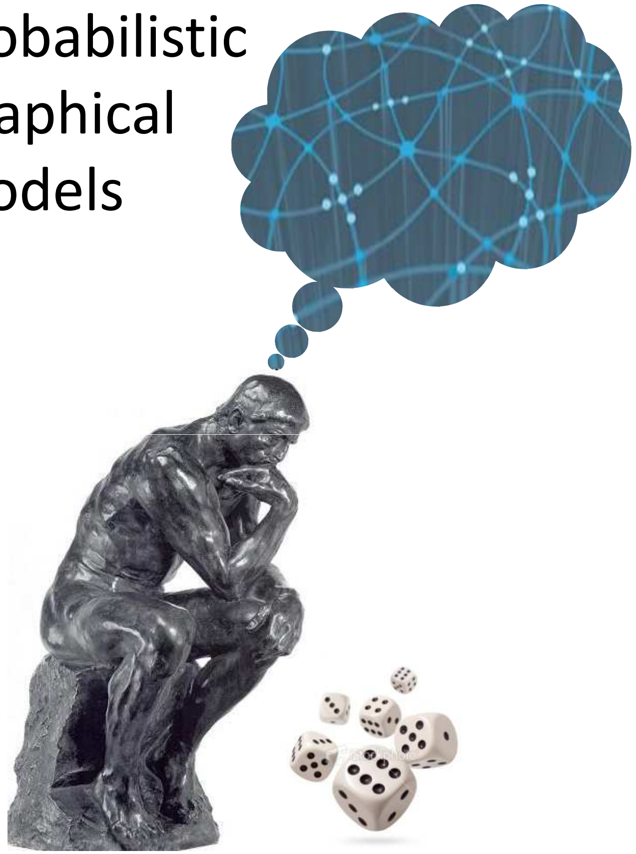
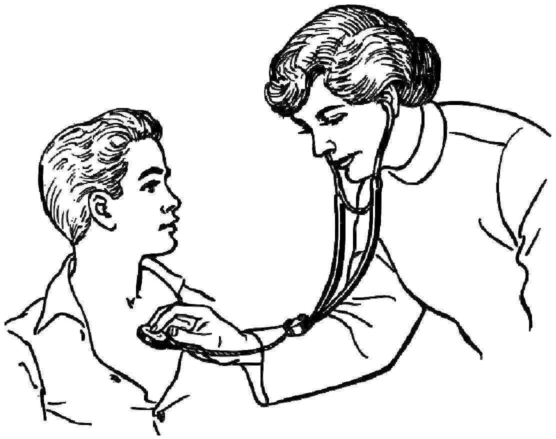
Probabilistic
Graphical
Models

