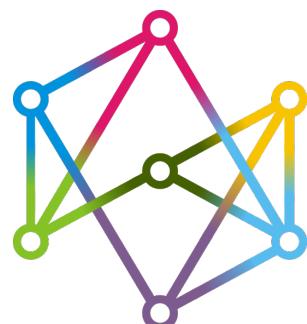


Towards Scalable Online Decision Making Under Uncertainty in Partially Observable Environments

Vadim Indelman



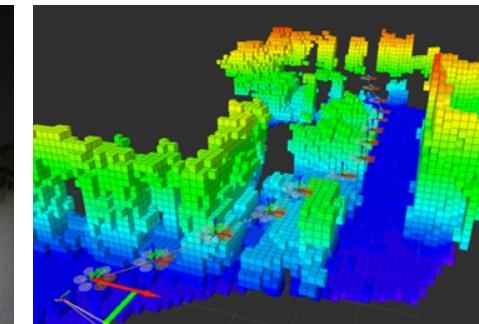
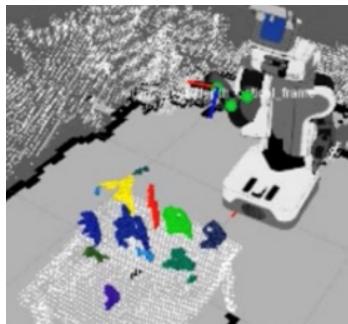
TECHNION
Israel Institute
of Technology



ANPL
Autonomous Navigation and
Perception Lab

Advanced Autonomy

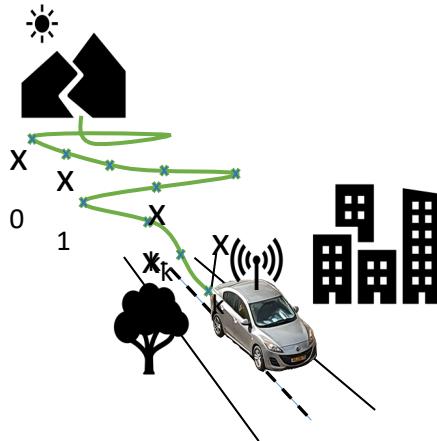
Involves autonomous navigation, active SLAM, informative gathering, active sensing, etc.



Advanced Autonomy

Perception and Inference

Where am I? What is the surrounding environment?



Decision-Making Under Uncertainty

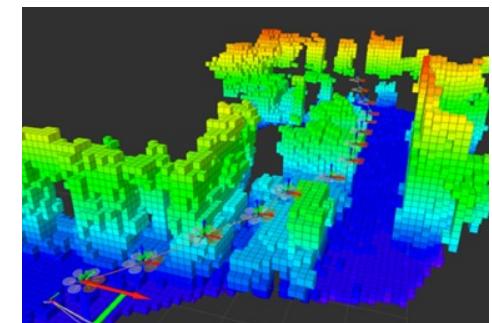
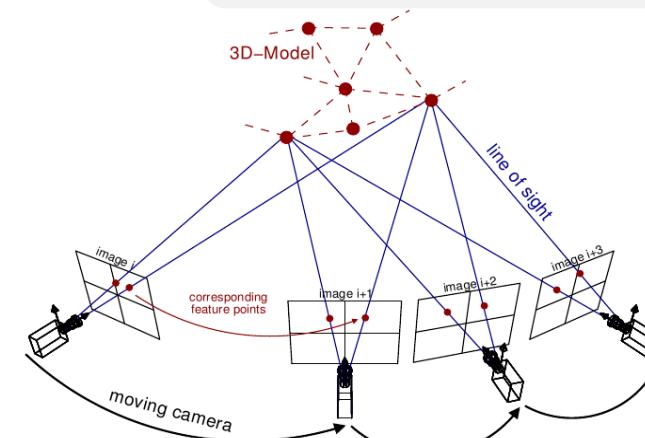
What should I be doing next?

Determine best action(s) to accomplish a task, account for different sources of uncertainty

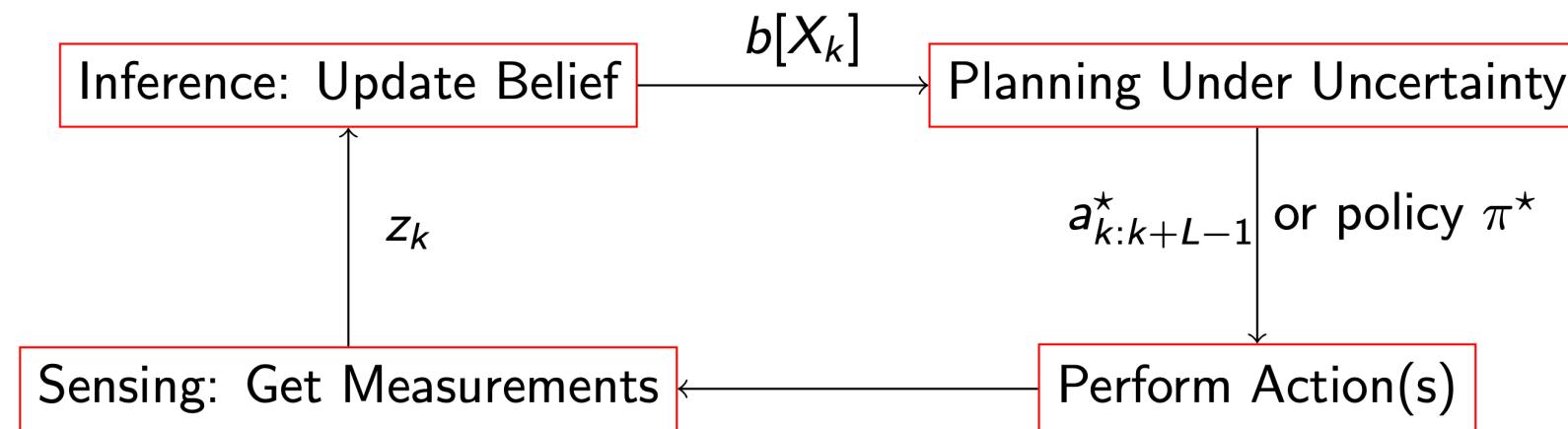
Perception and Inference



Decision-Making Under Uncertainty



Autonomy Loop

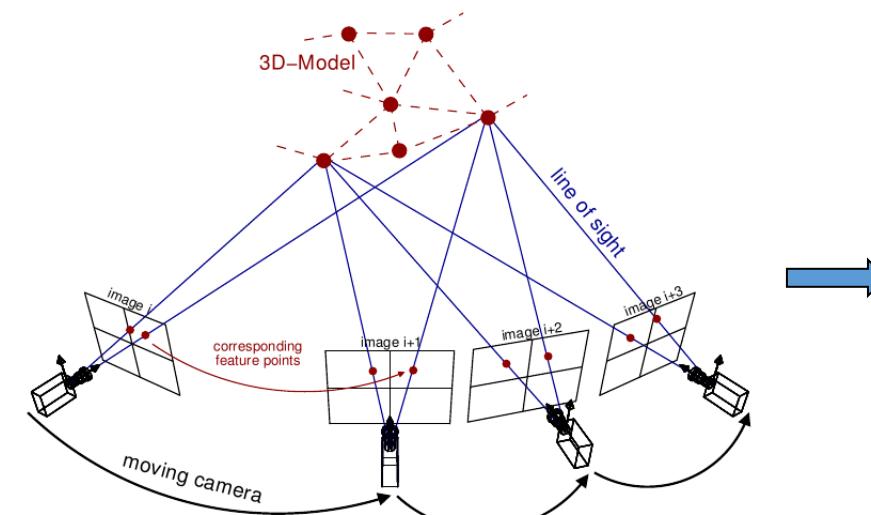
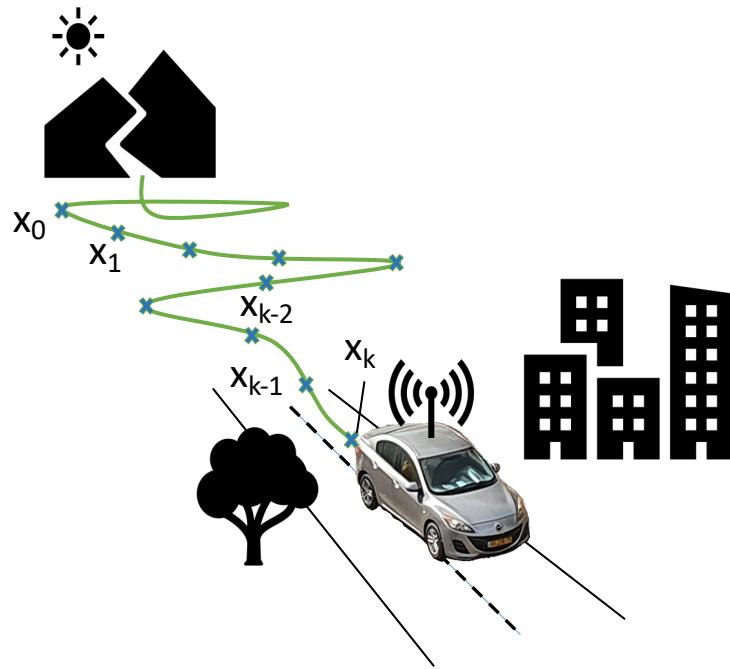


Perception and Inference

- Posterior belief at time k:

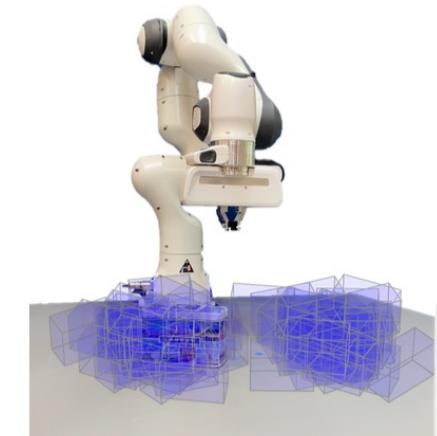
$$b_k \triangleq b[X_k] = \mathbb{P}(X_k \mid \underbrace{\mathcal{H}_k}_{\substack{\text{state/variables at} \\ \text{time instant } k}}, \underbrace{a_{0:k-1}}_{\text{actions}}, \underbrace{z_{1:k}}_{\text{observations}})$$

- Example:

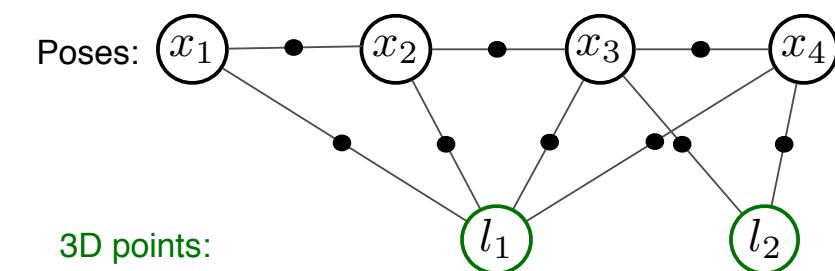


$$X_k \doteq \{x_0, \dots, x_k, L_k\}$$

Past & current robot states
Environment representation, e.g. Landmarks



Can be represented with graphical models, e.g. a Factor Graph

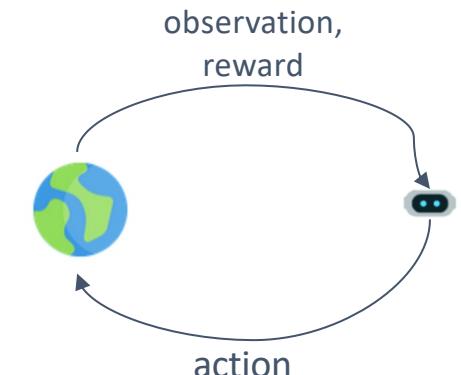


Partially Observable Markov Decision Process (POMDP)

- POMDP tuple:

$$\langle \mathcal{X}, \mathcal{Z}, \mathcal{A}, T, O, \rho, b_k \rangle$$

— state, observation, and action spaces
— transition and observation models
— Belief-dependent reward function
— Belief at planning time instant k



- Value function

$$V^\pi(b_k) = \mathbb{E}_{z_{k+1:k+L}} \left[\sum_{l=k}^{k+L} \rho(b_l, \pi_l(b_l)) \right]$$

Belief-dependent reward function

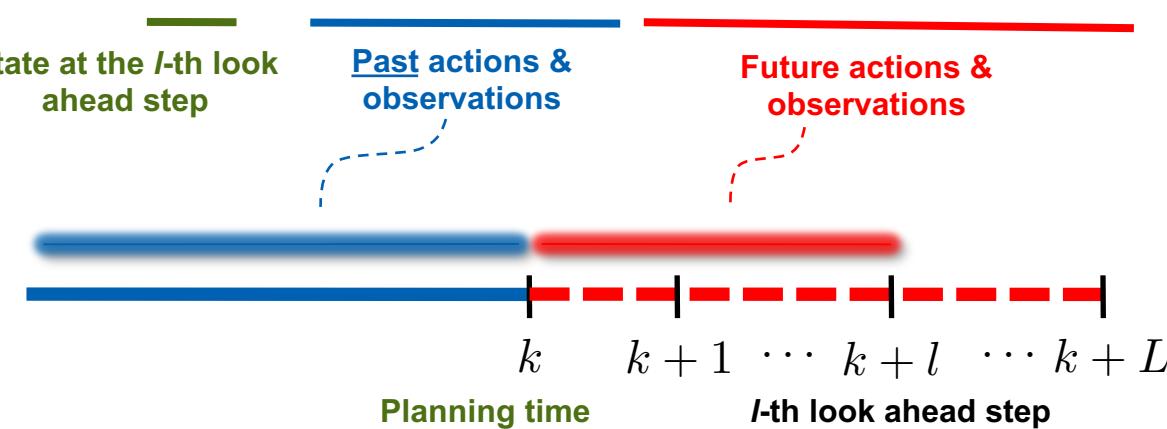
Partially Observable Markov Decision Process (POMDP)

- Value function

$$V^\pi(b_k) = \mathbb{E}_{z_{k+1:k+L}} \left[\sum_{l=k}^{k+L} \rho(b_l, \pi_l(b_l)) \right]$$

Belief-dependent reward function

- Belief at the ℓ -th look-ahead step: $b_{k+\ell} \triangleq b[X_{k+\ell}] = \mathbb{P}(X_{k+\ell} \mid a_{0:k-1}, z_{0:k}, a_{k:k+\ell-1}, z_{k+1:k+\ell})$



- Examples for reward function $\rho(b, a)$:

- Expected distance to goal (navigate to a goal)
- Information theoretic reward (reduce uncertainty)
- ...

Challenge

Probabilistic Inference

Maintain a distribution over the state given data

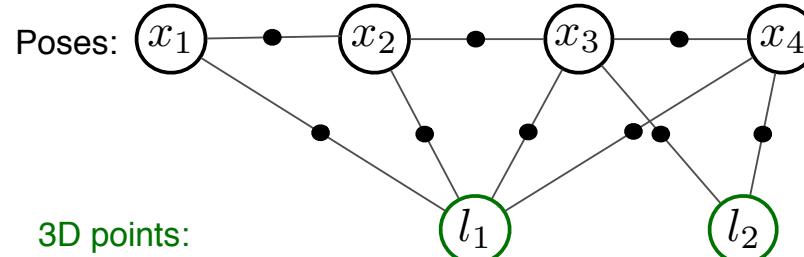
$$b_k \triangleq b[X_k] = \mathbb{P}(X_k \mid \underbrace{\text{state}}_{a_0:k-1}, \underbrace{\text{actions}}_{z_1:k})$$

Decision-making under uncertainty

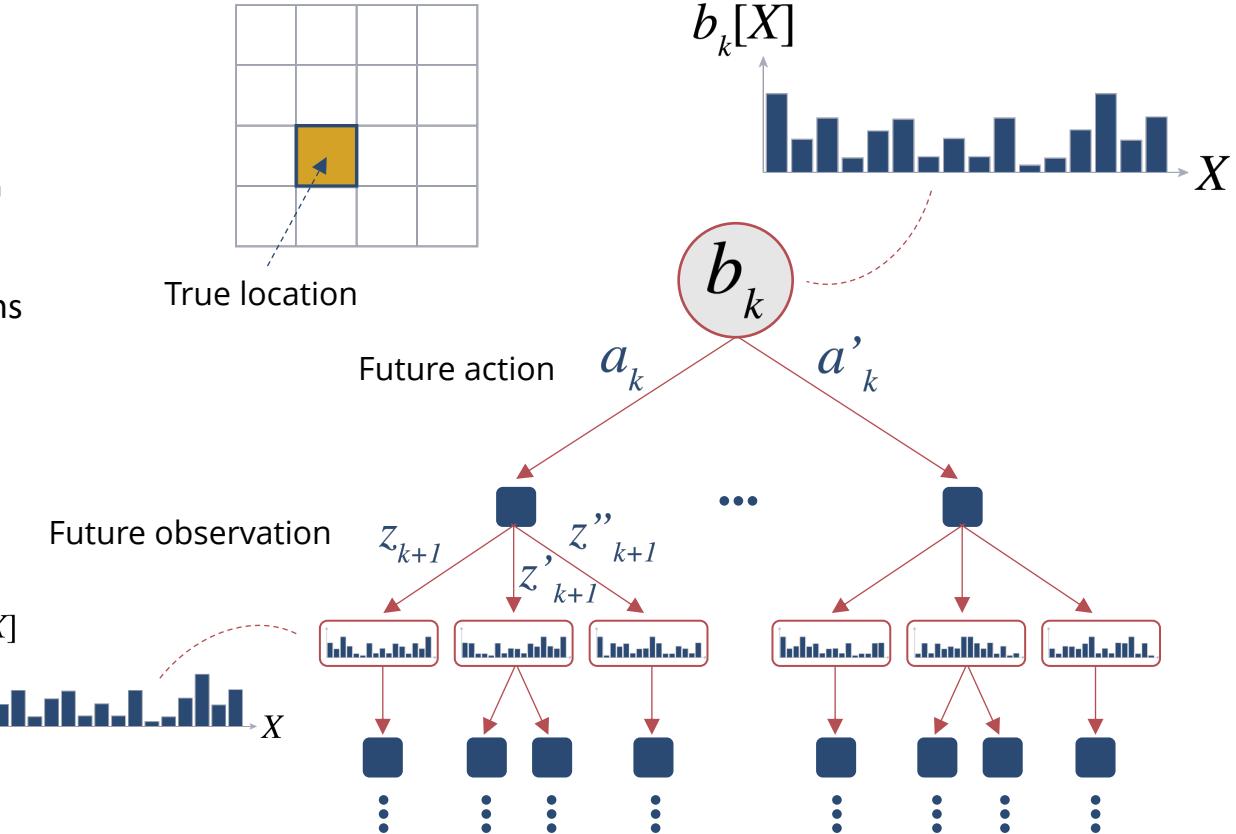
Involves reasoning about the entire observation and action spaces along planning horizon

Computationally intractable

More so, in high dimensional settings



Example - grid world



How can we act autonomously online and efficiently complete tasks in a safe and reliable fashion??

Agenda

Experience Reuse in POMDP Planning

POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

Agenda

Experience Reuse in POMDP Planning

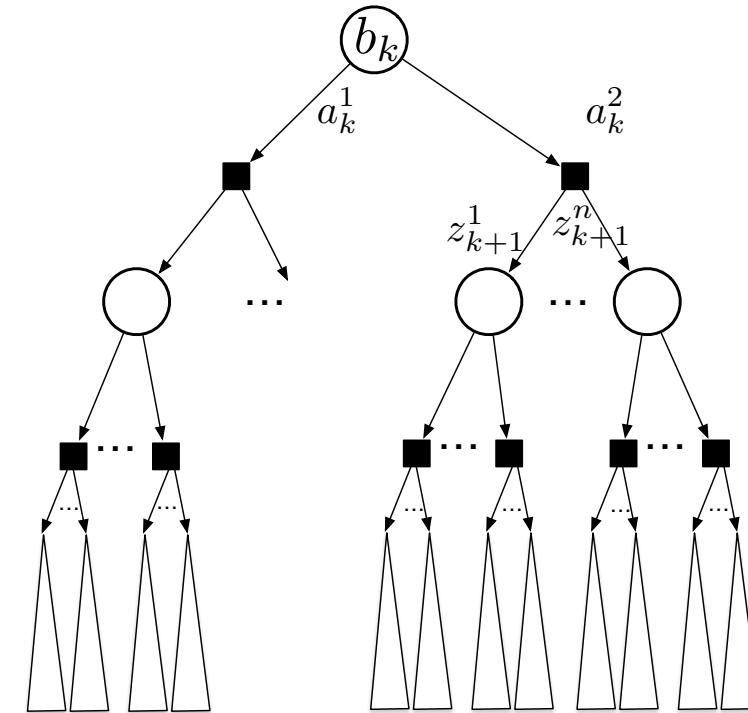
POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

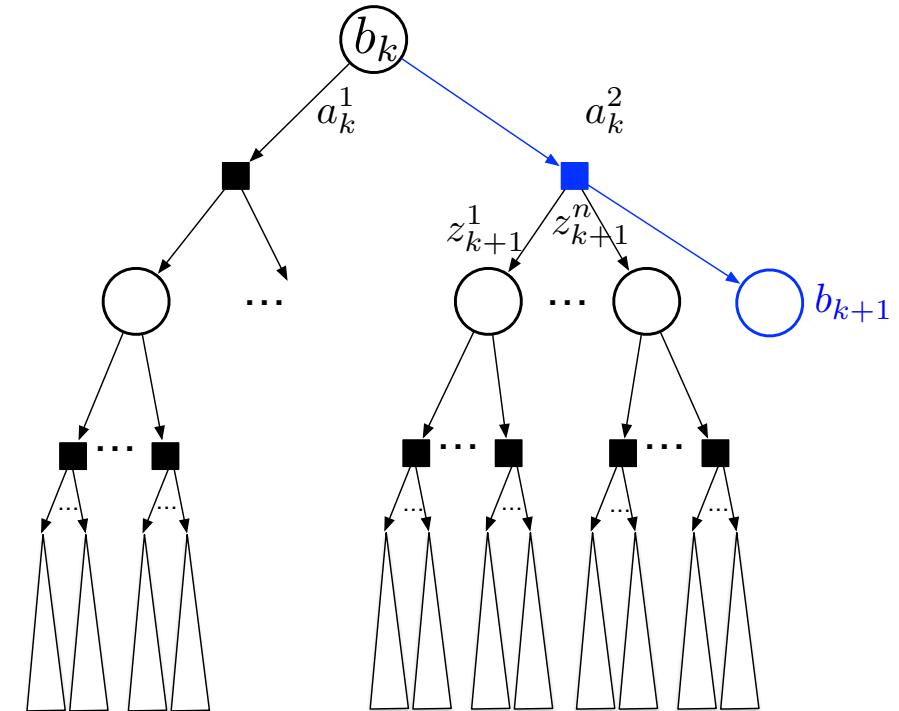
Experience Reuse in POMDP Planning

- Consider POMDPs with continuous state, action, and observation spaces



Experience Reuse in POMDP Planning

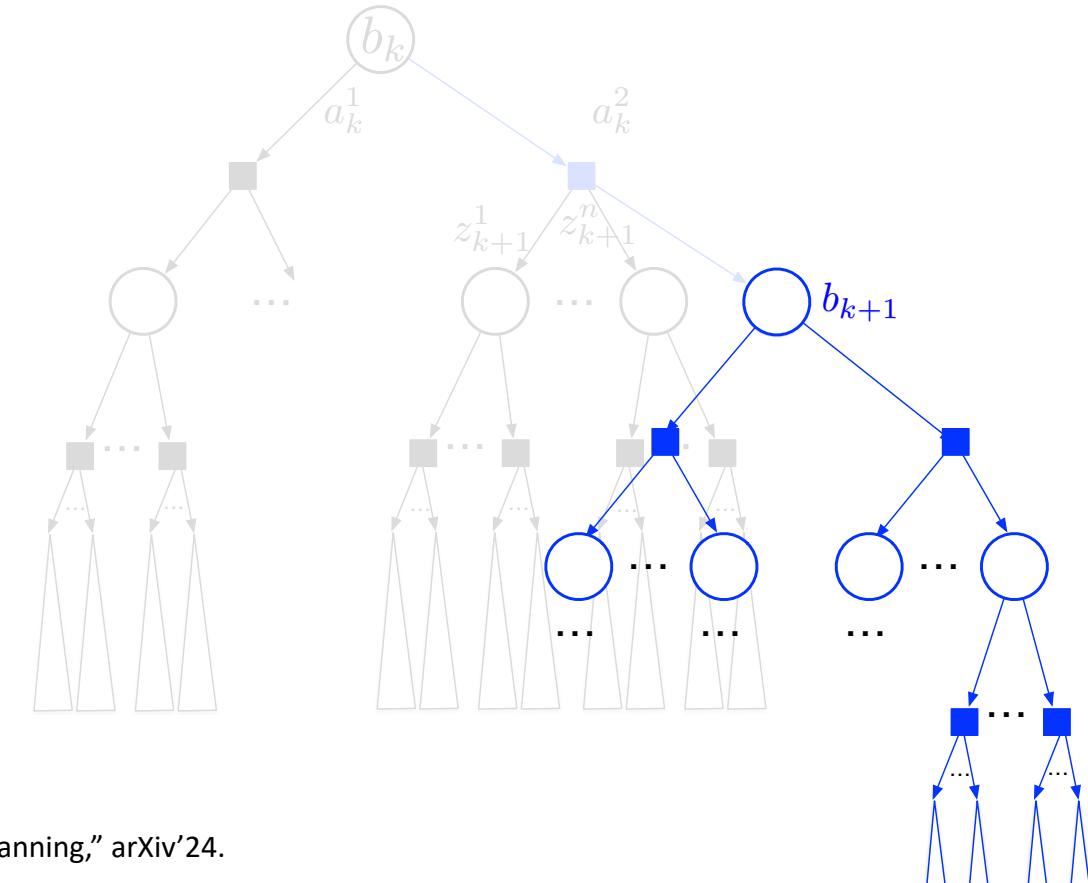
- Consider POMDPs with continuous state, action, and observation spaces
- The probability of sampling the same belief/observation twice is zero



Experience Reuse in POMDP Planning

- Consider POMDPs with continuous state, action, and observation spaces
- The probability of sampling the same belief/observation twice is zero

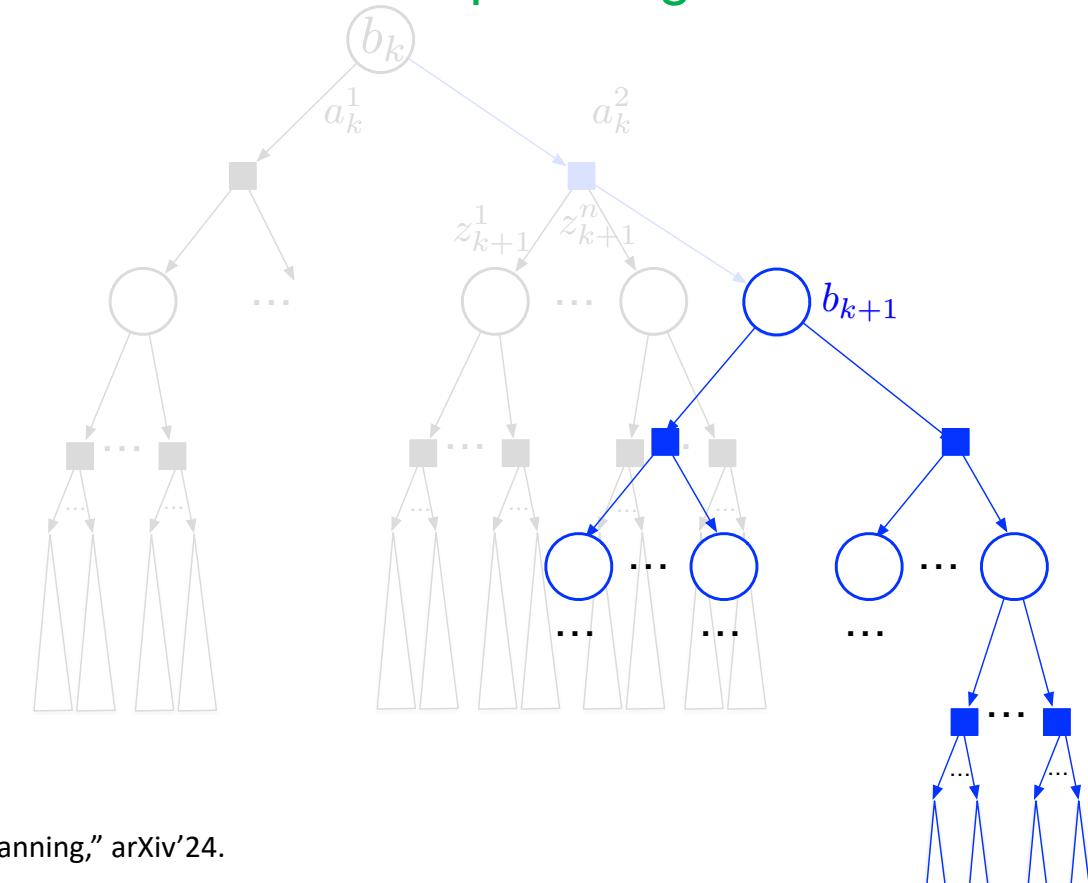
Online SOTA POMDP solvers typically perform calculations from **scratch at each planning session**



Experience Reuse in POMDP Planning

- Consider POMDPs with continuous state, action, and observation spaces
- The probability of sampling the same belief/observation twice is zero
- Previously sampled beliefs can still provide useful info in the current planning session

Online SOTA POMDP solvers typically perform calculations from **scratch at each planning session**



Experience Reuse in POMDP Planning

- Consider POMDPs with continuous state, action, and observation spaces
- The probability of sampling the same belief/observation twice is zero
- Previously sampled beliefs can still provide useful info in the current planning session

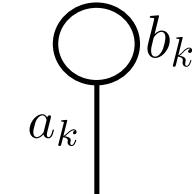
Key idea: Reuse previous trajectories/calculations to get an efficient estimation of

$$Q^\pi(b, a) = \mathbb{E}_\pi \left[\sum_{i=k}^{k+L-1} \gamma^{i-k} r(b_i, \pi_i(b_i), b_{i+1}) \mid b_k = b, a_k = a \right] \triangleq \mathbb{E}_\pi[G \mid b_k = b, a_k = a]$$

- Instead of calculating each planning session from scratch (state of the art)

