Part 1

Assigment 4 Part I. — Long Zhang — gstCN0342

Q1. (a). summation of probability equals to 1.

$\therefore k+2k+3k+4k = 1 = 10k \quad \therefore k = \frac{1}{10}$

(b) $P_{X,Y}(x,y) = P(X=X \cap Y=y)$, their joint probability distribution is:

X \ Y	1	2	3	4
1	1/100	2/100	3/100	4/100
2	2/100	4/100	6/100	8/100
3	3/100	6/100	9/100	12/100
4	4/100	8/100	12/100	16/100

therefore, their joint pmf is:

$P_{X,Y}(x,y) = P(X=x)P(Y=y) = \frac{1}{10}x \cdot \frac{1}{10}y$

$\therefore P_{X,Y}(x,y) = \frac{xy}{100} \quad x=1,2,3,4,\ y=1,2,3,4$

(c) From (b), we know $Z=\min(X,Y)$ takes these values:

Z \ X \ Y	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	2	3	3
4	1	2	3	4

$\therefore$ pmf of Z, $P_Z(Z)$

$P_Z(Z=1) = 7/16$

$P_Z(Z=2) = 5/16$

$P_Z(Z=3) = 3/16$

$P_Z(Z=4) = 1/16$

Q2. Say A,B,C means prize on behind door A,B,C; a,b,C means host open door A,B,C. w for win the prize.

known: contestant select door A, then host open door C.

⇒ prior probability of win the prize before door C open:

$$P(w) = \frac{1}{3}$$

⇒ posterior probability of win the prize after door C opened:

if prize on A:

$$P(A|c) = \frac{P(A)P(c|A)}{P(c)} = \frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{1}{2}} = \frac{1}{3}$$

if prize on B:

$$P(B|c) = \frac{P(B)P(c|B)}{P(c)} = \frac{\frac{1}{3}\cdot 1}{\frac{1}{2}} = \frac{2}{3}$$

$\because P(A|C) < P(B|c)$.

$\therefore$ The contestant should change his mind to select door B.

Q3. (a) suppose $0 \le t \le z < 1$

we have $\int_0^z \sum_{k=0}^{\infty} t^k dt = \sum_{k=0}^{\infty} \int_0^z t^k dt = \sum_{k=1}^{\infty} \frac{z^k}{k}$

$\because \sum_{k=0}^{\infty} t^k = \frac{1}{1-t}$ and $\int_0^z \frac{1}{1-t} = -\log(1-z) = \log\frac{1}{1-z}$

similar for $-1 < z \le 0 \Rightarrow \therefore \sum_{k=1}^{\infty} \frac{z^k}{k} = \log\frac{1}{1-z}$

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Q3. (b) If store have 0 costomers. then no one will win the prize, let's say, it's event A.

$$P(A) = \frac{1}{5}\left(\frac{4}{5}\right)^0 = \frac{1}{5}$$

The mutually exclusive event of A is costomer select randomly from the population of all costomers will win the prize.

$$P(A') = 1 - P(A) = \frac{4}{5}$$

(c). Let's call the event of a prize is pr.

~~Given a customer who has won a prize in a day in k customers~~

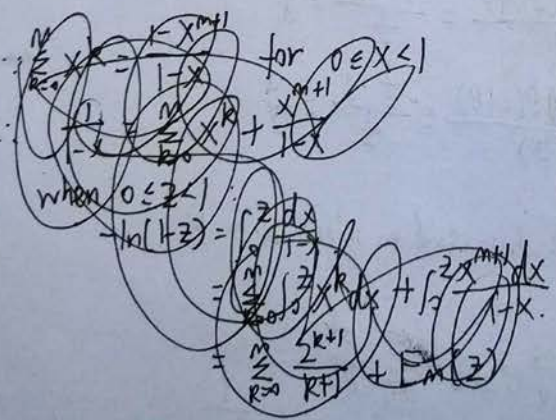
$\therefore P(pr \cap K=k) = \frac{1}{k}\frac{1}{5}\left(\frac{4}{5}\right)^k$ $k = 0, 1, 2, 3 \cdots$

