A Theory of Time and Temporal Incidence based on Instants and Periods*

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Abstract

Time is fundamental in representing and reasoning about changing domains. A proper temporal representation requires characterizing two notions: (1) time *itself, and (2)* temporal incidence, *i.e. the domain-independent properties for the truth-value of fluents and events throughout time. There are some problematic issues such as the expression of* instantaneous events *and* instantaneous holding of fluents, *the specification of the properties for the temporal holding of fluents and the* Dividing Instant Problem.

This paper presents a theory of time and temporal incidence which is more natural than its predecessors and satisfactorily addresses the issues above. Our theory of time, called $\mathcal{IP}$, is based on having instants and periods at equal level. We define a theory of temporal incidence *upon it whose main original feature is the distinction between continuous and discrete fluents.*

1 Introduction

In order for an intelligent system to interact with the real world it needs to be able to reason about the changes that happen in it and the events and actions that originate them. Consider, as an illustrative example, the so-called *hybrid systems* [10]. These are systems that involve both discrete and continuous change. For instance many electro-mechanical devices exhibit both continuous (e.g. the charge in a battery) and discrete behavior (e.g. a digital signal), and involve events that can be viewed as instantaneous (e.g. to close a relay) and others that take time (e.g. recharge the battery)[1].

The notion of *Time* has been recognized as a fundamental in formalizing change and action. Many theories for change and action are built upon a theory of time [17, 1, 14, 25, 20, 15, 4, 6, 7, 19, 13, 24]. In these systems, the domain at hand is formalized by expressing how propositions are true or false throughout time. Commonly there is a distinction between propositions describing the state of the world (*fluents*) and those describing the occurrences that make the world change (*events*). For these frameworks, that claim to have a wide range of practical applicationsin AI, an appropriate and precise theory of time is a fundamental component.

To enjoy a proper temporal representation, two notions need to be defined:

1. *time* itself, and
2. *temporal incidence*, i.e. the domain-independent properties for the truth-value of fluents and events throughout time.

The aim of this paper is to provide a natural theory of time and temporal incidence that supports the formalization of changing domains where discrete and continuous phenomena occur. There are a number of problematic issues that have been encountered by previous attempts. Namely, the expression of *instantaneous events* and *instantaneous holding of fluents*, the specification of the properties for the temporal holding of fluents and the *Dividing Instant Problem.*

This paper presents a theory of time and temporal incidence which can be used as a formal ground to model changing domains. Our theory of time is based on both instants and periods[2] together at equal level. We call it $\mathcal{IP}$. We define a theory of temporal incidence upon $\mathcal{IP}$. Its major original feature is the distinction between continuous and discrete fluents. Although the difference between them is something commonly agreed, there is no previous attempt were the specific features where they differ and its close relation with the theory of time have been accounted properly. It is a simple idea yet combined with the instant/period ontology turns out to be sufficient to satisfactorily address the problems encountered by previous approaches.

Section 2 presents some problematic issues to be considered. Section 3 discusses the shortcomings of previous approaches. Section 4 presents $\mathcal{IP}$. Section 5 presents our categorization of propositions and the theory of their temporal incidence. In section 6 we discuss how the above problems are satisfactorily addressed and section 7 presents an example where the

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[1]Hybrid systems are interesting because many real systems can be modeled as such.

[2]By period we merely mean a *time interval.*

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behavior a hybrid model is described using our approach. Finally section 8 summarizes our contribution.

2 Problematic Issues

Instantaneous events. There are many events like "turn off the light", "shoot the gun", "start moving" which intuitively are viewed as instantaneous. Modeling them can be controversial since null duration time elements seem to be needed to talk about them. Moreover, some problems arise when we need to model *complex sequences* of them occurring in presence of continuous change (we discuss it in detail in section 7).

Instantaneous fluent holding. Modeling *continuous change*, i.e. taking account of fluents whose value is continuously changing, involves representing parameters whose value may hold for only a single instant. A simple, representative example is the parameter "speed" of a ball tossed upwards in what we call it the *Tossed Ball Scenario* (TBS) (see figure 1).

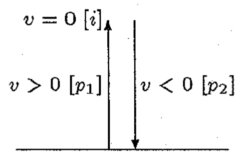

Figure 1: The Tossed Ball Scenario (TBS).

