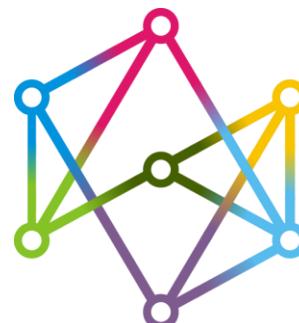


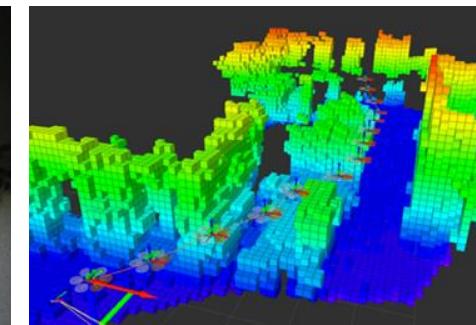
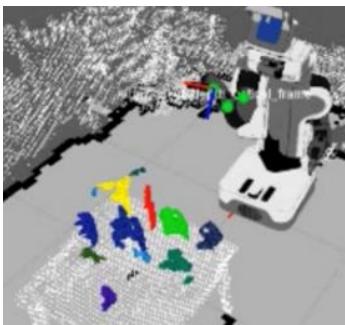
Towards Scalable Online Decision Making Under Uncertainty in Partially Observable Environments

Vadim Indelman



Advanced Autonomy

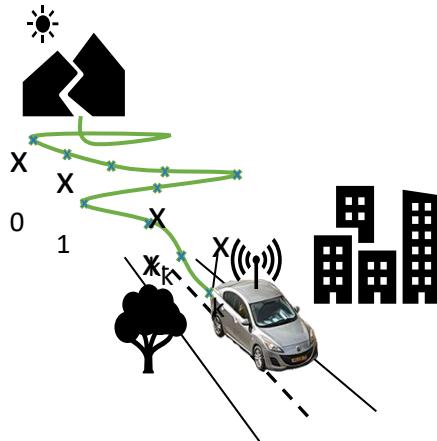
Involves autonomous navigation, active SLAM, informative gathering, active sensing, etc.



Advanced Autonomy

Perception and Inference

Where am I? What is the surrounding environment?



Decision-Making Under Uncertainty

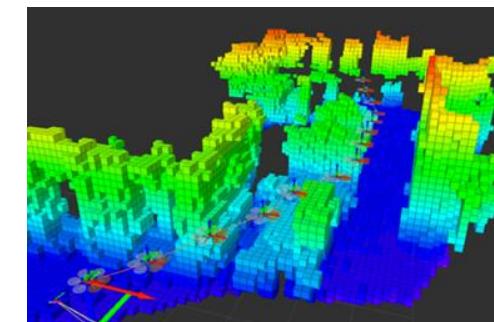
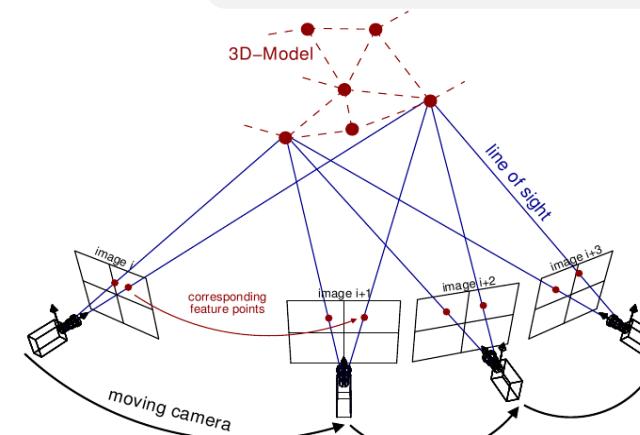
What should I be doing next?