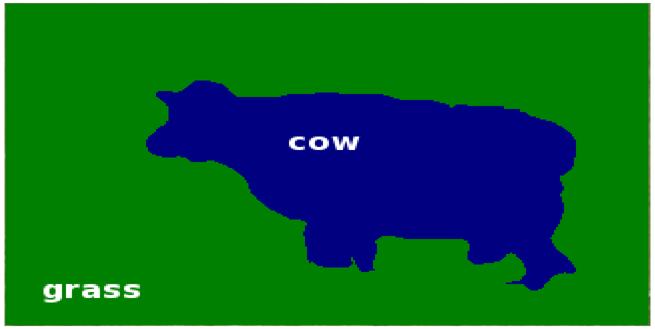


Introduction

Motivation and Overview



predisposing factors
symptoms
test results
diseases
treatment outcomes



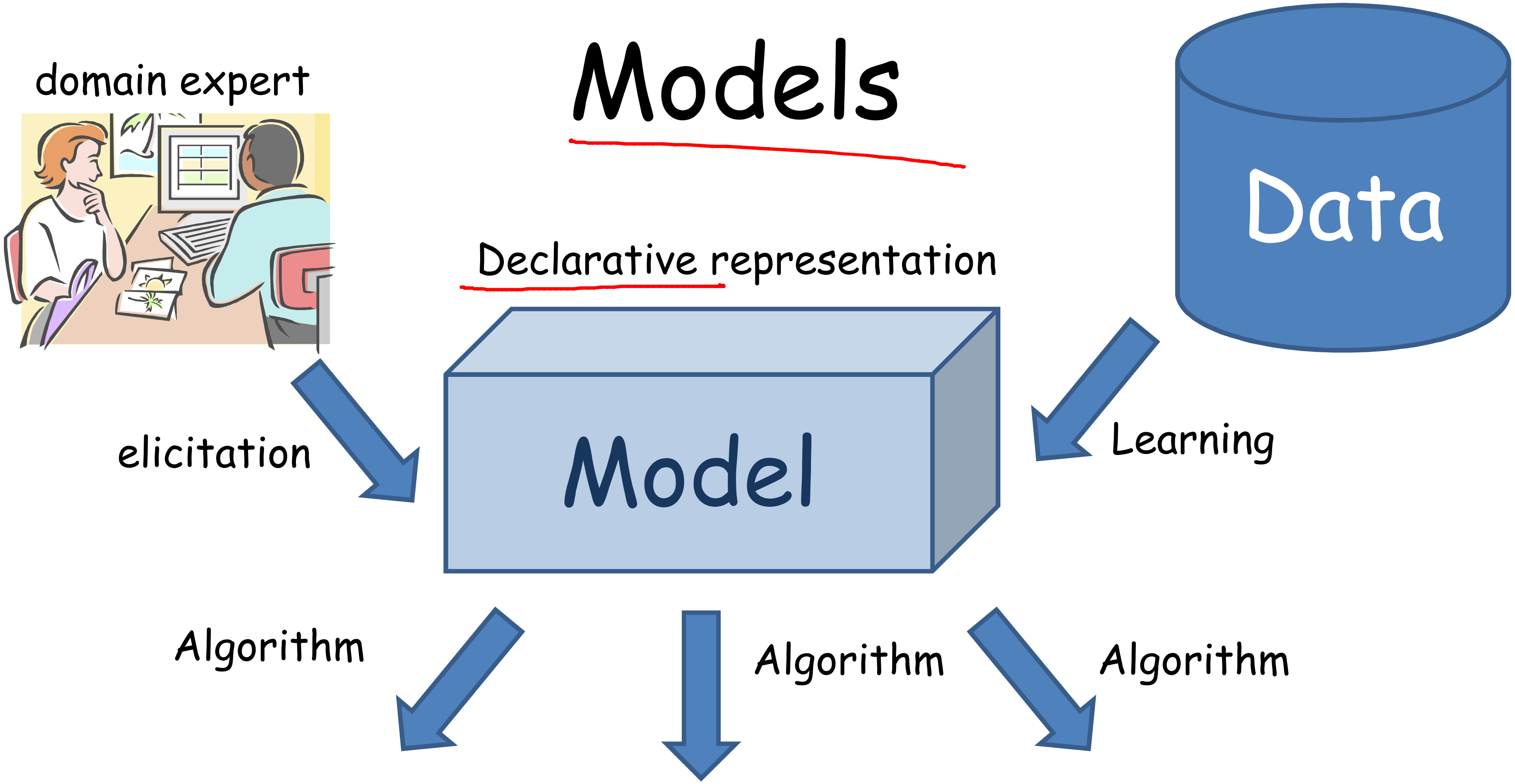
millions of pixels or
thousands of superpixels

each needs to be labeled
{grass, sky, water, cow, horse, …}

Probabilistic Graphical Models

Daphne Koller

Models


domain expert
Declarative representation
Data
elicitation
Model
Learning
Algorithm
Algorithm
Algorithm

Uncertainty

- Partial knowledge of state of the world
- Noisy observations
- Phenomena not covered by our model
- Inherent stochasticity

Probability Theory

- Declarative representation with clear semantics
- Powerful reasoning patterns conditioning decision making
- Established learning methods

Complex Systems

predisposing factors
symptoms
test results
diseases
treatment outcomes

class labels for
thousands of superpixels

Random variables $\boxed{X_1,\ldots, X_n}$ n binary valued

Joint distribution $\underline{P(X_1,\ldots, X_n)}$ distribution over 2^n possible states

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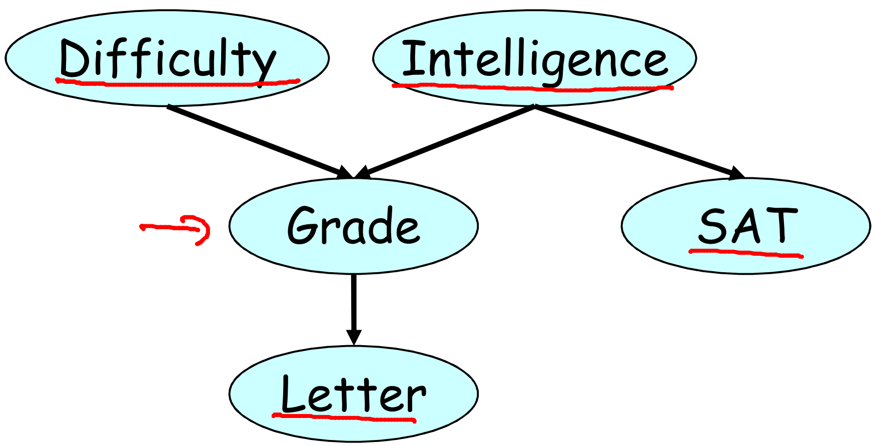
Graphical Models

