

Chinese Society of Aeronautics and Astronautics & Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn www.sciencedirect.com



Three-dimensional path planning for unmanned aerial vehicle based on interfered fluid dynamical system



Wang Honglun a,b,*, Lyu Wentao a,c, Yao Peng a,b, Liang Xiao d, Liu Chang a,b

Received 20 December 2013; revised 24 July 2014; accepted 29 September 2014 Available online 30 December 2014

KEYWORDS

Fluid dynamical system; Obstacle avoidance; Potential field; Three-dimensional path planning; UAV maneuverability; Unmanned aerial vehicle Abstract This paper proposes a method for planning the three-dimensional path for low-flying unmanned aerial vehicle (UAV) in complex terrain based on interfered fluid dynamical system (IFDS) and the theory of obstacle avoidance by the flowing stream. With no requirement of solutions to fluid equations under complex boundary conditions, the proposed method is suitable for situations with complex terrain and different shapes of obstacles. Firstly, by transforming the mountains, radar and anti-aircraft fire in complex terrain into cylindrical, conical, spherical, parallelepiped obstacles and their combinations, the 3D low-flying path planning problem is turned into solving streamlines for obstacle avoidance by fluid flow. Secondly, on the basis of a unified mathematical expression of typical obstacle shapes including sphere, cylinder, cone and parallelepiped, the modulation matrix for interfered fluid dynamical system is constructed and 3D streamlines around a single obstacle are obtained. Solutions to streamlines with multiple obstacles are then derived using weighted average of the velocity field. Thirdly, extra control force method and virtual obstacle method are proposed to deal with the stagnation point and the case of obstacles' overlapping respectively. Finally, taking path length and flight height as sub-goals, genetic algorithm (GA) is used to obtain optimal 3D path under the maneuverability constraints of the UAV. Simulation results show that the environmental modeling is simple and the path is smooth and suitable for UAV. Theoretical proof is also presented to show that the proposed method has no effect on the characteristics of fluid avoiding obstacles. © 2015 Production and hosting by Elsevier Ltd. on behalf of CSAA & BUAA.

E-mail address: hl_wang_2002@126.com (H. Wang).

Peer review under responsibility of Editorial Committee of CJA.



Production and hosting by Elsevier

1. Introduction

Path planning is one of the most important technologies for autonomous flight of unmanned aerial vehicle (UAV). Nowadays, the application of UAV is extending from high-altitude flight to low or super low-altitude, where the impact of terrain will be the key factor to be considered. Since 3D path planning

^a Science and Technology on Aircraft Control Laboratory, Beihang University, Beijing 100191, China

^b School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China

^c The 28th Research Institute of China Electronics Technology Group Corporation, Nanjing 210007, China

^d School of Automation, Shengyang Aerospace University, Shenyang 110136, China

^{*} Corresponding author at: School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China. Tel.: +86 10 82317546.

can give full play to the maneuverability of UAV and has an important role in military or civilian areas such as low altitude penetration, low altitude reconnaissance or disaster perception, many researchers in this area have carried out in-depth researches.

To generate a suitable path for UAV, path planning restricted by complex terrain should consider not only the potential impact of terrain on flight safety but also the performance constraints on UAV.

Probabilistic roadmap (PRM)¹ and rapidly-exploring random tree(RRT)² have been applied to 3D path planning for its efficiency, but both of them are carried out by spatial discretization. When facing to complex environment, the modeling is complex and the paths of these methods are composed of roadmaps or waypoints, which are not smooth enough for UAV. In addition, in a very cluttered environment, the RRT may fail to find a path.³

Methods based on intelligent computing like ant colony algorithm (ACO)⁴, evolutionary algorithm (EA)⁵ and particle swarm optimization (PSO)⁶ have strong search capabilities. However, the algorithm performance degrades with environmental complexity and may fall into local minima. Combination of these methods with other means to improve algorithm performance and handle more complex situations is a research hotspot at present.^{7–9}

In order to obtain 3D path which satisfies the maneuverability constraints on UAV, many new methods have been proposed recently. Frazzoil et al. 10 proposed a novel Maneuver-Based planning method, by which several motion primitives can be generated based on the dynamics of UAV and the path is obtained by selecting different motion primitives and connecting them up. This method is applied to real time path planning for its high efficiency¹¹, but the deficiency is restricting the flight maneuver within a finite number of motion primitives. Using the optimal control theory to solve path planning problem can easily transform various constraints and optimal index into mathematical expressions but the solution demands large computation. Although the application of some new solutions such as pseudospectral method¹² and nonlinear trajectory generation 13 can share the computation burden, for complex terrain constraints, the 3D path planning problem is still complex with this method. Nikolos et al. 14 used B-spline curves to simulate the 3D flight trajectory of aircraft, and then used an evolutionary algorithm to optimize the B-spline curve control points. This method can generate smooth path, but the terrain used for simulation is relatively simple, generated only by mathematical functions. Mattei and Blasi¹⁵ defined a weighted and oriented graph which indicates the minimum length trajectories between selected nodes in planning space and the optimal path between any two nodes can be optimized by searching algorithms. However, to build the graph, computational efficiency needs further improvement for multiple problems of quadratic programming.

Artificial potential field (APF)¹⁶ is initially proposed and used in collision avoidance for ground robot. In recent years, it is gradually applied to path planning for UAV.^{17,18} This method is simple in principle and has a small amount of calculation, but it may easily converge to local minima in global planning. Many researchers have proposed approaches to solve the problem^{19–21}, of which the most representative one is stream function.^{22,23} It uses the concept of hydrodynamic to establish potential filed in which local minima can be

avoided. The method is able to plan smooth path in a short time and shows good performance in the flight test.²⁴ However, the concept of stream function only exists in 2D flow and it cannot be applied to 3D path planning. Moreover, according to the current research result, obstacles can only be circular.²⁵

To expand the stream function method into three dimensions, the method based on fluid flow²⁶ is proposed recently. Based on the principles of a fluid flowing around objects and the phenomenon of stream flow from start to end, 3D smooth path that satisfies the maneuverability constraints on UAV can be generated in two ways, which are analytical method and numerical method. The analytical method has less calculation but can only handle spherical obstacles, which are not sufficient to model the complex terrain environment. The numerical method can handle more complex terrain and obstacles, but it needs more preprocessing and computational costs.

To overcome the shortcomings of analytical and numerical methods, on the basis of the method in Ref., ²⁶ in this paper we model the complex terrain environment as spherical, conical, cylindrical and parallelepiped geometry and combinations thereof and study the analytical path planning method in the presence of obstacles in these shapes. Simulation results demonstrate that the proposed method can plan smooth path which satisfies the maneuverability constraints on UAV in complex environment. The method not only keeps the advantages of analytical method with small amount of calculation, but also greatly expands the application of the method of fluid flow.