Determine best action(s) to accomplish a task, account for different sources of uncertainty

Perception and Inference

<.....>

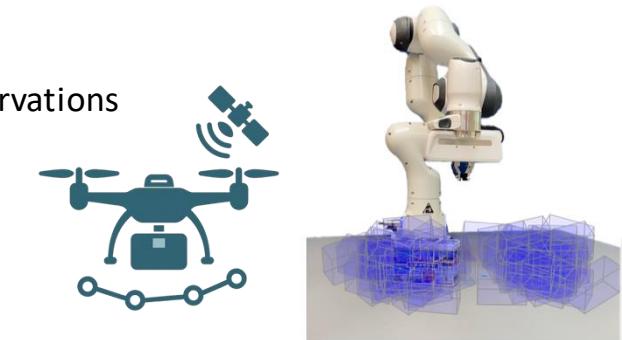
Decision-Making Under Uncertainty



Perception and Inference

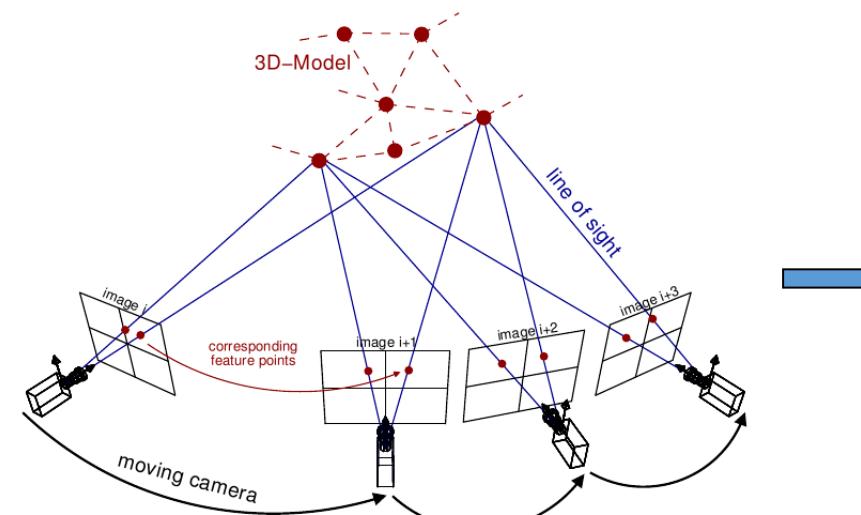
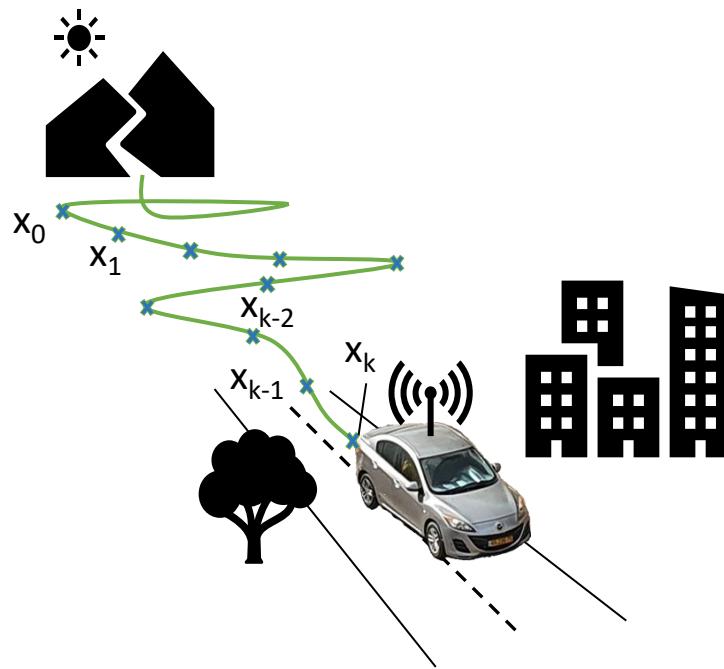
- Posterior belief at time instant k: $b_k \triangleq b[X_k] = \mathbb{P}(X_k \mid a_{0:k-1}, z_{1:k})$

state at time instant k
actions
observations

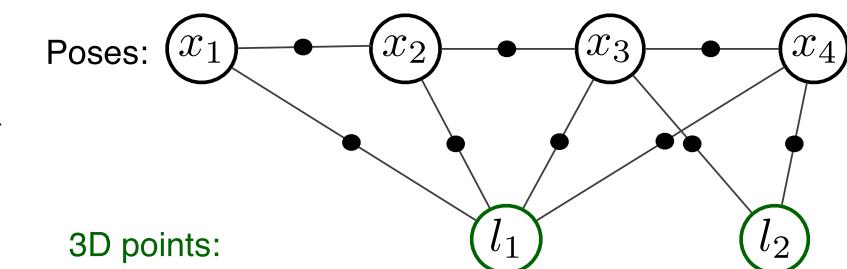


$$X_k \doteq \{x_0, \dots, x_k, L_k\}$$

Past & current robot states
Environment representation, e.g. landmarks



Can be represented with graphical models, e.g. a Factor Graph



Partially Observable Markov Decision Process (POMDP)