There must be an instant where the speed of the ball is zero, being not zero for a period before and a period after it. To model it we require the ability of talking about the holding of fluents at instants. However, it may lead to the *dividing instant problem* below.

Non-instantaneous fluent holding. Some difficulties arise when defining the properties of temporal incidence for non-instantaneous fluents such as the following:

- *Homogeneity*: If a fluent is true on a piece of time it must hold on any subtime [1, 7].
- *Concatenativity*[3]: If a fluent is true on two consecutive pieces of time it must be true on the piece of time obtained by concatenating them. Notice that there may be different views for the meaning of "consecutive".

[3] This property, in addition to being a semantic issue, may be important for computational efficiency since it allows to have a compact representation of fluents holding throughout numerous consecutive or overlaping periods.

Non-atomic fluents. In some cases, axiomatizing the holding of *non-atomic fluents* such as negation, conjunction or disjunction of atomic fluents may not be straight forward [20, 7].

Dividing Instant Problem (DIP). Assuming that time is made of instants and periods, we need to determine the truth-value of a fluent f (e.g. "the light is on") at an instant i, given that f is true on a period p_1 ending at i and it is false at a period p_2 beginning at it (see figure 2) [8, 21, 1, 7].

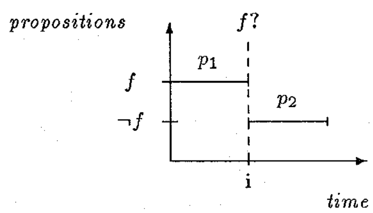

Figure 2: The Dividing Instant Problem (DIP).

If we want to be logically consistent the problem arises. If intervals are closed then f and $\neg f$ are both true at i. If they are open we might have a "truth gap" at i. The other two options are open/closed and closed/open intervals. An arbitrary decision here is artificial. Take, for instance, the fluent "being in contact with the floor". It seems that p_1 is closed/open interval whereas p_2 is open/closed.

3 Related Work

In this section we briefly summarize the main criticisms to previous approaches in terms of the issues above.

Instant-based approaches [17, 20, 4] They have been criticized for not being natural ("our direct experience is with phenomena that take time"), for being counterintuitive [8, 12] and for getting in trouble with the DIP [8, 21, 1].

Allen's interval-based approach [1]

- Neither *instantaneous holding* of fluents nor *instantaneous events* can be represented, the reason being that there are no instants.
- Characterizing holding of continuously changing fluents exhibits semantical problems. Specificly, the axiom for "homogeneity of fluent holding" (axiom H.2) and the axiom for "holding of negated fluents" (axiom H.4) do conflict. This and the previous problem are precisely presented in [7].

Allen and Hayes [2] They propose a theory of intervals but they suggest various ways of deriving instants from them. One type is defined as *nests* of intervals. These "instants", however, are only used for instantaneous events but not for fluents. They write ([2], section 4) "...resolutely refusing to allow fluents to hold at points. One could define a notion of a fluent X being true at a point p by saying that X is true at p just when there is some interval I containing p during which X is true". This is again not satisfactory for modeling continuous fluents (consider the "zero speed" fluent in the TBS).

The other type, called *moment*, is defined as an indivisible period. This is not adequate either because, although very short, it takes some time. In the TBS, for example, we cannot use a moment to talk about the time where the ball speed is zero because then the periods p1 and p2 would not meet. This would entail that the ball is hovering in the air for a while. The technique of *change of granularity*[9] applied to fix this shortcoming is problematic too since it leads to unintuitive and non-uniform models of time and introduces unnecessary technical complications [22].

Galton's instant-period theory [7] Galton's approach presents two central characteristics:

1. It is based on a time ontology with instants and periods *on the same footing*. The theory of time is not based on the classical set-theoretic interval constructions in order to avoid the DIP.
2. It diversifies fluents into instantaneous/durable and *states of position/states of motion*: a state of position can hold at isolated instants; if it holds during a period it holds at its limits (e.g. "a quantity taking a particular value"); a state of motion cannot hold at isolated instants (e.g. "a body's being at rest").

