A (Very Very) Brief Introduction to Language Models

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A Definition

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\begin{aligned}
P(X_1 ... X_n) &= P(X_1)P(X_2|X_1)P(X_3|X_{1:2}) \ldots P(X_n|X_{1:n-1}) \\
&= \prod_{k=1}^{n} P(X_k|X_{1:k-1})
\end{aligned}
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$$P(w_n|w_{n-N+1:n-1}) = \frac{C(w_{n-N+1:n-1}\, w_n)}{C(w_{n-N+1:n-1})}$$

$$P(w_{1:n}) \approx \prod_{k=1}^{n} P(w_k|w_{k-1})$$

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\begin{aligned}
& P(\texttt{<s> i want english food </s>}) \\
&\quad = P(\texttt{i}|\texttt{<s>})P(\texttt{want}|\texttt{i})P(\texttt{english}|\texttt{want}) \\
&\qquad\quad P(\texttt{food}|\texttt{english})P(\texttt{</s>}|\texttt{food}) \\
&\quad = .25 \times .33 \times .0011 \times 0.5 \times 0.68 \\
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Word Prediction Everywhere

An experiment:

- Open any chat/messaging app you use frequently
- Start typing

  I wish this lecture was ____

- What do you get after **was**?
- The same idea also applies also to full sentences!

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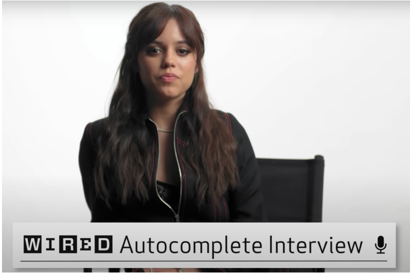

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- Humans do it too!

Please turn your homework ____

Why is automatizing this useful?

- speech recognition
- spell-checking/grammatical error correction
- machine translation
- maybe even more direct linguistics research ...

This is where Language Models (LMs) enter the picture!

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Tackling Next Word Prediction

A naive solution to next word prediction:

- We want the **most likely completion(s)**...

  Uhm, how do we figure out what is most likely?

- Idea! Most likely = Most frequent word
- **Approach:**
  1. Collect sufficiently large sample of texts (**corpus**)
  2. For each word (**type**), count how often it occurs in the entire sample (= its number of **tokens**).
  3. Calculate the **frequency** of the word in the sample:

$$\text{freq}(\textit{word}, \textit{sample}) = \frac{\text{number of tokens of } \textit{word}}{\text{word length of whole } \textit{sample}}$$

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Example calculation

Sample: 1000 words long

Words: be, bed, bee, bell

Type	be	bed	bee	bell
Tokens	**13**	**2**	**0**	**3**

$$\text{freq(be)} = \frac{\mathbf{13}}{1000} = 1.3\% \qquad \text{freq(bee)} = \frac{\mathbf{0}}{1000} = 0.0\%$$

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- BUT! Word usage varies by **context**!

Example

	tested	testing	testimony
I have			
I have been			
I have the			

- The frequency of words is not enough,
  we need frequencies of sequences of words ⇒ **n-grams**

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Defining n-Grams

n-gram a contiguous sequence of n words

n	Name	Example
1	unigram	John
2	bigram	John to
3	trigram	John to be
4	4-gram	John to be in
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String

John and Marie are not Bill and Sue

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Frequencies can be computed and used for n-grams, too.
→ we still need a representative corpus...

Example

- **Trigram frequencies**

bus is late	30%	train is late	15%
bus is lovely	25%	train is lovely	8%
bus is lazy	10%	train is lazy	2%

- **Input**
  I will text you if the train is __
- **Sorted completions**

- To predict a word w:
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The **preceding $n-1$ words** reliably predict the next word.

$$P(w_n|w_1 w_2 \dots w_{n-2} w_{n-1}) \approx P(w_n|w_{n-2} w_{n-1})$$

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This is where Neural Networks LMs come in handy...

Ok but, who cares?

LMs assign probabilities to sequences of words.

