



Mathematical Linguistics & Typological Complexity

Aniello De Santo

aniellodesanto.github.io
aniello.desanto@utah.edu
@AnyDs

Get the slides!
↓

SIGTYP Lecture Series
Nov 19, 2021



(Some) Big Questions

- ▶ Are there **laws** that govern linguistic knowledge?
- ▶ **Why** are those the laws?
- ▶ Do they relate to **typological gaps**, i.e. logically possible patterns we don't (seem to) find?
- ▶ What can we infer about **human learning processes**?

Cross-disciplinarity for the win

- ▶ Stand on the shoulders of giants.
- ▶ Cross-fertilization and multiple explanatory levels.
- ▶ Yields new generalizations and data.

(Some) Big Questions

- ▶ Are there **laws** that govern linguistic knowledge?
- ▶ **Why** are those the laws?
- ▶ Do they relate to **typological gaps**, i.e. logically possible patterns we don't (seem to) find?
- ▶ What can we infer about **human learning processes**?

Cross-disciplinarity for the win

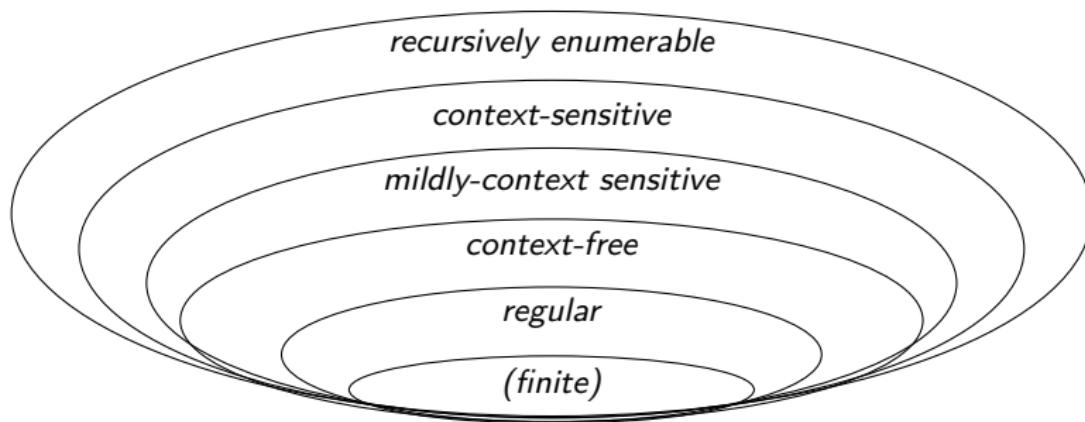
- ▶ Stand on the shoulders of giants.
- ▶ Cross-fertilization and multiple explanatory levels.
- ▶ Yields new generalizations and data.

Outline

- 1** Linguistics and Formal Language Theory
- 2** Refining the Hierarchy via Typological Insights
- 3** Artificial Grammar Learning
- 4** Summing Up & Future Directions

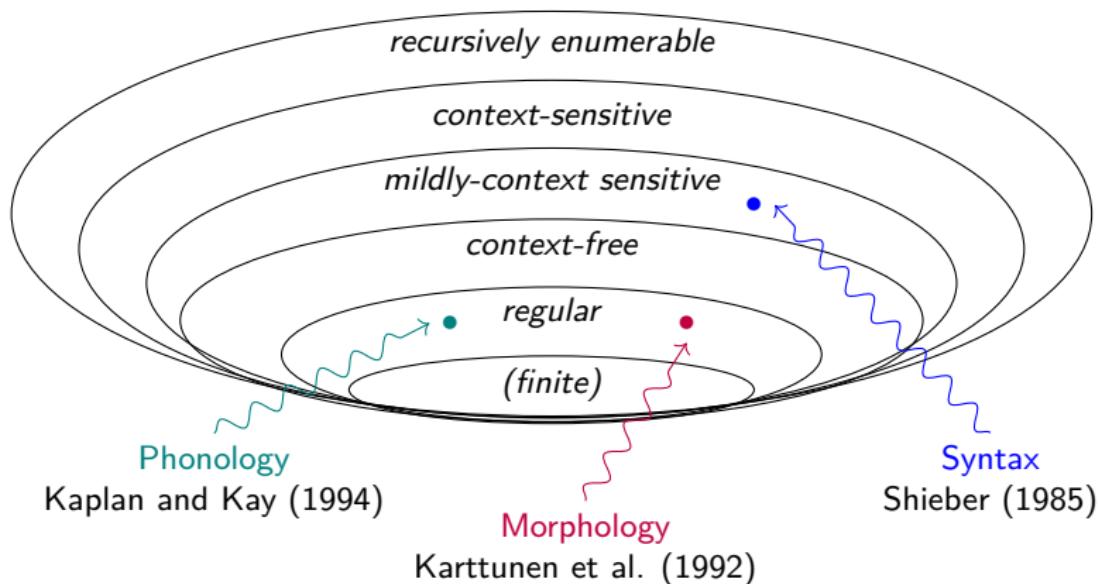
Computational Theories of Language

Languages (stringsets) can be classified according to the complexity of the grammars that generate them.

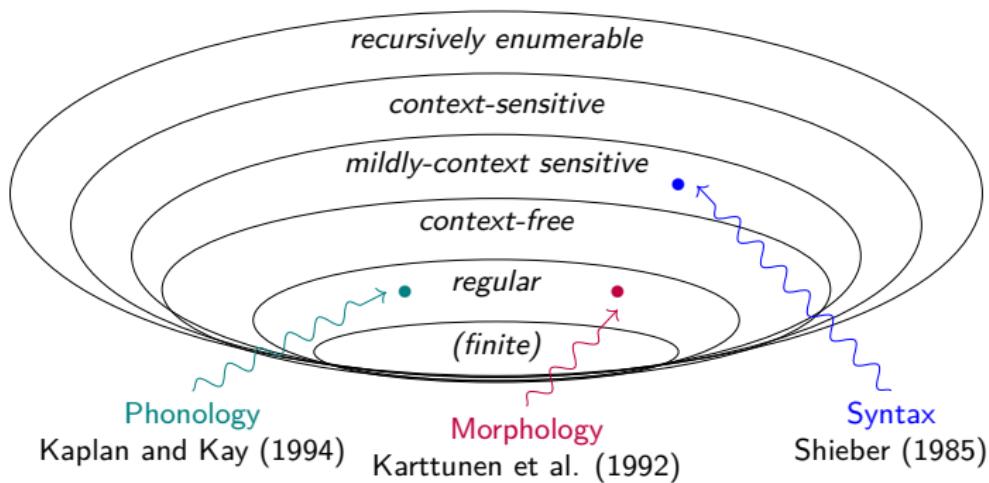


Computational Theories of Language

Languages (stringsets) can be classified according to the complexity of the grammars that generate them.



Precise Theories \Rightarrow Precise Predictions



Precise predictions for:

- ▶ typology \rightarrow e.g. no center embedding in phonology
- ▶ learnability \rightarrow e.g. no Gold learning for regular languages
- ▶ cognition \rightarrow e.g. finitely bounded working memory

Classifying Patterns

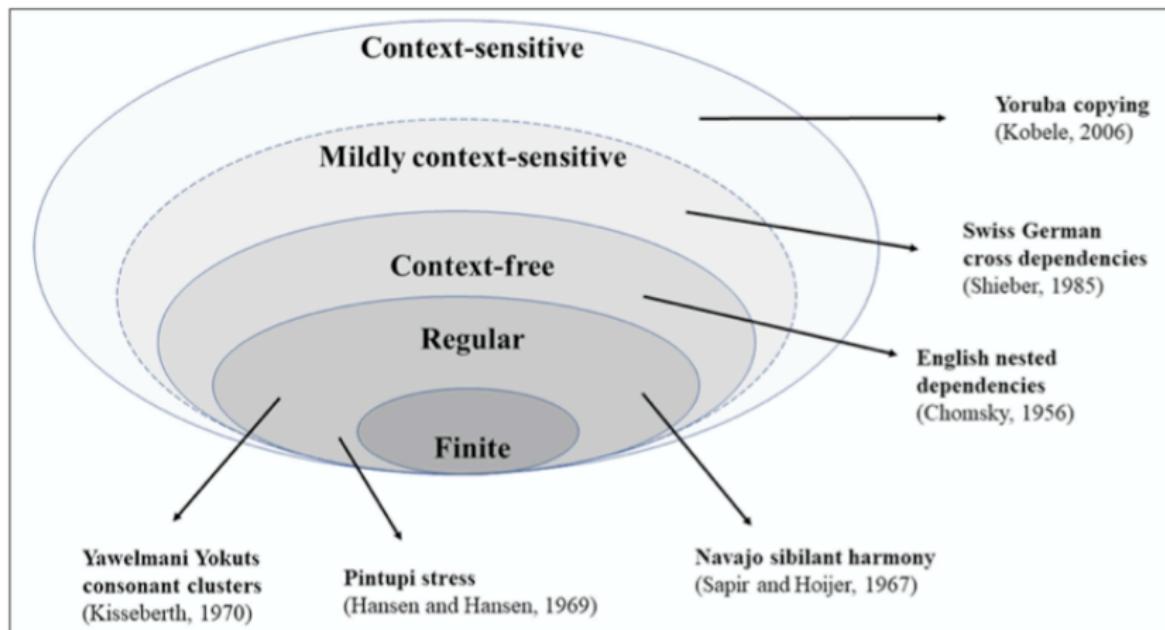
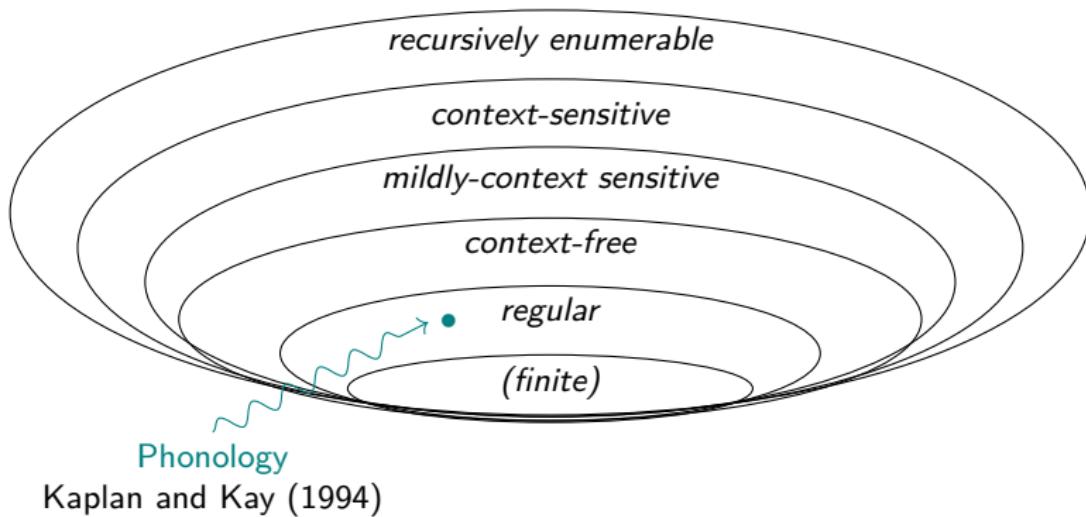
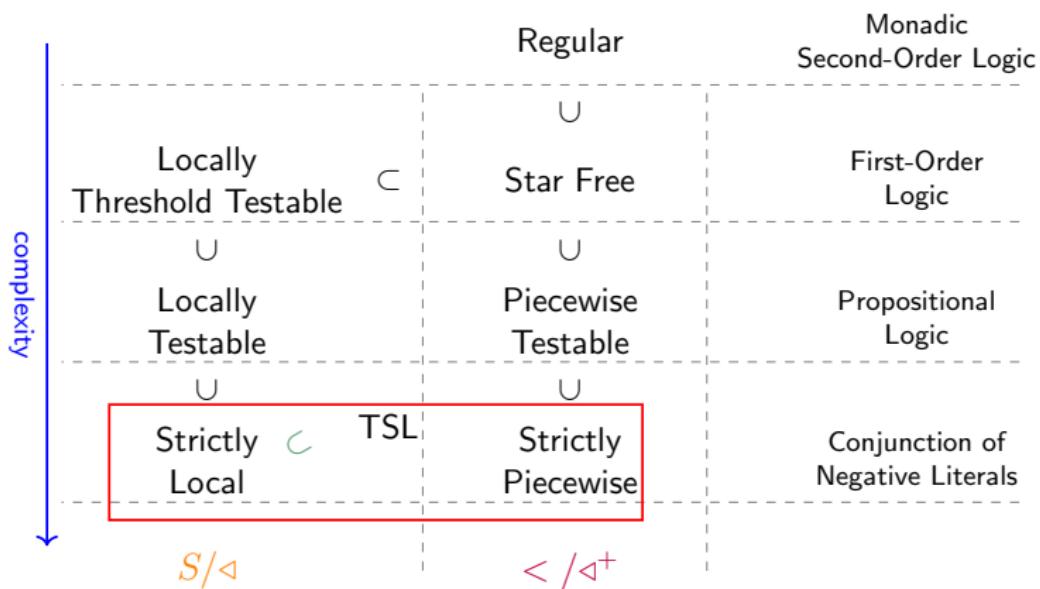


Figure 1: The Chomsky Hierarchy. Various features of natural language occupy different regions of the hierarchy. Figure reproduced from Figure 1 in Heinz (2010: 634) with permission.