Experience Reuse in POMDP Planning

- Consider a planning session at time instant k

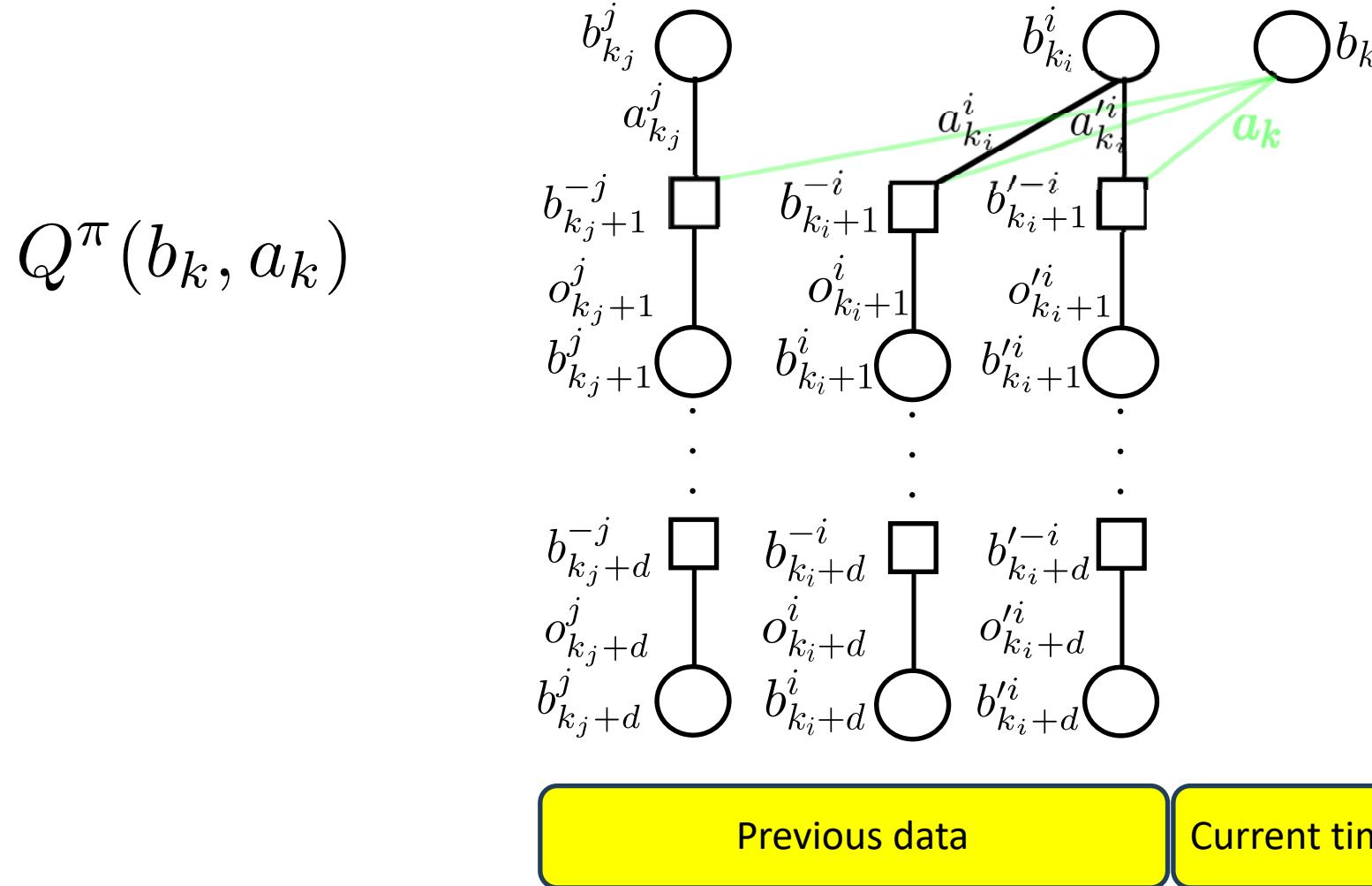


$$Q^\pi(b_k, a_k)$$

Current time

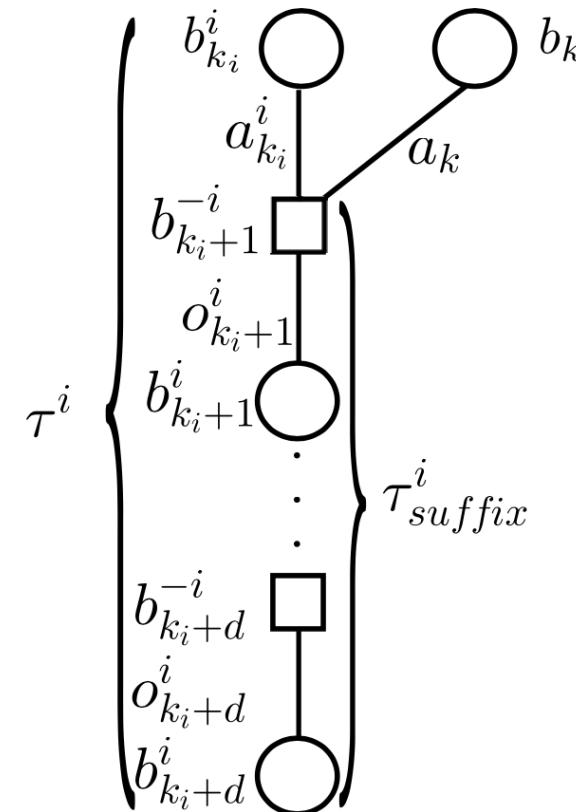
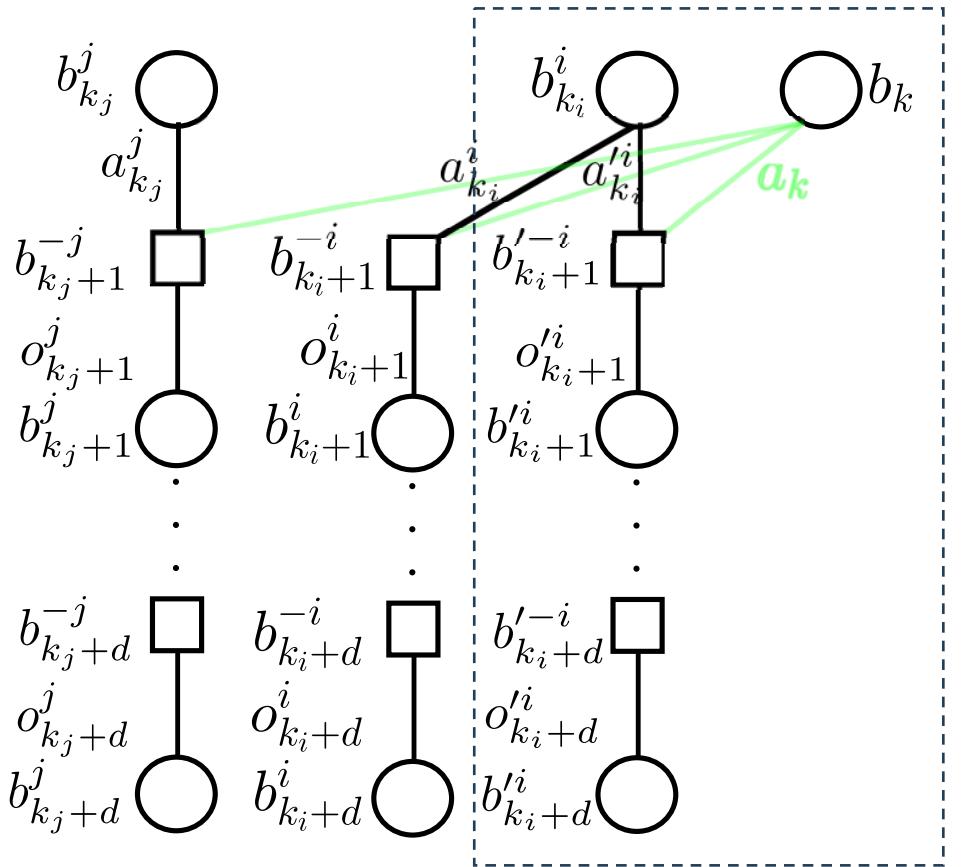
Experience Reuse in POMDP Planning

- Consider a planning session at time instant k



Experience Reuse in POMDP Planning

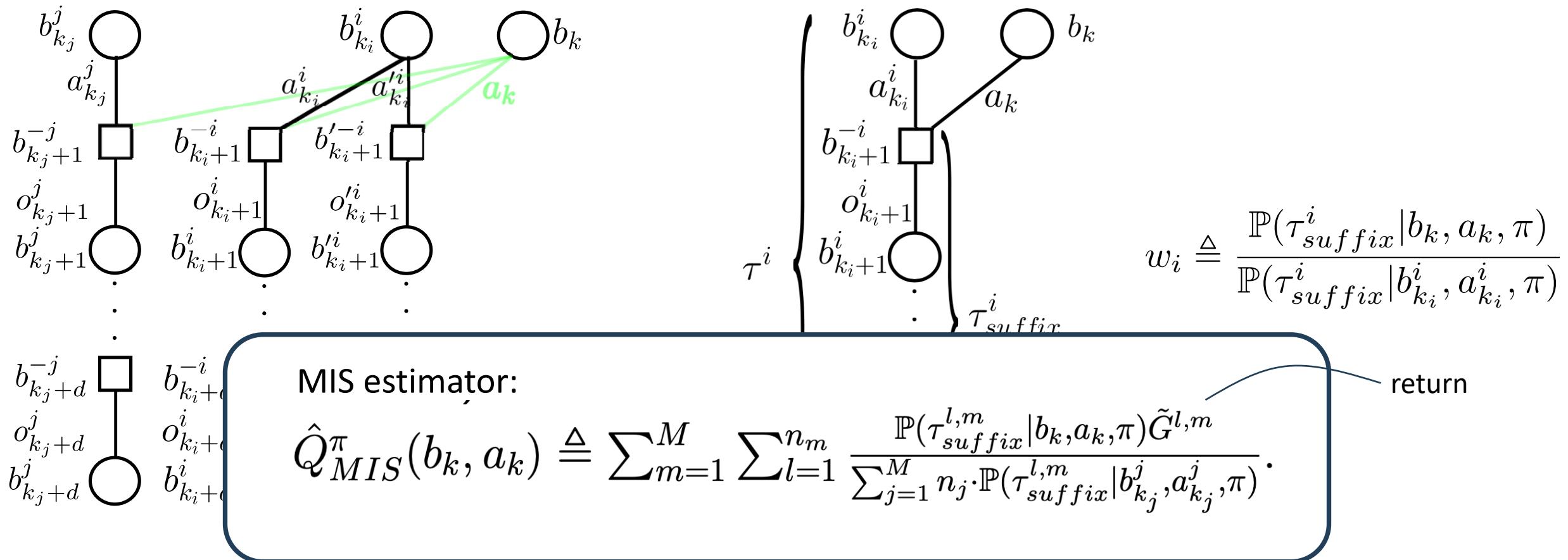
- **Key idea:** multiple importance sampling (MIS) estimator



$$w_i \triangleq \frac{\mathbb{P}(\tau_{suffix}^i | b_k, a_k, \pi)}{\mathbb{P}(\tau_{suffix}^i | b_{k_i}^i, a_{k_i}^i, \pi)}$$

Experience Reuse in POMDP Planning

- **Key idea:** multiple importance sampling (MIS) estimator



Experience-Based Value Function Estimation

MIS estimator:

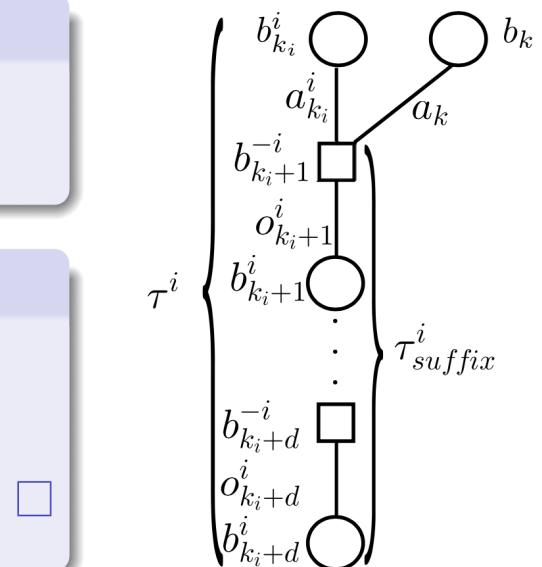
$$\hat{Q}_{MIS}^{\pi}(b_k, a_k) \triangleq \sum_{m=1}^M \sum_{l=1}^{n_m} \frac{\mathbb{P}(\tau_{suffix}^{l,m} | b_k, a_k, \pi) \tilde{G}^{l,m}}{\sum_{j=1}^M n_j \cdot \mathbb{P}(\tau_{suffix}^{l,m} | b_{k_j}^j, a_{k_j}^j, \pi)}.$$

Theorem 1

$$\frac{\mathbb{P}(\tau_{suffix}^i | b_k, a_k, \pi)}{\mathbb{P}(\tau_{suffix}^i | b_{k_i}^i, a_{k_i}^i, \pi)} = \frac{\mathbb{P}(b_{k_i+1}^{-i} | b_k, a_k)}{\mathbb{P}(b_{k_i+1}^{-i} | b_{k_i}^i, a_{k_i}^i)}$$

Proof.

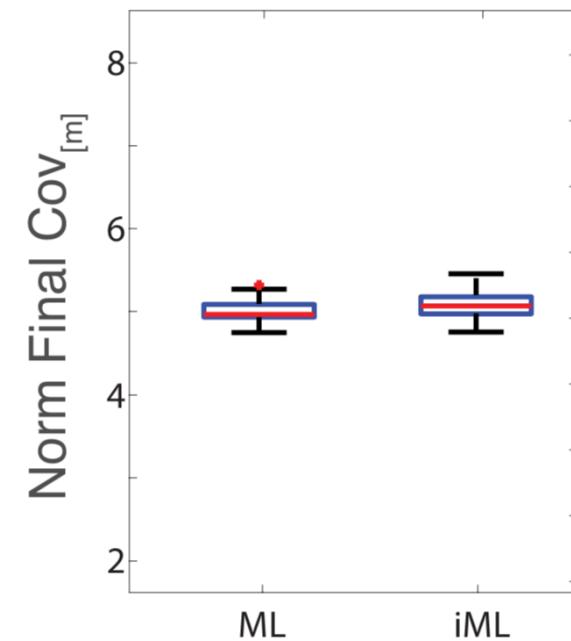
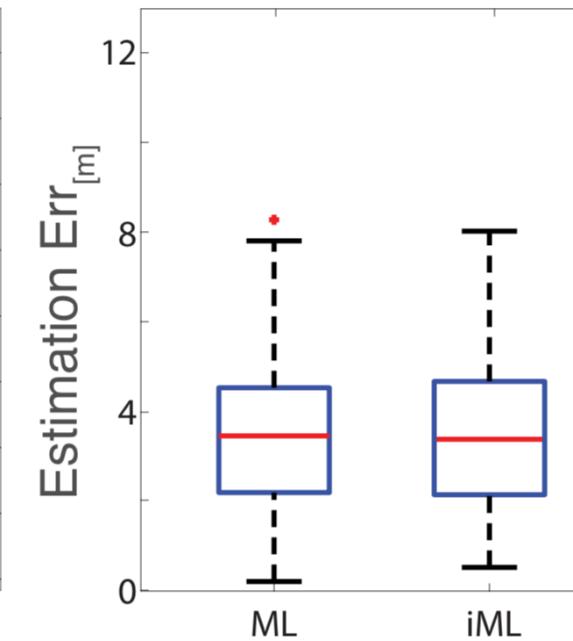
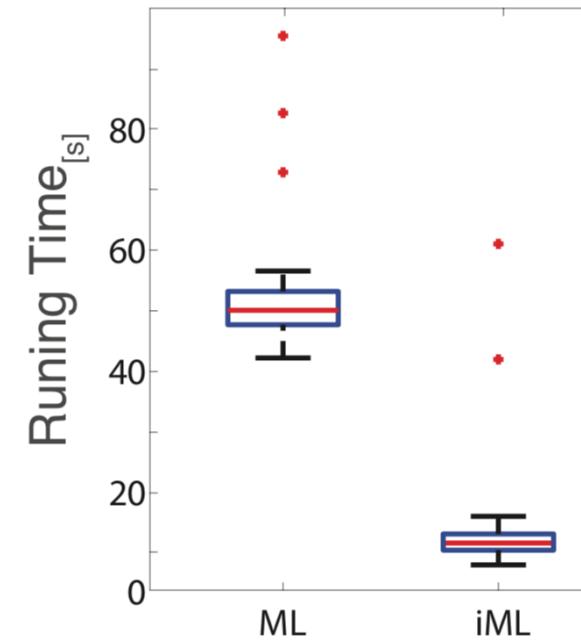
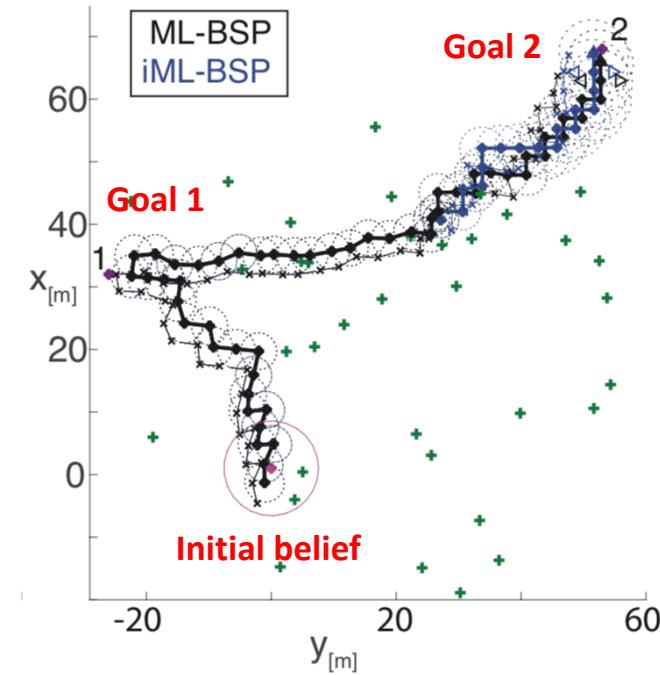
$$\begin{aligned} \frac{\mathbb{P}(\tau_{suffix}^i | b_k, a_k, \pi)}{\mathbb{P}(\tau_{suffix}^i | b_{k_i}^i, a_{k_i}^i, \pi)} &= \frac{\mathbb{P}(b_{k_i+1}^{-i}, o_{k_i+1}^i, \dots, b_{k_i+L}^i | b_k, a_k, \pi)}{\mathbb{P}(b_{k_i+1}^{-i}, o_{k_i+1}^i, \dots, b_{k_i+L}^i | b_{k_i}^i, a_{k_i}^i, \pi)} = \\ &\frac{\mathbb{P}(b_{k_i+1}^{-i} | b_k, a_k)}{\mathbb{P}(b_{k_i+1}^{-i} | b_{k_i}^i, a_{k_i}^i)} \cdot \frac{\cancel{\mathbb{P}(o_{k_i+1}^i, \dots, b_{k_i+L}^i | b_{k_i+1}^{-i}, \pi)}}{\cancel{\mathbb{P}(o_{k_i+1}^i, \dots, b_{k_i+L}^i | b_{k_i+1}^{-i}, \pi)}} = \frac{\mathbb{P}(b_{k_i+1}^{-i} | b_k, a_k)}{\mathbb{P}(b_{k_i+1}^{-i} | b_{k_i}^i, a_{k_i}^i)} \end{aligned}$$



Incremental Belief Space Planning

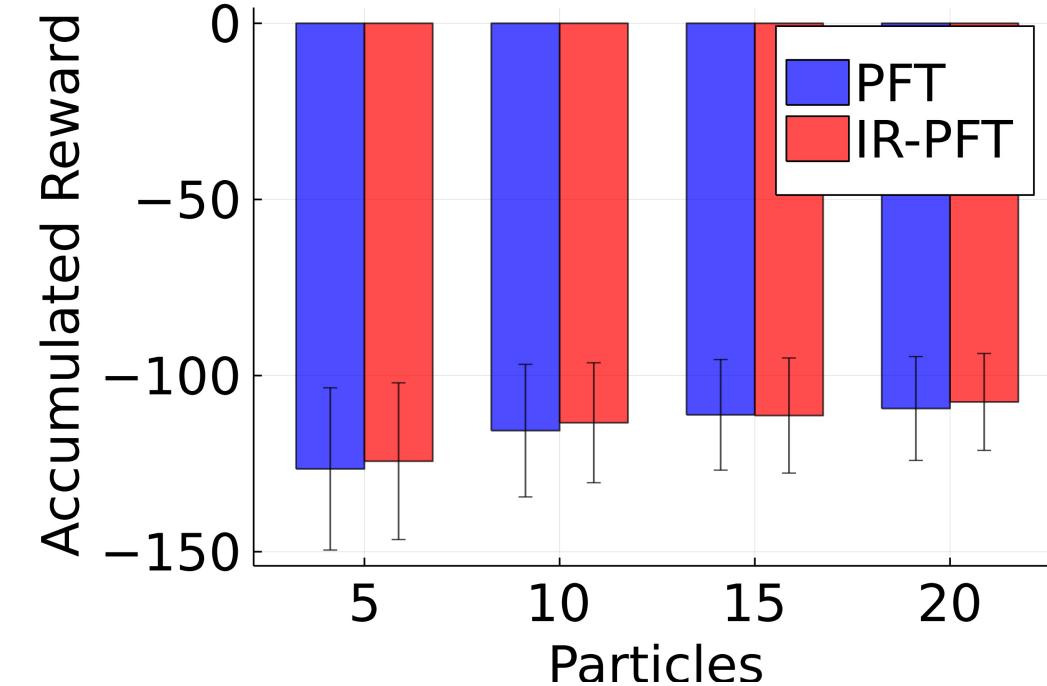
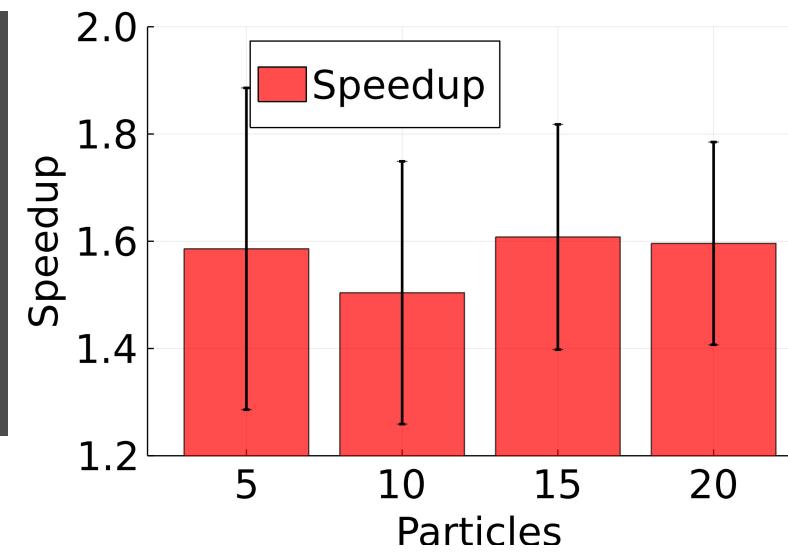
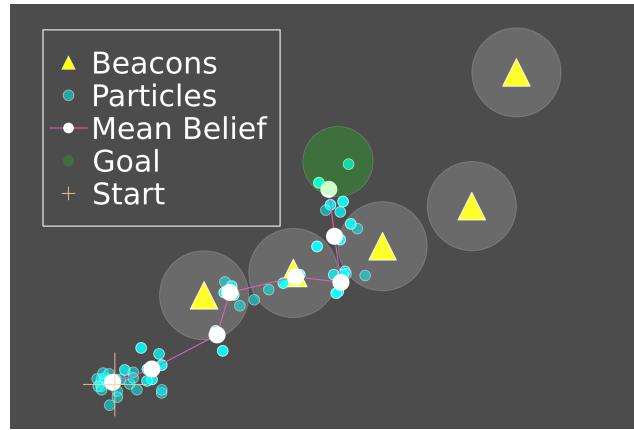
Basic simulation – autonomous navigation in unknown environments:

ML-BSP: BSP with ML observations
(one sample per look ahead step)



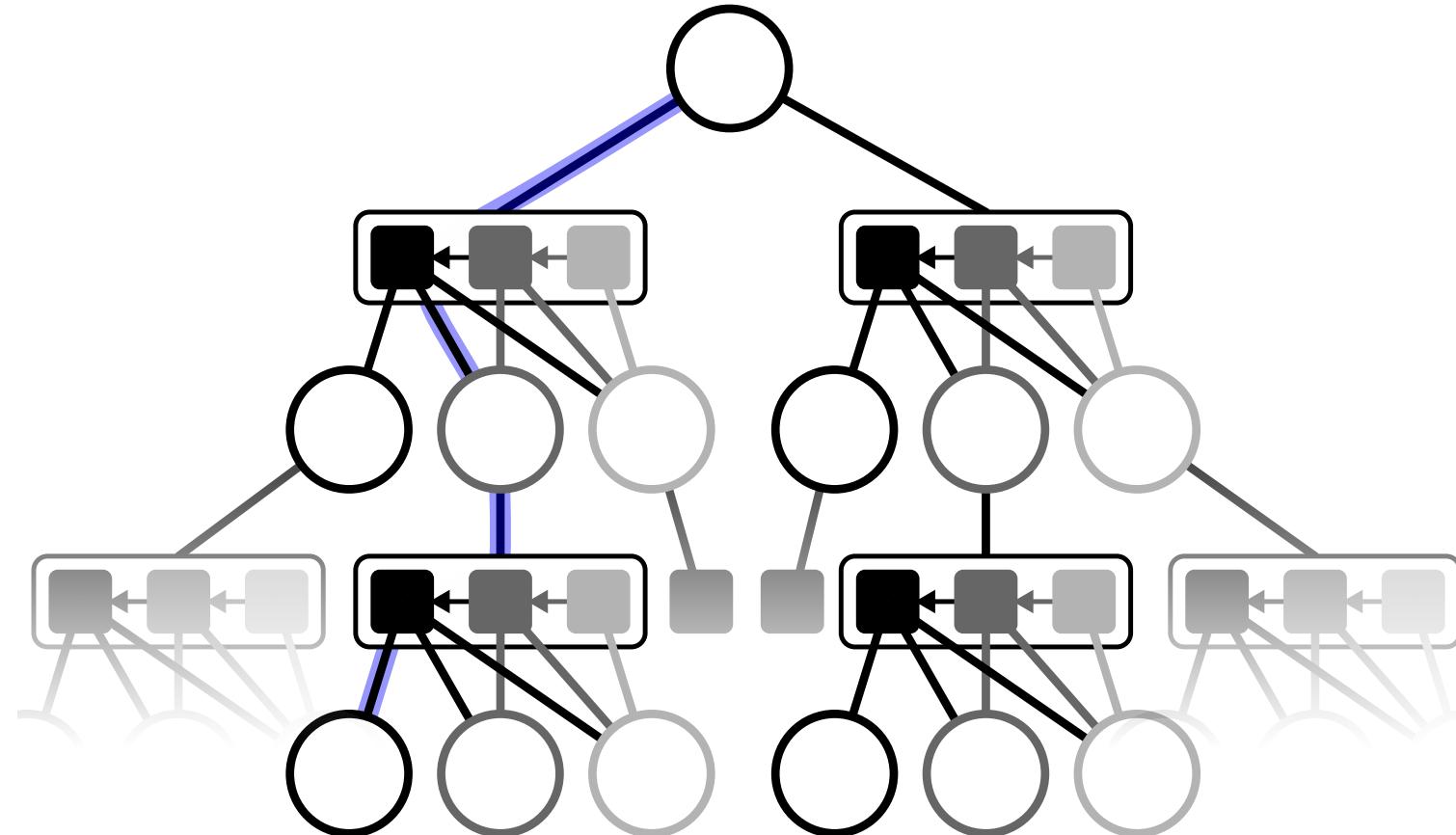
Incremental Reuse Particle Filter Tree (IR-PFT)

- Extend PFT-DPW¹, incorporating trajectories from previous planning sessions for fast estimation of $Q(b_k, a_k)$



¹Z. Sunberg and M. Kochenderfer. "Online algorithms for POMDPs with continuous state, action, and observation spaces." ICAPS, 2018.

Value Gradients with Action Adaptive Search Trees in Continuous (PO)MDPs



Agenda

Experience Reuse in POMDP Planning

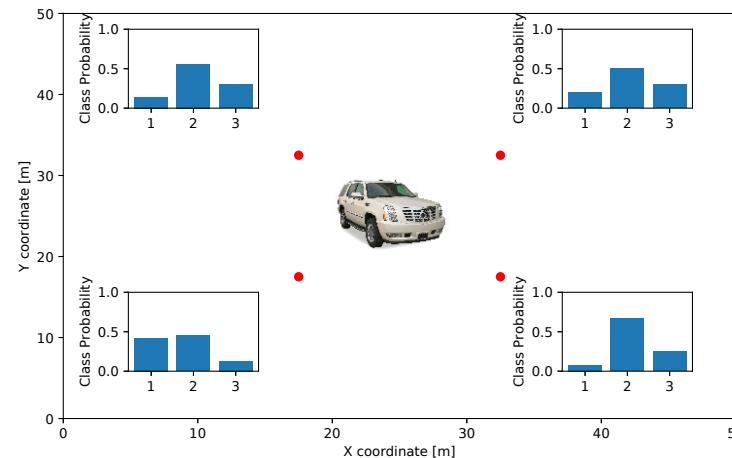
POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

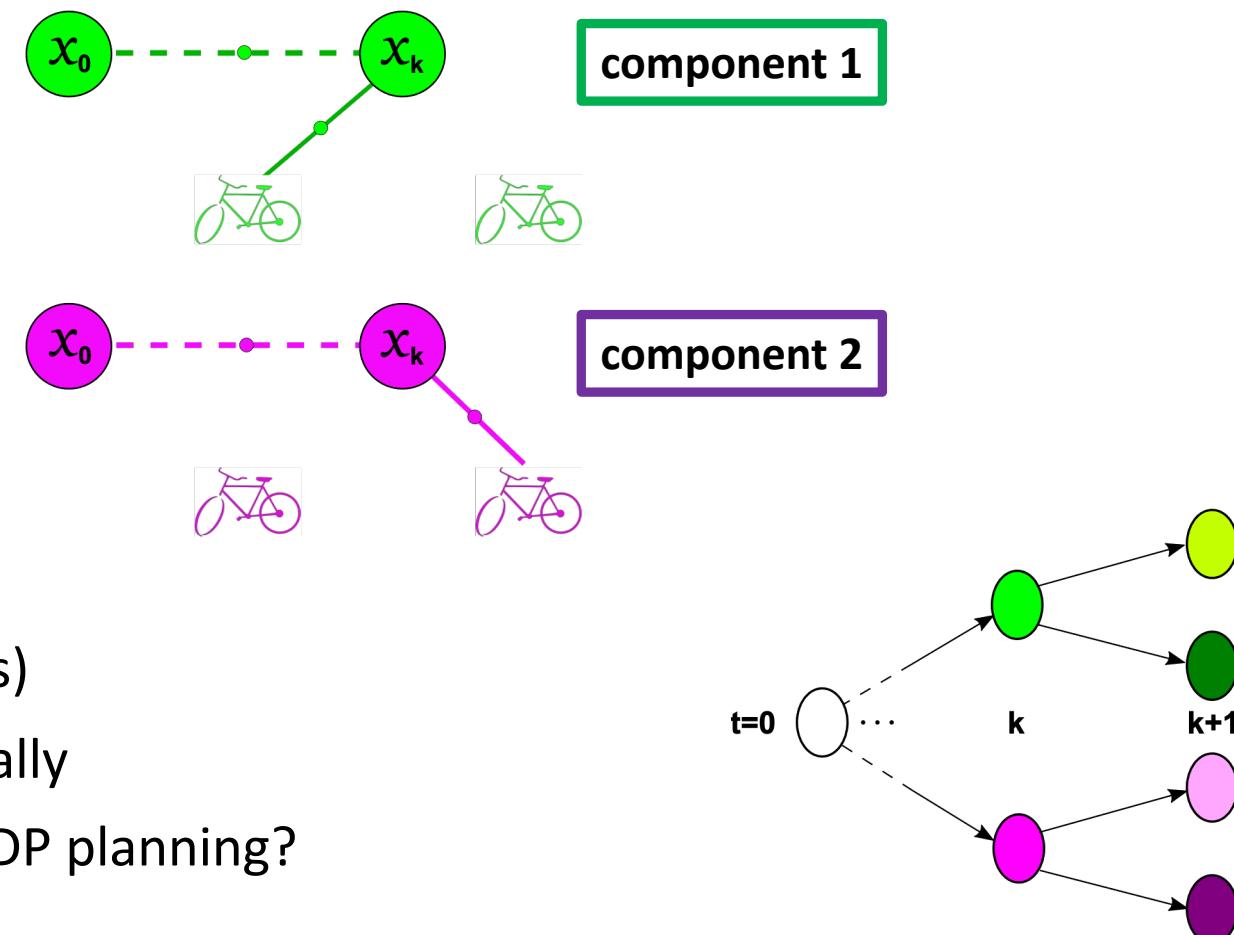
Multi-agent POMDP Planning with Inconsistent Beliefs

Autonomous Semantic Perception & Ambiguous Environments

Viewpoint dependent semantic models



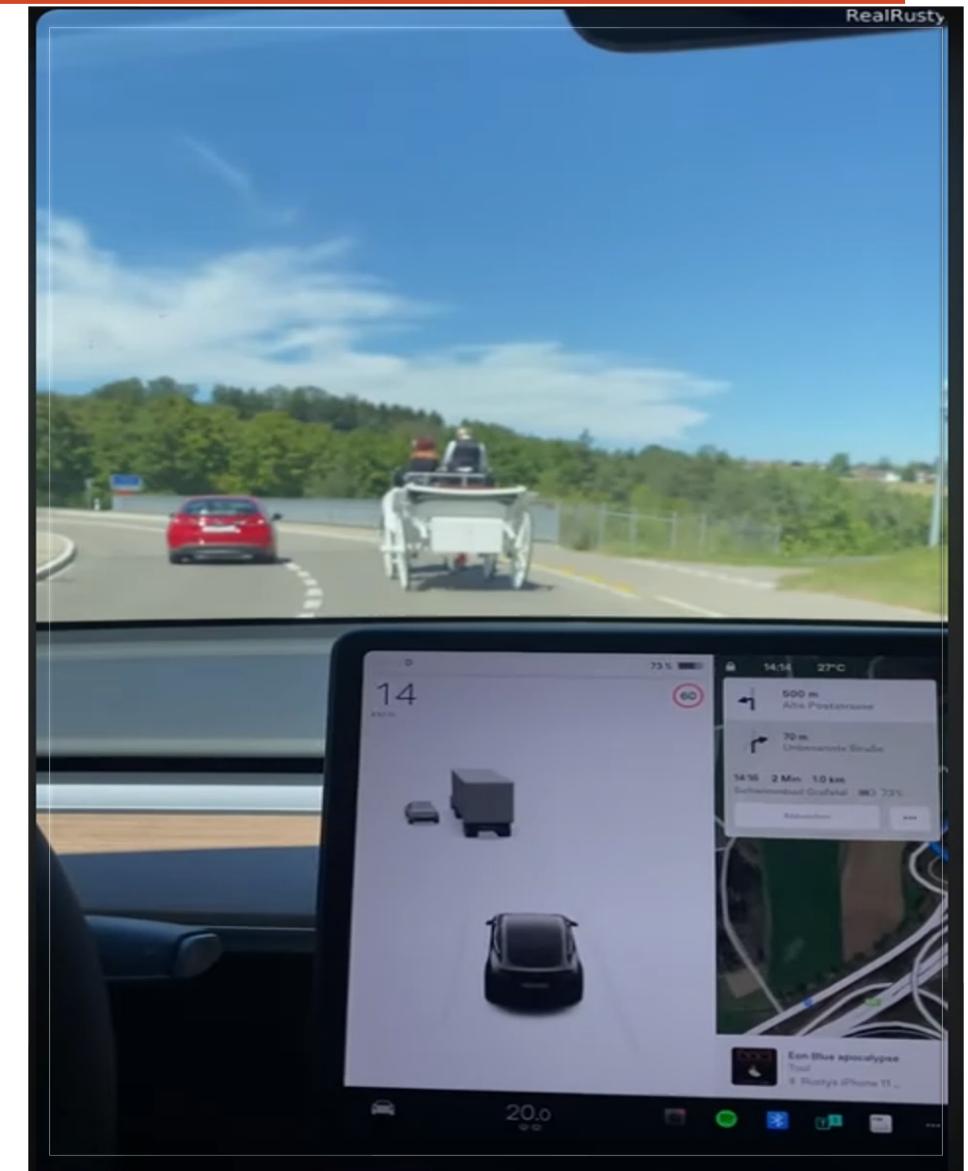
Data association hypotheses



- Hybrid beliefs (over continuous and discrete RVs)
- The number of hypotheses can grow exponentially
- How do we do probabilistic inference and POMDP planning?