The main shortcomings of this approach are the following:

1. The theory formed by the axioms is too weak for a proper account of the relations between instants and periods. We discuss it in detail in section 4.
2. It's not clear that Galton's new types of fluents are useful. Let us try to formalize the TBS: we consider the fluent speed $\neq 0$; if we model it as a state of position then we get in trouble; speed $\neq 0$ holds on p_1 and p_2 which must contain the limiting instants including i where speed$=0$; If we consider it an state of motion then we are not allowed to say that $\neg$speed $\neq 0$ is true at the isolated instant i.
3. While states of position are concatenable states of motion are not always. It looks somewhat counter-intuitive: it seems that states of position should not be concatenable since the quantity they represent may have a different value at the meeting point. Since it is not the case for states of motion it seems that they should be concatenable. In section 5 we shall follow this intuition.

4 Time

In this section we present our theory of time, called $\mathcal{IP}$, based on the idea of having instants and periods at equal level[4][5]. We are lead by the intuition that a period is characterized by its two endpoints.

Our language for time has two sorts of symbols, the *instants sort* ($\mathcal{I}$) and the *periods sort* ($\mathcal{P}$), which are formed by two infinite disjoint sets of symbols, and three primitive binary relation symbols, $\prec: \mathcal{I} \times \mathcal{I}$ and $\mathtt{begin}, \mathtt{end} : \mathcal{I} \times \mathcal{P}$.

The first-order logical formulation of $\mathcal{IP}$ consists of the following axioms:

$\mathbf{IP_1}$	$\neg(i \prec i)$
$\mathbf{IP_2}$	$i \prec i' \Rightarrow \neg(i' \prec i)$
$\mathbf{IP_3}$	$i \prec i' \wedge i' \prec i'' \Rightarrow i \prec i''$
$\mathbf{IP_4}$	$i \prec i' \vee i \prec i' \vee i = i'$
$\mathbf{IP_{5.1}}$	$\exists i' \ (i' \prec i)$
$\mathbf{IP_{5.2}}$	$\exists i' \ (i \prec i')$
$\mathbf{IP_6}$	$\mathtt{begin}(i,p) \wedge \mathtt{end}(i',p) \Rightarrow i \prec i'$
$\mathbf{IP_{7.1}}$	$\exists i \ \mathtt{begin}(i,p)$
$\mathbf{IP_{7.2}}$	$\exists i \ \mathtt{end}(i,p)$
$\mathbf{IP_{8.1}}$	$\mathtt{begin}(i,p) \wedge \mathtt{begin}(i',p) \Rightarrow i = i'$
$\mathbf{IP_{8.2}}$	$\mathtt{end}(i,p) \wedge \mathtt{end}(i',p) \Rightarrow i = i'$
$\mathbf{IP_9}$	$i \prec i' \Rightarrow \exists \ p \ (\mathtt{begin}(i,p) \wedge \mathtt{end}(i',p))$
$\mathbf{IP_{10}}$	$\mathtt{begin}(i,p) \wedge \mathtt{end}(i',p) \wedge$ $\wedge \ \mathtt{begin}(i,p') \wedge \mathtt{end}(i',p') \Rightarrow p = p'$

$IP_1 \div IP_4$ are the conditions for $\prec$ to be an *strict linear order* –namely irreflexive, asymmetric, transitive and linear– relation over the instants[6] IP_5 imposes unboundness on this ordered set. IP_6 is intended to *order* the extremes of a period. This axiom rules out durationless periods which are not necessary since we have instants as a primitive. Thus an instant cannot be identified as a null duration period. The pairs of axioms $IP_{7.\_}$ and $IP_{8.\_}$ formalize the intuition that the beginning and ending instants of a period always *exist* and are *unique* respectively. Conversely, axioms IP_9 and IP_{10} are intended to ensure the *existence* and *uniqueness* of periods to close the connection between instants and periods.

We now characterize the models by following the simple intuition of an interval being an ordered pair.