$X_1 \ldots X_n$ — nodes

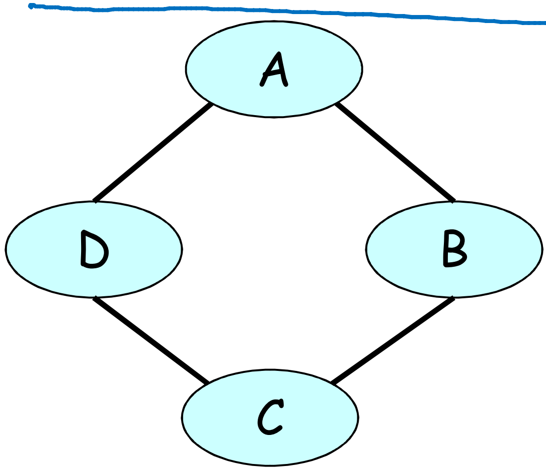
directed graph

Bayesian networks



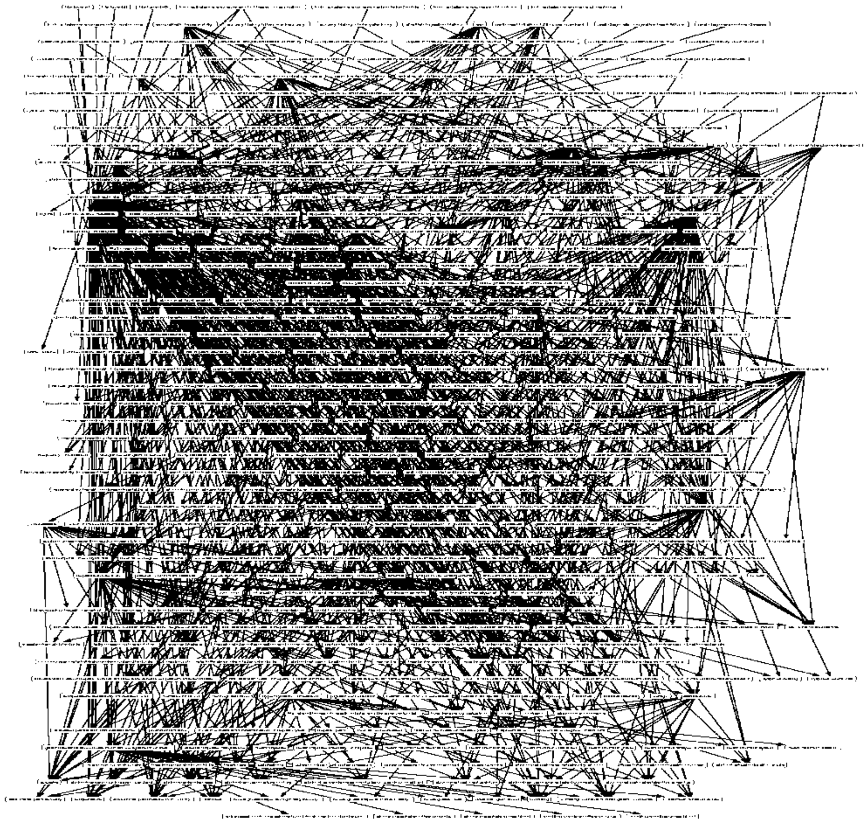
undirected graph

Markov networks

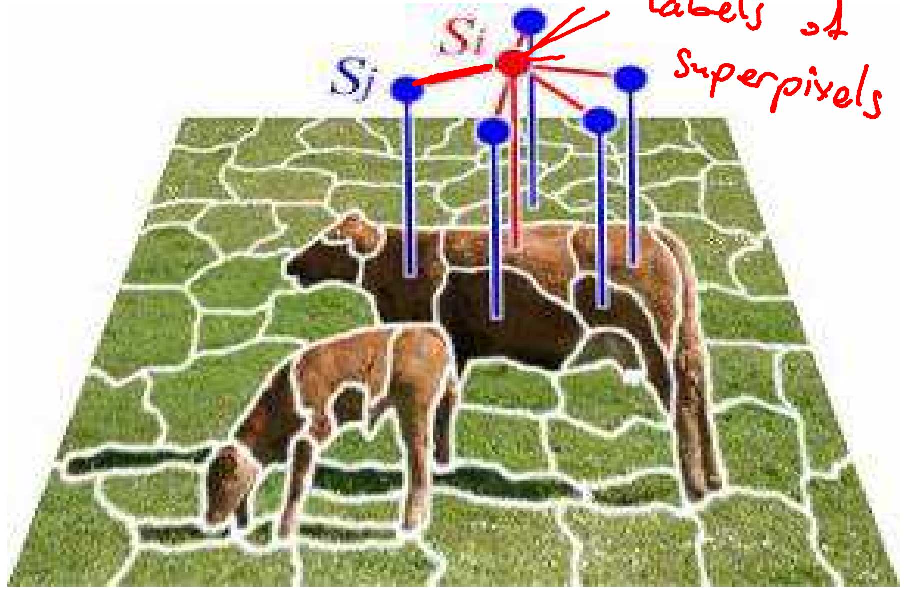


Graphical Models

diagnosis
CPCS



labels of superpixels



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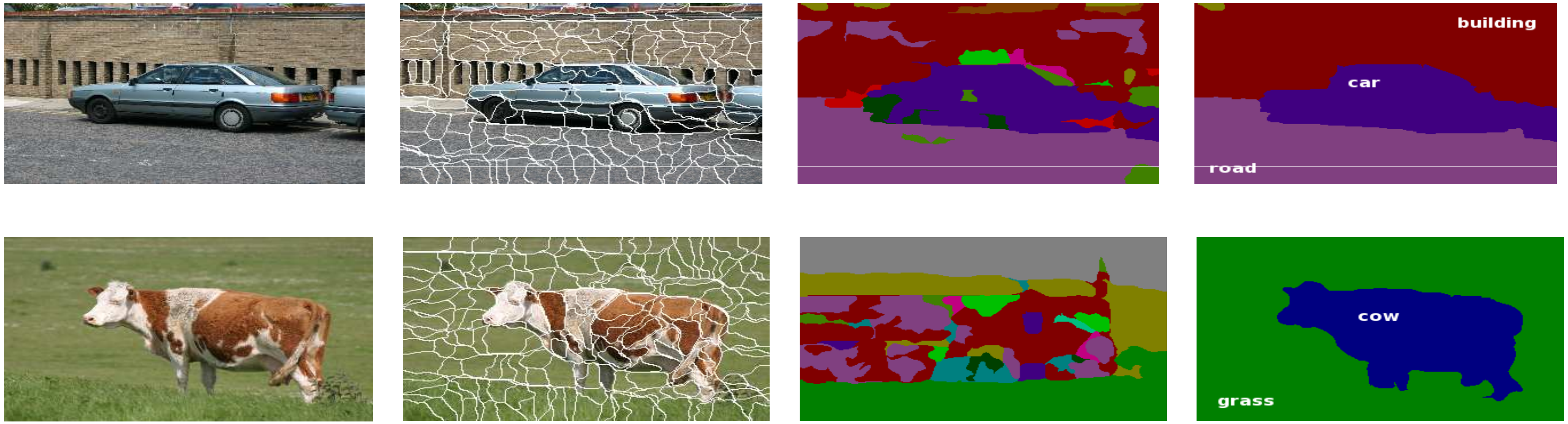
Graphical Representation

- Intuitive & compact data structure
- Efficient reasoning using general-purpose algorithms
- Sparse parameterization
  - feasible elicitation ← by hand
  - learning from data ← automatically

Many Applications

