

# Random Variables and Expectation

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## 1 Random variables

**Definition 1** (Random variable). *A function that assigns a real value  $X(w)$  to every outcome  $w$  in the sample space.*

Two types: discrete and continuous.

1. Distribution function for discrete RV:

$$F(x) = \sum_{i: x_i \leq x} p(x_i)$$

where  $p(x_i)$  is the probability mass function (basically probability value).

2. Distribution function for continuous RV:

$$F(x) = \int_{-\infty}^x f(x) dx$$

where  $f(x)$  is the probability density function.

## 2 Jointly distributed random variables

For two RVs  $X$  and  $Y$  associated with the same random experiment,

1. when  $X$  and  $Y$  are discrete:

Probability mass function:

$$p(x_i, y_i) = P(X = x_i, Y = y_i)$$

and

$$\sum_x \sum_y p(x_i, y_j) = 1$$

Marginal probability mass functions are given by:

$$p(x_i) = P(X = x_i) = \sum_j P(X = x_i, Y = y_j) = \sum_j p_{ij}$$

$$p(y_j) = P(Y = y_j) = \sum_i P(X = x_i, Y = y_j) = \sum_i p_{ij}$$

2. when  $X$  and  $Y$  are continuous:

$x$