Supplementary material

The supplementary file is divided into two parts. Subsection 1.1 illustrates didactic examples of solutions for a better understanding of the mathematical model proposed. Subsection 1.2 describes the data used in the computational tests carried out.

1.1 Illustrative example of solutions

For a better understanding of the proposed mathematical modeling and to facilitate the visualization of the decisions to be taken, Figure A shows a didactic example of programming and construction of a planting and harvesting calendar for four years, the first being for planting and the last three for harvesting (three cuts). The label 'I' represents the ideal harvesting month in the plot, while 'P' and 'H' mean, in that order, the months in which planting and harvesting took place. On the horizontal axis, '1' is the month of January, '2' is the month of February, and so on.

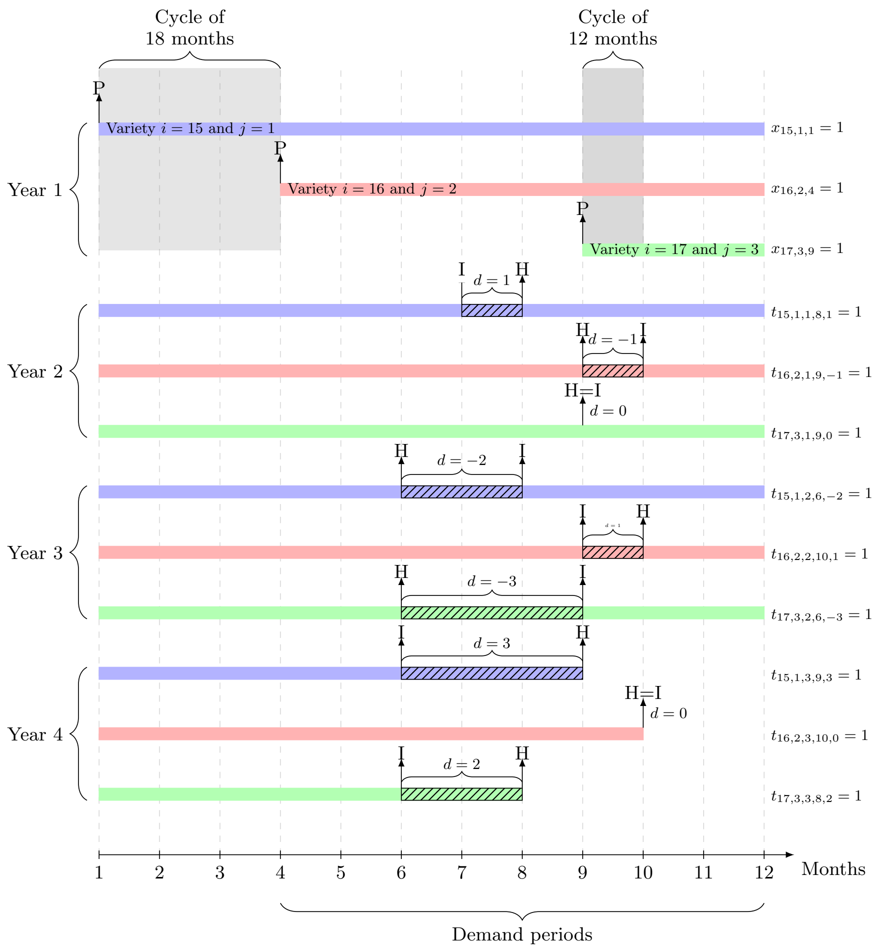

Fig. A Scheme with three plots being planted and harvested along four years of scheduling