Phonology as a Regular System

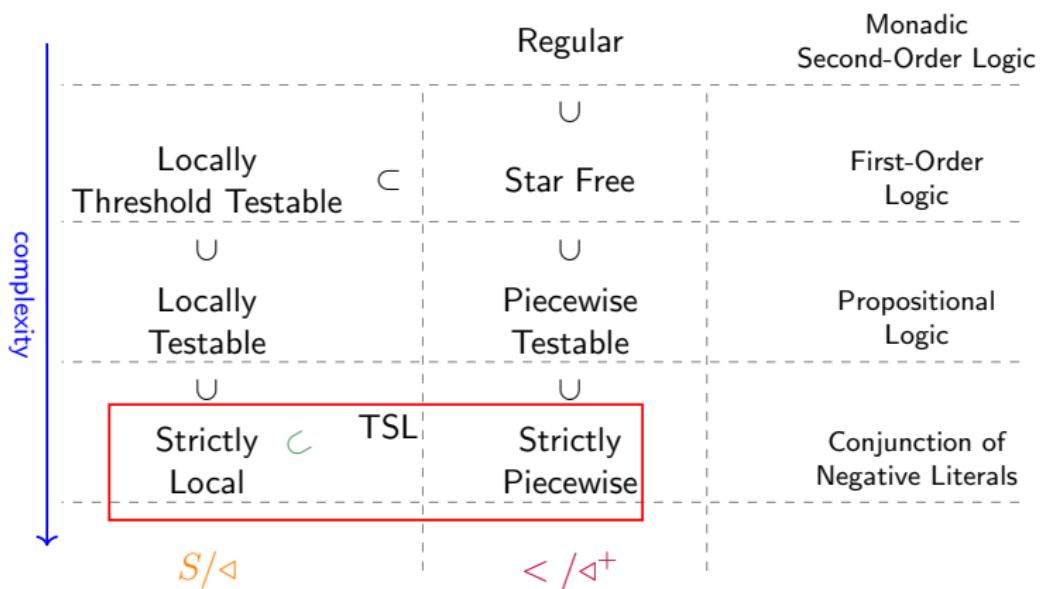


Beyond Monolithic Classes: Subregular Languages



- ▶ Multiple equivalent characterizations:
 \Rightarrow algebraic, logic, automata...

Beyond Monolithic Classes: Subregular Languages



- ▶ Multiple equivalent characterizations:
 \Rightarrow algebraic, logic, automata...

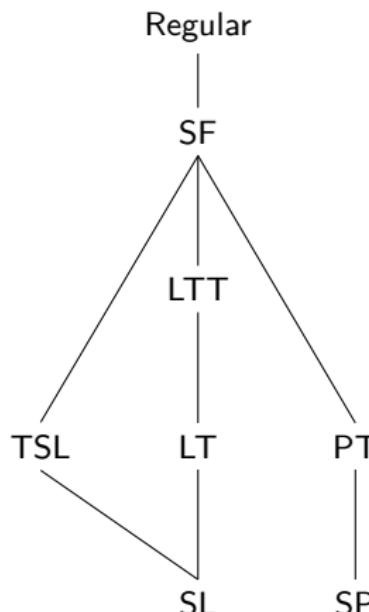
Phonology as a Subregular System

Subregular Phonotactics

- ▶ Majority of phonological patterns are **subregular**
(Heinz 2011a,b; Chandlee 2014; Graf 2017:a.o.).

Most phonological and morphological rules correspond to p-subsequential relations.

(Mohri 1997)



Phonology as a Subregular System

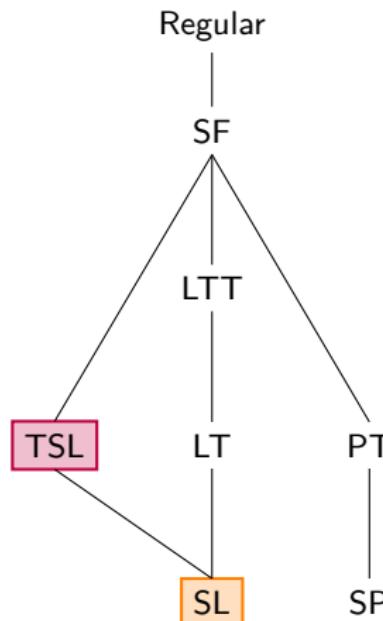
Subregular Phonotactics

- ▶ Majority of phonological patterns are **subregular** (Heinz 2011a,b; Chandlee 2014; Graf 2017:a.o.).

Most phonological and morphological rules correspond to p -subsequential relations.

(Mohri 1997)

A caveat:
Mostly phonotactics today!



Local Dependencies in Phonology

1 Word-final devoicing

Forbid voiced segments at the end of a word

- (1) a. * rad
- b. rat

1 Intervocalic voicing

Forbid voiceless segments in between two vowels

- (2) a. * faser
- b. fazer

These patterns can be described by **strictly local** (SL) constraints.

Local Dependencies in Phonology

1 Word-final devoicing

Forbid voiced segments at the end of a word

- (1) a. * rad
- b. rat

1 Intervocalic voicing

Forbid voiceless segments in between two vowels

- (2) a. * faser
- b. fazer

These patterns can be described by **strictly local** (SL) constraints.

Local Dependencies in Phonology are SL

Example: Word-final devoicing

- ▶ Forbid voiced segments at the end of a word: *[+voice]\$
- ▶ **German:** *z\$, *v\$, *d\$ (\$ = word edge).

\$ r a **d** \$ \$ r a t \$

Example: Intervocalic voicing

- ▶ Forbid voiceless segments in-between two vowels: *V[-voice]V
- ▶ **German:** *ase, *ise, *ese, *isi, ...

\$ f a **s** e r \$ \$ f a z e r \$

Local Dependencies in Phonology are SL

Example: Word-final devoicing

- ▶ Forbid voiced segments at the end of a word: *[+voice]\$
- ▶ **German:** *z\$, *v\$, *d\$ (\$ = word edge).

* \$ r a [d] \$ ok \$ r a [t] \$

Example: Intervocalic voicing

- ▶ Forbid voiceless segments in-between two vowels: *V[-voice]V
- ▶ **German:** *ase, *ise, *ese, *isi, ...

\$ f a s e r \$

\$ f a z e r \$

Local Dependencies in Phonology are SL

Example: Word-final devoicing

- ▶ Forbid voiced segments at the end of a word: *[+voice]\$
- ▶ **German:** *z\$, *v\$, *d\$ (\$ = word edge).

* \$ r a [d] \$ ok \$ r a [t] \$

Example: Intervocalic voicing

- ▶ Forbid voiceless segments in-between two vowels: *V[-voice]V
- ▶ **German:** *ase, *ise, *ese, *isi, ...

* \$ f [a s e] r \$ ok \$ f [a z e] r \$

Unbounded Dependencies Are Not SL

► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (3) a. * hasxintilawaʃ
- b. * haʃxintilawaſ
- c. haʃxintilawaʃ

Example: Samala

* \$ h a ſ x i n t i l a w a ſ \$

\$ h a ſ x i n t i l a w a ſ \$

Unbounded Dependencies Are Not SL

► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (3) a. * hasxintilawaʃ
- b. * haʃxintilawaſ
- c. haʃxintilawaʃ

Example: Samala

* \$ h a ſ x i n t i l a w a ſ \$

\$ h a ſ x i n t i l a w a ſ \$

Unbounded Dependencies Are Not SL

► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (3) a. * hasxintilawaʃ
- b. * haʃxintilawaſ
- c. haʃxintilawaʃ

Example: Samala

* \$ h aſ x i n t i l a w a ſ \$

\$ h aʃ x i n t i l a w a ſ \$

Unbounded Dependencies Are Not SL

► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (3) a. * hasxintilawaʃ
- b. * haʃxintilawaſ
- c. haʃxintilawaʃ

Example: Samala

* \$ h aſ x i n t i l a w a ſ \$
 |-----|
 |-----|

\$ h aſ x i n t i l a w a ſ \$
 |-----|
 |-----|

Unbounded Dependencies Are Not SL

► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (3) a. * hasxintilawaʃ
- b. * haʃxintilawaſ
- c. haʃxintilawaʃ

Example: Samala

* \$ h a [s x i n t i l a w a ſ] \$
 |-----|
 |-----|
\$ h a [ʃ x i n t i l a w a ſ] \$
 |-----|
 |-----|

► **But:** Sibilants can be arbitrarily far away from each other!

* \$ ſ t a j a n o w o n w a ſ \$

Unbounded Dependencies Are Not SL

► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (3) a. * hasxintilawaʃ
- b. * haʃxintilawaſ
- c. haʃxintilawaſ

Example: Samala

* \$ h aſ x i n t i l a w a ſ \$
\$ h aſ x i n t i l a w a ſ \$

► **But:** Sibilants can be arbitrarily far away from each other!

* \$ſ t a j a n o w o n w a ſ \$

Locality Over Tiers

* \$ **s** t a j a n o w o n w a **f** \$

- ▶ Sibilants can be arbitrarily far away from each other!
- ▶ **Problem:** SL limited to locality domains of size n ;

Tier-based Strictly Local (TSL) Grammars (Heinz et al. 2011)

- ▶ Projection of selected segments on a tier T (Goldsmith 1976)
- ▶ Strictly local constraints over T determine wellformedness
- ▶ Unbounded dependencies are local over **tiers**



Locality Over Tiers

* \$ **s** t a j a n o w o n w a **f** \$

- ▶ Sibilants can be arbitrarily far away from each other!
- ▶ **Problem:** SL limited to locality domains of size n ;

Tier-based Strictly Local (TSL) Grammars (Heinz et al. 2011)

- ▶ Projection of selected segments on a tier T (Goldsmith 1976)
- ▶ Strictly local constraints over T determine wellformedness
- ▶ Unbounded dependencies are local over **tiers**



Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (4) a. * hasxintilawaʃ
b. * haʃxintilawaš
c. haʃxintilawaʃ

- ▶ What do we need to project? [+strident]
- ▶ What do we need to ban? *[+ant][−ant], *[−ant][+ant]
I.E. *sʃ, *sʒ, *zʃ, *zʒ, *ʃs, *ʒs, *ʃz, *ʒz

Example: TSL Samala

s ʃ ʃ ʃ

.....

* \$ hasxintilawaʃ \$ ok \$ haʃxintilawaʃ \$

Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (4) a. * hasxintilawaʃ
b. * haʃxintilawaš
c. haʃxintilawaʃ

- ▶ What do we need to project? [+strident]
- ▶ What do we need to ban? *[+ant][−ant], * [−ant][+ant]
I.E. *sʃ, *sʒ, *zʃ, *zʒ, *ʃs, *ʒs, *ʃz, *ʒz

Example: TSL Samala



* \$ ha[s]xintilawaʃ \$



ok \$ ha[ʃ]xintilawaʃ \$

Interim Summary: SL and TSL for Phonology

- ▶ Linguistically natural (Goldsmith 1976)
- ▶ Captures wide range of phonotactic dependencies (McMullin 2016)
- ▶ Provably correct and efficient learning algorithms (Jardine and McMullin 2017)
- ▶ Rules out unattested patterns
(cf. Lai 2015, Aksanova et al. 2016, Graf & De Santo 2019, a.o.)

But:

- ▶ Typological variation is complex, knowledge is limited
- ▶ Can we truly gain cognitive insights?

Interim Summary: SL and TSL for Phonology

- ▶ Linguistically natural (Goldsmith 1976)
- ▶ Captures wide range of phonotactic dependencies (McMullin 2016)
- ▶ Provably correct and efficient learning algorithms (Jardine and McMullin 2017)
- ▶ Rules out unattested patterns
(cf. Lai 2015, Aksanova et al. 2016, Graf & De Santo 2019, a.o.)

