Evaluating Subregular Distinctions in the Complexity of Generalized Quantifiers

Aniello De Santo Thomas Graf John E. Drury

Stony Brook University
aniello.desanto@stonybrook.edu
aniellodesanto.github.io

QUAD: Quantifiers and Determiners
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The Talk in a Nutshell

Generalized Quantifiers and Semantic Complexity

Semantic automata as model of quantifier verification

- plausible link to cognitive results
- but: complexity linked to succinctness of automata

In This Talk

Coming back to formal language theory
→ subregular hierarchy & quantifier languages

Consequences

- complexity distinctions independent of the recognition mechanism
- cross-domain parallels, cognitive predictions and new experimental paradigms!

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Outline

Why Computational Complexity?

> *An algorithm is likely to be understood more readily by understanding* **the nature of the problem** *being solved than by examining the mechanism (and the hardware) in which it is embodied.*
>
> *(Marr 1983, p.27)*

Complexity in linguistics:

- boundaries on Natural Language
- competence vs performance debate
- cognitive predictions
  $\rightarrow$ resources
  $\rightarrow$ processing plausibility

Why should we care today?

- **meaning as truth-verification procedures**

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Generalized Quantifiers

Generalized quantifier $\mathbf{Q}(\mathbf{A}, \mathbf{B})$:

- two sets **A** and **B** as arguments
- returns truth value $(0, 1)$

Example

(1) Every student cheated.

- $\mathbf{every}(\mathbf{A}, \mathbf{B}) = 1$ iff $\mathbf{A} \subseteq \mathbf{B}$
- **student**: John, Mary, Sue
- **cheat**: John, Mary
- $\mathbf{student} \not\subseteq \mathbf{cheat} \Rightarrow \mathbf{every}(\mathbf{student}, \mathbf{cheat}) = 0$
- "Every student cheated" is false.

Binary Strings

- The language of **A** is the set of all permutations of **A**.

Example

student	John, Mary, Sue
L(**student**)	John Mary Sue, John Sue Mary
	Mary John Sue, Mary Sue John
	Sue John Mary, Sue Mary John

- Now replace every **a** $\in$ **A** by a truth value:
  1 if **a** $\in$ **B**
  0 if **a** $\notin$ **B**
- The result is the **binary string language** of **A** under **B**.

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student	John, Mary, Sue
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Quantifier Languages (van Benthem 1986)

- We can associate each quantifier Q with a language in $\{0, 1\}^*$
  $\Rightarrow$ Q accepts only binary strings of specific shape
- This is its **quantifier language**.

Example: *every*

- **every**(**A**, **B**) holds iff **A** $\subseteq$ **B**
- So every element of **A** must be mapped to 1.
- $L($**every**$) = \{1\}^*$

Example: *some*

- **some**(**A**, **B**) holds iff **A** $\cap$ **B** $\neq \emptyset$
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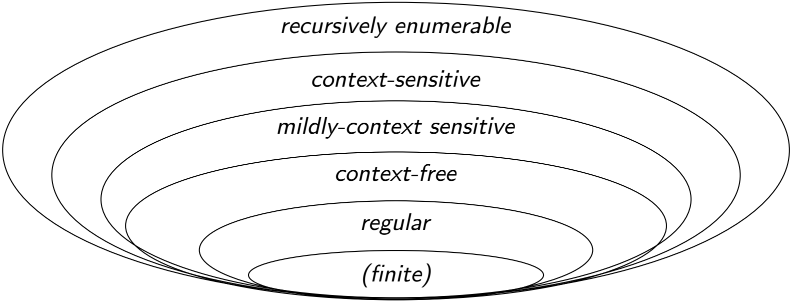
Languages (stringsets) can be classified according to the complexity of the grammars that generate them.



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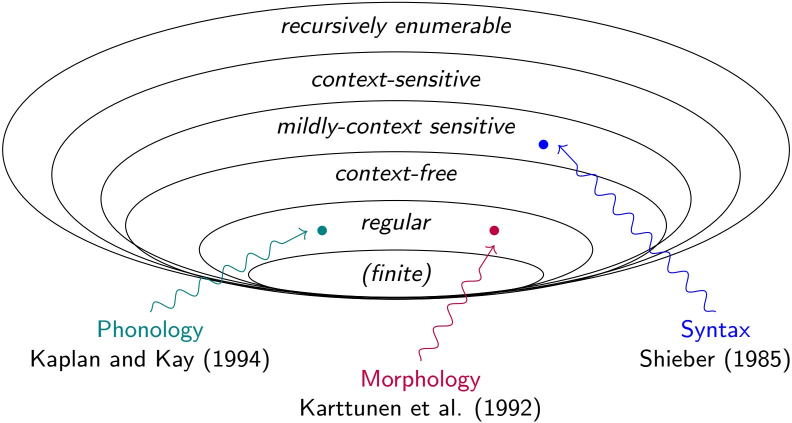
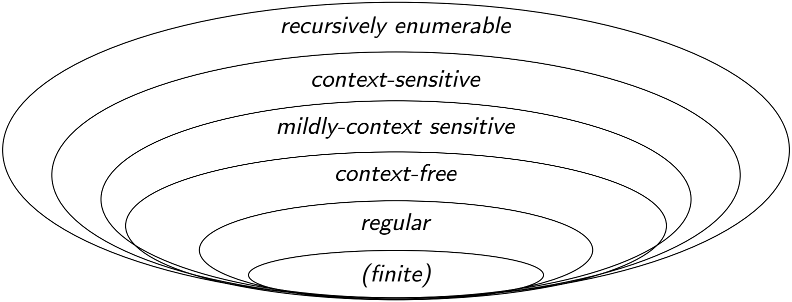