- **n-Gram LMs**: use local contexts for sequence prediction
- Spoilers: **Neural LMs**...

And?

- speech recognition
- spell-checking/grammatical error correction
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LMs as Tools for Psycholinguistics

Jacobs, De Santo, and Grobol (2023)

Zeugma	The architect bit the lime and **the dust**
Literal	The architect bit the lime and the apple

- We can use LMs to generate literal continuations

  The architect bit the ____
- Maze Task

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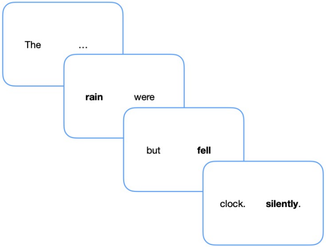
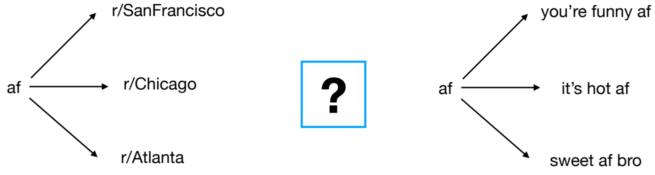
  The architect bit the ____
- Maze Task (Boyce & Levy, 2021):
  Use LMs to generate low probability foils



LMs as Tools for Sociolinguistics

Making "fetch" happen: The influence of social and linguistic context on nonstandard word growth and decline

Ian Stewart and Jacob Eisenstein
School of Interactive Computing
Georgia Institute of Technology



- Does the social context of a word influence its adoption more than its linguistic context?
- Use *unique* n-gram counts to measure *dissemination*: the diversity of linguistic contexts in which a word appears
- How do communities (e.g. *r/x,y,z*) predict word usage? (Lucy & Bamman, 2021)

LMs as Psycholinguistic Subjects

"Wait...Maybe I find **the models** interesting?"

- Can we use linguistic tests to understand them better?

LMs as Psycholinguistic Subjects

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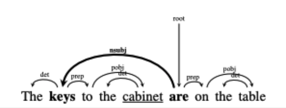
Agreement attraction errors

(1)
a. The **key is** on the table.
b. *The **key are** on the table.
c. *The **keys is** on the table.
d. The **keys are** on the table.



Assessing the Ability of LSTMs to Learn Syntax-Sensitive Dependencies

Tal Linzen[1,2] **Emmanuel Dupoux**[1]
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emmanuel.dupoux}@ens.fr

Yoav Goldberg
Computer Science Department
Bar Ilan University
yoav.goldberg@gmail.com

A Final Note: A Word of Caution

- LMs are sensitive to statistical regularities in language data...
  - Bias: treating language behavior as ground truth (Bolukbasi et al. 2016)
  - Exclusion/discrimination: what kind of data is included? (Bender et al. 2019)
  - Privacy: whose data and how do we get it? (Huang & Paul 2019)
  - Environmental and financial cost (Strubell et al. 2019)
  - And more!
- Reflect on **social impact** while conducting research!

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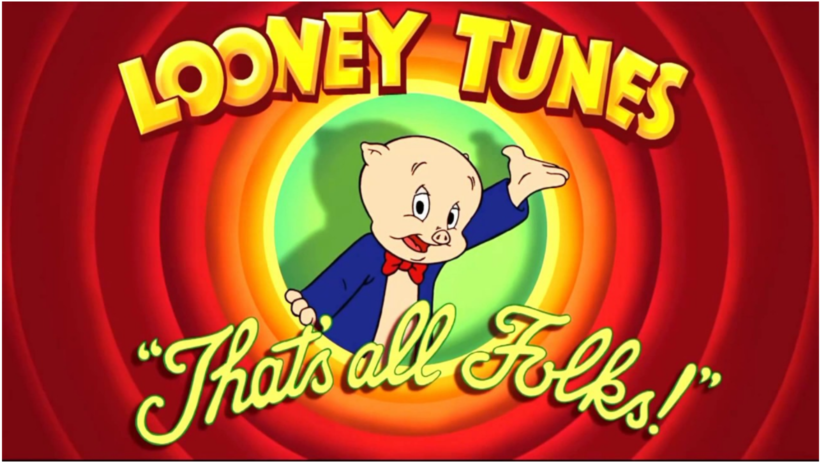
The End (?)