Definition 1 **($\mathcal{IP}$-structure)** *An $\mathcal{IP}$-structure is a tuple $\langle \mathcal{I}_d, \mathcal{P}_d, <_d, \mathtt{begin}_d, \mathtt{end}_d \rangle$ where $\mathcal{I}_d$ and $\mathcal{P}_d$ are sets of instants and periods respectively, $<_d$ is a binary relation on $\mathcal{I}_d$ and $\mathtt{begin}_d, \mathtt{end}_d$ are binary relations on $\mathcal{I}_d, \mathcal{P}_d$.*

We merely view periods as ordered pairs of instants. We show that the elements and the pairs of an unbounded linear order $\mathcal{S}$ form a model for $\mathcal{IP}$.

[4] Very much with in the same spirit of [7] and [5].

[5] Results on $\mathcal{IP}$ are a summary of those presented in [22, 23].

[6] Notice that IP_1 is actually redundant since it can be derived from IP_2. We include it for clarity.

Theorem 1 (a model) *Given an infinite set $\mathcal{S}$ and an unbounded strict linear order $<$ on it then the $\mathcal{IP}$-structure $\langle \mathcal{S}, \mathtt{pairs}(\mathcal{S}), <, \mathit{first}, \mathit{second}\rangle$ forms a model of $\mathcal{IP}$.*

Corollary 1 *Every model of $\mathcal{IP}$ is characterized by an infinite set $\mathcal{S}$ and a nunbounded strict linear order $<$ on it.*

Remark 1 *$\mathcal{IP}$ accepts both dense and discrete models of time.*

The subtheory of dense models, we call it $\mathcal{IP}_{dense}$, is axiomatized by adding the following *density* axiom:

$$\mathbf{IP_{11}} \quad i \prec i' \Rightarrow \exists i'' \, (i \prec i'' \prec i')$$

Corollary 2 (dense models) *The models of $\mathcal{IP}_{dense}$ are characterized by the set of elements and the set of ordered pairs of distinct elements of an unbounded, "dense", strict linearly ordered set. Hence $\mathcal{IP}_{dense}$ is an axiomatization of $Th(\langle INT(Q), Q\rangle)$.*

Let us see how our theory relates to previous ones. The original Allen and Hayes's theory (let us call it $\mathcal{I}_{\mathcal{AH}}$) [2] is exclusively based on intervals. For the sake of comparison, we can introduce instants using the following technique used by Ladkin [16]: we take pairs of intervals one meeting the other, apply the equivalence relation "having the same meeting point" and associate a instant to each class Let us call this extension $\mathcal{I}_{\mathcal{AH}_{\sim\mathcal{I}}}$.

We obtain a theory whose class of models is the same as our instant-period axiomatization, i.e. the theories are equivalent.

Theorem 2 $\mathcal{IP} \equiv \mathcal{I}_{\mathcal{AH}_{\sim\mathcal{I}}}$

Galton proposes [7] a theory of instants and periods "on the same footing". The axioms seem appropriate to avoid DIP-like criticisms and to specify the properties of the various types of fluents introduced. However, the theory looks very artificial. It is difficult to figure out the intuition that some of they intend to capture and it complicates the proofs of theorems. A more serious criticism is the fact that the theory is *too weak*. It is easy to identify examples of counter-intuitive models accepted by the theory:

Example 1 *Let us take a basic model M composed of an infinite set of periods and Allen's relations satisfying interval axioms plus an infinite set of instants which make M satisfy I_1 -for example $INT(Q)$ as periods and Q as instants:*

- *Example model 1: M plus a single instant $i \notin Q$ which limits a certain period p in M and only that one. In particular it does not limit any of those periods that meet or are met by p.*
- *Example model 2: M plus a single instant $i \notin Q$ which limits a certain period p in M and is not within any period. In particular it is not within any of those periods that overlap p.*

The reason of this weakness is the too loose connection between instants and periods. It can be properly extended to fit the idea of having an instant at the meeting points [22]. The resulting theory is equivalent to $\mathcal{IP}_{dense}$.

5 Temporal Incidence

In this section we present a theory of the ascription of fluents and occurrences in time. The key issue to consider is the temporal incidence of a fluent at the extremes of a period. Our approach is based on the following ideas:

1. *We allow fluents to hold at points.* We demonstrate that this does not, in fact, create any problem.
2. *We distinguish between continuous and discrete fluents.* We diversify fluents according to whether the change on the parameter they model is *continuous* or *discrete*.