Semantic Perception & SLAM

- Usually, semantics and geometry are considered **separately**
- Cannot use coupled observation models or priors
- Can lead to absurd results



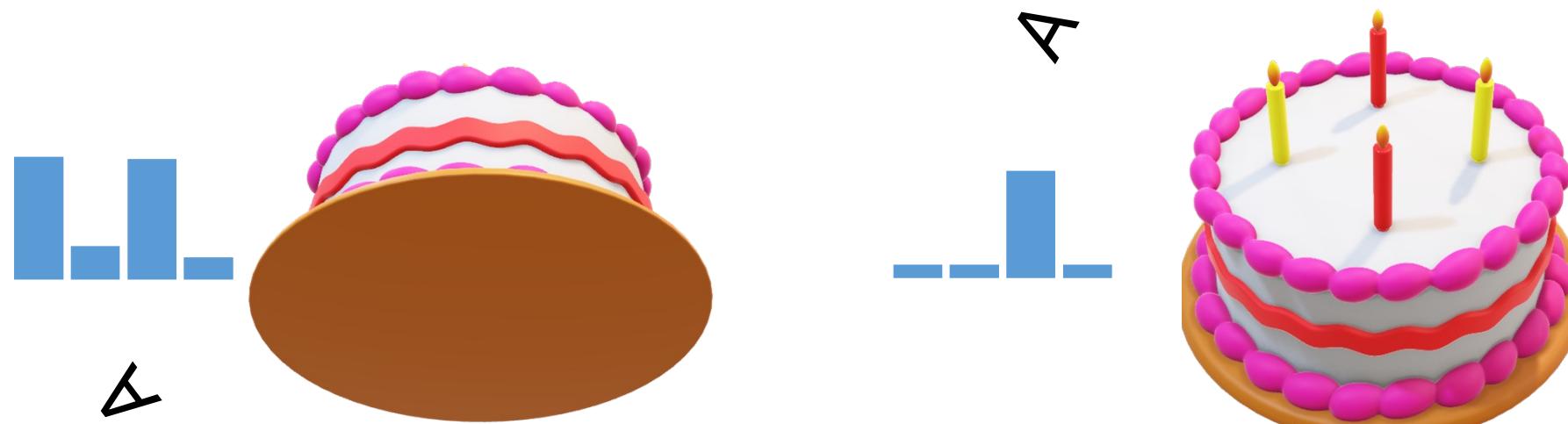
Class- and Viewpoint-Dependency

- Is it a floor or a roof?
- Depending on the viewpoint of the viewer!
 - Looking on the people below - it's a floor
 - Looking on the people above - it's a roof
- How do we know the viewpoint?



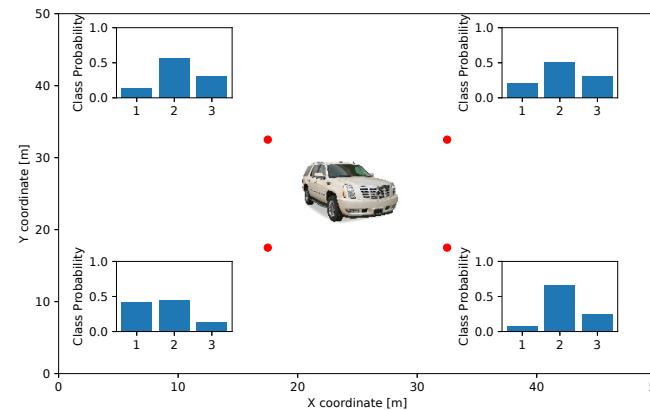
Class- and Viewpoint-Dependency

- Another example:



Coupled Models

- View-dependent semantic observation model:



$$\mathbb{P}(z^s \mid c, \mathcal{X}^{rel})$$

Semantic observation
(from a classifier)

Object class

Agent's viewpoint
relative to object

Find the cake.



- Class and poses can be coupled via learned prior probabilities.
- Reward/constraint can depend on both classes and poses (e.g., object search)

Y. Feldman and V. Indelman, "Bayesian Viewpoint-Dependent Robust Classification under Model and Localization Uncertainty," ICRA'18.

V. Tchuiiev, Y. Feldman, and V. Indelman, "Data Association Aware Semantic Mapping and Localization via a Viewpoint Dependent Classifier Model," IROS'19.

V. Tchuiiev and V. Indelman, "Epistemic Uncertainty Aware Semantic Localization and Mapping for Inference and Belief Space Planning," Artificial Intelligence, 2023.

T. Lemberg and V. Indelman, "Online Hybrid-Belief POMDP with Coupled Semantic-Geometric Models and Semantic Safety Awareness", arXiv'25.

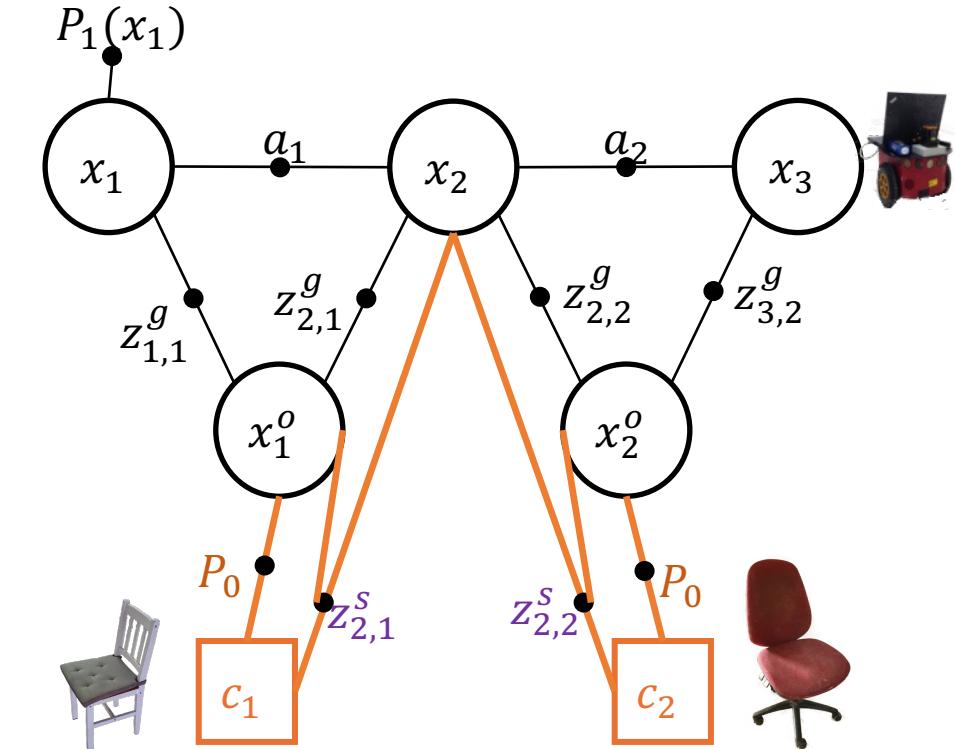
Hybrid Belief

- **Hybrid Belief** at time instant k:

$$b[X_k, C] = \mathbb{P}(X_k, C \mid \mathcal{H}_k)$$

Robot's and objects' poses Objects' classes History (actions, geometric & semantic observations)

- Classes and agent poses are dependent
- Classes of different objects are dependent
- As opposed to:
 - Per-frame classification
 - Modeling semantic observations as viewpoint **independent**



Y. Feldman and V. Indelman, "Bayesian Viewpoint-Dependent Robust Classification under Model and Localization Uncertainty," ICRA'18.

V. Tchouiev, Y. Feldman, and V. Indelman, "Data Association Aware Semantic Mapping and Localization via a Viewpoint Dependent Classifier Model," IROS'19.

V. Tchouiev and V. Indelman, "Epistemic Uncertainty Aware Semantic Localization and Mapping for Inference and Belief Space Planning," Artificial Intelligence, 2023.

T. Lemberg and V. Indelman, "Online Hybrid-Belief POMDP with Coupled Semantic-Geometric Models and Semantic Safety Awareness", arXiv'25.

POMDP Planning with Hybrid Semantic-Geometric Beliefs

- Value function

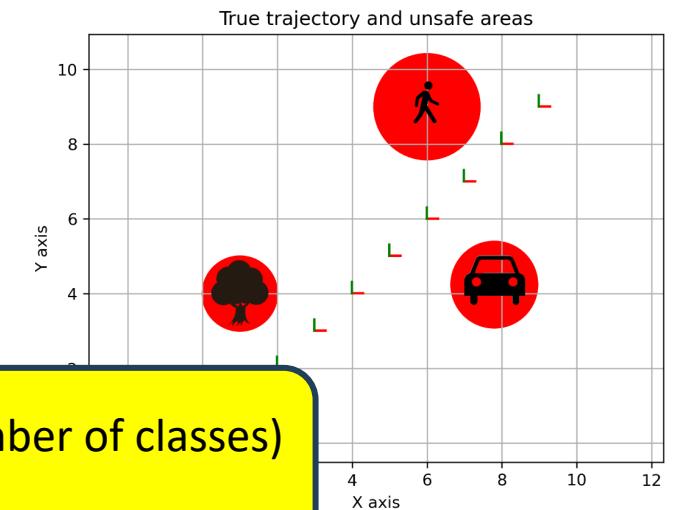
$$V^\pi(b_k) = \mathbb{E}_{z_{k+1:k+L}} \left[\sum_{l=k}^{k+L-1} \rho(b_l, \pi_l(b_l), b_{l+1}) \right]$$

- Semantic Risk Awareness

$$\mathbb{P}_{safe} \triangleq \mathbb{P}(\{\wedge_{t=k+1}^L x_t \notin \mathcal{X}_{unsafe}(C, X^o)\} \mid b_k[x_k, C, X^o], \pi)$$

Objects' classes Objects' poses

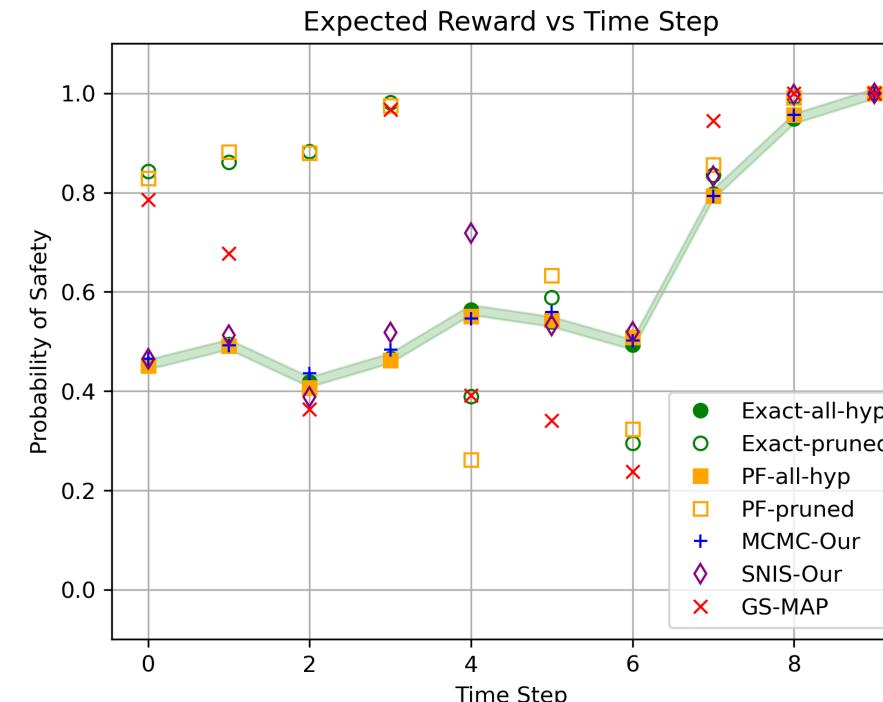
The number of classification hypotheses is M^N (N: number of objects, M: number of classes)
 How to sample w/o pruning hypotheses? How to estimate \mathbb{P}_{safe} ?



POMDP Planning with Hybrid Semantic-Geometric Beliefs

Experiments - Estimation of \mathbb{P}_{safe} with different methods

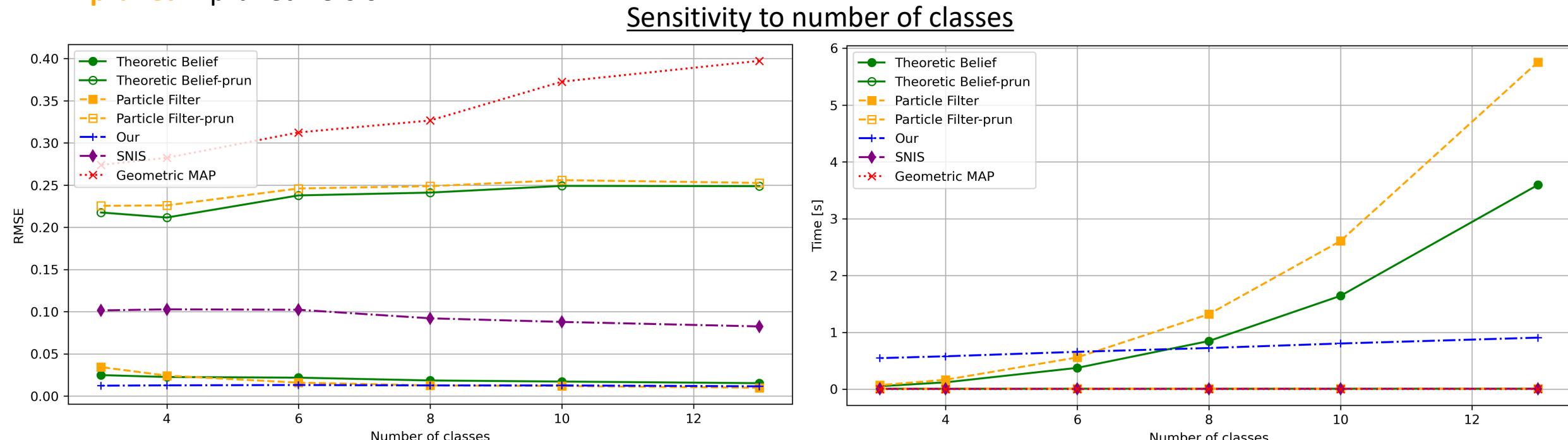
- **Exact-all-hyp** – belief computed exactly
 - **Exact-pruned** – pruned version
 - **PF-all-hyp** – Particle filter
 - **PF-pruned** – pruned version
- Our methods**
- **MCMC-Our** – MCMC samples
 - **SNIS-Our** – self-normalized importance sampling
 - **GS-MAP** – separate semantic and geometric



POMDP Planning with Hybrid Semantic-Geometric Beliefs

Experiments - Estimation of \mathbb{P}_{safe} with different methods

- **Exact-all-hyp** – belief computed exactly
 - **Exact-pruned** – pruned version
 - **PF-all-hyp** – Particle filter
 - **PF-pruned** – pruned version
- Our methods**
- **MCMC-Our** – MCMC samples
 - **SNIS-Our** – self-normalized importance sampling
 - **GS-MAP** – separate semantic and geometric

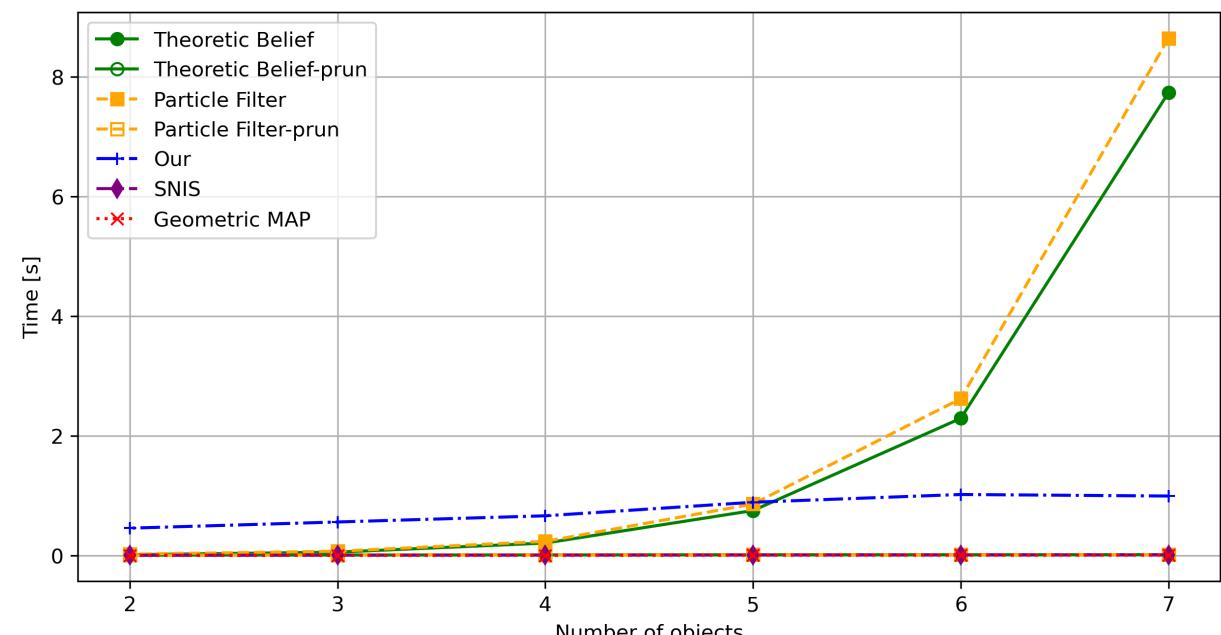
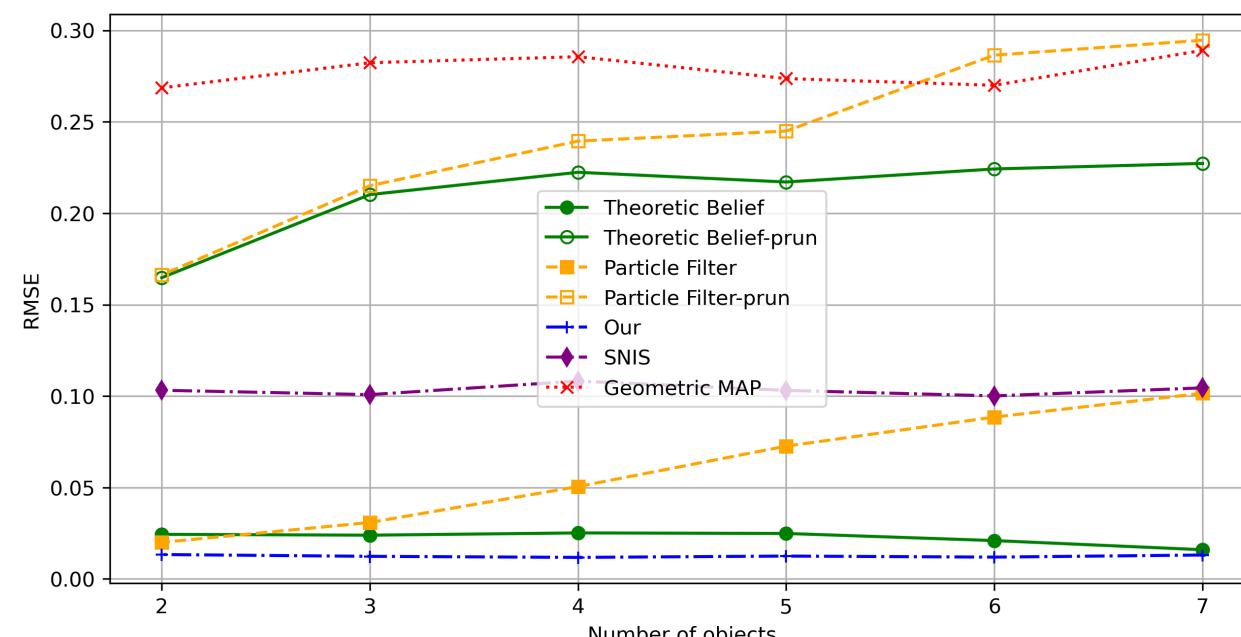


POMDP Planning with Hybrid Semantic-Geometric Beliefs

Experiments - Estimation of \mathbb{P}_{safe} with different methods

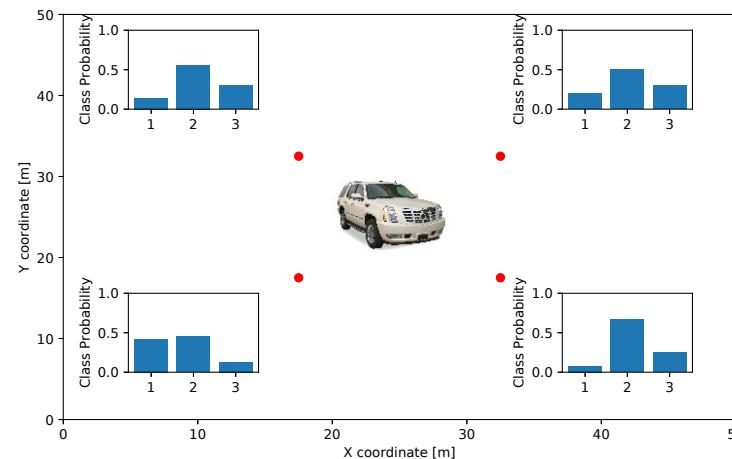
- **Exact-all-hyp** – belief computed exactly
 - **Exact-pruned** – pruned version
 - **PF-all-hyp** – Particle filter
 - **PF-pruned** – pruned version
- Our methods**
- **MCMC-Our** – MCMC samples
 - **SNIS-Our** – self-normalized importance sampling
 - **GS-MAP** – separate semantic and geometric

Sensitivity to number of objects

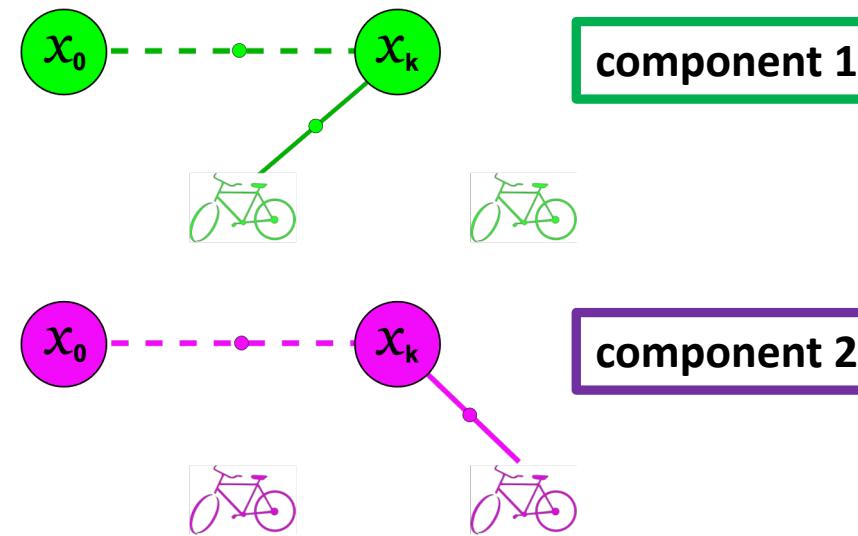


Autonomous Semantic Perception & Ambiguous Environments

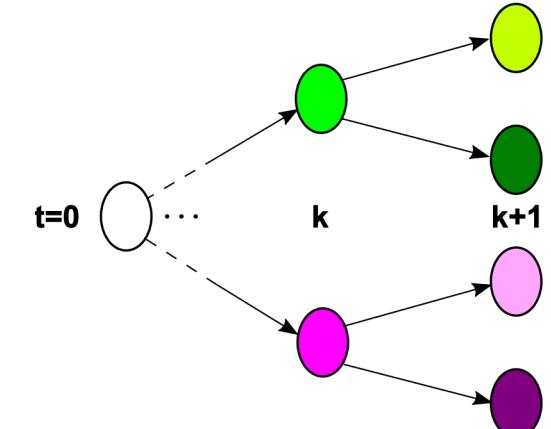
Viewpoint dependent semantic models



Data association hypotheses



- Hybrid beliefs (over continuous and discrete RVs)
- The number of hypotheses can grow exponentially
- How do we do probabilistic inference and POMDP planning?

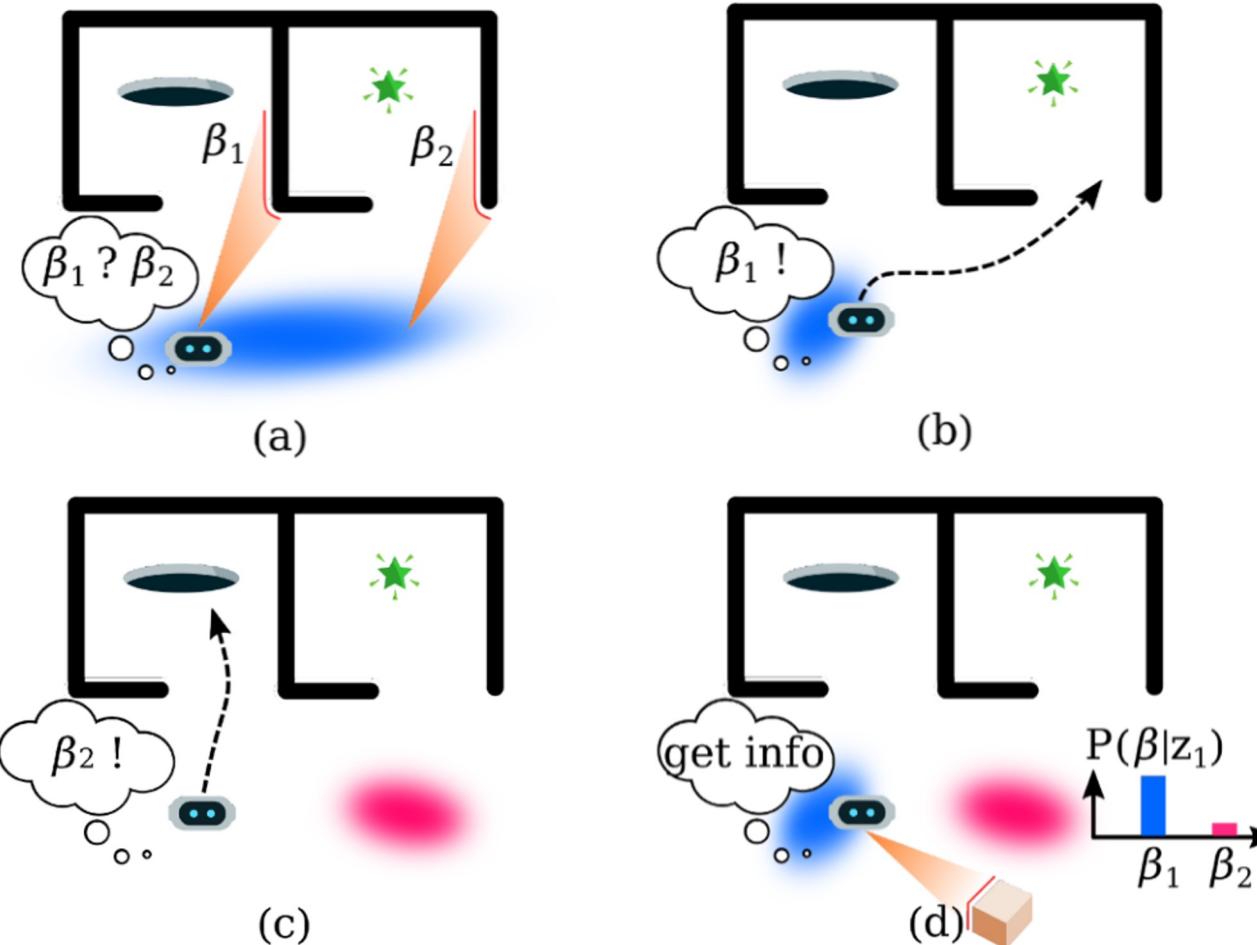


Ambiguous Scenarios

- Have to reason about data association hypotheses within inference and planning

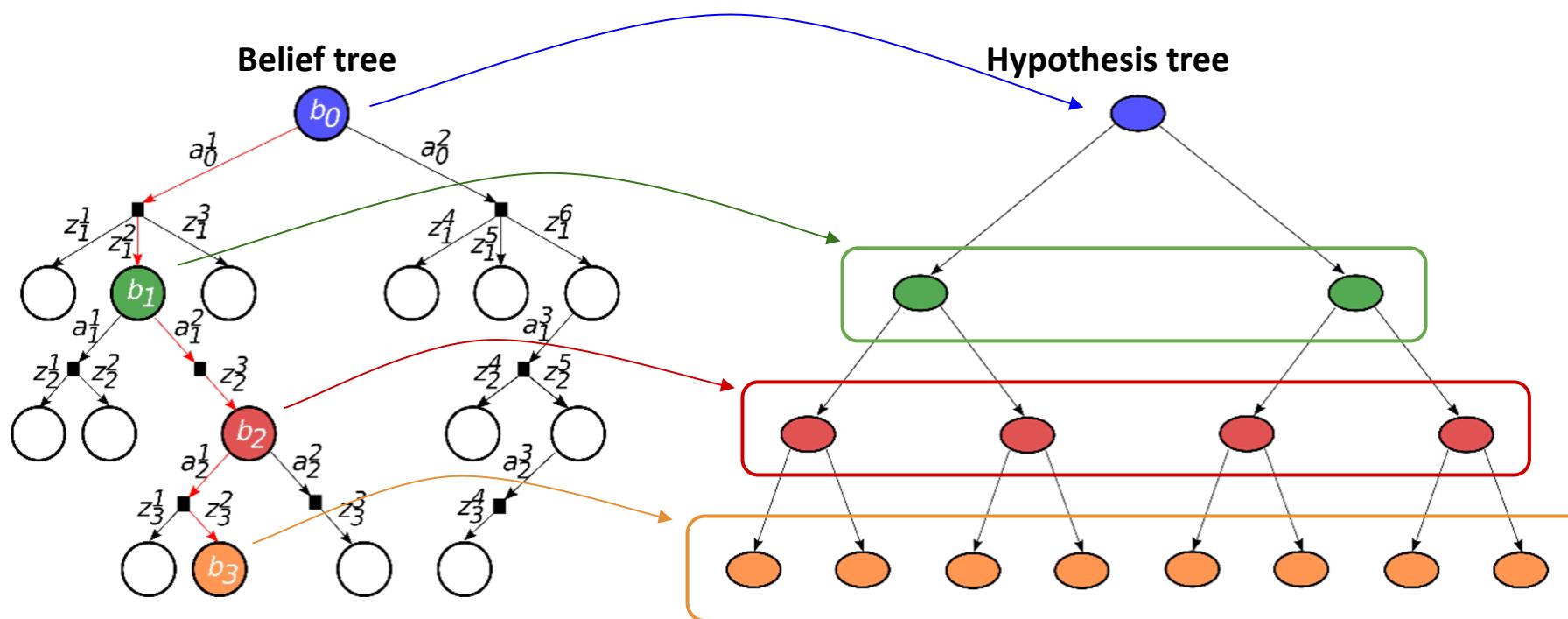
An observation:
(e.g. LIDAR)

How should the agent act?



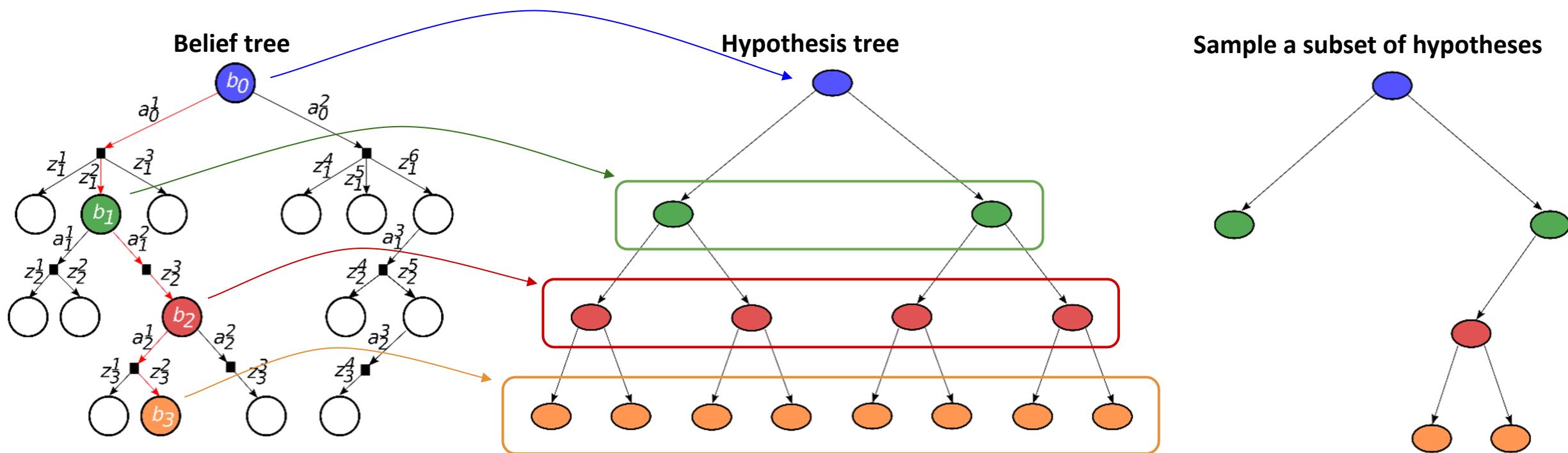
Continuous-Discrete State Spaces - the Challenge

- The number of hypotheses may grow **exponentially** with the planning horizon!