Appendix

N-Grams Limits

LMs assign probabilities to sequences of words.

- **n-Gram LMs**: use local contexts for sequence prediction.
  - Struggle to generalize to novel contexts
  - Struggle with long distance relations (Markov assumption)
- Spoilers: **Neural LMs**...
  - ...might help with these issues
    - Incorporate word similarity based on distributional information
    - More complex approximation of sentential dependencies

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Generalizing to Novel Contexts

Imagine our model has seen sequences like:

I have to make sure that the cat gets fed.
Pearl's parrot gets fed every day.

Then we want to complete the following:

I forgot to make sure that the dog gets ____

- It would be great if the model could take advantage of the similarity between *dog*,*cat*,*parrot* to predict *fed*!

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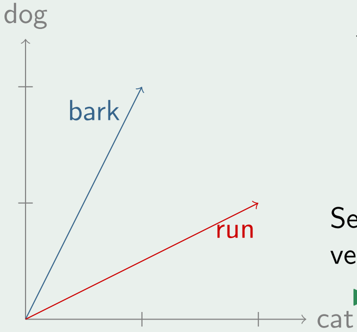
From Counts to Vector Spaces

The dog barked at the cat. The cat ran away. The dog ran after the cat. The dog kept barking. He also kept running.

2-Dimensional Vector Space with *dog* and *cat*



	dog	cat	bark	run
dog	-	2	2	1
cat	2	-	1	2
bark	$\boxed{2}$	$\boxed{1}$	-	0
run	$\boxed{1}$	$\boxed{2}$	0	-

Semantic similarity as angle between vectors:

- *bark* more closely related to *dog*
- *run* more closely related to *cat*

Long-distance Dependencies in Language

- Word choice can be influenced by words that are very far away.

Subject-verb agreement

- The key to the cabinet **is** on the table.
- The keys to the cabinets **are** on the table.
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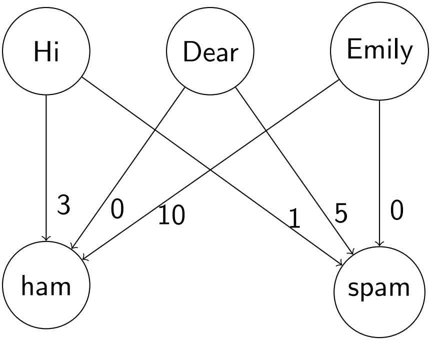
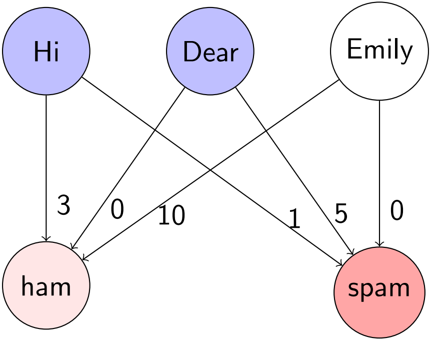
A Quick Excursus: The Perceptron

The Perceptron: A Mini-Version of a Neural Network

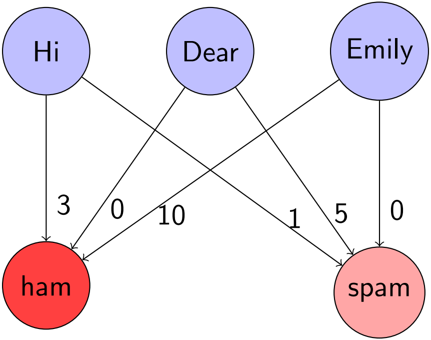
- **input layer:** neurons that are sensitive to input
- **output layer:** neurons that represent output values
- **connections:** weighted links between input and output layer
- most activated output neuron represents decision