Chomsky Hierarchy and Automata Theory



Semantic Automata (van Benthem 1986, Mostowski 1998)

We can rank quantifiers based on their quantifier languages and the complexity of the machine needed to recognize them.

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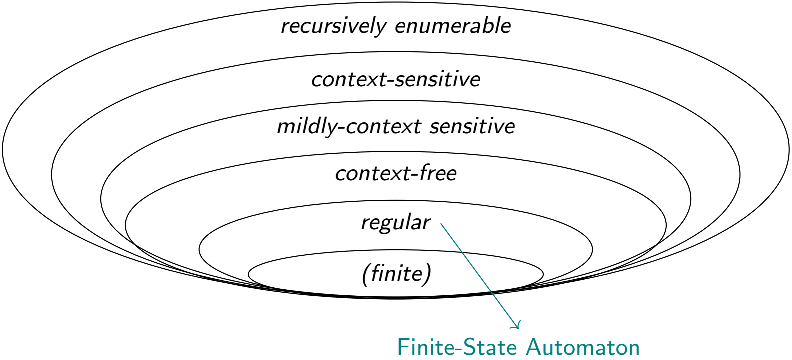

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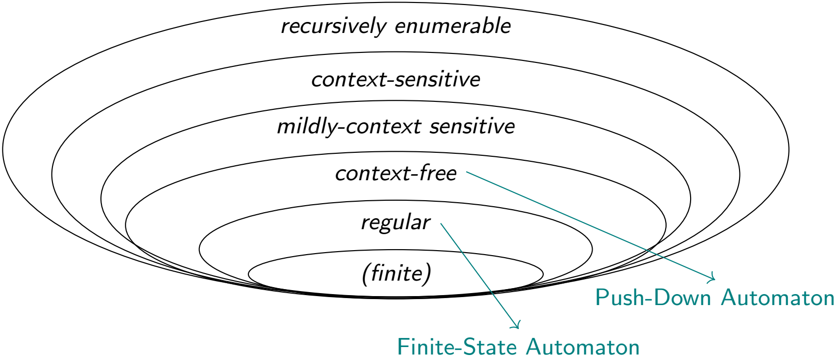

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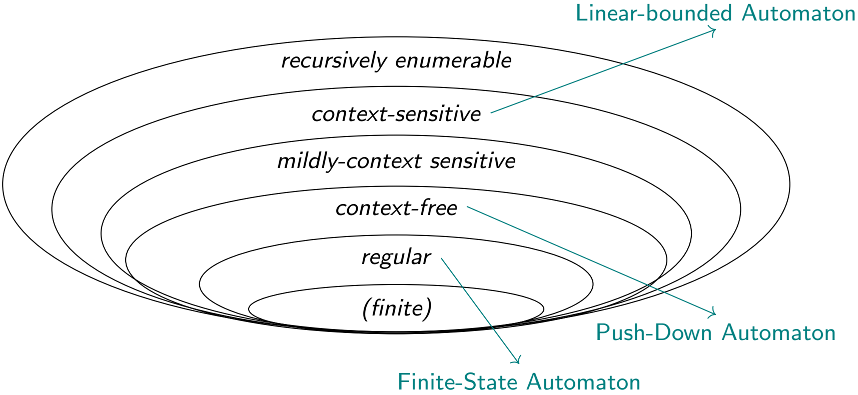