2. Path planning methods based on fluid computing

As two kinds of methods based on fluid computing, the basic idea of stream function and the method based on fluid flow is to transform the path planning problem into solving streamlines for fluid flow avoiding obstacles. Learning from the phenomenon of flow around obstacles, the terrain is considered as boundary conditions. Flow field distribution in the planning area is calculated by using fluid mechanics, then the streamlines in the field are taken as flight paths for UAV. In fluid mechanics, the problem of flow around obstacles is described by governing equations and boundary conditions,²⁷ and the controlling equation of potential flow is described as Eq. (1).

$$\nabla^2 \Phi = 0 \tag{1}$$

The boundary conditions of Eq. (1) are as follows:

On the surface of obstacles:
$$\frac{\partial \Phi}{\partial \mathbf{n}} = 0$$
 (2)

At infinite distance:
$$\nabla \Phi = \mathbf{u}_{\infty}$$
 (3)

where ∇ denotes the vector differential operator, Φ represents velocity potential, \mathbf{n} is unit normal vector along outward on the surface of obstacles and \mathbf{u}_{∞} is the velocity of flow at infinity. Eq. (1) is Laplace's equation, which describes the irrotational and incompressible characteristics of a potential flow. As the solution of equation, Φ is harmonic function and satisfies extremum principle. Because Φ can only reach the extremum on the boundary, it will avoid local minima. Eq. (2) is an equation of boundary and it shows impenetrable constraint of obstacles, which is the normal velocity on the surface of obstacles vanishes. According to Eq. (2), the normal compo-

nent of velocity on the surface of obstacles is 0 and only the tangential component exists. Eq. (3) is also an equation of boundary. It shows the range of interference caused by obstacles and it approaches 0 at infinity.

By solving Eqs. (1)–(3), the potential field will be obtained. Then velocity of flow is the derivative of velocity potential and the integral of velocity is the streamline. Finally the streamlines are taken as the paths. The paths calculated by this method have the following properties.²⁶

- (1) Except stagnation points where the velocity of the fluid is 0, paths can avoid local minima.
- (2) In the area far from obstacles, paths maintain the initial direction along straight line.
- (3) When close to obstacles, paths will change direction approximately alone the tangent of obstacles, which has the minimum deviation from initial direction.

Although the linear homogeneity of Eq. (1) provides a convenient solution to the problem, when the shapes of obstacles are irregular, the constraints of boundary will become very complex so that the analytical solution cannot be obtained. In this case, the problem can only be solved by numerical method, which needs complicated preprocessing and large computations. Therefore, both the Stream Function and method of fluid flow are now only suitable for the circular or spherical obstacles.

Recently, Khansari-Zadeh et al.²⁸ proposed a dynamical system approach of collision avoidance for robot. By their new dynamical equations, the movement of robot would avoid obstacles and the method maintains the stability of original system. Inspired by their approach, this paper studies analytical path planning method in the presence of spherical, conical, cylindrical and parallelepiped obstacles. Compared to the traditional one in Ref.²⁶, the proposed method not only handles multiple shapes of obstacles, but also avoids Laplace's equation with complex boundary conditions, which is suitable to deal with complex terrain environment.

3. Modeling of environment

When UAV is under the mission of penetration in low altitude, the threats to be considered include terrain obstacles, radar, and anti-aircraft fire (such as artillery and air defense missile system).

Preprocessing of the terrain environment is the first step in path planning for UAV. All the threats are treated as regular geometry, which reduces the complexity of the algorithm. Separate peaks and stretches of mountains are transformed to cones and parallelepipeds, respectively. The threat of artillery is taken as cylinder and its altitude is the radius of the fire range. According to air defense missile system, their fire ranges are represented by the spheres from near boundary to far boundary. Radar is treated as hemisphere whose radius is the scanning radius.

Eq. (4)¹⁶ is taken as the unified modeling of sphere, cylinder, cone and parallelepiped.

$$\Gamma(x, y, z) = \left(\frac{x - x_0}{a}\right)^{2p} + \left(\frac{y - y_0}{b}\right)^{2q} + \left(\frac{z - z_0}{c}\right)^{2r} = 1 \tag{4}$$

where x_0 , y_0 and z_0 represent the coordinate of the center point of the obstacle. a, b and c are constants and are used to control the size of the obstacle. Eq. (4) can be used to describe four geometries, according to different p, q and r.

When p = q = r = 1, obstacle is a sphere.

When p = q = 1 and r > 1, obstacle is approximately a cylinder.

When p = q = 1 and r < 1, obstacle is approximately a cone.

When p > 1, q > 1, and r > 1, obstacle is approximately a parallelepiped.

In order to simulate the real terrain environment, referring to Ref., ²⁹ Fig. 1 shows a complex terrain environment generated by Hill algorithm. ³⁰ One can get different types of terrain by changing the parameters provided with the algorithm. After adding the radar and enemy threats presented by red translucent geometry, the whole map is shown in Fig. 2.

Fig. 3 shows the terrain after preprocessed according to the described method. In order to ensure flight safety, the geometries should envelop the terrain threat and keep a certain margin.

4. Path planning method based on interfered fluid dynamical system

After preprocessing of terrain environment, flow field distribution needs to be simulated in planning space. In order to make the UAV move from the starting point to the target point, a sink flow is placed at the target point. Sink is a kind of fluid that flows from the surroundings to the target point. When there are no obstacles in the planning space, the UAV will reach the target point along a straight line from any starting point. As obstacles exist, the velocity field of the original flow is changed due to the interference of obstacles. This interfered flow filed satisfies impenetrable constraint of obstacles so that the streamlines can avoid the obstacles. The key to the IFDS method is to determine the velocity field of the interfered flow.

4.1. Avoidance of a single obstacle

Consider a flow field in 3D space with the velocity of $\mathbf{u}(x,y,z)$. When an obstacle whose boundary equation is defined by Eq. (4) is placed in the field, the velocity of interfered flow field is denoted as $\bar{\mathbf{u}}(x,y,z)$. Let $\boldsymbol{\xi} = [x-x_0,y-y_0,z-z_0]^T$, then the function $\Gamma(x,y,z)$ in Eq. (4) has continuous first derivative and increases monotonically with $||\boldsymbol{\xi}||$.

Theorem 1. For any point (x, y, z) outside the obstacle, define modulation matrix M as

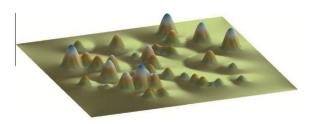


Fig. 1 Terrain generated through Hill algorithm.

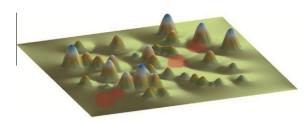


Fig. 2 Map after adding radar and enemy threats.

