



Structure Sensitive Tier Projection: Applications and Formal Properties

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The Paper in a Nutshell

The research program

- ▶ a tight upper bound to the complexity of natural language dependencies?

Subregular Hypothesis for Phonology

- ▶ Tier-based Strictly Local seems to be the right fit;
- ▶ But ... several outliers have been reported!

In This Talk

We explore (minimal) extensions to TSL

Outline

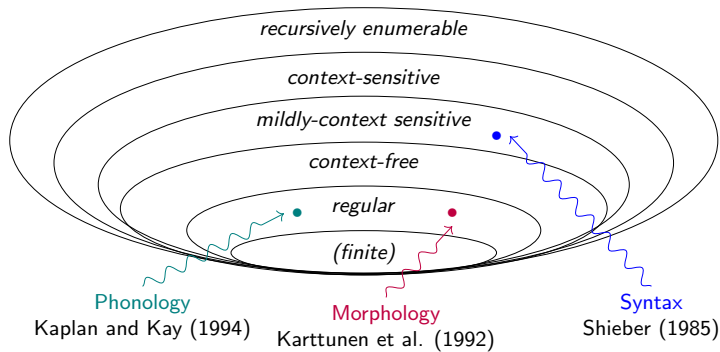
1 Preliminaries

2 Non-local Dependencies

3 ITSL

4 Conclusions

Computational Theories of Language



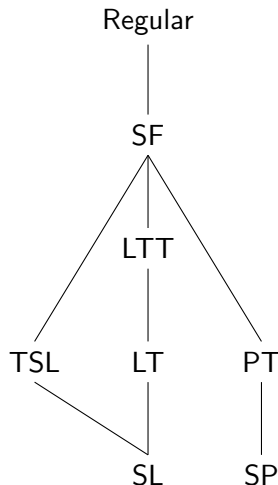
Precise predictions for:

- ▶ typology → e.g. no center embedding in phonology
- ▶ learnability → e.g. no Gold learning for regular languages
- ▶ cognition → e.g. finitely bounded working memory

Phonology as a Subregular System

Often forgotten: hierarchy of **subregular languages**

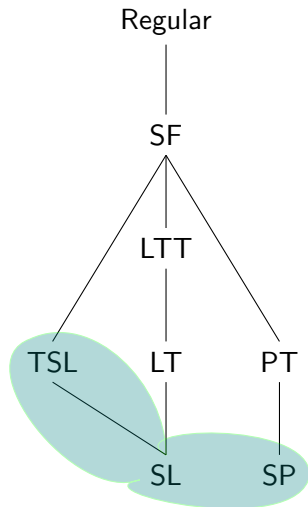
(McNaughton and Papert 1971; Rogers et al. 2010; Heinz et al. 2011; Graf 2017)



Phonology as a Subregular System

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Local Dependencies in Phonology are SL

1 Word-final devoicing

Forbid voiced segments at the end of a word

- (1) a. *rad
b. rat

Example: Word-final devoicing

- ▶ Forbid word final voiced segments: $*[+voice]\$$
- ▶ **German:** ***z**€, ***v**€, ***d**€ (€ = word edge).

\$ r a **d** \$ \$ r a t \$

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- ▶ Forbid word final voiced segments: $*[+voice]\$$
- ▶ **German:** ***z**€, ***v**€, ***d**€ (€ = word edge).

* € r a **d** € *ok* € r a t €

Unbounded Dependencies Are Not SL

► **Samala Sibilant Harmony**

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (2) a. * ha **s**xintilawa **ʃ**
b. * ha **ʃ**xintilawa **s**
c. ha **ʃ**xintilawa **ʃ**

Example: Samala

*\$ ha **s**xintilawa **ʃ**\$

\$ ha **ʃ**xintilawa **ʃ**\$

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 \$ ha^ʃxintilawa^ʃ\$

► **But:** Sibilants can be arbitrarily far away from each other!

*\$^stajanowonwa^ʃ\$

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► **But:** Sibilants can be arbitrarily far away from each other!

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Locality Over Tiers

* \$ **s** t a j a n o w o n w a **j** \$

- ▶ Sibilants can be arbitrarily far away from each other!
- ▶ **Problem:** SL limited to locality domains of size n ;
- ▶ **Solution:** locality over **tiers**. (Goldsmith 1976)

Tier-based Strictly Local (TSL) Grammars (Heinz et al. 2011)

- ▶ E_T : Projection of selected segments on a tier T ;
- ▶ Strictly local constraints over T determine wellformedness;

Locality Over Tiers

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- ▶ E_T : Projection of selected segments on a tier T ;
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Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. * ha^sxintilawa^ʃ
 b. * ha^ʃxintilawa^s
 c. ha^ʃxintilawa^ʃ

- ▶ What do we need to project? {^s, ^z, ^ʃ, ³}
- ▶ What do we need to ban?

*^{sʃ}, *^{s3}, *^{zʃ}, *^{z3}, *^{ʃs}, *^{3s}, *^{ʃz}, *^{3z}

Example: TSL Samala

* \$ha^sxintilawa^ʃ\$

ok \$ha^ʃxintilawa^ʃ\$

Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3)
- a. * ha **s**xintilawa **ʃ**
 - b. * ha **ʃ**xintilawa **s**
 - c. ha **ʃ**xintilawa **ʃ**

- ▶ What do we need to project? {**s**, **z**, **ʃ**, **ʒ**}
- ▶ What do we need to ban?

* **sʃ**, * **sʒ**, * **zʃ**, * **zʒ**, * **ʃs**, * **ʒs**, * **ʃz**, * **ʒz**

Example: TSL Samala

.....

* **ʃ**ha **s**xintilawa **ʃ**\$

ok \$ha **ʃ**xintilawa **ʃ**\$

Unbounded Dependencies are TSL

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*^sʃ, *^sʒ, *^ʃʃ, *^zʃ, *^ʃs, *^ʒs, *^ʃz, *^ʒz

Example: TSL Samala

* \$^ha^sxintilawa^ʃ\$

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Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3)
- a. * ha~~s~~xintilawa~~j~~
 - b. * ha~~j~~xintilawa~~s~~
 - c. ha~~j~~xintilawa~~j~~

- ▶ What do we need to project? {~~s~~, ~~z~~, ~~j~~, ~~3~~}
- ▶ What do we need to ban?

*~~s~~~~j~~, *~~s~~~~3~~, *~~j~~~~j~~, *~~z~~~~3~~, *~~j~~~~s~~, *~~3~~~~s~~, *~~j~~~~z~~, *~~3~~~~z~~

Example: TSL Samala

.....

* \$ha~~a~~s x i n t i l a w ~~j~~ \$

ok \$ha~~j~~ x i n t i l a w ~~j~~ \$

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Example: TSL Samala

^s

* \$ha^sxintilawa^ʃ\$

ok \$ha^ʃxintilawa^ʃ\$

Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. * ha^sxintilawa_f
- b. * ha_fxintilawa^s
- c. ha_fxintilawa_f

- ▶ What do we need to project? {^s, ^z, _f, ₃}
- ▶ What do we need to ban?

*^s_f, *^s₃, *_f^f, *^z₃, *_f^s, *₃^s, *_f^z, *₃^z

Example: TSL Samala

^s

* \$ha^s_fxintilawa_f\$

ok \$ha_fxintilawa_f\$

Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. * ha^sxintilawa^j
 b. * ha^jxintilawa^s
 c. ha^jxintilawa^j

- ▶ What do we need to project? {^s, ^z, ^j, ³}
- ▶ What do we need to ban?

*^s^j, *^s³, *^j^j, *^z³, *^j^s, *³^s, *^j^z, *³^z

Example: TSL Samala

^s

* \$ha^sxⁱntilawa^j\$

^{ok} \$ha^jxintilawa^j\$

Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

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 c. ha^ʃxintilawa^ʃ

- ▶ What do we need to project? {^s, ^z, ^ʃ, ^ʒ}
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*^sʃ, *^sʒ, *^ʃʃ, *^zʒ, *^ʃs, *^ʒs, *^ʃz, *^ʒz

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^s

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*^{sʃ}, *^{sʒ}, *^{ʃʃ}, *^{ʒʒ}, *^{ʃs}, *^{ʒs}, *^{ʃz}, *^{ʒz}

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*^sʃ, *^{s3}, *^zʃ, *^{z3}, *^ʃs, *³s, *^ʃz, *³z

Example: TSL Samala

^s

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ok \$ha^ʃxintilaw^ʃ\$

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Example: TSL Samala

^s _f

 * \$ha^sxintilaw_f_{\$}

ok \$ha_fxintilaw_f\$

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- ▶ What do we need to project? {**s**, **z**, **ʃ**, **ʒ**}
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* **sʃ**, * **sʒ**, * **zʃ**, * **zʒ**, * **ʃs**, * **ʒs**, * **ʃz**, * **ʒz**

Example: TSL Samala

s **ʃ**

* \$ha **s**xintilawa **ʃ**\$

ok \$ha **ʃ**xintilawa **ʃ**\$

Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

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Example: TSL Samala

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- c. ha **ʃ** x i n t i l a w a **ʃ**

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Example: TSL Samala

* \$ ha **s** x i n t i l a w **ʃ** \$



ok \$ ha **ʃ** x i n t i l a w **ʃ** \$



TSL: Why Do We Care?