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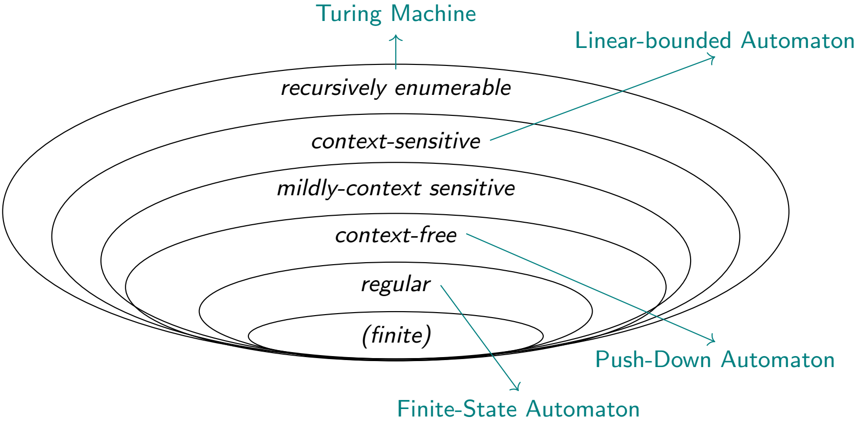

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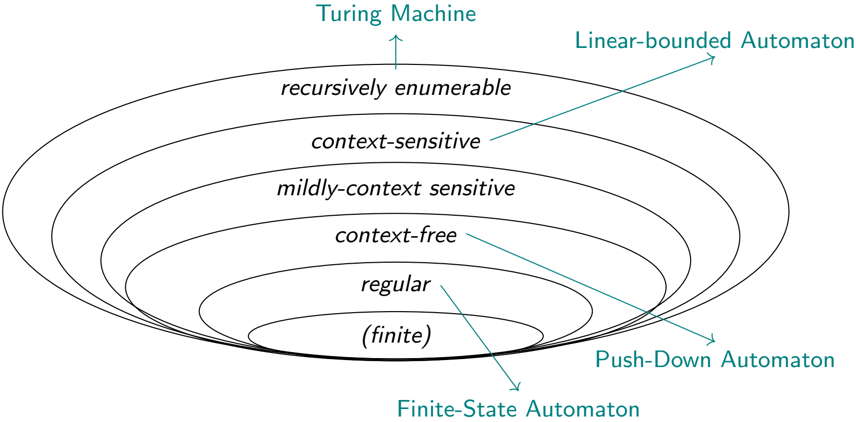

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Aristotelian Quantifiers are FSA-recognizable

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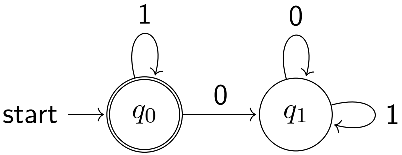

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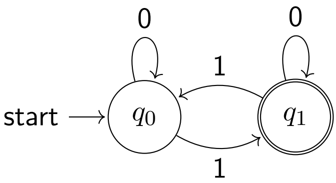
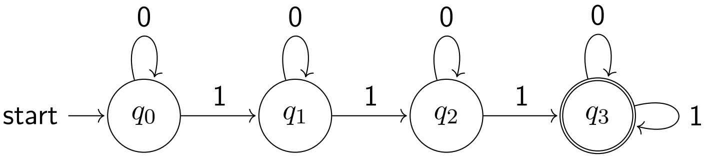
Other FSA-recognizable quantifiers

- Parity quantifiers: **An even number**



- Cardinal quantifiers: **At least 3**



Proportional Quantifiers

- **most**(**A**, **B**) holds iff $|\mathbf{A} \cap \mathbf{B}| > |\mathbf{A} - \mathbf{B}|$
- $L_{most} := \{w \in \{0,1\}^* : |1|_w > |0|_w\}$
- There is no finite automaton recognizing this language.
- We need internal memory.
  $\Rightarrow$ **push-down automata**: two states + a stack

A Hierarchy of Quantifiers' Complexity

FSA **PSA**

$\boxed{\{\textit{All, Some, Even, Odd, At least n, At most n}\}} < \boxed{\{\textit{Less than half, More than half, Most}\}}$

Are these all of equivalent complexity?

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Let's Look at the Automata One More Time

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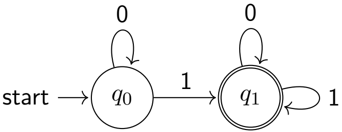

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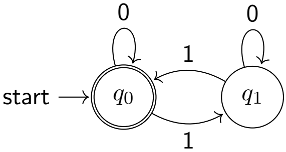

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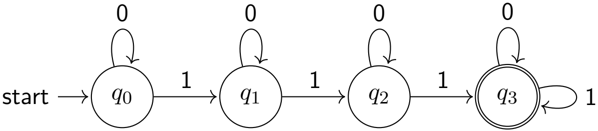

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Are these all of equivalent complexity?

- Cyclic vs acyclic automata
- The number of states matters
- But: Complexity = succinctness of automata?