Continuous-Discrete State Spaces - the Challenge

- The number of hypotheses may grow **exponentially** with the planning horizon!



Agenda

Experience Reuse in POMDP Planning

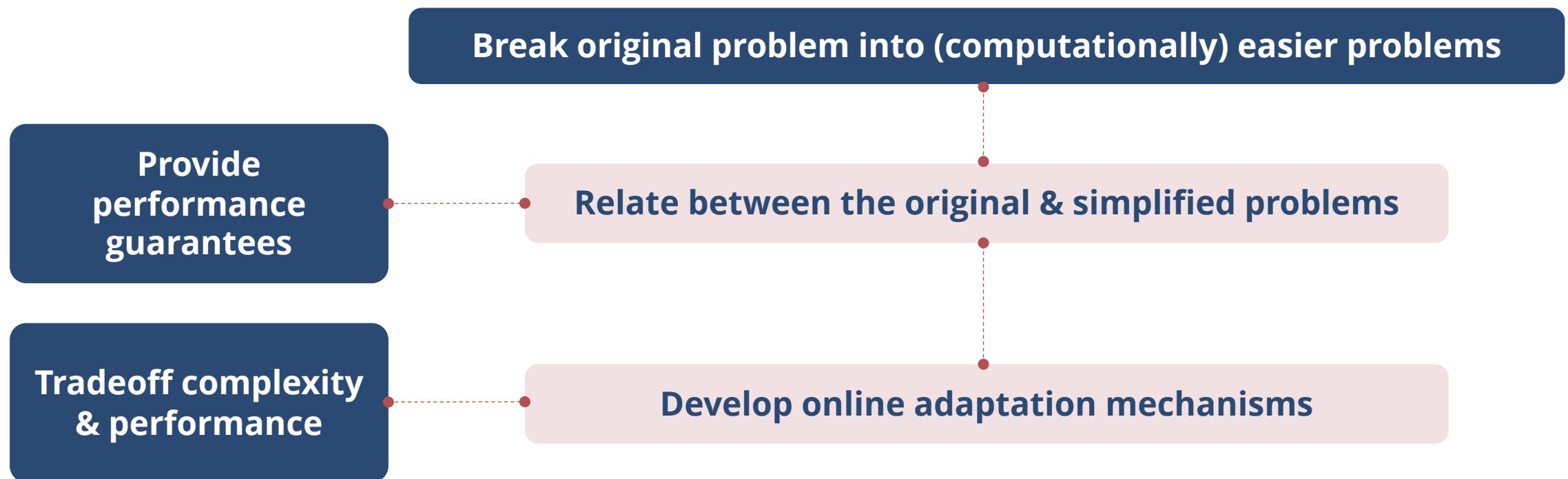
POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

Simplification Framework

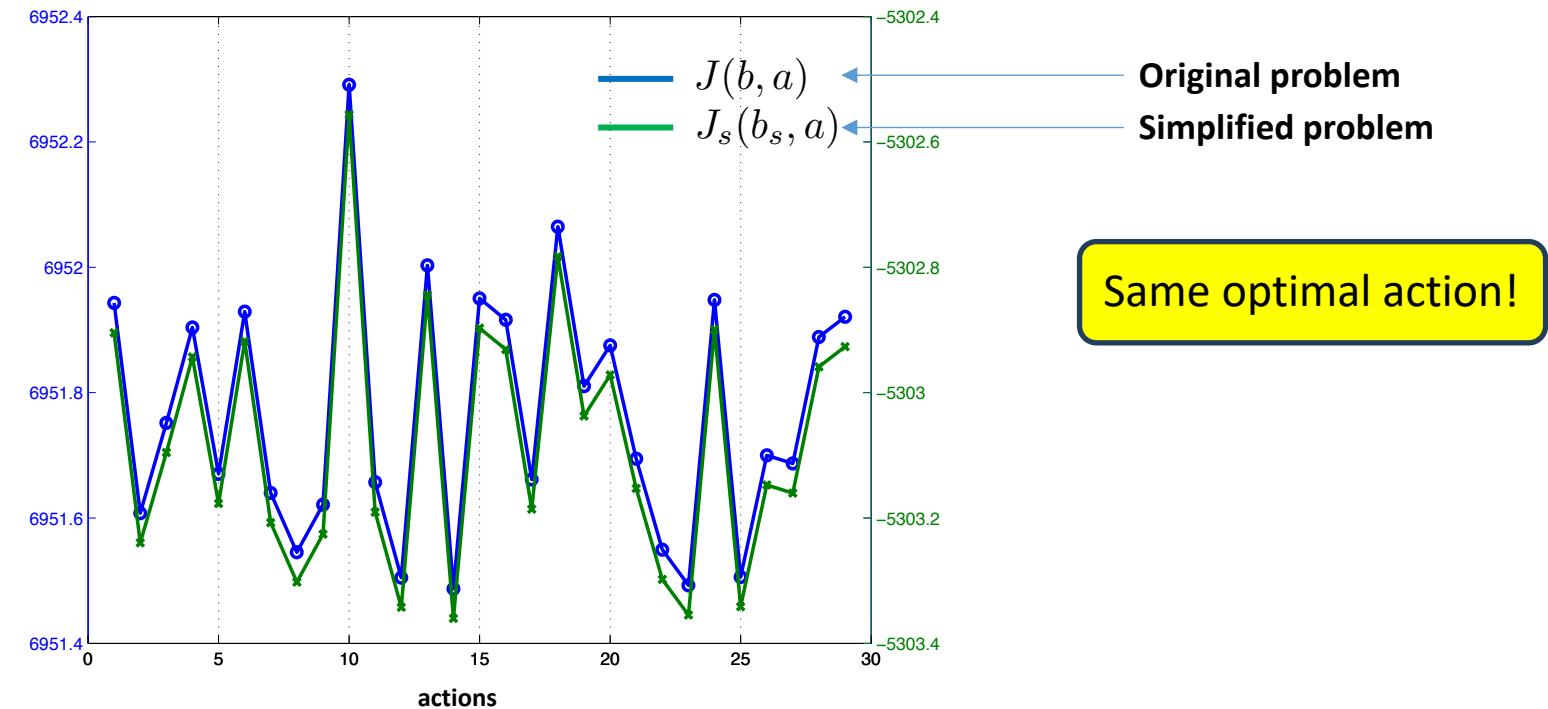
Accelerate decision making by adaptive simplification while providing performance guarantees



[Indelman RAL'16; Elimelech & Indelman IJRR'22; Sztynklic & Indelman IROS'22; Zhitnikov & Indelman AIJ'22, TRO'24; Shienman & Indelman ICRA'22; Barenboim & Indelman NIPS'23; Kitanov & Indelman IJRR'24; Zhitnikov et al. IJRR'24; Lev-Yehudi, Barenboim & Indelman AAAI'24, Yotam & Indelman TRO'24, Da & Indelman ISRR'24]

Simplification of Decision-Making Problems

- Each element of the decision-making problem can be simplified
- **Action-consistent** simplification preserves order between actions w.r.t. original problem



Simplification of Decision-Making Problems

$$\mathcal{LB}(b, a) \leq Q(b, a) \leq \mathcal{UB}(b, a)$$

Computationally cheap(er)
bounds

Simplification of Decision-Making Problems

Concept:

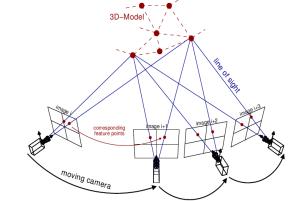
- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

Specific simplifications include:

- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of Risk-Averse POMDP Planning
- Simplification in a multi-agent setting

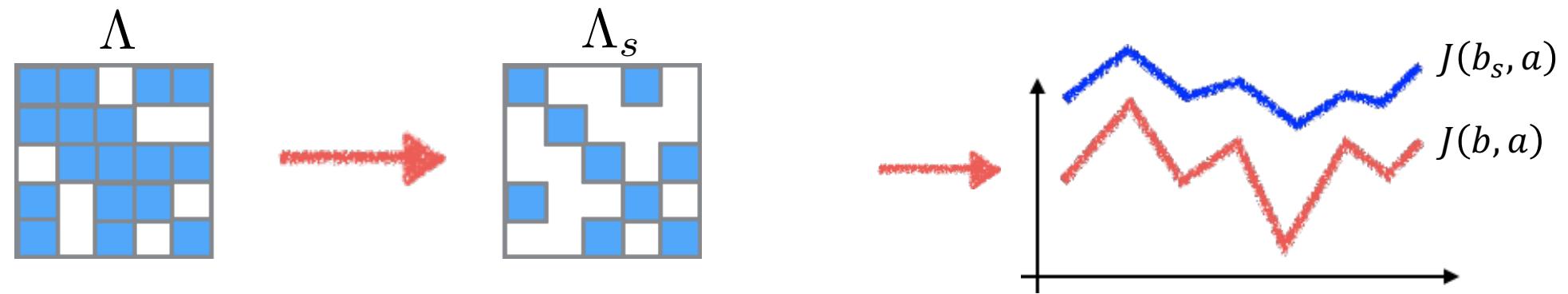
Belief Sparsification for Gaussian BSP

- Find an appropriate **sparsified** (square root) information matrix
- Perform decision making using that, rather the original, information matrix



Setting:

- Gaussian belief over high dim. state $X \in \mathbb{R}^n$: $b[X] = \mathcal{N}(X^\star, \Lambda^{-1}) = \mathcal{N}(X^\star, (R^T R)^{-1})$
- Information-theoretic reward (entropy): $H[X] = \frac{1}{2} \log((2\pi e)^n |\Lambda|^{-1})$



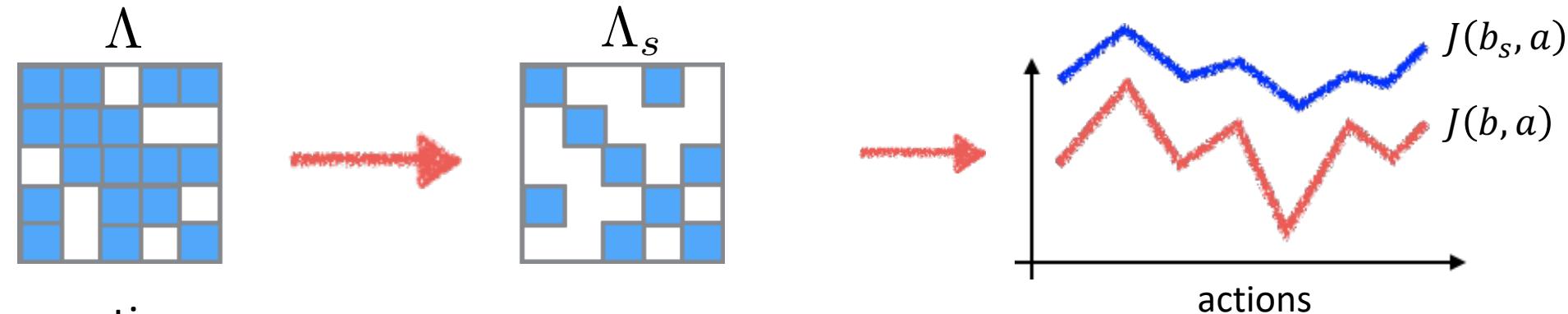
- Do we get the same performance (decisions), i.e. is it action consistent?

V. Indelman, "No Correlations Involved: Decision Making Under Uncertainty in a Conservative Sparse Information Space," IEEE RA-L'16.

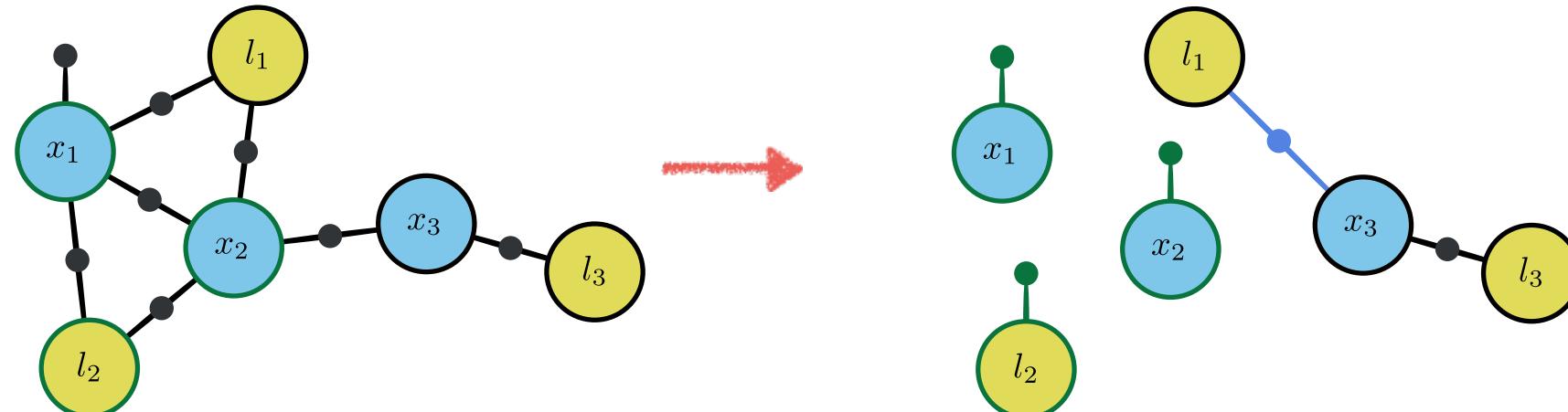
K. Elimelech and V. Indelman, "Simplified decision making in the belief space using belief sparsification," IJRR'22.

Belief Sparsification for Gaussian BSP

- Sparsification of (square root) information matrix



- Graphical models perspective:

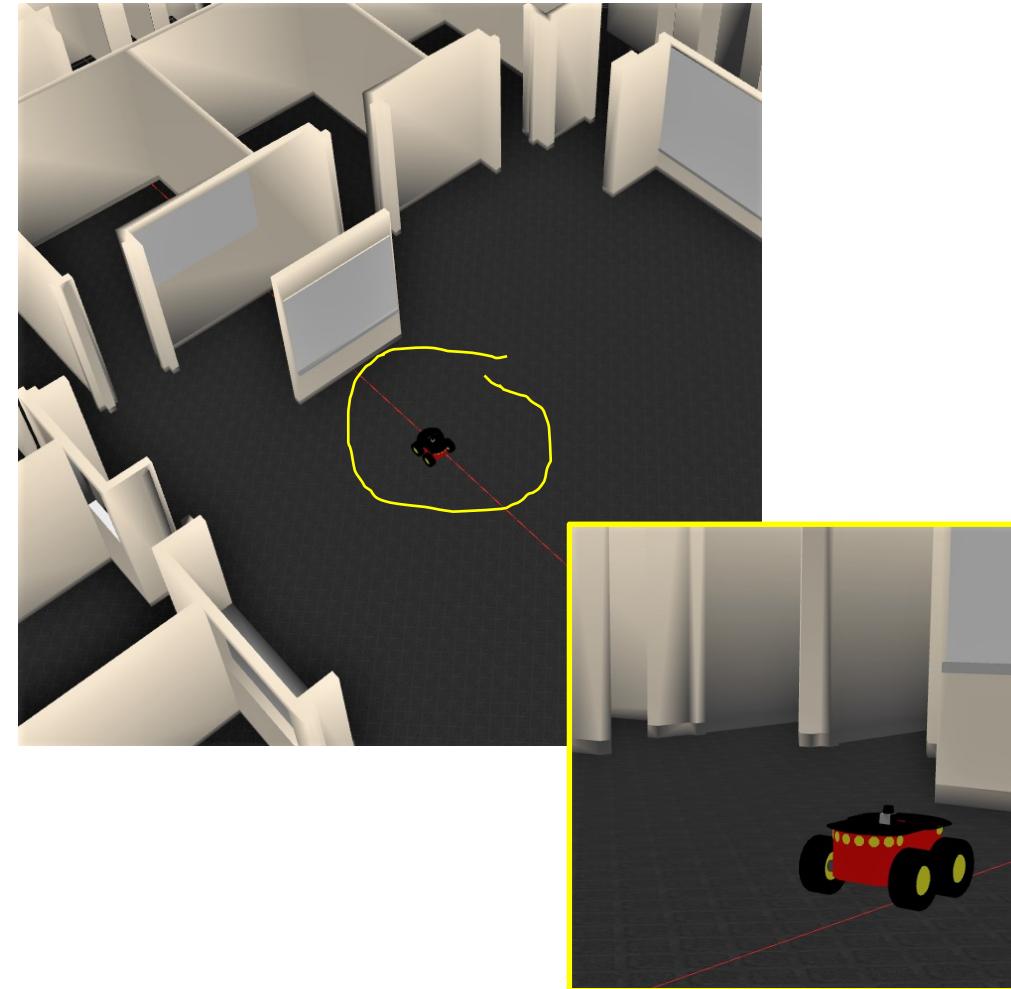
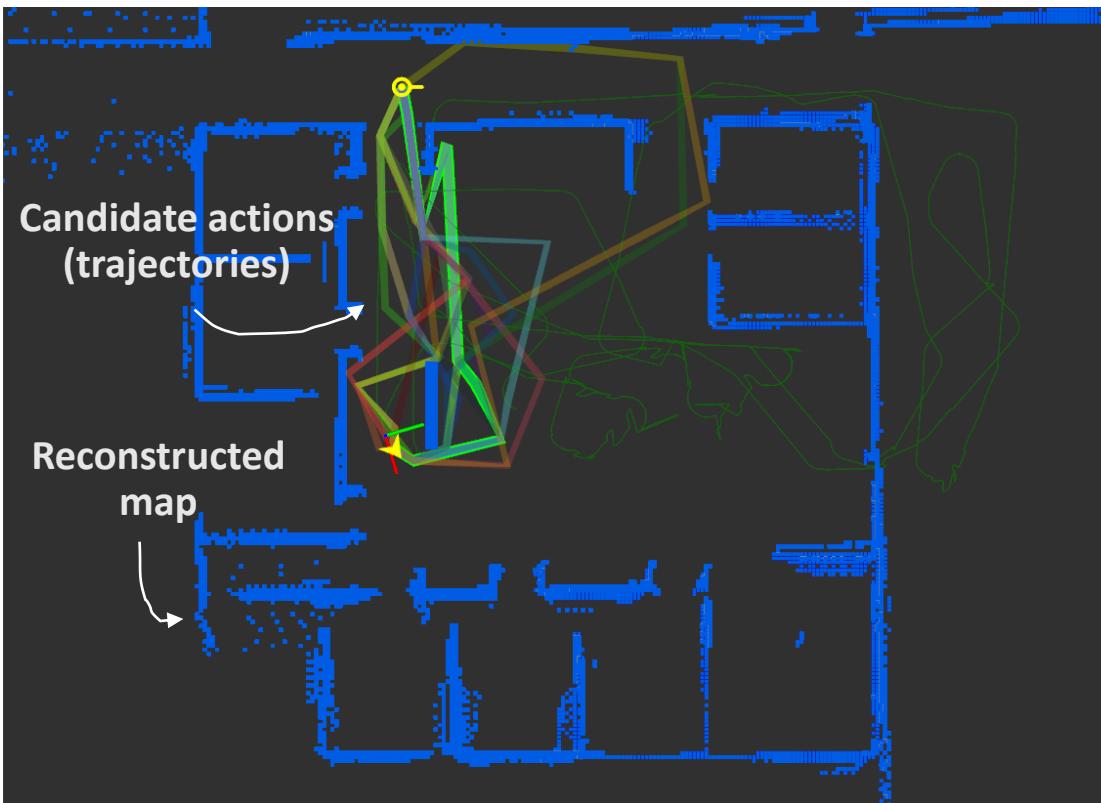


Belief Sparsification for Gaussian BSP

- Agent performs simultaneous localization and mapping
- Maintains a multivariate Gaussian belief

$$b[X] = \mathcal{N}(X^*, (R^T R)^{-1})$$

- Task: reach a goal with **minimum uncertainty**

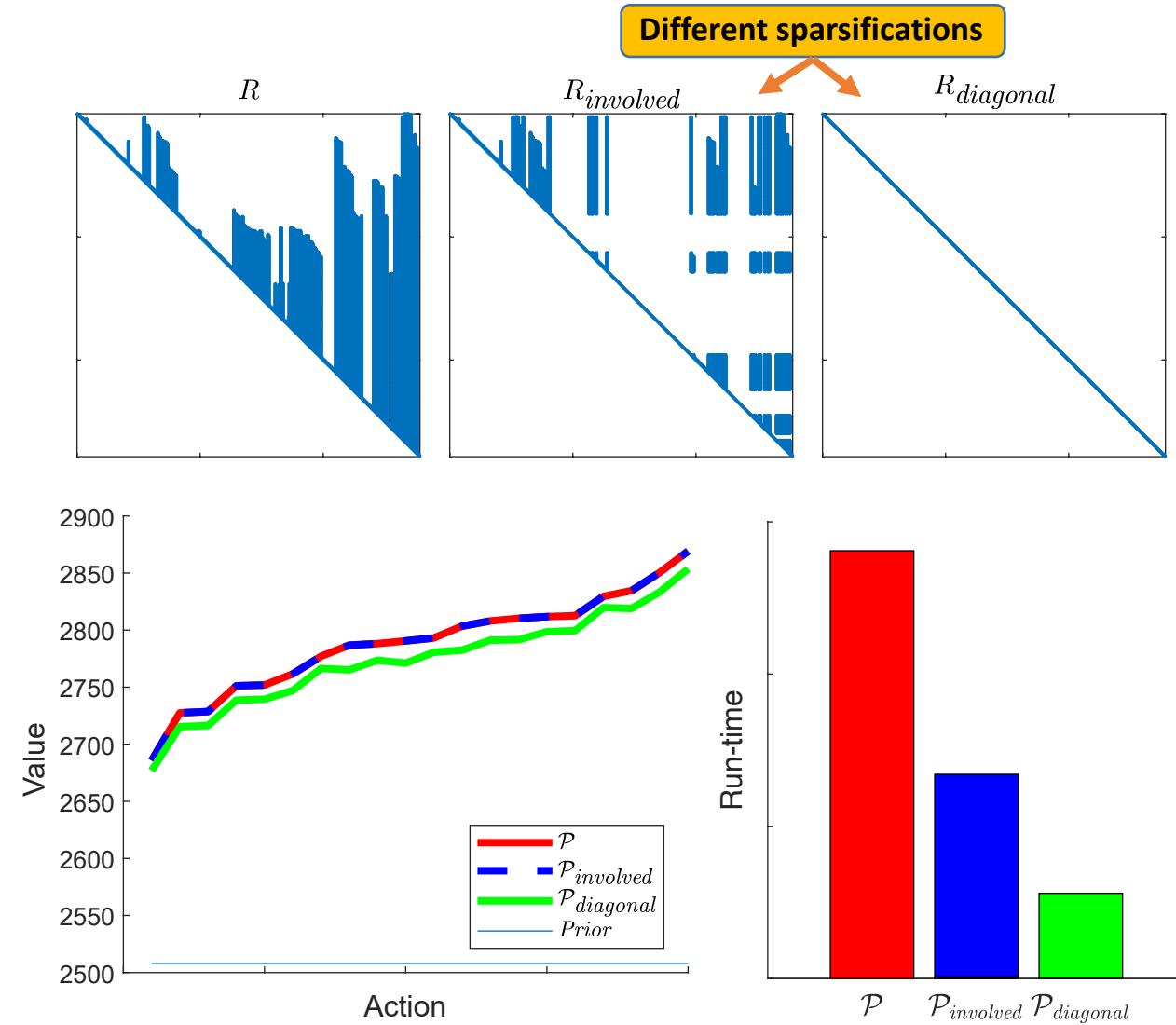
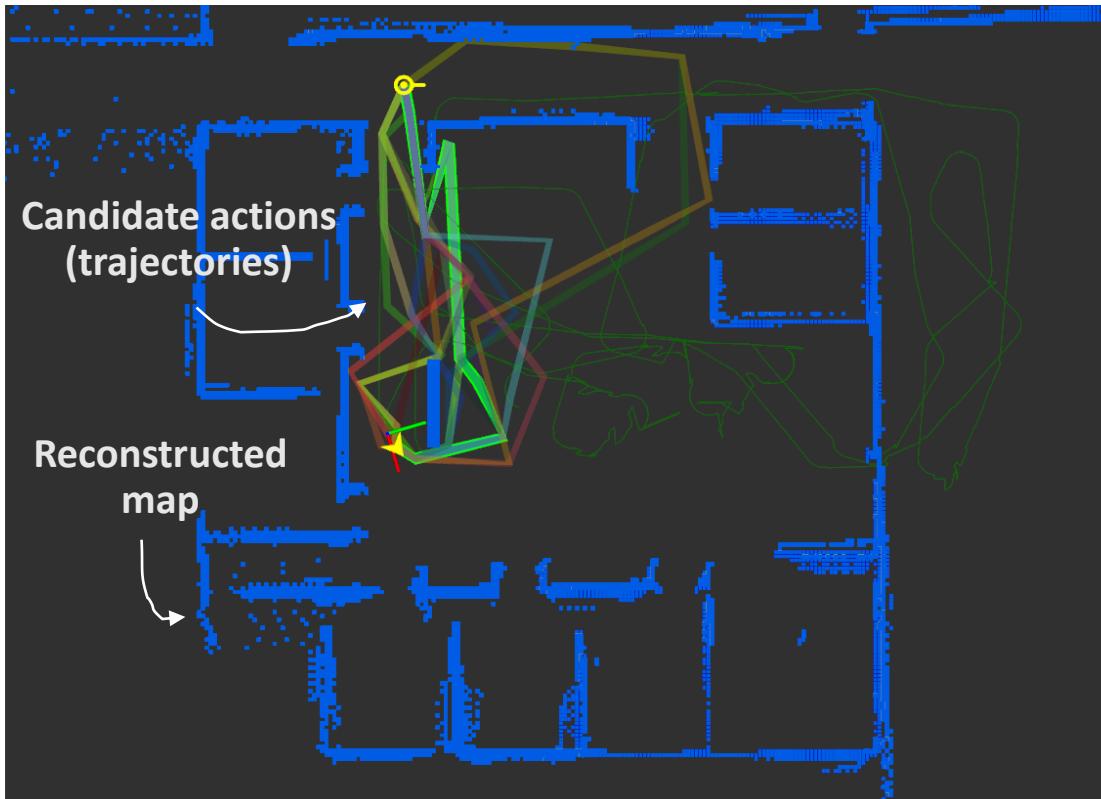


Belief Sparsification for Gaussian BSP

- Agent performs simultaneous localization and mapping
- Maintains a multivariate Gaussian belief

$$b[X] = \mathcal{N}(X^*, (R^T R)^{-1})$$

- Task: reach a goal with **minimum uncertainty**



Simplification of Decision-Making Problems

Concept:

- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

Specific simplifications include:

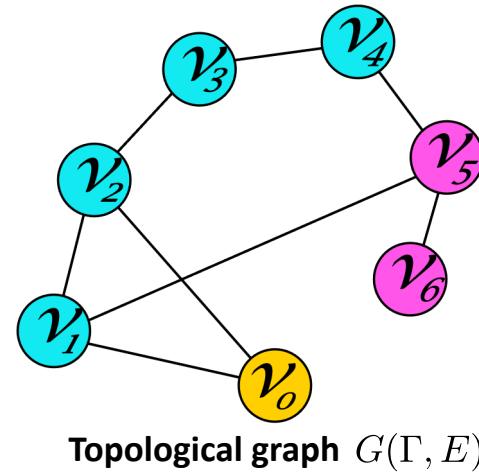
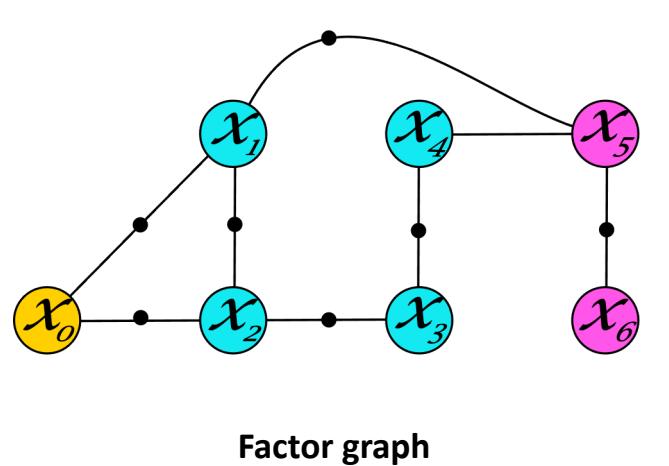
- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of Risk-Averse POMDP Planning
- Simplification in a multi-agent setting

Topological Gaussian Belief Space Planning (t-BSP)

- Topological properties of factor graphs dominantly determine estimation accuracy¹

Key idea:

- Design a metric of factor graph topology that is strongly correlated with entropy
- Determine best action using that topological metric (instead of entropy)
- **Does not require explicit inference, nor partial state covariance recovery**



topological
metric $s(G)$

graph signature

¹K. Khosoussi, et al. "Reliable graphs for SLAM", IJRR'19.

