School of Computing
National University of Singapore
Biometrics Course
July 2017

Assignment #4
Deadline: 2:00pm on 26 July 2017.

Part I

Q1. (5) A four-sided die (whose faces are numbered from 1 to 4) is biased such that the probability of getting the number n in a single roll is kn, where k is some constant.

(a). Deduce that $k = 1/10$.

(b). The die is rolled twice. Let X and Y denote the number of the first and second rolls respectively. Determine the joint probability mass function (pmf): $P_{X,Y}(x, y)$.

(c). Let $Z = \min(X, Y)$. Find the pmf of Z, $P_Z(z)$.

Q2. (4) A prize is hidden behind one of three doors A, B, and C. The contestant picks a door, say A, but it is left closed. The host (who knows the actual location of the prize, but will never reveal the prize at this stage of the game) opens door C and shows that there is no prize behind it. Should the contestant change his mind and select door B?

Formulate this problem as a Bayesian inference. What is the prior probability of the prize being behind each door (i.e., A, B, or C) before door C was opened? What is the posterior probability after door C has been opened? What should be the contestants decision?

Q3. (9) The probability that a store will have exactly k customers on any given day is

$$P_K(k) = \frac{1}{5}\left(\frac{4}{5}\right)^k, \quad k = 0, 1, 2, 3, \ldots$$

Every day, out of all the customers who purchased something from the store that day, one is randomly chosen to win a prize. Every customer that day has an equal chance of being chosen. Assume that no customer visits this store more than once a day, and further assume that the store can handle an infinite number of customers.

(a). Prove that

$$\sum_{k=1}^{\infty} \frac{1}{k} z^k = \log\left(\frac{1}{1-z}\right), \quad \text{if } |z| < 1$$

where log is the natural logarithm.

(b). What is the probability that a customer selected randomly from the population of all customers will win a prize?

(c). Given a customer who has won a prize, what is the probability that he was in the store on a day when it had a total of exactly k customers?

Since the store owner's birthday is in July, he decides to celebrate by giving two prizes each day in the month of July. At the end of each day, after one customer is randomly chosen to win the first prize, another winner is randomly chosen from the remaining $k-1$ customers, and given the second prize. No one can win both prizes.

Let X denote the customer number of the first winner, and let Y denote the customer number of the second winner.

(d) Determine the joint pdf, $P(x, y \mid k)$, for $1 \leq x, y \leq k$.

Part II

Face Detection (3)

In this task, you will learn face detection with OpenCV. OpenCV provides a Haar-cascade Classifier for face detection, which is a good start for you to explore face recognition. A function is defined in partII.py for you to perform face detection on an image. Read the code and understand it. Use the function to explore the capability of the Haar-cascade face detector.

Q4. The pose of the head can affect the performance of a face detector. Apply the face detection function on test1.png, test2.png and test3.png, and compare the results.

Q5. What other factors do you think can affect the performance of the face detector? Confirm your answers by using your own images, and showing the success or failure of the detector on your images. (At least two other factors).

Part III

Geometric Transform (4)

This task helps you understand the geometric transformation. Given a 2D point P at (P_x, P_y), you are now going to rotate it around the centre point C at (C_x, C_y). Follow these instructions:

1. Transform the original coordinates system to the one originated at C by $T_1 = \begin{bmatrix} 1 & 0 & -C_x \\ 0 & 1 & -C_y \\ 0 & 0 & 1 \end{bmatrix}$
2. Rotate the points by θ counter-clockwise by $R = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. Transform the altered coordinates system to original coordinates by $T_2 = \begin{bmatrix} 1 & 0 & C_x \\ 0 & 1 & C_y \\ 0 & 0 & 1 \end{bmatrix}$

4. Thus the transformation result for P would be computed by $\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = T_2 R T_1 \begin{bmatrix} P_x \\ P_y \\ 1 \end{bmatrix}$. The coordinates for P' is $(\frac{x'}{z'}, \frac{y'}{z'})$. Pay attention to the order of T_1, R and T_2.

Let coordinates for P, C be $(2, 3)$, $(3, 2)$, and $\theta = \frac{i}{4}\pi, i = 1, 2, \ldots, 7$. Use `matplotlib` to draw P and all its transformed results in a same figure. Save the figure and submit it.

Submission

For Part I, submit your handwritten, hardcopy solutions to the TA. Please write legibly. For Part II, submit your answers and images in a pdf file. For Part III, submit your Python code and figures (images). Zip all your files from Parts II and III into a single zip file, naming it: `XXX_assignment4.zip`, where `XXX` is your name in English. Please submit by the deadline stated at the start of this document.