



# Structure Sensitive Tier Projection: Applications and Formal Properties

Aniello De Santo & Thomas Graf

Stony Brook University  
[aniello.desanto@stonybrook.edu](mailto:aniello.desanto@stonybrook.edu)  
<https://aniellodesanto.github.io/>

FG 24th  
Riga, August 11, 2019

# The Paper in a Nutshell

## The research program

- ▶ a tight upper bound to the complexity of natural language dependencies?

## Subregular Hypothesis for Phonology

- ▶ Tier-based Strictly Local seems to be the right fit;
- ▶ But ... several outliers have been reported!

## In This Talk

We explore (minimal) extensions to TSL

# Outline

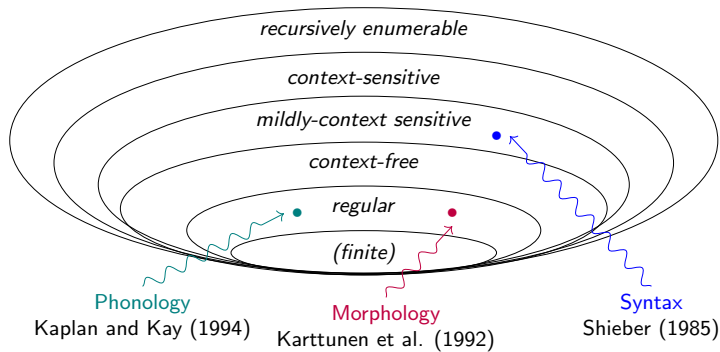
**1** Preliminaries

**2** Non-local Dependencies

**3** ITSL

**4** Conclusions

# Computational Theories of Language



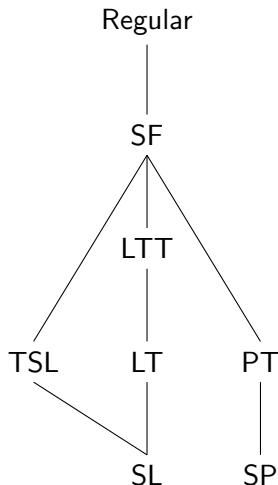
## Precise predictions for:

- ▶ typology → e.g. no center embedding in phonology
- ▶ learnability → e.g. no Gold learning for regular languages
- ▶ cognition → e.g. finitely bounded working memory

# Phonology as a Subregular System

Often forgotten: hierarchy of **subregular languages**

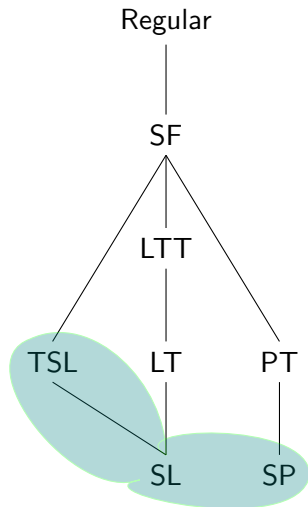
(McNaughton and Papert 1971; Rogers et al. 2010; Heinz et al. 2011; Graf 2017)



# Phonology as a Subregular System

Often forgotten: hierarchy of **subregular languages**

(McNaughton and Papert 1971; Rogers et al. 2010; Heinz et al. 2011; Graf 2017)



# Local Dependencies in Phonology are SL

## 1 Word-final devoicing

Forbid voiced segments at the end of a word

- (1) a. \*rad  
b. rat

### Example: Word-final devoicing

- ▶ Forbid word final voiced segments: \*[+voice]\$
- ▶ **German:** \***z**\$, \***v**\$, \***d**\$ (\$ = word edge).

\$ r a **d** \$            \$ r a t \$

# Local Dependencies in Phonology are SL

## 1 Word-final devoicing

Forbid voiced segments at the end of a word

- (1) a. \*rad  
b. rat

### Example: Word-final devoicing

- ▶ Forbid word final voiced segments:  $*[+voice]\$$
- ▶ **German:** \***z**€, \***v**€, \***d**€ (€ = word edge).

\*    €   r   a   **d** €    *ok*    €   r   a   t €



# Unbounded Dependencies Are Not SL

## ► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (2) a. \* ha **s**xintilawa **ʃ**  
b. \* ha **ʃ**xintilawa **s**  
c. ha **ʃ**xintilawa **ʃ**

Example: Samala

\*\$ ha **s**xintilawa **ʃ**\$

\$ ha **ʃ**xintilawa **ʃ**\$

# Unbounded Dependencies Are Not SL

## ► **Samala Sibilant Harmony**

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (2) a. \* ha **s**xintilawa **ʃ**  
 b. \* ha **ʃ**xintilawa **s**  
 c. ha **ʃ**xintilawa **ʃ**

Example: Samala

\*\$ ha **s**xintilawa **ʃ**\$

\$ ha **ʃ**xintilawa **ʃ**\$

# Unbounded Dependencies Are Not SL

## ► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (2) a. \* ha<sup>s</sup>xintilawa<sup>f</sup>  
 b. \* ha<sup>f</sup>xintilawa<sup>s</sup>  
 c. ha<sup>f</sup>xintilawa<sup>f</sup>

Example: Samala

\*\$ ha<sup>s</sup>xintilawa<sup>f</sup>\$  
 \$ ha<sup>f</sup>xintilawa<sup>f</sup>\$

# Unbounded Dependencies Are Not SL

## ► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (2) a. \* ha<sup>s</sup>xintilawa<sup>f</sup>  
b. \* ha<sup>f</sup>xintilawa<sup>s</sup>  
c. ha<sup>f</sup>xintilawa<sup>f</sup>

Example: Samala

\*\$ ha<sup>s</sup>xintilawa<sup>f</sup>\$  
\$ ha<sup>f</sup>xintilawa<sup>f</sup>\$

# Unbounded Dependencies Are Not SL

## ► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (2) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

### Example: Samala

\*\$ ha<sup>s</sup>xintilawa<sup>ʃ</sup>\$  
 \$ ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>\$

► **But:** Sibilants can be arbitrarily far away from each other!

\*\$<sup>s</sup>tajanowonwa<sup>ʃ</sup>\$

# Unbounded Dependencies Are Not SL

## ► Samala Sibilant Harmony

Sibilants must not disagree in anteriority.

(Applegate 1972)

- (2) a. \* ha<sup>s</sup>xintilawa<sup>f</sup>  
 b. \* ha<sup>f</sup>xintilawa<sup>s</sup>  
 c. ha<sup>f</sup>xintilawa<sup>f</sup>

### Example: Samala

\* \$ ha<sup>s</sup>xintilawa<sup>f</sup> \$

\$ ha<sup>f</sup>xintilawa<sup>f</sup> \$

► **But:** Sibilants can be arbitrarily far away from each other!

\* \$ <sup>s</sup>tajanowonwa<sup>f</sup> \$

# Locality Over Tiers

\* \$ **s** t a j a n o w o n w a **j** \$

- ▶ Sibilants can be arbitrarily far away from each other!
- ▶ **Problem:** SL limited to locality domains of size  $n$ ;
- ▶ **Solution:** locality over **tiers**. (Goldsmith 1976)

Tier-based Strictly Local (TSL) Grammars (Heinz et al. 2011)

- ▶  $E_T$ : Projection of selected segments on a tier  $T$ ;
- ▶ Strictly local constraints over  $T$  determine wellformedness;

# Locality Over Tiers

\* \$ **s** t a j a n o w o n w a **j** \$

- ▶ Sibilants can be arbitrarily far away from each other!
- ▶ **Problem:** SL limited to locality domains of size  $n$ ;
- ▶ **Solution:** locality over **tiers**. (Goldsmith 1976)

## Tier-based Strictly Local (TSL) Grammars (Heinz et al. 2011)

- ▶  $E_T$ : Projection of selected segments on a tier  $T$ ;
- ▶ Strictly local constraints over  $T$  determine wellformedness;



# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3)
- a. \* ha **s**xintilawa **ʃ**
  - b. \* ha **ʃ**xintilawa **s**
  - c. ha **ʃ**xintilawa **ʃ**

- ▶ What do we need to project? {**s**, **z**, **ʃ**, **ʒ**}
- ▶ What do we need to ban?

\* **sʃ**, \* **sʒ**, \* **zʃ**, \* **zʒ**, \* **ʃs**, \* **ʒs**, \* **ʃz**, \* **ʒz**

Example: TSL Samala

\* \$ha **s**xintilawa **ʃ**\$

*ok* \$ha **ʃ**xintilawa **ʃ**\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3)
- a. \* ha **s**xintilawa **ʃ**
  - b. \* ha **ʃ**xintilawa **s**
  - c. ha **ʃ**xintilawa **ʃ**

- ▶ What do we need to project? {**s**, **z**, **ʃ**, **ʒ**}
- ▶ What do we need to ban?

\* **sʃ**, \* **sʒ**, \* **zʃ**, \* **zʒ**, \* **ʃs**, \* **ʒs**, \* **ʃz**, \* **ʒz**

## Example: TSL Samala

.....

\* **ʃ**ha **s**xintilawa **ʃ**\$

*ok* \$ha **ʃ**xintilawa **ʃ**\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʃ, \*<sup>z</sup>ʃ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

\* \$<sup>h</sup>a<sup>s</sup>xintilawa<sup>ʃ</sup>\$

*ok* \$ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3)
- a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>
  - b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>
  - c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup><sup>ʃ</sup>, \*<sup>s</sup><sup>ʒ</sup>, \*<sup>ʃ</sup><sup>ʃ</sup>, \*<sup>z</sup><sup>ʃ</sup>, \*<sup>ʃ</sup><sup>s</sup>, \*<sup>ʒ</sup><sup>s</sup>, \*<sup>ʃ</sup><sup>z</sup>, \*<sup>ʒ</sup><sup>z</sup>

## Example: TSL Samala

\* \$ha<sup>a</sup>s xintilawa<sup>ʃ</sup>\$

*ok* \$ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʃ, \*<sup>z</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>xintilawa<sup>ʃ</sup>\$

*ok* \$ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʃ, \*<sup>z</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>xintilawa<sup>ʃ</sup>\$

*ok* \$ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʃ, \*<sup>z</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>x<sup>i</sup>ntilawa<sup>ʃ</sup>\$

<sup>ok</sup> \$ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3)
- a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>
  - b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>
  - c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʃ, \*<sup>z</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>xin<sup>n</sup>tilawa<sup>ʃ</sup>\$

<sup>ok</sup> \$ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>\$



# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʃ, \*<sup>z</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>xin<sup>t</sup>ilawa<sup>ʃ</sup>\$

<sup>ok</sup> \$ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʃ, \*<sup>z</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>xint<sup>i</sup>law<sup>ʃ</sup>\$

<sup>ok</sup> \$ha<sup>ʃ</sup>xintilaw<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>
- b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>
- c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʃ, \*<sup>z</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>xinti<sup>ʃ</sup>aw<sup>ʃ</sup>\$

<sup>ok</sup> \$ha<sup>ʃ</sup>xintilaw<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>
- b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>
- c.    ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʒ, \*<sup>z</sup>ʃ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>xintila<sup>a</sup>w<sup>ʃ</sup>\$

<sup>ok</sup> \$ha<sup>ʃ</sup>xintilaw<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3)
- a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>
  - b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>
  - c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʃ, \*<sup>z</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>xintila<sup>w</sup>ʃ\$

<sup>ok</sup> \$ha<sup>ʃ</sup>xintilawʃ\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>ʃ</sup>ʒ, \*<sup>z</sup>ʃ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

## Example: TSL Samala

<sup>s</sup>

\* \$ha<sup>s</sup>xintilaw<sup>ʃ</sup>\$

*ok* \$ha<sup>ʃ</sup>xintilaw<sup>ʃ</sup>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sub>f</sub>  
 b. \* ha<sub>f</sub>xintilawa<sup>s</sup>  
 c. ha<sub>f</sub>xintilawa<sub>f</sub>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sub>f</sub>, <sub>3</sub>}
- ▶ What do we need to ban?