- Medical diagnosis
- Fault diagnosis
- Natural language processing
- Traffic analysis
- Social network models
- Message decoding
- Computer vision
  - Image segmentation
  - 3D reconstruction
  - Holistic scene analysis
- Speech recognition
- Robot localization & mapping

Image Segmentation



superpixels

machine learning
to separate superpixels

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Medical Diagnosis

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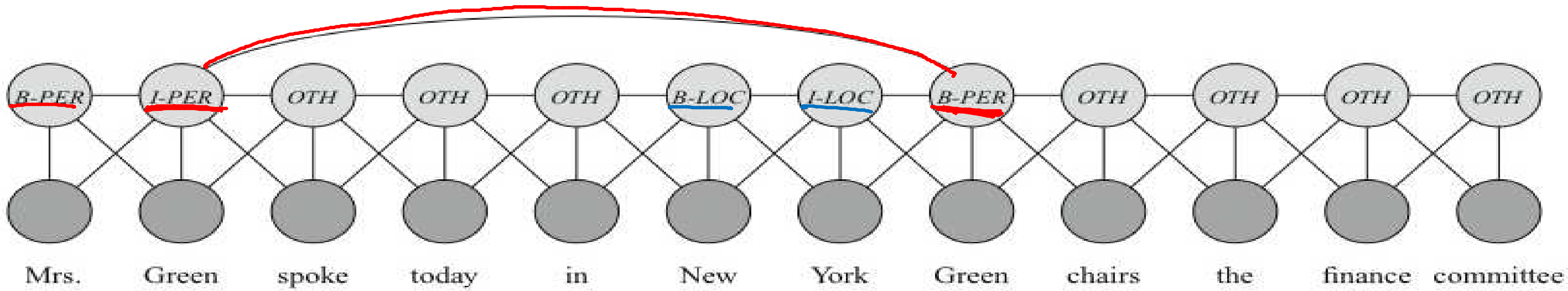
Textual Information Extraction

Mrs. Green spoke today in New York. Green chairs the finance committee.