- POMDP tuple:

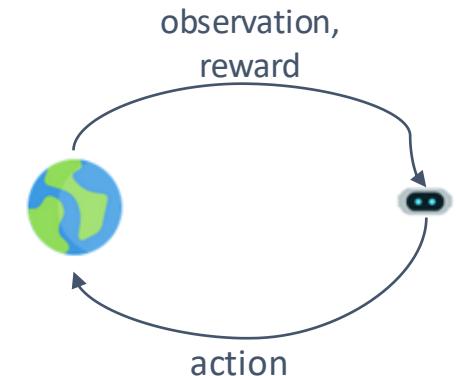
$$\langle \mathcal{X}, \mathcal{Z}, \mathcal{A}, T, O, \rho, b_k \rangle$$

State, observation, and action spaces

Transition and observation models

Belief-dependent reward function

Belief at planning time instant k



- Value function

$$V^\pi(b_k) = \mathbb{E}_{z_{k+1:k+L} | b_k, \pi} \left[\sum_{\ell=0}^L \underline{\rho}(b_{k+\ell}, \pi_{k+\ell}(b_{k+\ell})) \right]$$

Belief-dependent reward function

$$Q^\pi(b_k, a_k) = \rho(b_k, a_k) + \mathbb{E}_{z_{k+1} | b_k, a_k} [V^\pi(b_{k+1})]$$

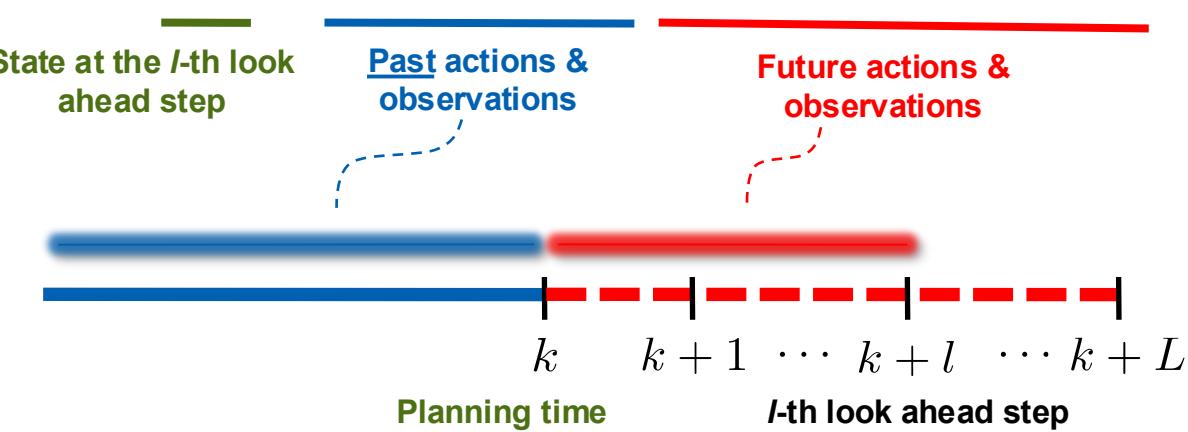
Partially Observable Markov Decision Process (POMDP)

- Value function

$$V^\pi(b_k) = \mathbb{E}_{z_{k+1:k+L} | b_k, \pi} \left[\sum_{\ell=0}^L \rho(b_{k+\ell}, \pi_{k+\ell}(b_{k+\ell})) \right]$$

Belief-dependent reward function

- Belief at the ℓ -th look-ahead step: $b_{k+\ell} \triangleq b[X_{k+\ell}] = \mathbb{P}(X_{k+\ell} | a_{0:k-1}, z_{0:k}, a_{k:k+\ell-1}, z_{k+1:k+\ell})$



- Examples for reward function $\rho(b, a)$:

- Expected distance to goal (navigate to a goal)
- Information theoretic reward (reduce uncertainty)
- ...

