Continuous Probability Distributions

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1 Continuous Uniform Distribution

This distribution has constant probability in a closed interval. Its p.d.f is given by

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & otherwise \end{cases}$$

2 Normal distribution

This is basically the most important distribution in the field of statistics. It can be defined as:

Definition 1 (Normal variate)**.** *A continuous RV with two parameters μ and σ with the p.d.f*

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2}, -\infty < x < \infty$$

Is called a normal variate and the distribution is called normal distribution.

The constants of normal distribution are as follows:
The mean is μ, the variance σ^2, the odd moments about mean are 0 and the even moments are given by

$$\mu_{2n} = (2n-1)\sigma^2\mu_{2n-2}$$

The M.G.F about origin is given by

$$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

The sum of two independent normal variates is also a normal variate with mean and variance as the sums of the individual means and variances.

2.1 Standard Normal Variate

If X is a normally distributed RV with mean μ and variance σ^2, and we define $Z = \frac{X-\mu}{\sigma}$, then Z is a normal variate with mean 0 and variance 1. This is called the *standard normal variate* and its p.d.f is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, -\infty < z < \infty$$

2.2 Area property of normal curve

The probability of a normal variate lying between two values $x_1 and x_2$ is given by the area under the normal curve from $x_1 to x_2$. That is,

$$P(x_1 \leq X \leq x_2) = P(z_2) - P(z_1)$$

where $P(z)$ is the *normal definite integral* and gives the area under standard normal curve between $Z = 0$ and $Z = z$.

Theorem 1. *68-95-99.7 rule: States that almost all values of a normal distribution lie within three standard deviations of the mean: 68% lie within the first stddev, 95% in the second, and 99.7% in the third.*

3 Exponential distribution

The exponential distribution is given by

$$f(x) = ae^{-ax}$$

The mean is given by $\frac{1}{a}$, variance $\frac{1}{a^2}$ and the M.G.F about origin is

$$M_X(t) = \sum_{r=0}^{\infty} (\frac{t}{a})^r, t < a$$

4 Weibull distribution

The Weibull distribution is basically the exponential distribution with parameters α and β. It is given by

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}}, x > 0, \alpha > 0, \beta > 0 \\ 0, otherwise \end{cases}$$

Setting $\beta = 1$ in this yields the exponential distribution.