Person Location Person organization



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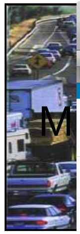
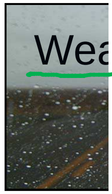
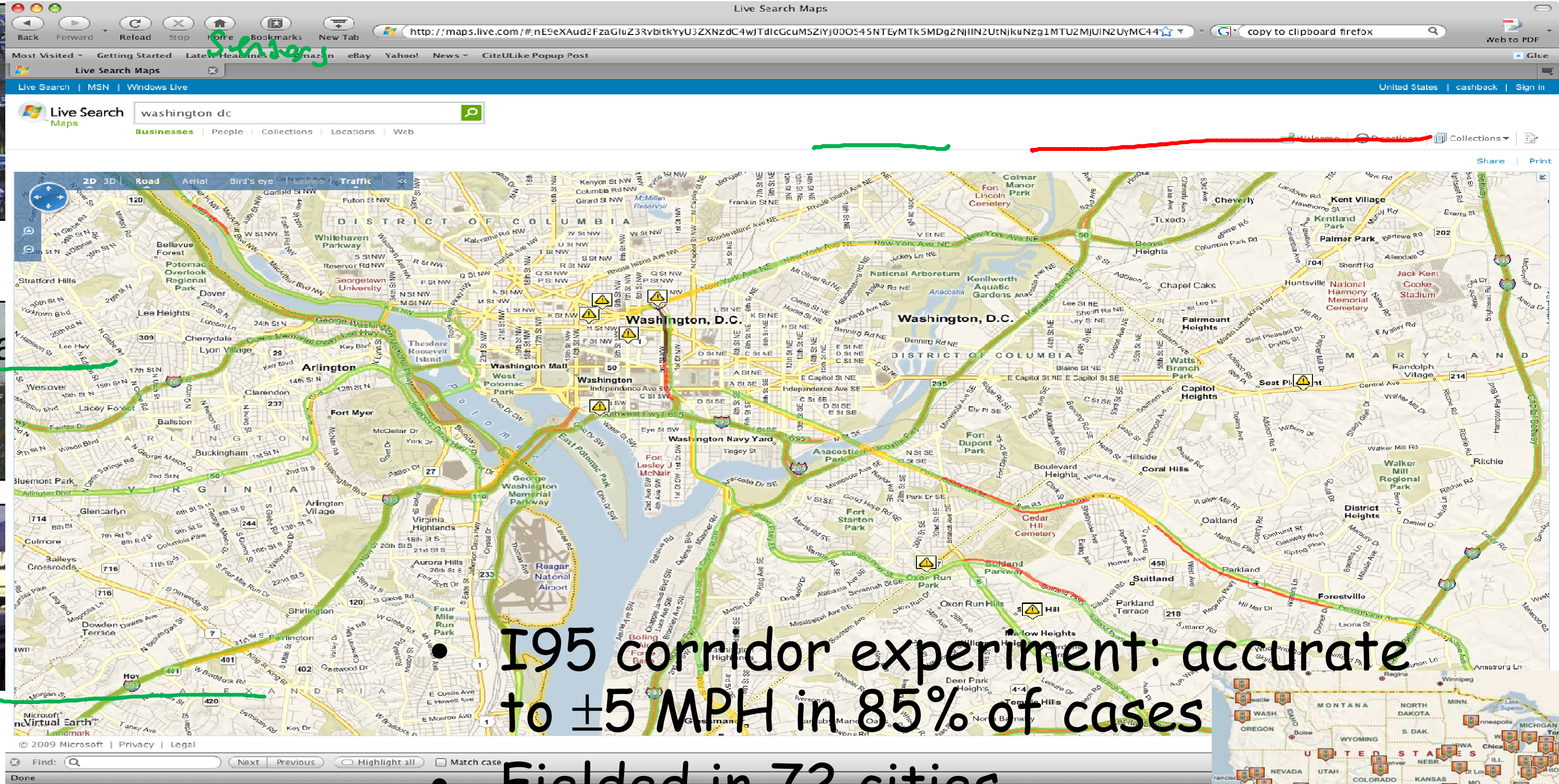
Multi-Sensor Integration: Traffic

M

uding

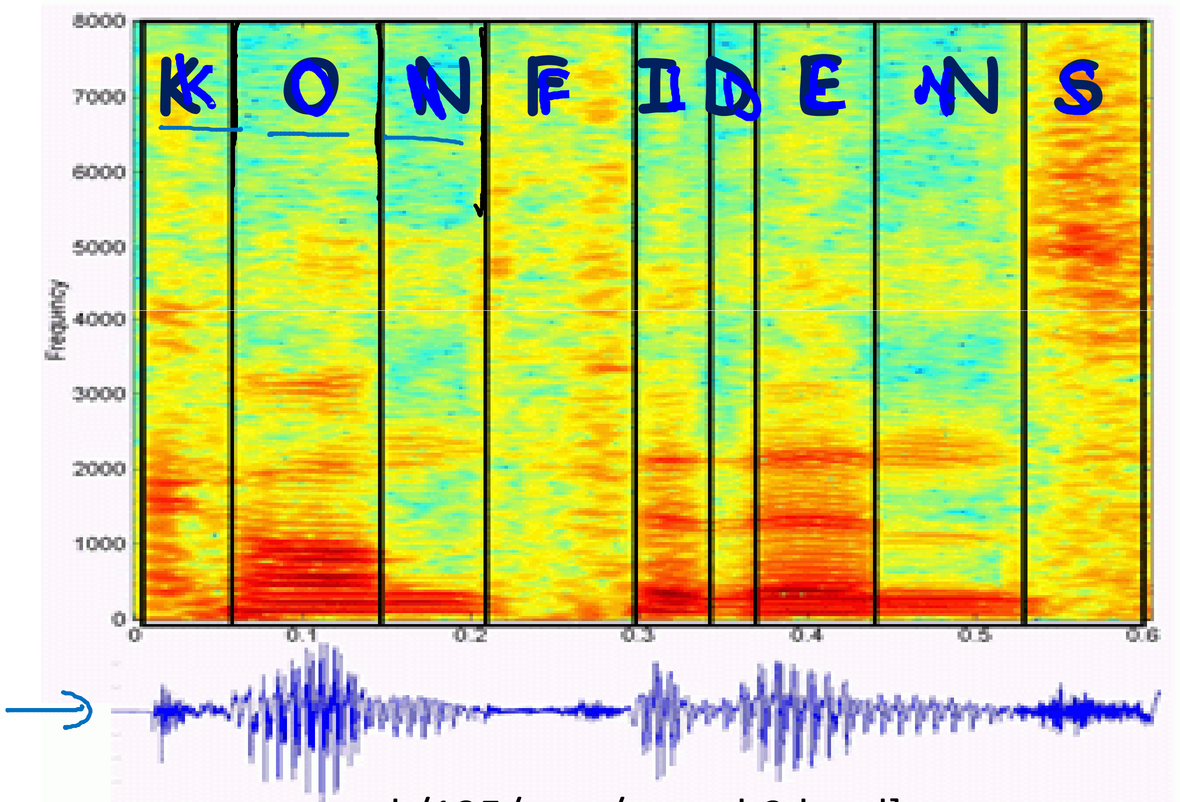
Wea

- I95 corridor experiment: accurate to ±5 MPH in 85% of cases
- Fielded in 72 cities

er

Thanks to: Eric Horvitz, Microsoft Research

Speech Recognition



[Source: http://www-g.eng.cam.ac.uk/125/now/speech2.html]