\*<sup>s</sup><sub>f</sub>, \*<sup>s</sup><sub>3</sub>, \*<sup>z</sup><sub>f</sub>, \*<sup>z</sup><sub>3</sub>, \*<sub>f</sub><sup>s</sup>, \*<sub>3</sub><sup>s</sup>, \*<sub>f</sub><sup>z</sup>, \*<sub>3</sub><sup>z</sup>

## Example: TSL Samala

<sup>s</sup>                      <sub>f</sub>  
 .....  
 \* \$ha<sup>s</sup>xintilawa<sub>f</sub><sub>\$</sub>

*ok* \$ha<sub>f</sub>xintilawa<sub>f</sub>\$

# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha<sup>s</sup>xintilawa<sup>ʃ</sup>  
 b. \* ha<sup>ʃ</sup>xintilawa<sup>s</sup>  
 c. ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>

- ▶ What do we need to project? {<sup>s</sup>, <sup>z</sup>, <sup>ʃ</sup>, <sup>ʒ</sup>}
- ▶ What do we need to ban?

\*<sup>s</sup>ʃ, \*<sup>s</sup>ʒ, \*<sup>z</sup>ʃ, \*<sup>z</sup>ʒ, \*<sup>ʃ</sup>s, \*<sup>ʒ</sup>s, \*<sup>ʃ</sup>z, \*<sup>ʒ</sup>z

Example: TSL Samala

.....  
 .....<sup>s</sup>.....<sup>ʃ</sup>.....

\* \$ha<sup>s</sup>xintilawa<sup>ʃ</sup>\$

*ok* \$ha<sup>ʃ</sup>xintilawa<sup>ʃ</sup>\$



# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony

- (3) a. \* ha **s** x i n t i l a w a **ʃ**
- b. \* ha **ʃ** x i n t i l a w a **s**
- c.    ha **ʃ** x i n t i l a w a **ʃ**

- ▶ What do we need to project? {**s**, **z**, **ʃ**, **ʒ**}
- ▶ What do we need to ban?

\* **sʃ**, \* **sʒ**, \* **zʃ**, \* **zʒ**, \* **ʃs**, \* **ʒs**, \* **ʃz**, \* **ʒz**

## Example: TSL Samala

\* \$ ha **s** x i n t i l a w a **ʃ** \$

*ok* \$ ha **ʃ** x i n t i l a w a **ʃ** \$



# Unbounded Dependencies are TSL

- ▶ Let's revisit Samala Sibilant Harmony


- (3) a. \* ha **s** x i n t i l a w **ʃ**
- b. \* ha **ʃ** x i n t i l a w a **s**
- c.    ha **ʃ** x i n t i l a w a **ʃ**

- ▶ What do we need to project? {**s**, **z**, **ʃ**, **ʒ**}
- ▶ What do we need to ban?


\* **sʃ**, \* **sʒ**, \* **zʃ**, \* **zʒ**, \* **ʃs**, \* **ʒs**, \* **ʃz**, \* **ʒz**

Example: TSL Samala

\* \$ ha **s** x i n t i l a w **ʃ** \$



*ok* \$ ha **ʃ** x i n t i l a w **ʃ** \$



# TSL: Why Do We Care?

- ▶ Linguistically natural (Goldsmith 1976)
- ▶ Captures wide range of phonotactic dependencies (McMullin 2016)
- ▶ Efficiently learnable from positive data (Jardine and Heinz 2016)
- ▶ Low resource demands
- ▶ Rules out unattested patterns (cf. Aksënova et al. 2016)

# TSL: Why Do We Care?

- ▶ Linguistically natural (Goldsmith 1976)
- ▶ Captures wide range of phonotactic dependencies (McMullin 2016)
- ▶ Efficiently learnable from positive data (Jardine and Heinz 2016)
- ▶ Low resource demands
- ▶ Rules out unattested patterns (cf. Aksënova et al. 2016)
- ▶ **But** not every long-distance pattern is TSL!  
(McMullin 2016; Mayer and Major 2018; Graf and Mayer 2018)

# Full Samala: A TSL Outlier

## Sibilant Harmony in SAMALA (McMullin 2016)

### 1) Unbounded sibilant harmony

- |   |                                   |                   |
|---|-----------------------------------|-------------------|
| a. /k- <b>s</b> u- <b>ʃ</b> ojin/           | k <b>ʃ</b> u <b>ʃ</b> ojin        | "I darken it"     |
| b. /k- <b>s</b> u-k'ili-mekeken- <b>ʃ</b> / | k <b>ʃ</b> uk'ilimekeket <b>ʃ</b> | "I straighten up" |

### 2) /s/ → [ʃ] when preceding (adjacent) [t, n, l]

- |                        |                 |              |
|------------------------|-----------------|--------------|
| a. / <b>s</b> -lok'in/ | <b>ʃ</b> lok'in | "he cuts it" |
| b. / <b>s</b> -tepuʔ/  | <b>ʃ</b> tepuʔ  | "he gambles" |

### 3) Long-distance agreement overrides local disagreement

- |   |               |                     |
|---|---------------|---------------------|
| a. / <b>s</b> -i <b>ʃ</b> t-i <b>ʃ</b> ti-jep-us/ | sististijepus | "they show him"     |
| b. / <b>s</b> -net-us/                            | snetus        | "he does it to him" |

Is there a TSL grammar for the complete pattern?

# Full Samala: A TSL Outlier

## Sibilant Harmony in SAMALA (McMullin 2016)

### 1) Unbounded sibilant harmony

- |   |                                   |                   |
|---|-----------------------------------|-------------------|
| a. /k- <b>su</b> - <b>ʃ</b> ojin/           | k <b>ʃ</b> u <b>ʃ</b> ojin        | “I darken it”     |
| b. /k- <b>su</b> -k’ili-mekeken- <b>ʃ</b> / | k <b>ʃ</b> uk’ilimekeket <b>ʃ</b> | “I straighten up” |

### 2) /s/ → [ʃ] when preceding (adjacent) [t, n, l]

- |                                 |                 |              |
|---------------------------------|-----------------|--------------|
| a. / <b>s</b> - <b>l</b> ok’in/ | <b>ʃ</b> lok’in | “he cuts it” |
| b. / <b>s</b> - <b>t</b> epuʔ/  | <b>ʃ</b> tepuʔ  | “he gambles” |

### 3) Long-distance agreement overrides local disagreement

- |   |               |                     |
|---|---------------|---------------------|
| a. / <b>s</b> -i <b>ʃ</b> t-i <b>ʃ</b> ti-jep-us/ | sististijepus | “they show him”     |
| b. / <b>s</b> - <b>n</b> et-us/                   | snetus        | “he does it to him” |

Is there a TSL grammar for the complete pattern?

# Full Samala: A TSL Outlier

## Sibilant Harmony in SAMALA (McMullin 2016)

### 1) Unbounded sibilant harmony

- |   |                                   |                   |
|---|-----------------------------------|-------------------|
| a. /k- <b>s</b> u- <b>ʃ</b> ojin/           | k <b>ʃ</b> u <b>ʃ</b> ojin        | “I darken it”     |
| b. /k- <b>s</b> u-k’ili-mekeken- <b>ʃ</b> / | k <b>ʃ</b> uk’ilimekeket <b>ʃ</b> | “I straighten up” |

### 2) /s/ → [ʃ] when preceding (adjacent) [t, n, l]

- |                        |                 |              |
|------------------------|-----------------|--------------|
| a. / <b>s</b> -lok’in/ | <b>ʃ</b> lok’in | “he cuts it” |
| b. / <b>s</b> -tepuʔ/  | <b>ʃ</b> tepuʔ  | “he gambles” |

### 3) Long-distance agreement overrides local disagreement

- |   |                       |                     |
|---|-----------------------|---------------------|
| a. / <b>s</b> -i <b>ʃ</b> t-i <b>ʃ</b> ti-jep-us/ | <b>s</b> ististijepus | “they show him”     |
| b. / <b>s</b> -net-us/                            | <b>s</b> netus        | “he does it to him” |

Is there a TSL grammar for the complete pattern?



# Full Samala: A TSL Outlier

## Sibilant Harmony in SAMALA (McMullin 2016)

### 1) Unbounded sibilant harmony

- |   |                                   |                   |
|---|-----------------------------------|-------------------|
| a. /k- <b>s</b> u- <b>ʃ</b> ojin/           | k <b>ʃ</b> u <b>ʃ</b> ojin        | “I darken it”     |
| b. /k- <b>s</b> u-k’ili-mekeken- <b>ʃ</b> / | k <b>ʃ</b> uk’ilimekeket <b>ʃ</b> | “I straighten up” |

### 2) /s/ → [ʃ] when preceding (adjacent) [t, n, l]

- |                        |                 |              |
|------------------------|-----------------|--------------|
| a. / <b>s</b> -lok’in/ | <b>ʃ</b> lok’in | “he cuts it” |
| b. / <b>s</b> -tepuʔ/  | <b>ʃ</b> tepuʔ  | “he gambles” |

### 3) Long-distance agreement overrides local disagreement

- |   |                       |                     |
|---|-----------------------|---------------------|
| a. / <b>s</b> -i <b>ʃ</b> t-i <b>ʃ</b> ti-jep-us/ | <b>s</b> ististijepus | “they show him”     |
| b. / <b>s</b> -net-us/                            | <b>s</b> netus        | “he does it to him” |

Is there a TSL grammar for the complete pattern?

# Sibilant Harmony in SAMALA (1/3)

## Generalization

- ▶ Anticipatory Sibilant harmony

## Grammar

$T = \{s, \int, z, ʒ\}$   $S = \{*s\int, *\int s, \dots\}$

*ok*    k    ∫    u    ∫    o    j    i    n                      \*    k    s    u    ∫    o    j    i    n

# Sibilant Harmony in SAMALA (1/3)

## Generalization

- ▶ Anticipatory Sibilant harmony

## Grammar

$T = \{s, \int, z, ʒ\}$   $S = \{*s\int, *\int s, \dots\}$

.....

*ok*   k   ∫   u   ∫   o   j   i   n                      \*   k   s   u   ∫   o   j   i   n

# Sibilant Harmony in SAMALA (1/3)

## Generalization

- ▶ Anticipatory Sibilant harmony

## Grammar

$T = \{s, \int, z, ʒ\}$   $S = \{*\int s, *\int s, \dots\}$

$\int \quad \int$   
 .....  
*ok* k ∫ u ∫ o j i n      \* k s u ∫ o j i n

# Sibilant Harmony in SAMALA (1/3)

## Generalization

- ▶ Anticipatory Sibilant harmony

## Grammar

$T = \{s, \int, z, ʒ\}$   $S = \{*s\int, *\int s, \dots\}$

$ok$   $\int$   $\int$   
 .....  
 $ok$  k  $\int$  u  $\int$  o j i n      \* k s u  $\int$  o j i n

# Sibilant Harmony in SAMALA (1/3)

## Generalization

- ▶ Anticipatory Sibilant harmony

## Grammar

$T = \{s, \int, z, ʒ\}$   $S = \{*s\int, *\int s, \dots\}$

$ok$   $\int$   $\int$   
 .....  
 $ok$  k  $\int$  u  $\int$  o j i n      \* k s u  $\int$  o j i n

# Sibilant Harmony in SAMALA (1/3)

## Generalization

- ▶ Anticipatory Sibilant harmony

## Grammar

$T = \{s, \int, z, ʒ\}$   $S = \{*s\int, *\int s, \dots\}$

*ok*

*ok* k ∫ u ∫ o j i n

s      ∫

\* k s u ∫ o j i n

# Sibilant Harmony in SAMALA (1/3)

## Generalization

- ▶ Anticipatory Sibilant harmony

## Grammar

$$T = \{s, \int, z, 3\} \quad S = \{^*s\int, ^*\int s, \dots\}$$

*ok*

*ok* k ∫ u ∫ o j i n

*\**

*\** k s u ∫ o j i n



# Sibilant Harmony in SAMALA (2/3)

## Generalization

- ▶ Anticipatory Sibilant harmony
- ▶ Local restriction against [ \*sn, \*st, \*sl]

## Grammar

$T = \{s, \int, z, ʒ, \textcolor{red}{n}, \textcolor{red}{t}, \textcolor{red}{l}\}$   $S = \{*\textcolor{red}{s}\int, *\textcolor{red}{j}s, *\textcolor{red}{s}n, *\textcolor{red}{s}t, *\textcolor{red}{s}l, \dots\}$

*ok*  $\int$  t e p u ?

