Model-Based Denton with application to National Accounts

Webinar on Benchmarking and temporal disaggregation

Corentin Lemasson NBB | 20 September 2023

BanqueNationaleBank
DE BELGIQUE VAN BELGIË
Eurosystem

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Introduction

- Assume we want to disaggregate annual series, referred as **benchmark**, on a quarterly basis, by means of a **quarterly indicator**.
- Various methods exist:
  - Chow-Lin and variants → recommended by Eurostat
  - Denton → recommended by IMF
  - But also, Cholette-Dagnum, cubic splines, pro-rata, etc.
- The **method defines how annual discrepancies** between the benchmark and the indicator **are distributed** over the quarters and how extrapolation is performed
  - Select the method that will give the closest estimate of the real unknown behavior of the benchmark on a quarterly basis, i.e., the **method with the most plausible underlying assumptions**
  - Keep the method simple as modeling is carried out on an annual basis reducing the number of observations

Model-Based Denton as a Temporal Disaggregation method

- (modified) Denton Proportional First Difference (PFD) method:

$$min_{y_t} \sum_{t=2}^{n} \left[\frac{y_t}{x_t} - \frac{y_{t-1}}{x_{t-1}}\right]^2 \text{ ; subject to } \sum_t y_t = Y_y$$

- Equivalently, it can be **expressed as a statistical model** considering the State Space representation:

$$y_t = \beta_t x_t$$
$$\beta_{t+1} = \beta_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

where the annual constraints are taken care of by the use of a cumulator (see Proietti (2005)[1])

- The **Benchmark-to-Indicator ratio** (BI ratio), $\frac{y_t}{x_t}$, gives valuable information on the (development) of the relationship between the benchmark and the indicator

[1]Proietti (2005): Temporal Disaggregation by State Space Methods: Dynamic Regression Methods Revisited

Comparison with Chow-Lin in the context of National Accounts

- Based on a selection of quality criteria retained from the literature:
  - Statistical soundness
  - Conformity between the disaggregated series and the indicator
  - Size of the revisions
  - Comparability
  - Ease of estimation

Statistical soundness: Chow-Lin and variants

- Chow-Lin model :

$$y_t = x_t\beta + u_t$$
$$u_t = \rho u_{t-1} + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2 I)\,; \quad \text{subject to } \sum_t y_t = Y_y$$

- **Underlying assumptions**: Fixed coefficient β overtime and σ^2 is constant
  - Unlikely that outside influences not captured in the model only affect u_t and not the coefficient β, especially in the context of National Accounts which are subject to
    - Many changes in methodology, sources and population
    - Variation in the relationship between the benchmark and the indicator over time

    → Sudden and continuous structural changes incompatible with Chow-Lin assumptions potentially leading to a disaggregated series with too little variability
  - Homoscedasticity implies sensitivity to outliers and errors in the benchmark or the indicator

Statistical soundness: model-based Denton

- Recall the model-based Denton equations (where the annual constraints are taken care of by replacing y_t by y_t^c)

$$y_t = \beta_t x_t$$
$$\beta_{t+1} = \beta_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

- **Underlying assumptions**: disaggregated BI ratio (β_t) evolves smoothly over time and σ_ε^2 is constant
  - Often constitutes more sensible assumptions in the context of National Accounts (especially for SA series in volume)
  - But, high sensitivity to distortions in the BI ratio, causing problematic wave effects
    - Model-based Denton PFD can be extended to deal with this issue by temporary increasing the variance, adding manual input and/or forecasting the annual BI ratio (see example)

