

Formalization of Requirements for CPS and NTSS

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1 Requirement Formalization

Autopilot:

R1.4.1:

Textual:

Steady state roll commands shall be tracked within 1 degree.

Formalisation:

```
(forall tc in [ 1 , T ]:
(
  ( Tk(tc) != Tk(tc-1) ) implies
  ( exists t2 in [ tc , T ]:
  (
    forall t in [ t2 , T ]:
    (
      Phi(t) - PhiRef(t) <= 1
    )
  ))));
```

R1.6:

Textual:

The maximum roll angle (Phi) allowed shall be 30 deg +/-10%

Formalisation:

```
G{0,T} (-33 <= Phi <= 33)
```

R12.1:

Textual:

When the autopilot is enabled, the aircraft altitude should reach the desired altitude within 500 seconds in calm air

Formalisation:

```
AP_Eng = 1 ==> F(G(alt - ALT_Ref ==> 0.0))
```

Finite State Machine:

R1:

Textual:

Exceeding sensor Limits shall latch an autopilot pullup when the pilot is not in control (not Standby) and the system is Supported without failures (not Apfail).

Formalisation:

```
G{0,T} (not standby and not apfail and supported and limits ==> pullup )
```

Non Linear Guidance:

R6:

Textual:

The change in the magnitude of the output over one frame of execution with T sample period shall not exceed the quantity of the combined velocity of the target plus the velocity of the vehicle multiplied by T

Regulator:

R7:

Textual:

The Inner Loop Pitch Regulator Shall not command transient changes in angular roll acceleration greater than 50 deg/sec²/sec

Formalisation:

$G\{0,T\} \text{ (mcvdt_cmd_fcs_dps2}(t) - \text{mcvdt_cmd_fcs_dps2}(t-1) \leq 0.5)$

Tustin:

R1:

Textual:

When Reset is True and the Initial Condition (ic) is bounded by the provided Top and Bottom Limits ($BL \leq ic \leq TL$),
the Output (yout) shall equal the Initial Condition (ic).

Formalisation:

$G\{0,T\} ((\text{Reset} = 1 \text{ and } Ic \leq tl \text{ and } Ic \geq bl \Rightarrow yout = Ic) \text{ and } \\ (\text{Reset} = 1 \text{ and } Ic \leq tl \text{ and } tl \geq bl \Rightarrow yout = tl) \text{ and } \\ (\text{Reset} = 1 \text{ and } Ic \leq bl \text{ and } tl \geq bl \Rightarrow yout = bl) \text{ and } \\ (\text{Reset} = 1 \text{ and } Ic \leq bl \text{ and } tl < bl \Rightarrow yout = bl) \\)$

R2:

Textual:

The Output (yout) shall be bounded by the provided Top and Bottom limits (TL and BL)

Formalisation:

$G\{0,T\} (TL \geq BL) \Rightarrow (BL \leq yout \text{ and } yout \leq TL) \text{ and } (TL < BL) \Rightarrow (TL \leq yout \text{ and } yout \leq BL)$
)

R4a:

Textual:

Over a 10 second computational duration at an execution frequency of 10 hz, the Output should equal the sine of time t, sin, where time is defined as a vector from 0 to 10 by increments of 0.1 seconds within a +/- 0.1 tolerance for an input equal to the cosine of time t, cos, with the sample delta time T = 0.1 seconds when in normal mode of operation.