\* s t e p u ?

# Sibilant Harmony in SAMALA (2/3)

## Generalization

- ▶ Anticipatory Sibilant harmony
- ▶ Local restriction against [ \*sn, \*st, \*sl]

## Grammar

$T = \{s, \int, z, ʒ, \textcolor{red}{n}, \textcolor{red}{t}, \textcolor{red}{l}\}$   $S = \{*\int s, * \int s, *sn, *st, *sl, \dots\}$

$\int \quad t$   
 .....  
 ok ∫ t e p u ?                      \* s t e p u ?

# Sibilant Harmony in SAMALA (2/3)

## Generalization

- ▶ Anticipatory Sibilant harmony
- ▶ Local restriction against [ \*sn, \*st, \*sl]

## Grammar

$T = \{s, \int, z, ʒ, \textcolor{red}{n}, \textcolor{red}{t}, \textcolor{red}{l}\}$   $S = \{*\textcolor{red}{s}\int, *\textcolor{red}{\int}s, *\textcolor{red}{s}n, *\textcolor{red}{s}t, *\textcolor{red}{s}l, \dots\}$

*ok*  $\int$   $t$  .....

*ok*  $\int$   $t$  e p u ?

\* s t e p u ?

# Sibilant Harmony in SAMALA (2/3)

## Generalization

- ▶ Anticipatory Sibilant harmony
- ▶ Local restriction against [ \*sn, \*st, \*sl]

## Grammar

$T = \{s, \int, z, ʒ, \textcolor{red}{n}, \textcolor{red}{t}, \textcolor{red}{l}\}$   $S = \{*\int s, *\int s, *sn, *st, *sl, \dots\}$

*ok*  $\int$   $t$   
 .....  
*ok*  $\int$   $t$  e p u ?

s t  
 .....  
 \* s t e p u ?

# Sibilant Harmony in SAMALA (2/3)

## Generalization

- ▶ Anticipatory Sibilant harmony
- ▶ Local restriction against [ \*sn, \*st, \*sl]

## Grammar

$T = \{s, \int, z, ʒ, \textcolor{red}{n}, \textcolor{red}{t}, \textcolor{red}{l}\}$   $S = \{*\int s, *\int s, *\textcolor{red}{s}n, *\textcolor{red}{s}t, *\textcolor{red}{s}l, \dots\}$

*ok*  $\int$   $t$  \_\_\_\_\_  
*ok*  $\int$   $t$  e p u ?

\*  $s$   $t$  \_\_\_\_\_  
 \*  $s$   $t$  e p u ?

# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$

*ok* s n e t u s

# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$

s  
.....  
ok s n e t u s

# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$

s   n  
.....  
*ok* s   n   e   t   u   s



# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*sʃ$ ,  $*sʃ$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, ʃ, n, t, l\}$   $S = \{*sʃ, *sʃ, *sn, *st, *sl\}$

s   n  
.....  
*ok*   s   n   e   t   u   s

# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*sʃ$ ,  $*sʃ$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, ʃ, n, t, l\}$   $S = \{*sʃ, *sʃ, *sn, *st, *sl\}$

$s \quad n \quad t$   
 .....  
*ok*  $s \quad n \quad e \quad t \quad u \quad s$

# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$

$s \quad n \quad t$   
 .....  
*ok*  $s \quad n \quad e \quad t \quad \boxed{u} \quad s$

# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$

$s \quad n \quad t \quad s$   
 .....  
*ok*  $s \quad n \quad e \quad t \quad u \quad s$

# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$

\* s n t s  
 .....  
*ok* s n e t u s

# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$

$\begin{array}{ccccccc} & & ok & & & & \\ * & & \text{---} & \text{---} & \text{---} & & \\ & s & n & & t & & s \\ & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \\ & & & & & & \end{array}$

$\begin{array}{ccccccc} ok & & & & & & \\ s & n & e & t & u & s & \end{array}$

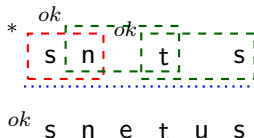
# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$



# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$

\* s n t s  
 .....  
*ok* s n e t u s



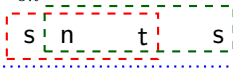
# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$

*ok*  
 $*$    
*ok* s n e t u s

# Sibilant Harmony in SAMALA (3/3)

## Generalization

- ▶ anticipatory sibilant harmony [ $*s\int$ ,  $*s\int$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

## Grammar

$T = \{s, \int, n, t, l\}$   $S = \{*s\int, *s\int, *sn, *st, *sl\}$



No TSL grammar can capture this pattern...

# Input-Sensitive TSL (ITSL) Languages

 $E_T$ 

Tier projection controlled by:

**1** label of segment

TSL

**1**

TSL languages are characterized by:

- ▶ a **1**-local projection function  $E_T$ ;
- ▶ strictly  $k$ -local constraints applied on  $T$ .

# Input-Sensitive TSL (ITSL) Languages

 $E_T$ 

Tier projection controlled by:

**1** label of segment

TSL

**1**

TSL languages are characterized by:

- ▶ a **1**-local projection function  $E_T$ ;
- ▶ strictly  $k$ -local constraints applied on  $T$ .

**Idea:**

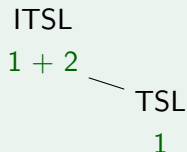
- ▶  $E_T$  is an input-strictly local transduction (Chandlee 2014)
- ▶ **What if:** the locality of  $E_T$  was **higher than 1**?

# Input-Sensitive TSL (ITSL) Languages

 $E_T$ 

Tier projection controlled by:

- 1 label of segment
- 2 local context



TSL languages are characterized by:

- ▶ a 1-local projection function  $E_T$ ;
- ▶ strictly  $k$ -local constraints applied on  $T$ .

**Idea:**

- ▶  $E_T$  is an input-strictly local transduction (Chandlee 2014)
- ▶ **What if:** the locality of  $E_T$  was higher than 1?

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*sʃ$ ,  $*sʃ$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

\$ s n e t u s \$

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*sʃ$ ,  $*sʃ$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

\$ s n e t u s \$

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*s\text{ʃ}$ ,  $*s\text{ʃ}$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

s  
.....  
\$ s n e t u s \$



# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*s\text{ʃ}$ ,  $*s\text{ʃ}$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

s n  
.....  
\$ s n e t u s \$

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*s\text{ʃ}$ ,  $*s\text{ʃ}$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

s   n  
.....  
\$ s n e t u s \$

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*s\text{ʃ}$ ,  $*s\text{ʃ}$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

s   n  
.....  
\$ s n e t u s \$

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*s\text{ʃ}$ ,  $*s\text{ʃ}$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

s   n  
.....  
\$ s n e t u s \$

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*s\text{ʃ}$ ,  $*s\text{ʃ}$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

s   n
s

.....

\$   s   n   e   t   u   s   \$

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*s\text{ʃ}$ ,  $*s\text{ʃ}$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

s   n                      s

.....

\$ s n e t u s \$

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*s\text{ʃ}$ ,  $*s\text{ʃ}$ ]
- ▶ palatalization to avoid local restriction [ $*sn$ ,  $*st$ ,  $*sl$ ]
- ▶ sibilant harmony overrides palatalization

*ok*  
 [ s   n                      s ]  
 .....  
 \$ s   n   e   t   u   s   \$

# An ITSL Account of Samala

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony [ $*s\text{f}$ ,  $*s\text{f}$ ]
- ▶ palatalization to avoid local restriction [ $*s\text{n}$ ,  $*s\text{t}$ ,  $*s\text{l}$ ]
- ▶ sibilant harmony overrides palatalization

$ok$   
 $\boxed{s \quad n \quad \quad \quad s}$   
 $\dots$   
 $\$ \quad s \quad n \quad e \quad t \quad u \quad s \quad \$$

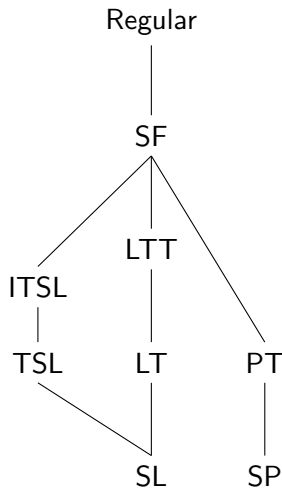
## Grammar

$$T = \{ \sigma : \sigma \in \{s, \text{f}\} \vee (\sigma \in \{n, t, l\} \wedge s \prec^+ \sigma) \}$$

$$S = \{ *s\text{f}, *s\text{f}, *s\text{n}(\neg s), *s\text{t}(\neg s), *s\text{l}(\neg s) \}$$



# ITSL: Relations to other Classes?



# Summing Up

## Tracing Back our Steps

- ▶ TSL as a strong upper bound to phonotactic complexity.
- ▶ but there are patterns that are unaccounted for!

## ITSL

- ▶ Natural generalization of TSL
- ▶ Covers a variety of patterns  
Korean vowel harmony, UTP, Yaka nasal harmony, ...
- ▶ Preserves TSL's computational properties
  - ▶ contained expressive power
  - ▶ Gold learnable  
Efficiently learnable? (cf. McMullin et al. 2019)
  - ▶ (lack of) closure properties

# Summing Up

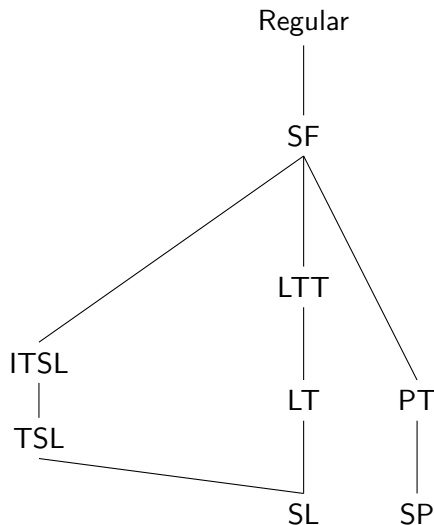
## Tracing Back our Steps

- ▶ TSL as a strong upper bound to phonotactic complexity.
- ▶ but there are patterns that are unaccounted for!

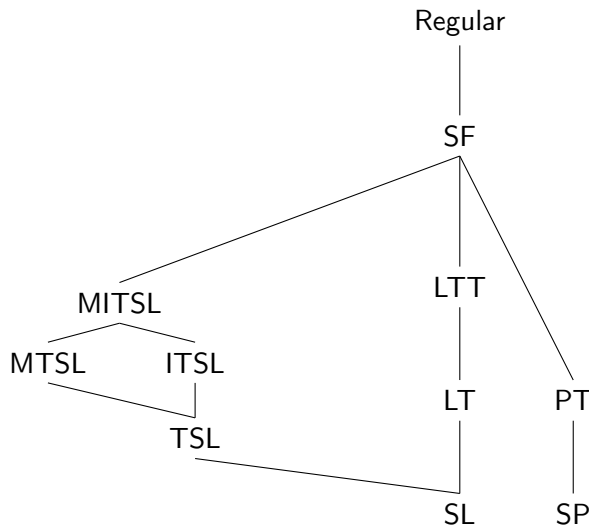
## ITSL

- ▶ Natural generalization of TSL
- ▶ Covers a variety of patterns  
Korean vowel harmony, UTP, Yaka nasal harmony, ...
- ▶ Preserves TSL's computational properties
  - ▶ contained expressive power
  - ▶ Gold learnable  
Efficiently learnable? (cf. McMullin et al. 2019)
  - ▶ (lack of) closure properties  
**Non-closure under intersection!**

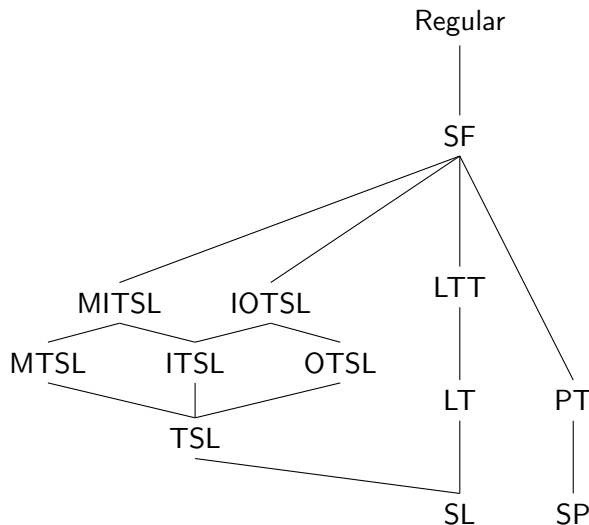
# Exploring the TSL Neighborhood



# Exploring the TSL Neighborhood



# Exploring the TSL Neighborhood



# Future Work

- ▶ Test typological predictions
- ▶ Cross-domain (syntax, semantics) generalizations  
(cf. Vu et al. 2019; Graf and Shafiei 2019; Graf and De Santo 2019)
- ▶ Further study of the TSL neighborhood  
(cf. Mayer and Major 2018; Graf and Mayer 2018)
- ▶ Learnability → learning algorithms, AGL experiments, NN?  
(Avcu 2017; McMullin et al. 2019; De Santo 2018)

# Future Work

- ▶ Test typological predictions
- ▶ Cross-domain (syntax, semantics) generalizations  
(cf. Vu et al. 2019; Graf and Shafiei 2019; Graf and De Santo 2019)
- ▶ Further study of the TSL neighborhood  
(cf. Mayer and Major 2018; Graf and Mayer 2018)
- ▶ Learnability → learning algorithms, AGL experiments, NN?  
(Avcu 2017; McMullin et al. 2019; De Santo 2018)

Thank you!