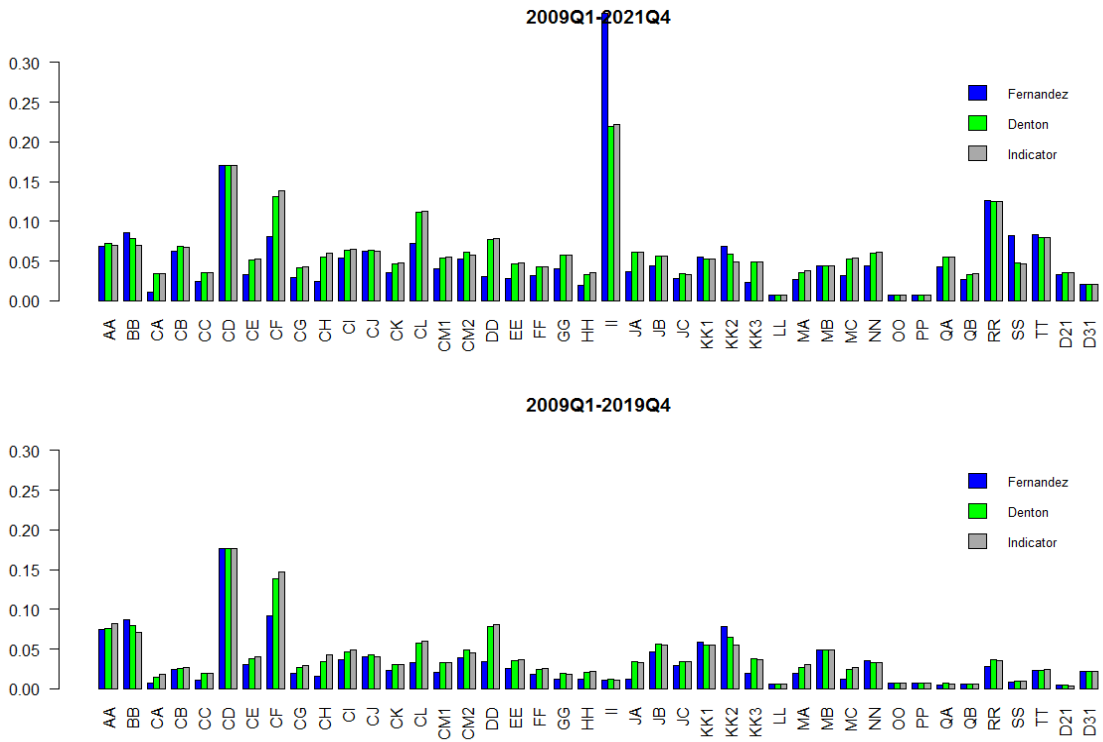
Conformity between the disaggregated series and the indicator

- Denton PFD achieves the goal of movement preservation by definition
- Chow-Lin and variants are composed of two parts:
  - A part related to the indicator ($x_t\beta$)
  - A part related to the smoothed residuals u_t and the constant

  → Only a part of the disaggregated series follows the movement of the indicator, the other part (which can be large if the overall fit is poor) is smoothed (except if constant < 0)

  - The question is whether the smoothing part is justified
    - If the indicator is the only information available and there is no evidence that the variability of the indicator is higher than that of the benchmark on an annual level, **adding an artificial smoothing pattern in the disaggregated series might be misleading** for the users.
    - In any case, care should be taken as the part of the disaggregated series being smoothed with Chow-Lin can be substantially upward biased by the failure of the model assumptions

Figure 3: Standard deviation of the QoQ growth rates (SA series)

