

## 200643 - MMIO - Models and Methods From Operations Research

Coordinating unit: 200 - FME - School of Mathematics and Statistics  
Teaching unit: 715 - EIO - Department of Statistics and Operations Research  
Academic year: 2017  
Degree: MASTER'S DEGREE IN STATISTICS AND OPERATIONS RESEARCH (Syllabus 2013). (Teaching unit Compulsory)  
ECTS credits: 5 Teaching languages: Spanish

### Teaching staff

Coordinator: ELENA FERNÁNDEZ AREIZAGA  
Others: Primer quadrimestre:  
MARIA ALBAREDA SAMBOLA - A, B  
ELENA FERNÁNDEZ AREIZAGA - A, B  
ALMA CRISTINA NÚÑEZ DEL TORO - B  
JESSICA RODRÍGUEZ PEREIRA - A

### Opening hours

Timetable: Previous appointment.

### Prior skills

Each student may choose between two different levels for the course: introductory or advanced, depending on her/his interest and previous knowledge on Operations Research. The introductory level is followed with topics 1-5. Alternatively, the students who choose the advanced level, will only follow the topic 6 (Advanced models and methods of Integer and Combinatorial Optimization).

The level of items 1-5 of the course, are basic and follow to a large extent the books

- Luenberger, David G; Ye, Yinyu. Linear and nonlinear programming [en línea]. 3rd ed. New York: Springer, cop. 2008. ISBN 978-0-387-74502-2.

- Ahuja, Ravindra K; Magnanti, Thomas L; Orlin, James B. Network flows : theory, algorithms, and applications. Englewood Cliffs, N.J.: Prentice Hall, cop. 1993. ISBN 013617549X.

The level of item 6, as well as its content follow, to a large extent, the text:

Laurence Wolsey. Integer Programming.

Wiley-Interscience series in discrete mathematics. John Wiley and Sons. New York. 1998. ISBN: 0-471-28366-5.

### Requirements

In order to follow properly this course and obtain its maximum output it is necessary to have previous basic knowledge on calculus with one and several variables, and to have basic knowledge of matrices and bases in vector spaces. It is highly recommended to know some basic programming techniques.

Topic 6 of the course has a higher level. In order to follow it properly and obtain its maximum output it is necessary either to have followed previously topics 1-5, or to have basic knowledge of modeling techniques and models in Operations Research and of Linear Programming.

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### Degree competences to which the subject contributes

#### Specific:

3. CE-2. Ability to master the proper terminology in a field that is necessary to apply statistical or operations research models and methods to solve real problems.
4. CE-3. Ability to formulate, analyze and validate models applicable to practical problems. Ability to select the method and / or statistical or operations research technique more appropriate to apply this model to the situation or problem.
5. CE-5. Ability to formulate and solve real problems of decision-making in different application areas being able to choose the statistical method and the optimization algorithm more suitable in every occasion.

Translate to english

6. CE-6. Ability to use appropriate software to perform the necessary calculations in solving a problem.

#### Transversal:

1. TEAMWORK: Being able to work in an interdisciplinary team, whether as a member or as a leader, with the aim of contributing to projects pragmatically and responsibly and making commitments in view of the resources that are available.
2. EFFECTIVE USE OF INFORMATION RESOURCES: Managing the acquisition, structuring, analysis and display of data and information in the chosen area of specialisation and critically assessing the results obtained.

### Teaching methodology

#### Theoretical sessions:

Lectures in which the topics of the syllabus are introduced and discussed. Slides will be used for some topics, while others will be dealt on the board. The faculty intranet will be used for making available teaching material related with the course: notes for some topics, resolved problems and previous exams.

#### Problem-solving sessions:

Classes in which numerical problems concerning the subjects studied in the theory sessions are posed and solved. Students are given a certain amount of time to solve problems themselves, and then the problems will be resolved and discussed collectively.

#### Lab:

There will be lab sessions in order to introduce students to practical implementation and solution of Operations Research models using available software.

#### Practicals:

Item 6 of the course is associated with a practical assignment that must be completed individually. A couple of sessions will be held in the computer hall to introduce students to practical procedures. The practical assignment consists of the implementation of some of the studied methods, when applied to the traveling salesman problem, and the computational study of its performance. The student will have to program some parts of the practical, although in other parts a standard software package will be used.

### Learning objectives of the subject

The objectives of the course depend on the choice of the student for the level of the level to attain.

#### BASIC LEVEL (Topics 1-5)

It is an introductory course on of Operations Research models and methods. The main objective is to give an overall view of the main classes of models and their main potential applications, as well as of the techniques that must be used in each case. Basic versions will be studied of the most usual techniques in non-linear, linear and integer programming. Without ignoring the formal aspects, special attention will be given to the interpretation and application of the studied concepts.