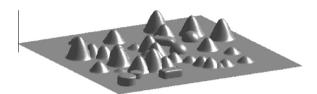


Fig. 3 Preprocessed map.

$$M(x, y, z) = I + \frac{1}{\Gamma_{P}^{\perp} n^{\mathsf{T}} n} (n^{\mathsf{T}} n I - 2n n^{\mathsf{T}})$$

$$(5)$$

where ${\bf I}$ is the identity matrix, $p \geq 0$ is constant, ${\bf n} = \left[\frac{\partial \Gamma}{\partial x}, \frac{\partial \Gamma}{\partial y}, \frac{\partial \Gamma}{\partial z}\right]^{\rm T}$. The interfered flow field $\bar{{\bf u}}(x,y,z)$ caused by the obstacle can be defined as

$$\bar{\boldsymbol{u}}(x,y,z) = \boldsymbol{M}(x,y,z)\boldsymbol{u}(x,y,z) \tag{6}$$

which has the following properties:

- (1) Impenetrable constraint of the obstacle is satisfied.
- (2) The disturbance caused by the obstacle approaches 0 at infinity, that is, when $||\xi|| \to \infty, \bar{u}|_{\infty} = u|_{\infty}$.

Proof (1). The impenetrability of the obstacle's boundary is ensured if the normal velocity at boundary point vanishes. Notice that n indicates the normal vector of the obstacle's boundary when $\Gamma(x,y,z)=1$. From Eq. (5), there is $M=2(I-\frac{1}{n-1}nn^{T})$, then

$$\boldsymbol{n}^{\mathrm{T}}\bar{\boldsymbol{u}} = \boldsymbol{n}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{u} = 2\left(\boldsymbol{n}^{\mathrm{T}} - \frac{\boldsymbol{n}^{\mathrm{T}}\boldsymbol{n}\boldsymbol{n}^{\mathrm{T}}}{\boldsymbol{n}^{\mathrm{T}}\boldsymbol{n}}\right)\boldsymbol{u} = 0 \tag{7}$$

(2) From Eq. (5), there is $\mathbf{M} = \left(1 + \frac{1}{\Gamma_{\rho}^{\perp}}\right)\mathbf{I} - 2\frac{1}{\Gamma_{\rho}^{\perp}}\frac{\mathbf{n}\mathbf{n}^{\mathrm{T}}}{\mathbf{n}^{\mathrm{T}}\mathbf{n}}$, where

$$\frac{\mathbf{n}\mathbf{n}^{T}}{\mathbf{n}^{T}\mathbf{n}} = \frac{1}{\mathbf{n}^{T}\mathbf{n}} \begin{vmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial \Gamma}{\partial y} \\ \frac{\partial \Gamma}{\partial z} \end{vmatrix} \begin{bmatrix} \frac{\partial \Gamma}{\partial x}, \frac{\partial \Gamma}{\partial y}, \frac{\partial \Gamma}{\partial z} \end{bmatrix}$$

$$= \frac{1}{\left(\frac{\partial \Gamma}{\partial x}\right)^{2} + \left(\frac{\partial \Gamma}{\partial y}\right)^{2} + \left(\frac{\partial \Gamma}{\partial z}\right)^{2}} \begin{bmatrix} \left(\frac{\partial \Gamma}{\partial x}\right)^{2} & \frac{\partial \Gamma}{\partial x} \frac{\partial \Gamma}{\partial y} & \frac{\partial \Gamma}{\partial x} \frac{\partial \Gamma}{\partial z} \\ \frac{\partial \Gamma}{\partial x} \frac{\partial \Gamma}{\partial y} & \left(\frac{\partial \Gamma}{\partial y}\right)^{2} & \frac{\partial \Gamma}{\partial y} \frac{\partial \Gamma}{\partial z} \\ \frac{\partial \Gamma}{\partial x} \frac{\partial \Gamma}{\partial z} & \frac{\partial \Gamma}{\partial y} \frac{\partial \Gamma}{\partial z} & \left(\frac{\partial \Gamma}{\partial z}\right)^{2} \end{bmatrix}.$$

The eigenvalue decomposition of $\frac{nn^T}{nTn}$ is

$$\frac{\mathbf{n}\mathbf{n}^{\mathrm{T}}}{\mathbf{n}^{\mathrm{T}}\mathbf{n}} = \begin{bmatrix} \frac{\partial \Gamma}{\partial x} & \frac{\partial \Gamma}{\partial y} & \frac{\partial \Gamma}{\partial z} \\ \frac{\partial \Gamma}{\partial y} & -\frac{\partial \Gamma}{\partial x} & 0 \\ \frac{\partial \Gamma}{\partial z} & 0 & -\frac{\partial \Gamma}{\partial x} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \Gamma}{\partial x} & \frac{\partial \Gamma}{\partial y} & \frac{\partial \Gamma}{\partial z} \\ \frac{\partial \Gamma}{\partial y} & -\frac{\partial \Gamma}{\partial x} & 0 \\ \frac{\partial \Gamma}{\partial z} & 0 & -\frac{\partial \Gamma}{\partial x} \end{bmatrix}^{-1}$$

Then the eigenvalue decomposition of the modulation matrix is given by

$$\mathbf{M} = \mathbf{E}\mathbf{D}\mathbf{E}^{-1} \tag{8}$$

where D is a 3×3 diagonal matrix composed of the eigenvalues²⁸:

$$\begin{cases} \lambda_{1} = 1 - \frac{1}{\Gamma_{\rho}^{\frac{1}{p}}} \\ \lambda_{2} = 1 + \frac{1}{\Gamma_{\rho}^{\frac{1}{p}}} \\ \lambda_{3} = 1 + \frac{1}{\Gamma_{\rho}^{\frac{1}{p}}} \end{cases}$$
(9)

and $E = [e^1, e^2, e^3]$ is the matrix of eigenvectors with²⁸

$$\begin{cases}
e^{1} = \mathbf{n} \\
e^{2} = \left[\frac{\partial \Gamma}{\partial y}, -\frac{\partial \Gamma}{\partial x}, 0\right]^{\mathrm{T}} \\
e^{2} = \left[\frac{\partial \Gamma}{\partial z}, 0, -\frac{\partial \Gamma}{\partial x}\right]^{\mathrm{T}}
\end{cases}$$
(10)

when $||\xi|| \to \infty$, as Γ increases monotonically with $||\xi||$, thus $\Gamma \to \infty$. According to Eq. (9), \boldsymbol{D} approaches identity matrix, thus $\bar{\boldsymbol{u}}|_{\infty} = \boldsymbol{u}|_{\infty}$.

