An Efficient Algorithm for Temporal Abduction

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Extended Abstract

Abduction is the process of generating explanations for an observation O, starting from a domain theory T [6]. Here we consider an extension of the "classical" definition of abduction in order to deal with temporal knowledge both in the theory and in the observations. This form of abduction is the basis of many problem solving activities such as temporal diagnosis and reasoning about actions and events.

In the paper we assume that T is a set of explanatory formulae of the form $a_1, \ldots, a_n$ *explains* $b_1, \ldots, b_m$ $\{C(t_{a_1}, \ldots, t_{a_n}, t_{b_1}, \ldots, t_{b_m})\}$ where a_i, b_j are atoms and t_a denotes a time interval in which a is true (an *episode* of a). An explanatory formula of this form represents the fact that the conjunction of a maximal episode for each one of $a_1, \ldots, a_n$ is a direct explanation of an episode of each b_i, where the set of temporal constraints $C(t_{a_1}, \ldots, t_{a_n}, t_{b_1}, \ldots, t_{b_m})$ is imposed on the temporal extents of the episodes.

Given a model T and an observation O (including a set C_O of temporal constraints), a **temporal explanation** is a set E of abducibles, i.e., atoms that occur only in antecedents of formulae, such that: the observed atoms in O are explained by E through chains of explanatory formulae in the model T; the set TE of temporal constraints, consisting of C_O plus the constraints associated with the explanatory formulae involved in the explanation, is consistent.

In our approach, the temporal constraints are expressed in terms of conjunctions of bounds on differences constraints between variables (called STP constraints in [4] and in the following). Different types of temporal constraints (e.g., continuous pointizable qualitative relations [8], precise or imprecise absolute temporal locations and durations, delays) can be represented using the STP framework [2], which is expressive enough for most applications [7].

The **consistency** of a set of STP constraints on a set K of variables can be checked in time cubic in the number of temporal variables in K. This computation also produces the **minimal network**, which is a compact representation of all the solutions to the set of constraints, consisting of the strictest constraints between all the variables [4].

In [3] we proved the following **locality property** concerning STP constraints: given a minimal network TCN_K on a set K of variables, checking whether a set of constraints TCN_A on a set A of variables is consistent with TCN_K can be done in time cubic in the number of variables in A (regardless of the number of variables in K), by propagating the constraints in TCN_K and TCN_A only on the variables in A (*local propagation* on A). Moreover, the temporal constraints on A obtained by the local propagation are the same (the strictest ones) that would be obtained by *global propagation* on all the variables in $K \cup A$.

In this paper, we show how this locality property can be used in order to compute temporal explanations efficiently. In fact, they could be computed using first an abductive reasoner as a generator of candidates and then a temporal reasoner for checking the consistency of each candidate. However, pruning temporally inconsistent candidate explanations as soon as possible during the generation process provides great focusing and computational advantages. One way to achieve such a goal is to perform, at each abductive step, *global* temporal constraint *propagation* on all the variables in the candidate being built (as, e.g., in [5]).

In the following we show how, thanks to the locality property, local temporal propagation can be used to prune efficiently temporally inconsistent candidates, obtaining the same pruning results that would be obtained by global propagation, both in case of singly connected and of non-singly connected explanation graphs.

Case (a). Consider the case where different observations (or different atoms in the antecedent of a formula) are explained with independent assumptions, so that the explanation is singly connected (as in Fig-

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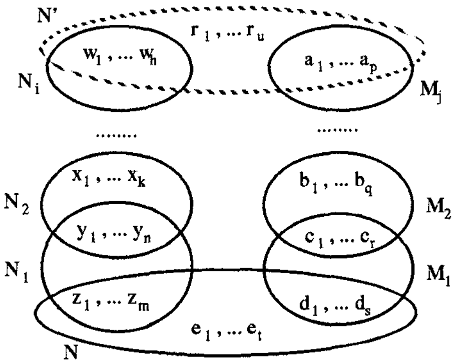

Figure 1: Two chains of explanatory formulae. For example, N_1 corresponds to the formula: F_1 : $y_1, \dots, y_n$ *explains* $z_1, \dots z_m$ $\{C_1\}$. The graph is non-singly connected if and only if the formula F' : $r_1, \dots r_u$ *explains* $w_1, \dots w_h$, $a_1, \dots a_p$ $\{C'\}$, corresponding to the dashed N', is included.

ure 1, without the dashed N', where $w_1 \dots w_h$ and $a_1 \dots a_p$ are abducible).

Let us first consider the chain of explanatory formulae $N, N_1, \dots, N_i$. Suppose that the minimal network for N containing (besides the others) the atoms $z_1 \dots z_m$ has been computed, and F_1 is selected in order to explain some of the atoms in $z_1, \dots, z_m$. The current explanation graph and temporal constraints must be extended using F_1 and its constraints C_1.