- ▶ Linguistically natural (Goldsmith 1976)
- ▶ Captures wide range of phonotactic dependencies (McMullin 2016)
- ▶ Efficiently learnable from positive data (Jardine and Heinz 2016)
- ▶ Low resource demands
- ▶ Rules out unattested patterns (cf. Aksënova et al. 2016)

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- ▶ Low resource demands
- ▶ Rules out unattested patterns (cf. Aksënova et al. 2016)
- ▶ **But** not every long-distance pattern is TSL!
(McMullin 2016; Mayer and Major 2018; Graf and Mayer 2018)

Full Samala: A TSL Outlier

Sibilant Harmony in SAMALA (McMullin 2016)

1) Unbounded sibilant harmony

- | | | |
|---|-----------------------------------|-------------------|
| a. /k- s u- ʃ ojin/ | k ʃ u ʃ ojin | “I darken it” |
| b. /k- s u-k’ili-mekeken- ʃ / | k ʃ uk’ilimekeket ʃ | “I straighten up” |

2) /s/ → [ʃ] when preceding (adjacent) [t, n, l]

- | | | |
|------------------------|-----------------|--------------|
| a. / s -lok’in/ | ʃ lok’in | “he cuts it” |
| b. / s -tepu?/ | ʃ tepu? | “he gambles” |

3) Long-distance agreement overrides local disagreement

- | | | |
|---|---------------|---------------------|
| a. / s -i ʃ t-i ʃ ti-jep-us/ | sististijepus | “they show him” |
| b. / s -net-us/ | snetus | “he does it to him” |

Is there a TSL grammar for the complete pattern?

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Is there a TSL grammar for the complete pattern?

Sibilant Harmony in SAMALA (1/3)

Generalization

- ▶ Anticipatory Sibilant harmony

Grammar

$T = \{s, \int, z, ʒ\}$ $S = \{*s\int, *\int s, \dots\}$

ok k ∫ u ∫ o j i n * k s u ∫ o j i n

Sibilant Harmony in SAMALA (1/3)

Generalization

- ▶ Anticipatory Sibilant harmony

Grammar

$T = \{s, \int, z, ʒ\}$ $S = \{*s\int, *\int s, \dots\}$

.....
^{ok} k ∫ u ∫ o j i n * k s u ∫ o j i n

Sibilant Harmony in SAMALA (1/3)

Generalization

- ▶ Anticipatory Sibilant harmony

Grammar

$T = \{s, \int, z, ʒ\}$ $S = \{*\int s, *\int s, \dots\}$

$\int \quad \int$

ok k ∫ u ∫ o j i n * k s u ∫ o j i n

Sibilant Harmony in SAMALA (1/3)

Generalization

- ▶ Anticipatory Sibilant harmony

Grammar

$T = \{s, \int, z, 3\}$ $S = \{^*s\int, ^*\int s, \dots\}$

ok \int \int

 ok k \int u \int o j i n * k s u \int o j i n

Sibilant Harmony in SAMALA (1/3)

Generalization

- ▶ Anticipatory Sibilant harmony

Grammar

$T = \{s, \int, z, ʒ\}$ $S = \{*s\int, *\int s, \dots\}$

ok
 \int \int

 ok k
 \int
 u
 \int
 o j i n $*$ k s u \int o j i n

Sibilant Harmony in SAMALA (1/3)

Generalization

- ▶ Anticipatory Sibilant harmony

Grammar

$$T = \{s, \int, z, 3\} \quad S = \{^*s\int, ^*\int s, \dots\}$$

ok

.....

ok k [u] u [o] o j i n

s ∫

.....

* k [s] u [∫] o j i n

Sibilant Harmony in SAMALA (1/3)

Generalization

- ▶ Anticipatory Sibilant harmony

Grammar

$$T = \{s, \int, z, 3\} \quad S = \{^*s\int, ^*\int s, \dots\}$$

ok

ok k ∫ u ∫ o j i n

*** k s u ∫ o j i n

Sibilant Harmony in SAMALA (2/3)

Generalization

- ▶ Anticipatory Sibilant harmony
- ▶ Local restriction against [*sn, *st, *sl]

Grammar

$T = \{s, \int, z, ʒ, \textcolor{red}{n}, \textcolor{red}{t}, \textcolor{red}{l}\}$ $S = \{*\textcolor{red}{s}\int, *\textcolor{red}{j}s, *\textcolor{red}{s}n, *\textcolor{red}{s}t, *\textcolor{red}{s}l, \dots\}$

ok \int t e p u ?

* s t e p u ?

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$\int \quad t$

 ok ∫ t e p u ? * s t e p u ?

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$T = \{s, \int, z, ʒ, \textcolor{red}{n}, \textcolor{red}{t}, \textcolor{red}{l}\}$ $S = \{*\textcolor{red}{s}\int, *\textcolor{red}{\int}s, *\textcolor{red}{s}n, *\textcolor{red}{s}t, *\textcolor{red}{s}l, \dots\}$

ok \int t

ok \int t e p u ?

* s t e p u ?

Sibilant Harmony in SAMALA (2/3)

Generalization

- ▶ Anticipatory Sibilant harmony
- ▶ Local restriction against [*sn, *st, *sl]

Grammar

$T = \{s, \int, z, ʒ, \textcolor{red}{n}, \textcolor{red}{t}, \textcolor{red}{l}\}$ $S = \{*\int s, *\int s, *sn, *st, *sl, \dots\}$

ok \int t

ok \int t e p u ?

s t

 * s t e p u ?

Sibilant Harmony in SAMALA (2/3)

Generalization

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- ▶ Local restriction against [*sn, *st, *sl]

Grammar

$T = \{s, \int, z, ʒ, \textcolor{red}{n}, \textcolor{red}{t}, \textcolor{red}{l}\}$ $S = \{*\int s, *\int s, *\textcolor{red}{s}n, *\textcolor{red}{s}t, *\textcolor{red}{s}l, \dots\}$

ok ∫ t _____
ok ∫ t e p u ?

* s t _____
 * s t e p u ?

Sibilant Harmony in SAMALA (3/3)

Generalization

- ▶ anticipatory sibilant harmony [$*s\int$, $*s\int$]
- ▶ palatalization to avoid local restriction [$*sn$, $*st$, $*sl$]
- ▶ sibilant harmony overrides palatalization

Grammar

$T = \{s, \int, n, t, l\}$ $S = \{*s\int, *s\int, *sn, *st, *sl\}$

ok s n e t u s

Sibilant Harmony in SAMALA (3/3)

Generalization

- ▶ anticipatory sibilant harmony [$*s\int$, $*s\int$]
- ▶ palatalization to avoid local restriction [$*sn$, $*st$, $*sl$]
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Grammar

$T = \{s, \int, n, t, l\}$ $S = \{*s\int, *s\int, *sn, *st, *sl\}$

s
.....
ok s n e t u s

Sibilant Harmony in SAMALA (3/3)

Generalization

- ▶ anticipatory sibilant harmony [$*s\int$, $*s\int$]
- ▶ palatalization to avoid local restriction [$*sn$, $*st$, $*sl$]
- ▶ sibilant harmony overrides palatalization

Grammar

$T = \{s, \int, n, t, l\}$ $S = \{*s\int, *s\int, *sn, *st, *sl\}$

s n
.....
ok s n e t u s

Sibilant Harmony in SAMALA (3/3)

Generalization

- ▶ anticipatory sibilant harmony [$*sʃ$, $*sʃ$]
- ▶ palatalization to avoid local restriction [$*sn$, $*st$, $*sl$]
- ▶ sibilant harmony overrides palatalization

Grammar

$T = \{s, ʃ, n, t, l\}$ $S = \{*sʃ, *sʃ, *sn, *st, *sl\}$

s n
.....
ok s n e t u s

Sibilant Harmony in SAMALA (3/3)

Generalization

- ▶ anticipatory sibilant harmony [$*s\int$, $*s\int$]
- ▶ palatalization to avoid local restriction [$*sn$, $*st$, $*sl$]
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Grammar