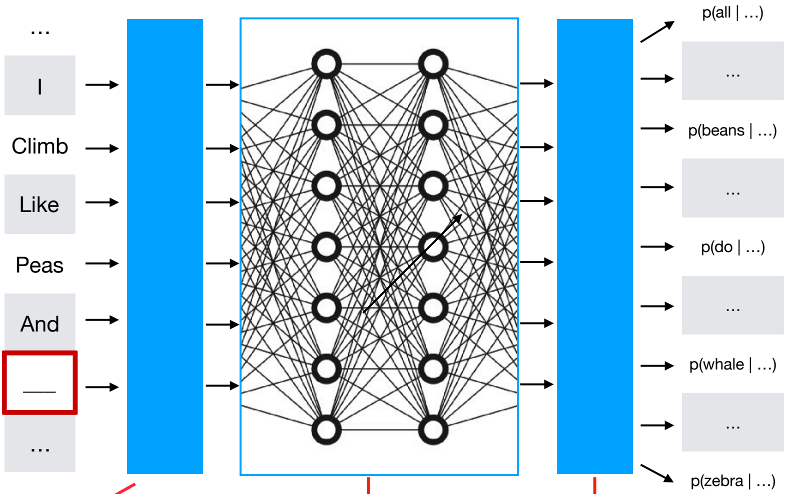
Perceptron Activation for *Hi Dear*



Perceptron Activation for *Hi Dear Emily*



Putting Things Together: Neural LMs



Embeddings

The input is now not words but their distributional representations

Hidden Layer

Derives approximations from more complex "contexts"

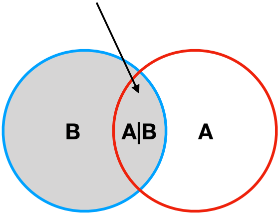
Output Layer

This makes probability predictions about upcoming word(s)

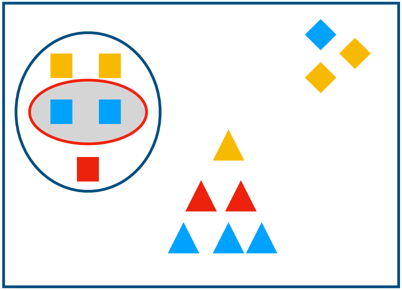
A Bit More on Conditional Probability

- We said we are interested in $P(\textbf{late}|\text{is})$

$$P(A|B) = \frac{P(A,B)}{P(B)}$$



- E.g. $P(\textbf{blue}|\blacksquare) = 2/5$



Estimating Bigram Probabilities: MLE

Ok but where do we get probabilities from?

- One possibility: Counts (Maximum Likelihood Estimate)!
  - For a unigram:

$$P(w_n) = \frac{count(w_n)}{\sum_{w \in V} count(w)}$$

- MLE of conditional probability for bigrams:

$$P(w_n|w_{n-1}) = \frac{count(w_n, w_{n-1})}{count(w_{n-1})}$$

- Note that the normalization factor is different than what we did for pure bigram frequency counts (which gave us an estimate of joint probability for each bigram)!

Frequencies for n-grams

Frequencies can be computed for n-grams, too.

Example: Calculating Bigram Frequencies

- **String**

  when buffalo buffalo buffalo buffalo buffalo buffalo

- **Bigram token list**

- **Bigram counts and frequencies**

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- **Bigram counts and frequencies**
  1. when buffalo: $1 \Rightarrow \frac{1}{6} = 16.7\%$
  2. buffalo buffalo: $5 \Rightarrow \frac{5}{6} = 83.3\%$

Ok, NOW Some Formulas

$$P(w_n|w_1 w_2 \cdots w_{n-1})$$

$P(\textbf{late}|\textit{I will text you if the train is})$ $P(\textbf{lazy}|\textit{I will text you if the train is})$

- Lots of possible sentences!
- Simplifying assumption:

$$P(w_n|w_1 w_2 \ldots w_{n-2} w_{n-1}) \approx P(w_n|w_{n-2} w_{n-1})$$

$$P(\textbf{late}|\textit{I will text you if the train is}) \approx P(\textbf{late}|\textit{train is})$$

The n-Gram Hypothesis (aka Markov Assumption)

One can reliably predict the next word based on the **preceding** $n-1$ **words**.