Challenge

Probabilistic Inference

Maintain a distribution over the state given data

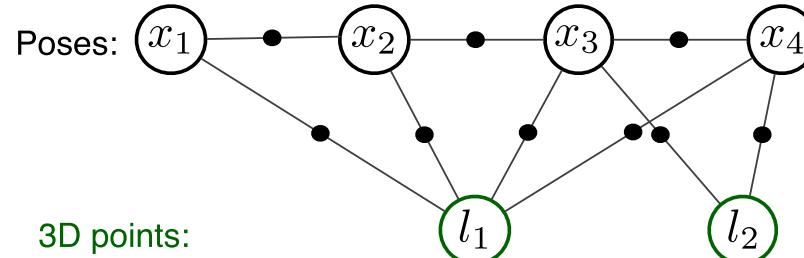
$$b_k \triangleq b[X_k] = \mathbb{P}(X_k \mid \underbrace{\text{state}}_{a_{0:k-1}}, \underbrace{\text{actions}}_{z_{1:k}})$$

Decision-making under uncertainty

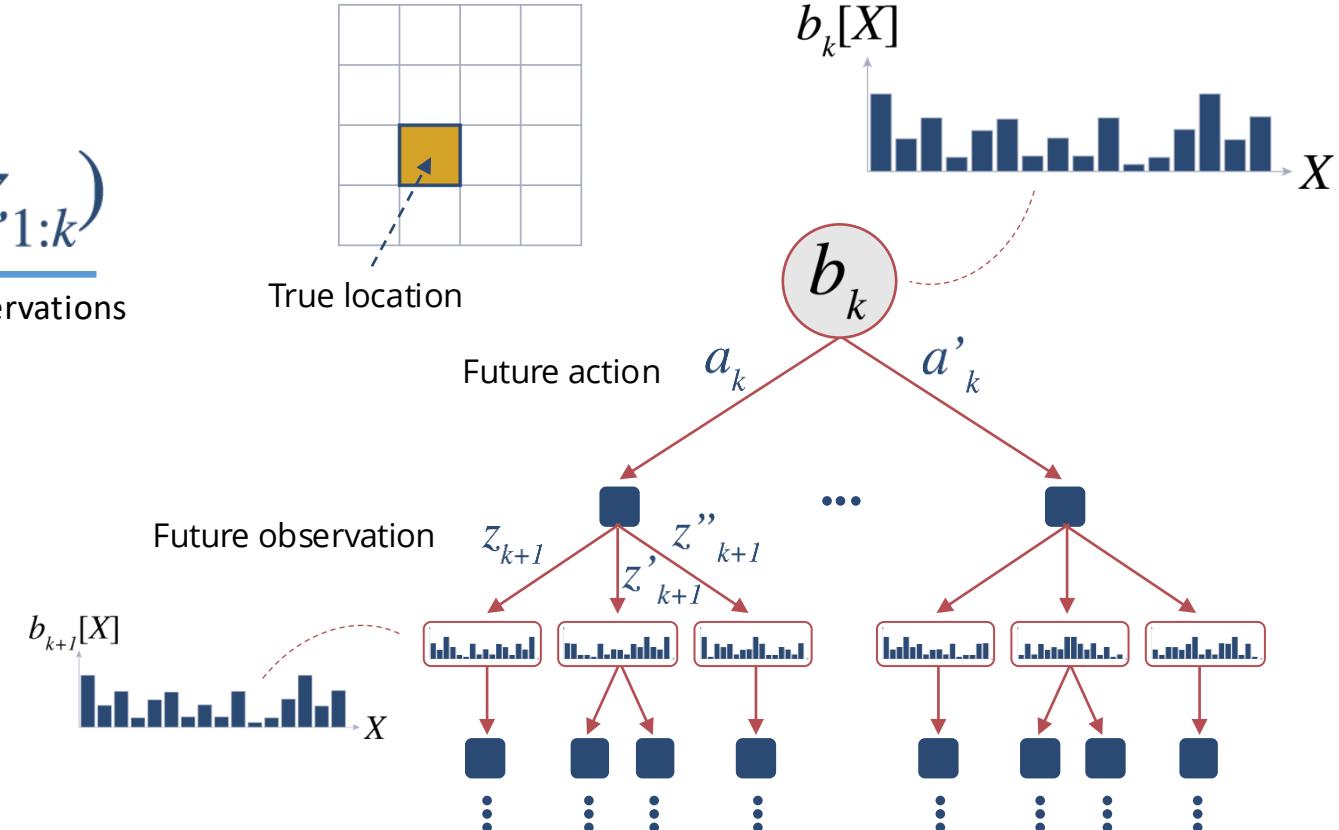
Involves reasoning about the entire observation and action spaces along planning horizon

Computationally intractable

More so, in high dimensional settings



Example - grid world



How can we plan and execute autonomously online and efficiently tasks in a safe and reliable fashion??