Reminder

It's all grounded in quantifier languages

- FSA recognizable quantifiers → Regular quantifier languages

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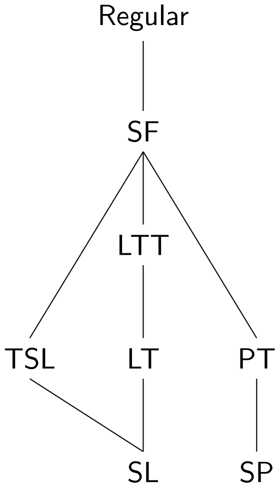
The Subregular Hierarchy

Often Forgotten:

- hierarchy of subregular languages (McNaughton&Papert 1971), (Rogers et al. 2010)

A Richness of Results

- Phonology is subregular (Heinz&Idsardi 2013, Heinz 2015)
- Morphotactics is subregular (Aksënova et al. 2016, Chandlee 2016)
- Morphology? (Aksënova&De Santo 2017)
- Syntax? (Graf&Heinz 2015)



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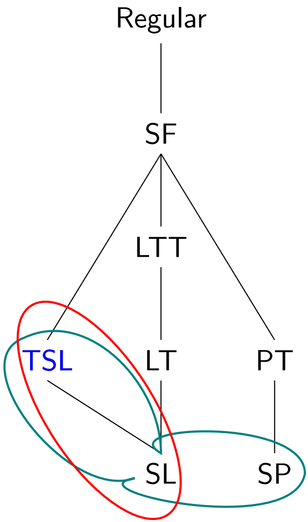

Strictly Local: Example

Word-Final Devoicing is SL

- SL grammars are lists of forbidden n-grams;
- Word-Final Devoicing: voiced segments at the end of a word are forbidden.

Word-Final Devoicing: German Example

- Grammar $S := \{{}^{*}z\ltimes,\ {}^{*}v\ltimes,\ {}^{*}d\ltimes\}$
- $\{\rtimes, \ltimes\} \rightarrow$ left and right word edge

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Tier-based Strictly Local (Heinz et al. 2011)

- TSL is a minimal expansion of SL
- Inspired by phonological tiers

TSL Grammars

- a projection function $E : \Sigma \rightarrow T \cup \lambda$, with $T \subseteq \Sigma$
- strictly local constraints over T

TSL Example: Sibilant harmony in Aari

- If multiple sibilants {s, z, ʒ, ʃ} occur in the same word, they must all be voiceless {s, ʃ} or voiced {z, ʒ} .

Grammar

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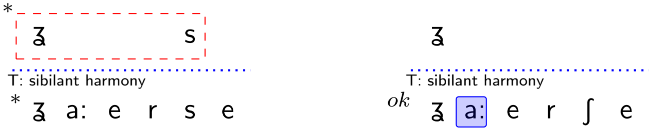

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ok ʒ a: e r ʃ e

TSL Example: Sibilant harmony in Aari

- If multiple sibilants {s, z, ʒ, ʃ} occur in the same word, they must all be voiceless {s, ʃ} or voiced {z, ʒ} .

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- T := {s, z, ʒ, ʃ}
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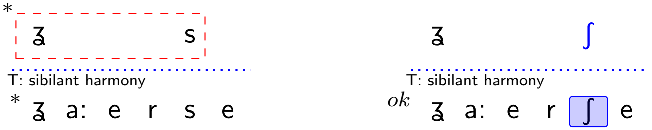

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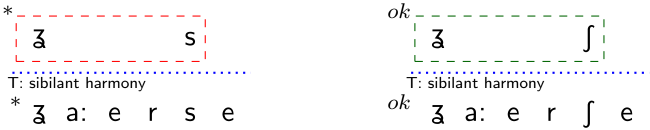

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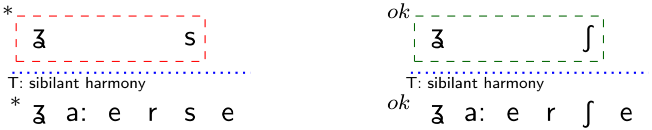

We can now use this perspective to look at quantifiers languages...

Subregular Quantifiers: *Every is SL*

Reminder: *Every*

- **every**(**A**, **B**) holds iff **A** $\subseteq$ **B**
- $L($**every**$) = \{1\}^*$
- Eg. *Every student cheated.*

False

student	John, Mary, Sue
cheat	John, Mary
binary strings	110, 101, 011
grammar	*0

True

student	John, Mary, Sue
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$\rtimes$ $\boxed{0\ 0\ 1}$ $\ltimes$

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cheat	
binary strings	000
grammar	??