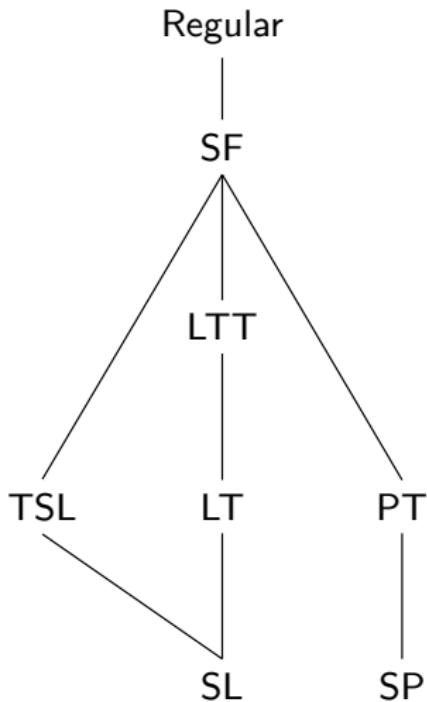
But:

- ▶ Typological variation is complex, knowledge is limited
- ▶ Can we truly gain cognitive insights?

Outline

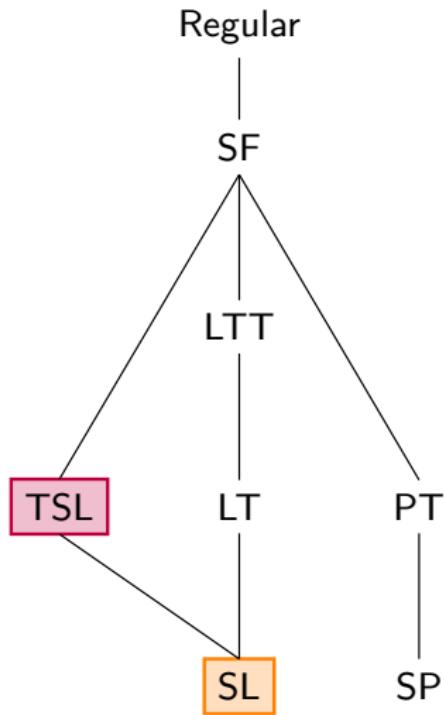
- 1** Linguistics and Formal Language Theory
- 2** Refining the Hierarchy via Typological Insights
- 3** Artificial Grammar Learning
- 4** Summing Up & Future Directions

SL and TSL: So What?



- ▶ **But** not every long-distance pattern is TSL!
(McMullin 2016, Mayer & Major 2018, De Santo & Graf 2019)

SL and TSL: So What?

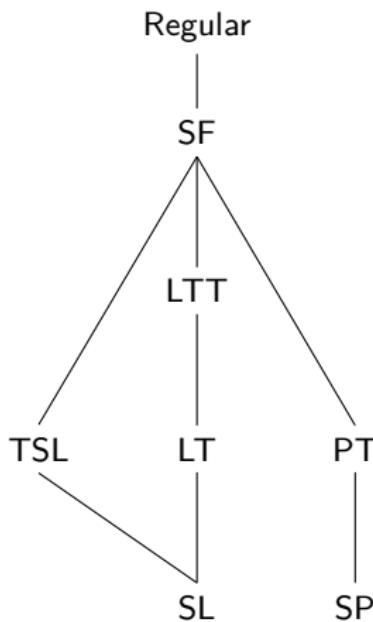


- ▶ **But** not every long-distance pattern is TSL!
(McMullin 2016, Mayer & Major 2018, De Santo & Graf 2019)

Concurrent Processes (De Santo and Graf, 2019)

Observation

- ▶ TSL is not closed under intersection



- ▶ We want to also account for multiple processes
So we can cover the complete phonotactics of a language
- ▶ Multiple non-interacting processes in attested patterns

A TSL Outlier

Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin2016)

- 1) Underlying causative prefix /s(:)-/

Base Causative

- a. uga **s**:uga "be evacuated"
- b. a**s**:twa **s**-as:twa "settle, be levelled"

A TSL Outlier

Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin2016)

- 1) Underlying causative prefix /s(:)-/

Base Causative

- a. uga **s**:uga "be evacuated"
- b. a**s**:twa **s**-as:twa "settle, be levelled"

- 2) Sibilant harmony

Base Causative

- a. fiaʃr ſ- fiaʃr "be full of straw, of discord"
- b. nza **z**:nza "be sold"

A TSL Outlier

Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin2016)

- 1) Underlying causative prefix /s(:)-/

Base Causative

- a. uga **s**:uga "be evacuated"
- b. a**s**:twa **s**-as:twa "settle, be levelled"

- 2) Sibilant harmony

Base Causative

- a. fiaʃr ſ- fiaʃr "be full of straw, of discord"
- b. nza **z**:nza "be sold"

- 3) Sibilant voicing harmony blocked

Base Causative

- a. ukz **s**:ukz "recognize"
- b. q:uʒ:i ſ- quʒ:i "be dislocated, broken"

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \emptyset, s, z, \emptyset \}$$

$$S = \{ *s\emptyset, *s\emptyset, *s\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset \}$$

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \emptyset, s, z, \emptyset \}$$

$$S = \{ *s\emptyset, *s\emptyset, *s\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset \}$$

z ʒ

* [z] m: [ʒ] d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \emptyset, s, z, \emptyset \}$$

$$S = \{ *s\emptyset, *s\emptyset, *s\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset \}$$

*
z ʒ

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \emptyset, s, z, \emptyset \}$$

$$S = \{ *s\emptyset, *s\emptyset, *s\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset \}$$

*
z z

* z m: z d a w |

z z

ok z m: z d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

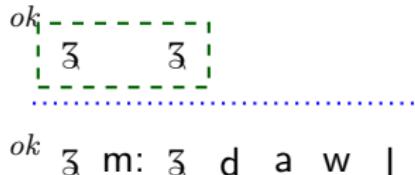
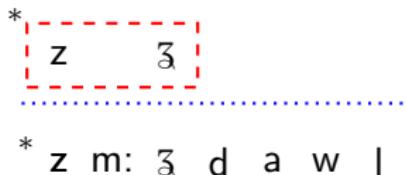
Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \emptyset, s, z, \emptyset \}$$

$$S = \{ *s\emptyset, *s\emptyset, *s\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset \}$$



Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \emptyset, s, z, \emptyset, q \}$$

$$S = \{ *s\emptyset, *s\emptyset, *s\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset \}$$

ok \emptyset q u \emptyset : i

* s q u \emptyset : i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, \text{s}, \text{z}, \textʃ, \text{q} \}$$

$$S = \{ *s\text{ʒ}, *s\text{z}, *s\textʃ, *ʒ\text{s}, *ʃ\text{s}, *z\text{s}, *z\textʃ, *ʒ\text{z}, *ʃ\text{z}, *ʒ\text{ʃ}, *ʒ\text{z} \}$$

ʃ q ʒ:

ok  q u  i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

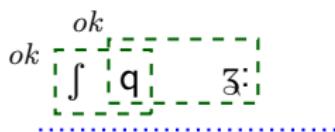
Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, \text{s}, \text{z}, \textʃ, \text{q} \}$$

$$S = \{ *s\text{ʒ}, *s\text{z}, *s\textʃ, *\text{ʒ}s, *\text{ʃ}s, *z\text{s}, *\text{z}\textʃ, *\text{ʃ}\text{ʒ}, *\text{ʃ}\text{z}, * \text{ʒ}\text{ʃ}, * \text{ʒ}\text{z} \}$$



ok \int q u ʒ: i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

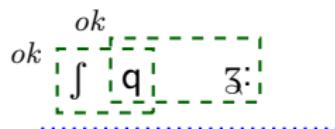
Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, \text{s}, \text{z}, \textʃ, \text{q} \}$$

$$S = \{ *s\text{ʒ}, *s\text{z}, *s\textʃ, *\text{ʒ}s, *\text{ʃ}s, *z\text{s}, *\text{z}\textʃ, *\text{ʃ}\text{ʒ}, *\text{ʃ}\text{z}, * \text{ʒ}\text{s}, * \text{ʒ}\text{ʃ}, * \text{ʒ}\text{z} \}$$



ok /ʃ q ʒ:/ i

s q ʒ:/

* [s q] u [ʒ:] i

Sibilant Harmony in IMDLAWN TASHLHIYT

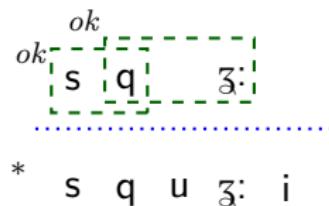
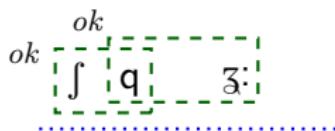
Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \emptyset, s, z, \emptyset, q \}$$

$$S = \{ *s\emptyset, *s\emptyset, *s\emptyset, *\emptyset s, *\emptyset s, *z\emptyset, *z\emptyset, *z\emptyset, *\emptyset z, *\emptyset z, *z\emptyset, *z\emptyset \}$$



Sibilant Harmony in IMDLAWN TASHLHIYT

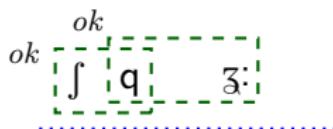
Generalization (2/2)

Voiceless obstruents block agreement in voicing.

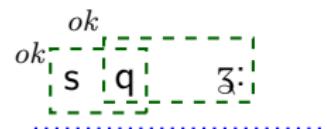
Grammar

$$T = \{ \emptyset, s, z, \emptyset, q \}$$

$$S = \{ *s\emptyset, *s\emptyset, *s\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset, *z\emptyset \}$$



ok \emptyset \emptyset \emptyset \emptyset



* $\textcolor{red}{s}$ $\textcolor{red}{q}$ $\textcolor{red}{u}$ $\textcolor{red}{ʒ}$ i

Multi-Tier Strictly Local (MTSL) Languages (1/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

ok \emptyset q u \emptyset i

Multi-Tier Strictly Local (MTSL) Languages (1/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

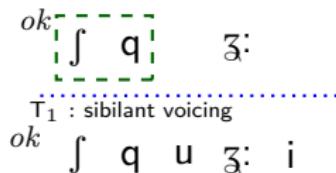
\emptyset q \emptyset :
.....
 ok $\boxed{\emptyset} \boxed{q}$ u $\boxed{\emptyset}: \quad i$

Multi-Tier Strictly Local (MTSL) Languages (1/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{*\emptyset s, *s \emptyset, *\emptyset z, *z \emptyset, *\emptyset \emptyset, *\emptyset \emptyset\}$

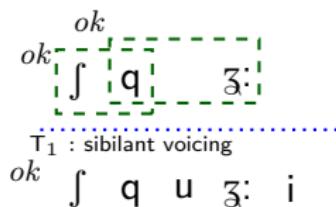


Multi-Tier Strictly Local (MTSL) Languages (1/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$



Multi-Tier Strictly Local (MTSL) Languages (1/2)

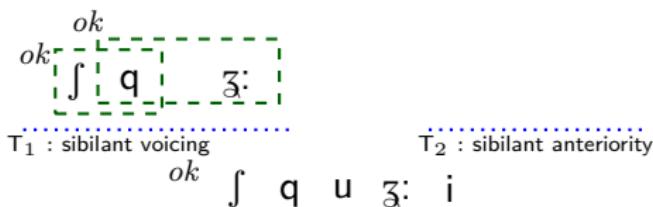
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

Unbounded agreement in anteriorsity:

- ▶ $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{^*s\emptyset, ^*\emptyset \emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$



Multi-Tier Strictly Local (MTSL) Languages (1/2)

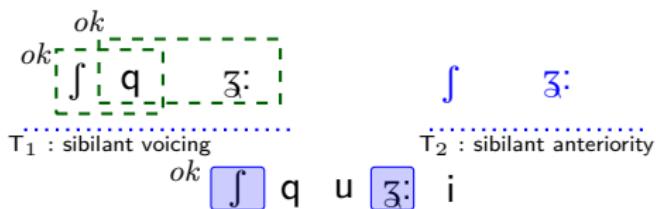
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

Unbounded agreement in anteriority:

- ▶ $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$



Multi-Tier Strictly Local (MTSL) Languages (1/2)

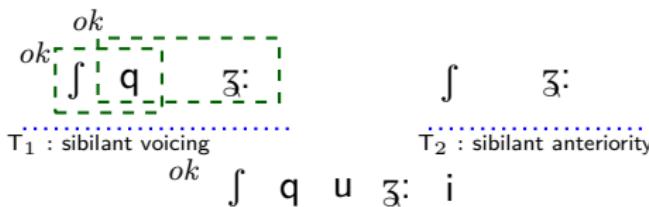
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{*\emptyset\emptyset, *s\emptyset, *\emptyset s, *z\emptyset, *\emptyset z, *s\emptyset, *\emptyset s\emptyset\}$

Unbounded agreement in anteriorsity:

- ▶ $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{*\emptyset\emptyset, *s\emptyset, *\emptyset s, *z\emptyset, *z\emptyset, *\emptyset z, *z\emptyset, *\emptyset z\}$



Multi-Tier Strictly Local (MTSL) Languages (1/2)