We use a standard reified temporal first-order language with equality (as in [17, 1, 7]). The decisions made regarding temporal representation are the following:

- **Time theory**: We take $\mathcal{IP}$. We define the instant-to-period (such as `within`) and period-to-period (such as `Meets`) relations upon $\leq$, `begin` and `end` for convenience.
- **Reified propositions**: We use standard first-order language forms although we shall not deal with quantified expressions. Propositions are classified into three classes: *continuous fluents*, *discrete fluents*[7] and *events*.
- **Temporal Occurrence Predicates (TOPs)**. We shall introduce a different TOP for each combination of temporal proposition and time unit (similar to [14, 7]):

$\textsc{Holds}^{\sim}_{on}(f,p) \equiv$	The continuous fluent f holds throughout the period p
$\textsc{Holds}^{\neg}_{on}(f,p) \equiv$	The discrete fluent f holds throughout the period p
$\textsc{Holds}^{\sim}_{at}(f,i) \equiv$	The continuous fluent f holds at the instant i
$\textsc{Holds}^{\neg}_{at}(f,i) \equiv$	The discrete fluent f holds at the instant i
$\textsc{Occurs}_{on}(e,p) \equiv$	The event e occurs on the period p
$\textsc{Occurs}_{at}(e,i) \equiv$	The event e occurs at the instant i

[7] We use the equality relation to express a fluent representing a parameter taking a certain value. E.g. the speed of a ball being positive on p is expressed as $\textsc{Holds}(\text{speed} = +, p)$. We omit necessary axioms imposing the exclusivity among the different values of a parameter.

Terminology. Henceforth we use the following notational shorthands. We may use **begin** and **end** in functional form (e.g. $i = \mathtt{begin}(p)$). $\textsc{Holds}_{on}$ stands for both $\textsc{Holds}_{\tilde{on}}$ and $\textsc{Holds}_{\overset{\neg}{on}}$ and $\textsc{Holds}_{at}$ stands for both $\textsc{Holds}_{\tilde{at}}$ and $\textsc{Holds}_{\overset{\neg}{at}}$. We define $\mathtt{within}(i,p) \equiv \mathtt{begin}(i',p) \wedge \mathtt{end}(i'',p) \Rightarrow i' \prec i \prec i''$ and, given p i p' such that $\mathtt{Meets}(p,p')$, $p'' = \mathtt{Concat}(p,p') \equiv \mathtt{begin}(p'') = \mathtt{begin}(p) \wedge \mathtt{end}(p'') = \mathtt{end}(p')$. We use its functional form too.

5.1 Properties of Temporal Incidence

Since instants and periods are defined at the same primitive ontological level in $\mathcal{IP}$, we are not forced to accept any assumption on the relation between the holding of a fluent on a period and its holding at the period endpoints. A fluent holds during a period if and only if it holds at the *inner* instants:

$$\textsc{Holds}_{on}(f,p) \iff (\mathtt{within}(i,p) \Rightarrow \textsc{Holds}_{at}(f,i)) \quad (1)$$

Nothing can be inferred about its holding at its endpoints. Either the fluent or its negation may hold at them. Our opinion is that this is domain-dependent. The fluent holding at the extreme point can go either way: (1) It can be the fluent finishing at that point (an example is perhaps "shoot the gun"), (2) the fluent starting at it (an example might be "start moving"), or (3) a different fluent representing the state of changing (the example here could be "turn on the light"). By keeping them independent we already avoid some of the problems in section 2 as we shall see in a moment.

Continuous fluents. A continuous fluent may hold both during a period and at a particular instant without any restriction. This is not the case for discrete ones.

Discrete fluents. The genuine property of discrete fluents is that *they cannot hold at an isolated instant*:

$$\textsc{Holds}_{\overset{\neg}{at}}(f,i) \Rightarrow \exists p\ (\textsc{Holds}_{\overset{\neg}{on}}(f,p) \wedge (\mathtt{within}(i,p) \vee \mathtt{begin}(i,p) \vee \mathtt{end}(i,p))) \quad (2)$$

Our distinction between continuous and discrete events do not fit with Galton's distinction between states of position and states of motion. However discrete fluents correspond to Shoham's *concatenable* proposition types. Identifying it as a key property in modeling changing domains is one of this paper's contributions.