Theorem 1 can be applied to any flow filed. However, as the original flow field in the IFDS method is a sink, the impact of Eq. (6) on the velocity field of the sink should be investigated in depth. Without loss of generality, assume that the sink is located at origin with strength of *C*. When there is no obstacle, the velocity of the sink is

$$\mathbf{u}(x, y, z) = \left[\frac{Cx}{x^2 + y^2 + z^2}, \frac{Cy}{x^2 + y^2 + z^2}, \frac{Cz}{x^2 + y^2 + z^2}\right]^{\mathrm{T}}$$
(11)

As the only singular point (velocity is infinite at this point) in the flow field, origin is the convergent point of all streamlines in the field. When an obstacle exists, the following conclusion can be drawn.

Theorem 2. Assuming a sink flow is located at the origin and the origin is outside the obstacle, the interfered flow filed \bar{u} calculated by Eq. (6) has the following properties:

- (1) The velocity field always has a non-zero component directing toward the origin outside the obstacle.
- (2) Origin is still the only singular point of the interfered filed

Proof.

(1) According to Eq. (6), the interfered flow filed can be written as

$$\bar{u} = Mu = u + v \tag{12}$$

$$v = (M - I)u \tag{13}$$

where u is the sink velocity, and v the additional velocity resulting from the obstacle. As the component of the velocity field due to the sink is always directing toward the origin, it is sufficient to show that ||v|| < ||u||. From Eq. (5), there is

$$\mathbf{v} = (\mathbf{M} - \mathbf{I})\mathbf{u} = \frac{1}{\Gamma_{\rho}^{\perp}} \left(\mathbf{u} - \frac{2\mathbf{n}\mathbf{n}^{\mathrm{T}}}{\mathbf{n}^{\mathrm{T}}\mathbf{n}} \mathbf{u} \right)$$
 (14)

where the geometric meaning of $\frac{nn^T}{n^Tn}u$ is the projection of u on n. u is decomposed in the coordinate system defined by n and its vertical vector t (When $\Gamma(x, y, z) = 1, t$ represents the tangent vector on the surface of obstacle). Let the components be e and e_1 respectively, thus

$$u = e_{\perp} + e \tag{15}$$

where $e = \frac{nn^T}{n^Tn}u$ and $e^Te_{\perp} = 0$, as depicted in Fig. 4. \square

Substituting Eq. (15) into Eq. (14) yields

$$v = \frac{1}{\Gamma_{\rho}^{\perp}} (e_{\perp} + e) - \frac{2}{\Gamma_{\rho}^{\perp}} e = \frac{1}{\Gamma_{\rho}^{\perp}} (e_{\perp} - e)$$
 (16)

Thus

$$\|\mathbf{v}\| = \frac{1}{\Gamma_{\nu}^{\frac{1}{p}}} \|\mathbf{u}\| \tag{17}$$

when outside the obstacle, $\Gamma > 1$, therefore ||v|| < ||u||.

(2) Obviously, according to Eq. (5), $\Gamma^{\perp}_{n} \mathbf{n}^{\mathsf{T}} \mathbf{n} \neq 0$ when outside the obstacle, modulation matrix \mathbf{M} would not bring in new singular point for the interfered flow field. Therefore, origin is still the only singular point.

From Fig. 4, in the area far from the obstacle, $\Gamma^{\bar{l}}_{\bar{l}}$ is large and ||v|| is small, then the angle between \bar{u} and u is small and the streamline maintains the initial direction approximately along straight line. The closer to the obstacle, the larger the ||v|| becomes (According to Eq. (17)), which also makes \bar{u} closer to the direction of t gradually. Therefore, the streamline will change direction along the approximate tangent of obstacle. From Theorems 1 and 2, it can be concluded that the interfered flow field has the general features of the phenomenon of flowing stream avoiding obstacles, and the streamline not only avoids the obstacle, but also flows from start to end.

In Eq. (5), the larger the constant ρ , the larger the amplitude of the deflection to the original flow, ²⁸ and consequently

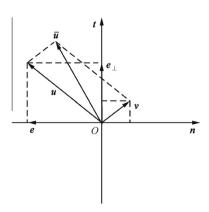


Fig. 4 Decomposition of flow velocity.

the earlier the UAV responds to the presence of an obstacle. Therefore ρ is called the reactivity parameter.

4.2. Avoidance of multiple obstacles

Consider there are K obstacles in the planning space, let the boundary equation of the kth obstacle be $\Gamma_k(x,y,z)=1$, $k=1,2,\ldots,K$. As in Theorem 1, the interfered velocity filed resulting from the kth obstacle alone can be calculated as

$$\bar{\mathbf{u}}_k(x, y, z) = \mathbf{M}_k(x, y, z)\mathbf{u}(x, y, z) \tag{18}$$

$$\boldsymbol{M}_{k}(x, y, z) = \boldsymbol{I} + \frac{1}{\Gamma_{k}^{\frac{1}{p_{k}}} \boldsymbol{n}_{k}^{\mathrm{T}} \boldsymbol{n}_{k}} (\boldsymbol{n}_{k}^{\mathrm{T}} \boldsymbol{n}_{k} \boldsymbol{I} - 2\boldsymbol{n}_{k} \boldsymbol{n}_{k}^{\mathrm{T}})$$

$$\tag{19}$$

where M_k and ρ_k are the modulation matrix and reactivity parameter of the kth obstacle, $n_k = \left[\frac{\partial \Gamma_k}{\partial x}, \frac{\partial \Gamma_k}{\partial y}, \frac{\partial \Gamma_k}{\partial z}\right]^T$. In order to guarantee the interfered flow filed to avoid all obstacles, weighted average of the velocity field is exploited which is similar to the stream function method. ²²

Theorem 3. Assuming a sink flow is located at the origin and the origin is outside all the obstacles, the velocity of interfered flow field $\bar{u}(x, y, z)$ caused by all the obstacles can be defined as

$$\bar{\boldsymbol{u}}(x,y,z) = \sum_{i=1}^{K} \omega_k \bar{\boldsymbol{u}}_k(x,y,z)$$
 (20)

$$\omega_k = \prod_{i=1, i \neq k}^K \frac{(\Gamma_i - 1)}{(\Gamma_k - 1) + (\Gamma_i - 1)}$$

$$\tag{21}$$

which has the following properties:

- (1) Impenetrable constraint of all the obstacles is satisfied.
- (2) The velocity field always has a non-zero component directed toward the origin outside the obstacles.
- (3) Origin is still the only singular point of the interfered filed.

Proof.

- (1) On the boundary of kth obstacle, $\Gamma_k(x,y,z) = 1$. From Eq. (21) there is $\omega_k = 1$ and $\omega_i = 0, i \neq k$, thus $\bar{\boldsymbol{u}}(x,y,z) = \bar{\boldsymbol{u}}_k(x,y,z)$. This indicates that $\bar{\boldsymbol{u}}(x,y,z)$ is exactly the same with the velocity field resulting from the kth obstacle alone. From the arbitrariness of k it can be seen that $\bar{\boldsymbol{u}}(x,y,z)$ satisfies impenetrable constraint of all the obstacles.
- (2) According to Theorem 2, for any point outside the obstacles, \bar{u}_k has a non-zero component directed toward the origin, and $\omega_k > 0$, which means \bar{u} also has a non-zero component directed toward the origin.
- (3) As $M_k(x, y, z)$ will not bring in new singular point for the interfered flow field, origin is still the only singular point.