The locality property guarantees that local propagation to N_1 produces a minimal network for the variables in N_1 only, such that the constraints in it are the same that would be obtained by (global) propagation to $N \cup N_1$. This allows us to localize constraint propagation to the atoms in a formula F when using F in an explanation step. This principle can be iteratively applied to the chain $N_1, \dots, N_i$ in figure 1: given the minimal network for N_1 (produced by local propagation), the minimal network for N_2 can be computed with local propagation to the atoms in N_2, and so on. When the abducible atoms $w_1 \dots w_h$ are reached, such a set of local propagations guarantees that the temporal constraints on such atoms are the strictest ones that would be obtained with global propagation. Thus, we have that inconsistencies are detected by local propagation when they would be detected with global propagation.

Since now N is no longer a minimal network (only the constraints between $w_1, \dots, w_h$ are now minimal), before moving to explaining $d_1, \dots, d_s$, the minimal constraints between them must be computed. We can thus proceed as follows:

1. backward visit from N_1 to N_i with local propagation to each N_k;
2. forward visit from N_i to N_1 with local propagation to each N_k;
3. local propagation to N, which ensures (after steps 1 and 2) that the strictest constraints on $d_1, \dots, d_s$, given the information on $N_1, \dots, N_i$ are computed;
4. backward visit of $M_1, \dots, M_j$, as in step 1.
5. forward visit of $M_j, \dots, M_1$, as in step 2.

Thus, in general, if the explanation graph is singly connected, a backward visit followed by a forward visit of each branch is sufficient to detect temporal inconsistencies, if any, as they would be detected with global propagation to all the visited atoms.

Case (b). Let us consider now the case of a non-singly connected explanation, as in Figure 1, including the dashed N', corresponding to:

$$F' : r_1, \dots r_u \; explains \; w_1, \dots w_h, \; a_1, \dots a_p \; \{C'\}$$

In order to apply the locality properties to F' at the end of step 4 of the strategy of case (a), we need to have the strictest constraints for $w_1, \dots w_h$, $a_1, \dots a_p$ before propagating C' to the atoms in F'. Unfortunately, this is not necessarily the case since the strategy for case (a) would only provide the strictest constraints for $a_1, \dots a_p$ (given $N_i, \dots N, \dots M_j$), but not those for $w_1, \dots w_h$, because the constraints collected during the backward visit of $M_1, \dots, M_j$ have not been propagated to $w_1, \dots, w_h$. It is possible to compute such strictest constraints if all the local propagations performed during the forward visit of $N_i, \dots, N$ and the backward visit of $M_1, \dots, M_j$ also involve the atoms which "join" the two chains (e.g., $w_1, \dots w_h$, $a_1, \dots a_p$); these atoms must be taken into account for the local propagations until $r_1, \dots, r_u$ can be reached again, after which they can be disregarded. Moreover, some further optimization is possible of the set of "junction" atoms.

We developed an explanation algorithm which follows the strategy discussed above, so that it uses local propagations only, and detects a temporal inconsistency if and only if it would be detected using global propagation (instead of local propagation). In particular, given a model T and an observation O, we compute an explanation by means of a depth-first backward nondeterministic search: the abducible atoms forming the explanation are searched starting from the atoms in O and applying the formulae in T backwards. Temporal consistency is checked whenever a new formula is selected. For the sake of efficiency, the algorithm assumes that there is at most one maximal episode of each atom a, and that T is acyclic.

We now analyze the overhead on the computation of one explanation due to temporal constraint propa-

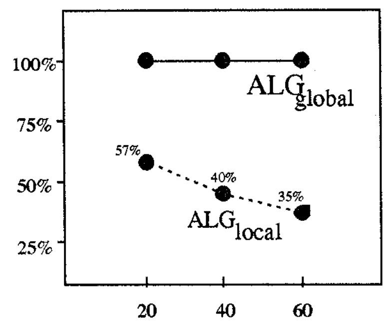

Figure 2: Ratio between computation time of ALG_{local} with respect to ALG_{global} for multiply connected explanation graphs.

gation. We consider the case where a solution is found without search, i.e., without backtracking on the non-deterministic choices. In case of search, we have similar advantages also on the failed branches.

In particular, we compare:

- the algorithm based on global propagation approach (called ALG_{global} in the following), which checks the temporal consistency by means of global propagation whenever a new formula is selected
- our algorithm (augmented with some optimizations for dealing with multiply connected graphs), which follows the stategy discussed above and performs local temporal propagations (ALG_{local}).

The algorithms have been implemented in C and Prolog; in particular, as regards the management of temporal information, we used LATER [1], an efficient manager of temporal information exploiting the locality property of STP mentioned above.

Figure 2 reports the ratio between the computation time for the two algorithms, as a function of the number of atoms in the explanation, in case of highly multiply connected explanations.

An analitycal evaluation of the time complexity of our approach shows that, although in the worst case the asymptotic complexity of our algorithm is the same as the global propagation one (i.e., the overhead on explanation is quartic in the number of nodes), in practice we have a significant improvement. Moreover, in case the theory plus the observations form a singly connected graph, we have also an improvement in the asymptotic complexity, since, in such a case, the local propagation algorithm adds a linear overhead. This is a further significant advantage of our approach wrt the global propagation one which is not sensitive to the topology.

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