Topological Gaussian Belief Space Planning (t-BSP)

Metric Space

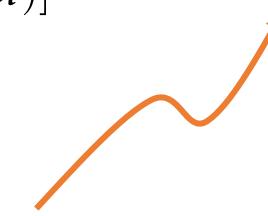
$$J(\mathcal{U}) = \frac{n}{2} \ln(2\pi e) + \frac{1}{2} \ln |\Sigma(X_{k+L})|$$

$$\mathcal{U}^* = \arg \min_{\mathcal{U}} J(\mathcal{U})$$

Topological space

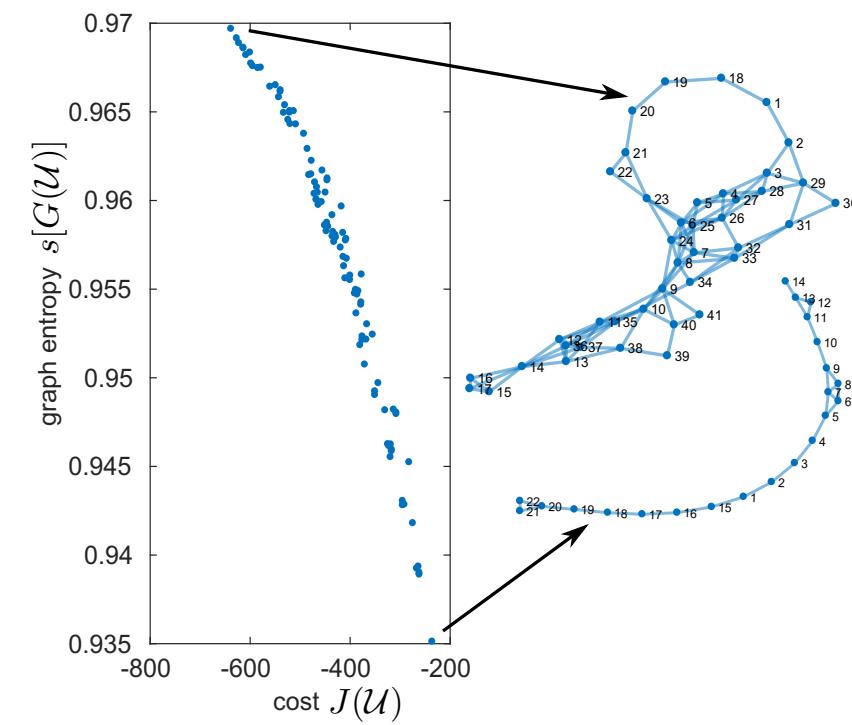
$$s(G) = H_{VN}(G) \approx 1 - \frac{1}{|\Gamma|} - \frac{1}{|\Gamma|^2} \sum_{(i,j) \in E} \frac{1}{d(i)d(j)}$$

$$\hat{\mathcal{U}}^* = \arg \max_{\mathcal{U}} s[G(\mathcal{U})]$$

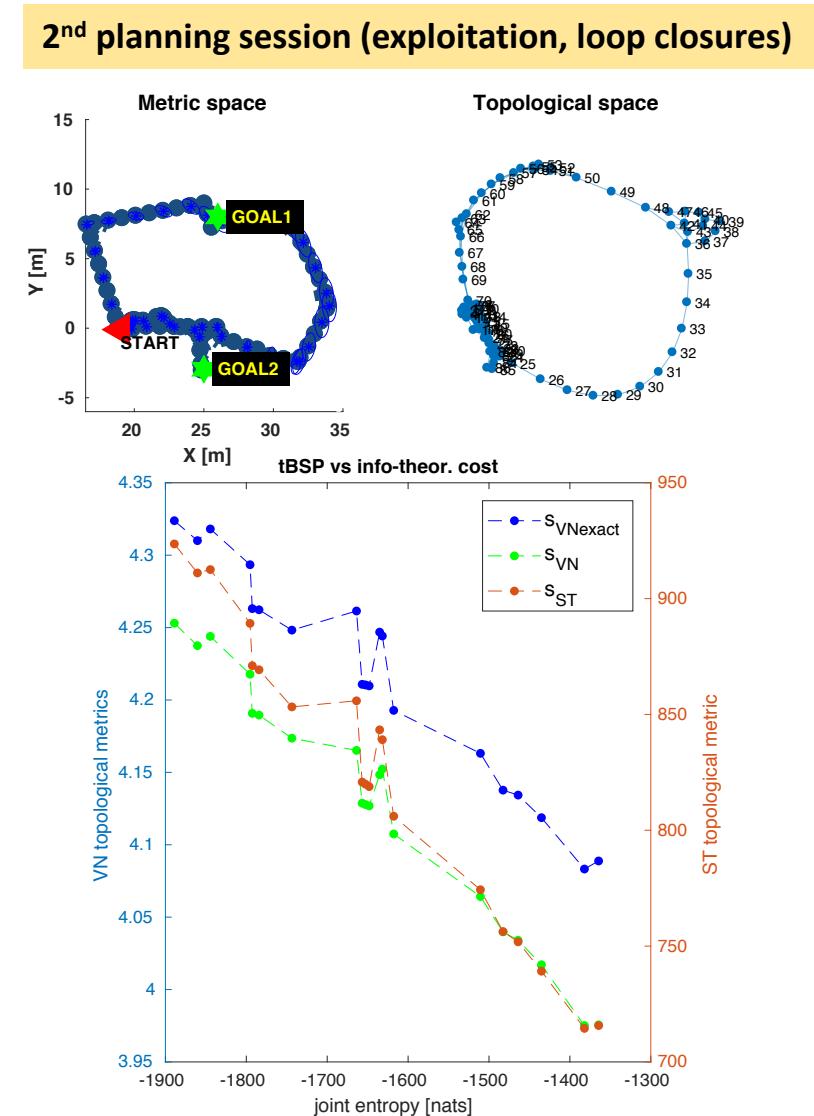
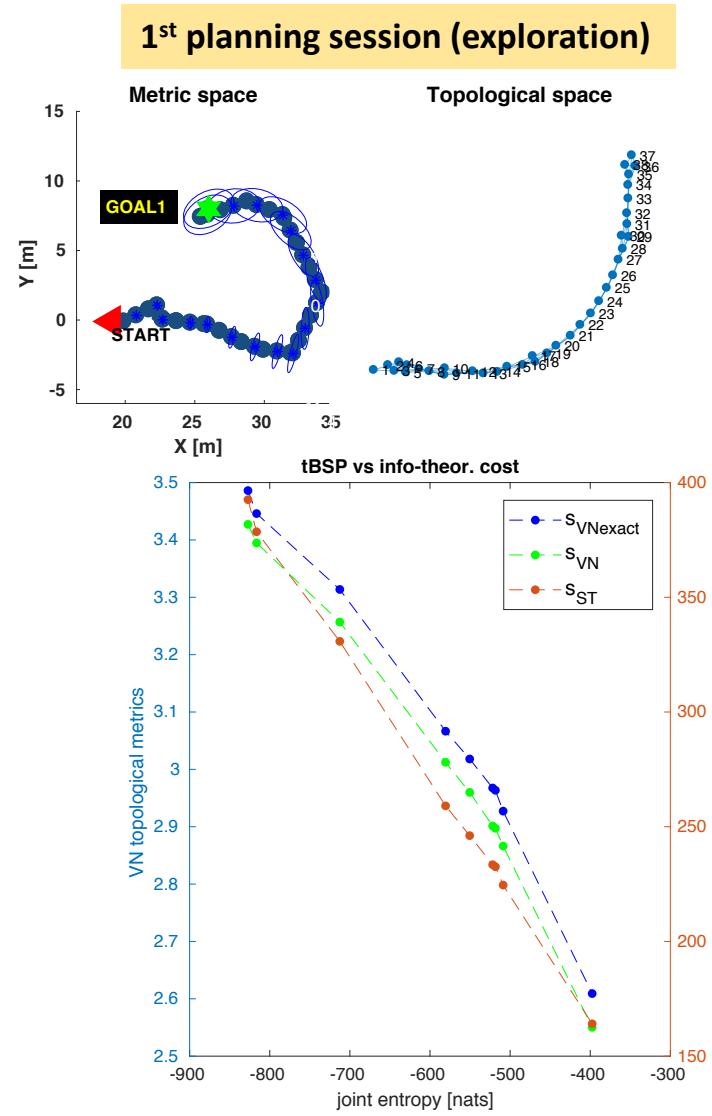
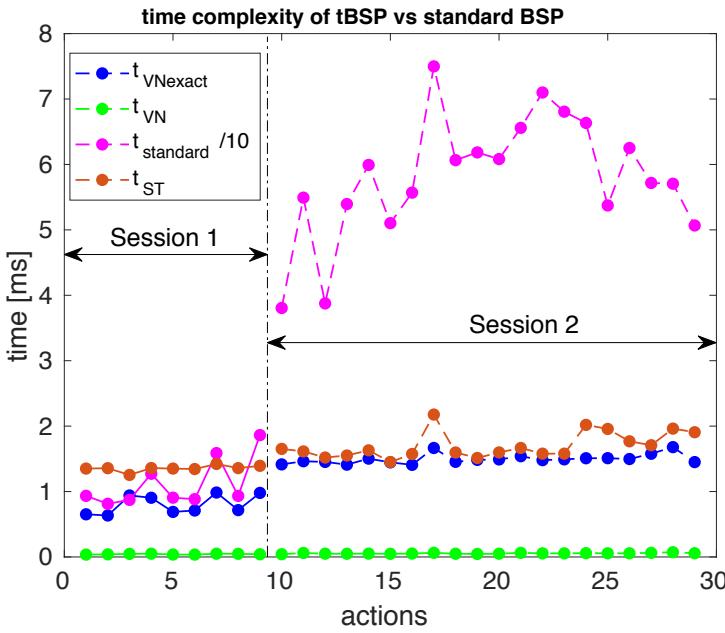
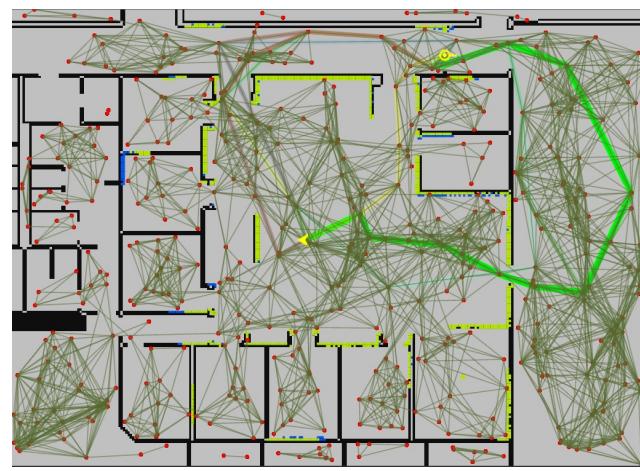


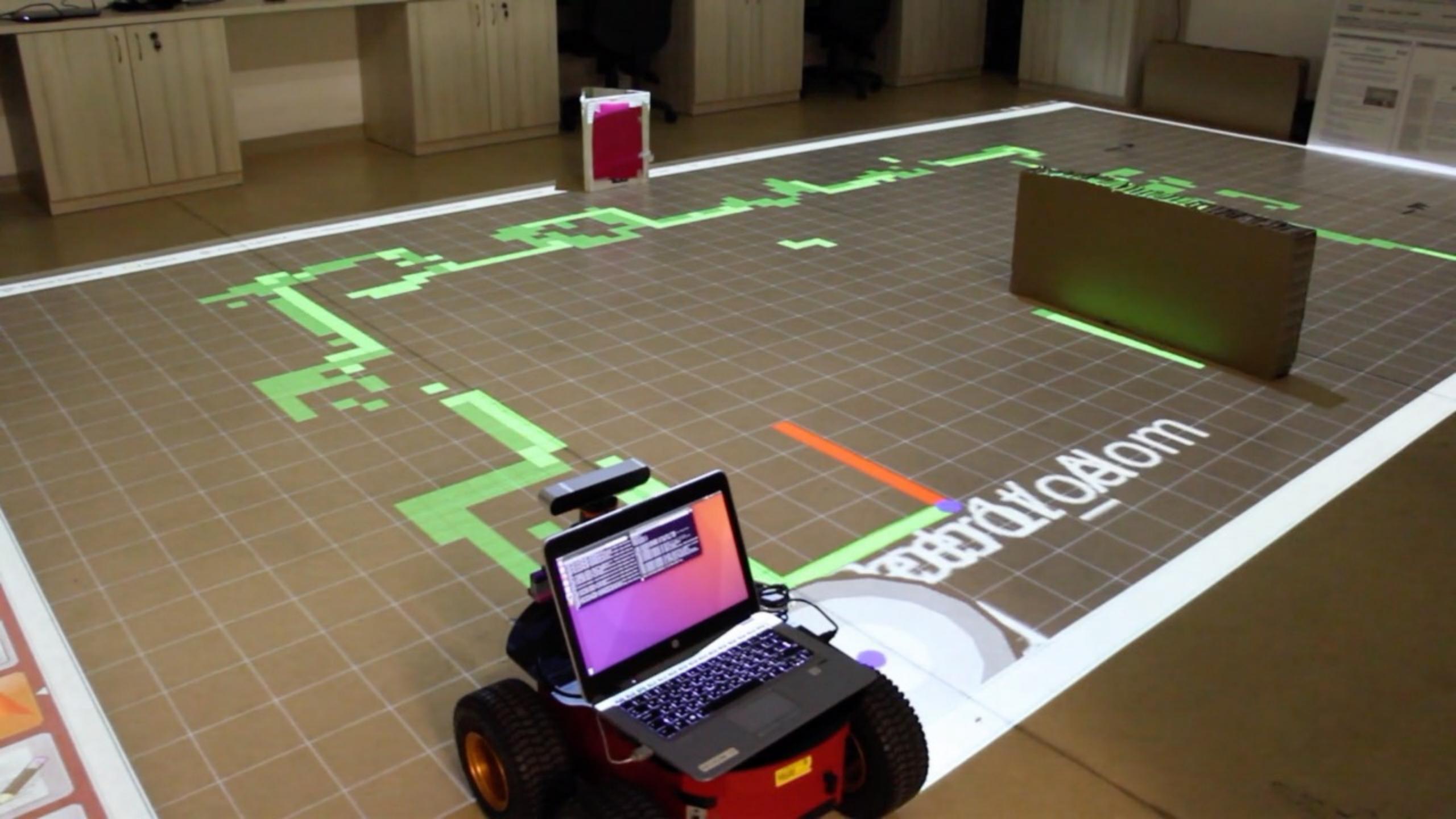
- Cheap to calculate, only a function of node degrees
- Supports incremental calculations
- Provided bounds on the error/loss $|J_k(\hat{\mathcal{U}}) - J_k(\hat{\mathcal{U}}^*)|$

Topological and info-theoretic metrics are strongly correlated



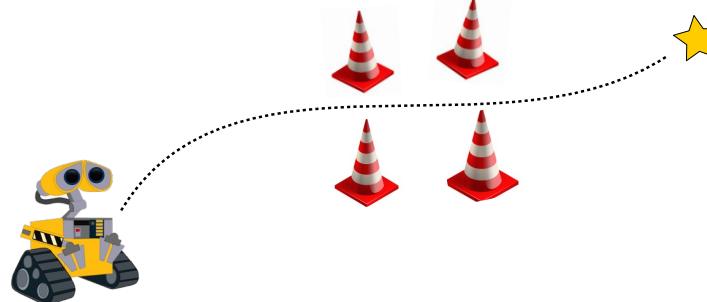
t-BSP: Gazebo Results





Focused Topological Gaussian Belief Space Planning (ft-BSP)

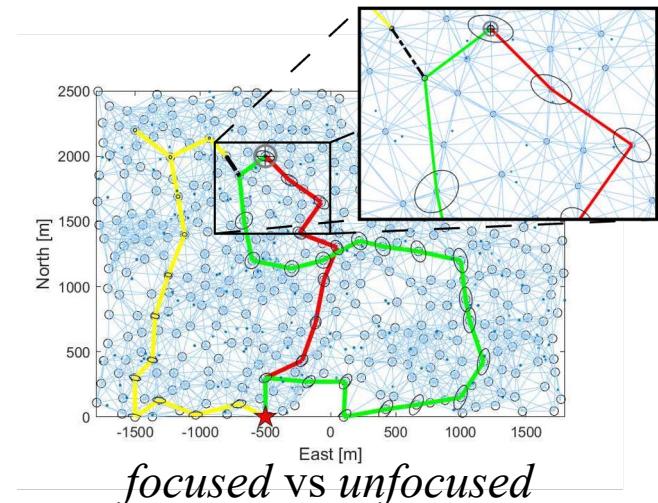
- **Unfocused** BSP – reduce uncertainty over all variables
- **Focused** BSP – reduce uncertainty over a predefined subset of variables (focused variables)



collision avoidance



focused reconstruction task



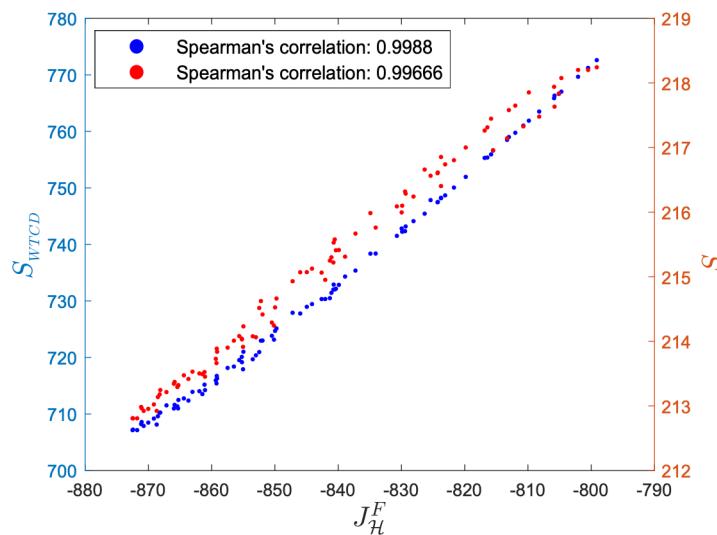
focused vs unfocused

$$J_{\mathcal{H}}^F(\mathcal{U}) = \frac{n^F}{2} \log(2\pi e) - \frac{1}{2} \log |\Lambda_{k+L}| + \frac{1}{2} \log |\Lambda_{k+L}^U| \quad \text{Expensive!}$$

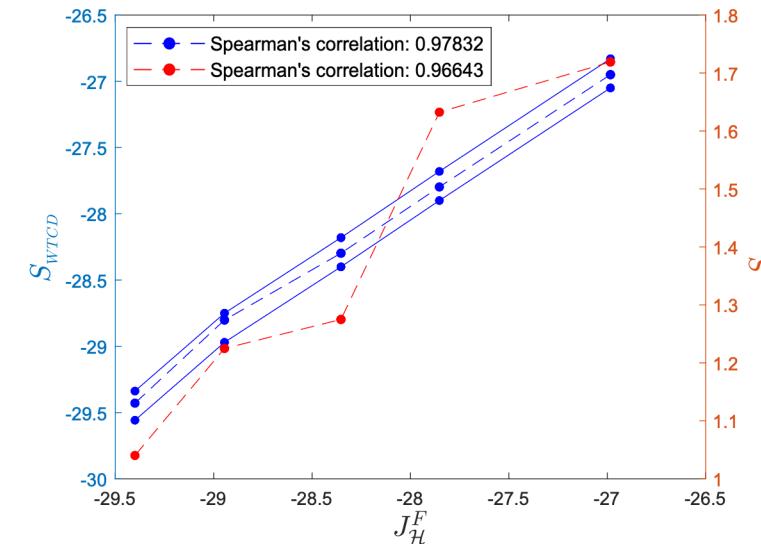
Topological signatures: $\begin{matrix} S_{WTCD} \\ S_{VNND} \end{matrix}$

Focused Topological Gaussian Belief Space Planning (ft-BSP)

- **Unfocused** BSP – reduce uncertainty over all variables
- **Focused** BSP – reduce uncertainty over a predefined subset of variables (focused variables)



Measurement Selection



Active 2D Pose SLAM

signature	measurement selection	active SLAM
S_{WTCD}	18.88	1.21
S_{VND}	12.02	0.14
$J_{\mathcal{H}}^F$	146.24	6.34

Average running time experiments in ms

Simplification of Decision-Making Problems

Concept:

- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

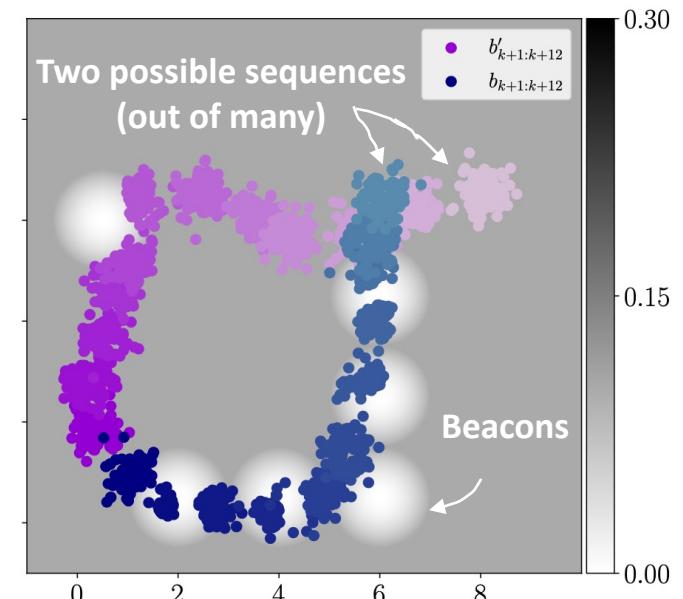
Specific simplifications include:

- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of Risk-Averse POMDP Planning
- Simplification in a multi-agent setting

Simplification of POMDPs with Nonparametric Beliefs

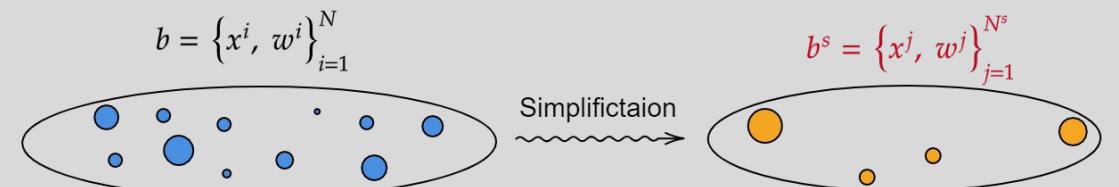
- Value function

$$V_k^\pi(b_k) \equiv J_k(b_k, \pi) = \mathbb{E}\left\{\sum_{l=0}^{L-1} r(b_{k+l}, \pi_{k+l}(b_{k+l})) + r(b_{k+L})\right\}$$



Simplification:

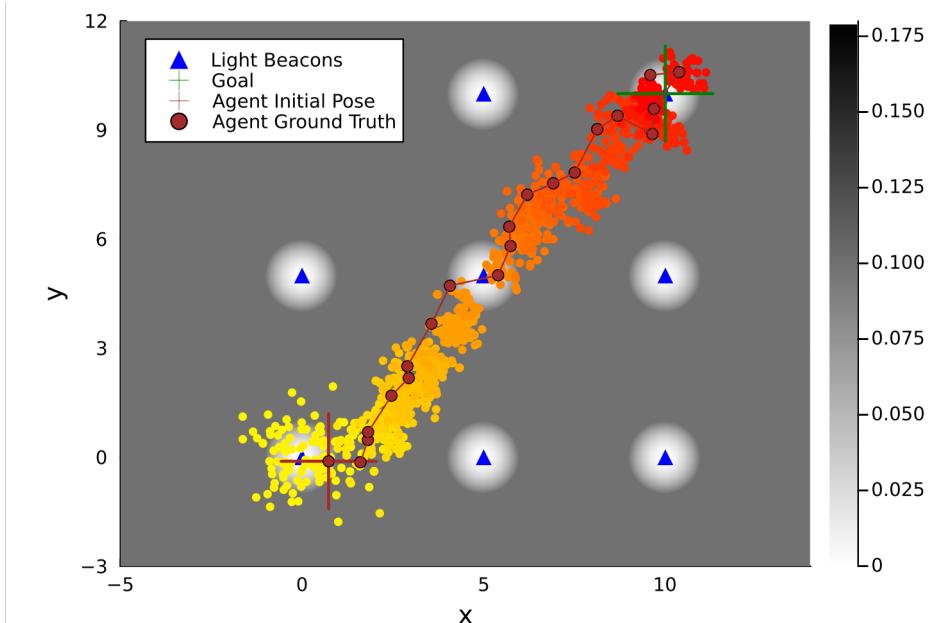
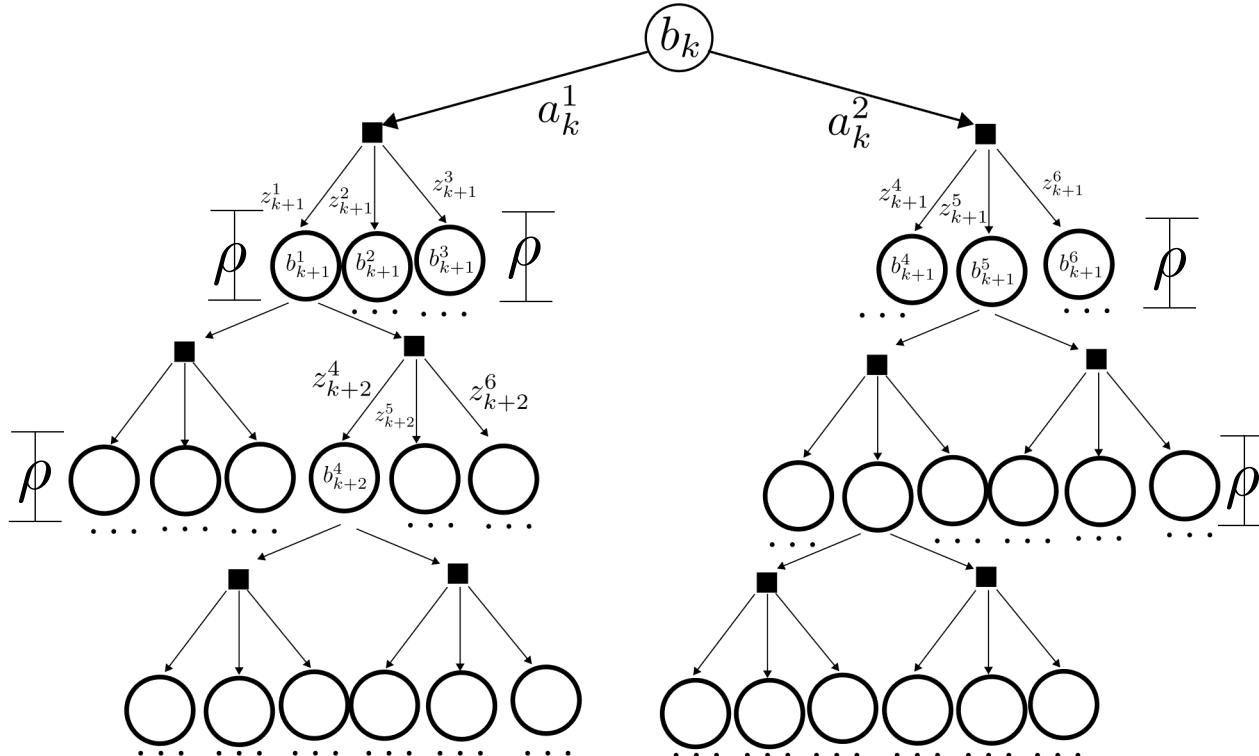
- Utilize a **subset** of samples for planning
- Information-theoretic reward (entropy)
- Analytical (**cheaper**) bounds over the reward



$$lb(b, b^s, a) \leq r(b, a) \leq ub(b, b^s, a)$$

Simplification of POMDPs with Nonparametric Beliefs

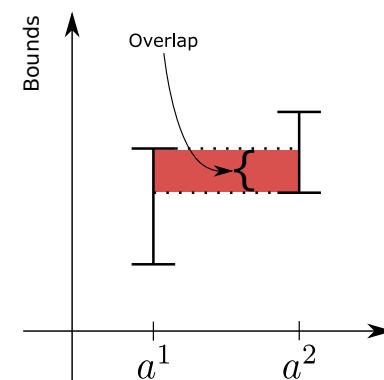
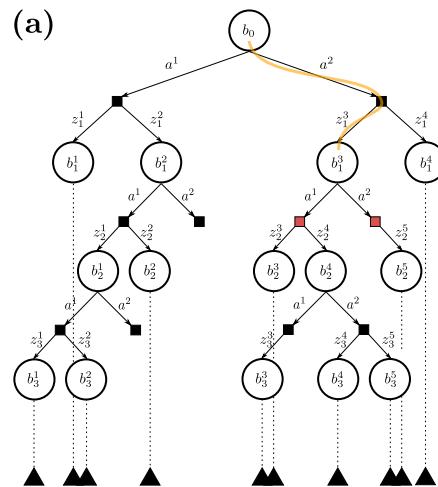
- Adaptive multi-level simplification in a Sparse Sampling setting:



Typical speedup of 20% - 50%,
Same performance!

Simplification of POMDPs with Nonparametric Beliefs

- Adaptive multi-level simplification in an MCTS setting:



Simplification of Decision-Making Problems

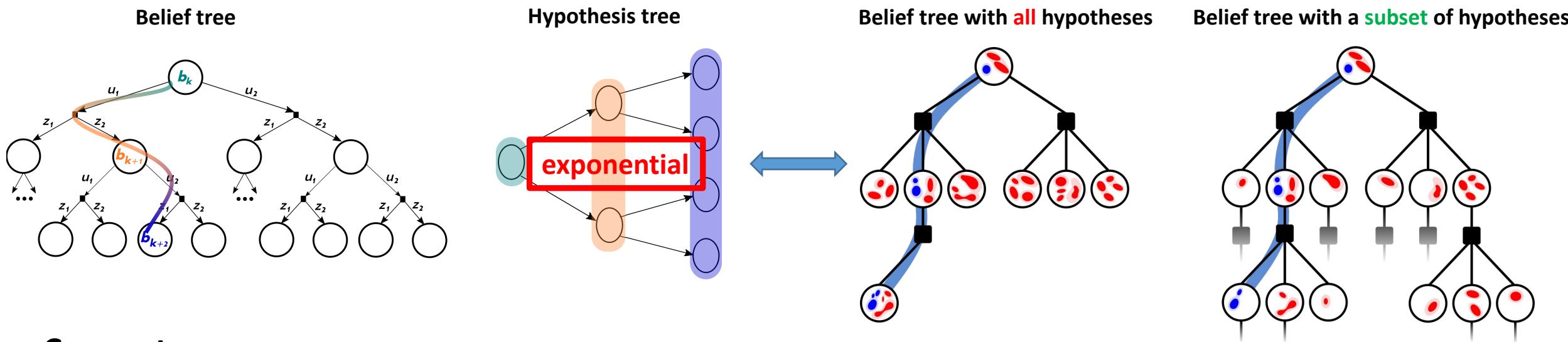
Concept:

- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

Specific simplifications include:

- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of Risk-Averse POMDP Planning
- Simplification in a multi-agent setting

Simplification of BSP/POMDP with Hybrid Beliefs



Concept:

- Instead, utilize only a **subset** of hypotheses
- Derive reward bounds, given planning task (reward)
 - Disambiguate between hypotheses
 - Navigate to a goal
 - ..

$$\mathcal{LB}(b_k, \pi) \leq V^\pi(b_k) \leq \mathcal{UB}(b_k, \pi)$$

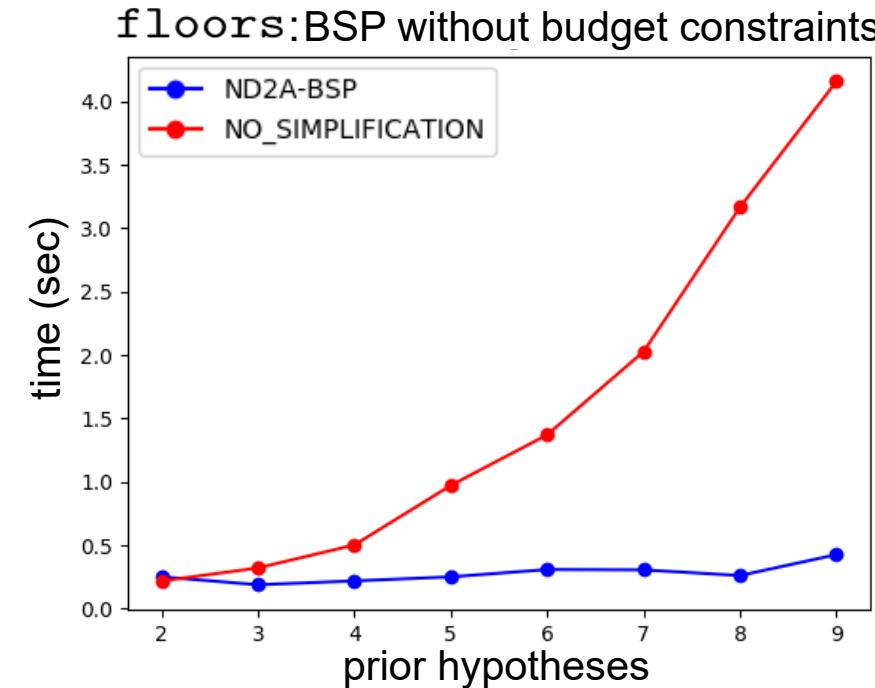
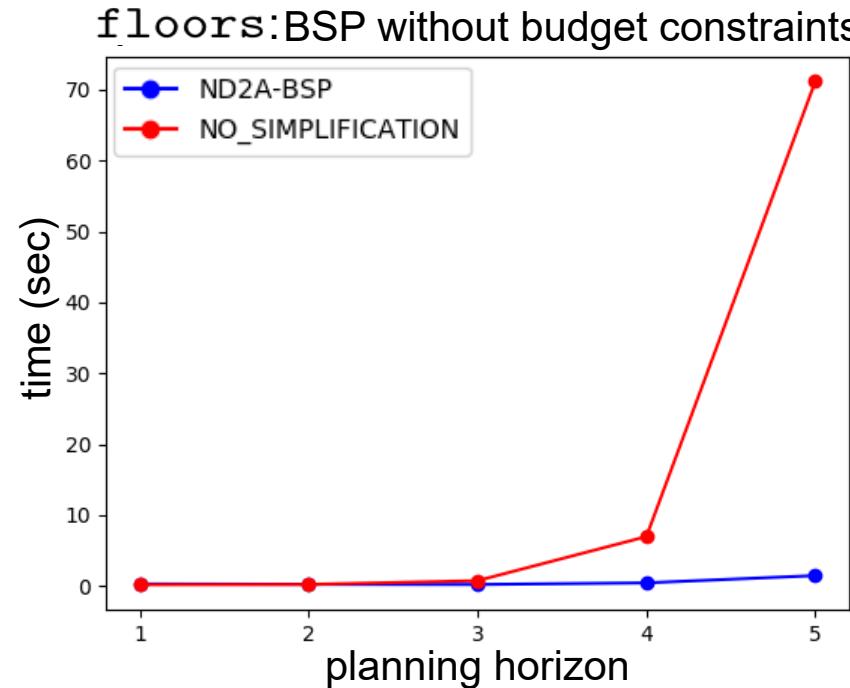
M. Shienman and V. Indelman, "D2A-BSP: Distilled Data Association Belief Space Planning with Performance Guarantees Under Budget Constraints," ICRA'22, **Outstanding Paper Award Finalist**.

M. Shienman and V. Indelman, "Nonmyopic Distilled Data Association Belief Space Planning Under Budget Constraints," ISRR'22.

M. Barenboim, M. Shienman, and V. Indelman, "Monte Carlo Planning in Hybrid Belief POMDPs," IEEE RA-L'23.

M. Barenboim, I. Lev-Yehudi, and V. Indelman, "Data Association Aware POMDP Planning with Hypothesis Pruning Performance Guarantees," IEEE RA-L'23.