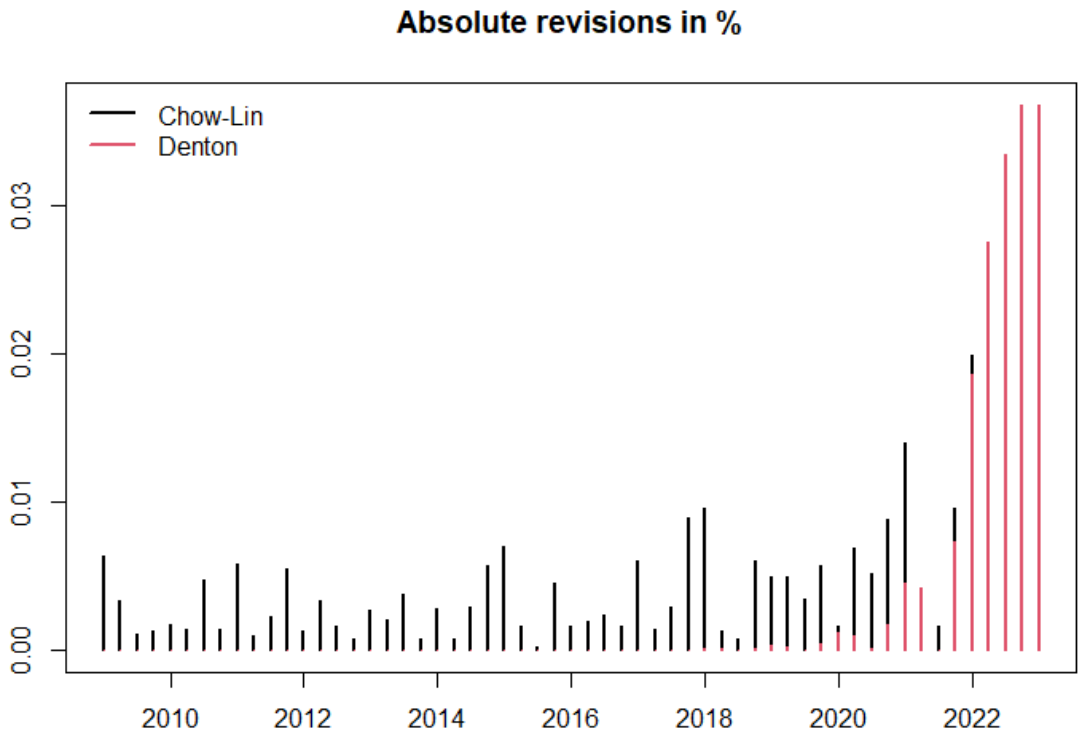


→ Roughly 20% of the overall variability of the indicators is lost when estimating Belgian quarterly GDP with Fernandez (Chow-Lin variant) for both periods considered. No loss of variability with the Denton PFD method. The percentage goes down to 10-15% when we remove branches where there is some evidence that the variability of the indicator tends to be higher than that of the benchmark over the whole period.

Size of the revisions

- Cross-validation considering the production side of GDP to compare the quality of extrapolation
  - Denton PFD shows smaller MAE and RMSE than Fernandez for 2/3 of the branches (! small sample size)
- Fundamental difference between the two methods
  - Denton PFD method works **on a local basis**
    - Distribution: the impact of revision in the latest benchmark/indicator values of the series decreases quickly as we go back in time
    - Extrapolation: quality of extrapolation relies on the stability of the annual BI ratio over the few previous years
  - Chow-Lin and variants work **on a global basis**
    - Distribution: impact of revision in the latest benchmark/indicator values of the series modifies the estimate of the coefficient, thus also the whole historical series
    - Extrapolation: performs well when there is globally a high negative correlation (cst>0) between the annual BI ratio and the level of the indicator

Example: Revisions related to the (mock) release of annual figure in the construction sector for 2022 displaying a difference in growth rate of 3 percentage points with the quarterly indicator



Comparability and ease of estimation

- The fact that Chow-Lin tends to significantly reduce the variability of the indicator causes issues of comparability at different levels:
  - Within the disaggregated series since Covid-19
  - Between aggregates
  - Between countries
- Ease of estimation
  - Model-based Denton is now easily accessible thanks to JD+
  - It might require more adjustments than Chow-Lin, which, in a way, can be a good thing as it forces the producer to investigate the relationship between the benchmark and the indicator more in details
  - It also offers more flexibility (use of ad-hoc method at some periods, freezing past, etc.)

Example

Estimation of quarterly value added in catering industry with a turnover indicator using a model-based Denton approach

Input

	(scaled) indicator (x)	cumulated series (yc)	(scaled) annual BI ratio	relative std error of β_t (σ_ε)
...	...	...	...	...
2018Q1	84.4	NA		1
2018Q2	97.9	NA		1
2018Q3	98.7	NA		1
2018Q4	99.9	7885	99.8	1
2019Q1	88.9	NA		1
2019Q2	102.8	NA		1
2019Q3	103.8	NA		1
2019Q4	104.5	8297	100	1
2020Q1	77.2	1510*(e)*		10
2020Q2	40.8	2200*(e)*		1
2020Q3	88.5	3922*(e)*		1
2020Q4	50.1	4793	90.1	1
2021Q1	40.9	NA		1
2021Q2	70.4	NA		1
2021Q3	112.2	NA		1
2021Q4	102.0	6544	96.9	1
2022Q1	92.4	NA		1
2022Q2	121.2	NA		1
2022Q3	125.2	NA		1
2022Q4	123.9	9597*(e)*	100*(e)*	1
2023Q1	113.3	NA		1