$\rtimes$ $\boxed{0 \; 0^n \; 0}$ $\ltimes$

True

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False

student	John, Mary, Sue
cheat	
binary strings	000
grammar	$T = \{1\}$
	$S = \{{}^*\rtimes\ltimes\}$

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$$\rtimes \ 0 \ 0 \ 0 \ \ltimes$$

True

student	John, Mary, Sue
cheat	John,
binary strings	100, 010, 001
grammar	$T = \{1\}$
	$S = \{^* \rtimes \ltimes\}$

$$\boxed{\rtimes \quad 1} \quad \ltimes$$

$$\rtimes \ 0 \ 1 \ 0 \ \ltimes$$

Subregular Quantifiers: *Some* is TSL

Reminder: *some*

- **some**(**A**, **B**) holds iff **A** $\cap$ **B** $\neq \emptyset$
- $L($**some**$) = \{0,1\}^* \, 1 \, \{0,1\}^*$
- Eg. *Some student cheated.*

False

student	John, Mary, Sue
cheat	
binary strings	000
grammar	$T = \{1\}$
	$S = \{{}^*\rtimes\ltimes\}$

F $\rtimes \quad \ltimes$

$\rtimes \; 0 \; 0 \; 0 \; \ltimes$

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$\rtimes$ 0 1 0 $\ltimes$

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F $\boxed{\rtimes \qquad \ltimes}$

$\rtimes \quad 0 \quad 0^n \quad 0 \quad \ltimes$

True

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cheat	John,
binary strings	100, 010, 001
grammar	$T = \{1\}$
	$S = \{{}^*\rtimes\ltimes\}$

T $\boxed{\rtimes \quad 1}\boxed{1 \quad \ltimes}$

$\rtimes \quad 0^n \quad 1 \quad 0^n \quad \ltimes$

Parity Quantifiers?

An even number

- **An even number**(**A**, **B**) holds iff $|\mathbf{A} \cap \mathbf{B}| \geq 2n$, with $n > 0$
- $L(\textbf{even}) = \{w \in 0, 1^* s.t. |1|_w \geq 2n$, with $n > 0\}$

Is $L(\textbf{even})$ a TSL language?

F 1 1 1 0 0 T 1 1 1 1 0 F 1 1 1 1 1

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F 1 1 1 0 0   T 1 1 1 1 0   F 1 1 1 1 1

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....

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Is $L(\textbf{even})$ a TSL language?

1

F $\boxed{1}$ 1 1 0 0

T 1 1 1 1 0

F 1 1 1 1 1

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1 1

F 1 $\boxed{1}$ 1 0 0

T 1 1 1 1 0

F 1 1 1 1 1

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1 1 1

[F] 1 1 $\boxed{1}$ 0 0 [T] 1 1 1 1 0 [F] 1 1 1 1 1

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Is $L(\textbf{even})$ a TSL language?

1 1 1

F 1 1 1 $\boxed{0}$ 0

T 1 1 1 1 0

F 1 1 1 1 1

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1 1 1

F 1 1 1 0 $\boxed{0}$ T 1 1 1 1 0 F 1 1 1 1 1

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F $\boxed{1 \quad 1 \quad 1}$

F 1 1 1 0 0

T 1 1 1 1 0

F 1 1 1 1 1

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F 1 1 1 0 0

1

T $\boxed{1}$ 1 1 1 0

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F $\boxed{1\ 1\ 1}$

F 1 1 1 0 0

1 1

T 1 $\boxed{1}$ 1 1 0

F 1 1 1 1 1

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T 1 1 $\boxed{1}$ 1 0

F 1 1 1 1 1

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F $\boxed{1 \quad 1 \quad 1}$ 1 1 1 1

F 1 1 1 0 0   T 1 1 1 $\boxed{1}$ 0   F 1 1 1 1 1

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F $\boxed{1\ 1\ 1}$ $\qquad$ 1 1 1 1

F 1 1 1 0 0 $\qquad$ T 1 1 1 1 $\boxed{0}$ $\qquad$ F 1 1 1 1 1

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F $\boxed{1 \quad 1 \quad 1}$ F $\boxed{1 \quad 1 \quad 1}$ 1

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F $\boxed{1\ 1\ 1}$

T $\boxed{1\ 1\ 1\ 1}$

F 1 1 1 0 0

T 1 1 1 1 0

F 1 1 1 1 1

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F 1 1 1	T 1 1 1 1	1
F 1 1 1 0 0	T 1 1 1 1 0	F $\boxed{1}$ 1 1 1 1

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F $\boxed{1\ 1\ 1}$	T $\boxed{1\ 1\ 1\ 1}$	1 1
F 1 1 1 0 0	T 1 1 1 1 0	F 1 $\boxed{1}$ 1 1 1

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F $\boxed{1\ 1\ 1}$	T $\boxed{1\ 1\ 1\ 1}$	1 1 1
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F $\boxed{1\ 1\ 1}$	T $\boxed{1\ 1\ 1\ 1}$	1 1 1 1
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F $\boxed{1\ 1\ 1}$	T $\boxed{1\ 1\ 1\ 1}$	1 1 1 1 1
F 1 1 1 0 0	T 1 1 1 1 0	F 1 1 1 1 $\boxed{1}$

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An even number

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Is $L($**even**$)$ a TSL language?