Three plots $j = 1, 2, 3$, each with a different color, receive the crop varieties $i = 15, 16, 17$ in Months $1, 4, 9$, respectively. Then $x_{15,1,1} = x_{16,2,4} = x_{17,3,9} = 1$. Thus, for plot $j = 1$, the cane life cycle is 18 months ($y_1 = 0$). Therefore, the ideal harvesting month for this plot (with maximum sucrose) is July in Year 2. However, the harvesting was done in August, one month after the ideal month. In other words, harvesting variety $i = 15$ from plot $j = 1$, in cut $c = 1$, occurred in month $h = 8$ with deviation $d = 1$. Therefore, $t_{15,1,1,8,1} = 1$. Next, in Year 3, the ideal harvesting time for this same plot is the month of August (12 months after the first cut) but the operation is performed two months before ($d = -2$) the ideal period. Then, $t_{15,1,2,6,-2} = 1$, indicating that variety $i = 15$ is harvested in plot $j = 1$ at cut $c = 2$ in month $h = 6$ with deviation $d = -2$. Finally, in Year 4, the harvesting is in $h = 9$, with $d = 3$ months of delay ($t_{15,1,3,9,3} = 1$). Note that the decisions in one cut in the previous year directly affect the calendar for the next year and need to be adjusted at each new cut. Further interpretations of decision variables are presented in the same figure.

For a better understanding of the relationship between planting and harvesting decisions and deviations from cane maturation, Table A shows the complete compromise solution $\mathbf{s}^*_{0.5}$ with $|J| = 30$ plots of the real case. This table shows the plots j (column 1), the designated varieties (column 2) and the month p they were planted (column 3). Depending on the planting season, the growing cycle can be 12 or 18 months (column 4). The other columns illustrate the harvest planning for the four cuts, where the harvesting months (h) and the indicator whether the harvest was done early ($d < 0$), in the correct period ($d = 0$) or late ($d > 0$). It should be noted that the majority of the 20 first plots (dedicated to sucrose cane) are harvested with a deviation of $d = 0$ along the different cuts. Table 6 shows objective values for this solution as 138,162.00 tons of sucrose and fiber production volumes at the cost of R\$ 31,817,100.00.

1.2 Data for the computational tests

1.2.1 Real-life instances

All the other parameters necessary for the computational experiments for the real-life instances (cf. Subsection 7.1) are presented as follows. The number of plots dedicated to the cultivation of sugarcane (κ^s) and energy-cane (κ^e) in each instance is presented in columns 2 and 3 of Table B. In addition, the monthly demands for sucrose (σ^s_{mc}) and fiber (σ^e_{mc}) and the milling capacity (δ^s_{mc}) were generated using a uniform distribution whose ranges are shown in columns 6–8. As the total number of plots increases, a higher sucrose and fiber yield is expected (based on average cane yield and plot area) and a higher milling capacity of the mill in charge.

Considering that a sugarcane field can have at least three to five different varieties cultivated, to avoid the proliferation of pests and risking the whole production of sugarcane, the maximum percentage of planting of the same variety (η) was 30%, adopted by the mills in Brazil. Each plot area (ℓ_j, in ha) was uniformly generated in the range $[25, 30]$, based on reference values in cultivation areas in Brazil. The data for the 25 varieties considered (index i), being $n^e = 5$ varieties of energy-cane and $n^s = 20$ of sucrose cane. The values for the productivity (ρ^s_i and ρ^e_i) for the first cut and the percentage of sucrose (ζ^s_i and ζ^e_i) and fiber (ω^s_i and ω^e_i) are presented in Tables C and D, respectively. This data source is based on agronomic studies in [51] and [9]. The costs of cultivating, harvesting, and transportation of the varieties of sugarcane and energy-cane were based on [61]. A value of R\$ 94.00 ($\beta^e_{jc} = \beta^s_{jc}$ for all $j \in J$ and $c = 1$) per ton for soil preparation, planting, cultivation, harvesting and transportation of the planted cane (cane harvested in the first cut). In the case of ratoon cane (cane harvested from the second cut), a cost of R\$ 65.00 ($\beta^e_{jc} = \beta^s_{jc}$ for all $j \in J$ and