$\therefore$ and we also have $P(pr \mid K=k) = \frac{1}{k}$

$$\therefore P(K=k \mid pr) = \frac{P(pr \mid K=k)P(K=k)}{\sum_{i=0}^{\infty} P(pr \mid k=i)P(K=i)} = \frac{\frac{1}{k}\frac{1}{5}\left(\frac{4}{5}\right)^k}{\sum_{i=1}^{\infty} \frac{1}{i}\frac{1}{5}\left(\frac{4}{5}\right)^i}$$

(d). $P(x,y \mid k) = P(X=x, Y=y \mid K=k)$ is only non-zero when $k \geq 2$ and $x \neq y$ and $1 \leq x, y \leq k$.

$$\therefore P(x,y \mid k) = \frac{1}{k}\frac{1}{k-1} \qquad k \geq 2,\ x \neq y,\ 1 \leq x, y \leq k.$$



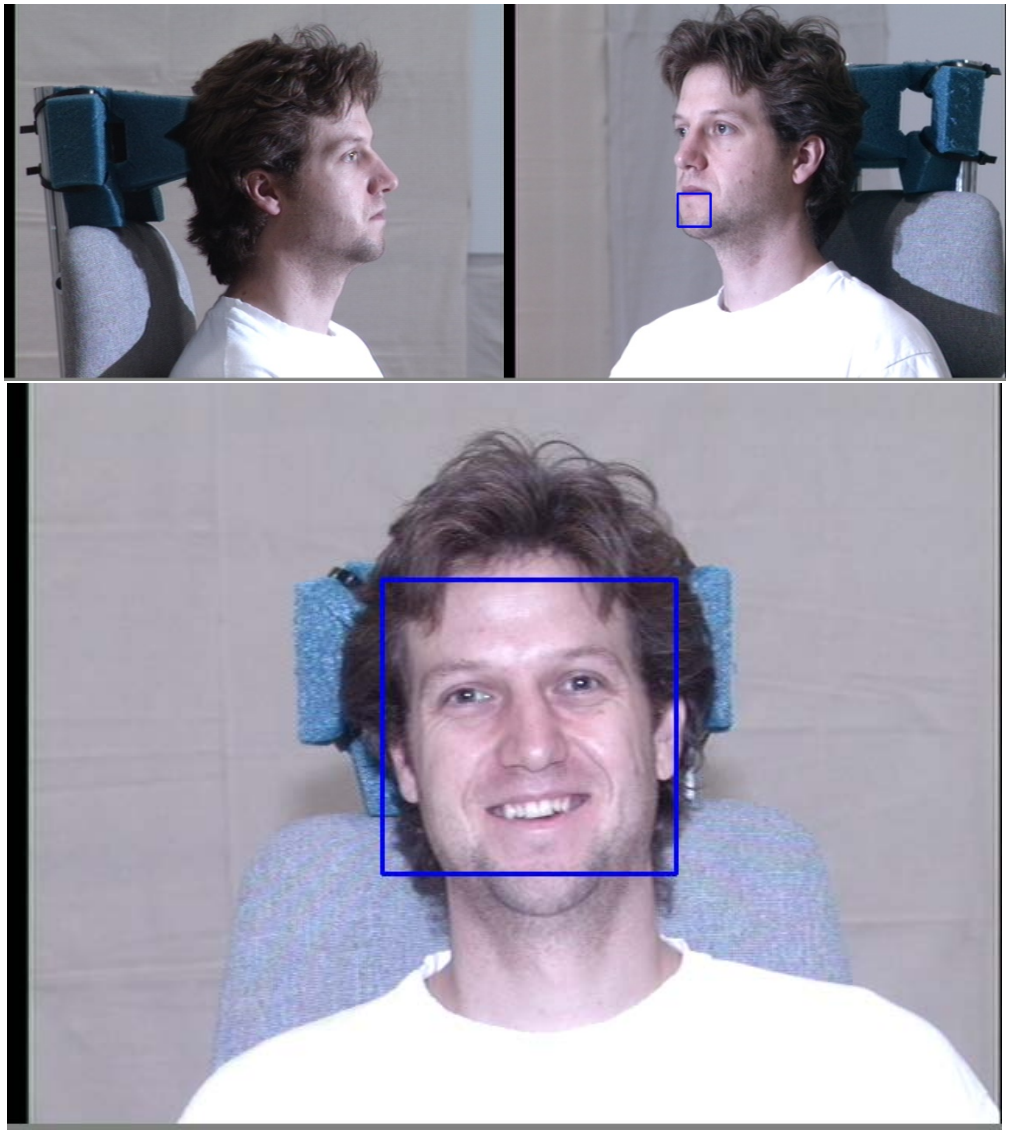
Part 2

Q4. The pose of the head can affect the performance of a face detector. Apply the face detection function on test1.png, test2.png and test3.png, and compare the results.