Events. Unlike previous approaches, our proposal does not include any specific axiom governing the occurrence of events. It corresponds to the intuition that events represent an *accomplishment* which may concurrently happen. For example, one may accomplish producing a software module on the period "Jan/1st-Dec/31st" and accomplish producing another module on the period "Jul/1st-Jul/31st".

Non-atomic fluents.

Negation:

$$\textsc{Holds}_{at}(f,i) \iff \neg\textsc{Holds}_{at}(\neg f,i) \quad (3)$$

Conjunction:

$$\textsc{Holds}_{at}(f \wedge f',i) \iff \textsc{Holds}_{at}(f,i) \wedge \textsc{Holds}_{at}(f',i) \quad (4)$$

Disjunction:

$$\textsc{Holds}_{at}(f \vee f',i) \iff \textsc{Holds}_{at}(f,i) \vee \textsc{Holds}_{at}(f',i) \quad (5)$$

6 Revisiting the Problems

Let us now see how the problems presented in section 2 are addressed.

Instantaneous events. Since we have instants in our ontology, we can express them straight forwardly using $\textsc{Occurs}_{at}$ at an instant. In the DIP, for instance, we have $\textsc{Occurs}_{at}(\text{switchoff}, i)$. In section 7 we discuss the issues regarding sequences of events.

Instantaneous holding. Having instants in our ontology allows us to talk about instantaneous holding of a fluent. The underlying view of "open intervals" (axiom 1) ensures that there will be no conflict with the holding of that fluent on meeting periods. For discrete fluents it only can happen if it holds on a meeting period (as enforced by axiom 2). In the case of a continuous fluent there is no restriction and it can be expressed using the $\textsc{Holds}_{\tilde{at}}$ predicate. In the TBS, for instance, the speed of the ball is a continuous fluent and the scenario is simply expressed as:

$$\begin{array}{ll} \textsc{Holds}_{\tilde{on}}(\text{speed} = +, p_1) & \\ \textsc{Holds}_{\tilde{at}}(\text{speed} = 0, i) & \mathtt{end}(p_1) = i = \mathtt{begin}(p_2) \\ \textsc{Holds}_{\tilde{on}}(\text{speed} = -, p_2) & \end{array}$$

Non-instantaneous fluent holding. Some of the fundamental properties of previous approaches are either theorems of our theory or they are very easy to specify. For instance, *Allen's Homogeneity* $\textsc{Holds}_{\tilde{on}}(f,p) \iff \mathtt{In}(p',p) \Rightarrow \textsc{Holds}_{\tilde{on}}(f,p')$ is easy to prove. The properties for concatenativity are as follows:

Theorem 3 (Concatenativity of discrete fluents)
If $\mathtt{Meets}(p,p')$ *then*

$$\textsc{Holds}_{\overset{\neg}{on}}(f,p) \wedge \textsc{Holds}_{\overset{\neg}{on}}(f,p') \iff \textsc{Holds}_{\overset{\neg}{on}}(f,\mathtt{Concat}(p,p')) \quad (6)$$

Theorem 4 (Concatenativity of continuous fluents)
If $\texttt{Meets}(p,p')$ *then*

$$\textsc{Holds}^{\sim}_{on}(f,p) \wedge \textsc{Holds}^{\sim}_{on}(f,p') \wedge \textsc{Holds}^{\sim}_{at}(f,\mathrm{end}(p)) \iff \textsc{Holds}^{\sim}_{on}(f,\mathtt{Concat}(p,p')) \tag{7}$$

The Dividing Instant Problem. The proposition in the DIP can be regarded as a discrete fluent. Then the scenario is formalized as:

$$\textsc{Holds}^{\neg}_{on}(\mathrm{light}=\mathrm{on},p_1) \quad \mathtt{Meets}(p_1,p_2)$$
$$\textsc{Holds}^{\neg}_{on}(\mathrm{light}=\mathrm{off},p_2)$$

Note that given this information only, the query $\textsc{Holds}^{\neg}_{on}(\mathrm{light}=\mathrm{on},\mathbf{end}(p_2))$ cannot be answered. Answering it requires additional domain-dependent information. In some cases we will like to regard period p_1 as closed at the end. For instance, imagine the fluent f="being in contact with the floor" for a ball being lifted up. At the dividing instant one probably want to have f=true. There are other examples for which the closed period is p_2. In the light example, it might be the case that the most appropriate is viewing the fluent light as having three possible values {on, changing, off}. We believe that this is a domain-dependent issue and our approach does not make any commitment about that. However it provides the means to specify "safely" what happens *at* the dividing instant.