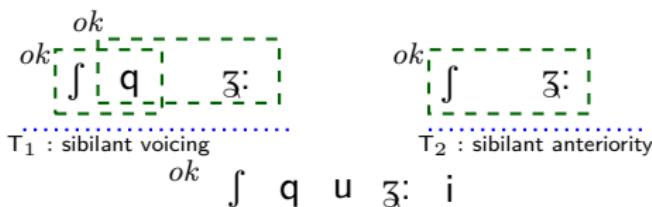
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

Unbounded agreement in anteriority:

- ▶ $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$



Multi-Tier Strictly Local (MTSL) Languages (2/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

Unbounded agreement in anteriorsity:

- ▶ $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

* s q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (2/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{*\emptyset\emptyset, *s\emptyset, *\emptyset s, *z\emptyset, *\emptyset z, *s\emptyset, *\emptyset s\}$

Unbounded agreement in anteriorsity:

- ▶ $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{*\emptyset\emptyset, *s\emptyset, *\emptyset s, *z\emptyset, *z\emptyset, *\emptyset z, *s\emptyset, *\emptyset s\}$



Multi-Tier Strictly Local (MTSL) Languages (2/2)

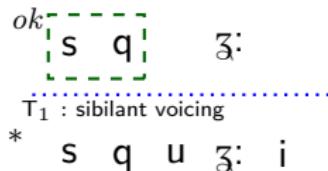
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{*\emptyset\emptyset, *s\emptyset, *\emptyset s, *z\emptyset, *\emptyset z, *s\emptyset, *\emptyset s\}$

Unbounded agreement in anteriorsity:

- ▶ $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{*\emptyset\emptyset, *s\emptyset, *\emptyset s, *z\emptyset, *z\emptyset, *\emptyset z, *z\emptyset, *\emptyset z\}$



Multi-Tier Strictly Local (MTSL) Languages (2/2)

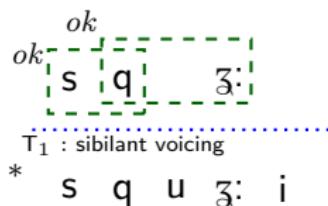
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

Unbounded agreement in anteriorsity:

- ▶ $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$



Multi-Tier Strictly Local (MTSL) Languages (2/2)

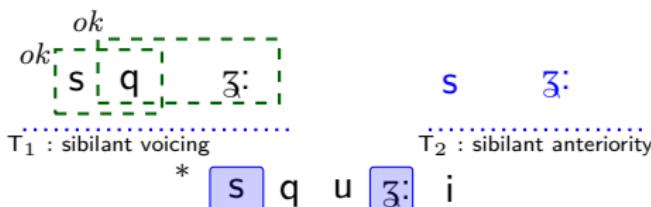
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

Unbounded agreement in anteriority:

- $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{^*s\emptyset, ^*\emptyset\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$



Multi-Tier Strictly Local (MTSL) Languages (2/2)

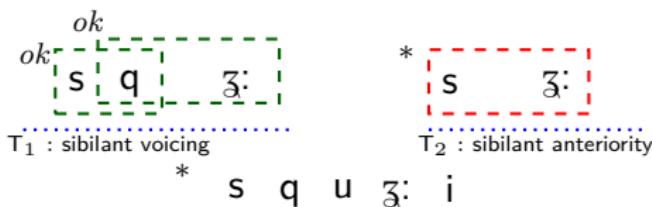
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

- ▶ $T_1 = \{\emptyset, s, z, \emptyset, q\}$ $S_1 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$

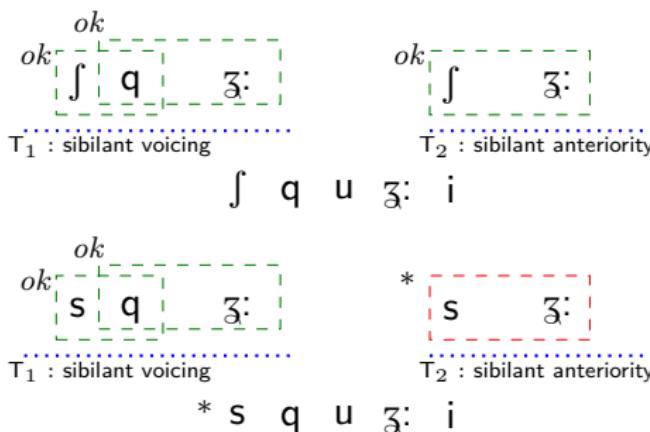
Unbounded agreement in anteriorsity:

- ▶ $T_2 = \{\emptyset, s, z, \emptyset\}$ $S_2 = \{^*s\emptyset, ^*s\emptyset, ^*\emptyset s, ^*\emptyset z, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset, ^*\emptyset \emptyset\}$



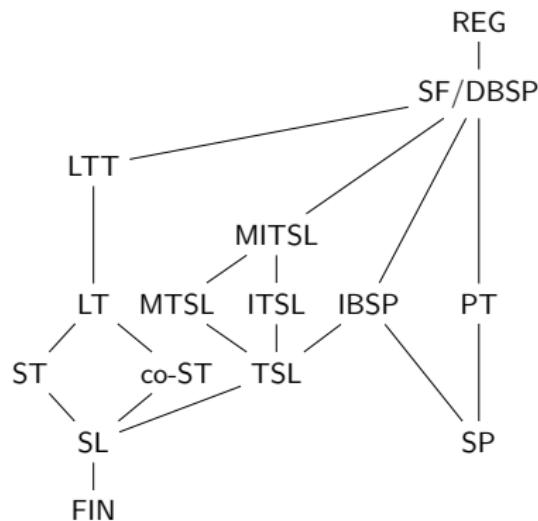
Accounting for Concurrent Processes

- ▶ MTSL: TSL closure under intersection
(De Santo & Graf, 2019)



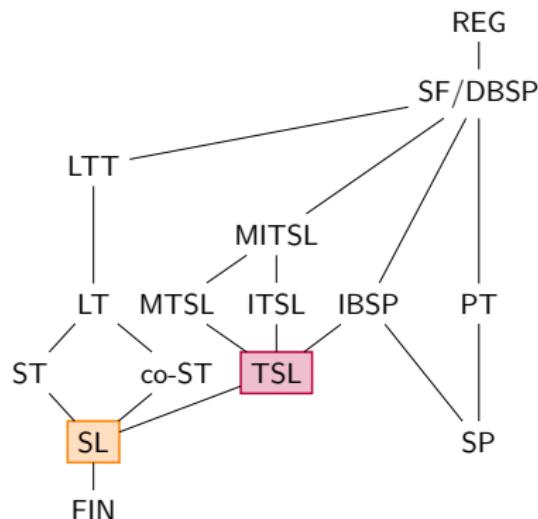
- ▶ Intersection closure accounts for multiple concurrent processes
- ▶ Can characterize the complete phonotactics of a language

A Plethora of Combination



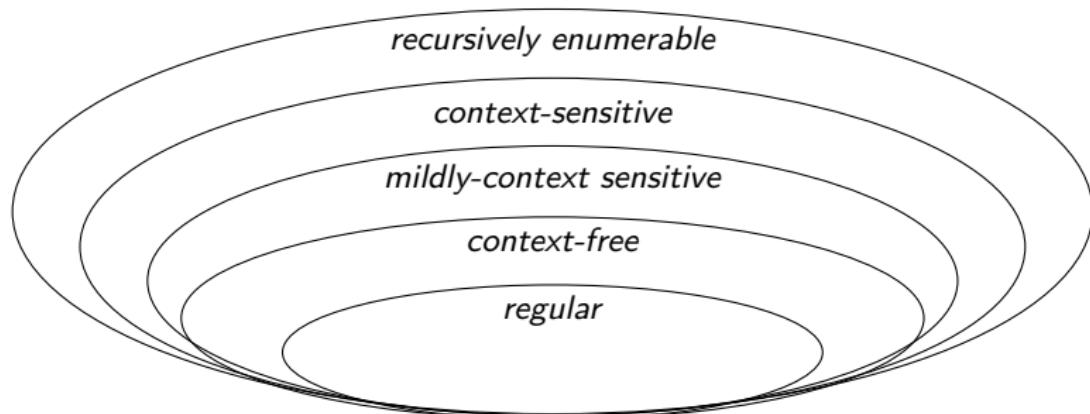
- ▶ The goal is **not** identifying a single “correct” class
- ▶ Pinpoint fundamental properties of the patterns:
SL: \triangleleft , TSL: \triangleleft_T , ...

A Plethora of Combination

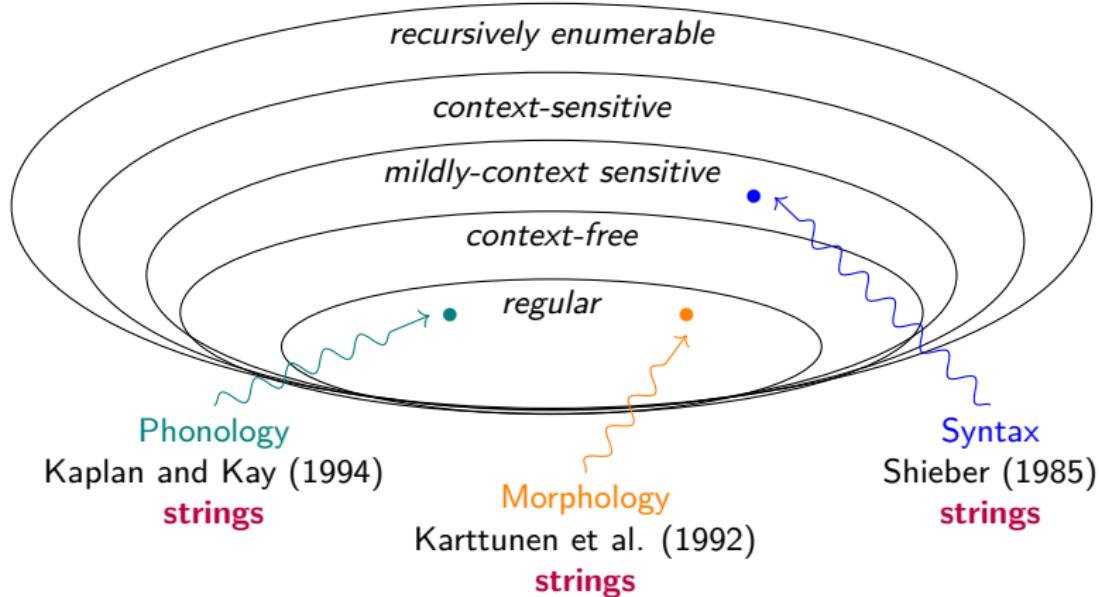


- ▶ The goal is **not** identifying a single “correct” class
- ▶ Pinpoint fundamental properties of the patterns:
SL: \triangleleft , TSL: \triangleleft_T , ...

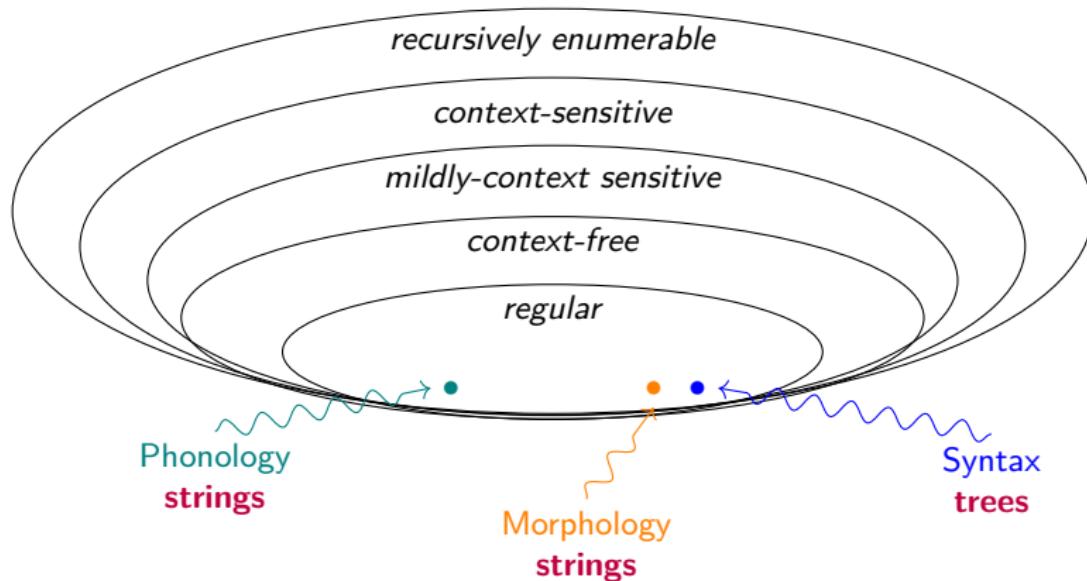
Cross-domain Parallels



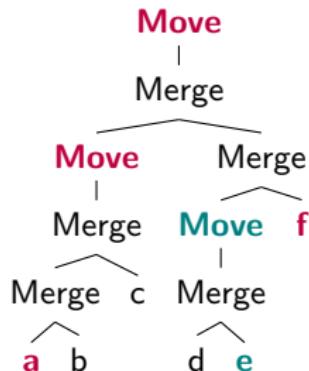
Cross-domain Parallels



Cross-domain Parallels

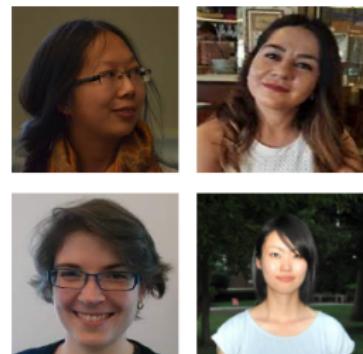


Subregular Syntax

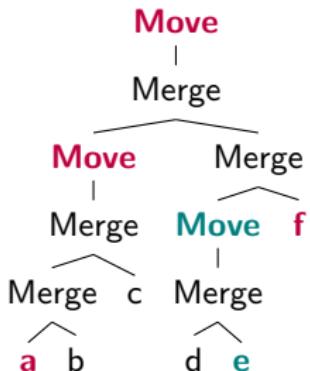


► Some results for syntax

- ▶ regular tree languages
(Michaelis 2004; Kobele et al. 2007)
- ▶ subregular operations (Graf 2018)
- ▶ subregular dependencies/constraints
(Laszakovits 2018; Vu et al. 2019)
- ▶ tree automata and parsing restrictions
(Graf & De Santo 19, Ikawa et al. 20)