Simplification of BSP/POMDP with Hybrid Beliefs



- Significant speed-up in planning
- Same planning performance is **guaranteed** (no overlap between bounds)

Simplification of BSP/POMDP with Hybrid Beliefs

- Derived a deterministic bound to relate the full set of hypotheses to a subset thereof,

Corollary

For any policy π , and selection of hypotheses set $\{\beta_{0:\tau}^i\}_{i=0}^{|\mathcal{B}|}$ the following holds,

$$|V^\pi(b_0) - \bar{V}^\pi(\bar{b}_0)| \leq \mathcal{R}_{max} \left[\mathcal{T} \delta_0^\beta + \sum_{k=1}^{\mathcal{T}} \sum_{\tau=1}^k \mathbb{E}_{z_{1:\tau}} [\delta_\tau^\beta] \right].$$

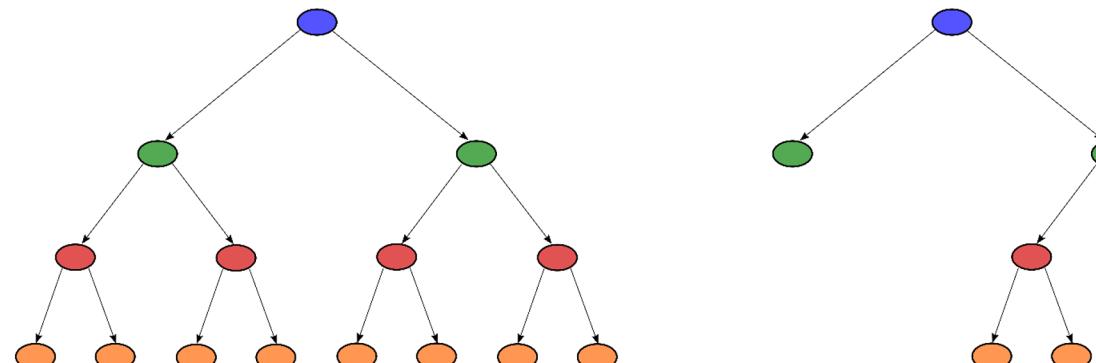
Full tree

Any subset

Importantly, the bound relies on the available hypotheses

Can bound the theoretical value with access only to the simplified tree

Bounds can be evaluated online



Simplification of Decision-Making Problems

Concept:

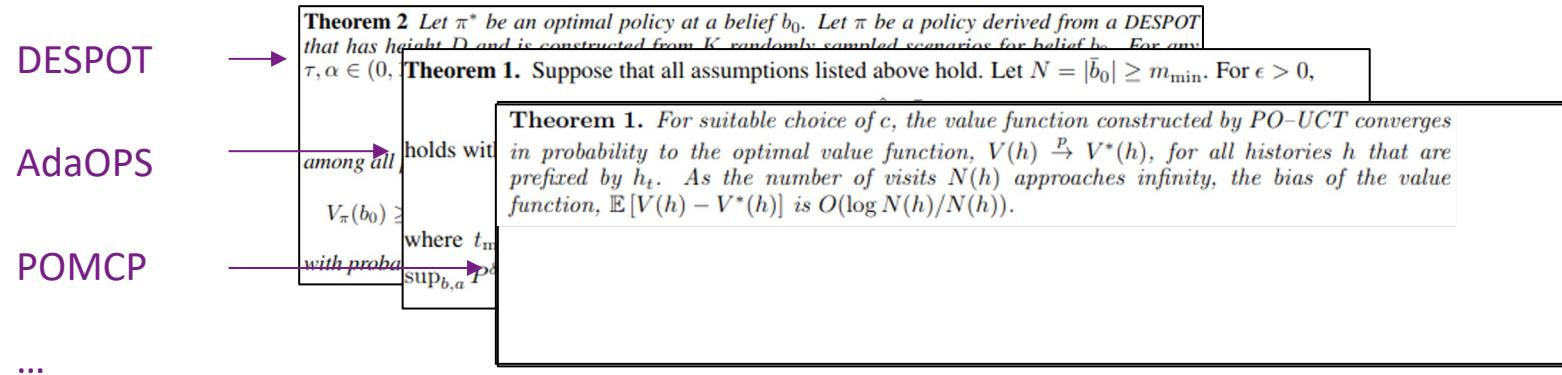
- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

Specific simplifications include:

- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of Risk-Averse POMDP Planning
- Simplification in a multi-agent setting

POMDPs with Deterministic Guarantees

SOTA sampling based approaches come with probabilistic theoretical guarantees



Can we get deterministic guarantees?

We show that deterministic guarantees are indeed possible!

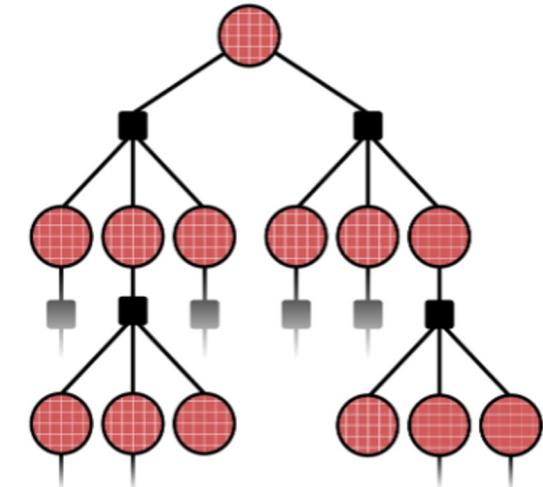
Online POMDP Planning with Anytime Deterministic Guarantees

Concept:

Instead of solving the original POMDP, consider a simplified version of that POMDP.

$$\mathcal{M} \xrightarrow{\text{wavy arrow}} \bar{\mathcal{M}}$$

Derive a mathematical relationship between the solution of the simplified, and the theoretical POMDP.



$$V(b) \xleftarrow{\text{?}} \bar{V}(b)$$

Online POMDP Planning with Anytime Deterministic Guarantees

- Given a POMDP: $\mathcal{M} = \langle \mathcal{X}, \mathcal{Z}, \mathcal{A}, b_0, \mathcal{P}_T, \mathcal{P}_Z, \rho, \gamma \rangle$
- Define a **simplified** POMDP,

$$\bar{\mathcal{M}} = \langle \bar{\mathcal{X}}, \bar{\mathcal{Z}}, \mathcal{A}, \bar{b}_0, \bar{\mathcal{P}}_T, \bar{\mathcal{P}}_Z, \rho, \gamma \rangle$$



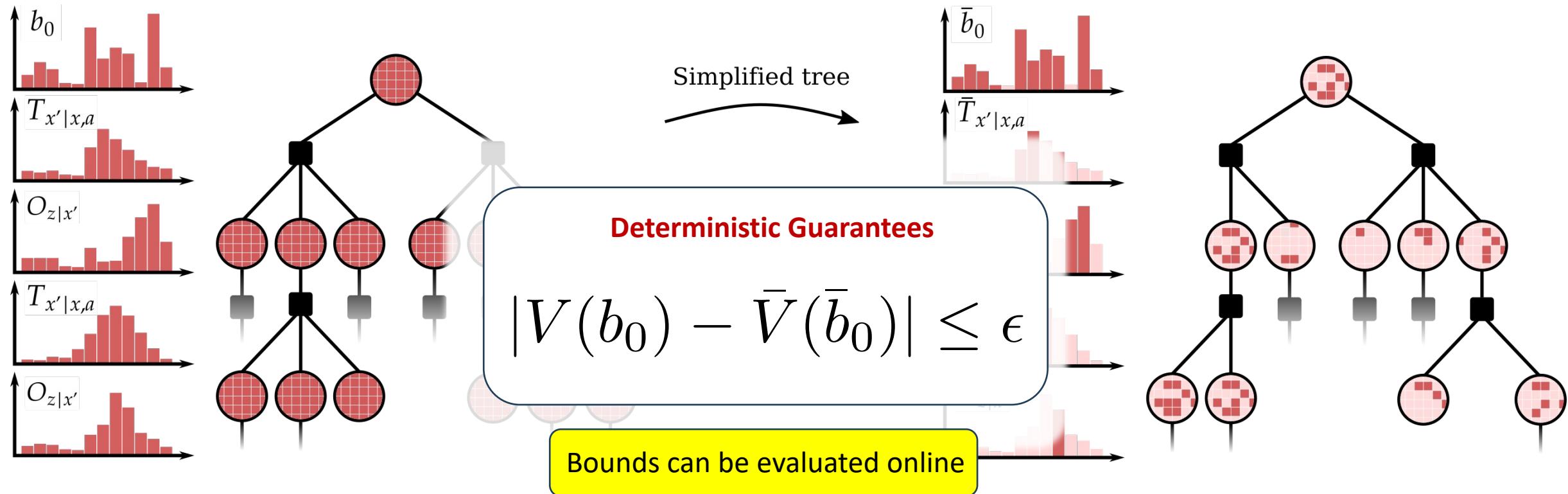
$$\begin{aligned}\bar{\mathcal{X}}(H_t) &\subset \mathcal{X} & \bar{b}_0(x) &\triangleq \begin{cases} b_0(x) & , x \in \bar{\mathcal{X}}_0 \\ 0 & , \text{otherwise} \end{cases} \\ \bar{\mathcal{Z}}(H_t) &\subset \mathcal{Z} & \bar{\mathbb{P}}(x_{t+1} | x_t, a_t) &\triangleq \begin{cases} \mathbb{P}(x_{t+1} | x_t, a_t) & , x_{t+1} \in \bar{\mathcal{X}}(H_{t+1}^-) \\ 0 & , \text{otherwise} \end{cases} \\ && \bar{\mathbb{P}}(z_t | x_t) &\triangleq \begin{cases} \mathbb{P}(z_t | x_t) & , z_t \in \bar{\mathcal{Z}}(H_t) \\ 0 & , \text{otherwise} \end{cases}\end{aligned}$$

- Simplified value function

$$\bar{V}^\pi(\bar{b}_t) \triangleq r(\bar{b}_t, \pi_t) + \bar{\mathbb{E}}_{z_{t+1:\mathcal{T}}} [\bar{V}^\pi(\bar{b}_{t+1})]$$

Online POMDP Planning with Anytime Deterministic Guarantees

- Deterministic guarantees (assuming discrete spaces)



Online POMDP Planning with Anytime Deterministic Guarantees

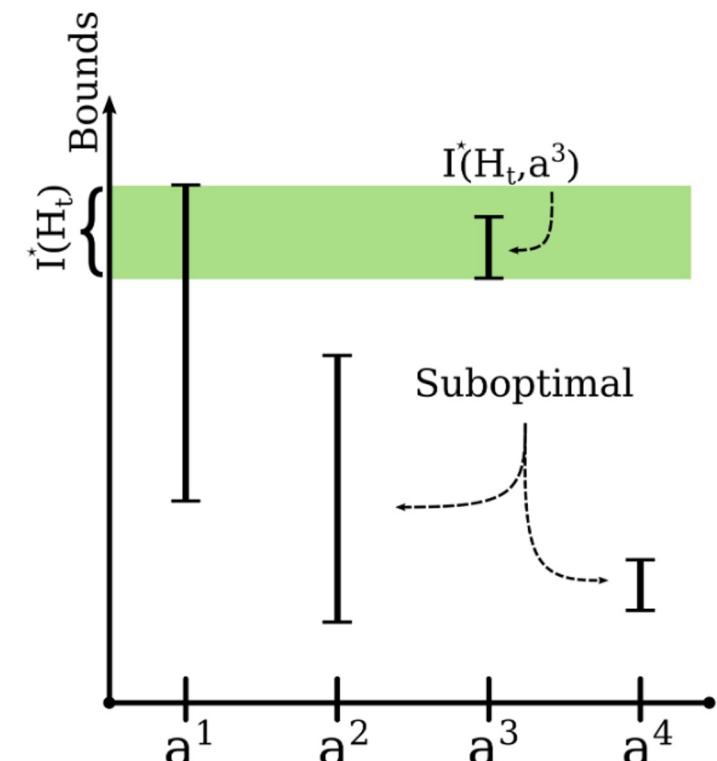
Importantly, the bounds can be calculated during planning.

How can we use them?

- **Pruning of sub-optimal branches**
 - Made possible by the deterministic guarantees
- **Stopping criteria for the planning phase**
 - Made possible by the deterministic guarantees
- **Finding the optimal solution in finite time**
 - Without recovering the theoretical tree

Deterministic Guarantees

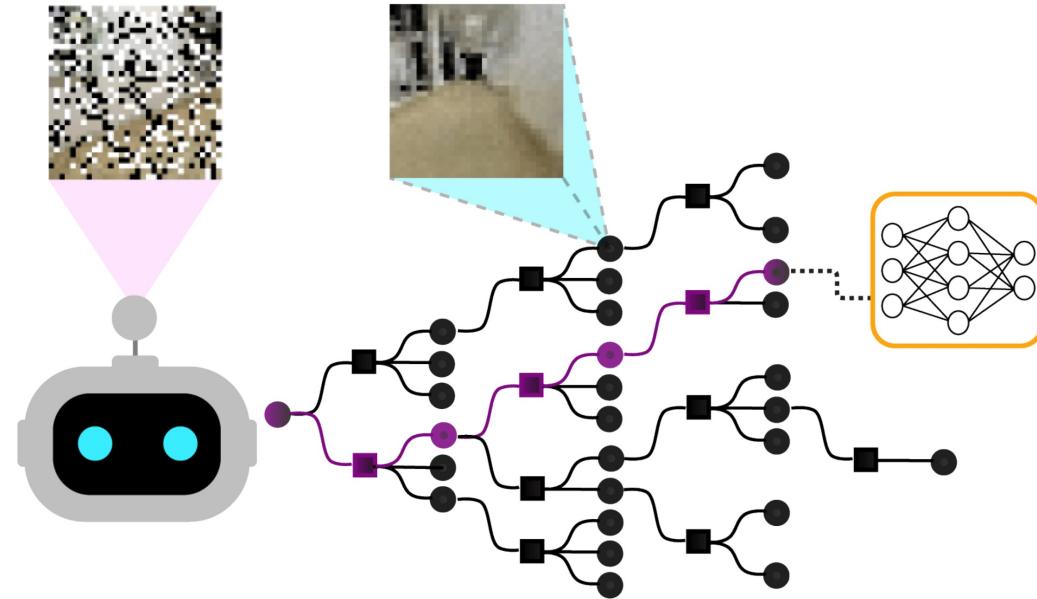
$$|V(b_0) - \bar{V}(\bar{b}_0)| \leq \epsilon$$



Simplifying Complex Observation Models with Probabilistic Guarantees

Visual POMDP planning

- Visual observations are complex to model in planning^{1,2}
- Learned observation models are (often) impractical for solving POMDPs in real time



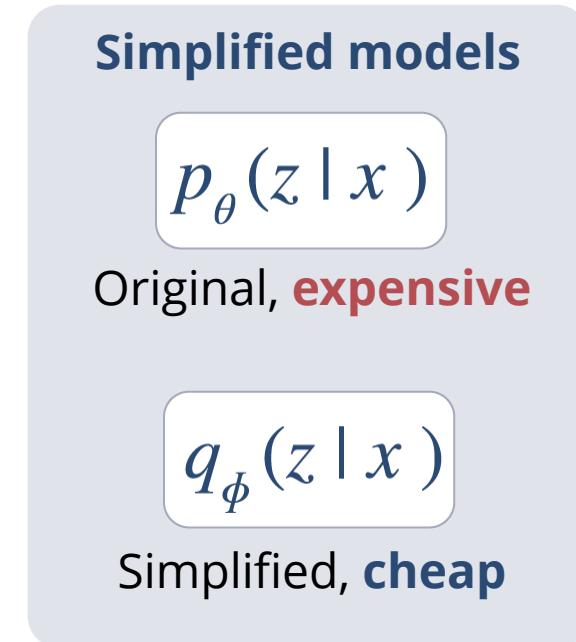
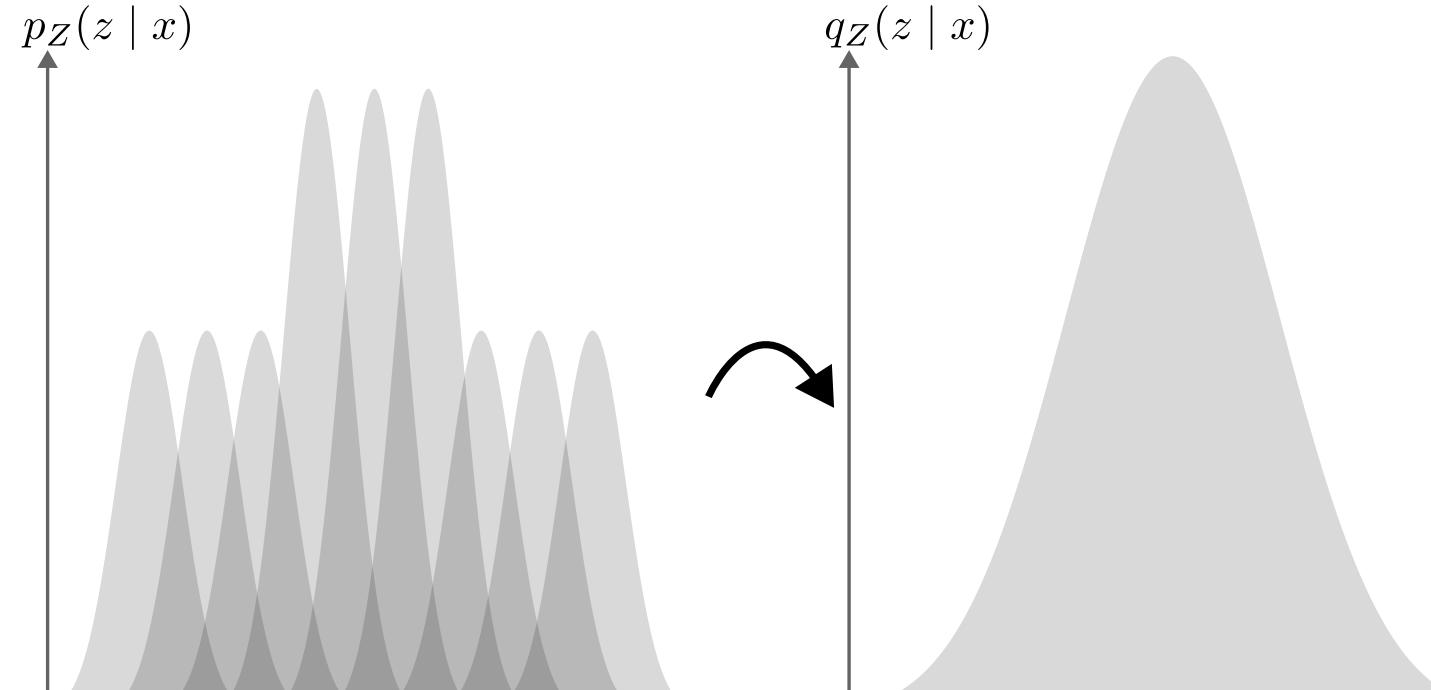
**Can we simplify the learned models?
What is the impact on planning performance?**

¹Wang et al., “DualSMC: Tunneling Differentiable Filtering and Planning under Continuous POMDPs”.

²Deglurkar et al., “Compositional Learning-based Planning for Vision POMDPs”.

Simplifying Complex Observation Models with Probabilistic Guarantees

- We replace the (learned) observation model p_Z with a cheaper model q_Z
 - Simpler GMM, Shallower Neural Network, etc.
 - Example:



Simplifying Complex Observation Models with Probabilistic Guarantees

- We replace the (learned) observation model p_Z with a cheaper model q_Z
- Simplified action-value function: $Q_P^{q_Z}$

Corollary 3

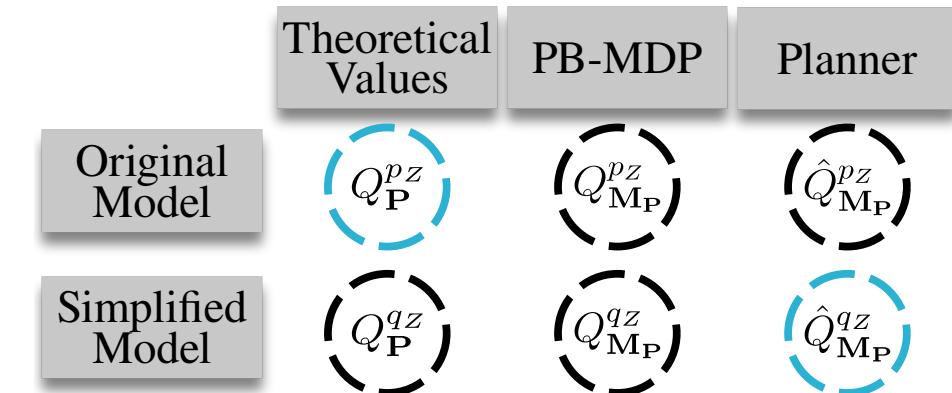
For arbitrary $\varepsilon, \delta > 0$ there exists a number of particles for which

$$\underline{|Q_P^{p_Z}(b_t, a) - \hat{Q}_{M_P}^{q_Z}(\bar{b}_t, a)|} \leq \Phi_{M_P}(\bar{b}_t, a) + \varepsilon$$

with probability of at least $1 - \delta$ for any guaranteed planner

Theoretical Q function
of the POMDP, with
original models

Estimator of the Q function of a
particle-belief POMDP, with
simplified models



- Importance sampling
- Separate calculations to offline/online

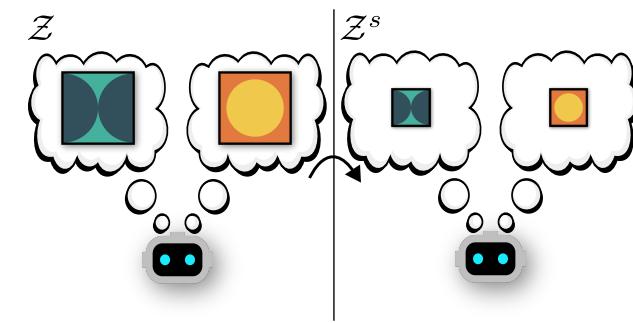
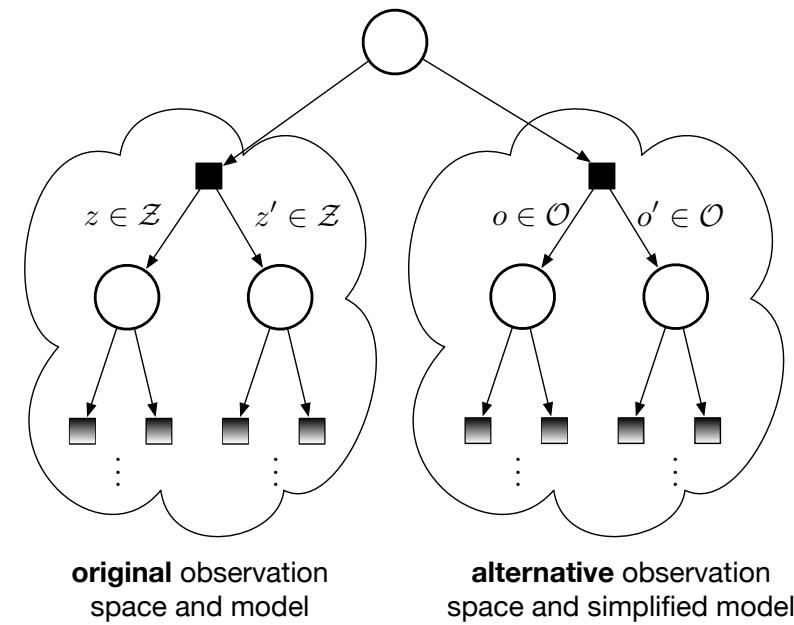
Simplified POMDP Planning with an Alternative Observation Space

- Switch to an alternative observation space and model

Model Definition

POMDP tuple: $\langle \mathcal{X}, \mathcal{A}, \mathcal{Z}, \mathbb{P}_T, \mathbb{P}_Z, b_k, r \rangle \rightarrow \langle \mathcal{X}, \mathcal{A}, \mathcal{O}, \mathbb{P}_T, \mathbb{P}_O, b_k, r \rangle$

- Only at certain levels and branches of the tree



Simplified POMDP Planning with an Alternative Observation Space

- Switch to an alternative observation space and model

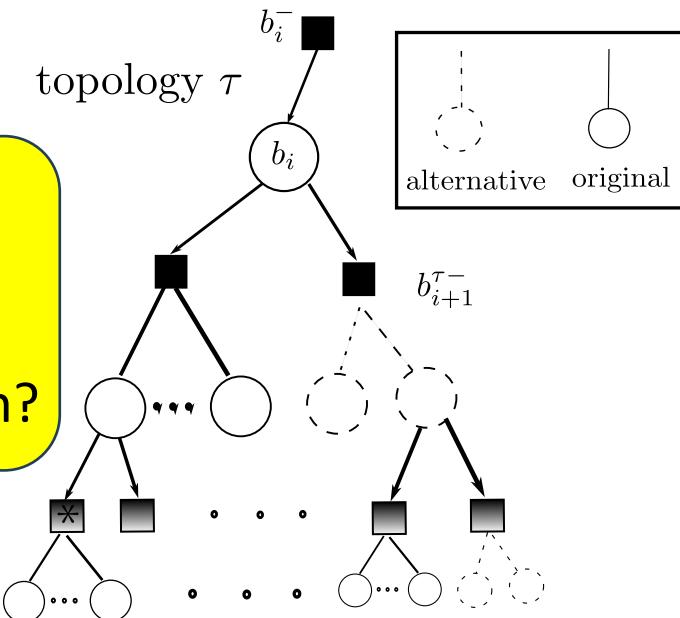
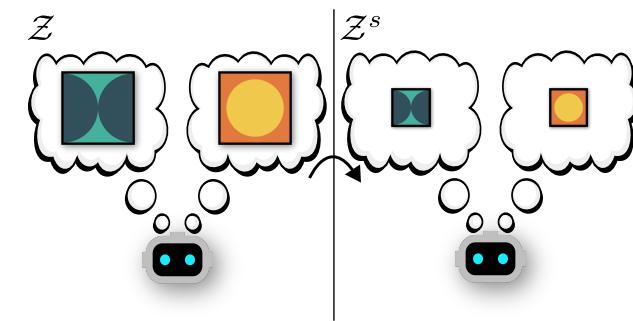
Model Definition

POMDP tuple: $\langle \mathcal{X}, \mathcal{A}, \mathcal{Z}, \mathbb{P}_T, \mathbb{P}_Z, b_k, r \rangle \rightarrow \langle \mathcal{X}, \mathcal{A}, \mathcal{O}, \mathbb{P}_T, \mathbb{P}_O, b_k, r \rangle$

- Only at certain levels and branches of the tree

Main questions addressed:

- How to decide online where to simplify in belief tree?
- How to provide formal performance guarantees?
- How to adaptively transition between the different levels of simplification?



Simplification of Decision-Making Problems

Concept:

- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

Specific simplifications include:

- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of Risk-Averse POMDP Planning
- Simplification in a multi-agent setting

Partitioning of a Multivariate Observation Space

- Value function: $V^\pi(b_k) = R(b_k, \pi_k(b_k)) + \mathbb{E}_{z_{k+1:k+\ell}} \left[\sum_{i=k+1}^{k+\ell} R(b_i, \pi_i(b_i)) \right]$

- Belief-dependent reward: entropy

$$R(b, \pi(b)) \triangleq -\mathcal{H}(X) \equiv \mathbb{E}_{X \sim b} (\log b[X])$$

- The expected reward at each i th look-ahead step:

$$\mathbb{E}_{Z_{k+1:i}} [R(b_i, a_{i-1})] = -\mathcal{H}(X_i | Z_{k+1:i})$$

- Future observations are drawn from the distribution $\mathbb{P}(Z_{k+1:i} | b_k, \pi)$

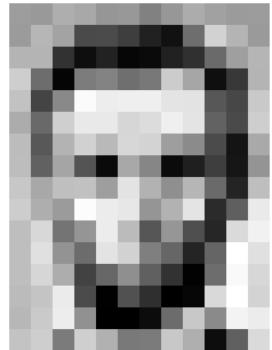
Partitioning of a Multivariate Observation Space

- Consider a multivariate random variable $Z \in \mathcal{Z}$, that represents future observations:

$$Z = (Z^1, Z^2, \dots, Z^m)$$

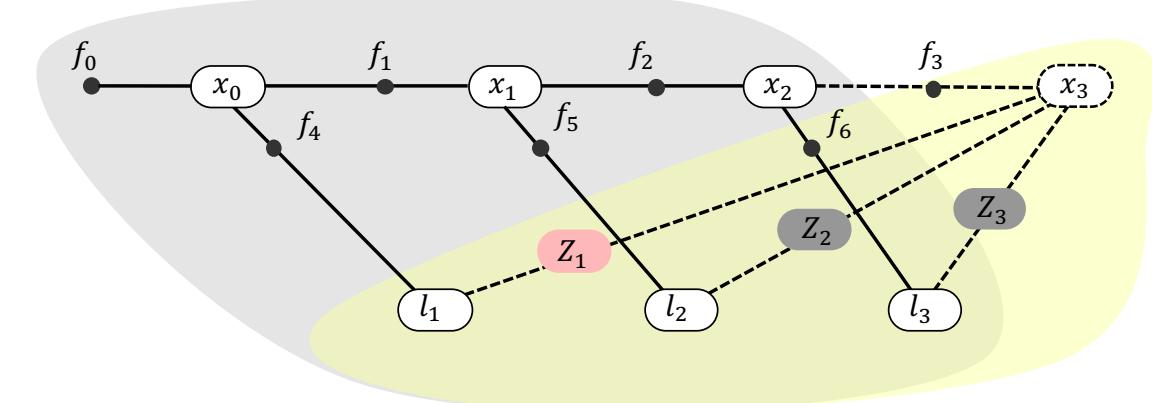
- Examples:

Raw measurement of an image sensor



157	153	174	168	150	152	129	151	172	161	155	156
155	182	163	74	75	62	93	17	110	210	180	154
180	180	50	14	54	6	10	33	48	106	159	181
209	109	5	124	131	111	120	204	166	15	66	180
194	68	137	251	237	239	228	227	87	71	201	
172	105	207	233	235	214	220	239	228	98	74	206
188	68	179	209	186	215	211	158	139	75	20	169
180	97	165	84	10	158	134	11	51	62	22	148
199	168	191	193	159	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	85	156	79	38	218	241
190	224	147	108	227	210	127	103	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	236	75	1	81	47	0	6	217	255	211
183	202	237	149	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

Factor graph



Partitioning of a Multivariate Observation Space

- Consider a multivariate random variable $Z \in \mathcal{Z}$, that represents future observations:

$$Z = (Z^1, Z^2, \dots, Z^m)$$

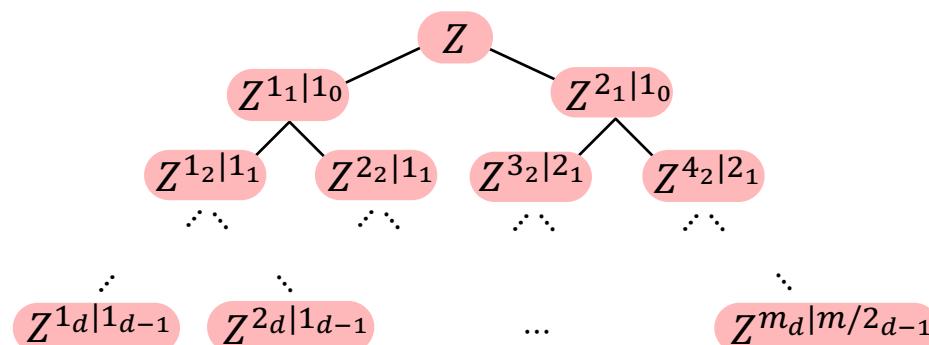
- We can partition $Z \in \mathcal{Z}$ into different subsets/components, e.g.

$$Z^s = \{Z^1, Z^2, \dots, Z^n\}$$

$$Z^{\bar{s}} = \{Z^{n+1}, Z^{n+2}, \dots, Z^m\}$$

$$Z = Z^s \cup Z^{\bar{s}}$$

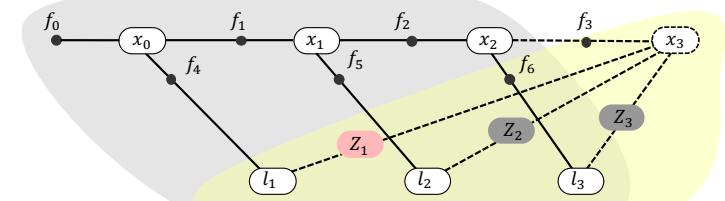
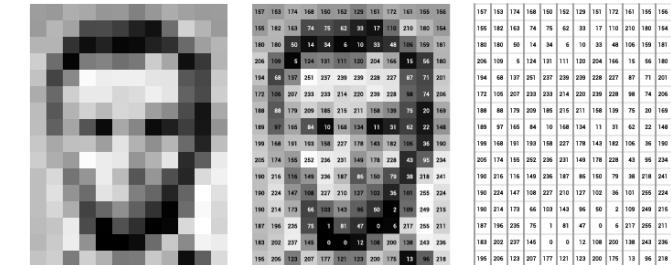
- Hierarchical Partitioning:



Partitioning of a Multivariate Observation Space

But why is this a good idea?