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An Observation on Frequencies: Zipf's Law

- Word models care about word frequency.
- But there is a problem...

Zipf's Law

The frequency of a type is inversely proportional to its rank.

In Plain English

The most frequent word is

- 2 times as common as the 2nd most frequent word,
- 3 times as common as the 3rd most frequent word,
- and so on.

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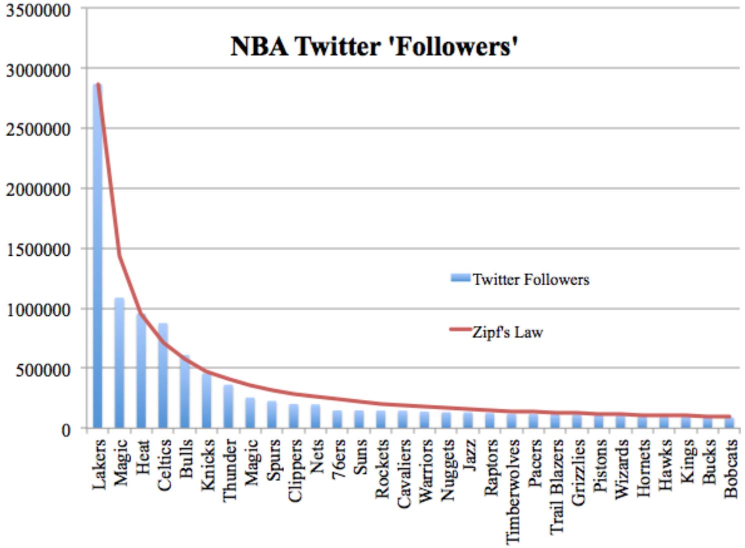
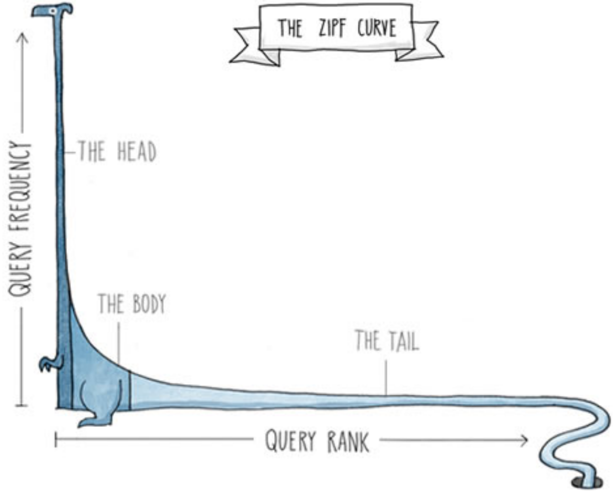
An Example from. . . the NBA?



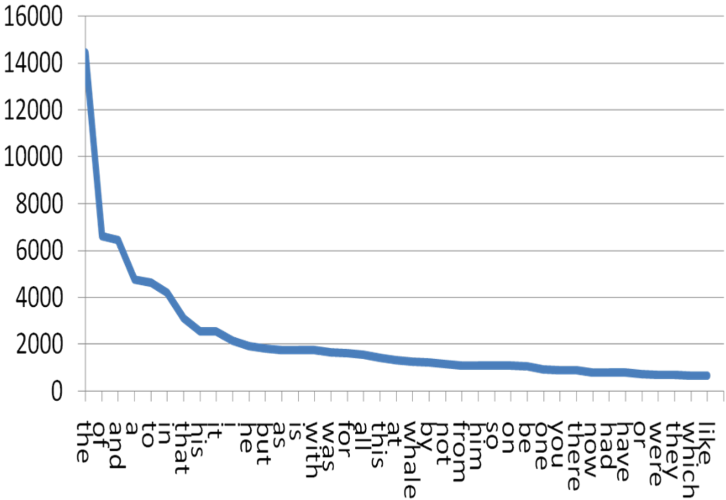
Visualizing Zipf Distributions



Zipf's Law is Everywhere...