According to Theorem 1, when $||\tilde{\xi}|| \to \infty$, $\bar{u}_k|_{\infty} = u|_{\infty}$, thus $\bar{u}|_{\infty} = \sum_{i=1}^K \omega_k u|_{\infty}$. Although $\bar{u}|_{\infty} \neq u|_{\infty}$ in this condition, the direction of interfered velocity filed is the same as the original one, therefor the streamline will maintain the initial direction. \square

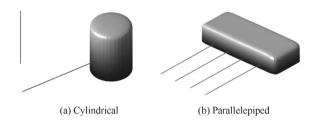


Fig. 5 Stagnation phenomenon in cylindrical and parallelepiped obstacles.

4.3. Extra control force method

Although the IFDS method can avoid local minima of an APF, stagnation point problem still exists, which means the velocity is 0 at some points in the flow field. When the UAV reaches these points, it will always stop there.

According to Theorem 3, as the velocity is nonzero in the area outside all the obstacles, stagnation points exist only on the boundaries of the obstacles. In addition, since the normal velocity vanishes at the boundary points, another condition of the stagnation point is \bar{u} aligned with the normal vector of the obstacle at a boundary point, namely $\cos\langle n, \bar{u} \rangle = \pm 1$. These two conditions are similar to stagnation point problem in the stream function. For a cylindrical obstacle, as shown in Fig. 5(a), if the straight line which connects the target point and the current point of UAV passes through the symmetry plane of the cylinder, stagnation point will be produced. For a parallelepiped obstacle, as shown in Fig. 5(b), as long as the UAV moves toward the direction vertical to the parallelepiped, the streamlines will not reach the target.

The stagnation point has similarities with local minima of an APF. In the APF, when the UAV, obstacle and target are aligned, the attractive force of the target and repulsive force

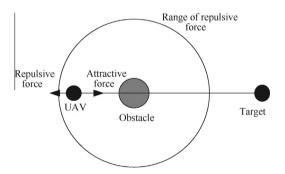
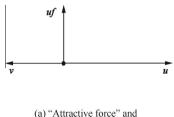


Fig. 6 Local minimal in APF.



"repulsive force" are aligned

of the obstacle are collinear at the opposite direction. As the repulsive force increases gradually, the two forces will be equal in magnitude and opposite in direction at some point, when local minimal happens, as shown in Fig. 6.

An effective way to solve local minima problem is that³¹: as long as the repulsive force aligns with the attractive force, extra control force is added to change the direction of the resultant force, escaping from the collinear state.

Based on the above idea, Eq. (12) can be analyzed from the perspective of "force": In this equation, u is the sink velocity, which can be seen as the "attractive force" pulling from the target; v is the addition velocity resulting from the obstacle. From Eq. (17), ||v|| has a maximum value on the obstacle boundary and vanishes in the infinite, which can be seen as the "repulsive force" repelling from the obstacle. As depicted in Fig. 7(a), in the planning process, as long as v and u are aligned, an extra control force is added represented by uf, which satisfies the following conditions:

$$uf \perp u$$
 (22)

$$||uf|| = ||v|| \tag{23}$$

Eq. (22) ensures that the component of extra control force along the direction of attractive force is always 0, avoiding penetration of the obstacle due to the increase of attractive force. Eq. (23) ensures that the further UAV is away from the obstacle, the smaller the extra control force is, avoiding large curve of the streamline. Another function of these two equations is: Once uf is added, v and u will not be aligned. Suppose the component of v along uf is vf, from Fig. 7(b), there is $||vf|| = ||v|| \cos \theta = ||uf|| \cos \theta < ||uf||$, which ensures that the resultant force of these three forces is always nonzero. uf is applied until the streamline avoids the obstacle.

Due to the particularity of 3D path planning, **uf** is located in the plane perpendicular to **u**. The direction of **uf** determines the direction of obstacle avoidance for UAV, that is whether to fly over or around the obstacle. As shown in Fig. 8, the direction of **uf** can be determined by the relative distance between

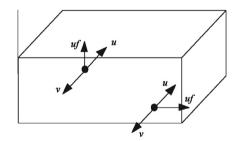
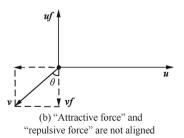


Fig. 8 Direction of extra control force.



Extra control force.

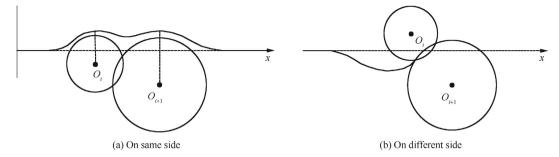


Fig. 9 Flow direction is on same and different sides of overlapped obstacles.

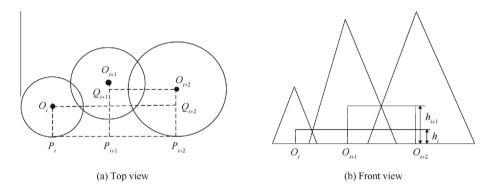


Fig. 10 Virtual obstacles in the case of three connecting cones.

the UAV and obstacle boundary: if the UAV is closer to the left and right boundaries, then *uf* is along the horizontal direction; if the UAV is closer to the top and bottom boundaries, then *uf* is along the vertical direction.

4.4. Virtual obstacles method

When the two obstacles O_i and O_{i+1} overlap, at the intersection points, $\Gamma_i = 1$ and $\Gamma_{i+1} = 1$. From Eq. (21), both the numerator and denominator of ω_i and ω_{i+1} become zero. Therefore for the case obstacles overlap, further processing is needed.

As shown in Fig. 9(a), when the flow direction is on the same side of obstacles, both O_i and O_{i+1} cause upward deformation to the streamline so that the streamline will not pass through the intersection points. Therefore Eq. (21) is still available and ensures that the streamline avoids these two obstacles.

As shown in Fig. 9(b), when the flow direction is on the different sides of obstacles, O_i causes downward deformation to streamline while O_{i+1} causes upward deformation. It makes the streamline moving toward the concave trap area and the streamline will be stuck at the intersection points. For this case, a virtual obstacle method is introduced. The general idea of setting virtual obstacles is to fill the concave trap area to prevent the streamline flow into it on the premise of trying not to waste flyable space. Taking the case with three connecting cones as an example, the virtual obstacle method works as follows.