$T = \{s, \int, n, t, l\}$ $S = \{*s\int, *s\int, *sn, *st, *sl\}$

$s \quad n \quad t$

ok $s \quad n \quad e \quad \boxed{t} \quad u \quad s$

Sibilant Harmony in SAMALA (3/3)

Generalization

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- ▶ palatalization to avoid local restriction [$*sn$, $*st$, $*sl$]
- ▶ sibilant harmony overrides palatalization

Grammar

$T = \{s, ʃ, n, t, l\}$ $S = \{*sʃ, *sʃ, *sn, *st, *sl\}$

s n t

ok s n e t u s

Sibilant Harmony in SAMALA (3/3)

Generalization

- ▶ anticipatory sibilant harmony [$*sʃ$, $*sʃ$]
- ▶ palatalization to avoid local restriction [$*sn$, $*st$, $*sl$]
- ▶ sibilant harmony overrides palatalization

Grammar

$T = \{s, ʃ, n, t, l\}$ $S = \{*sʃ, *sʃ, *sn, *st, *sl\}$

$s \quad n \quad t \quad s$

ok $s \quad n \quad e \quad t \quad u \quad s$

Sibilant Harmony in SAMALA (3/3)

Generalization

- ▶ anticipatory sibilant harmony [$*s\int$, $*s\int$]
- ▶ palatalization to avoid local restriction [$*sn$, $*st$, $*sl$]
- ▶ sibilant harmony overrides palatalization

Grammar

$T = \{s, \int, n, t, l\}$ $S = \{*s\int, *s\int, *sn, *st, *sl\}$

*
s n t s

ok s n e t u s

Sibilant Harmony in SAMALA (3/3)

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Grammar

$T = \{s, \int, n, t, l\}$ $S = \{*s\int, *s\int, *sn, *st, *sl\}$

$\begin{array}{ccccccc}
 & & ok & & & & \\
 * & & \text{---} & \text{---} & \text{---} & & \\
 & \boxed{s} & \boxed{n} & & \boxed{t} & & s \\
 & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \\
 ok & s & n & e & t & u & s
 \end{array}$

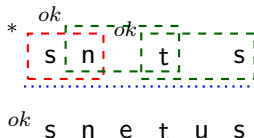
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$T = \{s, \int, n, t, l\}$ $S = \{*s\int, *s\int, *sn, *st, *sl\}$

* s n t s

ok s n e t u s

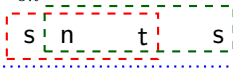
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Grammar

$T = \{s, \int, n, t, l\}$ $S = \{*s\int, *s\int, *sn, *st, *sl\}$

ok
 $*$ 

ok s n e t u s

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- ▶ sibilant harmony overrides palatalization

Grammar

$T = \{s, \int, n, t, l\}$ $S = \{*s\int, *s\int, *sn, *st, *sl\}$



No TSL grammar can capture this pattern...

Input-Sensitive TSL (ITSL) Languages

 E_T

Tier projection controlled by:

1 label of segment

TSL

1

TSL languages are characterized by:

- ▶ a **1**-local projection function E_T ;
- ▶ strictly k -local constraints applied on T .

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 E_T

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TSL

1

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Idea:

- ▶ E_T is an input-strictly local transduction (Chandlee 2014)
- ▶ **What if:** the locality of E_T was **higher than 1**?

Input-Sensitive TSL (ITSL) Languages

 E_T

Tier projection controlled by:

- 1 label of segment
- 2 local context

ITSL

 $1 + 2$

TSL

1

TSL languages are characterized by:

- ▶ a 1-local projection function E_T ;
- ▶ strictly k -local constraints applied on T.

Idea:

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An ITSL Account of Samala

SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [$*s\text{ʃ}$, $*s\text{ʃ}$]
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\$ s n e t u s \$

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s

.....

\$ s n e t u s \$

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s n
.....
\$ s n e t u s \$

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s n
.....
\$ s n e t u s \$

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s n
.....
\$ s n e t u s \$

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s n
.....
\$ s n e t u s \$

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s n
s

.....

\$ s n e t u s \$

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s n s

 \$ s n e t u s \$

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- ▶ sibilant harmony overrides palatalization

ok
 [s n s]

 \$ s n e t u s \$

An ITSL Account of Samala

SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [$*s\text{f}$, $*s\text{f}$]
- ▶ palatalization to avoid local restriction [$*s\text{n}$, $*s\text{t}$, $*s\text{l}$]
- ▶ sibilant harmony overrides palatalization

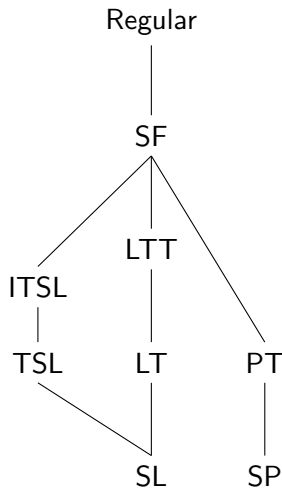
ok
 $\boxed{s \quad n \quad \quad \quad s}$
 \dots
 $\$ \quad s \quad n \quad e \quad t \quad u \quad s \quad \$$

Grammar

$$T = \{ \sigma : \sigma \in \{s, \text{f}\} \vee (\sigma \in \{n, t, l\} \wedge s \prec^+ \sigma) \}$$

$$S = \{ *s\text{f}, *s\text{f}, *s\text{n}(\neg s), *s\text{t}(\neg s), *s\text{l}(\neg s) \}$$

ITSL: Relations to other Classes?



Summing Up

Tracing Back our Steps

- ▶ TSL as a strong upper bound to phonotactic complexity.
- ▶ but there are patterns that are unaccounted for!

ITSL

- ▶ Natural generalization of TSL
- ▶ Covers a variety of patterns
Korean vowel harmony, UTP, Yaka nasal harmony, ...
- ▶ Preserves TSL's computational properties
 - ▶ contained expressive power
 - ▶ Gold learnable
Efficiently learnable? (cf. McMullin et al. 2019)
 - ▶ (lack of) closure properties

Summing Up

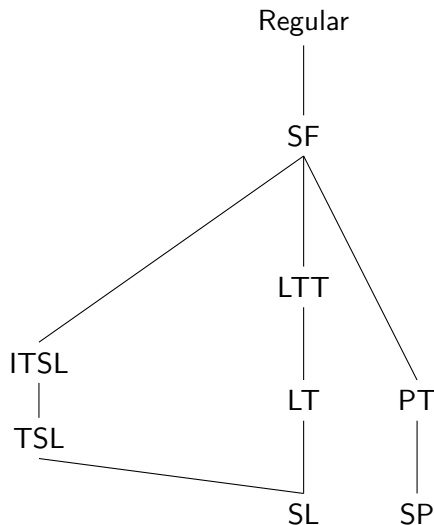
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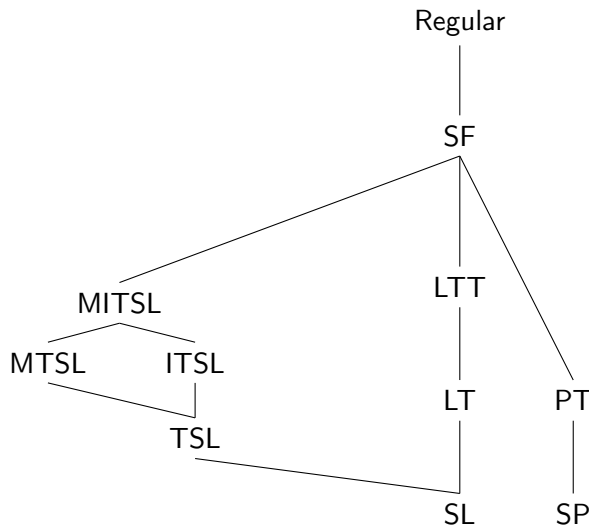
ITSL

- ▶ Natural generalization of TSL
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Korean vowel harmony, UTP, Yaka nasal harmony, ...
- ▶ Preserves TSL's computational properties
 - ▶ contained expressive power
 - ▶ Gold learnable
Efficiently learnable? (cf. McMullin et al. 2019)
 - ▶ (lack of) closure properties
Non-closure under intersection!