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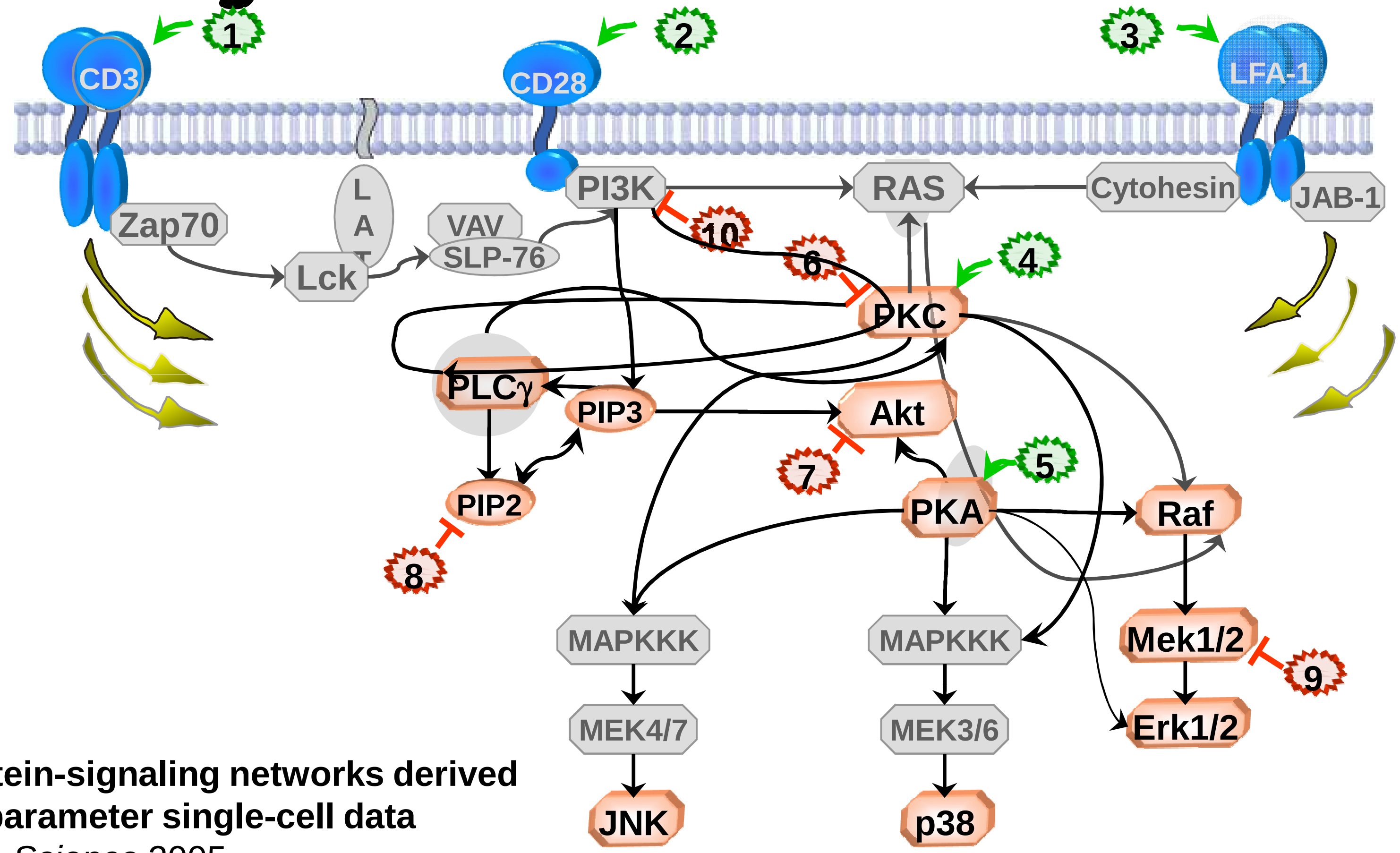
Biological Network Reconstruction