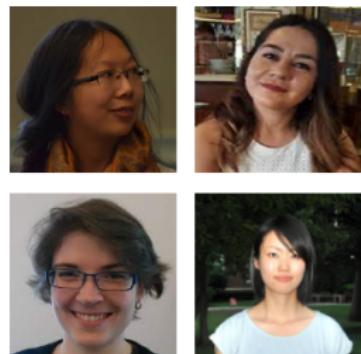


Subregular Syntax

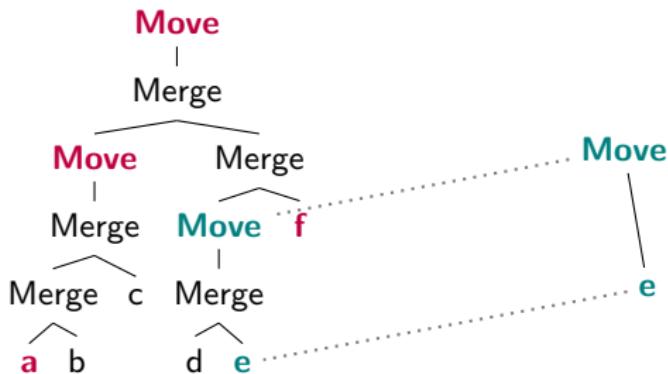


► Some results for syntax

- regular tree languages
(Michaelis 2004; Kobele et al. 2007)
- subregular operations (Graf 2018)
- subregular dependencies/constraints
(Laszakovits 2018; Vu et al. 2019)
- tree automata and parsing restrictions
(Graf & De Santo 19, Ikawa et al. 20)

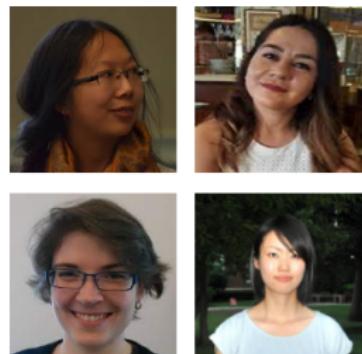


Subregular Syntax

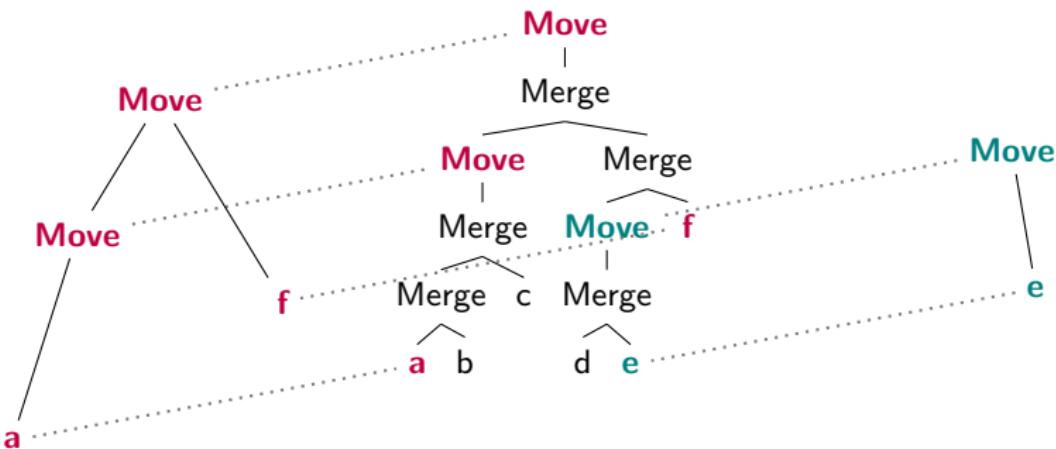


► Some results for syntax

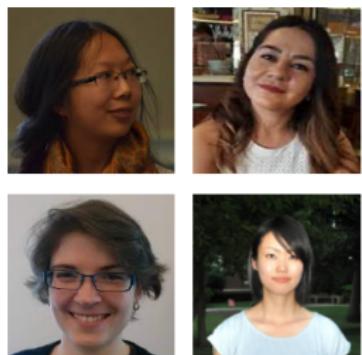
- ▶ regular tree languages
(Michaelis 2004; Kobele et al. 2007)
- ▶ subregular operations (Graf 2018)
- ▶ subregular dependencies/constraints
(Laszakovits 2018; Vu et al. 2019)
- ▶ tree automata and parsing restrictions
(Graf & De Santo 19, Ikawa et al. 20)



Subregular Syntax



- ▶ Some results for syntax
 - ▶ regular tree languages
(Michaelis 2004; Kobele et al. 2007)
 - ▶ subregular operations (Graf 2018)
 - ▶ subregular dependencies/constraints
(Laszakovits 2018; Vu et al. 2019)
 - ▶ tree automata and parsing restrictions
(Graf & De Santo 19, Ikawa et al. 20)



Interim Summary: Again, So What?

Strong Parallelism

Subregular dependencies in phonology, (morphology), and syntax
subregular over their respective **structural representations**.

We gain a unified perspective on:

- ▶ Attested and unattested typology
- ▶ learnability?

Interim Summary: Again, So What?

Strong Parallelism

Subregular dependencies in phonology, (morphology), and syntax
subregular over their respective **structural representations**.

We gain a unified perspective on:

- ▶ Attested and unattested typology
 - ✗ Intervocalic Voicing iff applied **an even times** in the string
 - ✗ Have a CP iff it dominates ≥ 3 TPs
- ▶ learnability?

Interim Summary: Again, So What?

Strong Parallelism

Subregular dependencies in phonology, (morphology), and syntax
subregular over their respective **structural representations**.

We gain a unified perspective on:

- ▶ Attested and unattested typology
 - ✗ Intervocalic Voicing iff applied **an even times** in the string
 - ✗ Have a CP iff it dominates ≥ 3 TPs
- ▶ learnability?
 - Learnable from positive examples of strings/trees.
 - Which information primitives are we sensitive to?

Interim Summary: Again, So What?

Strong Parallelism

Subregular dependencies in phonology, (morphology), and syntax
subregular over their respective **structural representations**.

We gain a unified perspective on:

- ▶ Attested and unattested typology
 - ✗ Intervocalic Voicing iff applied **an even times** in the string
 - ✗ Have a CP iff it dominates ≥ 3 TPs

- ▶ learnability?

Learnable from positive examples of strings/trees.
Which information primitives are we sensitive to?

But:

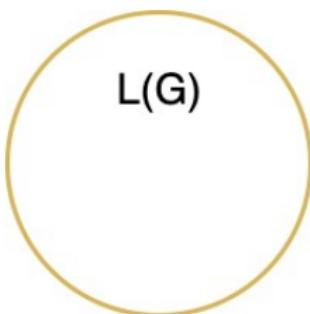
- ▶ Typological variation is complex
- ▶ Our knowledge of attested pattern is limited

Outline

- 1** Linguistics and Formal Language Theory
- 2** Refining the Hierarchy via Typological Insights
- 3** Artificial Grammar Learning
- 4** Summing Up & Future Directions

Artificial Grammar Learning (AGL)

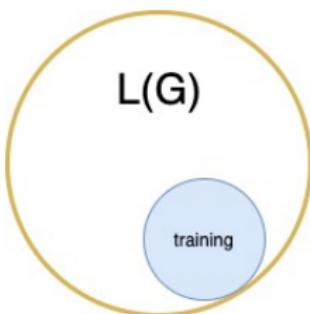
- ▶ Can be used to test implicit learning abilities (Reber, 1976)



- ▶ Possible vs. impossible rules (Musso et al. 01, Culbertson 21)
- ▶ Child language acquisition (Nowal and Baggio 2017, a.o.)
→ but careful with test sets (De Santo 2017)
- ▶ Animal cognition (Wilson et al. 2020, a.o.)
→ cf. (De Santo and Rawski 2020)

Artificial Grammar Learning (AGL)

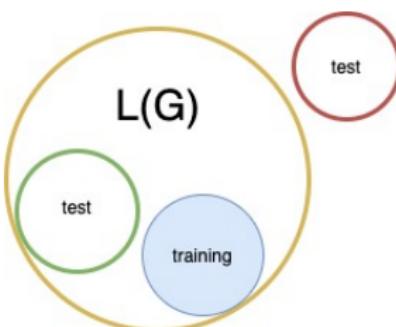
- ▶ Can be used to test implicit learning abilities (Reber, 1976)



- ▶ Possible vs. impossible rules (Musso et al. 01, Culbertson 21)
- ▶ Child language acquisition (Nowal and Baggio 2017, a.o.)
→ but careful with test sets (De Santo 2017)
- ▶ Animal cognition (Wilson et al. 2020, a.o.)
→ cf. (De Santo and Rawski 2020)

Artificial Grammar Learning (AGL)

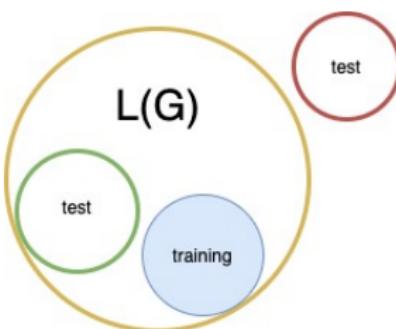
- ▶ Can be used to test implicit learning abilities (Reber, 1976)



- ▶ Possible vs. impossible rules (Musso et al. 01, Culbertson 21)
- ▶ Child language acquisition (Nowal and Baggio 2017, a.o.)
→ but careful with test sets (De Santo 2017)
- ▶ Animal cognition (Wilson et al. 2020, a.o.)
→ cf. (De Santo and Rawski 2020)

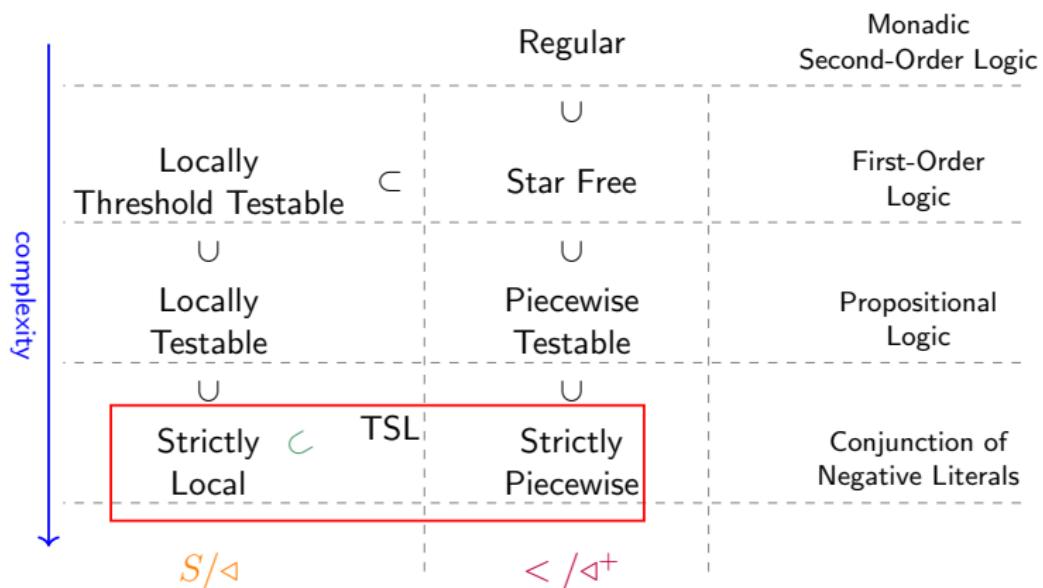
Artificial Grammar Learning (AGL)

