Math 337 Homework 09

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1. Solve the BVP
$$y'' - \frac{2y}{(1+x)^2} = -\frac{4}{(1+x)^2}, \qquad y(0) = 0, \quad y(1) = 1$$
using the collocation method with $\phi_j = \sin(j\pi x)$ for $j = 1, \ldots, M$ and $M = 10$. Use the equidistant collocation points $x_k = x_0 + k \cdot h$. Compare you finite-element solution with the exact solution $y_{\text{exact}} = 2x/(1+x)$ by plotting them together, and also by plotting, in a separate figure, the error for $x \in [0, 1]$.

   Solution:

2. Obtain Eqs. (9.20) and (9.22) of the notes.

   Solution:

3. Solve the BVP in Problem 1 by the Galerkin method with the hat functions and $M = 10$. Compare your result with the exact solution. As in Problem 1, investigate how the error of the Galerkin method scales with $(1/M)$.

   Solution:

Appendix 1: ODE Functions

Appendix 2: Numerical Methods