## Acknowledgments



This work was supported by the National Science Foundation under Grant No. BCS-1845344.

# References I

- Aksënova, Alëna, Thomas Graf, and Sedigheh Moradi. 2016. Morphotactics as tier-based strictly local dependencies. In *Proceedings of SIGMorPhon 2016*.
- Applegate, R.B. 1972. *Ineseno chumash grammar*. Doctoral Dissertation, UC Berkeley.
- Avcu, Enes. 2017. Experimental investigation of the subregular hierarchy. In *Proceedings of the 35th West Coast Conference on Formal Linguistics at Calgary, Alberta Canada*.
- Chandlee, Jane. 2014. *Strictly local phonological processes*. Doctoral Dissertation, U. of Delaware.
- De Santo, Aniello. 2018. Commentary: Developmental constraints on learning artificial grammars with fixed, flexible, and free word order. *Front. in Psychology* 9:276.
- Goldsmith, John. 1976. *Autosegmental phonology*. Doctoral Dissertation, MIT, Cambridge, MA.
- Graf, Thomas. 2017. The power of locality domains in phonology. *Phonology* 34:385–405. URL <https://dx.doi.org/10.1017/S0952675717000197>.
- Graf, Thomas, and Aniello De Santo. 2019. Sensing tree automata as a model of syntactic dependencies. In *Proceedings of the 16th Meeting on the Mathematics of Language*, 12–26.
- Graf, Thomas, and Connor Mayer. 2018. Sanskrit n-retroflexion is input-output tier-based strictly local. In *Proceedings of SIGMorPhon 2018*.
- Graf, Thomas, and Nazila Shafiei. 2019. C-command dependencies as TSL string constraints. In *Proceedings of SCiL 2019*, 205–215.

## References II

- Heinz, Jeffrey, Chetan Rawal, and Herbert Tanner. 2011. Tier-based strictly local constraints for phonology. In *Proceedings of the ACL 49th: Human Language Technologies: Short Papers - Volume 2*, 58–64. URL <http://dl.acm.org/citation.cfm?id=2002736.2002750>.
- Jardine, Adam, and Jeffrey Heinz. 2016. Learning tier-based strictly 2-local languages. *Transactions of the ACL* 4:87–98. URL <https://aclweb.org/anthology/Q/Q16/Q16-1007.pdf>.
- Mayer, Connor, and Travis Major. 2018. A challenge for tier-based strict locality from Uyghur backness harmony. In *Formal Grammar 2018. Lecture Notes in Computer Science*, vol. 10950, ed. A. Foret, G. Kobele, and S. Pogodalla, 62–83. Springer, Berlin, Heidelberg.
- McMullin, Kevin. 2016. *Tier-based locality in long-distance phonotactics?: learnability and typology*. Doctoral Dissertation, U. of British Columbia.
- McMullin, Kevin, Alëna Aksënova, and Aniello De Santo. 2019. Learning phonotactic restrictions on multiple tiers. *Proccedings of SCiL 2019* 2:377–378.
- McNaughton, Robert, and Seymour Papert. 1971. *Counter-free automata*. Cambridge: MIT Press.
- Rogers, James, Jeffrey Heinz, Gil Bailey, Matt Edlefsen, Molly Visscher, David Wellcome, and Sean Wibel. 2010. On languages piecewise testable in the strict sense. In *Lecture notes in artificial intelligence*, ed. G. Jäger C. Ebert and J. Michaelis, volume 6149, 255–265. Berlin: Springer.
- Vu, Mai Ha, Nazila Shafiei, and Thomas Graf. 2019. Case assignment in TSL syntax: A case study. In *Proccedings of SCiL 2019*, 267–276.

# Appendix

# Closure Properties of Subregular Classes

	SL	TSL	MTSL	ITSL	IOTSL	SF	Reg
$\cup$	×	×	×	×	×	✓	✓
$\cap$	✓	×	✓	×	×	✓	✓
Relabeling	×	×	×	×	×	×	✓
Complement	×	×	×	×	×	✓	✓

# (I)TSL vs. SP

- **Strictly piecewise** (SP) grammars: forbidden **subsequences**.  
Precedence (not successor) as the core relation.

## Sibilant harmony

► (I)TSL



► SP

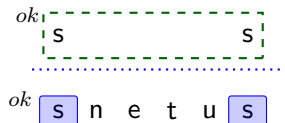
Diagram illustrating SP for sibilant harmony. The word "okenet" is shown. The "ok" prefix and the final "s" are highlighted in blue boxes. The first "s" and the final "s" are each enclosed in dashed green boxes, indicating that they are treated as separate, non-interfering units.

# (I)TSL vs. SP

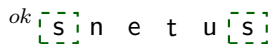
- **Strictly piecewise** (SP) grammars: forbidden **subsequences**.  
Precedence (not successor) as the core relation.

## Sibilant harmony

### ► (I)TSL



### ► SP



# (I)TSL vs. SP: Incomparability

(I)TSL  $\not\subseteq$  SP

SAMALA's harmony is ITSL but not SP

*ok* [s][n] e t u [s]

*ok* [s] n e [t][u] s

SP  $\not\subseteq$  ITSL

$S = \{^*ab, ^*cd\} \Rightarrow L(S) \in SP$  but  $L(S) \notin ITSL$

\* a c e b e d

\* a c e b e d



# (I)TSL vs. SP: Incomparability

(I)TSL  $\not\subseteq$  SP

SAMALA's harmony is ITSL but not SP

*ok* [s][n] e t u [s]

*ok* [s] n e [t][u] s

SP  $\not\subseteq$  ITSL

$S = \{^*ab, ^*cd\} \Rightarrow L(S) \in SP$  but  $L(S) \notin ITSL$

\* a c e b e d

\* a c e b e d

# (I)TSL vs. SP: Incomparability

(I)TSL  $\not\subseteq$  SP

SAMALA's harmony is ITSL but not SP

*ok* [s][n] e t u [s]

*ok* [s] n e [t][u] s

SP  $\not\subseteq$  ITSL

$S = \{^*ab, ^*cd\} \Rightarrow L(S) \in SP$  but  $L(S) \notin ITSL$

\* [a] c e [b] e d

\* a c e b e d

# (I)TSL vs. SP: Incomparability

(I)TSL  $\not\subseteq$  SP

SAMALA's harmony is ITSL but not SP

*ok* [s][n] e t u [s]

*ok* [s] n e [t][u] s

SP  $\not\subseteq$  ITSL

$S = \{^*ab, ^*cd\} \Rightarrow L(S) \in SP$  but  $L(S) \notin ITSL$

\* a [c] e b e [d]

\* a c e b e d

# (I)TSL vs. SP: Incomparability

(I)TSL  $\not\subseteq$  SP

SAMALA's harmony is ITSL but not SP

*ok* [s][n] e t u [s]

*ok* [s] n e [t][u] s

SP  $\not\subseteq$  ITSL

$S = \{^*ab, ^*cd\} \Rightarrow L(S) \in SP$  but  $L(S) \notin ITSL$

\* a [c] e b e [d]

\* [a] [c] e [b] e [d]

# (I)TSL vs. SP: Incomparability

(I)TSL  $\not\subseteq$  SP

SAMALA's harmony is ITSL but not SP

*ok* [s][n] e t u [s]

*ok* [s] n e [t][u] s

SP  $\not\subseteq$  ITSL

$S = \{^*ab, ^*cd\} \Rightarrow L(S) \in SP$  but  $L(S) \notin ITSL$

\* a [c] e b e [d]

a c b d  
.....  
\* [a] [c] e [b] e [d]

# Another TSL Outlier

## Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin 2016)

### 1) Underlying causative prefix /s(:)-/

	<i>Base</i>	<i>Causative</i>	
a.	uga	s:-uga	"be evacuated"
b.	as:twā	s-as:twā	"settle, be levelled"

### 2) Sibilant harmony

	<i>Base</i>	<i>Causative</i>	
a.	fiaʃr	ʃ- fiaʃr	"be full of straw, of discord"
b.	nza	z:-nza	"be sold"

### 3) Sibilant voicing harmony blocked

	<i>Base</i>	<i>Causative</i>	
a.	ukz	s:-ukz	"recognize"
b.	q:uɰi	ʃ- q:uɰi	"be dislocated, broken"

Can we write a TSL grammar to capture this pattern?

# Another TSL Outlier

## Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin 2016)

### 1) Underlying causative prefix /s(:)-/

*Base*      *Causative*

- |    |        |          |                       |
|----|--------|----------|-----------------------|
| a. | uga    | s:-uga   | "be evacuated"        |
| b. | as:twā | s-as:twā | "settle, be levelled" |

### 2) Sibilant harmony

*Base*      *Causative*

- |    |       |          |                                |
|----|-------|----------|--------------------------------|
| a. | fiaʃr | ʃ- fiaʃr | "be full of straw, of discord" |
| b. | nza   | z:-nza   | "be sold"                      |

### 3) Sibilant voicing harmony blocked

*Base*      *Causative*

- |    |        |           |                         |
|----|--------|-----------|-------------------------|
| a. | ukz    | s:-ukz    | "recognize"             |
| b. | q:uʒ:i | ʃ- q:uʒ:i | "be dislocated, broken" |

Can we write a TSL grammar to capture this pattern?

# Another TSL Outlier

## Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin 2016)

1) Underlying causative prefix /s(:)-/

*Base*      *Causative*

- |    |        |          |                       |
|----|--------|----------|-----------------------|
| a. | uga    | s:-uga   | "be evacuated"        |
| b. | a:-twa | s-a:-twa | "settle, be levelled" |

2) Sibilant harmony

*Base*      *Causative*

- |    |      |         |                                |
|----|------|---------|--------------------------------|
| a. | fiaɹ | ɹ- fiaɹ | "be full of straw, of discord" |
| b. | nza  | z:-nza  | "be sold"                      |

3) Sibilant voicing harmony blocked

*Base*      *Causative*

- |    |       |          |                         |
|----|-------|----------|-------------------------|
| a. | ukz   | s:-ukz   | "recognize"             |
| b. | q:uɹi | ɹ- q:uɹi | "be dislocated, broken" |

Can we write a TSL grammar to capture this pattern?