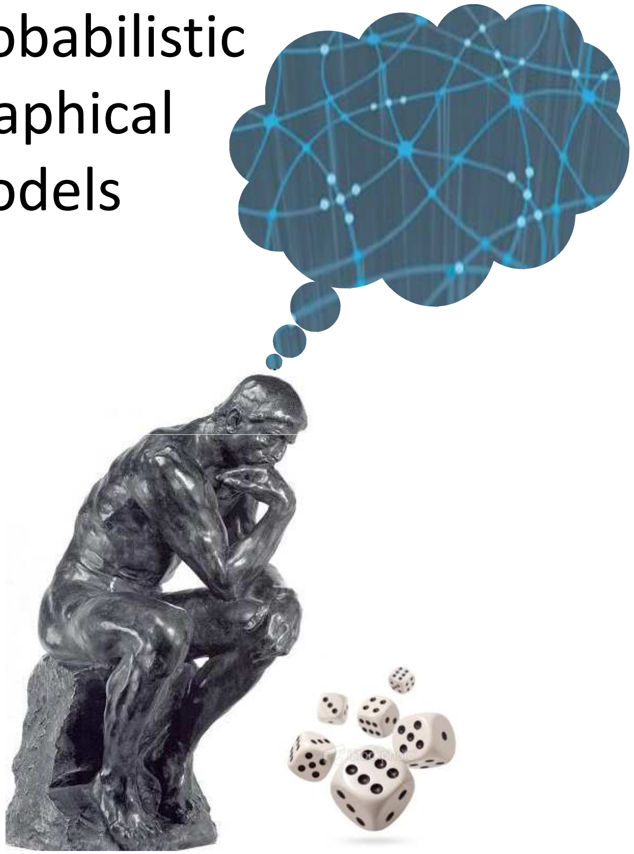


Causal protein-signaling networks derived from multiparameter single-cell data
Sachs et al., *Science* 2005

Overview

- Representation
  - Directed and undirected
  - Temporal and plate models
- Inference reasoning
  - Exact and approximate
  - Decision making
- Learning
  - Parameters and structure
  - With and without complete data

Probabilistic
Graphical
Models



Introduction

Preliminaries: Distributions

Joint Distribution

$P(I, D, G)$

- Intelligence (I) ← 2
  - i^0 (low), i^1 (high),
- Difficulty (D) ← 2
  - d^0 (easy), d^1 (hard)
- Grade (G) ← 3
  - g^1 (A), g^2 (B), g^3 (C)

parameters
$2 \times 2 \times 3 = 12$

independent params
11

I	D	G	Prob.
i^0	d^0	g^1	0.126
i^0	d^0	g^2	0.168
i^0	d^0	g^3	0.126
i^0	d^1	g^1	0.009
i^0	d^1	g^2	0.045
i^0	d^1	g^3	0.126
i^1	d^0	g^1	0.252
i^1	d^0	g^2	0.0224
i^1	d^0	g^3	0.0056
i^1	d^1	g^1	0.06
i^1	d^1	g^2	0.036
i^1	d^1	g^3	0.024

1

Conditioning

condition on g^1

I	D	G	Prob.
i^0	d^0	g^1	0.126
~~i^0~~	~~d^0~~	~~g^2~~	~~0.168~~
~~i^0~~	~~d^0~~	~~g^3~~	~~0.126~~
i^0	d^1	g^1	0.009
~~i^0~~	~~d^1~~	~~g^2~~	~~0.045~~
~~i^0~~	~~d^1~~	~~g^3~~	~~0.126~~
i^1	d^0	g^1	0.252
~~i^1~~	~~d^0~~	~~g^2~~	~~0.0224~~
~~i^1~~	~~d^0~~	~~g^3~~	~~0.0056~~
i^1	d^1	g^1	0.06
~~i^1~~	~~d^1~~	~~g^2~~	~~0.036~~
~~i^1~~	~~d^1~~	~~g^3~~	~~0.024~~

Conditioning: Reduction

I	D	G	Prob.
i^0	d^0	g^1	0.126
i^0	d^1	g^1	0.009
i^1	d^0	g^1	0.252
i^1	d^1	g^1	0.06

Conditioning: Renormalization

I	D	G	Prob.
i^0	d^0	g^1	0.126 /0.447
i^0	d^1	g^1	0.009
i^1	d^0	g^1	0.252
i^1	d^1	g^1	0.06

$P(I, D, g^1)$

unnormalized measure

Σ 0.447

I	D	Prob.
i^0	d^0	0.282
i^0	d^1	0.02
i^1	d^0	0.564
i^1	d^1	0.134

$P(I, D \mid g^1)$

Marginalization

Marginalize I

P(I, D)

I	D	Prob.
i^0	d^0	0.282
i^0	d^1	0.02
i^1	d^0	0.564
i^1	d^1	0.134

D	Prob.
d^0	0.846
d^1	0.154

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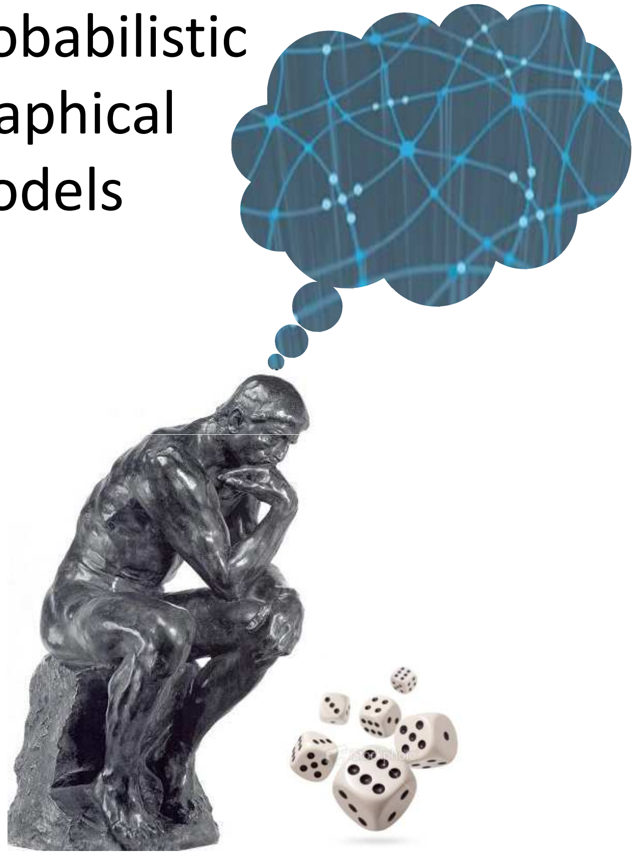