For test1.png, algorithm didn't recognize the face in this picture. It's a complete side face. Easy for human, seems hard for computers.

For test2.png, face turned outside a little, but algorithm recognized a wrong position.

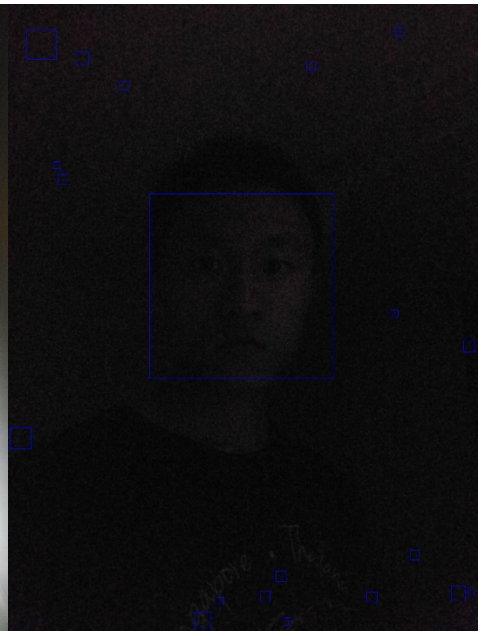
For test3.png, algorithm successfully recognized the face, the front face, most clearly.

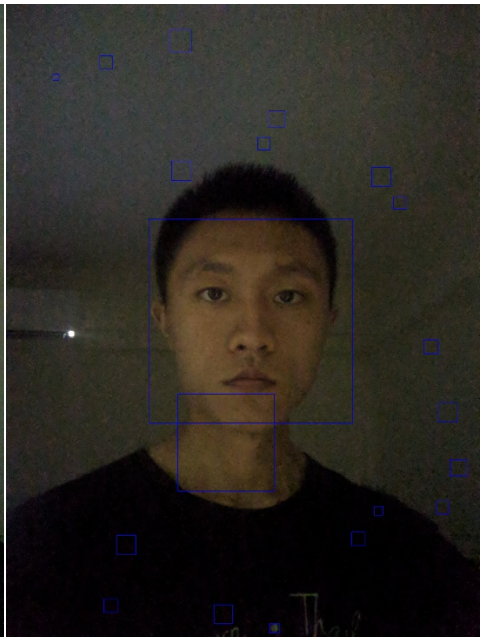


Q5. What other factors do you think can affect the performance of the face detector? Confirm your answers by using your own images, and showing the success or failure of the detector on your images. (At least two other factors).

The light will affect. (use my photos below to test)

Seems algorithm perform well in dark environment, but worse in highlight environment.

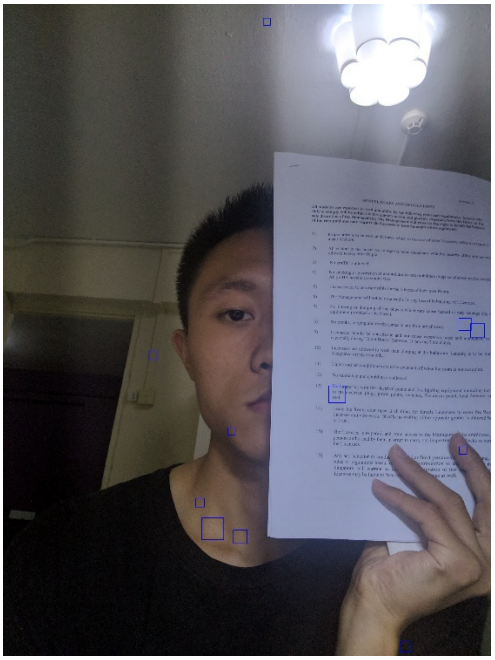




Although, there are noisy in dark room, but algorithm still recognize the most important part--face