Table A Schedule of planting and harvesting along the four cuts of the instance with 30 plots in the compromise solution obtained with exact method

Planting (x^*_{ijp})				Harvesting (t^*_{ijchd})							
				$c = 1$		$c = 2$		$c = 3$		$c = 4$	
j	i	p	Cycle (months)	h	d	h	d	h	d	h	d
1	9	1	18	7	0	7	0	6	−1	6	0
2	14	2	18	8	0	8	0	8	0	8	0
3	9	10	12	10	0	10	0	10	0	10	0
4	14	4	18	10	0	10	0	10	0	10	0
5	16	2	18	8	0	8	0	8	0	8	0
6	9	1	18	6	−1	6	0	6	0	6	0
7	9	1	18	6	−1	6	0	5	−1	5	0
8	14	2	18	8	0	8	0	8	0	8	0
9	16	1	18	7	0	7	0	7	0	7	0
10	6	1	18	7	0	7	0	7	0	7	0
11	18	2	18	8	0	8	0	8	0	8	0
12	14	2	18	8	0	8	0	8	0	8	0
13	16	9	12	9	0	9	0	9	0	9	0
14	16	9	12	9	0	9	0	9	0	9	0
15	9	4	18	10	0	10	0	10	0	10	0
16	14	10	12	10	0	10	0	10	0	10	0
17	16	9	12	9	0	9	0	9	0	9	0
18	16	1	18	7	0	7	0	7	0	7	0
19	9	4	18	11	1	11	0	11	0	11	0
20	14	2	18	8	0	8	0	8	0	8	0
21	25	1	18	4	−3	4	0	4	0	6	2
22	21	2	18	11	3	11	0	11	0	11	0
23	25	1	18	4	−3	4	0	5	1	5	0
24	23	1	18	6	−1	5	−1	8	3	11	3
25	21	1	18	5	−2	5	0	5	0	4	−1
26	21	1	18	4	−3	4	0	4	0	4	0
27	23	1	18	4	−3	6	2	8	2	11	3
28	24	1	18	5	−2	4	−1	6	2	5	−1
29	25	9	12	11	2	11	0	11	0	11	0
30	23	1	18	5	−2	5	0	8	3	11	3

$c > 1$) per ton was considered for cultivation, harvesting and transportation. Due to the climate, precipitation and temperature characteristics of the south-central region of Brazil, the months of planting (index p) were defined as $T_P = \{1, 2, 3, 4, 9, 10\}$, being 1 for January, 2 for February, and so on. The harvesting months depend on the climate in the center-south region of Brazil and were defined as $T_H = \{4, 5, \ldots, 11\}$ (from April to November) because in this period, the precipitation index allows this operation (index h). The set of possible deviations (index d) from the ideal harvest period was defined as $D = \{-3, -2, -1, 0, 1, 2, 3\}$ to avoid harvesting in plots with cane still in formation or that is extremely old (causing buds hard). In addition, the formulas for calculating production volumes estimates (1)–(3) can be imprecise for high d values. Finally, the rates of decrease in sugarcane production and increase in energy-cane production were set to $\alpha^s = \alpha^e = 6\%$ from the first cut, based on [7, 8].

1.2.2 Semi-random instances

The ranges in which the parameters were generated in the experiment carried out in Subsection 7.2 are shown in Table E (vary with $|J|$) and F (do not change with $|J|$). The other parameters and the Julia random generator computer, software and seed settings were kept as described in subsection 7.1.