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The learning objectives of the course are:

- To provide a basic knowledge in the main models and techniques in Operations Research, as well as of the main applications. To familiarize students with basic methods that allow solving some practical applications.
- To know the possible modeling alternatives and the nature of the different classes of problems in Operations Research and their potential applications, with special emphasis in those related to statistical problems.
- To know the basic concepts and methodology of non-linear optimization with or without constraints: line search, gradient and Newton Methods, and Karush-Kuhn-Tucker conditions.
- To know the basic concepts and methodology of linear programming, duality and sensitivity analysis.
- To know the main Network Flow models, as well as their applications, including shortest paths and spanning trees.
- To know some basic concepts related to integer programming and, in particular, those related to cutting planes and basic enumerative methods.

Skills to achieve:

- The ability to formulate a suitable model for an specific mathematical optimization problem, and to implement it using a suitable modeling language.
- The ability to apply correctly the basic versions of the gradient and Newton methods.
- The ability to formulate the Karush-Kuhn-Tucker conditions for an optimization problema with constraints, and to manually solve simple examples.
- The ability to solve with the Simplex Algorithm small linear programming problems, and to answer simple sensitivity analysis questions.
- The ability to solve simple Network Flow models, including shortest paths and minimum spanning trees.
- The ability to apply basic integer programming techniques.

### ADVANCED LEVEL (Topic 6):

This course studies models and techniques of Integer Programming. Special attention is given to the potential applications of the models and their relation to combinatorial optimization. The main techniques that are studied are enumerative methods (branch-and-bound), methods related to cutting planes and Lagrangean relaxation. Basic concepts related to the description of polyhedra are also introduced. The application to classical combinatorial optimization models, like the traveling salesman problem or the knapsack problem, is also presented.

The main learning objectives of this course are:

- To provide a basic background in operations research, particularly in the field of Integer Programming. To familiarize students with methods for solving some practical applications of integer programming and combinatorial optimization problems.
- To know the possible modeling alternatives for the different types of problems of discrete optimization as well as their potential applications.
- To know the basic methodology of integer programming and, in particular, enumerative and cutting plane methods, as well as possible combinations of the above.
- To know results of duality theory and their implications in discrete programming. To exploit the properties of duality and the characteristics of the structure of a problem for solving discrete problems.
- To know the properties of the Lagrangean Dual for the case of discrete optimization.
- To know some basic heuristic methods for some combinatorial optimi

### Study load

Total learning time: 125h	Hours large group:	30h	24.00%
	Hours small group:	15h	12.00%
	Self study:	80h	64.00%



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### Content

#### Topic 1: Introduction to models and formulations of Operations Research

Learning time: 17h

Theory classes: 5h  
Laboratory classes: 2h  
Self study : 10h

##### Description:

Introduction to the course, highlighting the potential applications as well as the relevance in the discipline of models and mathematical optimization formulations.

#### Topic 2: Mathematical Optimization

Learning time: 45h

Theory classes: 10h  
Laboratory classes: 5h  
Self study : 30h

##### Description:

- 2.1. Existence and characterization of solutions to general optimization problems. Karush-Kuhn-Tucker conditions.
- 2.2. Solution methods
  - i. Line search.
  - ii. Gradient method. Newton method.
  - iii. Optimization methods for statistical modeling: LARS.

#### Topic 3: Linear Programming methods and their properties

Learning time: 21h 20m

Theory classes: 5h  
Laboratory classes: 3h  
Self study : 13h 20m

##### Description:

- 3.1 Bases and extreme points.
- 3.2 Basic concepts of duality and sensitivity analysis.

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Topic 4: Network flow models: max flow, min-cost flow	Learning time: 21h 20m Theory classes: 5h Laboratory classes: 3h Self study : 13h 20m
Description: 4.1 Flow balance in a network. 4.2 Properties of linear formulations and their solutions. 4.3 Shortest path problems. 4.4 Spanning trees.	
Topic 5: Basic models in integer programming and their properties	Learning time: 20h 20m Theory classes: 5h Laboratory classes: 2h Self study : 13h 20m
Description: 5.1 Cutting planes: Gomory cuts 5.2 Enumerative methods: branch-and-bound, branch-and-cut.	