7 Example: Modelling Hybrid Systems

In this section we illustrate the application of our time theory by means of an example in qualitative modeling. A (qualitative) model of a system is usually the result of an abstraction intended to simplify the analysis. In hybrid models this abstraction produces discontinuous or discrete behaviors together with continuous ones. There are many instances of *Hybrid systems*: most of electro-mechanical devices are, e.g. photocopiers, cars, stereo sets, video cameras, etc. There have been several attempts of introducing discrete changes into a continuous model in the area of qualitative modeling [18, 6, 11, 10]. Some semantical problems have been encountered because of the different nature of discrete changes and continuous change. We shall see that an adequate theory of time and temporal incidence help in overcoming them.

Let's consider a particular example to illustrate these problems and how are they handled in our proposal. The following example, the qualitative model, the intended envisionment and the tentative solutions are from [10]. Figure 3 shows a simple circuit in which electric power is provided to a load either by a solar array or a rechargeable battery.

Its behavior is described as follows:

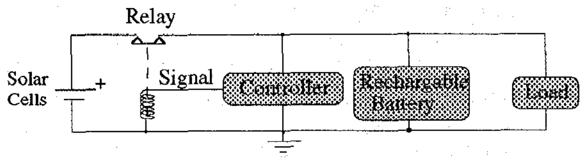

Figure 3: The hybrid circuit example.

C0: "If the sun is shining and the relay is closed, the solar array is a source of current and the battery accumulates charge."

D1: "If the relay is closed, when the signal from the controller goes high, then the relay opens."

D2: "If the relay is open, when the signal from the controller goes low, then the relay closes."

D3: "If the signal is low, when the controller detects that the charge level in the battery has reached the threshold q_2, then the controller turns on the signal to the relay."

Now, let us consider a particular envisionment or predicted qualitative behaviour. Typically it is described as a sequence of states that hold alternatively at a point and during an interval. When discrete events happen "...we would like to model them as being instantaneous" (Iwasaki). The problems arise at the point where sequences of them need to be described, for instance "the signal goes high and *immediately after* the relay closes". Given the initial state of our example where the *signal* is *low*, the *relay* is *closed* and the sun is shining, the intended envisionment would be the following sequence of states:

S1	(t_1,t_2)	$Q_{BA} < q_2$	*signal=low*	*relay=closed*
S2	t_2	$Q_{BA} = q_2$	*signal=low*	*relay=closed*
S2.1	$t_{2.1}$	$Q_{BA} =?$	*signal=high*	*relay=closed*
S2.2	$t_{2.2}$	$Q_{BA} =?$	*signal=high*	*relay=open*
S3	$(t_{2.2},\text{-})$	$Q_{BA} < q_2$	*signal=high*	*relay=open*

The state S2.1 is produced by the signal going high and S2.2 by the relay closing. It is not clear how to model the times of S2, S2.1 and S2.2 or the time spans and the discrete events between them. Assuming that discrete events and changes take no time leads to logical contradiction because of their specification. It is common for a change to take place only if the effect is not already in place. In the case of **D1**, for example, the signal is low and goes high, but if the change is instantaneous both values for the signal would be true at the same time which is obviously contradictory.

The alternative is to assume that discrete changes take a very little time interval. It is problematic too since the value of continuous variables changing concurrently becomes unknown after a sequence of actions. In the example, the charge would keep continuously increasing for a short while. After a number of discrete events these small variations accumulate and complicate the value computation.

There are several solutions proposed to solve this quandary based on complicating the model of time: by introducing mythical time ([18], *direct method*), by extending the real numbers with infinitessimals ([18], *approximation method*)[3], or by using non-standard analysis [10]. We next show that it is not necessary. We use our theory of instants/periods and events/continuous fluents/discrete fluents as follows:

- Discrete events are modeled as instantaneous events.
- Continuous (discrete) quantities are modeled as continuous (discrete) fluents.