- A distribution is probably Zipfian if
  - there is a **long neck**:
    a few types make up the majority of tokens,
  - there is a **long tail**:
    most types only have 1 token (**hapax legomenon**)
- Surprisingly, Zipf's Law shows up in tons of places:
  - size of large cities in a country
  - citations for academic papers
  - frequencies of last names
  - frequencies of weekdays in text

. . . Even in Language!



An Important Consequence of Zipf's Law

- Texts mostly consist of stop words.
- Hence it can be difficult to get representative counts for non-stop words.

Sparse Data Problem

- Most of the data is not informative.
- You need tons of data to have enough useful data.

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Example

- Most models require corpora with at least a few million sentences.
- Really good models (e.g. Google translate) use billions of data points.

Defining Larger n-Grams

n-gram a contiguous sequence of n words

n	Name	Example
1	unigram	John
2	bigram	John to
3	trigram	John to be
4	4-gram	John to be in
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Example

String

John and Marie are not Bill and Sue

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How long can n-grams be?

- It is tempting to move to longer and longer n-grams in order to handle long-distance dependencies.
- But this has **two problems**:

  data sparsity longer n-grams require too much data
  storage needs longer n-grams require lots of storage
- Data sparsity is much more severe than storage needs.

Sparse data: A simple calculation

Words	**bigrams**	**trigrams**	**5-grams**	**6-grams**
10	100	1000	10,000	100,000
100	10,000	1,000,000	10,000,000,000	**1,000,000,000,000**
10,000	10^8	$\mathbf{10^{12}}$	10^{20}	10^{24}
25,000	6.3×10^8	1.6×10^{13}	9.7×10^{21}	2.4×10^{26}

Some comparison values

4.3×10^{17}	number of seconds since the Big Bang
5×10^{22}	number of stars in observable universe
10^{24}	milliliters of water in the Earth's oceans
8.8×10^{26}	diameter of observable universe, in meters
10^{80}	number of atoms in observable universe

- **Conclusion:** with large n, most n-grams are **never encountered** in a corpus $\Rightarrow$ frequency 0

Things get worse: A more realistic estimate

- The Linux dictionary american-english-insane has 650,000 entries.
- This makes the numbers much worse.
  Can you guess how many 5-grams there are then?

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$$116 \text{ octillion} \approx \mathbf{10^{29}}$$

10^{29} is larger than the number of shotglasses it takes to drain the Earth's oceans over 2000 times.

Evaluating Language Models: Perplexity

The **perplexity** of a language model is defined as the inverse of the probability of the test set, normalized by the number of tokens (N) in the test set.

$$PP(w_1 ... w_N) = \sqrt[N]{\frac{1}{P(w_1 ... w_N)}}$$

A LM with lower perplexity is better because it assigns a higher probability to the unseen test corpus. But note that two LMs can be compared wrt to perplexity iff they use the same vocabulary!

- Trigram models have lower perplexity than bigram models, etc.

Intrinsic vs. Extrinsic Evaluation

Perplexity tells us which LM assigns a higher probability to unseen text.
This doesn't necessarily tell us which LM is better for a specific task.
Task-based evaluation:

- Train model A, plug it into your system for performing task T
- Evaluate performance of system A on task T
- Train model B, plug it in, evaluate system B on same task T
- Compare scores of system A and system B on task T.

Extrinsic Evaluation: Word Error rate [1]

Originally developed for speech recognition.

How much does the *predicted* sequence of words differ from the *actual* sequence of words in the correct transcript?

$$\mathrm{WER} = \frac{\mathrm{Insertions} + \mathrm{Deletions} + \mathrm{Substitutions}}{\mathrm{Actual\ words\ in\ transcript}}$$

Insertions: "eat lunch" → "eat **a** lunch"
Deletions: "see **a** movie" → "see movie"
Substitutions: "drink **ice** tea" → "drink **nice** tea"

[1] slide adapted from J. Hockenmaier