F $\boxed{1 \quad 1 \quad 1}$

F 1 1 1 0 0

T $\boxed{1 \quad 1 \quad 1 \quad 1}$

T 1 1 1 1 0

T $\boxed{1 \quad 1 \quad 1 \quad 1}$ 1

F 1 1 1 1 1

Parity Quantifiers?

An even number

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Is $L($**even**$)$ a TSL language?

F $\boxed{1 \ 1 \ 1}$ | T $\boxed{1 \ 1 \ 1 \ 1}$ | F $\boxed{1 \ 1 \ 1 \ 1 \ 1}$

F 1 1 1 0 0 | T 1 1 1 1 0 | F 1 1 1 1 1

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F 1 1 1	T 1 1 1 1	F 1 1 1 1 1
F 1 1 1 0 0	T 1 1 1 1 0	F 1 1 1 1 1

Since n is arbitrary, there is **no general TSL grammar** that can generate $L($**even**$)$.

Characterization of Quantifier Languages: Summary

Language	Constraint	Complexity	Subregular Grammar
every	$\lvert 0\rvert_w = 0$	SL-1	**S** $:= \{\neg 0\}$
no	$\lvert 1\rvert_w = 0$	SL-1	**S** $:= \{\neg 1\}$
some	$\lvert 1\rvert_w \geq 1$	TSL-2	**T** $:= \{1\}$, **S** $:= \{\neg \rtimes \ltimes\}$
not all	$\lvert 0\rvert_w \geq 1$	TSL-2	**T** $:= \{0\}$, **S** $:= \{\neg \rtimes \ltimes\}$
(at least) n	$\lvert 1\rvert_w \geq n$	TSL-$(n+1)$	**T** $:= \{1\}$, **S** $:= \left\{\neg \rtimes 1^k \ltimes\right\}_{k \leq n}$
(at most) n	$\lvert 1\rvert_w \leq n$	TSL-$(n+1)$	**T** $:= \{1\}$, **S** $:= \left\{\neg 1^{k+1}\right\}$
all but n	$\lvert 0\rvert_w = n$	TSL-$(n+1)$	**T** $:= \{0\}$, **S** $:= \left\{\neg 0^{n+1}, \neg \rtimes 0^k \ltimes\right\}_{k \leq n}$
even number	$\lvert 1\rvert_w = 2n,\ n \geq 0$	regular	**impossible**
most	$\lvert 1\rvert_w \geq \lvert 0\rvert_w$	context-free	**impossible**

A Complexity Hierarchy (Revisited)

- Semantic Automata predictions

FSA | **PSA**

$\boxed{\{\textit{All, Some}\} < \{\textit{Even, Odd}\} < \{\textit{At least n, At most n}\}} < \boxed{\{\textit{Less than half, More than half, Most}\}}$

- Subregular characterization predictions

SL | **TSL** | **REG** | **CF**

$\boxed{\{\textit{All}\}} < \boxed{\{\textit{Some, At least n, At most n}\}} < \boxed{\{\textit{Even, Odd}\}} < \boxed{\{\textit{Less than half, More than half, Most}\}}$

Automata vs Quantifier Languages

- cardinal < parity;
- complexity independent of the specific recognition machine

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- Semantic Automata predictions

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$\{$*All, Some*$\} < \{$*Even, Odd*$\} < \{$*At least n, At most n*$\}$	$< \{$*Less than half, More than half, Most*$\}$

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$\{All, Some\} < \{Even, Odd\} < \{At\ least\ n, At\ most\ n\}$			$< \{Less\ than\ half, More\ than\ half, Most\}$

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SL	TSL	REG	CF
$\{All\}$	$< \{Some, At\ least\ n, At\ most\ n\}$	$< \{Even, Odd\}$	$< \{Less\ than\ half, More\ than\ half, Most\}$

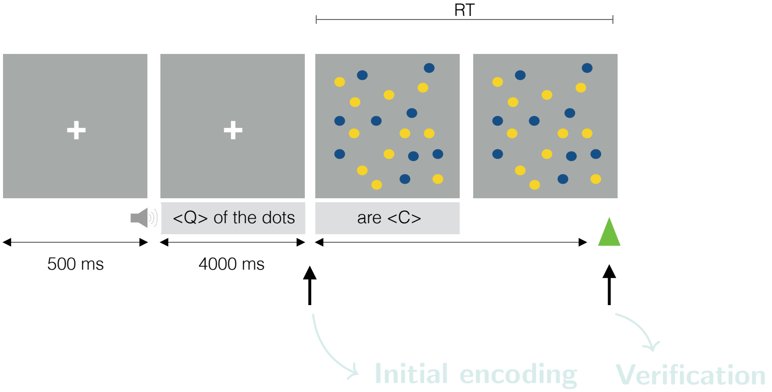
Automata vs Quantifier Languages

- cardinal < parity;
- complexity independent of the specific recognition machine
- **what's the cognitive reality of this predictions?**