As shown in Fig. 10, line P_iP_{i+2} is the common tangent of O_i and O_{i+2} , P_{i+1} is the perpendicular foot of O_{i+1} on P_iP_{i+2} , and Q_{i+1} and Q_{i+2} are the perpendicular foot of O_i and O_{i+2} on $O_{i+1}P_{i+1}$ and $O_{i+2}P_{i+2}$ respectively. The maximum height of the overlap region of O_i and O_{i+1} is h_i and the maximum

height of the overlap region of O_{i+1} and O_{i+2} is h_{i+1} . In order to avoid the streamline entering the concave trap area, two virtual parallelepiped obstacles are designed. The first one takes $O_iP_iP_{i+2}Q_{i+2}$ as bottom face and its height is h_i ; the second one takes $O_{i+2}P_{i+2}P_{i+1}Q_{i+1}$ as bottom face and its height is h_{i+1} . These two virtual obstacles can not only fill the concave trap effectively, but also ensure that the area between the cones is still available for UAV.

5. Optimization of reactivity parameter based on genetic algorithm

When there are multi-obstacles in environment, each obstacle has a reactivity parameter. Although the sizes and shapes of obstacles are determined before planning, a variety of streamlines will be obtained by changing the reactivity parameter of each obstacle. Some of the streamlines are not suitable for UAV and some are unreasonable. Therefore, genetic algorithm is employed to optimize the reactivity parameters.

5.1. Fitness function

Genetic algorithm is a kind of natural selection with stochastic method and it is an imitation of biological evolution. It uses the fitness function to connect the method and object, and this function is influenced by the following three factors.

(1) Length of path *D*. The fuel constraint can be transformed into this factor. To reduce the calculations, the length of path will not be calculated precisely, instead, the 3D Manhattan distance between discrete points will be used.

- (2) Altitude of flight *H*. Altitude of flight reflects the ability to follow the terrain. The lower the altitude is, the better the terrain following gets UAV constraints. The paths should satisfy UAV constraints. Here, the maximum climb altitude, maximum climb angle and maximum horizontal turning angle will be considered:
 - (1) Maximum climb altitude: Extract the heights (z coordinates) of the points in each streamline and compare them with the constraint of maximum altitude H. If all of the z coordinates satisfy $z \le H$, the streamline satisfies the maximum altitude constraint.
 - (2) Maximum climb angle: Assume θ_z is the angle between each route segment (composed of each two adjacent discrete points) and the horizontal plane (plane xy). Compare these angles with the constraint of the maximum climb angle α . If all of the θ_z satisfy $\theta_z \leq \alpha$, the streamline satisfies the maximum climb angle constraint.
 - (3) Maximum horizontal turning angle: Assume θ_{xy} is the angle between the projections of each two adjacent route segments in horizontal plane (plane xy). Compare these angles with the constraint of the maximum horizontal turning angle γ . If all of the θ_{xy} satisfy $\theta_{xy} \leq \gamma$, the streamline satisfies the maximum horizontal turning angle constraint.

In terms of UAV constraints, the goal of optimization is to find the path with the minimum cost. The fitness function is shown in Eq. (24).

$$\begin{cases} F = 1/J \\ J = \begin{cases} s_1 D^* + s_2 H^* & \text{Satisfy UAV constraints} \\ L + s_1 D^* + s_2 H^* & \text{Not satisfy UAV constraints} \end{cases}$$

The superscript * denotes the normalized value and J is the cost of path. s_1 and s_2 are the positive weighting factors and $s_1 + s_2 = 1$. L is a large positive number and it is the penalty term of UAV constraints. Then the evaluation of paths is transformed into a problem of unconstrained optimization. From Eq. (24), the greater the value of fitness function gets, the better the path is.

5.2. Algorithm procedure

The procedure of optimization by GA is as follows^{32,33}:

- **Step 1**. Determine the maximum number of iterations and the range of $\rho^1, \rho^2, \dots, \rho^K$.
- **Step 2**. Determine the coding scheme. Each parameter uses binary characters with *codel* bit to encode. Therefore, the total length of chromosome is $K \times codel$, which is denoted by $P = p_1 p_2 \cdots p_{codel}$.
- **Step 3**. Initial population formed by a size of individuals is randomly generated.
- **Step 4.** Calculate the fitness function F_i of each individual in the population.
- **Step 5**. Use fitness proportional selection method to choose the individual. According to the proportion of individual's fitness F_i in total fitness $\sum F$, the probability of copy of the individual is determined.

Step 6. Single-point crossover and Gaussian mutation operators³² are engaged in the genetic manipulation of the populations $P_i(k)$ to produce the next generation $P_i(k+1)$. **Step 7.** Repeat Step 4 to Step 6 until the parameter is not changed in 10 generations or reaches a preset maximum number of iterations.

6. Simulation and analysis

6.1. Obstacle avoidance streamline

Fig. 11 shows the streamline when the four different obstacles are presented alone. The analytical expressions of these four obstacles are as follows:

$$\begin{cases} x^{2} + y^{2} + \left(\frac{z}{1.5}\right)^{0.6} = 1\\ x^{2} + y^{2} + z^{2} = 1\\ x^{10} + y^{10} + \left(\frac{z}{2.5}\right)^{10} = 1\\ x^{6} + \left(\frac{y}{3}\right)^{6} + z^{6} = 1 \end{cases}$$
(25)

After the flow velocity through Eq. (6) is implemented in MATLAB, the streamline can be calculated by using the Euler method for numerical integration. In Fig. 11(c) and Fig. 11(d), the dotted line is the streamline without the extra control force method and the solid line is the streamline after using the method. The figure shows that the streamlines avoid the stagnation problem smoothly.

Fig. 12 shows the result when obstacles overlap and three cone obstacles are used to imitate connected mountains. The solid line is the streamline using the virtual obstacles method,

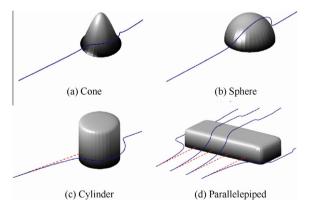


Fig. 11 Streamlines avoiding single obstacle.

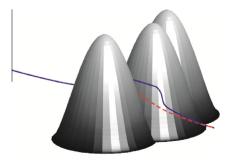


Fig. 12 Streamline when obstacles overlap.

while the dotted line is on the contrary. In this figure one can see that the streamline in solid line can cross the area between the mountains and avoid collision with them.

6.2. Multiple obstacles avoidance of flight path

To verify the effectiveness of the IFDS method, analytical method and numerical method in Ref.²⁶ are employed for comparison. In this section, the reactivity parameters are not going to be optimized, only one path under a set of reactivity parameters which satisfies the maneuverability constraints is selected.