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Topic 6: Advanced models and methods of integer programming and combinatorial optimization

Learning time: 125h

Self study (distance learning): 80h

Theory classes: 30h

Laboratory classes: 15h

### Description:

- 6.1 Combinatorial Optimization and its relation with integer Programming. Matching problems; sequencing; packing, covering and partitioning. Facility location problems, vehicle routing and network design.
- 6.2 Characterization of polyhedra associated with problems with integer variables.
- 6.3 Exact solution methods.
  - i. Valid inequalities. The separation problem and cutting plane methods.
  - ii. Enumerative methods: implicit enumeration, branch-and-bound and branch-and-cut. Particular cases: Gomory cuts, Chvátal-Gomory, Benders cuts, ...
- 6.4 Heuristic methods. Constructive methods (greedy, GRASP, ...), improving methods. Metaheuristics and math-heuristics.
- 6.5 Lagrangean Relaxation in integer programming.
  - i. The Lagrangean Dual. Relation between dualization and convexification.
  - ii. The solution of the Lagrangean Dual: Non-smooth optimization, subgradient optimization.
- 6.6 Some combinatorial optimization problems.
  - i. The Knapsack Problem. Valid inequalities and facets: cover cuts. Separation and lifting.
  - ii. The Traveling Salesman Problem (TSP). Basic Properties and modeling alternatives. Valid inequalities and their separation: subtour elimination, 2-matching, comb inequalities.

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### Qualification system

#### A) EVALUATION VIA Topics 1-5:

##### A.1. Continuous evaluation:

- \* Partial exam of topics 1 and 2. Weight for the continuous evaluation: 0.25
- \* Individual exercises to be issued in dates that will be announced, of each of the Topics 3, 4 and 5.
- \* Final exam

The final result will be:  $0.25 N1 + 0.15(N2 + N3 + N4) + 0.3 F$ , where

N1: Grade of the partial exam of Topics 1 and 2.

N2-N4: Grades of the individual exercises of Topics 3, 4 and 5, respectively.

F: Grade of the final exam.

##### A.2. Single act evaluation:

There will be a final exam of Topics 1-5 of the course.

#### B) EVALUATION VIA TOPIC 6:

##### B.1. Continuous evaluation

Exams: There will be a partial exam (in which a minimum grade of 5 releases from repetition of this part in the final exam), and a final exam.

Practical: Completion of an assigned individual piece of work.

Optional: To issue a collection of solved exercises.

Active participation in class will be assessed

In order to pass the course by means of the continuous evaluation it is necessary to score a minimum of 4 in both the exam and the practical. The final course result is calculated as follows:

$$0.4 \text{ (exam grade)} + 0.4 \text{ (practical grade)} + 0.1 \text{ (optional exercises)} + 0.1 \text{ (participation in class)}$$

##### B.2. Single act evaluation:

There will be an exam covering Topic 6 as well as a practical assignment. The final course result for the single act evaluation call is computed as follows:

$$0.7 \text{ (exam grade)} + 0.3 \text{ (practical grade)}$$

For the single act evaluation, an score of at least 7 in the practical assignment of the continuous evaluation will release from repeating the practical project. Otherwise the student will be assigned a new practical.



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### Bibliography

#### Basic:

Ahuja, Ravindra K; Magnanti, Thomas L; Orlin, James B. Network flows : theory, algorithms, and applications. Englewood Cliffs, N.J.: Prentice Hall, cop. 1993. ISBN 013617549X.

Luenberger, David G; Ye, Yinyu. Linear and nonlinear programming [on line]. 3rd ed. New York: Springer, cop. 2008Available on: <<https://web.stanford.edu/class/msande310/310trialtext.pdf>>. ISBN 978-0-387-74502-2 .

Wolsey, L. A. Integer programming. New York: John Wiley & Sons, 1998. ISBN 0471283665.

#### Complementary:

Bazaraa, M. S; Sherali, Hanif D; Shetty, C. M. Nonlinear programming : theory and algorithms. 3rd ed. Hoboken, N.J.: Wiley-Interscience, cop. 2006. ISBN 978-0-471-48600-8.

Bertsekas, Dimitri P. Nonlinear programming. 2nd ed. Belmont: Athena Scientific, cop. 1999. ISBN 1886529000.

Cook, W. [et al.]. Combinatorial optimization. New York: Wiley, 1998. ISBN 047155894X.

Fourer, Robert; Gay, David M; Kernighan, Brian W. AMPL : a modeling language for mathematical programming. 2nd ed. Pacific Grove, CA: Thomson/Brooks/Cole, cop. 2003. ISBN 0-534-38809-4.

Nemhauser, G.L.; Wolsey, L.A. Integer and combinatorial optimization. New York: John Wiley and Sons, 1988. ISBN 047182819X.

Padberg, M. Linear optimization and extensions. 2nd, revised and expanded ed. New York: Springer-Verlag, 1999. ISBN 3540658335.

#### Others resources:

##### Computer material

###### CPLEX

Software for the solution of integer programming problems

###### AMPL

Modeling language for mathematical optimization