Formal Complexity and Cognition

- FO-quantifiers vs higher order quantifiers
  - neuroimaging (McMillan et al. 2005, Clark & Grossman 2007)
  - patient literature (McMillan et al. 2009, Troiani et al 2009,)
- Psycholinguistic evidence for semantic automata
  - many behavioral findings (Szymanik & Zajenkowsky 2009, 2010, Steinert-Threlkeld & Icard 2013, i.a.)
  - for a survey: Szymanik (2016)
- Subregular hierarchy and cognition
  - general discussion(Rogers et al. 2013)
  - animal vs human cognition (Pulum & Rogers 2006, Rogers & Pullum 2011)
  - learnability and acquisition (Lai 2015, Avcu 2017)

Testing the Subregular Predictions

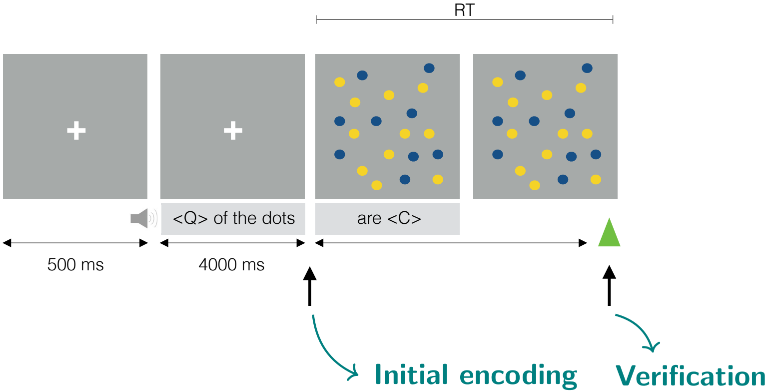


- McMillan et al. (2005)

Disentangling encoding from verification

- Pupil size ← ongoing...
- ERP, fMRI, MEG, ...

Testing the Subregular Predictions



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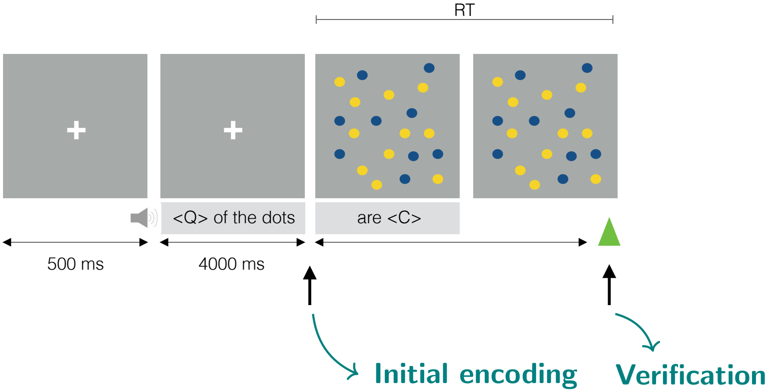

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Disentangling **encoding** from **verification**

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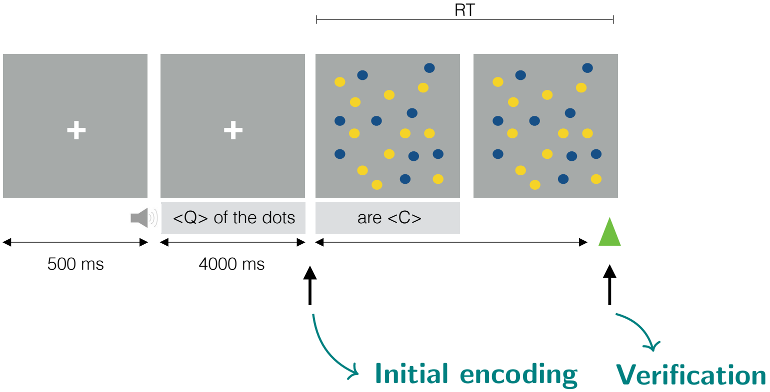
Testing the Subregular Predictions



- McMillan et al. (2005)

Disentangling encoding from verification

- Pupil size ← **ongoing...**
- ERP, fMRI, MEG, ...

Conclusions

Tracing Back our Steps

- SA as a plausible model of quantifier complexity
- Refined by looking at weaker classes in the Chomsky hierarchy
  $\Rightarrow$ **subregular characterization of generalized quantifiers**
- fine-grained complexity distinctions intrinsic to quantifiers' specification

Outcomes & Future Work

- Computational complexity and cognition
  - strong, cross-domain linking hypothesis
  - new experimental paradigms
- Support for cross-domain subregular generalizations
  - typological predictions (Graf 2017)
  - insights on learnability/acquisition
- New theoretical questions
  - e.g. permutation closure & subregular languages?