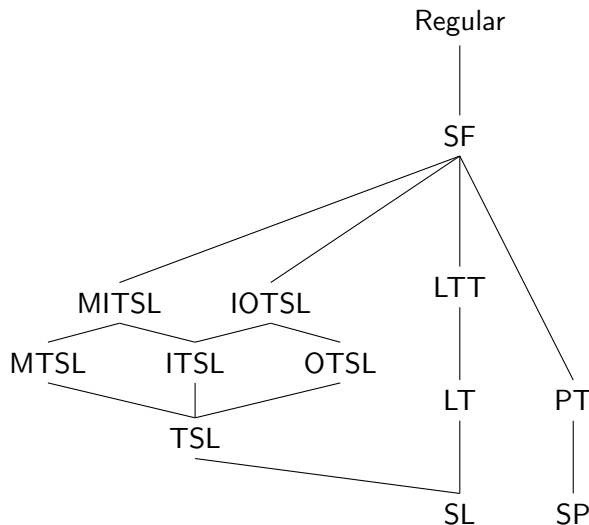
Exploring the TSL Neighborhood



Exploring the TSL Neighborhood



Exploring the TSL Neighborhood



Future Work

- ▶ Test typological predictions
- ▶ Cross-domain (syntax, semantics) generalizations
(cf. Vu et al. 2019; Graf and Shafiei 2019)
- ▶ Further study of the TSL neighborhood
Important: lack of a good abstract characterization
- ▶ Learnability → learning algorithms, AGL experiments, NN?
E.g., which expression of locality is more natural?

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Thank you!

Acknowledgments



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Appendix

Closure Properties of Subregular Classes

	SL	TSL	MTSL	ITSL	IOTSL	SF	Reg
\cup	×	×	×	×	×	✓	✓
\cap	✓	×	✓	×	×	✓	✓
Relabeling	×	×	×	×	×	×	✓
Complement	×	×	×	×	×	✓	✓

(I)TSL vs. SP

- **Strictly piecewise** (SP) grammars: forbidden **subsequences**.
Precedence (not successor) as the core relation.

Sibilant harmony

► (I)TSL



► SP

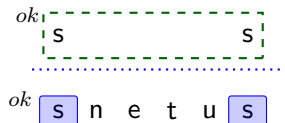


(I)TSL vs. SP

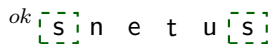
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Sibilant harmony

► (I)TSL



► SP



(I)TSL vs. SP: Incomparability

(I)TSL $\not\subseteq$ SP

SAMALA's harmony is ITSL but not SP

ok [s][n] e t u [s]

ok [s] n e [t][u] s

SP $\not\subseteq$ ITSL

$S = \{^*ab, ^*cd\} \Rightarrow L(S) \in SP$ but $L(S) \notin ITSL$

* a c e b e d

* a c e b e d

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* [a] c e [b] e d

* a c e b e d

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* a [c] e b e [d]

* a c e b e d

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* a [c] e b e [d]

* [a] [c] e [b] e [d]

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ok [s][n] e t u [s]

ok [s] n e [t][u] s

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$S = \{^*ab, ^*cd\} \Rightarrow L(S) \in SP$ but $L(S) \notin ITSL$

* a [c] e b e [d]

a c b d
.....
* [a][c] e [b] e [d]

Another TSL Outlier

Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin 2016)

1) Underlying causative prefix /s(:)-/

	<i>Base</i>	<i>Causative</i>	
a.	uga	s:-uga	"be evacuated"
b.	as:twā	s-as:twā	"settle, be levelled"

2) Sibilant harmony

	<i>Base</i>	<i>Causative</i>	
a.	fiaʃr	ʃ- fiaʃr	"be full of straw, of discord"
b.	nza	z:-nza	"be sold"

3) Sibilant voicing harmony blocked

	<i>Base</i>	<i>Causative</i>	
a.	ukz	s:-ukz	"recognize"
b.	q:uɰi	ʃ- quɰi	"be dislocated, broken"

Can we write a TSL grammar to capture this pattern?

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1) Underlying causative prefix /s(:)-/

Base *Causative*

- | | | | |
|----|----------------------|------------------------|-----------------------|
| a. | uga | s:-uga | "be evacuated" |
| b. | a ^s :-twa | s-a ^s :-twa | "settle, be levelled" |

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|----|------------------|---------------------|--------------------------------|
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| b. | n ^z a | z:-n ^z a | "be sold" |

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Base *Causative*

- | | | | |
|----|--------------------|-----------------------|-------------------------|
| a. | uk ^z | s:-uk ^z | "recognize" |
| b. | q:u ^z i | ʃ- q:u ^z i | "be dislocated, broken" |

Can we write a TSL grammar to capture this pattern?

Another TSL Outlier

Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin 2016)

1) Underlying causative prefix /s(:)-/

Base *Causative*

- | | | | |
|----|----------------------|------------------------|-----------------------|
| a. | uga | s:-uga | "be evacuated" |
| b. | a ^s :-twa | s-a ^s :-twa | "settle, be levelled" |

2) Sibilant harmony

Base *Causative*

- | | | | |
|----|------------------|---------------------|--------------------------------|
| a. | fia ^r | ʃ- fia ^r | "be full of straw, of discord" |
| b. | n ^z a | z:-n ^z a | "be sold" |

3) Sibilant voicing harmony blocked

Base *Causative*

- | | | | |
|----|--------------------|------------------------------------|-------------------------|
| a. | uk ^z | s:-uk ^z | "recognize" |
| b. | q:u ^z i | ʃ- q ^z u ^z i | "be dislocated, broken" |

Can we write a TSL grammar to capture this pattern?

Another TSL Outlier

Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin 2016)

1) Underlying causative prefix /s(:)-/

Base *Causative*

- | | | | |
|----|--------|----------|-----------------------|
| a. | uga | s:-uga | "be evacuated" |
| b. | a:-twa | s-a:-twa | "settle, be levelled" |

2) Sibilant harmony

Base *Causative*

- | | | | |
|----|-------|----------|--------------------------------|
| a. | fiaʃr | ʃ- fiaʃr | "be full of straw, of discord" |
| b. | nza | z:-nza | "be sold" |

3) Sibilant voicing harmony blocked

Base *Causative*

- | | | | |
|----|--------|-----------|-------------------------|
| a. | ukz | s:-ukz | "recognize" |
| b. | q:uʒ:i | ʃ- q:uʒ:i | "be dislocated, broken" |

Can we write a TSL grammar to capture this pattern?

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *ʒs, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *f\text{ʒ}, *ʒz \}$$

z

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

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Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *\text{ʒ}s, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *f\text{ʒ}, *\text{ʒ}z \}$$

z

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *\text{ʒ}s, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *f\text{ʒ}, *\text{ʒ}z \}$$

z ʒ

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *fs, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$

z ʒ

.....

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

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z ʒ

.....

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *\text{ʒ}s, *\text{ʒ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *\text{ʒ}\text{ʃ}, *\text{ʒ}z \}$$

z ʒ

.....

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

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$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

z ʒ

.....

* z m: ʒ d a w l

ok ʒ m: ʒ d a w l

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

*
z ʒ

 *

* z m: ʒ d a w |

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

*
z ʒ

 * z m: ʒ d a w |

ʒ

 ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

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$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

*
z ʒ

 *

z m: ʒ d a w |

ʒ

 ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT


Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

*


 * z m: ʒ d a w l

ʒ

 ok ʒ m: ʒ d a w l

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

*
z ʒ

 *

z m: ʒ d a w |

ʒ ʒ

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f s, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$

*
z ʒ

 *

z m: ʒ d a w |

ʒ ʒ

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

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$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}f, {}^*f\text{ʒ}, {}^*f\text{ʒ}, {}^*\text{ʒ}z \}$$

*
z ʒ

 * z m: ʒ d a w l

ʒ ʒ

ok ʒ m: ʒ d a w l

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}f, {}^*f\text{ʒ}, {}^*fz, {}^*f\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

*
z ʒ

 * z m: ʒ d a w l

ʒ ʒ

ok ʒ m: ʒ d a w l

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (1/2)

Sibilants must agree in anteriority and voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

* z ʒ

* z m: ʒ d a w |

ok ʒ ʒ

ok ʒ m: ʒ d a w |

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, * \text{ʒ}f, * \text{ʒ}z \}$$

ok ʃ q u ʒ: i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *\text{ʒ}s, *\text{ʃ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *\text{ʒ}\text{ʃ}, *\text{ʒ}z \}$$

ʃ

.....

ok ʃ q u ʒ: i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *ʒs, *\text{ʃ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *ʒ\text{ʃ}, *ʒz \}$$

ʃ q

ok ʃ q u ʒ: i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *ʒs, *\text{ʃ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *ʒ\text{ʃ}, *ʒz \}$$

ʃ q

.....

ok ʃ q u ʒ: i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *\text{ʒ}s, *\text{ʃ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *\text{ʒ}\text{ʃ}, *\text{ʒ}z \}$$

ʃ q ʒ:

.....

ok ʃ q u ʒ: i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ʃ q ʒ:

ok

ʃ q u ʒ: i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *ʒs, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ok f q ʒ:

ok f q u ʒ: i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f\text{ʒ}, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *z\text{ʒ} \}$$

ok

ok ʃ q u ʒ: i

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ok

ok

ʃ q ʒ:

ʃ q u ʒ: i

s

*

s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ok

ok

ʃ q ʒ:

ok

ʃ q u ʒ: i

s q

*

s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ok

ok

ʃ q ʒ:

ok

ʃ q u ʒ: i

s q

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f\text{ʒ}, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$

ok

ok

ʃ q ʒ:

ok

ʃ q u ʒ: i

s q ʒ:

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ok

ʃ q ʒ:

ok ʃ q u ʒ: i

s q ʒ:

* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f s, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$

^{ok}
^{ok} [ʃ] [q] [ʒ:]

^{ok} ʃ q u ʒ: i

^{ok}
^{ok} [s] [q] [ʒ:]

 * s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

Generalization (2/2)

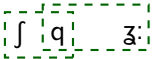
Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

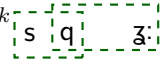
$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f\text{ʒ}, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$

ok



ok ʃ q u ʒ: i

ok



* s q u ʒ: i

Sibilant Harmony in IMDLAWN TASHLHIYT

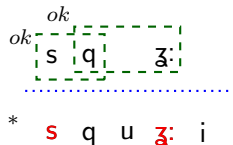
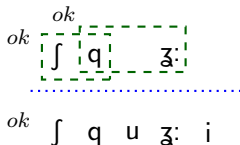
Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f\text{ʒ}, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$



Sibilant Harmony in IMDLAWN TASHLHIYT

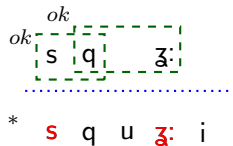
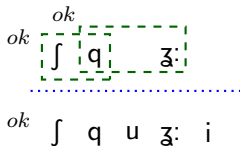
Generalization (2/2)

Voiceless obstruents block agreement in voicing.

Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *\text{ʒ}s, *\text{ʃ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *\text{ʒ}\text{ʃ}, *\text{ʒ}z \}$$



No TSL grammar can block voicing and enforce anteriority!

Multi-Tier Strictly Local (MTSL) Languages (1/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \text{f}, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\text{f}z, ^*\text{f}\text{ʒ}, ^*\text{ʒ}\text{f}\}$$

ok ʃ q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (1/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \text{f}, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\text{f}z, ^*\text{f}\text{ʒ}, ^*\text{ʒ}\text{f}\}$$

ʃ

.....

T₁ : sibilant voicing

ok ʃ q u ʒ: i

$\int q$

 T₁ : sibilant voicing
 ok \int q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (1/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʃ, s, z, f, q}\} \quad S_1 = \{*\text{ʃʃ}, *\text{sz}, *\text{ʃs}, *\text{zs}, *\text{fz}, *\text{fʃ}, *\text{ʃf}\}$$

ʃ q

.....

T_1 : sibilant voicing

ok ʃ q u ʃ: i

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\int q ʃ:
 $\overline{T_1}$: sibilant voicing
ok \int q u ʃ: i

Multi-Tier Strictly Local (MTSL) Languages (1/2)

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\int q ʒ:
 $\overset{\text{ok}}{\text{T}_1} : \text{sibilant voicing}$
 \int q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (1/2)

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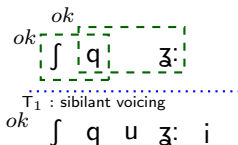
ok $\boxed{\text{f} \text{ q}}$ ʒ:
 T_1 : sibilant voicing
 ok $\text{f} \text{ q} \text{ u} \text{ ʒ} \text{ i}$

Multi-Tier Strictly Local (MTSL) Languages (1/2)

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Multi-Tier Strictly Local (MTSL) Languages (1/2)

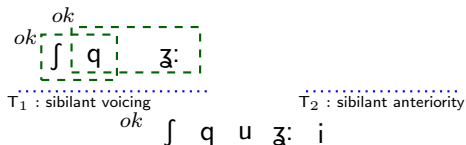
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

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Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



Multi-Tier Strictly Local (MTSL) Languages (1/2)

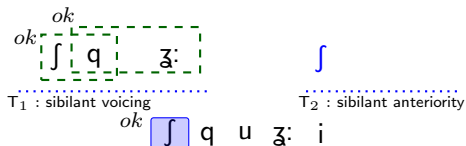
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

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Unbounded agreement in anteriority:

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Multi-Tier Strictly Local (MTSL) Languages (1/2)

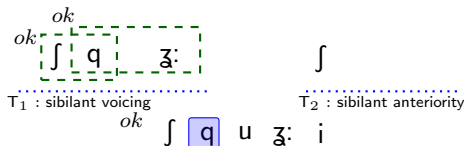
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

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Multi-Tier Strictly Local (MTSL) Languages (1/2)

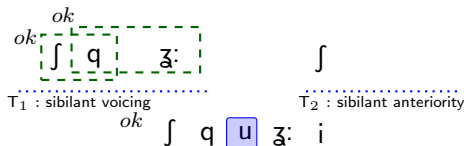
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

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Unbounded agreement in anteriority:

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Multi-Tier Strictly Local (MTSL) Languages (1/2)

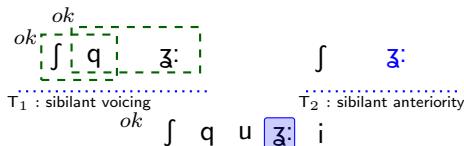
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

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Multi-Tier Strictly Local (MTSL) Languages (1/2)

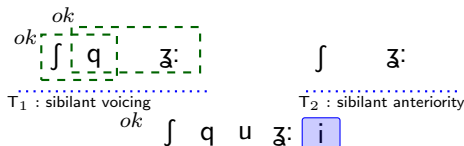
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, \text{s}, \text{z}, \text{f}, \text{q}\} \quad S_1 = \{*\text{sʒ}, *\text{sz}, *\text{ʒs}, *\text{zs}, *\text{fz}, *\text{fʒ}, *\text{ʒf}\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, \text{s}, \text{z}, \text{f}\} \quad S_2 = \{*\text{sʒ}, *\text{sʃ}, *\text{ʒs}, *\text{ʃs}, *\text{zs}, *\text{zf}, *\text{zʒ}, *\text{fz}, *\text{ʒz}\}$$



Multi-Tier Strictly Local (MTSL) Languages (1/2)

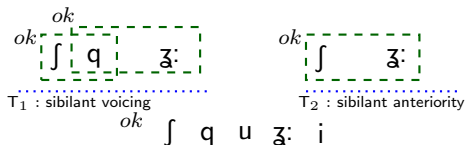
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



Multi-Tier Strictly Local (MTSL) Languages (2/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*\text{ʒ}z, ^*fz, ^*\text{ʒ}z\}$$

* s q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (2/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

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Unbounded agreement in anteriority:

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S

.....
 T_1 : sibilant voicing
 * s q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (2/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$

s q

T_1 : sibilant voicing
 * s q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (2/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

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Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$

s q

T_1 : sibilant voicing

* s q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (2/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

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Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$

s q ʒ:

.....

T₁ : sibilant voicing

* s q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (2/2)

Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

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s q ʒ:

.....

T₁ : sibilant voicing

* s q u ʒ: i

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Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*\text{ʒ}z, ^*fz, ^*\text{ʒ}z\}$$

ok s q ʒ:

 T_1 : sibilant voicing
 * s q u ʒ: i

Multi-Tier Strictly Local (MTSL) Languages (2/2)

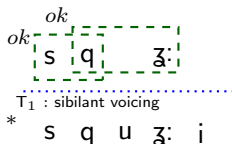
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*f\text{ʃ}\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*s\text{ʃ}, ^*\text{ʒ}s, ^*f\text{ʃ}, ^*zs, ^*z\text{ʃ}, ^*\text{ʒ}\text{ʒ}, ^*f\text{ʒ}, ^*\text{ʒ}\text{ʒ}\}$$



Multi-Tier Strictly Local (MTSL) Languages (2/2)

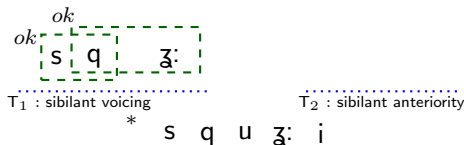
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

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Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



Multi-Tier Strictly Local (MTSL) Languages (2/2)

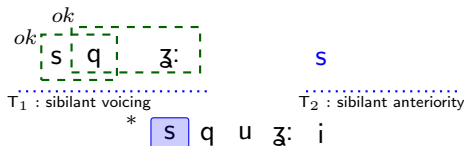
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

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$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

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Multi-Tier Strictly Local (MTSL) Languages (2/2)