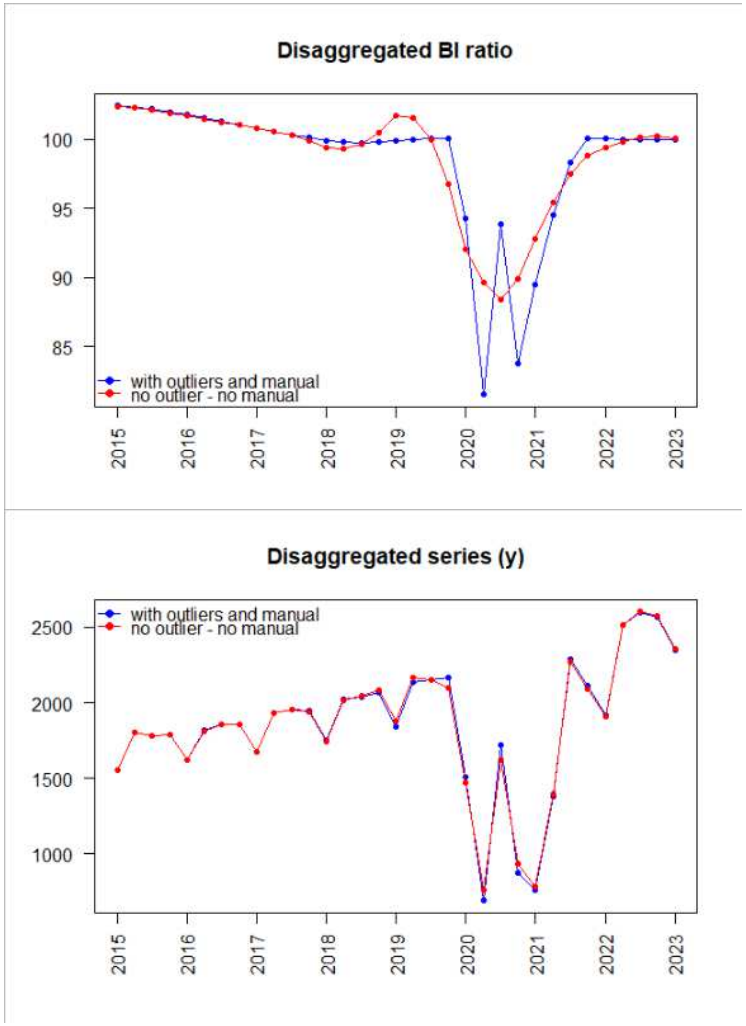
R Code

```
library(rjd3sts)

# Import and format input (yt and xt)
# ...

# Define components
# note: if more than one, we'd just need to defined them and use the
# aggregation function.
regressor<-rjd3sts::var_reg("x", xt, relative_stderr_beta, scale=1, fixed=T)

# Introduce cumulator
cumulator<-rjd3sts::cumul("cumulator", regressor, frequency_ratio)

# Define the model
model_mbdenton<-rjd3sts::model()
rjd3sts::add(model_mbdenton, cumulator)

# Build equations
equations<-rjd3sts::equation("equations")
rjd3sts::add_equation(equations, "cumulator")
rjd3sts::add(model_mbdenton, equations)

# Estimate the model
rslt<-rjd3sts::estimate(model_mbdenton, yc, marginal = T,
                        initialization = "Augmented_Robust")

# Some results
beta_t<-rjd3toolkit::result(rslt, "ssf.smoothing.states")[,2]
stderr_beta_t<-sqrt(rjd3toolkit::result(rslt, "ssf.smoothing.vstates")[,2])
```

Estimation of quarterly value added in catering industry with a turnover indicator using a model-base Denton approach

Output

Period	cumulated series (yc)	disaggregated series (y)	(scaled) disaggregated BI ratio (y/x)
...	...	...	...
01/01/2018	1750	1750	99.9
04/01/2018	3775	2025	99.8
07/01/2018	5817	2042	99.7
10/01/2018	7885	2068	99.8
01/01/2019	1842	1842	99.9
04/01/2019	3975	2133	100.0
07/01/2019	6128	2153	100.0
10/01/2019	8297	2168	100.0
01/01/2020	1510	1510	94.3
04/01/2020	2200	690	81.5
07/01/2020	3922	1723	93.9
10/01/2020	4793	871	83.8
01/01/2021	758	758	89.5
04/01/2021	2138	1380	94.5
07/01/2021	4426	2288	98.3
10/01/2021	6544	2117	100.1
01/01/2022	1918	1918	100.0
04/01/2022	4432	2514	100.0
07/01/2022	7028	2596	100.0
10/01/2022	9597	2570	100.0
01/01/2023	2349	2349	100.0



Tools

- RJDemetra https://github.com/orgs/rjdemetra/repositories
  - **rjd3bench** already implements the model-based Denton
    - Already include the possibility to add outliers (i.e., LS in the BI ratio)
    - Fixing values of the disaggregated series at some periods and explicit forecast of BI ratio not yet included
  - **rjd3sts**
    - Examples

    https://github.com/palatej/test_rjd3sts/blob/main/R%20files/bench.R or https://github.com/clemasso/nbbTD/blob/main/R/mbDenton.R)

- **nbbTD**: a R tool based on rjd3sts for multiprocessing temporal disaggregation of time series (tailored for the production of official statistics) https://github.com/clemasso/nbbTD