# Another TSL Outlier

## Sibilant Harmony in IMDLAWN TASHLHIYT (McMullin 2016)

1) Underlying causative prefix /s(:)-/

*Base*      *Causative*

- |    |        |          |                       |
|----|--------|----------|-----------------------|
| a. | uga    | s:-uga   | "be evacuated"        |
| b. | a:-twa | s-a:-twa | "settle, be levelled" |

2) Sibilant harmony

*Base*      *Causative*

- |    |       |          |                                |
|----|-------|----------|--------------------------------|
| a. | fiaʃr | ʃ- fiaʃr | "be full of straw, of discord" |
| b. | nza   | z:-nza   | "be sold"                      |

3) Sibilant voicing harmony blocked

*Base*      *Causative*

- |    |        |           |                         |
|----|--------|-----------|-------------------------|
| a. | ukz    | s:-ukz    | "recognize"             |
| b. | q:uʒ:i | ʃ- q:uʒ:i | "be dislocated, broken" |

Can we write a TSL grammar to capture this pattern?

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

\* z m: ʒ d a w |

*ok* ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \mathfrak{z}, s, z, f \}$$

$$S = \{ *s\mathfrak{z}, *sz, *s\mathfrak{f}, *\mathfrak{z}s, *f s, *zs, *z\mathfrak{f}, *z\mathfrak{z}, *fz, *f\mathfrak{z}, *\mathfrak{z}f, *\mathfrak{z}z \}$$

$z$

\*  $z$  m:  $\mathfrak{z}$  d a w |

*ok*  $\mathfrak{z}$  m:  $\mathfrak{z}$  d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}z \}$$

z

\* z m: ʒ d a w |

*ok* ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

z      ʒ

.....

\* z m: ʒ d a w |

*ok* ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

z      ʒ

.....

\* z m: ʒ d a w |

*ok* ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

z      ʒ

.....

\* z m: ʒ d a w |

*ok* ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*s\text{f}, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*z\text{f}, {}^*z\text{ʒ}, {}^*\text{f}z, {}^*\text{f}\text{ʒ}, {}^*\text{ʒ}\text{f}, {}^*\text{ʒ}z \}$$

z      ʒ

.....

\* z m: ʒ d a w |

*ok* ʒ m: ʒ d a w |



# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}z \}$$

z      ʒ

.....

\* z m: ʒ d a w l

*ok* ʒ m: ʒ d a w l

# Sibilant Harmony in IMDLAWN TASHLHIYT


## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

\*  


\* z m: ʒ d a w |

*ok* ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

\*  
z      ʒ  
 .....  
 \* z m: ʒ d a w |

ʒ  
 .....  
 ok ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT


## Generalization (1/2)



Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *fs, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$

\*  
  
 .....  
 \* z m: ʒ d a w |

ʒ  
  
 .....  
 ok ʒ  ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

\*  
z      ʒ  
 .....  
 \* z m: ʒ d a w |

ʒ  
 .....  
*ok* ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

\*  
z      ʒ  
 .....  
 \*

z m: ʒ d a w |

ʒ      ʒ  
 .....  
*ok* ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *ʒs, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *f\text{ʒ}, *z\text{ʒ} \}$$

\*  
z      ʒ  
 .....  
 \*

z m: ʒ d a w |

ʒ      ʒ  
 .....  
*ok* ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT


## Generalization (1/2)

Sibilants must agree in anteriority and voicing.


## Grammar

$$T = \{ \mathfrak{z}, s, z, \mathfrak{f} \}$$

$$S = \{ {}^*s\mathfrak{z}, {}^*sz, {}^*s\mathfrak{f}, {}^*\mathfrak{z}s, {}^*\mathfrak{f}s, {}^*zs, {}^*z\mathfrak{f}, {}^*z\mathfrak{z}, {}^*\mathfrak{f}z, {}^*\mathfrak{f}\mathfrak{z}, {}^*\mathfrak{z}\mathfrak{f}, {}^*\mathfrak{z}z \}$$

\*  


\* z m:  $\mathfrak{z}$  d a w |

$\mathfrak{z}$   $\mathfrak{z}$   
  
*ok*  $\mathfrak{z}$  m:  $\mathfrak{z}$  d a w |



# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}f, {}^*f\text{ʒ}, {}^*f\text{ʒ}, {}^*\text{ʒ}z \}$$

\*  
z      ʒ  
 .....  
 \* z m: ʒ d a w l

ʒ      ʒ  
 .....  
*ok* ʒ m: ʒ d a w l

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (1/2)

Sibilants must agree in anteriority and voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f \}$$

$$S = \{ {}^*s\text{ʒ}, {}^*sz, {}^*sf, {}^*\text{ʒ}s, {}^*\text{ʒ}s, {}^*zs, {}^*zf, {}^*z\text{ʒ}, {}^*\text{ʒ}z, {}^*\text{ʒ}\text{ʒ}, {}^*\text{ʒ}f, {}^*\text{ʒ}z \}$$

\* z      ʒ

\* z m: ʒ d a w |

ok ʒ      ʒ

ok ʒ m: ʒ d a w |

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *z\text{ʒ}, *z\text{ʃ}, *z\text{ʒ}, *fz, *f\text{ʒ}, * \text{ʒ}\text{ʃ}, * \text{ʒ}z \}$$

*ok*    ʃ    q    u    ʒ:    i

\*    s    q    u    ʒ:    i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *\text{ʒ}s, *\text{ʃ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *\text{ʒ}\text{ʃ}, *\text{ʒ}z \}$$

ʃ

.....

ok ʃ q u ʒ: i

\* s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *\text{ʒ}s, *\text{ʒ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *\text{ʒ}\text{ʃ}, *q\text{ʒ} \}$$

ʃ q

ok ʃ q u ʒ: i

\* s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *\text{ʒ}s, *\text{ʃ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *\text{ʒ}\text{ʃ}, *\text{ʒ}z \}$$

ʃ q

.....

*ok* ʃ q u ʒ: i

\* s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *\text{ʒ}s, *\text{ʃ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *\text{ʒ}\text{ʃ}, *\text{ʒ}z \}$$

ʃ q ʒ:

.....

*ok* ʃ q u ʒ: i

\* s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ʃ q ʒ:

*ok*

ʃ q u ʒ: i

\* s q u ʒ: i



# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *ʒs, *zs, *zf, *zʒ, *fz, *fʒ, *ʒf, *ʒz \}$$

*ok* f q ʒ:

*ok* f q u ʒ: i

\* s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

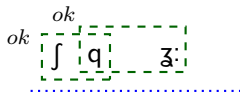
Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f\text{ʒ}, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *z\text{ʒ} \}$$

*ok*



ʃ q ʒ:

*ok* ʃ q u ʒ: i

\* s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ok

ok

ʃ q ʒ:

ʃ q u ʒ: i

s

\*

s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f s, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *z\text{ʒ} \}$$

ok

ok

ʃ q ʒ:

ok ʃ q u ʒ: i

s q

\* s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ok

ok

ʃ q ʒ:

---

ok

ʃ q u ʒ: i

s q

---

\* s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f\text{ʒ}, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$

$ok$   $\int$   $q$   $\text{ʒ}:$   
 .....  
 $ok$   $\int$   $q$   $u$   $\text{ʒ}:$   $i$

$s$   $q$   $\text{ʒ}:$   
 .....  
 $*$   $s$   $q$   $u$   $\text{ʒ}:$   $i$

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *ʒs, *fz, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *ʒf, *ʒz \}$$

ok

ok

ʃ q ʒ:

---

ok

ʃ q u ʒ: i

s q ʒ:

---

\* s q u ʒ: i

# Sibilant Harmony in IMDLAWN TASHLHIYT

## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f s, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$

<sup>ok</sup>  
<sup>ok</sup> [ʃ] [q] [ʒ:]  
 .....  
<sup>ok</sup> ʃ q u ʒ: i

<sup>ok</sup>  
<sup>ok</sup> [s] [q] [ʒ:]  
 .....  
 \* s q u ʒ: i



# Sibilant Harmony in IMDLAWN TASHLHIYT

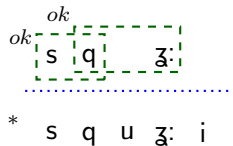
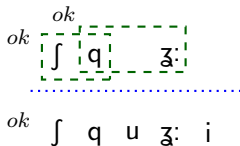
## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f\text{ʒ}, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$



# Sibilant Harmony in IMDLAWN TASHLHIYT

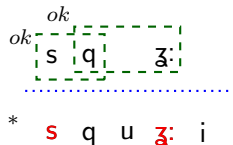
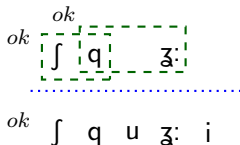
## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, f, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *sf, *\text{ʒ}s, *f\text{ʒ}, *zs, *zf, *z\text{ʒ}, *fz, *f\text{ʒ}, *\text{ʒ}f, *\text{ʒ}z \}$$



# Sibilant Harmony in IMDLAWN TASHLHIYT

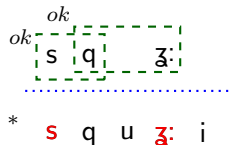
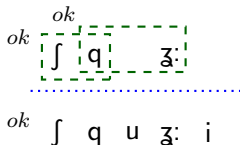
## Generalization (2/2)

Voiceless obstruents block agreement in voicing.

## Grammar

$$T = \{ \text{ʒ}, s, z, \text{ʃ}, q \}$$

$$S = \{ *s\text{ʒ}, *sz, *s\text{ʃ}, *\text{ʒ}s, *\text{ʃ}s, *zs, *z\text{ʃ}, *z\text{ʒ}, *\text{ʃ}z, *\text{ʃ}\text{ʒ}, *\text{ʒ}\text{ʃ}, *\text{ʒ}z \}$$



No TSL grammar can block voicing and enforce anteriority!

# Multi-Tier Strictly Local (MTSL) Languages (1/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \text{f}, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\text{f}z, ^*\text{f}\text{ʒ}, ^*\text{ʒ}\text{f}\}$$

*ok*    ʃ    q    u    ʒ:    i

# Multi-Tier Strictly Local (MTSL) Languages (1/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʃ, s, z, f, q}\} \quad S_1 = \{^*s\text{ʃ}, ^*sz, ^*\text{ʃ}s, ^*zs, ^*\text{ʃ}z, ^*\text{ʃ}\text{ʃ}, ^*\text{ʃ}\text{f}\}$$

ʃ

.....

T<sub>1</sub> : sibilant voicing

ok ʃ q u ʒ: i

$\int q$   
 $T_1$  : sibilant voicing  
*ok*  $\int q$  u ʒ: i

# Multi-Tier Strictly Local (MTSL) Languages (1/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʃ, s, z, f, q}\} \quad S_1 = \{*\text{ʃʃ}, *\text{sz}, *\text{ʃs}, *\text{zs}, *\text{fz}, *\text{fʃ}, *\text{ʃf}\}$$

ʃ   q

.....

$T_1$  : sibilant voicing

*ok*   ʃ   q   u   ʃ:   i

# Multi-Tier Strictly Local (MTSL) Languages (1/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʃ, s, z, ʒ, q}\} \quad S_1 = \{*\text{sʃ}, *\text{sz}, *\text{ʃs}, *\text{zs}, *\text{ʒz}, *\text{ʃʃ}, *\text{ʃʒ}\}$$

ʃ q ʒ:

.....