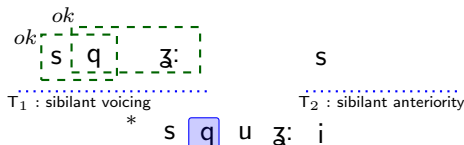
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

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Unbounded agreement in anteriority:

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Multi-Tier Strictly Local (MTSL) Languages (2/2)

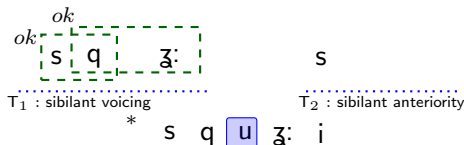
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

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Multi-Tier Strictly Local (MTSL) Languages (2/2)

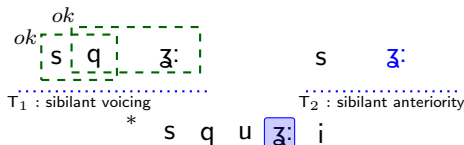
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

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Multi-Tier Strictly Local (MTSL) Languages (2/2)

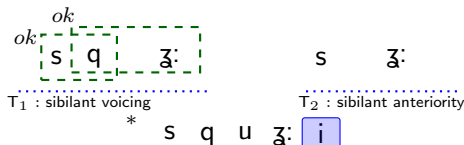
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

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$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



Multi-Tier Strictly Local (MTSL) Languages (2/2)

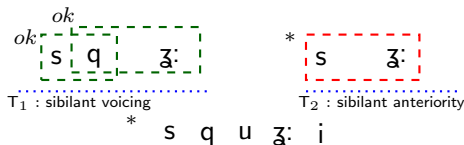
Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

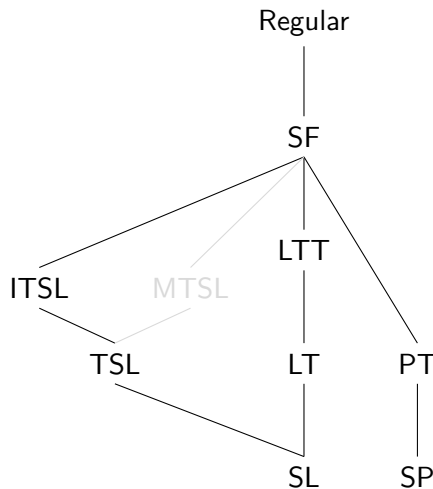
$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

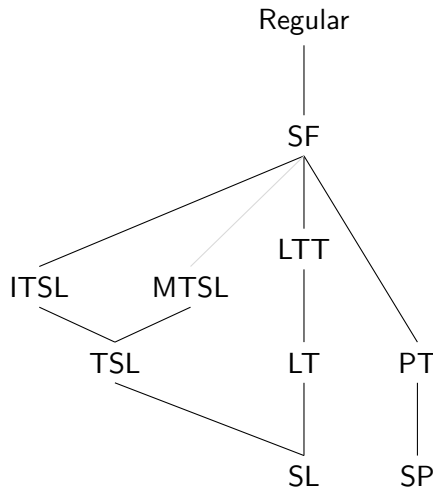
$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



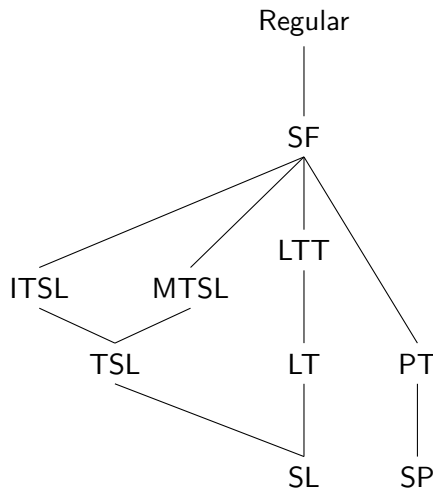
MTSL: Relations to other Classes



MTSL: Relations to other Classes



MTSL: Relations to other Classes



Incomparability of ITSL and MTSL (1/2)

MTSL $\not\subseteq$ ITSL

We already have an example

- ▶ IMDLAWN TASHLHIYT's harmony is MTSL but not ITSL

ITSL \subseteq MTSL?

- ▶ Is every ITSL language also MTSL?

IMDLAWN TASHLHIYT's Sibilant Harmony \notin ITSL

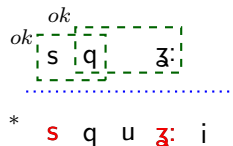
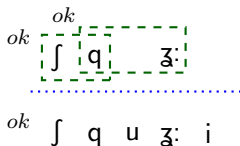
Reminder: Sibilant Harmony in IMDLAWN TASHLHIYT

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T = \{\text{ʒ}, s, z, f, q\} \quad S = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T = \{\text{ʒ}, s, z, f\} \quad S = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



This pattern is not ITSL!

IMDLAWN TASHLHIYT's Sibilant Harmony \notin ITSL

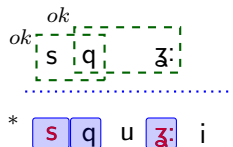
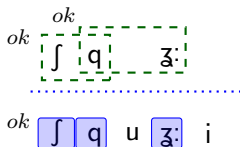
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IMDLAWN TASHLHIYT's Sibilant Harmony \notin ITSL

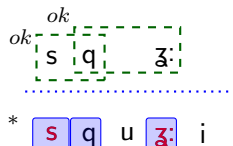
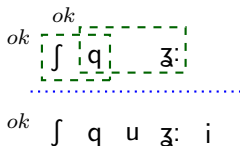
Reminder: Sibilant Harmony in IMDLAWN TASHLHIYT

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T = \{\text{ʒ}, s, z, f, q\} \quad S = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

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This pattern is not ITSL!

Incomparability of ITSL and MTSL (2/2)

MTSL $\not\subseteq$ ITSL

IMDLAWN TASHLHIYT's harmony is MTSL but not ITSL.

ITSL $\not\subseteq$ MTSL

We have already seen an example:

- ▶ SAMALA's harmony is ITSL but not MTSL.

An MTSL Account for SAMALA?

SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

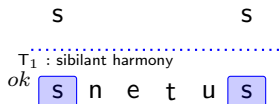
ok s n e t u s

This pattern is not MTSL!

An MTSL Account for SAMALA?

SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
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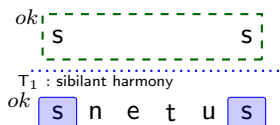


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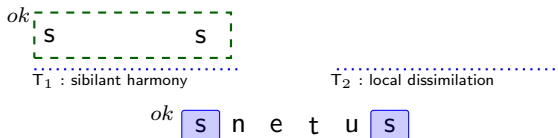


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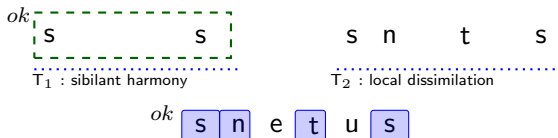


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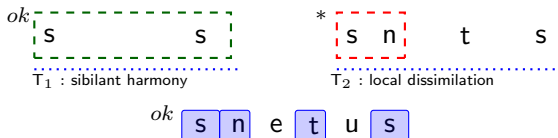


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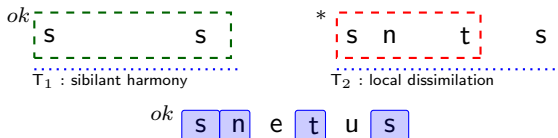


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- ▶ sibilant harmony overrides palatalization



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An MTSL Account for SAMALA?

SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

ok s s

.....
 T_1 : sibilant harmony

*ok*₁ s n t s

.....
 T_2 : local dissimilation

ok s n e t u s

This pattern is not MTSL!

An MTSL Account for SAMALA?

SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

ok s s
 T₁ : sibilant harmony

*ok*₁ s n t s
 T₂ : local dissimilation

ok s n e t u s

This pattern is not MTSL!

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int

.....

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int

.....

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int

.....

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int $t\int^h$

.....

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int $t\int^h$

.....

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int $t\int^h$

.....

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int $t\int^h$

.....

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int $t\int^h$ \int

.....
T₁: anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int $t\int^h$ \int

.....

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int **w** a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int $t\int^h$ \int

.....

T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

\int $t\int^h$ \int \int

.....

