldots, rel_{13}[\alpha_{13}])$$

where α_i is the preference degree of rel_i $(i = 1, \ldots, 13)$.

In this framework, called IA^{fuz}, different types of temporal constraints can be represented *e.g.* soft constraints, that allow expressing preferences among solutions, prioritized constraints, where the priority indicates how essential it is that a constraint be satisfied, or uncertain constraints. Meiri's Qualitative Algebra has been extended to the fuzzy case in a similar way in [2]; this extension is called QA^{fuz}.

4.2 Tractable Fuzzy Algebras

The tractable fragments of the classical QA identified in [14] can be extended to the fuzzy case exploiting the fact that in fuzzy theory a property holds if it is valid for each α-cut [26], whose definition is:

Definition 15. Given a fuzzy set defined by the membership function over a generic domain D $\mu : D_1 \times \ldots \times D_k \rightarrow [0, 1]$, and given a real number $\alpha \in [0, 1]$, the α-cut of R^{fuz} is the crisp set $R_\alpha = \{d \in D_1 \times \ldots \times D_k : \mu(d) \geq \alpha\}$.

By defining the tractable fragments in such a way that their α-cuts are tractable classical subsets the tractability property is ensured [3].

5 Neighborhood Relations of the Fuzzy Temporal Qualitative Algebra

To develop a mode general model for representing imperfect temporal information it is now useful to add a vertical dimension taking into account other forms of imperfection different from coarseness. To this aim it is possible to give a fuzzy extension of nQA while maintaining tractability. This new subclass of QA^{fuz} allows combining the "vertical" and the "horizontal" aspects of imperfect information.

The previously cited method based on α-cuts is exploited, however a particular attention has to be devoted when assigning the preference degrees to the relations: in fact, it is not sufficient to restrict all the α-cuts of a given set to belong to the classical tractable nQA, as in the case of IA^{fuz}, where all combinations of atomic relations are possible. In nQA^{fuz} not all combinations are allowed, otherwise sets belonging to upper α-cuts could have relations which are not present in lower α-cuts (see Figure 3). It is necessary to guarantee that upper α-cuts are subsets of lower α-cuts.

This problem was not addressed in a similar work proposed by Guesgen's [11], since there preference degrees were assigned by the user. Here, instead, an additional condition is needed, and to avoid this problem the definition for nQA^{fuz} becomes:

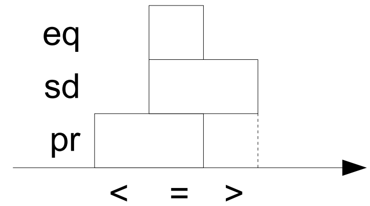

Figure 3. problematic hypothetical nQA^{fuz} relation

Definition 16. nQA^{fuz} is a set of fuzzy relations $\{R^{fuz} : \forall \alpha_i, \alpha_j \in [0, 1], \alpha_i \geq \alpha_j \quad (R_{\alpha i}, R_{\alpha j} \in nQA) \wedge (R_{\alpha i} \subseteq R_{\alpha j})\}$.

Definition 17. nIA^{fuz} the subset of nQA^{fuz} involving interval-interval relations.

nIA^{fuz} is semantically equivalent to the SA_c^{fuz} algebra defined in [4], the only difference is that inferences in the first "interpretation" can be performed *directly* on coarse relations, and not composing disjunctions of atomic relations.

Definition 18. nPI^{fuz} is the subset of nQA^{fuz} involving point-interval and interval-point relations.

Definition 19. nPA^{fuz} is the subset of nQA^{fuz} involving point-point relations.

Proposition 20. *The algebra nQA^{fuz} is tractable.*

Proof. nQA^{fuz} is a subset of the tractable algebra JK_{18}^{fuz} identified in [3]. □

A method to ensure the inclusion of upper α-cuts in lower α-cuts is to create a hierarchy of nQA relations, Schilder has proposed such a hierarchy for SA_c exploiting relations between endpoints [20].

In this paper the hierarchies for nPA and nPI are proposed; they have been represented in Figures 4 and 5.

A well formed nQA^{fuz} relation can be always be interpreted as an IA^{fuz} relation, but the vertical bars that represent preference degrees can, in this case, also be drawn as horizontal bars that represent neighborhood relations (Figure 6).

Such a bi-dimensional representation is sufficient for visualizing nQA^{fuz} subsets involving point-point or point-interval relations, that is nPI^{fuz} and nPA^{fuz} relations, while the subset involving interval-interval relations (*i.e.* nIA^{fuz} relations) needs a 3D graph, which, for example, could be based on the neighborhood structure (Figure 7). A nQA^{fuz} relation can therefore be defined by means of its α-cuts, and preference degrees can refer to these, instead of being associated to each atomic relation.