- ▶ Can be used to test implicit learning abilities (Reber, 1976)



- ▶ Possible vs. impossible rules (Musso et al. 01, Culbertson 21)
- ▶ Child language acquisition (Nowal and Baggio 2017, a.o.)
→ but careful with test sets (De Santo 2017)
- ▶ Animal cognition (Wilson et al. 2020, a.o.)
→ cf. (De Santo and Rawski 2020)

Testing Subregular Predictions



Example: Attested vs. Unattested Patterns

Attested: Unbounded Sibilant Harmony

- ▶ Every sibilant needs to harmonize



* \$hasxintilawf\$



ok \$hafxintilawf\$

Unattested: First-Last Harmony

- ▶ Harmony only holds between initial and final segments

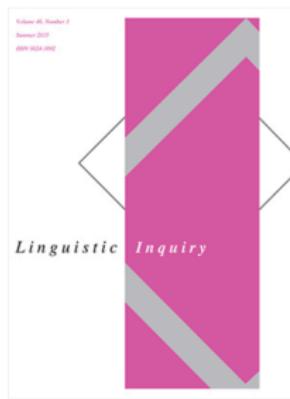


ok \$hasxintilawf\$



* \$satxintilawf\$

Lai (2015)



Learnable vs. Unlearnable Harmony Patterns

Regine Lai

Posted Online July 09, 2015

https://doi.org/10.1162/LING_a_00188

© 2015 Massachusetts Institute of Technology

Linguistic Inquiry

Volume 46 | Issue 3 | Summer 2015
p.425-451

Keywords: phonotactics, learnability, computational phonology,
formal theory, typology, dependencies

Lai (2015): Stimuli

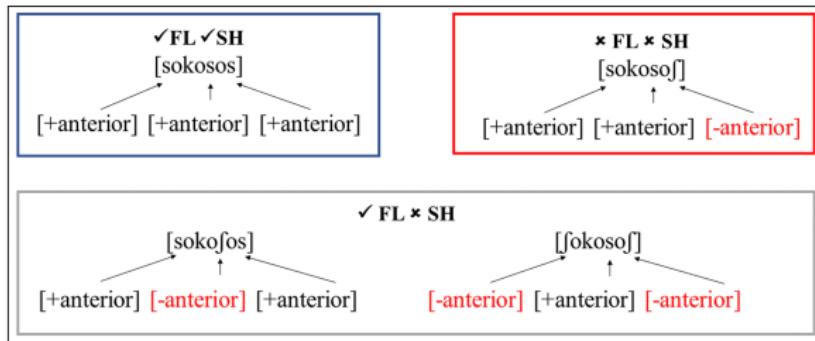


Figure 3: Comparison of SH and FL stimuli.

Lai (2015): Stimuli

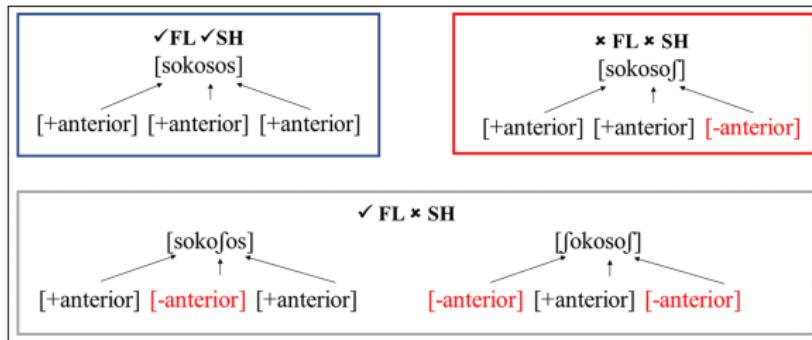


Figure 3: Comparison of SH and FL stimuli.

Table 6

Predicted results with respect to the control group for each test pairing if Sibilant Harmony and First-Last Assimilation grammars were internalized

	Pairs		
Conditions	FL/*SH vs. *FL/*SH (e.g., [s . . . ſ . . . s] vs. [s . . . s . . . ſ])	FL/SH vs. *FL/*SH (e.g., [s . . . s . . . s] vs. [s . . . s . . . ſ])	FL/SH vs. FL/*SH (e.g., [s . . . s . . . s] vs. [s . . . ſ . . . s])
	Rate of FL/*SH	Rate of FL/SH	Rate of FL/SH
SH	~ Control	> Control	> Control
FL	> Control	> Control	~ Control

Lai (2015): Results

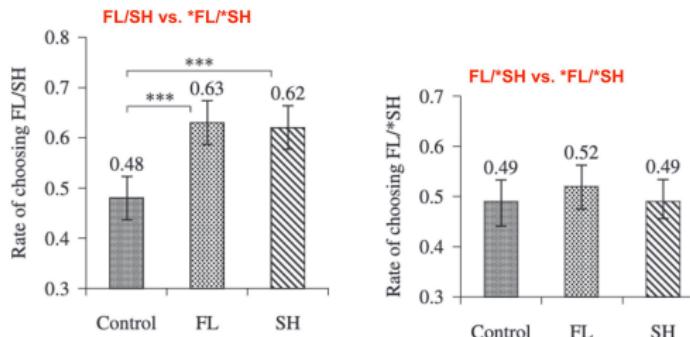


Table 6

Predicted results with respect to the control group for each test pairing if Sibilant Harmony and First-Last Assimilation grammars were internalized

Conditions	Pairs		
	FL/*SI vs. *FL/*SI (e.g., [s . . . f . . . s] vs. [s . . . s . . . f])	FL/SI vs. *FL/*SI (e.g., [s . . . s . . . s] vs. [s . . . s . . . f])	FL/SI vs. FL/*SI (e.g., [s . . . s . . . s] vs. [s . . . f . . . s])
SH	~ Control	> Control	> Control
FL	> Control	> Control	~ Control

- ▶ See Avcu and Hestvik (2020), Avcu et al. (2019) for replications

Lai (2015): Results

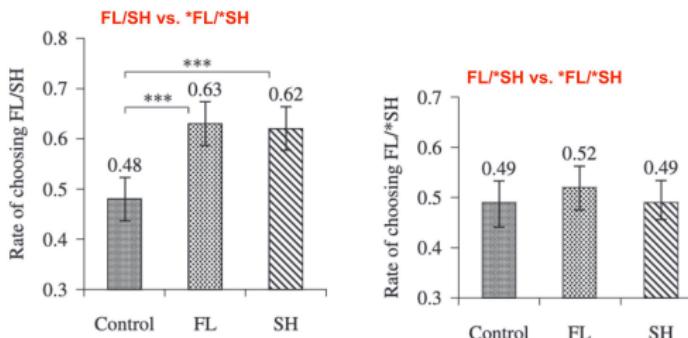


Table 6

Predicted results with respect to the control group for each test pairing if Sibilant Harmony and First-Last Assimilation grammars were internalized

Conditions	Pairs		
	FL/*SIH vs. *FL/*SIH (e.g., [s . . . f . . . s] vs. [s . . . s . . . f])	FL/SIH vs. *FL/*SIH (e.g., [s . . . s . . . s] vs. [s . . . s . . . f])	FL/SIH vs. FL/*SIH (e.g., [s . . . s . . . s] vs. [s . . . f . . . s])
SH	~ Control	> Control	> Control
FL	> Control	> Control	~ Control

- ▶ See Avcu and Hestvik (2020), Avcu et al. (2019) for replications

A Plethora of Testable Predictions

Observation

- ▶ Attested patterns **A** and **B** are TSL.
- ▶ But combined pattern **A+B** is not TSL.

Prediction

- ▶ **A+B** should be harder to learn than **A** and **B**

Example: Compounding Markers

Morphotactics as Tier-Based Strictly Local Dependencies

Alëna Aksënova Thomas Graf Sedigheh Moradi

- ▶ Russian has an infix **-o-** that may occur between parts of compounds.
- ▶ Turkish has a single suffix **-sı** that occurs at end of compounds.

(5) vod **-o-** voz **-o-** voz
water -COMP- carry -COMP- carry
'carrier of water-carriers'

(6) türk bahçe kapı **-sı** (***-Sİ**)
turkish garden gate -COMP (*-COMP)
'Turkish garden gate'



Example: Compounding Markers [cont.]

- ▶ Russian and Turkish are TSL.

Tier₁ COMP affix and stem edges #

Russian *n*-grams **oo**, **\$o**, **o\$**

Turkish *n*-grams **sisi**, **\$si**, **si#**

- ▶ The combined pattern would yield **Ruskish**: stem^{*n+1*}-si^{*n*}
- ▶ This pattern is not regular and hence **not TSL either**.

Testable Predictions

- ▶ Can naive subjects learn Russian-like, Turkis-like, and Ruskish-like compounding?

Outline

- 1** Linguistics and Formal Language Theory
- 2** Refining the Hierarchy via Typological Insights
- 3** Artificial Grammar Learning
- 4** Summing Up & Future Directions

Of Black Swans and Flying Pigs



Of Black Swans and Flying Pigs



Of Black Swans and Flying Pigs



- ▶ Not a single data point, but classes of phenomena
- ▶ Value of restrictive theories: predictive and explanatory
- ▶ We learn from falsifying them too!

Complexity as a Magnifying Lens

- ▶ We can compare patterns and predictions across classes
- ▶ We can also compare patterns within a same class

Proceedings of the Society for Computation in Linguistics

Volume 1

Article 8

2018

Formal Restrictions On Multiple Tiers

Alena Aksenova

Stony Brook University, alena.aksenova@stonybrook.edu

Sanket Deshmukh

Stony Brook University, sanket.deshmukh@stonybrook.edu



Testing Harmony Systems

Reminder:

- ▶ MTSL's multiple-tier idea...

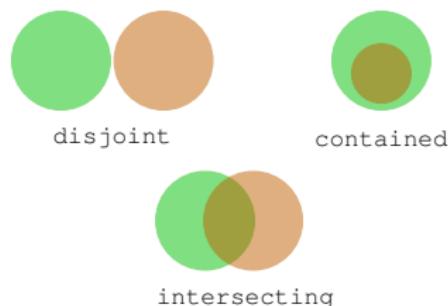
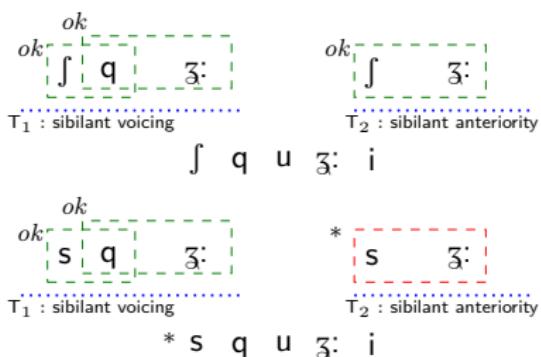


Figure 2: Theoretically possible tier alphabet relations

Testing Harmony Systems (cont.)

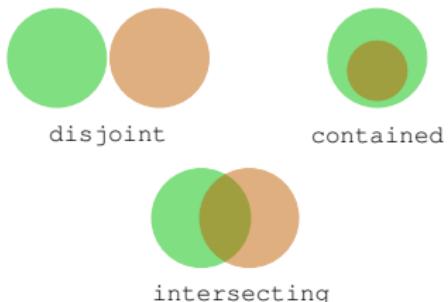


Figure 2: Theoretically possible tier alphabet relations

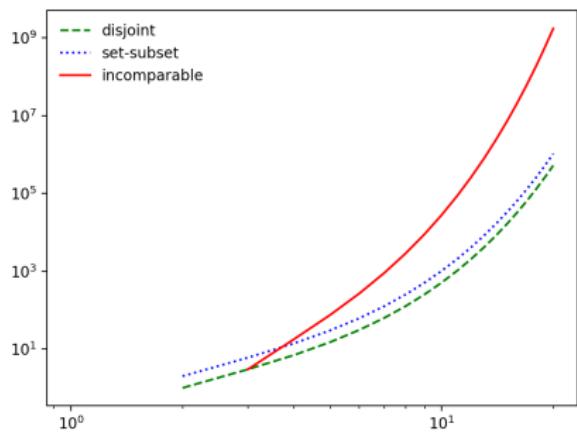
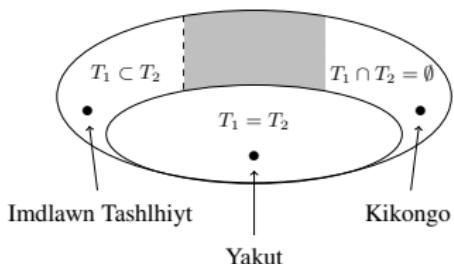


Figure 7: Growth of number of partitions of sets containing up to 20 elements (loglog scale)

Learnability Generalizations

Learning Interactions of Local and Non-Local Phonotactic Constraints from Positive Input

Aniello De Santo

Dept. of Linguistics

University of Utah

aniello.desanto@utah.edu

Alëna Aksënova

Google NYC

alenaks@google.com

- ▶ Efficiently learn MITSL₂ grammars from positive data

Unlearnable Patterns

- ▶ No overlapping tiers with the same ${}^*\rho_1\rho_2$ restriction
e.g. $T_1 = \{a, b, c\}$, $T_2 = \{a, b, d\}$, $G_1 = G_2 = \{{}^*ab\}$
- ▶ This is *predicted* from the structure of the grammar
(see also Lambert et al. 2021)

From Blackbox to Blackbox

Multi-Element Long Distance Dependencies: Using SP k Languages to Explore the Characteristics of Long-Distance Dependencies

Abhijit Mahalunkar

Applied Intelligence Research Center
Technological University Dublin
Dublin, Ireland
abhijit.mahalunkar@mydit.ie

John D. Kelleher

ADAPT Research Center
Technological University Dublin
Dublin, Ireland
john.d.kelleher@dit.ie

- ▶ Strictly-piecewise Languages
 - ▶ Basically: Skip-gram models
 - ▶ Capture long distance dependencies over strings
 - ▶ Modulate parameters of variation:
e.g., length of the dependency, alphabet size, etc.