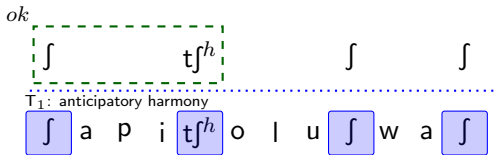
T_1 : anticipatory harmony

\int a p i $t\int^h$ o l u \int w a \int

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

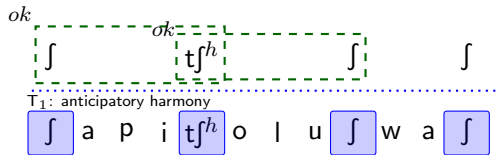
$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$



Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

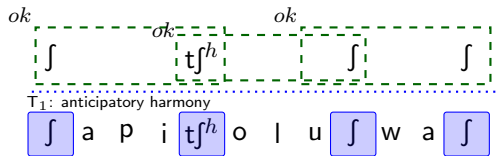
$$T = \{ s, \int, t f^h \}, S = \{ * s \int, * \int s, * s t f^h, * t f^h \}$$



Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

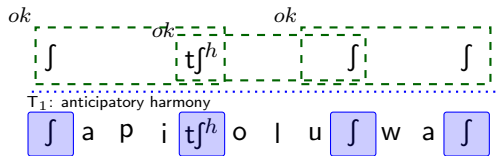
$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$



Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



First/Last Harmony in PSEUDO-SAMALA

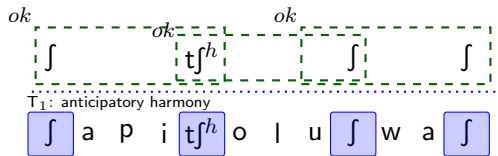
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$

∫ a p i t f^h o l u s w a ∫

Structure-Sensitive TSL: Overgeneration

Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



First/Last Harmony in PSEUDO-SAMALA

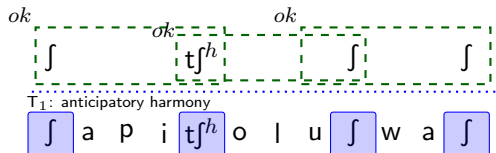
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$

$\bowtie \int a p i \text{tf}^h o l u s w a \int \bowtie$

Structure-Sensitive TSL: Overgeneration

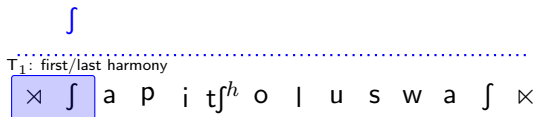
Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$



First/Last Harmony in PSEUDO-SAMALA

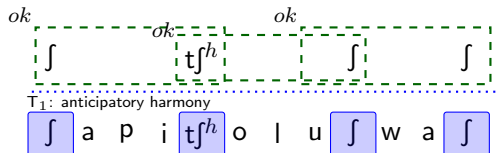
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Structure-Sensitive TSL: Overgeneration

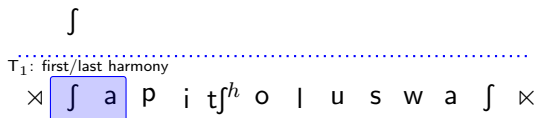
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First/Last Harmony in PSEUDO-SAMALA

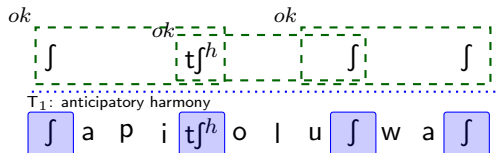
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Structure-Sensitive TSL: Overgeneration

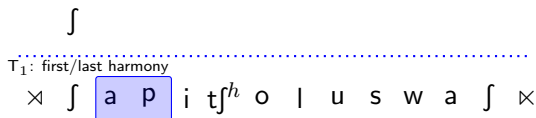
Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



First/Last Harmony in PSEUDO-SAMALA

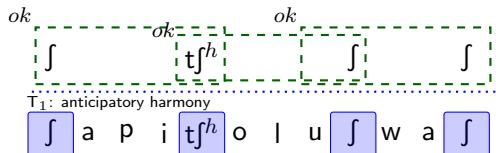
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Structure-Sensitive TSL: Overgeneration

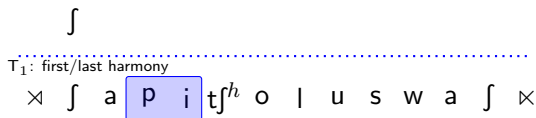
Anticipatory Harmony in SAMALA

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First/Last Harmony in PSEUDO-SAMALA

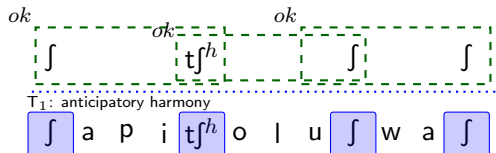
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Structure-Sensitive TSL: Overgeneration

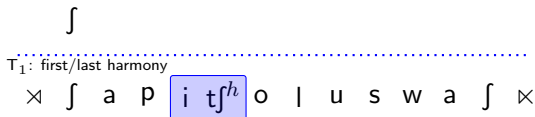
Anticipatory Harmony in SAMALA

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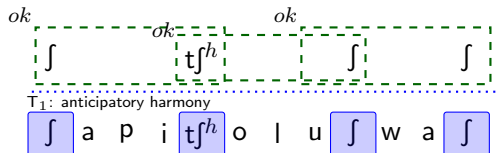
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Structure-Sensitive TSL: Overgeneration

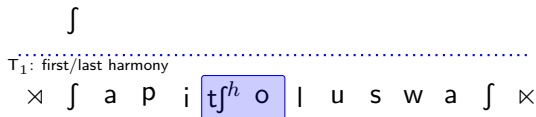
Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



First/Last Harmony in PSEUDO-SAMALA

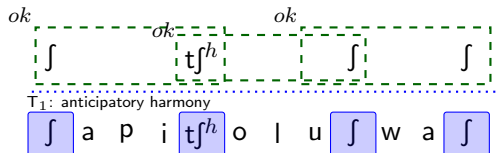
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Structure-Sensitive TSL: Overgeneration

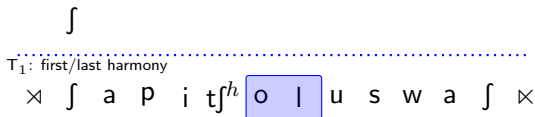
Anticipatory Harmony in SAMALA

$$T = \{s, \int, tf^h\}, S = \{^*sf, ^*fs, ^*stf^h, ^*tf^h\}$$



First/Last Harmony in PSEUDO-SAMALA

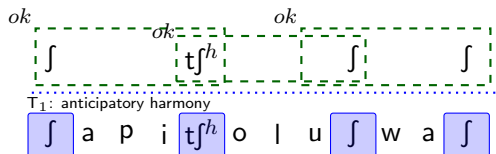
$$T = \{\sigma: \sigma \in \{s, \int, tf^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} S = \{^*sf, ^*fs, ^*stf^h, ^*tf^h s\}$$



Structure-Sensitive TSL: Overgeneration

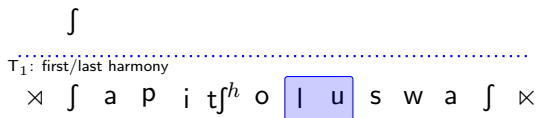
Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



First/Last Harmony in PSEUDO-SAMALA

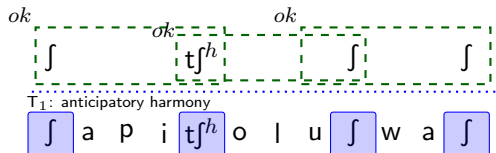
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Structure-Sensitive TSL: Overgeneration

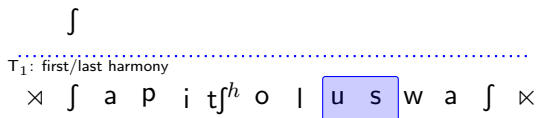
Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



First/Last Harmony in PSEUDO-SAMALA

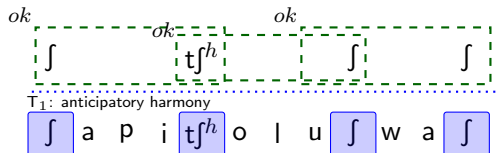
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Structure-Sensitive TSL: Overgeneration

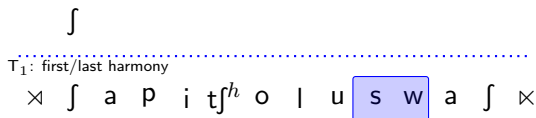
Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



First/Last Harmony in PSEUDO-SAMALA

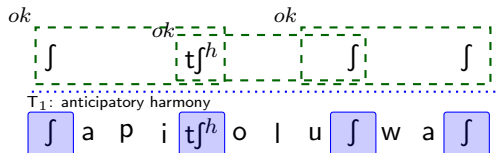
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