Since $\textsc{Holds}_{on}$ is defined as holding at the inner points only, the values of fluent being changed by an instantaneous event will not be defined at the time the event occurs unless there is some specific knowledge that allows to infer it.

The sequence of states representing the intended envisionment becomes simpler:

S1	(t_1, t_2)	$Q_{BA} < q_2$	*signal=low*	*relay=closed*
S2	t_2	$Q_{BA} = q_2$	*signal=?*	*relay=?*
S3	$(t_2, \_)$	$Q_{BA} < q_2$	*signal=high*	*relay=open*

Its formalization in terms of events occurring and holding of continuous and discrete fluents is as follows:

$\widetilde{\textsc{Holds}}_{on}(Q_{BA} < q_2, p_1)$	
$\textsc{Holds}^{\urcorner}_{on}(\mathit{signal} = \mathit{low}, p_2)$	
$\textsc{Holds}^{\urcorner}_{on}(\mathit{relay} = \mathit{closed}, p_3)$	
$\widetilde{\textsc{Holds}}_{at}(Q_{BA} = q_2, \mathrm{end}(p_1))$	
$\textsc{Occurs}_{at}(\mathit{turn\_on}(\mathit{signal}), \mathrm{end}(p_1))$	$\mathrm{end}(p_1) = \mathrm{end}(p_2)$
$\textsc{Holds}^{\urcorner}_{on}(\mathit{signal} = \mathit{high}, p_4)$	$\mathtt{Meets}(p_2, p_4)$
$\textsc{Occurs}_{at}(\mathit{open}(\mathit{relay})), \mathrm{end}(p_3))$	$\mathrm{end}(p_2) = \mathrm{end}(p_3)$
$\textsc{Holds}^{\urcorner}_{on}(\mathit{relay} = \mathit{open}, p_5)$	$\mathtt{Meets}(p_3, p_5)$

The behavior the system in our example can also be formalized using our theory and this modelling method. The rules are given in appendix A.

8 Conclusion

We discussed some problematic issues that arise in temporal representations of changing domains. Namely, the expression of *instantaneous events* and *instantaneous holding of fluents*, the specification of the properties for the temporal holding of fluents and the *Dividing Instant Problem*.

We presented a simple theory of time and temporal incidence that satisfactorily overcomes the problems encountered by previous approaches. Its main features are:

- The time ontology is composed of both instants and periods, where a period is merely interpreted as an ordered pair of points. Having instants is said to cause semantical problems due to the DIP. Nevertheless, we have seen that instants are required to express instantaneous holding of fluents and instantaneous events. We have shown that the DIP is not necessarily a problem.
- We diversify fluents according to whether the change on the parameter they model is *continuous* or *discrete*.

We have shown that both features are required to adequately model discrete and continuous change.

The theory here presented should be of interest as a firm foundation for time to theoretical works on representation of time and to practical works on modeling changing domains.

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A Rules formalizing the Behavior of the Circuit Example

D1 $\textsc{Holds}^{\neg}_{on}(relay = closed, p) \wedge$
$\wedge\ \textsc{Occurs}_{at}(open(relay), \mathbf{end}(p)) \Rightarrow$
$\Rightarrow \textsc{Holds}^{\neg}_{on}(relay = open), p') \wedge \mathtt{Meets}(p, p')$

D2 $\textsc{Holds}^{\neg}_{on}(relay = closed, p) \wedge$
$\wedge\ \textsc{Occurs}_{at}(close(relay), \mathbf{end}(p)) \Rightarrow$
$\Rightarrow \textsc{Holds}^{\neg}_{on}(relay = closed), p') \wedge \mathtt{Meets}(p, p')$

D3 $\textsc{Holds}^{\neg}_{on}(signal = low, p) \wedge$
$\wedge\ \textsc{Holds}^{\sim}_{at}(Q_{BA} \geq q_2, \mathbf{end}(p)) \Rightarrow$
$\Rightarrow \textsc{Occurs}_{at}(turn\_on(signal), \mathbf{end}(p))$