- Apply partitioning to a raw image measurement of size 20x20 binary pixels
- Consider all of the different permutations for each pixel, 2^{400} in total
- If we partition $Z^s \triangleq \{Z^{x,y} \mid y \leq 10\}$ and $Z^{\bar{s}} \triangleq \{Z^{x,y} \mid y > 10\}$
 - Need to consider 2^{200} permutations for each
 - Overall, 2^{201} vs 2^{400} permutations



Partitioning of a Multivariate Observation Space

Lemma 2

Given two sets of expected measurements $(Z^s, Z^{\bar{s}})$, the conditional Entropy can be factorized as

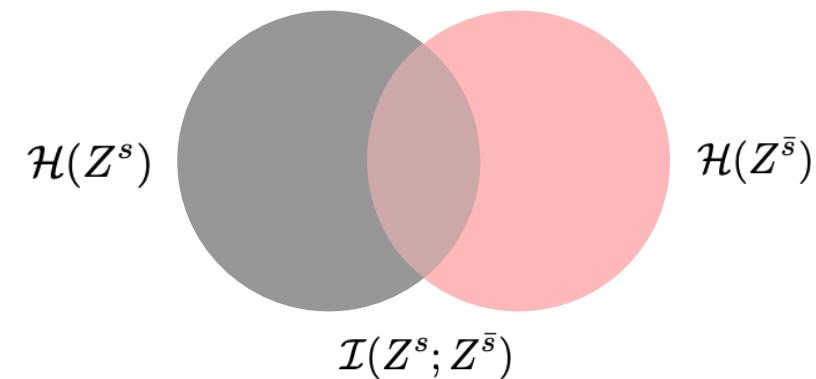
$$\mathcal{H}(X|Z) = \mathcal{H}(Z^s|X) + \mathcal{H}(Z^{\bar{s}}|X) - \mathcal{H}(Z^s, Z^{\bar{s}}) + \mathcal{H}(X)$$

$$\mathcal{H}(Z^s, Z^{\bar{s}}) = \mathcal{H}(Z^s) + \mathcal{H}(Z^{\bar{s}}) - \mathcal{I}(Z^s; Z^{\bar{s}})$$

$$\mathcal{LB} \leq \mathcal{H}(X|Z) \leq \mathcal{UB}$$

$$\mathcal{LB} \triangleq \mathcal{H}(Z^s | X) + \mathcal{H}(Z^{\bar{s}}|X) - \mathcal{H}(Z^s) - \mathcal{H}(Z^{\bar{s}}) + \mathcal{H}(X)$$

$$\mathcal{UB} \triangleq \mathcal{H}(Z^s|X) + \mathcal{H}(X) - \mathcal{H}(Z^s)$$



Partitioning of a Multivariate Observation Space

Multivariate Gaussian Belief

- Posterior information matrix:

$$\Lambda_i = \Lambda_k^{\text{Aug}} + A_i^T A_i$$

- Prior work^{1,2} - application of the matrix determinant lemma:

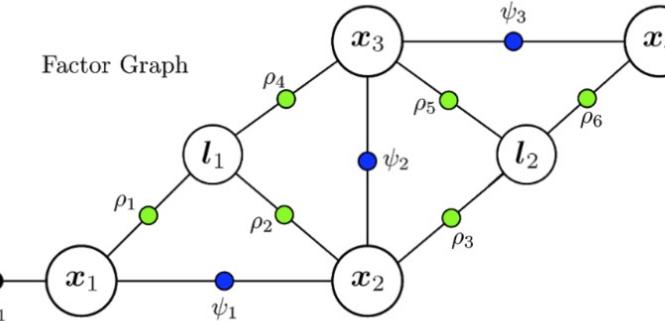
entropy $\propto |\Lambda_k + A^T \cdot A| = |\Lambda_k| \cdot |I_m + A \cdot \Sigma_k \cdot A^T|$

posterior info matrix

For information gain:

$$\ln \frac{|\Lambda_k + A^T A|}{|\Lambda_k|} = \ln |I_m + A \cdot \Sigma_k \cdot A^T| = \ln |I_m + {}^I A \cdot \Sigma_k^{M, {}^I X} \cdot ({}^I A)^T|$$

One-time calculations for all candidate actions
Recover entries only for the involved variables in any of the actions



Measurement Jacobian

l_1	l_2	x_1	x_2	x_3	x_4	
X	X	X	X	X	X	ρ_1
X	X	X	X	X	X	ρ_2
X	X	X	X	X	X	ρ_4
X	X	X	X	X	X	ρ_5
X	X	X	X	X	X	ρ_6
						ϕ_1
						ψ_1
						ψ_2
						ψ_3

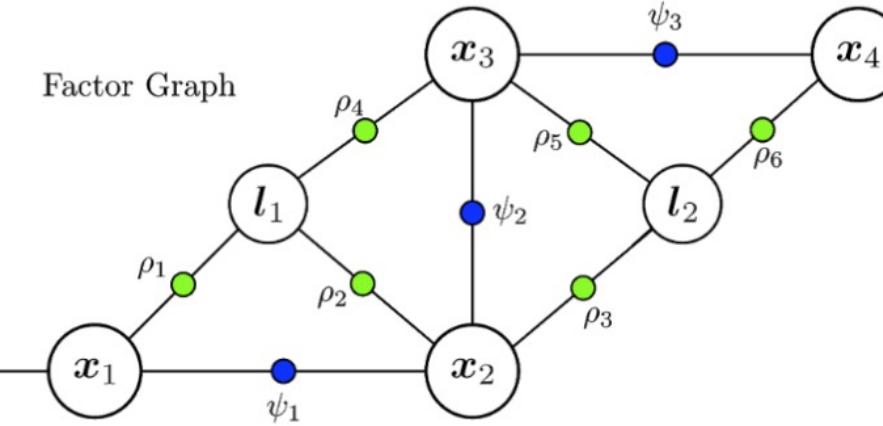
¹D. Kopitkov and V. Indelman, "No belief propagation required: belief space planning in high-dimensional state spaces via factor graphs, the matrix determinant lemma, and re-use of calculation", IJRR'17.

²D. Kopitkov and V. Indelman, "General-purpose incremental covariance update and efficient belief space planning via a factor-graph propagation action tree", IJRR'19.

Partitioning of a Multivariate Observation Space

Partitioning of a Gaussian Belief

$$\Lambda_i = \Lambda_k^{\text{Aug}} + A_i^T A_i$$



Measurement Jacobian

	l_1	l_2	x_1	x_2	x_3	x_4	
ρ_1	X						
ρ_2	X						
ρ_3	X						
ρ_4		X					
ρ_5		X					
ρ_6		X					
ϕ_1			X				
ψ_1			X				
ψ_2			X	X			
ψ_3			X	X	X		

Observation partitioning corresponds to splitting the Jacobian into blocks

$$Z_i \mapsto (Z_i^s, Z_i^{\bar{s}})$$

$$A_i \mapsto \begin{bmatrix} A_i^s \\ A_i^{\bar{s}} \end{bmatrix}$$

Bounds¹:

$$\mathcal{LB} = C - \frac{1}{2Z_{k+1:i}} \mathbb{E} \left[\ln \frac{f \left(\Lambda_k^{\text{Aug}-}, A_i^s \right) \cdot f \left(\Lambda_k^{\text{Aug}-}, A_i^{\bar{s}} \right)}{|\Lambda_k^{\text{Aug}-}|} \right]$$

$$\mathcal{UB} = C - \frac{1}{2Z_{k+1:i}} \mathbb{E} \left[\ln f \left(\Lambda_k^{\text{Aug}-}, A_i^s \right) \right],$$

$$f(\Lambda, A) \triangleq |\Lambda + A^T A|$$

Reduced complexity wrt rAMDL²:
 $O\left(\frac{m^3}{4}\right)$ vs $O(m^3)$

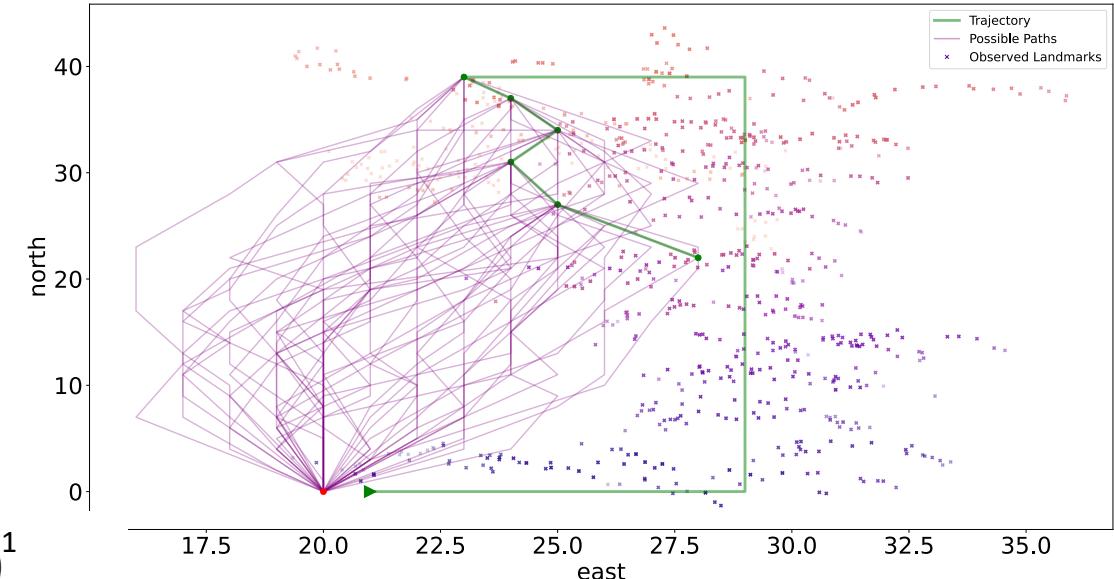
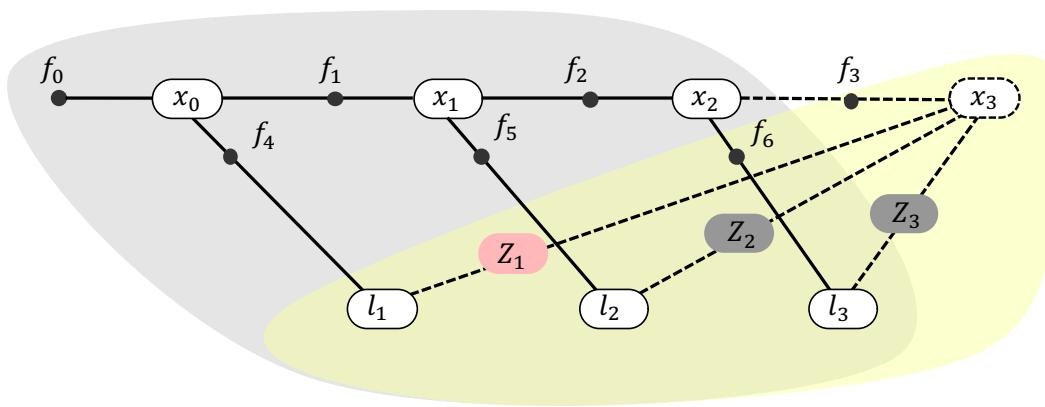
¹T. Yotam and V. Indelman, "Measurement Simplification in ρ -POMDP with Performance Guarantees," IEEE T-RO'24.

²D. Kopitkov and V. Indelman, "No Belief Propagation Required: Belief Space Planning in High-Dimensional State Spaces via Factor Graphs, Matrix Determinant Lemma and Re-use of Calculation," IJRR'17.

²D. Kopitkov and V. Indelman, "General-purpose incremental covariance update and efficient belief space planning via a factor-graph propagation action tree", IJRR'19.

Partitioning of a Multivariate Observation Space

Application to Active SLAM



# Paths	# Factors	RP	rAMDL ²	MP (ours) ¹
100	2956	No	11.521 ± 0.537	6.888 ± 0.155
100	2956	Yes	24.636 ± 1.381	11.758 ± 0.372
100	5904	Yes	84.376 ± 14.458	32.069 ± 4.913

Table: Total planning time in seconds (lower is better)

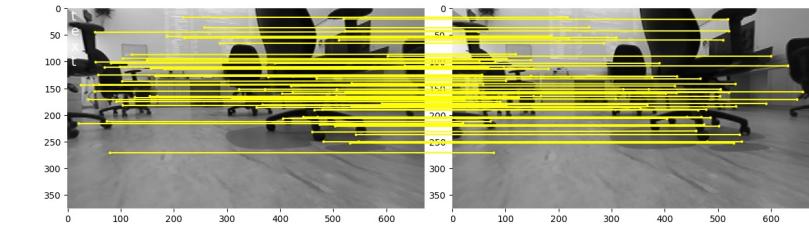
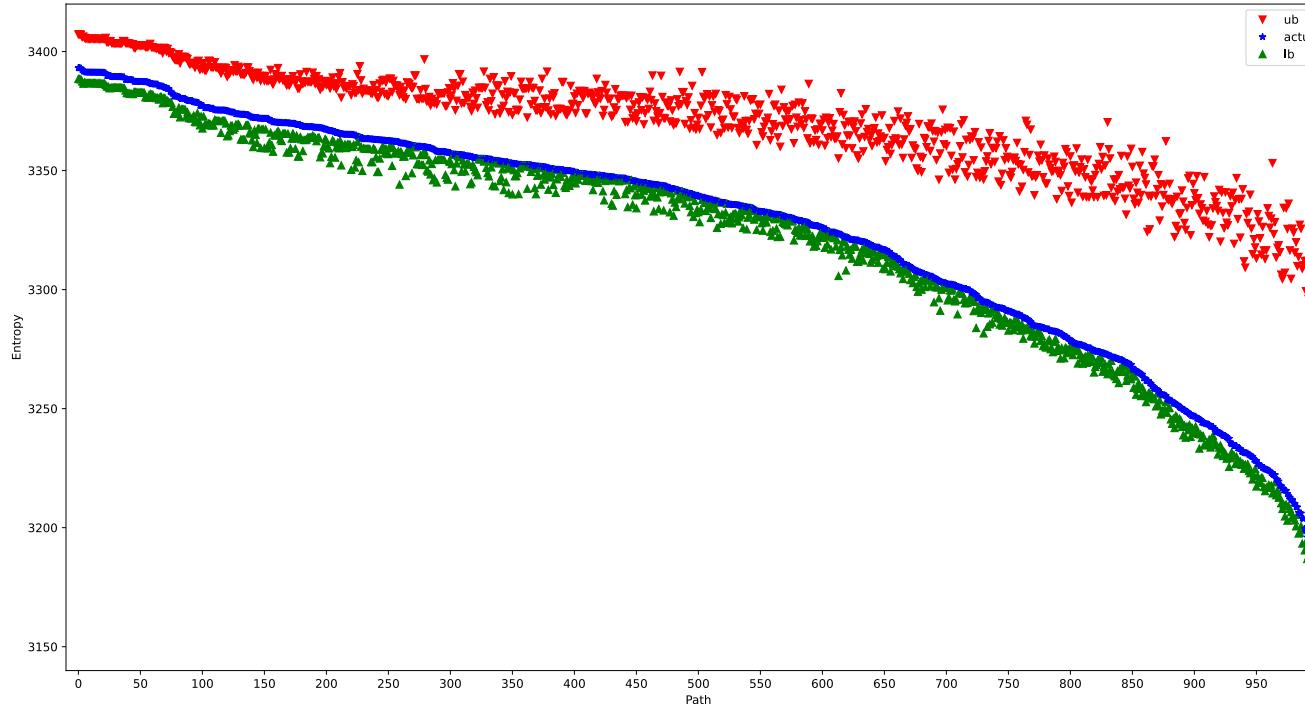
¹T. Yotam and V. Indelman, "Measurement Simplification in ρ -POMDP with Performance Guarantees," IEEE T-RO'24.

²D. Kopitkov and V. Indelman, "No Belief Propagation Required: Belief Space Planning in High-Dimensional State Spaces via Factor Graphs, Matrix Determinant Lemma and Re-use of Calculation," IJRR'17.

²D. Kopitkov and V. Indelman, "General-purpose incremental covariance update and efficient belief space planning via a factor-graph propagation action tree", IJRR'19.

Partitioning of a Multivariate Observation Space

Application to Active SLAM



Method	time [sec]
MP (ours) ¹	585.507 ± 27.153
rAMD ²	802.545 ± 25.651
iSAM2 ³	1764.835 ± 26.521

Table: Total planning time in seconds (lower is better)

¹T. Yotam and V. Indelman, "Measurement Simplification in ρ -POMDP with Performance Guarantees," IEEE T-RO'24.

²D. Kopitkov and V. Indelman, "No Belief Propagation Required: Belief Space Planning in High-Dimensional State Spaces via Factor Graphs, Matrix Determinant Lemma and Re-use of Calculation," IJRR'17.

²D. Kopitkov and V. Indelman, "General-purpose incremental covariance update and efficient belief space planning via a factor-graph propagation action tree", IJRR'19.

³M. Kaess, et al., "iSAM2: Incremental smoothing and mapping using the Bayes tree," IJRR'12.

Simplification of Decision-Making Problems

Concept:

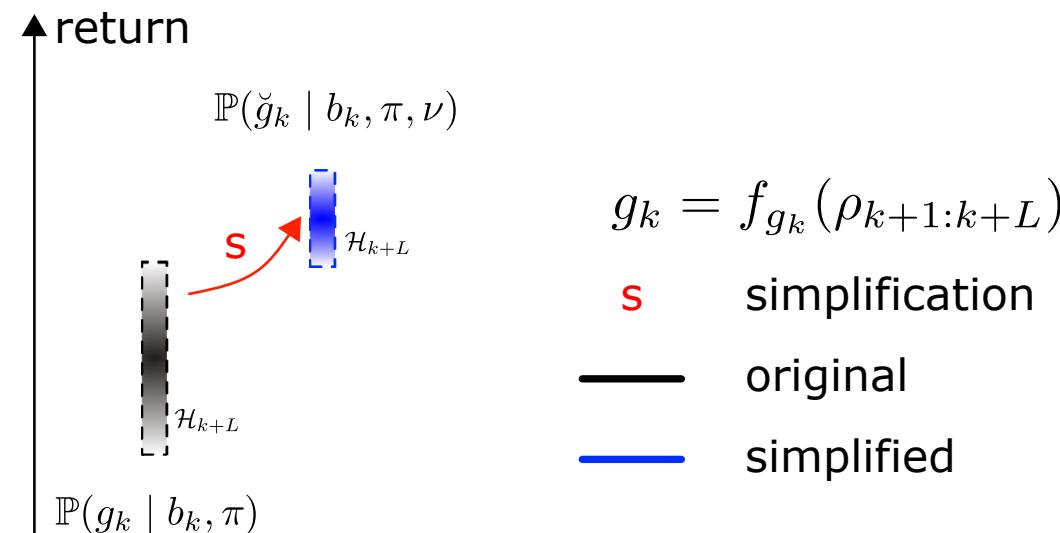
- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

Specific simplifications include:

- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of Risk-Averse POMDP Planning
- Simplification in a multi-agent setting

Simplification of Risk Averse POMDP Planning

- Impact of simplification on **distribution** over returns/rewards
- Simplified **risk aware** decision making with belief-dependent rewards



$$V^\pi(b_k) = \varphi \left(\mathbb{P}(\rho_{k+1:k+L} \mid b_k, \pi_{k:k+L-1}), g_k \right)$$

Probabilistically Constrained Belief Space Planning

$$\max_{\pi_{k+}} \mathbb{E} \left[\sum_{\ell=k}^{k+L-1} \rho_{\ell+1} \middle| b_k, \pi_{k+} \right]$$

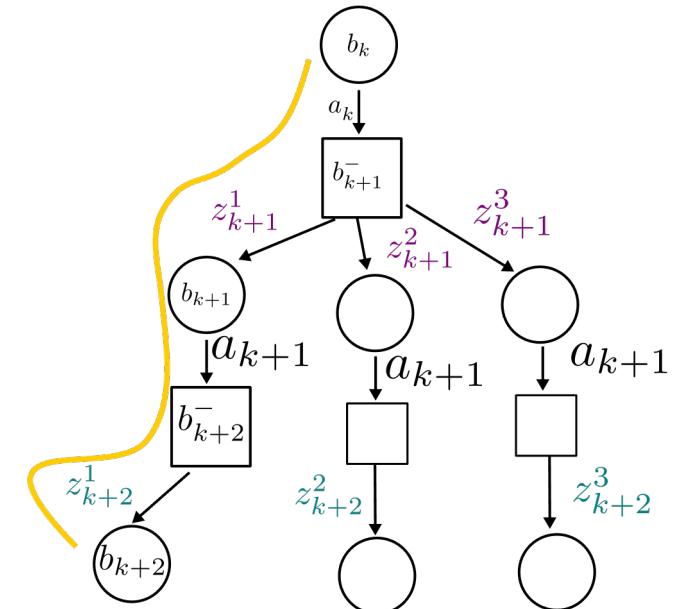
subject to $P(c(b_{k:k+L}; \phi, \delta) = 1 | b_k, \pi_{k+}) \geq 1 - \epsilon$

Information gain¹:

$$c(b_{k:k+L}; \phi, \delta) \triangleq \mathbf{1}_{\{\left(\sum_{\ell=k}^{k+L-1} \phi(b_\ell, b_{\ell+1}) \right) \geq \delta\}}(b_{k:k+L})$$

Safety²:

$$c(b_{k:k+L}; \phi, \delta) \triangleq \prod_{\ell=k}^{k+L} \mathbf{1}_{\{b_\ell: \phi(b_\ell) \geq \delta\}}(b_\ell)$$



¹A. Zhitnikov and V. Indelman, "Simplified Continuous High Dimensional Belief Space Planning with Adaptive Probabilistic Belief-dependent Constraints," T-RO'24.

²A. Zhitnikov and V. Indelman, "Anytime Probabilistically Constrained Provably Convergent Online Belief Space Planning," arXiv'24.

Agenda

Experience Reuse in POMDP Planning

POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

Simplification of Decision-Making Problems

Concept:

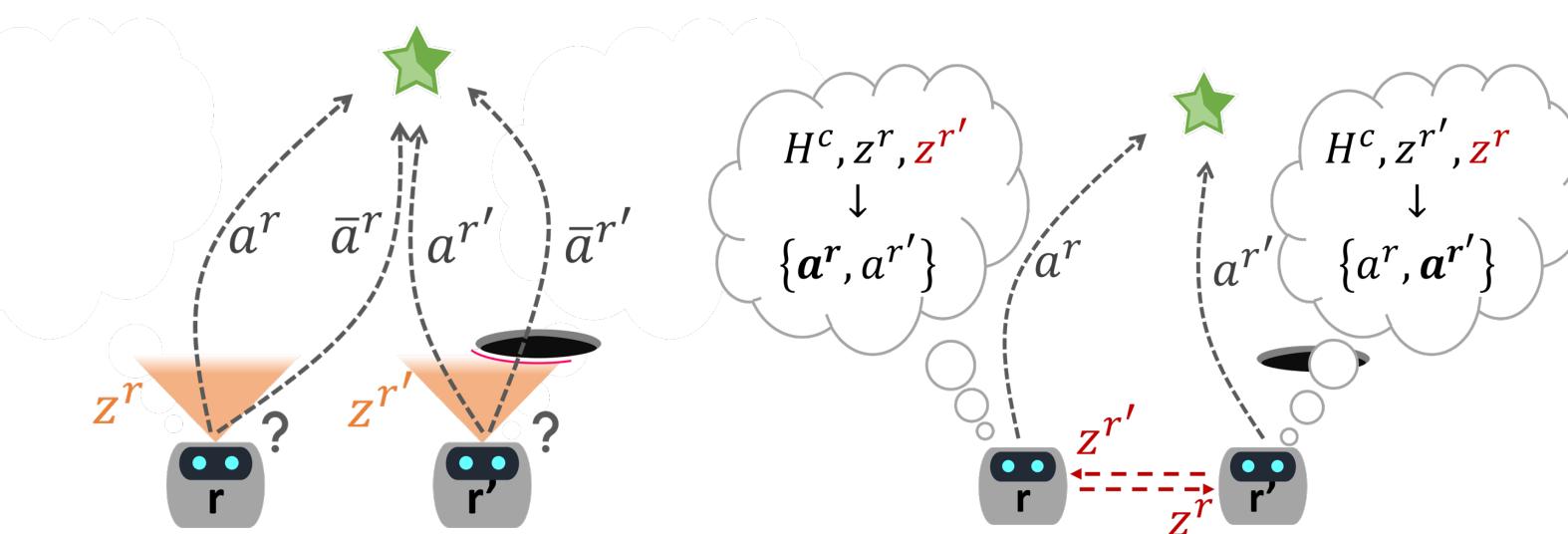
- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

Specific simplifications include:

- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of Risk-Averse POMDP Planning
- Simplification in a multi-agent setting

Multi-Robot Belief Space Planning

- **A common assumption:** Beliefs of different robots are consistent at planning time
- Requires prohibitively frequent data-sharing capabilities!



Multi-Robot Cooperative BSP with Inconsistent Beliefs

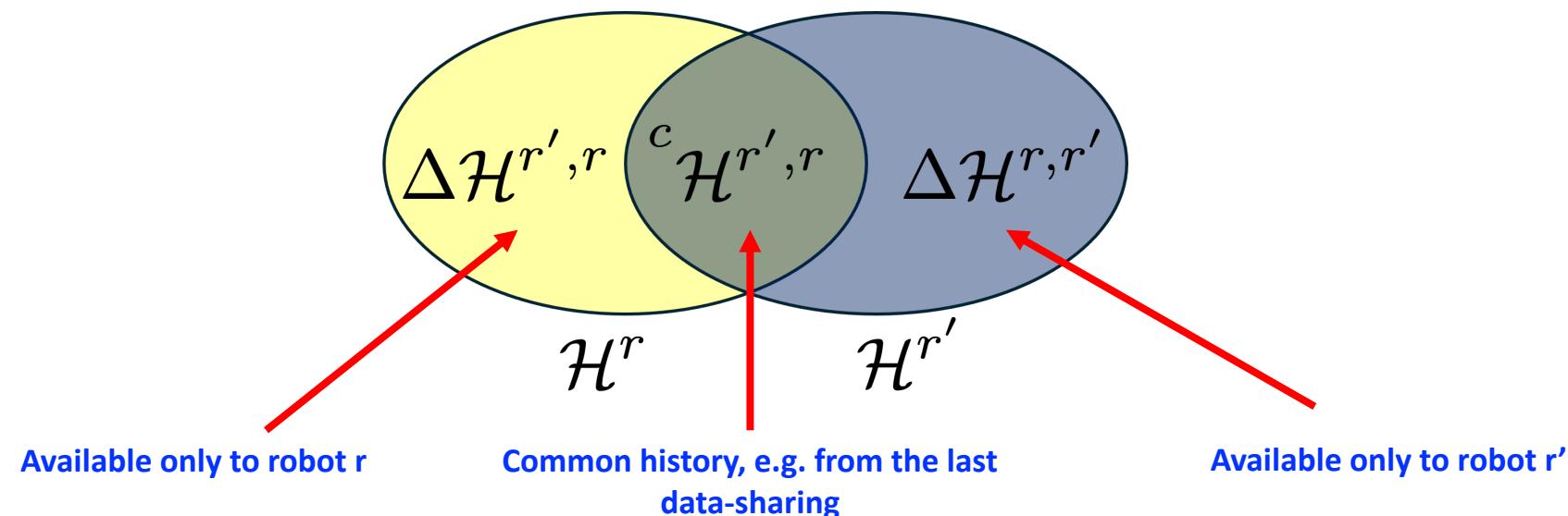
What happens when data-sharing capabilities between the robots are limited?

- Histories & beliefs of the robots may **differ** due to limited data-sharing capabilities

$$b_k^r = \mathbb{P}(x_k \mid \mathcal{H}_k^r)$$

$$b_k^{r'} = \mathbb{P}(x_k \mid \mathcal{H}_k^{r'})$$

$$\mathcal{H}_k^r \neq \mathcal{H}_k^{r'}$$



Multi-Robot Cooperative BSP with Inconsistent Beliefs

What happens when data-sharing capabilities between the robots are limited?

- Histories & beliefs of the robots may **differ** due to limited data-sharing capabilities

$$b_k^r = \mathbb{P}(x_k \mid \mathcal{H}_k^r) \quad b_k^{r'} = \mathbb{P}(x_k \mid \mathcal{H}_k^{r'}) \quad \mathcal{H}_k^r \neq \mathcal{H}_k^{r'}$$

- Decentralized POMDP tuple from the perspective of robot r:

$$\langle \mathcal{X}, \mathcal{Z}, \mathcal{A}, T, O, \rho, b_k^r \rangle$$

- Objective function:

$$J(b_k^r, a_{k+}) = \mathbb{E}_{z_{k+1:k+L}} \left[\sum_{l=0}^{L-1} \rho(b_{k+l}^r, a_{k+l}) + \rho(b_{k+L}^r) \right]$$

Multi-Robot Cooperative BSP with Inconsistent Beliefs

What happens when data-sharing capabilities between the robots are limited?

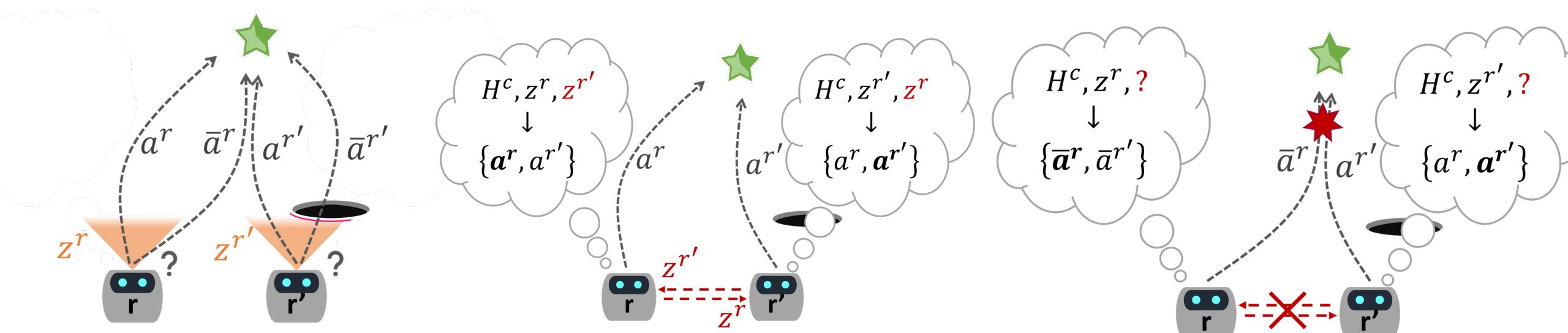
- Histories & beliefs of the robots may **differ** due to limited data-sharing capabilities

$$b_k^r = \mathbb{P}(x_k \mid \mathcal{H}_k^r)$$

$$b_k^{r'} = \mathbb{P}(x_k \mid \mathcal{H}_k^{r'})$$

$$\mathcal{H}_k^r \neq \mathcal{H}_k^{r'}$$

- Can lead to a lack of coordination and unsafe and sub-optimal actions



T. Kundu, M. Rafaeli, and V. Indelman, "Multi-Robot Communication-Aware Cooperative Belief Space Planning with Inconsistent Beliefs: An Action-Consistent Approach," IROS'24.

T. Kundu, M. Rafaeli, A. Gulyaev, and V. Indelman, "Action-Consistent Decentralized Belief Space Planning with Inconsistent Beliefs and Limited Data Sharing: Framework and Simplification Algorithms with Formal Guarantees," arXiv'25.

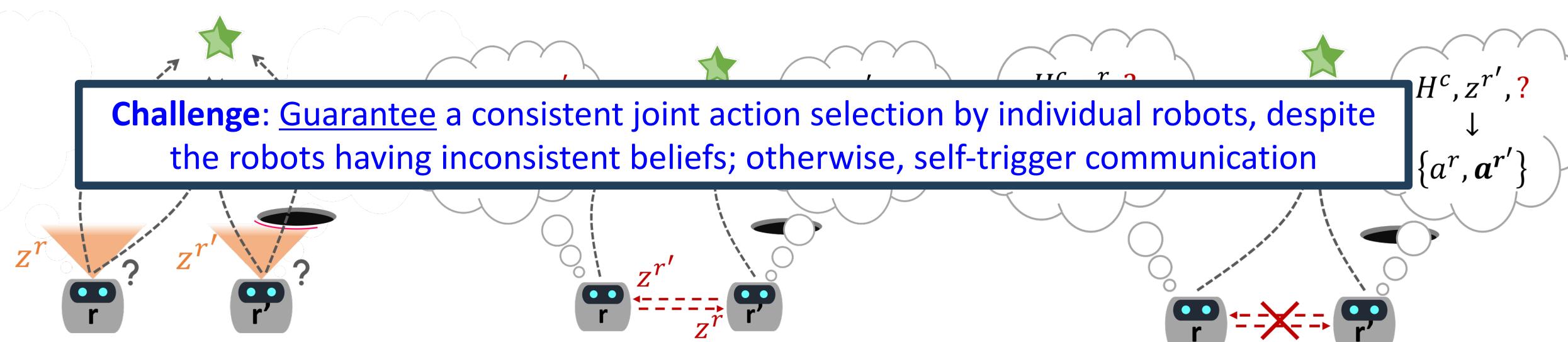
Multi-Robot Cooperative BSP with Inconsistent Beliefs

What happens when data-sharing capabilities between the robots are limited?

- Histories & beliefs of the robots may **differ** due to limited data-sharing capabilities

$$b_k^r = \mathbb{P}(x_k \mid \mathcal{H}_k^r) \quad b_k^{r'} = \mathbb{P}(x_k \mid \mathcal{H}_k^{r'}) \quad \mathcal{H}_k^r \neq \mathcal{H}_k^{r'}$$

- **Can lead to a lack of coordination and unsafe and sub-optimal actions**

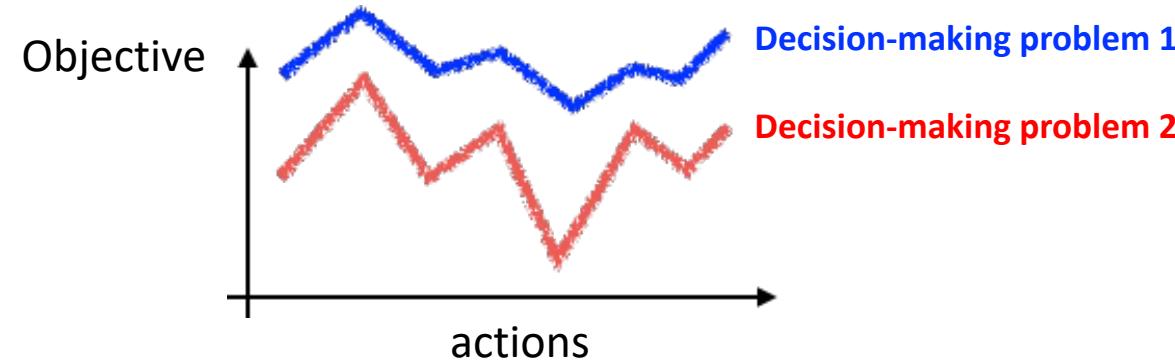


T. Kundu, M. Rafaeli, and V. Indelman, "Multi-Robot Communication-Aware Cooperative Belief Space Planning with Inconsistent Beliefs: An Action-Consistent Approach," IROS'24.