Formalisation:

$G\{0,T\} (TL \geq BL \ \&\& \ y \geq BL \ \&\& \ y \leq TL) || (BL \geq TL \ \&\& \ y \geq TL \ \&\& \ y \leq BL) \\ \Rightarrow (\text{abs}(yout100-10) \leq 0.1)$

R4b:

Textual:

Over a 10 second computational duration at an execution frequency of 10 hz, the Output should equal the sine of time t, sin, where time is defined as a vector from 0 to 10 by increments of 0.1 seconds within a +/- 0.1 tolerance for an input equal to the cosine of time t, cos, with the sample delta time T = 0.1 seconds when in normal mode of operation.
Formalisation:

$G\{0,T\} \ (\ TL \geq BL \ \&\& \ y \geq BL \ \&\& \ y \leq TL) || (BL \geq TL \ \&\& \ y \geq TL \ \&\& \ y \leq BL)$
 $\Rightarrow \text{abs}(y - \sin \leq 0.1)$

NTSS:

Textual:

Good network connectivity should be maintained even when high traffic flows through different priority classes.

Formalization:

R =

$$\begin{aligned}
& \overline{mos}_n \quad \text{if} \quad \bigwedge_{i=1..n} \overline{mos}_i < \overline{mosTh}_i, \\
& 1 + \overline{mos}_{n-1} \quad \text{if} \quad \overline{mos}_n \geq \overline{mosTh}_n \quad \wedge \quad \bigwedge_{i=1..n-1} \overline{mos}_i < \overline{mosTh}_i, \\
& 2 + \overline{mos}_{n-2} \quad \text{if} \quad \bigwedge_{i=\{n-1,n\}} \overline{mos}_i \geq \overline{mosTh}_i \quad \wedge \quad \bigwedge_{i=\{1..n-2\}} \overline{mos}_i < \overline{mosTh}_i, \\
& \dots \\
& n \quad \text{if} \quad \bigwedge_{i=1..n} \overline{mos}_i \geq \overline{mosTh}_i
\end{aligned}$$

2 Input Specification for CPS and NTSS

Table 1: Input Specification for CPS and NTSS

Subject: TU1...TU9		
Input Name	Type	Range
Xin	Double	[-20,20]
TL	Double	[-10,10]
BL	Double	[-10,10]
IC	Double	[-20,20]

Subject: NLG		
Input Name	Type	Range
X_{targ}	Double	[-100,100]
X_v	Double	[-100,100]
V_v	Double	[-100,100]
V_{targ}	Double	[-100,100]
r	Double	[0,100]

Subject: REG		
Input Name	Type	Range
beta_adc_deg	Double	[0,5]
vtas_adc_kts	Double	[0,5]
lev_md_fos_dps	Double	[0,5]
hdg_des_deg	Double	[0,5]
mev_emd_fes_dps	Double	[0,5]
alt_des_ft	Double	[0,5]
nev_cmd_fcs_dps	Double	[0,5]
xev_cmd_fcs_fps	Double	[0,5]
airspeed_des_fps	Double	[0,5]
hcv_cmd_fcs_fps	Double	[0,5]
lcv_fcs_dps	Double	[0,5]
mcv_fcs_dps	Double	[0,5]
ncv_fcs_dps	Double	[0,5]
dcv_fcs_fps	Double	[0,5]
zcv_cmd_fcs_fps	Double	[0,5]
betadot	Double	[0,5]

Subject: FSM		
Input Name	Type	Range
standby	Boolean	{0,1}
apfail	Boolean	{0,1}
supported	Boolean	{0,1}
limits	Boolean	{0,1}

Subject: AP1...AP3		
Input Name	Type	Range
AP Eng	Boolean	{0,1}
HDG Mode	Boolean	{0,1}
ALT Mode	Boolean	{0,1}
HDG Ref	Double	[-180,180]
Turn knob	Double	[0,45]
ALT Ref	Double	[0,1000]
Pitch wheel	Double	[-30,30]
Throttle	Double	[0,1]

Subject: NTSS		
Input Name	Type	Range
Class0	Integer	[0,thresh ₀]
Class1	Integer	[0,thresh ₁]
Class2	Integer	[0,thresh ₂]
Class3	Integer	[0,thresh ₃]
Class4	Integer	[0,thresh ₄]
Class5	Integer	[0,thresh ₅]
Class6	Integer	[0,thresh ₆]
Class7	Integer	[0,thresh ₇]