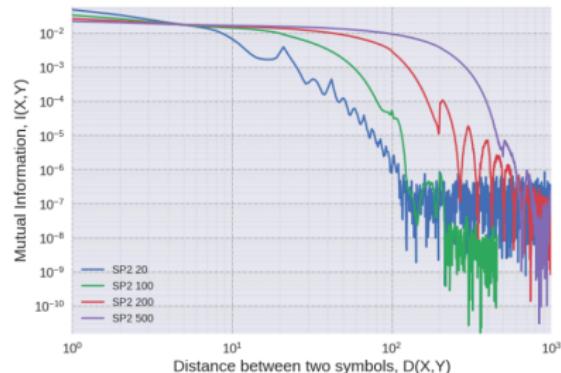


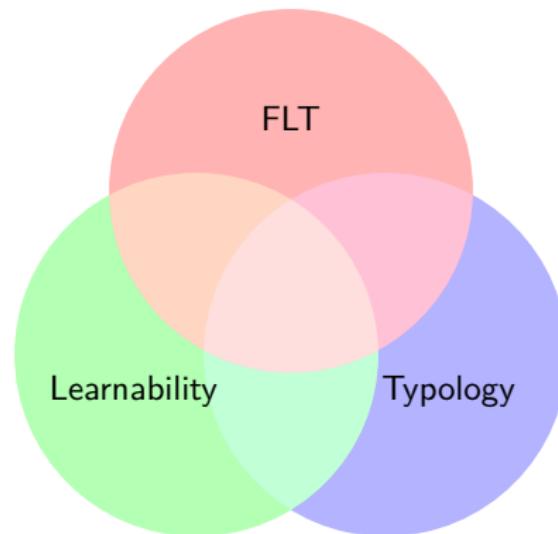
Figure 3: LDD characteristics of datasets of SP2 grammar exhibiting LDDs of length 20, 100, 200 and 500.

Theory Building

*The problem that we cannot deduce [...] theories from data is a limitation, or **perhaps an attribute**, of all empirical science [...] Still, one may abduce hypotheses [...] Abduction is **reasoning from observations** [...] It consists of two steps: generating candidate **hypotheses** (abduction proper), and selecting the “best” explanatory one (inference to the **best explanation**).*

(van Roji & Baggio 2020, pg. 9)

A Collaborative Enterprise!



Thank you!



Mathematical Linguistics and Cognitive Complexity
Aniello De Santo, Jonathan Rawski

Selected References I

- Applegate, R.B. 1972. *Ineseno chumash grammar*. Doctoral Dissertation, University of California, Berkeley.
- Avcu, Enes, and Arild Hestvik. 2020. Unlearnable phonotactics. *Glossa: a journal of general linguistics* 5.
- Chandlee, Jane. 2014. *Strictly local phonological processes*. Doctoral Dissertation, University of Delaware. URL <http://udspace.udel.edu/handle/19716/13374>.
- Graf, Thomas. 2017. The power of locality domains in phonology. *Phonology* In press.
- Graf, Thomas. 2018. Why movement comes for free once you have adjunction. In *Proceedings of CLS 53*, ed. Daniel Edmiston, Marina Ermolaeva, Emre Hakgüder, Jackie Lai, Kathryn Montemurro, Brandon Rhodes, Amara Sankhagowit, and Miachel Tabatowski, 117–136.
- Heinz, Jeffrey. 2011a. Computational phonology — part I: Foundations. *Language and Linguistics Compass* 5:140–152.
- Heinz, Jeffrey. 2011b. Computational phonology — part II: Grammars, learning, and the future. *Language and Linguistics Compass* 5:153–168.
- Heinz, Jeffrey, Chetan Rawal, and Herbert G. Tanner. 2011. Tier-based strictly local constraints in phonology. In *Proceedings of the 49th Annual Meeting of the Association for Computational Linguistics*, 58–64. URL <http://www.aclweb.org/anthology/P11-2011>.

Selected References II

- Kobele, Gregory M., Christian Retoré, and Sylvain Salvati. 2007. An automata-theoretic approach to Minimalism. In *Model Theoretic Syntax at 10*, ed. James Rogers and Stephan Kepser, 71–80.
- Lai, Regine. 2015. Learnable vs. unlearnable harmony patterns. *Linguistic Inquiry* 46:425–451.
- Laszakovits, Sabine. 2018. Case theory in minimalist grammars. In *International Conference on Formal Grammar*, 37–61. Springer.
- Michaelis, Jens. 2004. Observations on strict derivational minimalism. *Electronic Notes in Theoretical Computer Science* 53:192–209.
- Vu, Mai Ha, Nazila Shafiei, and Thomas Graf. 2019. Case assignment in TSL syntax: A case study. In *Proceedings of the Society for Computation in Linguistics (SCiL) 2019*, ed. Gaja Jarosz, Max Nelson, Brendan O'Connor, and Joe Pater, 267–276.

From Blackbox to Blackbox

Multi-Element Long Distance Dependencies: Using SP k Languages to Explore the Characteristics of Long-Distance Dependencies

Abhijit Mahalunkar

Applied Intelligence Research Center
Technological University Dublin
Dublin, Ireland
abhijit.mahalunkar@mydit.ie

John D. Kelleher

ADAPT Research Center
Technological University Dublin
Dublin, Ireland
john.d.kelleher@dit.ie

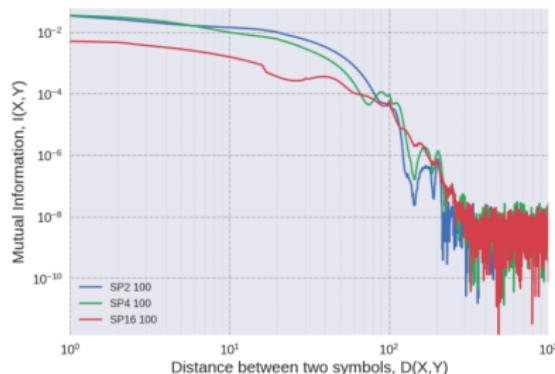


Figure 2: LDD characteristics of datasets of SP2, SP4 and SP16 grammar exhibiting LDD of length 100.

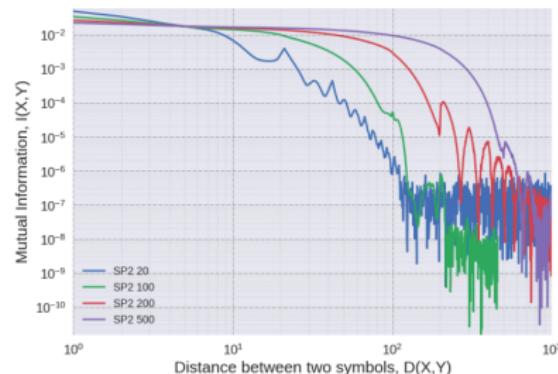


Figure 3: LDD characteristics of datasets of SP2 grammar exhibiting LDDs of length 20, 100, 200 and 500.

Example: Circumfixation in Indonesian

- ▶ Indonesian has circumfixation with no upper bound on the distance between the two parts of the circumfix.

(7) maha siswa
big pupil
'student'

(8) *(ke-) maha siswa *(-an)
NMN- big pupil -NMN
'student affairs'

- ▶ Requirements: exactly one *ke-* and exactly one *-an*

Tier ₁	contains all NMN affixes	\$	an	ke	ke	\$		
Tier ₀	contains all morphemes							
<i>n</i> -grams	\$an, ke\$, keke, anan	\$	an	m	s	ke	ke	\$

Example: Circumfixation in Indonesian

- ▶ Indonesian has circumfixation with no upper bound on the distance between the two parts of the circumfix.

(7) maha siswa
big pupil
'student'

(8) *(ke-) maha siswa *(-an)
NMN- big pupil -NMN
'student affairs'

- ▶ Requirements: exactly one *ke-* and exactly one *-an*

Tier₁	contains all NMN affixes	\$	an	ke	ke	\$		
Tier₀	contains all morphemes							
n-grams	\$an, ke\$, keke, anan	\$	an	m	s	ke	ke	\$

Example: Swahili *vyo*

Swahili *vyo* is either a prefix or a suffix,
depending on presence of negation. (?)

- (9) a. a- vi- **soma -vyo**
SBJ:CL.1- OBJ:CL.8- read -REL:CL.8
'reads'
- b. a- si- **vyo-** vi- **soma**
SBJ:CL.1- NEG- REL:CL.8- read -OBJ:CL.8
'doesn't read'

Example: Swahili *vyo* [cont.]

- (10) a. * a- **vyo-** vi- **soma**
 SBJ:CL.1- REL:CL.8- OBJ:CL.8- read
- b. * a- **vyo-** vi- **soma -vyo**
 SBJ:CL.1- REL:CL.8- OBJ:CL.8- read -REL:CL.8
- c. * a- si- **vyo-** vi- **soma**
 SBJ:CL.1- NEG- REL:CL.8- OBJ:CL.8- read
 -vyo
 REL:CL.8-
- d. * a- si- vi- **soma -vyo**
 SBJ:CL.1- NEG- OBJ:CL.8- read REL:CL.8-

Example: Swahili *vyo* [cont.]

Generalizations About *vyo*

- ▶ may occur at most once
- ▶ must follow negation prefix *si-* if present
- ▶ is a prefix iff *si-* is present

Tier₁ contains *vyo*, *si*, and stem edges #

Tier₀ contains all morphemes

n*-grams** ***vyo**vyo, ***vyo*##*vyo*** “at most one *vyo*”

vyo**si, ***vyo*##*si*** “*vyo* follows *si*”

si*##*vyo, **\$*vyo*##** “*vyo* is prefix iff *si* present”

TSL Phonology: Accounting for Context

► Unbounded Tone Plateauing in Luganda (UTP)

No L may occur within an interval spanned by H.
(Hyman 2011)

- (11) a. **LHLLLL**
b. **LLLLHL**
c. * **LHLLHL**
d. **LHHHHHL**

Example

TSL Phonology: Accounting for Context

► Unbounded Tone Plateauing in Luganda (UTP)

No L may occur within an interval spanned by H.
(Hyman 2011)

- (11) a. **LHLLLL**
b. **LLLLHL**
c. * **LHLLHL**
d. **LHHHHHL**

Example

TSL Phonology: Accounting for Context

► Unbounded Tone Plateauing in Luganda (UTP)

No L may occur within an interval spanned by H.
(Hyman 2011)

- (11) a. **LHLLLL**
b. **LLLLHL**
c. * **LHLLHL**
d. **LHHHHHL**

Example

* **LHLLHL**

TSL Phonology: Accounting for Context

► Unbounded Tone Plateauing in Luganda (UTP)

No L may occur within an interval spanned by H.
(Hyman 2011)

- (11) a. **LHLLLL**
b. **LLLLHL**
c. * **LHLLHL**
d. **LHHHHHL**

Example

L H L L H L
.....
***L H L L H L**

TSL Phonology: Accounting for Context

► Unbounded Tone Plateauing in Luganda (UTP)

No L may occur within an interval spanned by H.
(Hyman 2011)

- (11) a. **LHLLLL**
b. **LLLLHL**
c. * **LHLLHL**
d. **LHHHHHL**

Example

The diagram illustrates the constraints of Unbounded Tone Plateauing in Luganda (UTP). It shows two rows of tone sequences. The top row contains the sequence **LHLLHL**, where the second tone (H) spans a long interval, indicated by a dashed red box around the first four tones (L, H, L, L). The bottom row shows the same sequence with a asterisk (*) preceding it, indicating it is grammatically incorrect according to UTP rules.