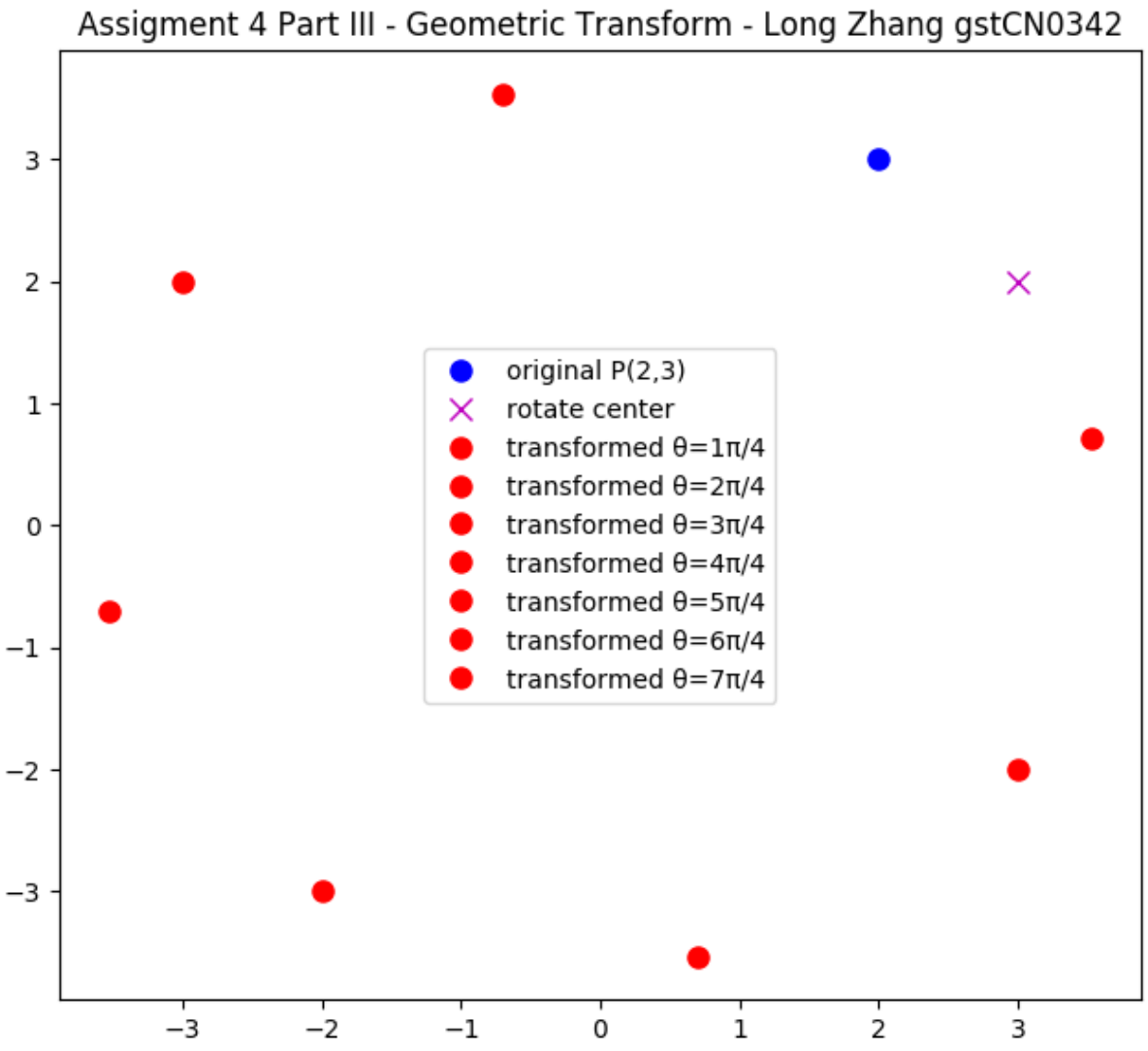
The other thing that will affect face reorganization are wearing objects, ornaments, etc. These objects that will block the facial features.

Like my picture below, when I wear sunglasses and a hat, wow, it didn't recognize it is me!



For the others(untested), I think people's emotion on their face will also matters.

Part 3


Assigment 4 Part III - Geometric Transform - Long Zhang gstCN0342
original P(2,3)
rotate center
transformed θ=1π/4
transformed θ=2π/4
transformed θ=3π/4
transformed θ=4π/4
transformed θ=5π/4
transformed θ=6π/4
transformed θ=7π/4
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3