Conclusions

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Selected References I

1 **Aksënova, Alëna, Thomas Graf, and Sedigheh Moradi**. 2016. Morphotactics as tier-based strictly local dependencies. In Proceedings of SIGMorPhon 2016.

2 **Aksënova, Alëna and Aniello De Santo**. 2017. Strict Locality in morphological derivations. In Proceedings of CLS53 2017.

3 **Avcu Enes**. 2017. Experimental investigation of the subregular hierarchy. In Proceedings of PLC41, 2017.

4 **van Benthem, Johan**. 1986. Semantic automata. In Essays in logical semantics, 151?176. Dordrecht: Springer.

5 **Chandlee, Jane**. 2016. Computational locality in morphological maps. Ms., Haverford College.

6 **Graf, Thomas**. 2017. The subregular complexity of monomorphemic quantifiers. Ms., Stony Brook University.

7 **Graf, Thomas and Heinz, Jeffrey**. 2016. *Tier-based strictly locality in phonology and syntax*. Ms., Stony Brook University and University of Delaware.

8 **Heinz, Jeffrey**. 2015. *The Computational Nature of Phonological Generalizations*. Ms., University of Delaware.

9 **Heinz, Jeffrey**, Chetan Rawal, and Herbert G. Tanner. 2011.Tier-based strictly local constraints in phonology. In *Proceedings of ACL 49th*, 58–64.

10 **Kaplan, Ronald M., and Martin Kay**. 1994. Regular models of phonological rule systems. *Computational Linguistics* 20:331?378.

11 **Karttunen, Lauri, Ronald M. Kaplan, and Annie Zaenen**. 1992. Two-level morphology with composition. In *COLING'92,* 141-148.

12 **Just, M. A., P. A. Carpenter, and A. Miyake**. 2003. Neuroindices of cognitive workload: Neuroimaging, pupillometric and event-related potential.

13 **Lai, Regine**. 2015. Learnable vs. unlearnable harmony patterns. Linguistic Inquiry 46:425?451.

Selected References II

14 **Marr, David** (1983). Vision: A Computational Investigation into the Human Representation and Processing Visual Information. San Francisco: W. H. Freeman.

15 **McMillan Corey T. , Robin Clark, Peachie Moore, Christian Devita, and Murray Grossman**. 2005. Neural basis for generalized quantifier comprehension. Neuropsychologia, 43(12):1729?1737, Jan2005.

16 **McNaughton, Robert, and Seymour Papert**. 1971. Counter-free automata. Cambridge, MA: MIT Press.

17 **Mostowski, M.** 1998. Computational semantics for monadic quantifiers. Journal of Applied Non-Classical Logics, 8, 107–121.

18 **Pullum, Geoffrey K., and James Rogers**. 2006. Animal pattern-learning experiments: Some mathematical background. Ms., Radcliffe Institute for Advanced Study, Harvard University.

19 **Rogers, J., J. Heinz, M. Fero, J. Hurst, D. Lambert, and S. Wibel** (2013). Cognitive and Subregular Complexity, Chapter Formal Grammar, pp. 90?108. Springer.

20 **Rogers, James and Geoffrey Pullum**. 2007. Aural Pattern Recognition Experiments and the Subregular Hierarchy. In Proc. of *Mathematics of Language 10*, 1 –16.

21 **Shieber, Stuart M.** 1985. Evidence against the context-freeness of natural language. *Linguistics and Philosophy*. 8:333–345.

22 **Steinert-Threlkeld Shane and Thomas F. Icard III**. 2013. Iterating Semantic Automata. Linguistics and Philosophy, 2013.

23 **Szymanik Jakub and Marcin Zajenkowski**. 2010. Comprehension of simple quantifiers: empirical evaluation of a computational model. Cognitive Science, 34(3):521–532, April 2010.

24 **Szymanik Jakub and Marcin Zajenkowski**. 2011. Contribution of working memory in parity and proportional judgments. Belgian Journal of Linguistics, 25(1):176–194, January 2011.

25 **Szymanik, Jacub**. 2016. Quantifiers and Cognition: Logical and Computational Perspectives. Springer International Publishing.

26 **Troiani, V., Peelle, J., Clark, R., and Grossman, M.** 2009. Is it logical to count on quantifiers?

Dissociable neural networks underlying numerical and logical quantifiers. Neuropsychologia, 47, 104–111.

Appendix

Logical Definability of Subregular Classes

	Regular	Monadic Second-Order Logic
	$\cup$	
Locally Threshold Testable $\subset$	Star Free	First-Order Logic
$\cup$	$\cup$	
Locally Testable	Piecewise Testable	Propositional Logic
$\cup$	$\cup$	
Strictly Local $\subset$ TSL	Strictly Piecewise	Conjunction of Negative Literals
$S/\triangleleft$	$</\triangleleft^+$	