Introduction

Preliminaries: Factors

Factors

- A factor $\phi(X_1,\ldots,X_k)$

  $$\phi : \underbrace{Val(X_1,\ldots,X_k)} \rightarrow \mathbb{R}$$

- Scope = $\{X_1,\ldots,X_k\}$

Joint Distribution

$P(I,D,G)$

I	D	G	Prob.
i^0	d^0	g^1	0.126
i^0	d^0	g^2	0.168
i^0	d^0	g^3	0.126
i^0	d^1	g^1	0.009
i^0	d^1	g^2	0.045
i^0	d^1	g^3	0.126
i^1	d^0	g^1	0.252
i^1	d^0	g^2	0.0224
i^1	d^0	g^3	0.0056
i^1	d^1	g^1	0.06
i^1	d^1	g^2	0.036
i^1	d^1	g^3	0.024

Unnormalized measure $P(I,D,g^1)$

Scope = {I, D}

$P(I,D,g^1)$

I	D	G	Prob.
i^0	d^0	g^1	0.126
i^0	d^1	g^1	0.009
i^1	d^0	g^1	0.252
i^1	d^1	g^1	0.06

Conditional Probability Distribution (CPD)

$P(G \mid I, D)$

	g^1	g^2	g^3
i^0, d^0	0.3	0.4	0.3
i^0, d^1	0.05	0.25	0.7
i^1, d^0	0.9	0.08	0.02
i^1, d^1	0.5	0.3	0.2
context	A	B	C

General factors

Scope = {A, B}

A	B	ϕ
a^0	b^0	30
a^0	b^1	5
a^1	b^0	1
a^1	b^1	10

Factor Product

$\phi(A,B)$

a^1	b^1	0.5
a^1	b^2	0.8
a^2	b^1	0.1
a^2	b^2	0
a^3	b^1	0.3
a^3	b^2	0.9

$\phi_2(B,C)$

b^1	c^1	0.5
b^1	c^2	0.7
b^2	c^1	0.1
b^2	c^2	0.2

A,B,C

a^1	b^1	c^1	$0.5 \cdot 0.5 = 0.25$
a^1	b^1	c^2	$0.5 \cdot 0.7 = 0.35$
a^1	b^2	c^1	$0.8 \cdot 0.1 = 0.08$
a^1	b^2	c^2	$0.8 \cdot 0.2 = 0.16$
a^2	b^1	c^1	$0.1 \cdot 0.5 = 0.05$
a^2	b^1	c^2	$0.1 \cdot 0.7 = 0.07$
a^2	b^2	c^1	$0 \cdot 0.1 = 0$
a^2	b^2	c^2	$0 \cdot 0.2 = 0$
a^3	b^1	c^1	$0.3 \cdot 0.5 = 0.15$
a^3	b^1	c^2	$0.3 \cdot 0.7 = 0.21$
a^3	b^2	c^1	$0.9 \cdot 0.1 = 0.09$
a^3	b^2	c^2	$0.9 \cdot 0.2 = 0.18$

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Factor Marginalization

Scope A,B,C

a^1	b^1	c^1	0.25
a^1	b^1	c^2	0.35
a^1	b^2	c^1	0.08
a^1	b^2	c^2	0.16
a^2	b^1	c^1	0.05
a^2	b^1	c^2	0.07
a^2	b^2	c^1	0
a^2	b^2	c^2	0
a^3	b^1	c^1	0.15
a^3	b^1	c^2	0.21
a^3	b^2	c^1	0.09
a^3	b^2	c^2	0.18

A,C

a^1	c^1	0.33
a^1	c^2	0.51
a^2	c^1	0.05
a^2	c^2	0.07
a^3	c^1	0.24
a^3	c^2	0.39

Factor Reduction

a^1	b^1	c^1	0.25
a^1	b^1	c^2	0.35
a^1	b^2	c^1	0.08
a^1	b^2	c^2	0.16
a^2	b^1	c^1	0.05
a^2	b^1	c^2	0.07
a^2	b^2	c^1	0
a^2	b^2	c^2	0
a^3	b^1	c^1	0.15
a^3	b^1	c^2	0.21
a^3	b^2	c^1	0.09
a^3	b^2	c^2	0.18

$\Rightarrow$

a^1	b^1	c^1	0.25
a^1	b^2	c^1	0.08
a^2	b^1	c^1	0.05
a^2	b^2	c^1	0
a^3	b^1	c^1	0.15
a^3	b^2	c^1	0.09

A, B

Why factors?

- Fundamental building block for defining distributions in high-dimensional spaces
- Set of basic operations for manipulating these probability distributions