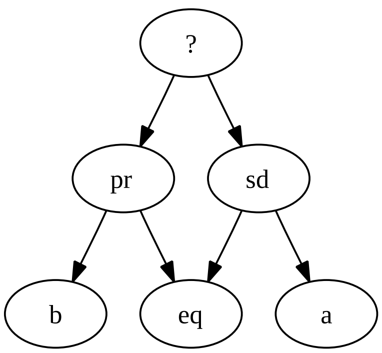

Figure 4. a hierarchy for nPA_c relations

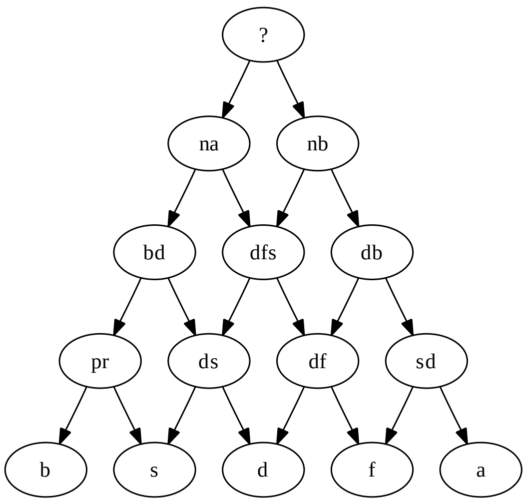

Figure 5. a hierarchy for nPI relations

Example 21. The PA^{fuz} relation $\{<[\alpha_1], =[\alpha_2], >[\alpha_3]\}, \alpha_1 > \alpha_2 > \alpha_3$ represented on the left of Figure 6 is a well-formed nPA^{fuz} relation and it is written as $\{<[\alpha'_1], pr[\alpha'_2], ?[\alpha'_3]\}, \alpha'_i = \alpha_i \ \forall i \in \{1, 2, 3\}$. This relation is represented on the right of the same Figure.

These hierarchies have the additional advantage that they allow, in a sense, to identify multiple levels of coarseness (*e.g.:* relation bd is coarser than relation ds, which in turn is coarser than relation s) and to reason using these different levels grouped by the preference degrees. Therefore, the preference degrees themselves can be assigned referring to the hierarchies.

Example 22. In the classical Meiri's scenario about John and Fred there are 3 types of qualitative relations involved: s, f and $\{s, si, d, di, f, fi, o, oi, eq\}$. The last set is semantically equivalent to the coarse relation ct. It is possible to relax these qualitative constraints in lower α-cuts by following the links in the hierarchies. For instance, s can be relaxed in $\{s[1.0], bd[0.7]\}$, f in $\{f[1.0,]db[0.7]\}$ and ct in

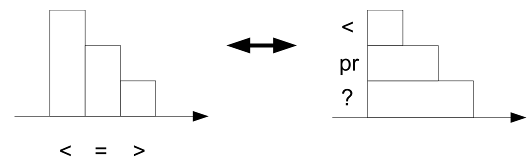

Figure 6. from fuzzy relations to fuzzy neighbor relations

$\{ct[1.0,]nb \wedge na[0.8]\}$ (for this hierarchy refer to Figure 5 of Schilder's paper [20]); in this way the problem becomes less "rigid" and additional less preferred solutions can be possibly obtained.

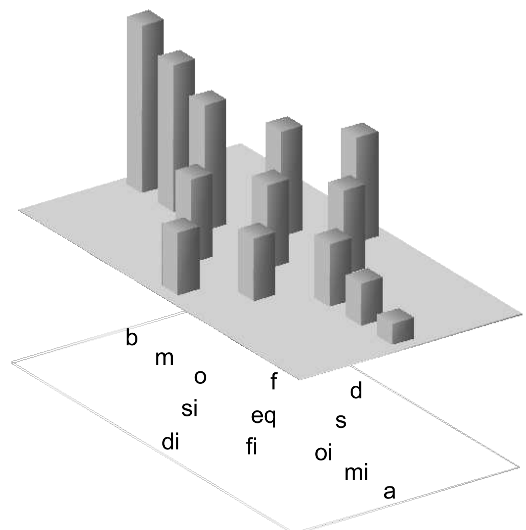

Figure 7. visualization of a nIA^{fuz} relation $\{?[0.1], nb[0.2], nsb[0.3], yo[0.4], ys[0.5], pr[0.6], b[0.7]\}$ using the neighborhood structure.

6 Conclusions

The concepts of neighborhood and fuzzy relation have been combined in this study and two new tractable subalgebras, nQA and its fuzzy extension nQA^{fuz}, have been defined. nQA allows reasoning with intervals and points in a coarse way, nQA^{fuz} adds to it the possibility to perform both a "horizontal" reasoning about similar relations and a "vertical" reasoning connecting concepts with their definitions.

Now, a possible direction is to consider how such rich fuzzy relations can be expressed in a more human friendly way, in order to exploit their full potential, for example using simple sentences which could possibly be re-interpreted back in terms of nQA^{fuz} relations. Another direction, which is currently under consideration, is the integration of the new coarse relations into the system FTR [2] and their interactions with fuzzy metric constraints.

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