T. Kundu, M. Rafaeli, A. Gulyaev, and V. Indelman, "Action-Consistent Decentralized Belief Space Planning with Inconsistent Beliefs and Limited Data Sharing: Framework and Simplification Algorithms with Formal Guarantees," arXiv'25.

Action Consistency

- If two decision-making problems have the **same action preference**, this implies both have the **same best action regardless of the actual objective/value function values**



- Key idea:** to guarantee consistent multi-robot decision-making, each robot
 - reasons about its own and other robots' action preferences while accounting for the **missing information** between the robots
 - checks if for all these realizations, we get the same best joint action

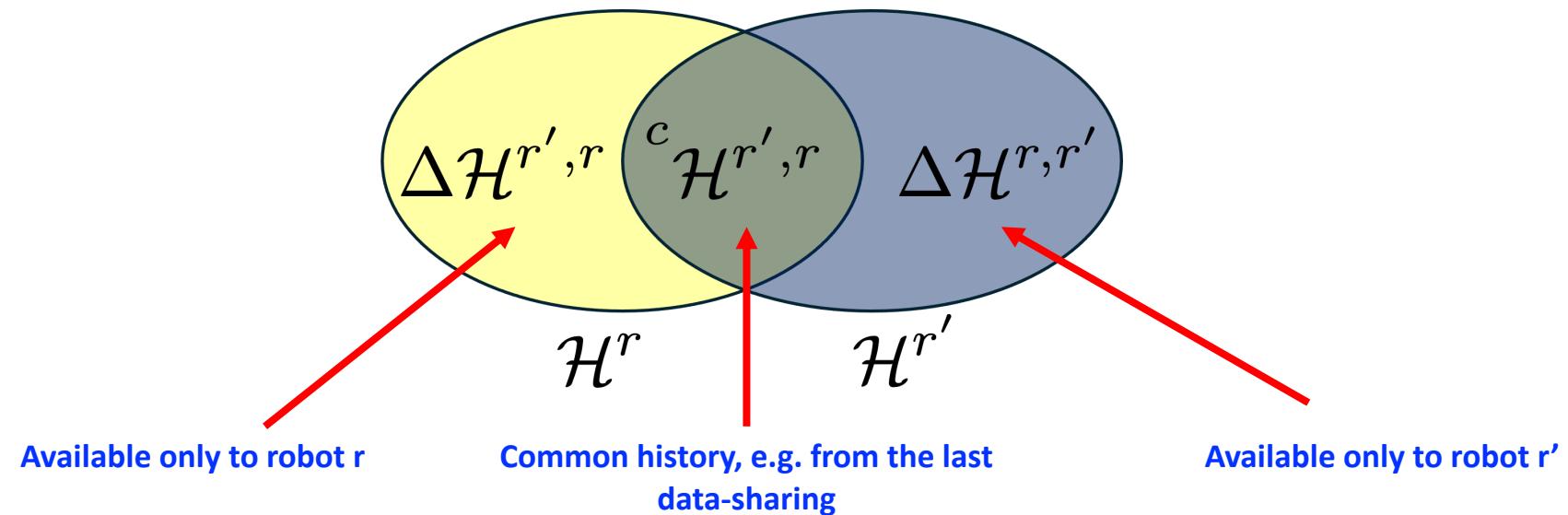
V. Indelman, "No Correlations Involved: Decision Making Under Uncertainty in a Conservative Sparse Information Space," IEEE RA-L'16.

K. Elimelech and V. Indelman, "Simplified decision making in the belief space using belief sparsification," IJRR'22.

A. Kitanov and V. Indelman, "Topological Belief Space Planning for Active SLAM with Pairwise Gaussian Potentials and Performance Guarantees," IJRR'24.

Decentralized Verification of Multi-Robot Action Consistency (MR-AC)

- From the perspective of robot r , MR-AC holds if the selected joint actions are the same based on:
 - Its local information
 - What it perceives about the reasoning of the other robot r'
 - What it perceives about the reasoning of itself perceived by the other robot r'

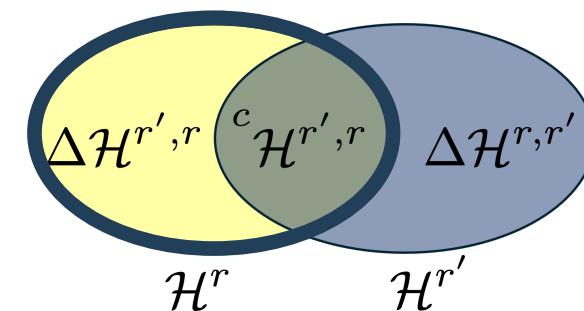


T. Kundu, M. Rafaeli, and V. Indelman, "Multi-Robot Communication-Aware Cooperative Belief Space Planning with Inconsistent Beliefs: An Action-Consistent Approach," IROS'24.

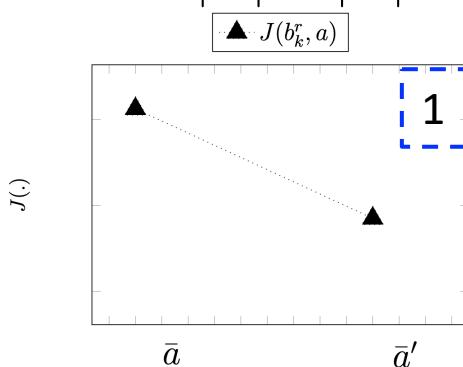
T. Kundu, M. Rafaeli, A. Gulyaev, and V. Indelman, "Action-Consistent Decentralized Belief Space Planning with Inconsistent Beliefs and Limited Data Sharing: Framework and Simplification Algorithms with Formal Guarantees," arXiv'25.

Decentralized Verification of Multi-Robot Action Consistency (MR-AC)

- From the perspective of robot r , MR-AC holds if the selected joint actions are the same based on:
 - Its local information

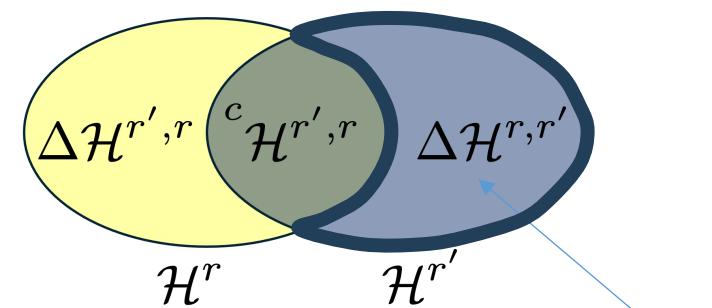


Toy example for $|\mathcal{A}| = |\mathcal{Z}| = 2$:

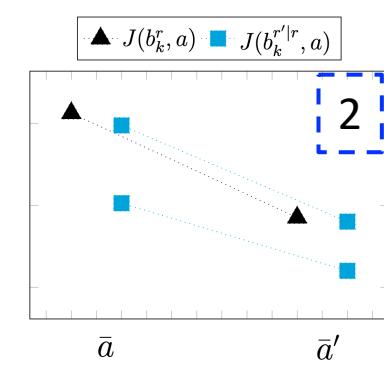
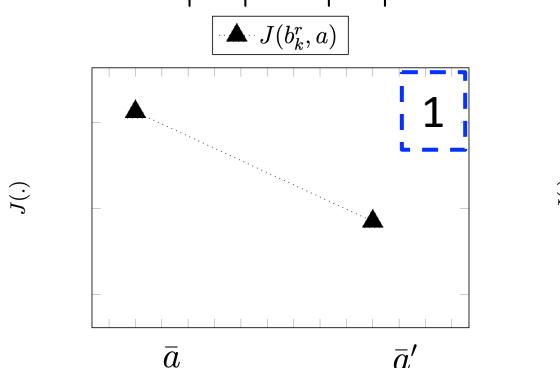


Decentralized Verification of Multi-Robot Action Consistency (MR-AC)

- From the perspective of robot r , MR-AC holds if the selected joint actions are the same based on:
 - Its local information
 - What it perceives about the reasoning of the other robot r'



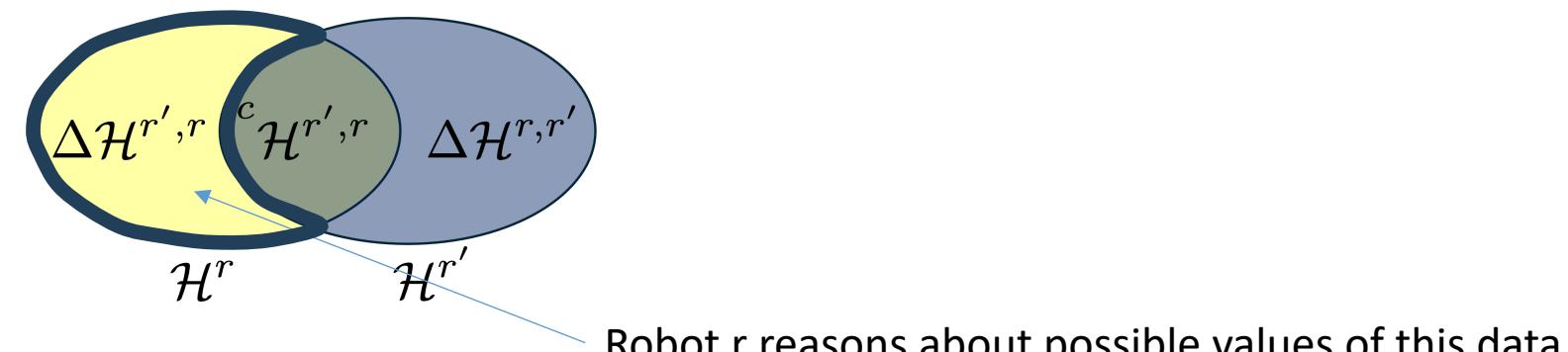
Toy example for $|\mathcal{A}| = |\mathcal{Z}| = 2$:



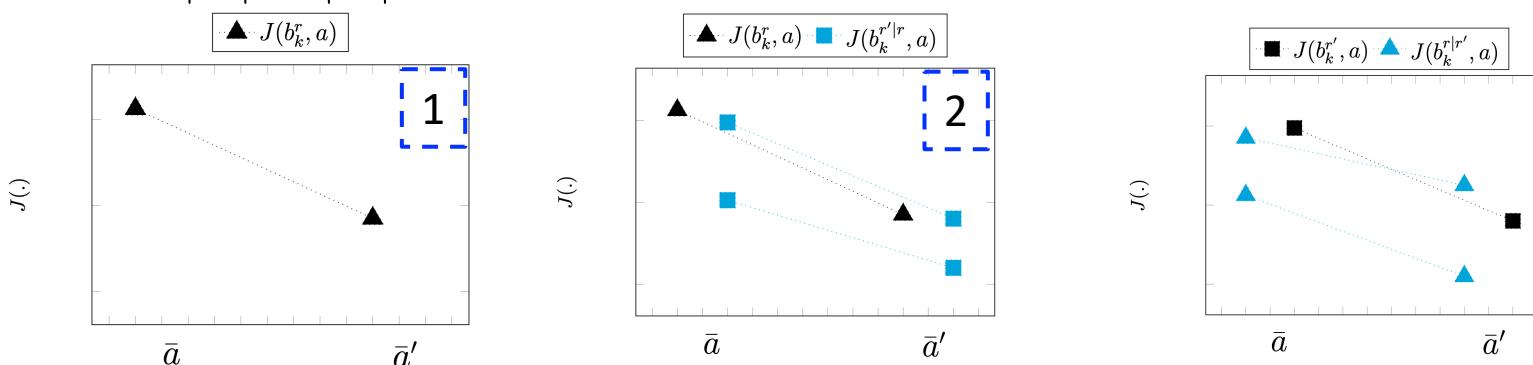
Robot r reasons about possible values of this data

Decentralized Verification of Multi-Robot Action Consistency (MR-AC)

- From the perspective of robot r , MR-AC holds if the selected joint actions are the same based on:
 - Its local information
 - What it perceives about the reasoning of the other robot r'
 - What it perceives about the reasoning of itself perceived by the other robot r'**



Toy example for $|\mathcal{A}| = |\mathcal{Z}| = 2$:



Decentralized Verification of Multi-Robot Action Consistency (MR-AC)

- From the perspective of robot r , MR-AC holds if the selected joint actions are the same based on:
 - Its local information
 - What it perceives about the reasoning of the other robot r'
 - What it perceives about the reasoning of itself perceived by the other robot r'**

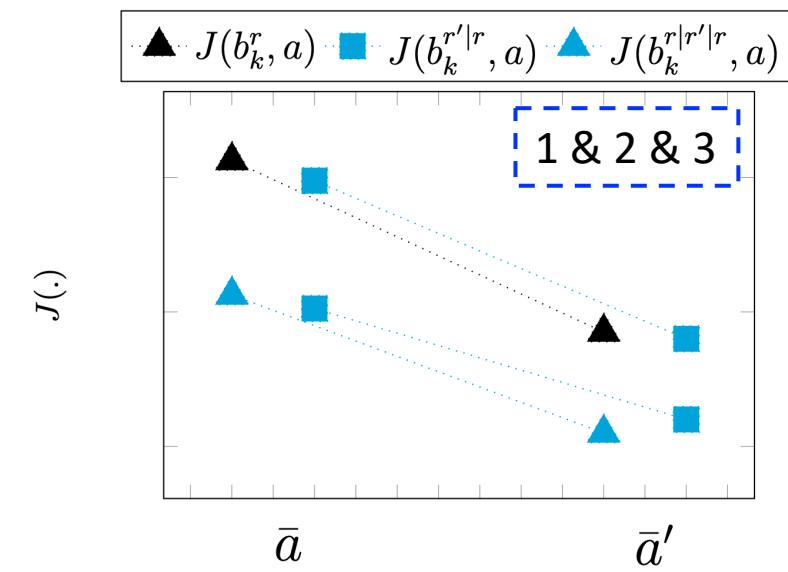
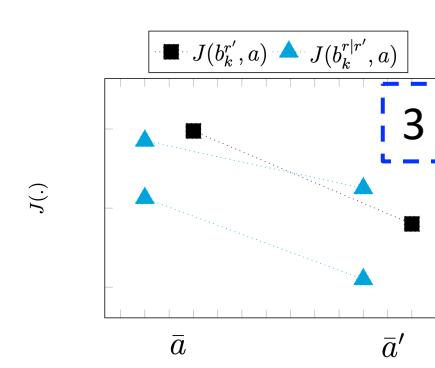
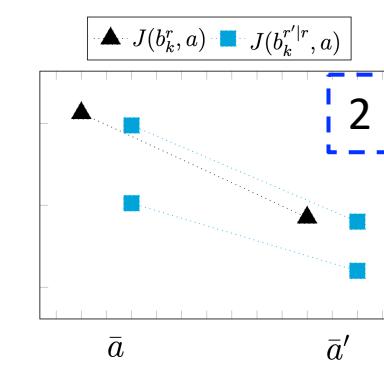
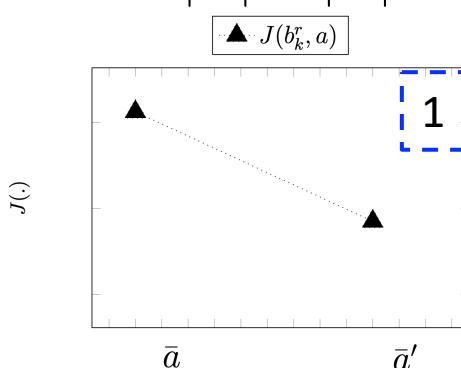
For each possible observation of r' , $\tilde{z}^r \in \Delta \mathcal{Z}_k^{r',r}$, robot r

constructs a plausible belief of robot r' : $b_k^{r|r'|r}(\tilde{z}^r) \triangleq \mathbb{P}(x_k \mid {}^c\mathcal{H}_k^{r',r}, \tilde{z}^r)$

evaluates $J(b_k^{r|r'|r}(\tilde{z}^r), a) \quad \forall a \in \mathcal{A}$

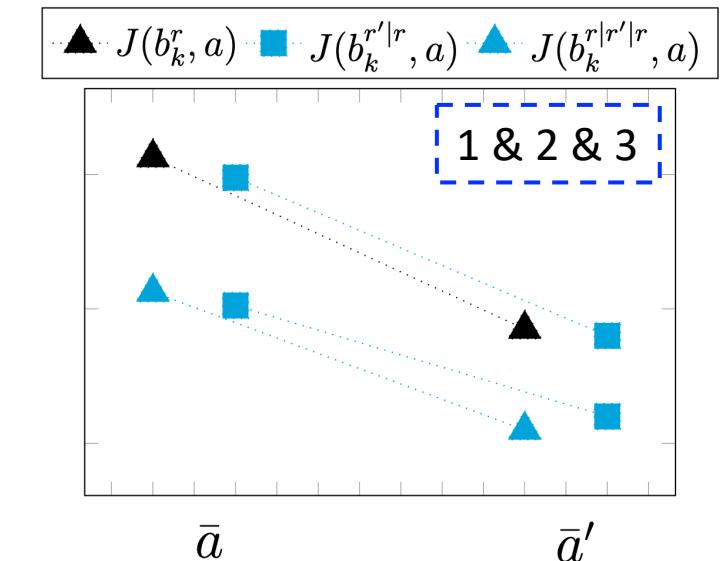
Checks if \bar{a} is selected

Toy example for $|\mathcal{A}| = |\mathcal{Z}| = 2$:



Decentralized Verification of Multi-Robot Action Consistency (MR-AC)

- From the perspective of robot r , MR-AC holds if the selected joint actions are the same based on:
 1. Its local information
 2. What it perceives about the reasoning of the other robot r'
 3. What it perceives about the reasoning of itself perceived by the other robot r'
- Same best action in all cases?
 - Yes:** MR-AC is guaranteed to be satisfied
 - Robots are **guaranteed** to choose the same joint action
 - No further data sharing** is needed!
 - No:** self-trigger communication, share some data, repeat Steps 1-3

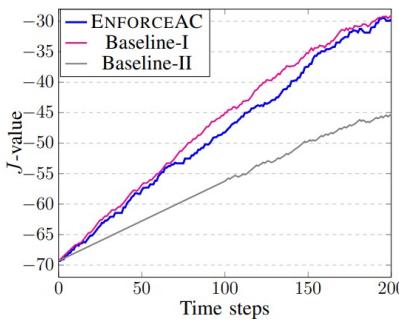
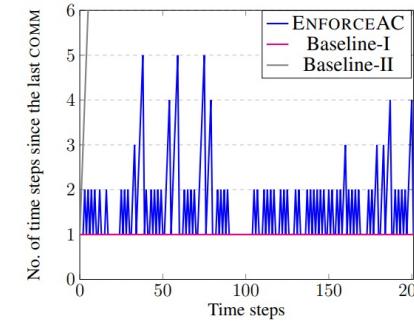
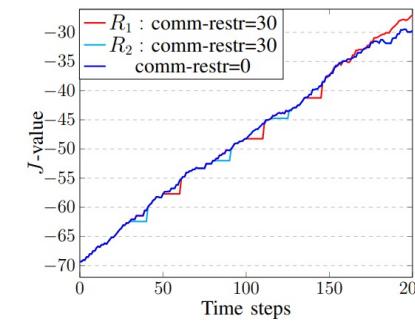


T. Kundu, M. Rafaeli, and V. Indelman, "Multi-Robot Communication-Aware Cooperative Belief Space Planning with Inconsistent Beliefs: An Action-Consistent Approach," IROS'24.

T. Kundu, M. Rafaeli, A. Gulyaev, and V. Indelman, "Action-Consistent Decentralized Belief Space Planning with Inconsistent Beliefs and Limited Data Sharing: Framework and Simplification Algorithms with Formal Guarantees," arXiv'25.

Simulation Results (Search & Rescue Scenario)

- EnforceAC: our approach
- Baseline I: always communicate all data
- Baseline II: never communicate

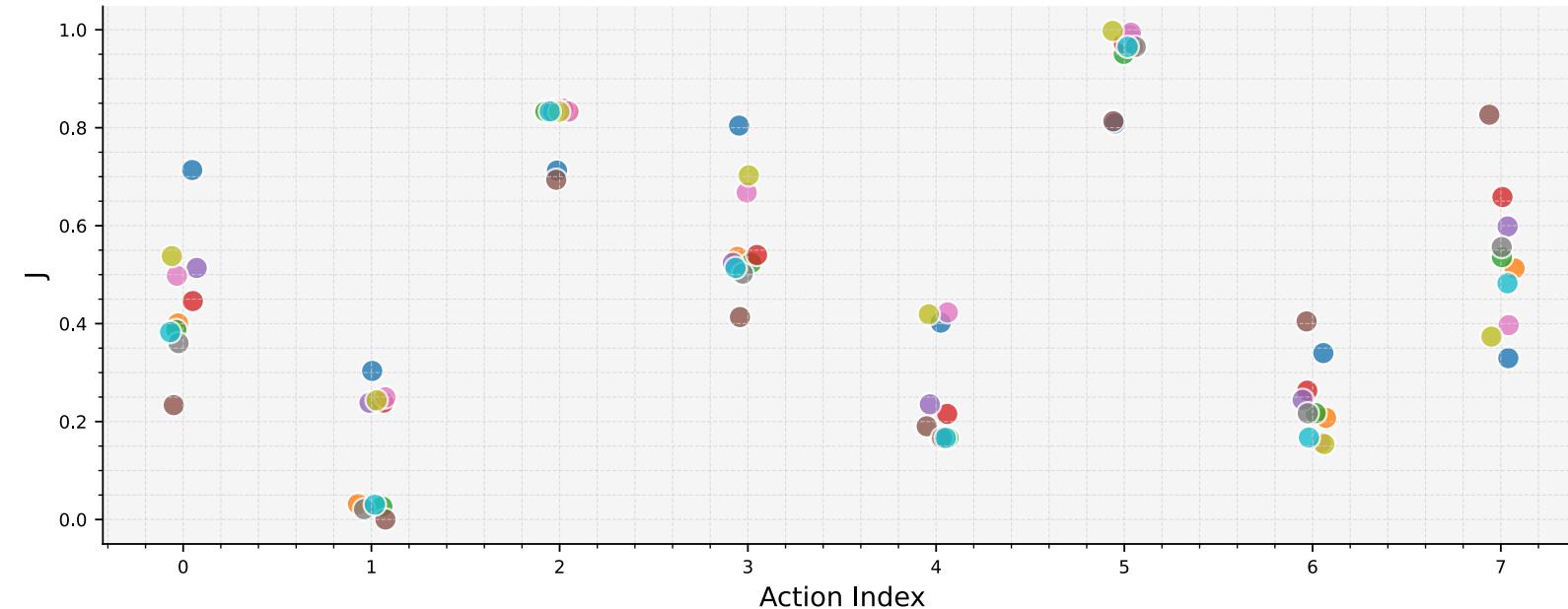
(a) $comm\text{-}restr = 0$ (b) $comm\text{-}restr = 0$ (c) $comm\text{-}restr = 30$

NOT-AC (ACTION INCONSISTENCY), COMMS AND TIME FOR $E = 200$.

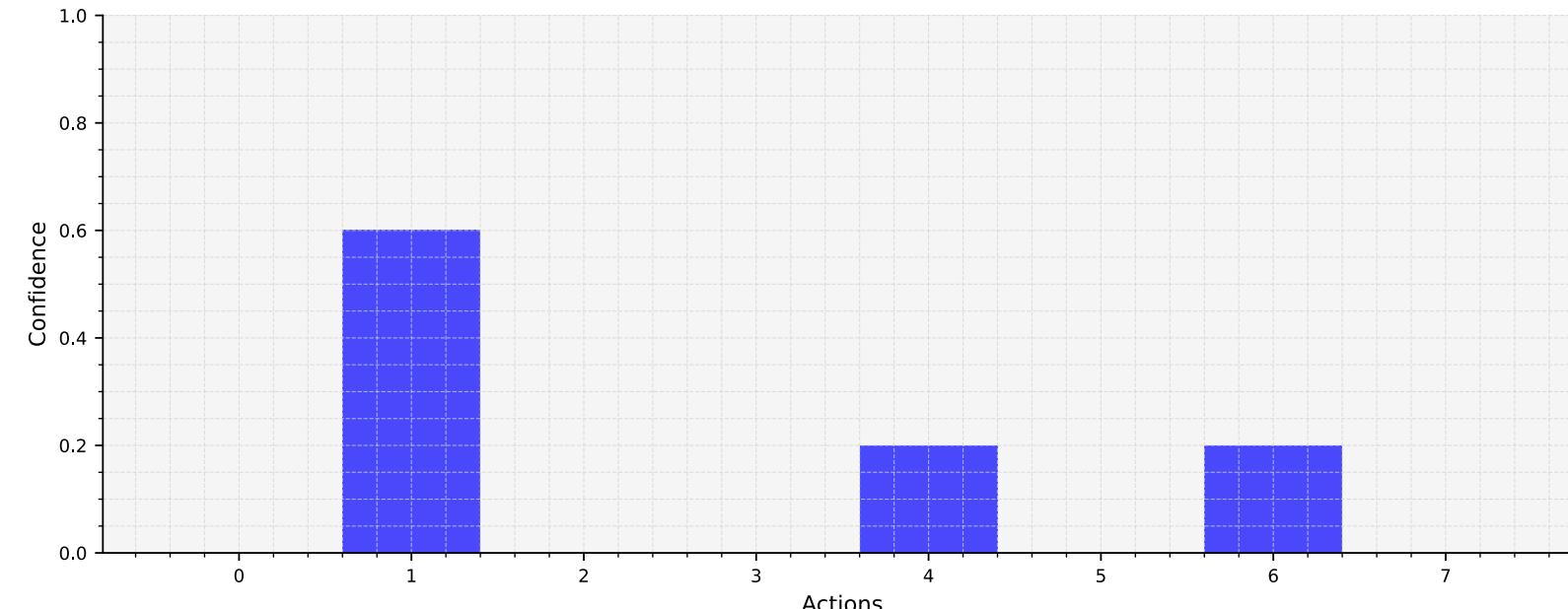
Input	Algorithm	Not-AC	COMM	Time
$comm\text{-}restr = 0$	Baseline-II	181	0	1.3s
$Motion\ prim. = 4$	Baseline-I	0	400	1.3s
ENFORCEAC	ENFORCEAC	0	238	12.4s
$comm\text{-}restr = 0$	Baseline-II	185	0	1.3s
$Motion\ prim. = 4$	Baseline-I	0	400	1.4s
Entropy-Init	ENFORCEAC	0	268	8.7s
$comm\text{-}restr = 0$	Baseline-II	194	0	3.6s
$Motion\ prim. = 8$	Baseline-I	0	400	3.5s
MaxEntropy-Init	ENFORCEAC	0	248	36.4s
$comm\text{-}restr = 0$	Baseline-II	188	0	3.6s
$Motion\ prim. = 8$	Baseline-I	0	400	3.6s
Entropy-Init	ENFORCEAC	0	278	31.1s
$comm\text{-}restr = 20$	Baseline-II	194	0	3.3s
$Motion\ prim. = 8$	Baseline-I	14	360	4.3s
MaxEntropy-Init	ENFORCEAC	13	224	94.9s
$comm\text{-}restr = 20$	Baseline-II	188	0	3.2s
$Motion\ prim. = 8$	Baseline-I	14	360	3.6s
Entropy-Init	ENFORCEAC	10	251	31.2s
$comm\text{-}restr = 30$	Baseline-II	188	0	3.4s
$Motion\ prim. = 8$	Baseline-I	22	340	4.0s
MaxEntropy-Init	ENFORCEAC	20	238	46.9s

Probabilistic MRAC

Step 2:
Sampled measurement realizations



MRAC probability for each action

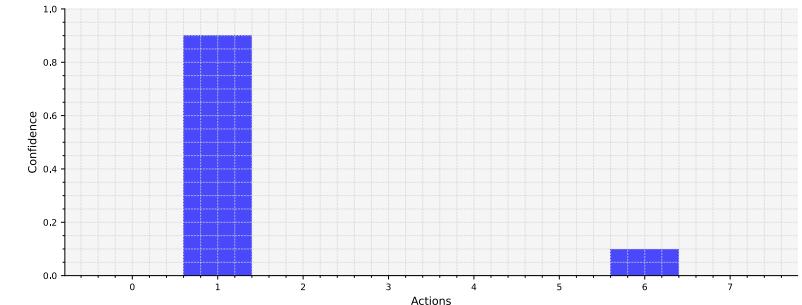
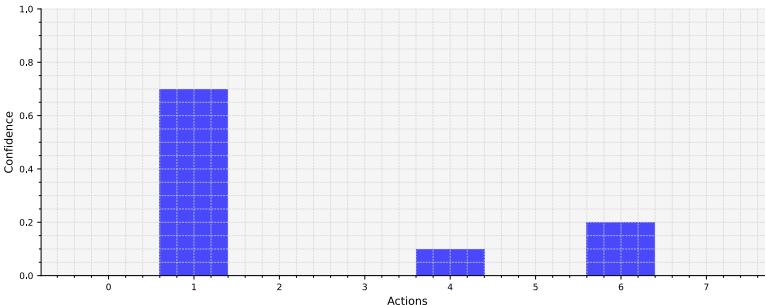
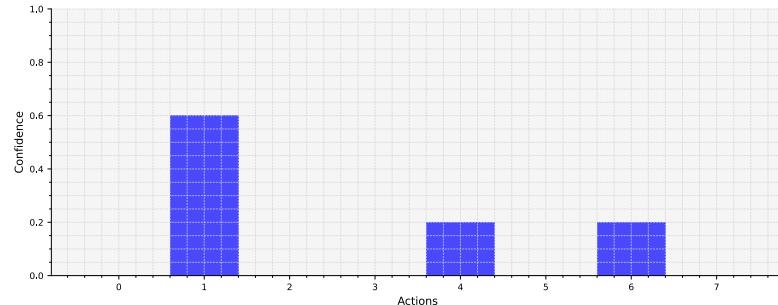
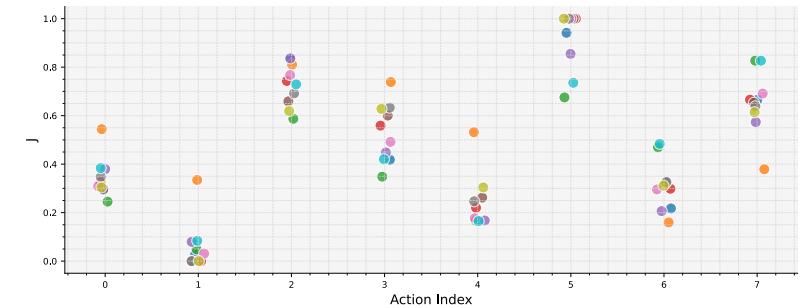
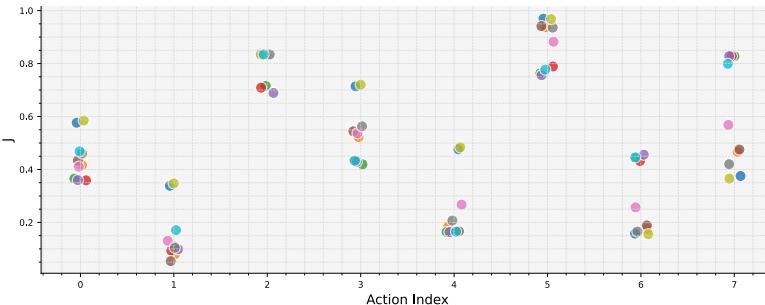
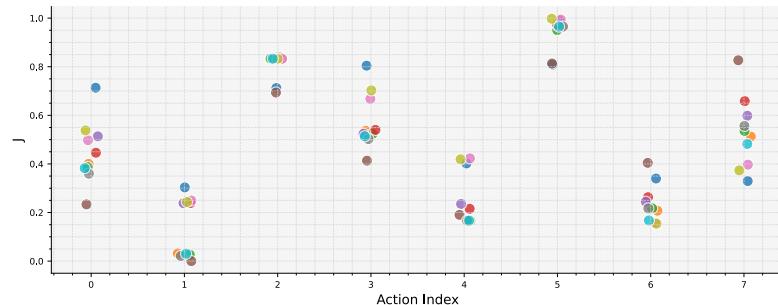


T. Kundu, M. Rafaeli, and V. Indelman, "Multi-Robot Communication-Aware Cooperative Belief Space Planning with Inconsistent Beliefs: An Action-Consistent Approach," IROS'24.

T. Kundu, M. Rafaeli, A. Gulyaev, and V. Indelman, "Action-Consistent Decentralized Belief Space Planning with Inconsistent Beliefs and Limited Data Sharing: Framework and Simplification Algorithms with Formal Guarantees," arXiv'25.

Probabilistic MRAC

data sharing



T. Kundu, M. Rafaeli, and V. Indelman, "Multi-Robot Communication-Aware Cooperative Belief Space Planning with Inconsistent Beliefs: An Action-Consistent Approach," IROS'24.

T. Kundu, M. Rafaeli, A. Gulyaev, and V. Indelman, "Action-Consistent Decentralized Belief Space Planning with Inconsistent Beliefs and Limited Data Sharing: Framework and Simplification Algorithms with Formal Guarantees," arXiv'25.

Agenda

Towards Scalable Online Decision Making Under Uncertainty in Partially Observable Environments

Experience Reuse in POMDP Planning

POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

Thank You