Table B Data for the five considered instances

$\lvert J \rvert$	κ^s	κ^e	# of binary variables	# of constraints	σ^s_{mc} (ton.)	σ^e_{mc} (ton.)	δ_{mc} (ton.)
30	20	10	142,530	571	$[0.9, 1.1] \cdot 10^3$	$[0.9, 1.1] \cdot 10^3$	$[1.5, 1.7] \cdot 10^4$
65	50	15	373,815	1,096	$[2.9, 3.1] \cdot 10^3$	$[2.9, 3.1] \cdot 10^3$	$[3.9, 4.1] \cdot 10^4$
150	125	25	862,650	2,371	$[5.9, 6.1] \cdot 10^3$	$[5.9, 6.1] \cdot 10^3$	$[7.9, 8.1] \cdot 10^4$
300	250	50	1,725,300	4,621	$[1.1, 1.3] \cdot 10^4$	$[1.1, 1.3] \cdot 10^4$	$[1.5, 1.7] \cdot 10^5$
500	400	100	2,875,500	7,621	$[1.9, 2.1] \cdot 10^4$	$[1.9, 2.1] \cdot 10^4$	$[2.6, 2.8] \cdot 10^5$

Table C Data of sugarcane varieties in the first cut

Variety i	Name	ρ^s_i (ton·ha^{-1})	ω^s_i (%)	ζ^s_i (%)
1	CTC15	132.80	12.40	14.50
2	CTC9	121.93	12.38	14.00
3	RB925211	137.20	11.50	15.00
4	CTC6	89.29	12.30	14.80
5	RB855156	121.70	11.30	14.80
6	CTC2	140.60	12.90	15.00
7	RB857515	129.10	12.20	14.30
8	SP80-1842	130.90	12.00	13.54
9	SP83-2847	100.00	12.34	15.84
10	SP80-3280	112.30	12.38	15.00
11	RB928062	165.00	11.50	13.50
12	RB966928	117.80	12.40	14.50
13	CTC20	148.20	12.00	14.00
14	CTC17	113.00	12.40	15.60
15	SP81-3250	123.10	12.00	13.32
16	CTC4	142.40	12.93	15.70
17	RB92579	132.12	12.38	14.00
18	RB855453	112.80	12.90	14.90
19	SP813250	126.70	11.74	13.20
20	RB86755	136.00	11.16	15.00

Source: [51]

Table D Data of energy-cane varieties in the first cut

Variety i	ρ^e_i (ton·ha^{-1})	ω^e_i (%)	ζ^e_i (%)
21	236	23.0	8.00
22	215	24.0	6.00
23	200	32.0	7.00
24	181	22.0	9.00
25	179	25.0	8.00

Source: [7]

Table E Intervals for demand and milling capacity generated in the semi-random instances

$\lvert J\rvert$	σ^s_{mc} (ton)	σ^e_{mc} (ton)	δ_{mc} (ton)
30	$[0.8, 1.2] \times 10^3$	$[0.8, 1.2] \times 10^3$	$[1.5, 1.9] \times 10^4$
65	$[2.8, 3.2] \times 10^3$	$[2.9, 3.1] \times 10^3$	$[3.8, 4.2] \times 10^4$
150	$[5.8, 6.2] \times 10^3$	$[5.7, 6.1] \times 10^3$	$[7.8, 8.2] \times 10^4$
300	$[1.0, 1.4] \times 10^4$	$[1.1, 1.3] \times 10^4$	$[1.5, 1.8] \times 10^5$
500	$[1.8, 2.2] \times 10^4$	$[1.9, 2.1] \times 10^4$	$[2.6, 2.9] \times 10^5$

Table F Other parameters generated in the semi-random instances

n^s	n^e	ρ^s_i (ton)	ρ^e_i (ton)	ω^s_i (%)	ω^e_i (%)	ζ^s_i (%)	ζ^e_i (%)	ℓ_j (ha)	β^s_{mc} R\$$\cdot$ha^{-1}	β^e_{mc} R\$$\cdot$ha^{-1}	η
$[20, 30]$	$[10, 20]$	$[80, 140]$	$[200, 240]$	$[0.11, 0.13]$	$[0.25, 0.35]$	$[0.13, 0.15]$	$[0.06, 0.09]$	$[20, 30]$	$[80, 100]$	$[60, 80]$	$[0.25, 0.35]$