$T_1$  : sibilant voicing

ok ʃ q u ʒ: i



# Multi-Tier Strictly Local (MTSL) Languages (1/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʃ, s, z, f, q}\} \quad S_1 = \{^*s\text{ʃ}, ^*sz, ^*\text{ʃ}s, ^*zs, ^*\text{ʃ}z, ^*\text{ʃ}\text{ʃ}, ^*\text{ʃ}\text{f}\}$$

$\int$  q      ʒ:  
 $\overset{\text{ok}}{\text{T}_1} : \text{sibilant voicing}$   
 $\int$  q u ʒ: i

# Multi-Tier Strictly Local (MTSL) Languages (1/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \text{f}, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\text{f}z, ^*\text{f}\text{ʒ}, ^*\text{ʒ}\text{f}\}$$

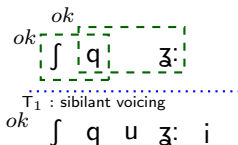
$ok$   $\boxed{\text{f} \text{ q}}$  ʒ:  
 $T_1$  : sibilant voicing  
 $ok$   $\text{f} \text{ q} \text{ u} \text{ ʒ} \text{ i}$

# Multi-Tier Strictly Local (MTSL) Languages (1/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \text{ʃ}, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\text{ʃ}z, ^*\text{ʃ}\text{ʒ}, ^*\text{ʒ}\text{ʃ}\}$$



# Multi-Tier Strictly Local (MTSL) Languages (1/2)

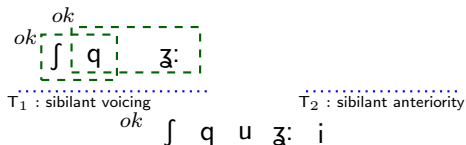
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (1/2)

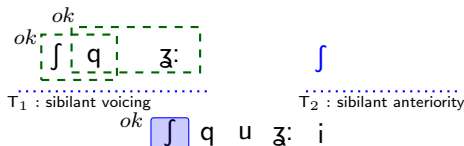
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (1/2)

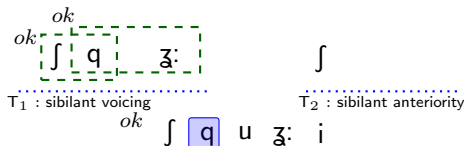
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, \text{f}, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*\text{f}z, ^*\text{f}\text{ʒ}, ^*\text{ʒ}\text{f}\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, \text{f}\} \quad S_2 = \{^*s\text{ʒ}, ^*s\text{f}, ^*\text{ʒ}s, ^*\text{ʒ}\text{f}, ^*zs, ^*z\text{f}, ^*\text{ʒ}\text{z}, ^*\text{f}z, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (1/2)

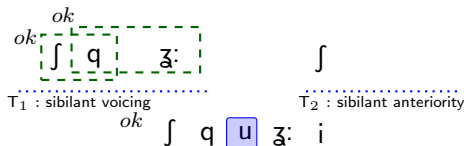
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (1/2)

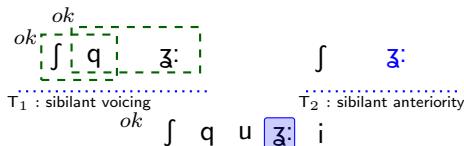
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$





# Multi-Tier Strictly Local (MTSL) Languages (1/2)

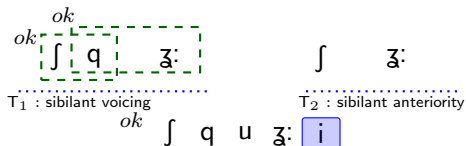
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, \text{s}, \text{z}, \text{f}, \text{q}\} \quad S_1 = \{*\text{sʒ}, *\text{sz}, *\text{ʒs}, *\text{zs}, *\text{fz}, *\text{fʒ}, *\text{ʒf}\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, \text{s}, \text{z}, \text{f}\} \quad S_2 = \{*\text{sʒ}, *\text{sʃ}, *\text{ʒs}, *\text{ʃs}, *\text{zs}, *\text{zf}, *\text{zʒ}, *\text{fz}, *\text{ʒz}\}$$



# Multi-Tier Strictly Local (MTSL) Languages (1/2)

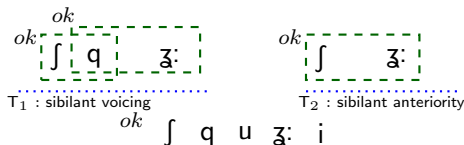
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (2/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*\text{ʒ}z, ^*fz, ^*\text{ʒ}z\}$$

\* s q u ʒ: i

# Multi-Tier Strictly Local (MTSL) Languages (2/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$

S

.....  
 $T_1$  : sibilant voicing  
 \* s q u ʒ: i

# Multi-Tier Strictly Local (MTSL) Languages (2/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*\text{ʒ}z, ^*fz, ^*\text{ʒ}z\}$$

s q

$T_1$  : sibilant voicing  
 \* s q u ʒ: i

# Multi-Tier Strictly Local (MTSL) Languages (2/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*\text{ʒ}z, ^*fz, ^*\text{ʒ}z\}$$

s q

$T_1$  : sibilant voicing

\* s q u ʒ: i

# Multi-Tier Strictly Local (MTSL) Languages (2/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$

s   q   ʒ:

.....  
 $T_1$  : sibilant voicing  
 \*   s   q   u   ʒ:   i

# Multi-Tier Strictly Local (MTSL) Languages (2/2)

## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*\text{ʒ}z, ^*fz, ^*\text{ʒ}z\}$$

s   q   ʒ:

.....  
T<sub>1</sub> : sibilant voicing

\*   s   q   u   ʒ:   i



# Multi-Tier Strictly Local (MTSL) Languages (2/2)

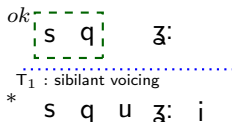
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (2/2)

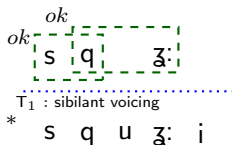
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (2/2)

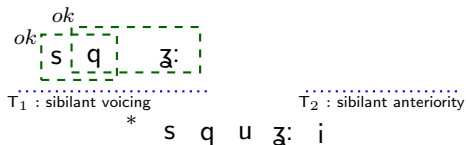
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (2/2)

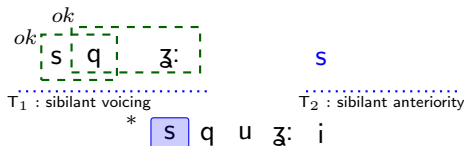
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (2/2)

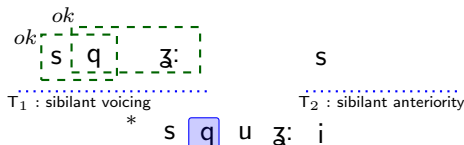
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (2/2)

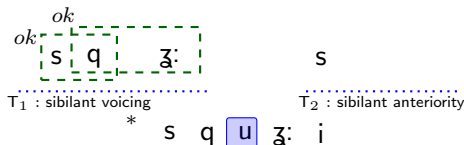
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (2/2)

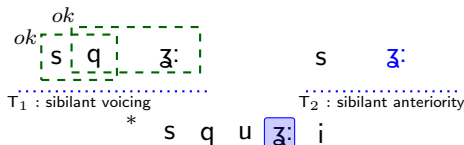
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



# Multi-Tier Strictly Local (MTSL) Languages (2/2)

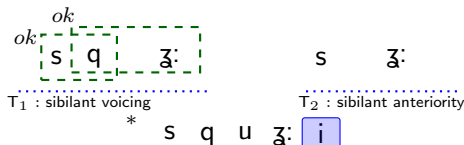
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$





# Multi-Tier Strictly Local (MTSL) Languages (2/2)

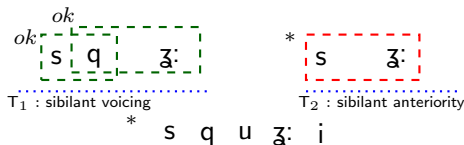
## Sibilant Harmony in IMDLAWN TASHLHIYT (Revisited)

Voiceless obstruents block agreement in voicing:

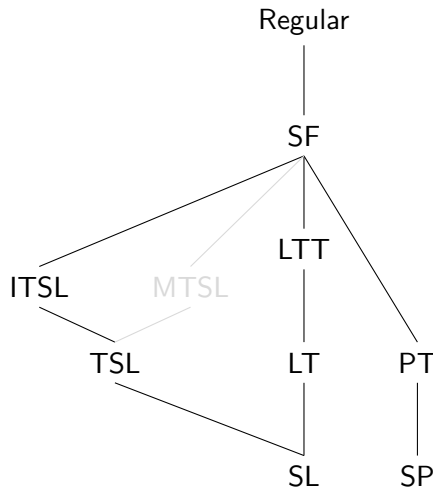
$$\blacktriangleright T_1 = \{\text{ʒ}, s, z, f, q\} \quad S_1 = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

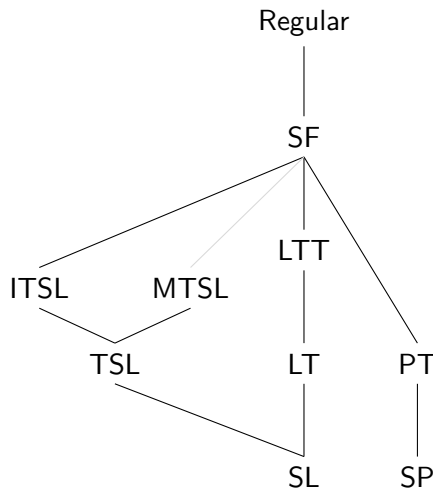
$$\blacktriangleright T_2 = \{\text{ʒ}, s, z, f\} \quad S_2 = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



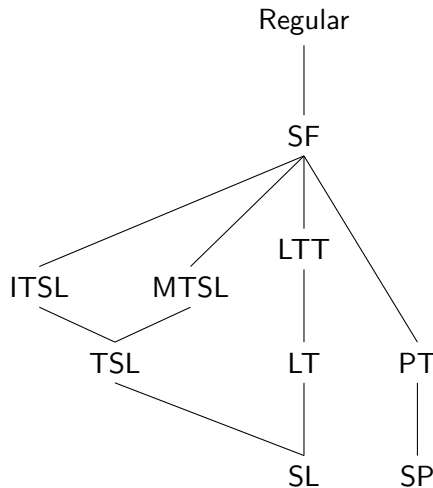
# MTSL: Relations to other Classes



# MTSL: Relations to other Classes



# MTSL: Relations to other Classes



# Incomparability of ITSL and MTSL (1/2)

MTSL  $\not\subseteq$  ITSL

We already have an example

- ▶ IMDLAWN TASHLHIYT's harmony is MTSL but not ITSL

ITSL  $\subseteq$  MTSL?

- ▶ Is every ITSL language also MTSL?

# IMDLAWN TASHLHIYT's Sibilant Harmony $\notin$ ITSL

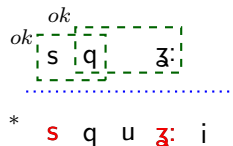
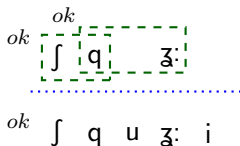
## Reminder: Sibilant Harmony in IMDLAWN TASHLHIYT

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T = \{\text{ʒ}, s, z, f, q\} \quad S = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T = \{\text{ʒ}, s, z, f\} \quad S = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



This pattern is not ITSL!

# IMDLAWN TASHLHIYT's Sibilant Harmony $\notin$ ITSL

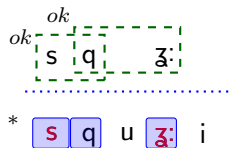
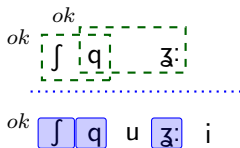
## Reminder: Sibilant Harmony in IMDLAWN TASHLHIYT

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T = \{\text{ʒ}, s, z, f, q\} \quad S = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T = \{\text{ʒ}, s, z, f\} \quad S = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*\text{ʒ}z, ^*fz, ^*\text{ʒ}z\}$$



This pattern is not ITSL!

# IMDLAWN TASHLHIYT's Sibilant Harmony $\notin$ ITSL

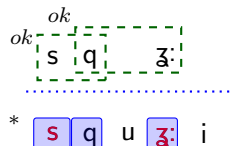
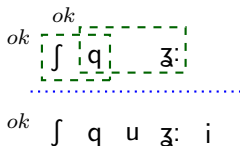
## Reminder: Sibilant Harmony in IMDLAWN TASHLHIYT

Voiceless obstruents block agreement in voicing:

$$\blacktriangleright T = \{\text{ʒ}, s, z, f, q\} \quad S = \{^*s\text{ʒ}, ^*sz, ^*\text{ʒ}s, ^*zs, ^*fz, ^*f\text{ʒ}, ^*\text{ʒ}f\}$$

Unbounded agreement in anteriority:

$$\blacktriangleright T = \{\text{ʒ}, s, z, f\} \quad S = \{^*s\text{ʒ}, ^*sf, ^*\text{ʒ}s, ^*fs, ^*zs, ^*zf, ^*z\text{ʒ}, ^*fz, ^*\text{ʒ}z\}$$



This pattern is not ITSL!