Structure-Sensitive TSL: Overgeneration

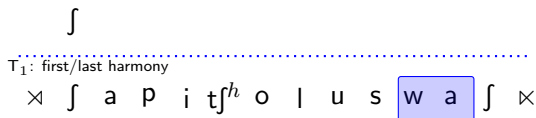
Anticipatory Harmony in SAMALA

$$T = \{s, \int, tf^h\}, S = \{^*sf, ^*fs, ^*stf^h, ^*tf^h\}$$



First/Last Harmony in PSEUDO-SAMALA

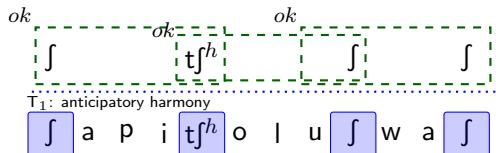
$$T = \{\sigma: \sigma \in \{s, \int, tf^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*sf, ^*fs, ^*stf^h, ^*tf^h s\}$$



Structure-Sensitive TSL: Overgeneration

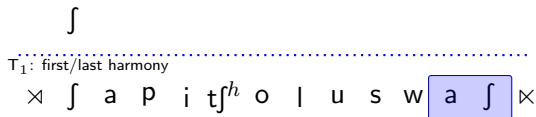
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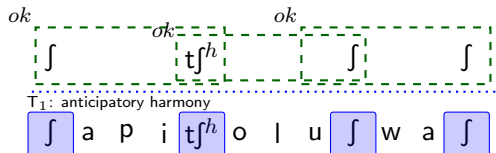
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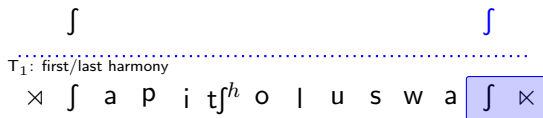
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First/Last Harmony in PSEUDO-SAMALA

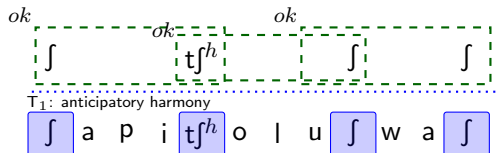
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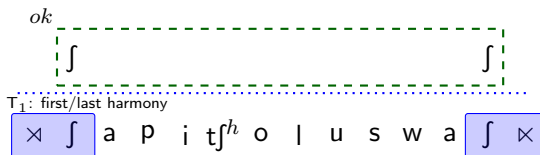
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TESL: Example

Tier Grammar(s) for L

$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\} \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

a)

⋈ c a c d b ⋈

b)

⋈ c a d d b ⋈

TESL: Example

Tier Grammar(s) for L

$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

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a)

\bowtie c a c d b \bowtie

b)

\bowtie c a d d b \bowtie

TESL: Example

Tier Grammar(s) for L

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a)

$\bowtie \quad c \quad a \quad c \quad \quad b \quad \bowtie$
..... T_1
 \bowtie c a c d b \bowtie

b)

$\bowtie \quad c \quad a \quad d \quad d \quad b \quad \bowtie$

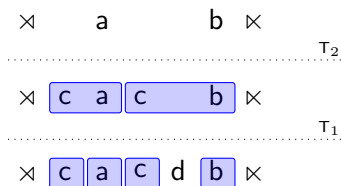
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a)



b)

$\bowtie \text{c a d d b} \bowtie$

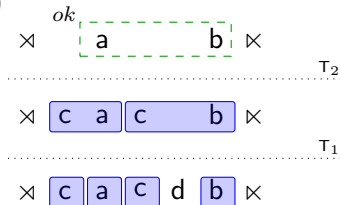
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a)



b)

$\bowtie c a d d b \bowtie$

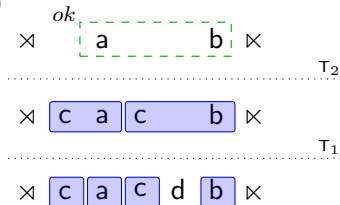
TESL: Example

Tier Grammar(s) for L

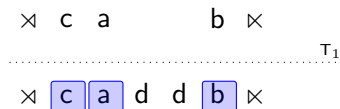
$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\} \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

a)



b)



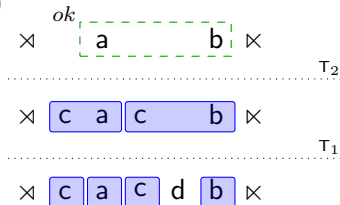
TESL: Example

Tier Grammar(s) for L

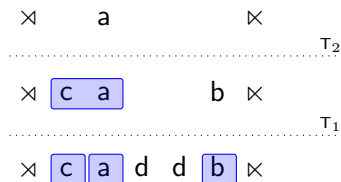
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a)



b)



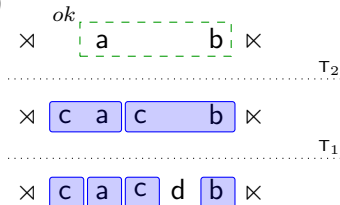
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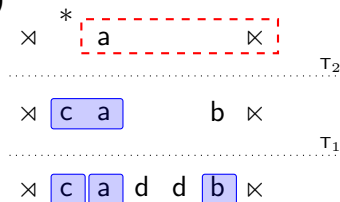
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a)



b)



TSL Syntax (of the Intuition)

TSL syntax

Merge and Move dependencies are TSL! (Graf and Heinz 2016)

- ▶ We move from strings to trees.
- ▶ We project tiers for trees.
- ▶ We enforce n -local tree constraints.

Tree n -gram grammars:

- ▶ Patterns are described by forbidden tree n -gram(s).
- ▶ A derivational tree is well formed iff no tier T contains any forbidden n -gram(s).

TSL Syntax (of the Intuition)

TSL syntax

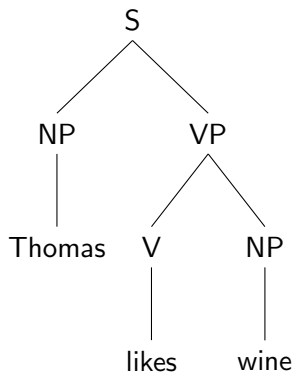
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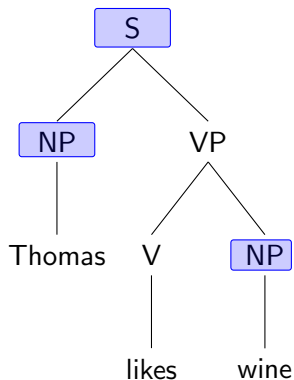
TSL Syntax: Example (of the Intuition)



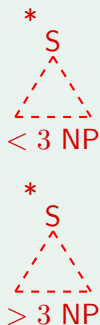
Tier-grammar



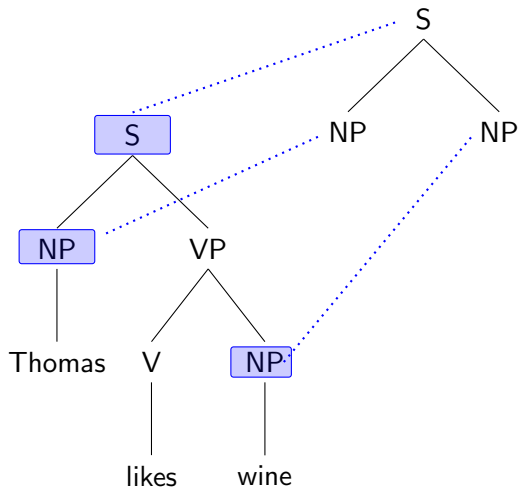
TSL Syntax: Example (of the Intuition)



Tier-grammar



TSL Syntax: Example (of the Intuition)

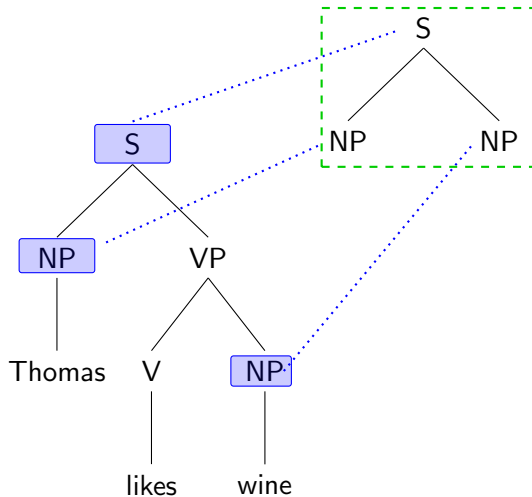


Tier-grammar

*
S
△
< 3 NP

*
S
△
> 3 NP

TSL Syntax: Example (of the Intuition)



Tier-grammar

*
S
△
< 3 NP

*
S
△
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