Input-Sensitive TSL (ITSL) Languages

Defining Tier Projection

Tier projection controlled by:

- | | | |
|---|------------------|-----|
| 1 | label of segment | TSL |
| | | 1 |

TSL languages are characterized by:

- ▶ a 1-local projection function;
- ▶ strictly k -local constraints applied on T .

Input-Sensitive TSL (ITSL) Languages

Defining Tier Projection

Tier projection controlled by:

- | | | |
|----------|------------------|----------|
| 1 | label of segment | TSL |
| | | 1 |

TSL languages are characterized by:

- ▶ a **1**-local projection function;
- ▶ strictly k -local constraints applied on T .

Idea:

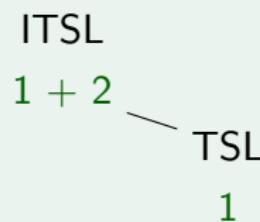
- ▶ Projection is an input-strictly local transduction (Chandee 2015)
- ▶ **What if:** the locality of E_T was **higher than 1**?

Input-Sensitive TSL (ITSL) Languages

Defining Tier Projection

Tier projection controlled by:

- 1 label of segment
- 2 local context



TSL languages are characterized by:

- ▶ a 1-local projection function;
- ▶ strictly k -local constraints applied on T .

Idea:

- ▶ Projection is an input-strictly local transduction (Chandee 2015)
- ▶ **What if:** the locality of E_T was higher than 1?

Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example

ok **L H L L L L**

* **L H L L H L**

Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example

H

.....

ok **L** **H** **L** **L** **L** **L**

* **L** **H** **L** **L** **H** **L**

Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example

H L

.....

ok **L** **H L** **L L L**

* **L H L L H L**

Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example

H L

.....

ok **L H L L L L**

* **L H L L H L**

Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example

H L

.....

ok **L H L L L L**

* **L H L L H L**

Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example

H L

.....

ok **L H L L L L**

* **L H L L H L**

Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example



ok **L H L L L L**

* **L H L L H L**

Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example



Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example



Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example



Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example



Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example



Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example



Accounting for Context [cont.]

A ITSL analysis for UTP (DeSanto & Graf 2019):

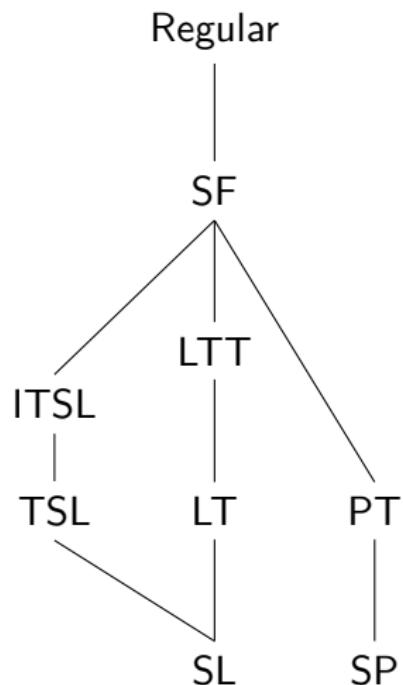
- ▶ Project every **H**; project **L** iff immediately follows **H**
- ▶ Ban: **HLH**

Example

ok  **L H L L L L**

*  **L H L L H L**

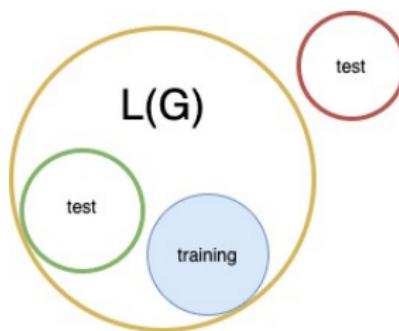
Finer Granularity



Outline

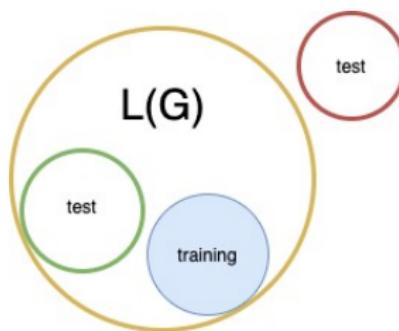
5 Limits of AGL

Testing Predictions with AGL



- ▶ It is a powerful technique
- ▶ We must be careful in drawing inferences from laboratory behavior
- ▶ Importantly: Common fallacies in experimental design

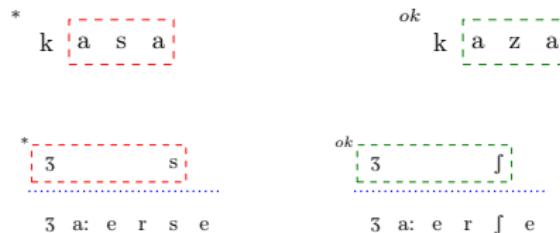
Testing Predictions with AGL



- ▶ It is a powerful technique
- ▶ We must be careful in drawing inferences from laboratory behavior
- ▶ Importantly: Common fallacies in experimental design

The Fallacy of Generalization

- ▶ Imagine we want to test the ability to learn long-distance dependencies:



- ▶ Assuming an alphabet $\Sigma = \{a, b, c, d, e\}$, the training samples could look like the following:

$$L_{loc} = \{abcd, aabcd, baacd, bcaaе, \dots\}$$

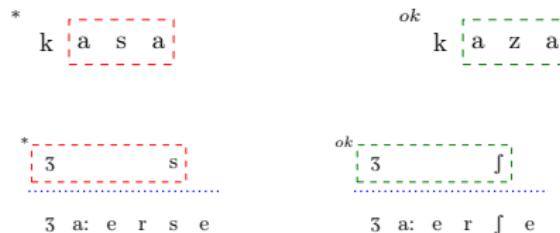
$$L_{dist} = \{abacd, bacad, bcada, bcaea, \dots\}$$

What happens if we test on stimuli with similar distances?

$$L_{test} = \{abcad, abcad, bacda, abcea, \dots\}$$

The Fallacy of Generalization

- ▶ Imagine we want to test the ability to learn long-distance dependencies:



- ▶ Assuming an alphabet $\Sigma = \{a, b, c, d, e\}$, the training samples could look like the following:

$$L_{loc} = \{abcd, aabcd, baacd, bcaaе, \dots\}$$

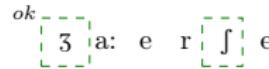
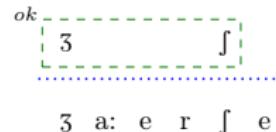
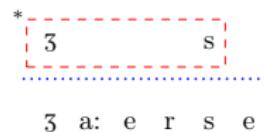
$$L_{dist} = \{abacd, bacad, bcada, bcaea, \dots\}$$

What happens if we test on stimuli with similar distances?

$$L_{test} = \{abcad, abcad, bacda, abcea, \dots\}$$

Picking the Right Primitives

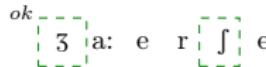
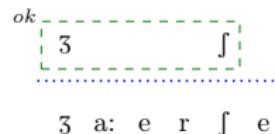
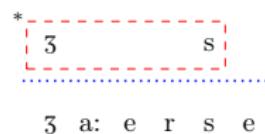
Long-distance relations?



- ▶ Stimuli are often ambiguous between overlapping classes
- ▶ Distinguishing between representation requires care

Picking the Right Primitives

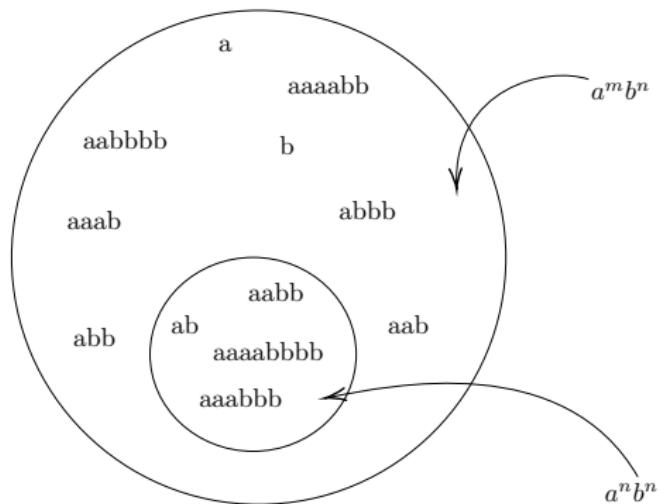
Long-distance relations?



- ▶ Stimuli are often ambiguous between overlapping classes
- ▶ Distinguishing between representation requires care

The Set/Subset Problem: Case 1

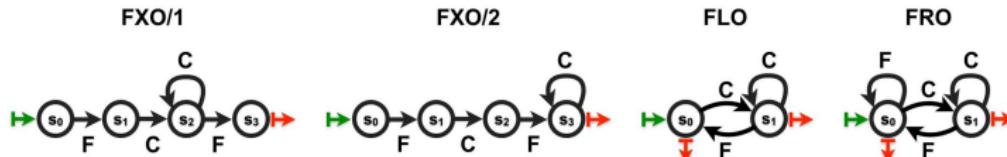
- ▶ Can participants learn $a^n b^n$?
- ▶ We must beware of $a^m b^n$



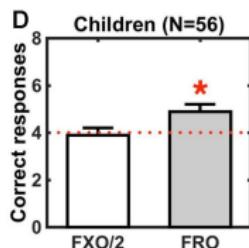
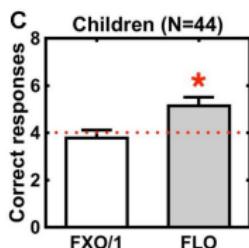
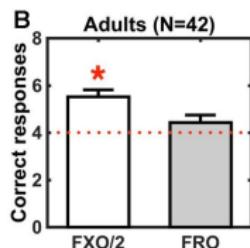
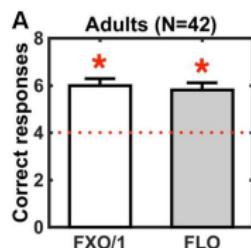
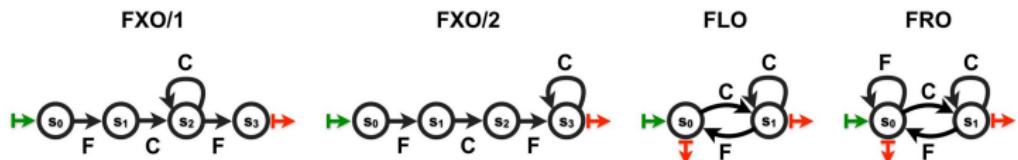
Evaluating Contrasts

Developmental Constraints on Learning Artificial Grammars with Fixed, Flexible and Free Word Order

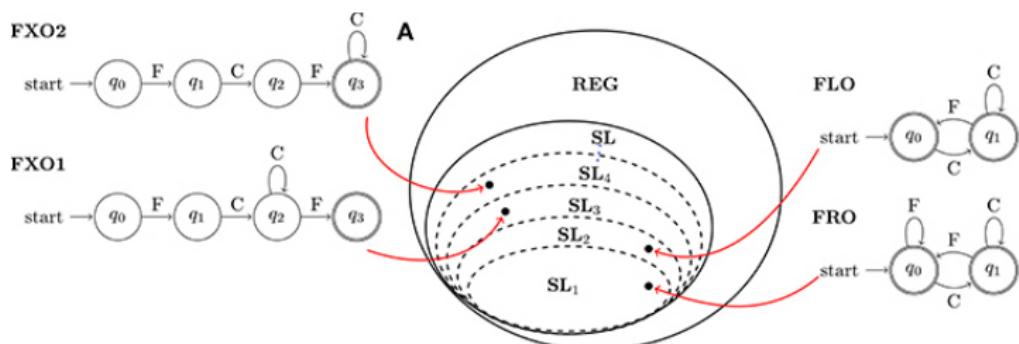
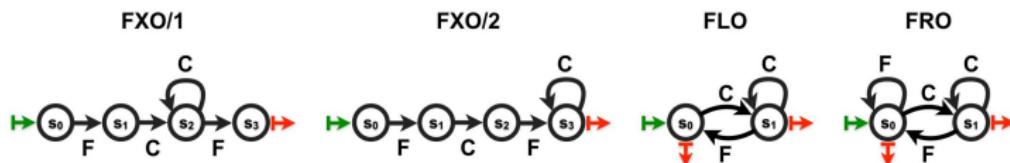
 Iga Nowak^{1,2} and  Giosuè Baggio^{2,3*}



Nowak and Baggio (2017): Results



Complexity Measures and Other Issues (De Santo, 2017)



The Set/Subset Problem: Case 2

- ▶ Can participants learn a truly free-word order language?