# Incomparability of ITSL and MTSL (2/2)

MTSL  $\not\subseteq$  ITSL

IMDLAWN TASHLHIYT's harmony is MTSL but not ITSL.

ITSL  $\not\subseteq$  MTSL

We have already seen an example:

- ▶ SAMALA's harmony is ITSL but not MTSL.

# An MTSL Account for SAMALA?

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

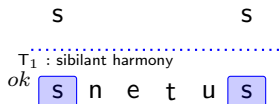
*ok* s n e t u s

This pattern is not MTSL!

# An MTSL Account for SAMALA?

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

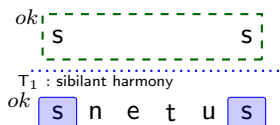


This pattern is not MTSL!

# An MTSL Account for SAMALA?

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

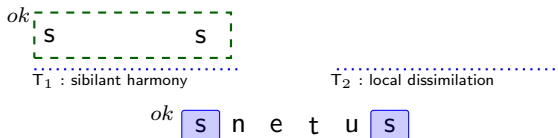


This pattern is not MTSL!

# An MTSL Account for SAMALA?

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

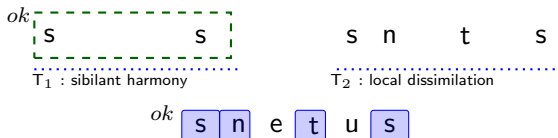


This pattern is not MTSL!

# An MTSL Account for SAMALA?

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

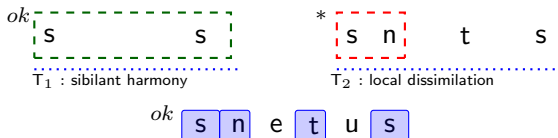


This pattern is not MTSL!

# An MTSL Account for SAMALA?

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

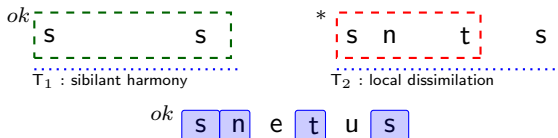


This pattern is not MTSL!

# An MTSL Account for SAMALA?

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization



This pattern is not MTSL!



# An MTSL Account for SAMALA?

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

*ok* s                      s  
 T<sub>1</sub> : sibilant harmony

*ok*<sub>1</sub> s   n                      t                      s  
 T<sub>2</sub> : local dissimilation

*ok* s n e t u s

This pattern is not MTSL!

# An MTSL Account for SAMALA?

## SAMALA Sibilant Harmony (Revisited)

- ▶ anticipatory sibilant harmony
- ▶ palatalization to avoid local restrictions
- ▶ sibilant harmony overrides palatalization

*ok* s                      s  
 T<sub>1</sub> : sibilant harmony

*ok*<sub>1</sub> s   n                      t                      s  
 T<sub>2</sub> : local dissimilation

*ok* s n e t u s

This pattern is not MTSL!

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$  a p i  $t\int^h$  o l u  $\int$  w a  $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$

$T_1$ : anticipatory harmony

$\int$  a p i  $t\int^h$  o l u  $\int$  w a  $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$

.....

$T_1$ : anticipatory harmony

$\int$  a p i  $t\int^h$  o l u  $\int$  w a  $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$

.....

$T_1$ : anticipatory harmony

$\int$  a p i  $t\int^h$  o l u  $\int$  w a  $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$

.....

$T_1$ : anticipatory harmony

$\int$  a p i  $t\int^h$  o l u  $\int$  w a  $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$   $t\int^h$

.....

$T_1$ : anticipatory harmony

$\int$  a p i  $t\int^h$  o l u  $\int$  w a  $\int$



# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$                        $t\int^h$

.....

$T_1$ : anticipatory harmony

$\int$    a   p   i    $t\int^h$    o   l   u    $\int$    w   a    $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$   $t\int^h$

.....

$T_1$ : anticipatory harmony

$\int$  a p i  $t\int^h$  o l u  $\int$  w a  $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$                        $t\int^h$

.....

$T_1$ : anticipatory harmony

$\int$    a   p   i    $t\int^h$    o   l   u    $\int$    w   a    $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$                        $t\int^h$                        $\int$

.....  
T<sub>1</sub>: anticipatory harmony

$\int$    a   p   i    $t\int^h$    o   l   u    $\int$    w   a    $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$                        $t\int^h$                        $\int$

.....

$T_1$ : anticipatory harmony

$\int$    a   p   i    $t\int^h$    o   l   u    $\int$    **w**   a    $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$                        $t\int^h$                        $\int$

.....

$T_1$ : anticipatory harmony

$\int$    a   p   i    $t\int^h$    o   l   u    $\int$    w   a    $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

$\int$                        $t\int^h$                        $\int$                        $\int$

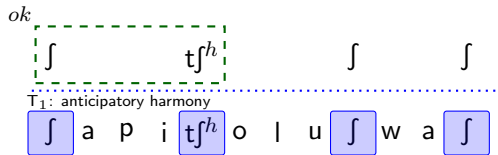
.....  
T<sub>1</sub>: anticipatory harmony

$\int$    a   p   i    $t\int^h$    o   l   u    $\int$    w   a    $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$

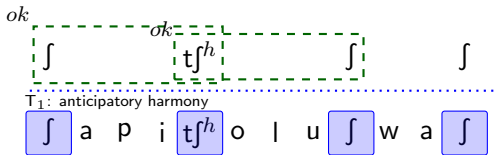




# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

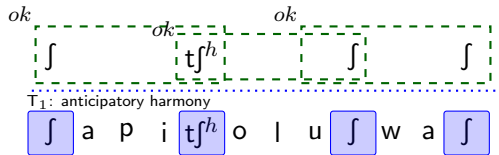
$$T = \{ s, \int, t f^h \}, S = \{ * s \int, * \int s, * s t f^h, * t f^h \}$$



# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

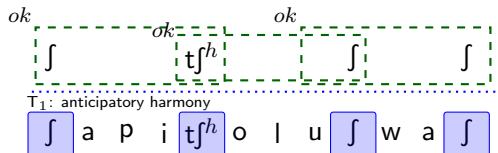
$$T = \{ s, \int, t\int^h \}, S = \{ * s\int, * \int s, * st\int^h, * t\int^h \}$$



# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

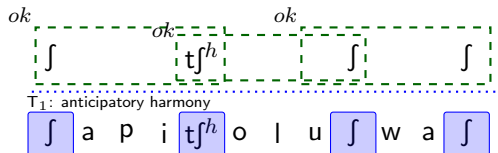
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$

$\int$  a p i  $\text{tf}^h$  o l u s w a  $\int$

# Structure-Sensitive TSL: Overgeneration

## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

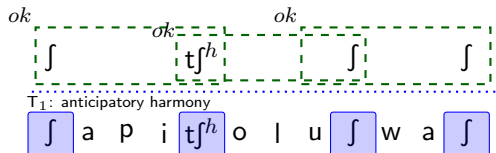
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$

$\bowtie \int a p i \text{tf}^h o l u s w a \int \bowtie$

# Structure-Sensitive TSL: Overgeneration

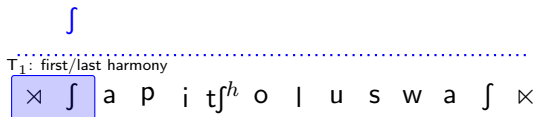
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t f^h\}, S = \{^* s \int, ^* \int s, ^* s t f^h, ^* t f^h\}$$



## First/Last Harmony in PSEUDO-SAMALA

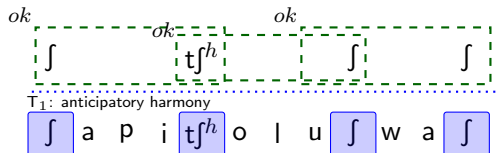
$$T = \{\sigma: \sigma \in \{s, \int, t f^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} S = \{^* s \int, ^* \int s, ^* s t f^h, ^* t f^h s\}$$



# Structure-Sensitive TSL: Overgeneration

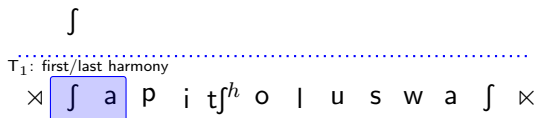
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$



## First/Last Harmony in PSEUDO-SAMALA

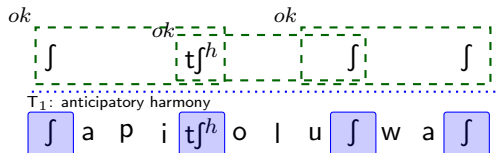
$$T = \{\sigma: \sigma \in \{s, \int, t\int^h\} \wedge (\bowtie\sigma \vee \sigma\bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h s\}$$



# Structure-Sensitive TSL: Overgeneration

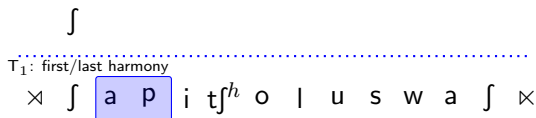
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

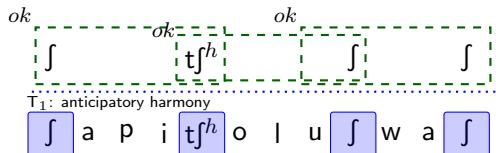
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



# Structure-Sensitive TSL: Overgeneration

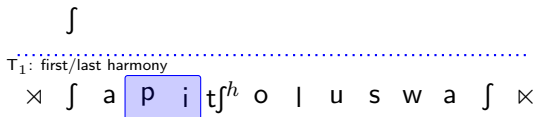
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, t\int^h\}, S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h\}$$



## First/Last Harmony in PSEUDO-SAMALA

$$T = \{\sigma: \sigma \in \{s, \int, t\int^h\} \wedge (\bowtie\sigma \vee \sigma\bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*st\int^h, ^*t\int^h s\}$$

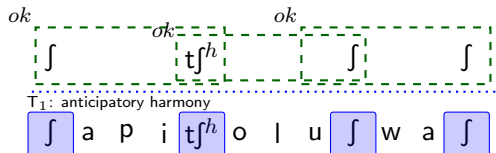




# Structure-Sensitive TSL: Overgeneration

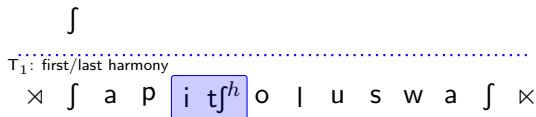
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

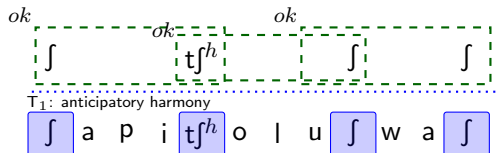
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



# Structure-Sensitive TSL: Overgeneration

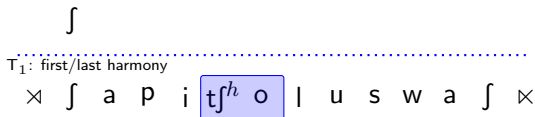
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

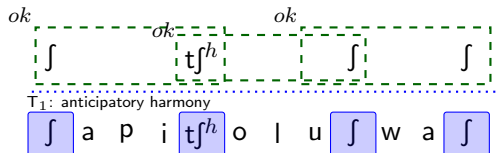
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



# Structure-Sensitive TSL: Overgeneration

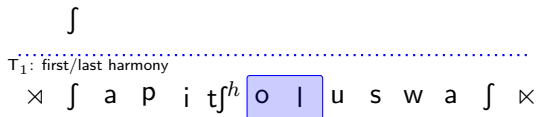
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, tf^h\}, S = \{^*sf, ^*fs, ^*stf^h, ^*tf^h\}$$



## First/Last Harmony in PSEUDO-SAMALA

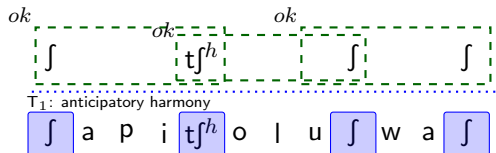
$$T = \{\sigma: \sigma \in \{s, \int, tf^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*sf, ^*fs, ^*stf^h, ^*tf^h s\}$$