Fig. 13(a) shows the paths calculated by the analytical method and the proposed method, the range for path planning is $40 \times 40 \times 20$, the starting point is (0,0), the finishing point is (30,30) and the initial flight height is 2. As the analytical method can only handle spherical obstacles, the planning environment is composed of 5 spherical obstacles with different radii. The time consumptions of these two methods in MAT-LAB are 0.04 s and 0.11 s respectively, both showing high computation efficiency. As shown in Fig. 13(a), the paths generated by the two methods are basically the same. The paths change gently when close to the obstacles. After passing by

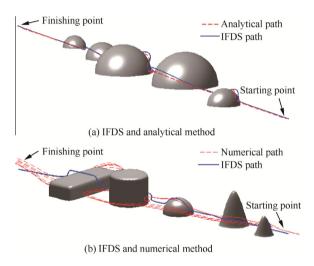


Fig. 13 Flight paths generated from IFDS and analytical/numerical method.

the obstacles, they maintain the initial direction along straight line and finally reach the destination smoothly. The result implies that even though the path generated by our proposed method is not obtained by solving the Laplace's Equation, it still has the properties of the analytical path.

Fig. 13(b) shows the paths calculated by the numerical method and the proposed method. As the numerical method can handle more complex terrain and obstacles, the spheres in Fig. 13(a) are substituted into obstacles in different shapes. In addition, as the computation domain in numerical method is the whole planning environment, there would be multiple paths in it. After eliminating the streamlines which disconnect between the starting point and finishing point and those obviously not suitable for UAV, all feasible paths are shown in dotted line. One can observe that there is a big difference between the paths generated by each method. The shapes of the obstacles are more complex than the spherical ones; after passing by the first two cones, the streamlines in numerical method cannot return to the initial flow direction quickly, making the latter

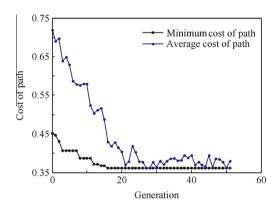


Fig. 15 Change of cost of path in optimization process.

Table 1	Cost of each path.				
Type	Path I	Path II	Path III	Path IV	Path V
D	0.4953	0.3771	0.5645	0.3814	0.4639
H	0.5352	0.5387	0.2210	0.5340	0.1214
J	0.5073	0.4256	0.4614	0.4272	0.3612

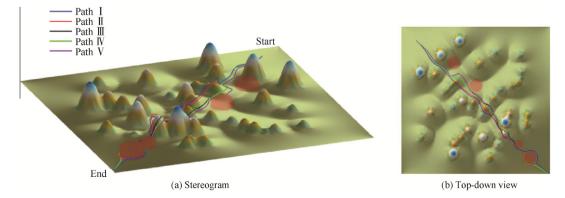


Fig. 14 3D flight path with different reactivity parameters.

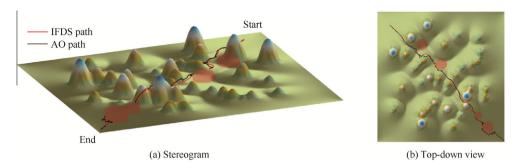


Fig. 16 Comparison between IFDS path and AO path.

half of the streamlines pass the obstacles from the side. This leads to long paths, and the terrain is not fully exploited either. Meanwhile, the presence of the parallelepiped obstacle makes the streamlines hard for UAV to follow. Thus, although the numerical method can better reflect the characteristics of the fluid, from the perspective of terrain following and path optimality, the path proposed in this paper is more suitable for UAV compared to the path by numerical method.

6.3. Optimal flight path in complex environment

In this section, the proposed method is studied in complex environment for full verification. Here the terrain map in Section 3 is still used.

The 3D flight paths in this environment are shown in Fig. 14. There are five paths in it obtained by combining different reactivity parameters. Path I–IV are the typical 3D path without optimizing the reactivity parameters and path V is the optimal path after optimization of the reactivity parameters by GA. The settings of the GA are as follows: population size = 40, chromosome length = 5, the range of reactivity parameter is 0.5–30, and the weighting factors in fitness function are $s_1 = 0.7$, $s_2 = 0.3$.

Fig. 15 shows the change of cost in the process of optimization where the black line represents the minimal cost in each generation and the blue line represents the average cost in each generation. The cost of path is normalized which ranges from 0 to 1. After 51 steps of iteration, the algorithm converges, and the minimal cost of path decreases (0.45-0.36)/0.45=20%. The calculation corresponds to 2.8 min computation time in a Intel Core 2 (E6600 2.4 GHz) PC, which is fast enough for off-line planning in such a complex terrain. Actually, with some feasible solutions, the convergence speed of the GA algorithm can be further improved.

For the convenience of comparing each path in Fig. 14, the length of path, the altitude of flight and the cost of path are listed in Table 1. The simulation result shows that using GA to optimize the reactivity parameters can guarantee the flying quality and achieve optimality.

6.4. Comparison with ant optimization algorithm

As a new path planning method, the proposed method has great advantages over other ones. In this section, modified ant optimization (AO) algorithm⁴ is chosen for comparison. The AO algorithm works well on known terrain and it can also provide optimal path under the maneuverability constraints of UAV.

The performance index of AO algorithm represents the trends of searching. To ensure the comparability, Eq. (24) is still used. The parameters in simulation are: number of ants is 300, number of searches is 500 and other parameters in AO algorithm are the same as those in Ref.⁴

As shown in Fig. 16, the red line is the optimal IFDS path (Path V) in Fig. 14 and the black line is the AO path. The cost values of AO path are D=0.4677, H=0.1383 and J=0.3689. These two paths are very similar but the IFDS path is smoother than the AO one, which is more suitable for UAV. On the other hand, AO algorithm is able to find the global optimum theoretically. However, the process will consume more time compared with the proposed method (The AO algorithm takes 4.7 min in this terrain). Based on the simulation results, the IFDS method demonstrates its superiority over the AO method.

7. Conclusions

- (1) Inspired by natural principles of stream avoiding obstacles, a path planning method based on interfered dynamical system is proposed. This method can obtain optimal 3D path which not only satisfies the maneuverability constraints of UAV, but also has the natural characteristics of flow around obstacles.
- (2) The proposed method is simple in environmental modeling and shows great advantages in dealing with complex terrain. In addition, it maintains the path feature and the advantages of a small amount of computation in analytical method. Therefore, this method greatly expands the application of the method of fluid flow and is well positioned to meet the requirements of 3D path planning.

The presented work is focused on off-line path planning under static obstacles. Our future work will concentrate on applying this method to on-line path planning under uncertain terrain/threat and extending it to cooperative path planning of UAVs.

Acknowledgment

This research was supported by the National Natural Science Foundation of China (No. 61175084).

References

 Yang J, Dymond P, Jenkin M. Practicality-based probabilistic roadmaps method. 2011 Canadian conference on computer and robot vision. NL, Canada: IEEE; 2011. p. 102–8.