# Structure-Sensitive TSL: Overgeneration

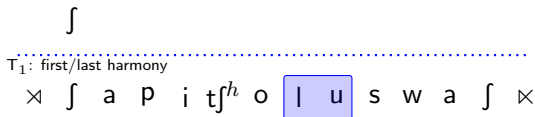
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

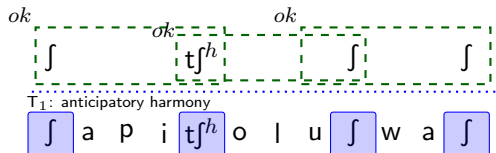
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



# Structure-Sensitive TSL: Overgeneration

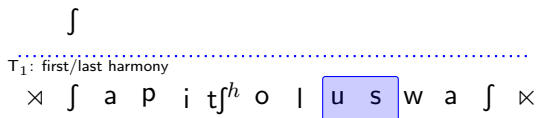
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

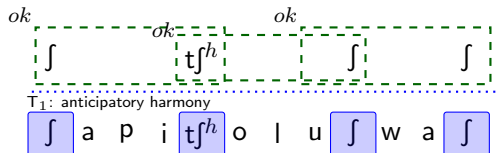
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



# Structure-Sensitive TSL: Overgeneration

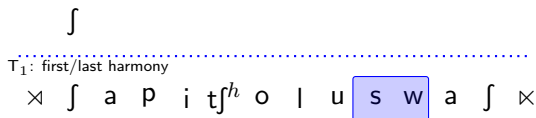
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

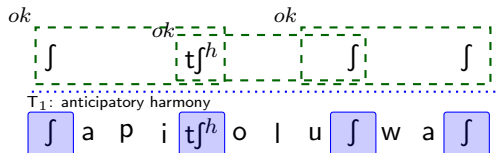
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



# Structure-Sensitive TSL: Overgeneration

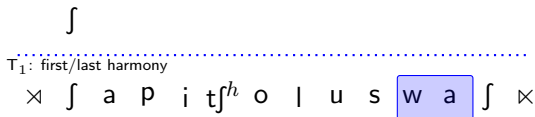
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

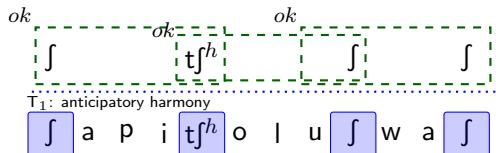
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



# Structure-Sensitive TSL: Overgeneration

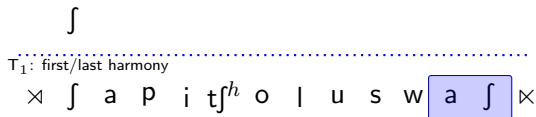
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$

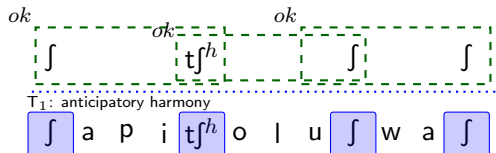




# Structure-Sensitive TSL: Overgeneration

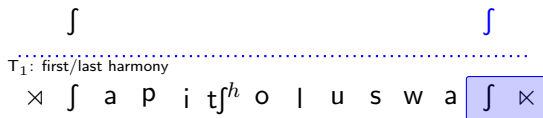
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



## First/Last Harmony in PSEUDO-SAMALA

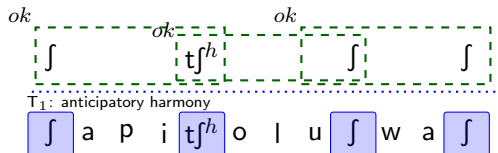
$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie \sigma \vee \sigma \bowtie)\} \quad S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



# Structure-Sensitive TSL: Overgeneration

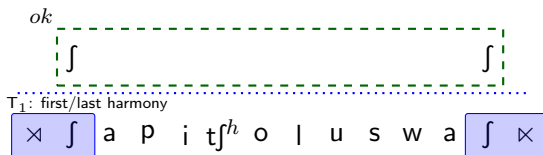
## Anticipatory Harmony in SAMALA

$$T = \{s, \int, \text{tf}^h\}, S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h\}$$



## First/Last Harmony in PSEUDO-SAMALA

$$T = \{\sigma: \sigma \in \{s, \int, \text{tf}^h\} \wedge (\bowtie\sigma \vee \sigma\bowtie)\} S = \{^*s\int, ^*\int s, ^*s\text{tf}^h, ^*\text{tf}^h s\}$$



# TESL: Example

Tier Grammar(s) for  $L$

$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\} \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

a)

⋈ c a c d b ⋈

b)

⋈ c a d d b ⋈

# TESL: Example

Tier Grammar(s) for  $L$

$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\} \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

**a)**

$\bowtie$  c a c d b  $\bowtie$

**b)**

$\bowtie$  c a d d b  $\bowtie$

# TESL: Example

## Tier Grammar(s) for $L$

$$T_1 = \{\sigma : \sigma \in \{a, b, c\}^*\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\}^* \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

a)

$\bowtie \quad c \quad a \quad c \quad \quad b \quad \bowtie$   
..... $T_1$   
 $\bowtie \quad \boxed{c} \quad \boxed{a} \quad \boxed{c} \quad d \quad \boxed{b} \quad \bowtie$

b)

$\bowtie \quad c \quad a \quad d \quad d \quad b \quad \bowtie$

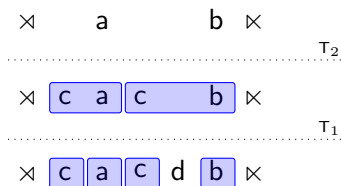
# TESL: Example

## Tier Grammar(s) for $L$

$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\} \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

**a)**



**b)**

$\bowtie \text{ c a d d b } \bowtie$

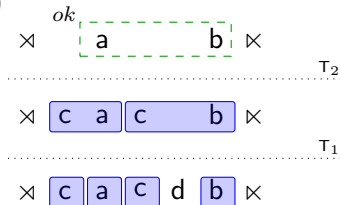
# TESL: Example

## Tier Grammar(s) for $L$

$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\} \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

a)



b)

$\bowtie \text{ } c \text{ } a \text{ } d \text{ } d \text{ } b \text{ } \bowtie$

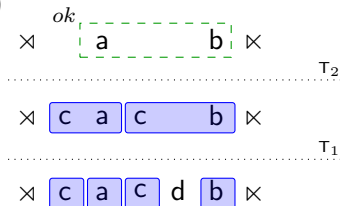
# TESL: Example

## Tier Grammar(s) for $L$

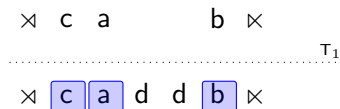
$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\} \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

a)



b)





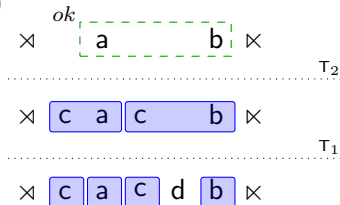
# TESL: Example

## Tier Grammar(s) for $L$

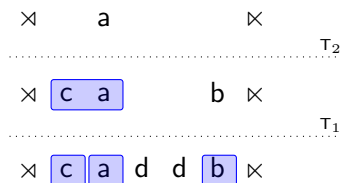
$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\} \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

a)



b)



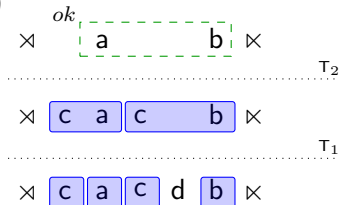
# TESL: Example

## Tier Grammar(s) for $L$

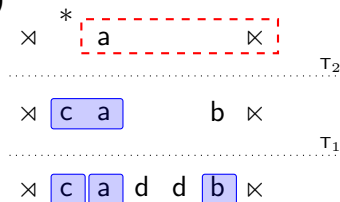
$$T_1 = \{\sigma : \sigma \in \{a, b, c\}\}, S_1 = \{\}$$

$$T_2 = \{\sigma : \sigma \in \{a, b\} \wedge (c \prec^+ \sigma)\}, S_2 = \{^* \bowtie b, ^* bb, ^* aa, ^* a \bowtie\}$$

a)



b)



# TSL Syntax (of the Intuition)

## TSL syntax

Merge and Move dependencies are TSL! (Graf and Heinz 2016)

- ▶ We move from strings to trees.
- ▶ We project tiers for trees.
- ▶ We enforce  $n$ -local tree constraints.

## Tree $n$ -gram grammars:

- ▶ Patterns are described by forbidden tree  $n$ -gram(s).
- ▶ A derivational tree is well formed iff no tier  $T$  contains any forbidden  $n$ -gram(s).

# TSL Syntax (of the Intuition)

## TSL syntax

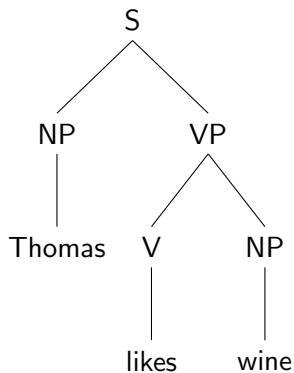
Merge and Move dependencies are TSL! (Graf and Heinz 2016)

- ▶ We move from strings to trees.
- ▶ We project tiers for trees.
- ▶ We enforce  $n$ -local tree constraints.

## Tree $n$ -gram grammars:

- ▶ Patterns are described by forbidden tree  $n$ -gram(s).
- ▶ A derivational tree is well formed iff no tier  $T$  contains any forbidden  $n$ -gram(s).

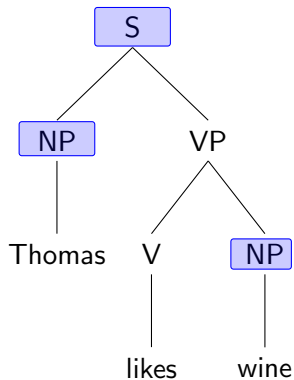
## TSL Syntax: Example (of the Intuition)



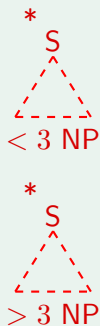
### Tier-grammar



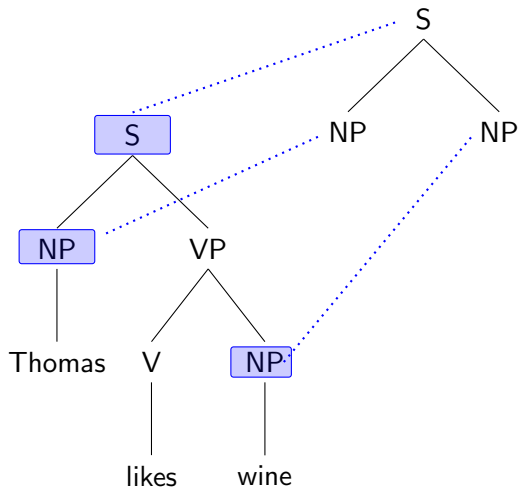
## TSL Syntax: Example (of the Intuition)



### Tier-grammar



# TSL Syntax: Example (of the Intuition)

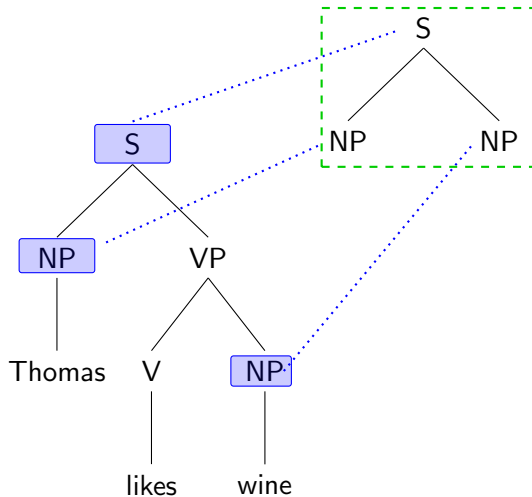


## Tier-grammar

\*  
S  
△  
< 3 NP

\*  
S  
△  
> 3 NP

## TSL Syntax: Example (of the Intuition)



### Tier-grammar

\*  
S  
△  
< 3 NP

\*  
S  
△  
> 3 NP