- Zucker M, Kuffner J, Branicky M. Multipartite RRTs for rapid replanning in dynamic environments. 2007 IEEE international conference on robotics and automation. Roma, Italy: IEEE; 2007. p. 1603-9
- 3. Mujumdar A, Padhi R. Evolving philosophies on autonomous obstacle/collision avoidance of unmanned aerial vehicles. *J Aerosp Comput Inform Commun* 2011;8(2):17–41.
- 4. Chen M, Wu Q, Jiang C. A modified ant optimization algorithm for path planning of UCAV. *Appl Soft Comput* 2008;8(4):1712–8.
- Zhao L, Murthy VR. Optimal flight path planner for an unmanned helicopter by evolutionary algorithms. *AIAA guidance,* navigation and control conference and exhibit. South Carolina, USA: AIAA; 2007. p. 3716–39.
- Hao Y, Zu W, Zhao Y. Real-time obstacle avoidance method based on polar coordination particle swarm optimization in dynamic environment. 2007 2nd IEEE conference on industrial electronics and applications. Piscataway, NJ, USA: IEEE; 2007. p. 1612–17.
- Foo JL, Knutzon JS, Kalivarapu V, Oliver JH, Winer EH. Threedimensional path planning of unmanned aerial vehicles using particle swarm optimization. 11th AIAA/ISSMO multidisciplinary analysis and optimization conference; Portsmouth, Virginia, USA: AIAA; 2006. p. 992–1001.
- Duan HB, Yu YX, Zhang XY. Three-dimension path planning for UCAV using hybrid meta-heuristic ACO-DE algorithm. Proceedings of Asia simulation conference-7th international conference on system simulation and scientific computing. Beijing, China: IEEE; 2009. p. 919–24.
- Darrah MA, Niland WM. Increasing UAV task assignment performance through parallelized genetic algorithms. AIAA 2007 infotech@aerospace conference and exhibit. California, USA: AIAA; 2007. p. 7–10.
- Frazzoil E, Dahleh MA, Feron E. Maneuver-based motion planning for nonlinear systems with symmetries. *IEEE Trans Rob* 2005;21(6):1077–91.
- Karimi J, Pourtakdoust SH. Optimal maneuver-based motion planning over terrain and threats using a dynamic hybrid PSO algorithm. *Aerosp Sci Technol* 2013;26(1):60–71.
- Gong Q, Lewis R, Ross M. Pseudospectral motion planning for autonomous vehicles. J Guid Control Dynam 2009;32(3):1039–45.
- Dever C, Mettler B, Feron E, Popovic J, McConley M. Nonlinear trajectory generation for autonomous vehicles via parameterized maneuver classes. *J Guid Control Dynam* 2006;29(2):289–302.
- Nikolos IK, Valavanis KP, Tsourveloudis NC, Kostaras AN. Evolutionary algorithm based offline/online path planner for UAV navigation. *IEEE Trans Syst Man Cybern Part B: Cybern* 2003;33(6):898–912.
- Mattei M, Blasi L. Smooth flight trajectory planning in the presence of no-fly zones and obstacles. J Guid Control Dynam 2010;33(2):454–62.
- 16. Khaitib O. Real-time obstacle avoidance for manipulators and mobile robots. *Int J Robot Res* 1986;**5**(1):90–8.
- Paul T, Krogstad TR, Gravdahl JT. Modeling of UAV formation flight using 3D potential field. Simul Model Pract Theory 2008;16(9):1453–62.
- Scherer S, Singh S, Chamberlain L, Elgersma M. Flying fast and low among obstacles: methodology and experiments. *Int J Robot Res* 2008;27(5):549–74.

- Kim J, Khosla P. Real-time obstacle avoidance using harmonic potential functions. *IEEE Trans Robot Autom* 1992;8(3):338–49.
- Park MG, Jeon JH, Lee MC. Obstacle avoidance for mobile robots using artificial potential field approach with simulated annealing. *IEEE international symposium on industrial electronics*. Pusan, Korea: IEEE; 2001. p. 1530–5.
- Agirrebeitia J, Aviles R, Bustos IF, Ajuria G. A new APF strategy for path planning in environments with obstacles. *Mech Mach Theory* 2005;40(6):645–58.
- Waydo S, Murray RM. Vehicle motion planning using stream functions. *Proceedings of the IEEE international conference on robotics and automation*; Taipei, Taiwan: IEEE; 2003. p. 2484–91.
- Sullivan J, Waydo S, Campbell M. Using stream functions for complex behavior and path generation. *AIAA guidance, navigation* and control conference and exhibit. Austin, Texas, USA: AIAA; 2003. p. 1–9.
- 24. Campbell ME, Lee J, Scholte E, Rathbun D. Simulation and flight test of autonomous aircraft estimation, planning, and control algorithms. *J Guid Control Dynam* 2007;30(6):1597–609.
- Daily R, Bevly DM. Harmonic potential field path planning for high speed vehicles. *Proceedings of 2008 American control conference.* Seattle, WA, USA: IEEE; 2008. p. 4609–14.
- 26. Liang X, Wang HL, Li DW, Lv WT. Three-dimensional path planning for unmanned aerial vehicles based on principles of stream avoiding obstacles. *Acta Aeronaut Astronaut Sin* 2013;34(7):1670–81 Chinese.
- Philip JP. Fluid mechanics: SI version. 8th ed. Washington: Wiley; 2011. p. 401–5.
- Khansari-Zadeh SM, Billard A. A dynamical system approach to realtime obstacle avoidance. *Auton Robot* 2012;32(4):433–54.
- 29. Isil H, Haluk RT, Murat E. 3-D path planning for the navigation of unmanned aerial vehicles by using evolutionary algorithms. Proceedings of the 10th annual conference on genetic and evolutionary computation. New York, USA, ACM; 2008. p. 1499–1506.
- Terrain Generation Tutorial [Internet]. Available from: http://www.stuffwithstuff.com/robot-frog/3d/index.html.
- Zhang JY, Zhao ZP, Liu D. A path planning method for mobile robot based on artificial potential field. J Harbin Inst Technol 2006;38(8):1306–9 Chinese.
- Min HQ, Bi S, Chen YP. Motion planning of robot collision avoidance based on multi-objective genetic algorithm. *J Harbin Inst Technol* 2005;37(7):915–7 Chinese.
- Li Q, Wang LJ, Chen B. An improved artificial potential field method with parameters optimization based on genetic algorithms. J Univ Sci Technol Beijing 2012;34(2):202–6 Chinese.

Wang Honglun is a professor and Ph.D. supervisor at School of Automation Science and Electrical Engineering, Beihang University. He received the Ph.D. degree from Northwestern Polytechnical University in 1998. His main research interests include autonomous flight control for unmanned aerial vehicles, sense and avoidance technology for unmanned aerial vehicles, fire control and automated aerial refueling.