Agenda

Experience Reuse in POMDP Planning

POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

Agenda

Experience Reuse in POMDP Planning

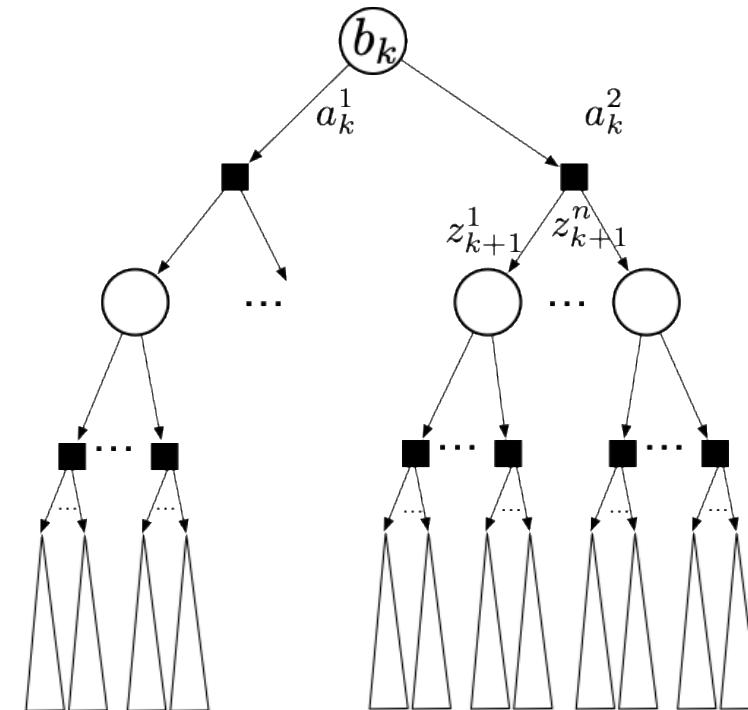
POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

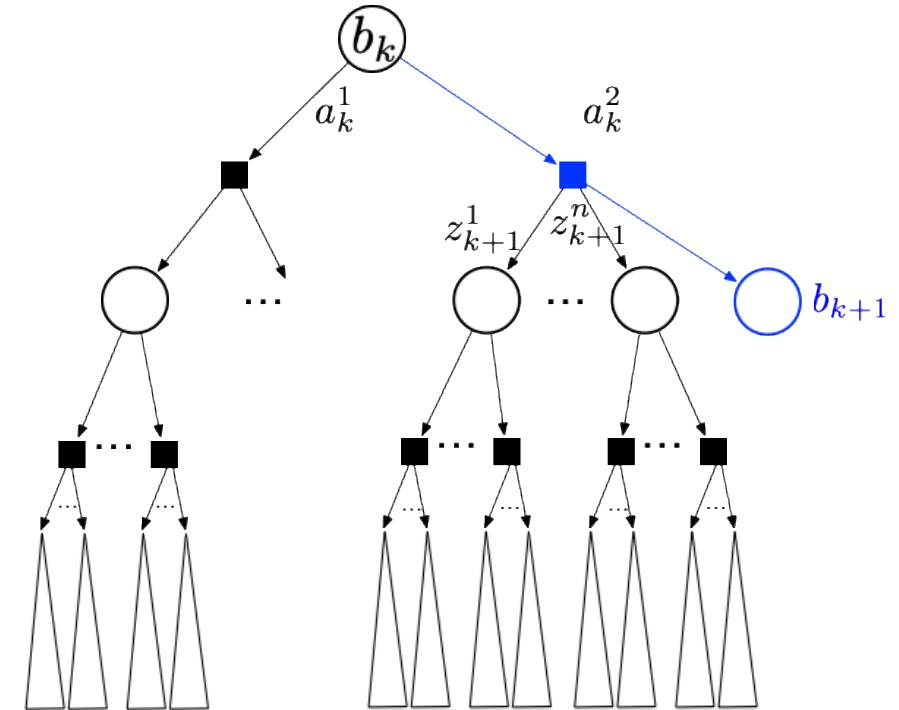
Experience Reuse in POMDP Planning

- Consider POMDPs with continuous state, action, and observation spaces



Experience Reuse in POMDP Planning

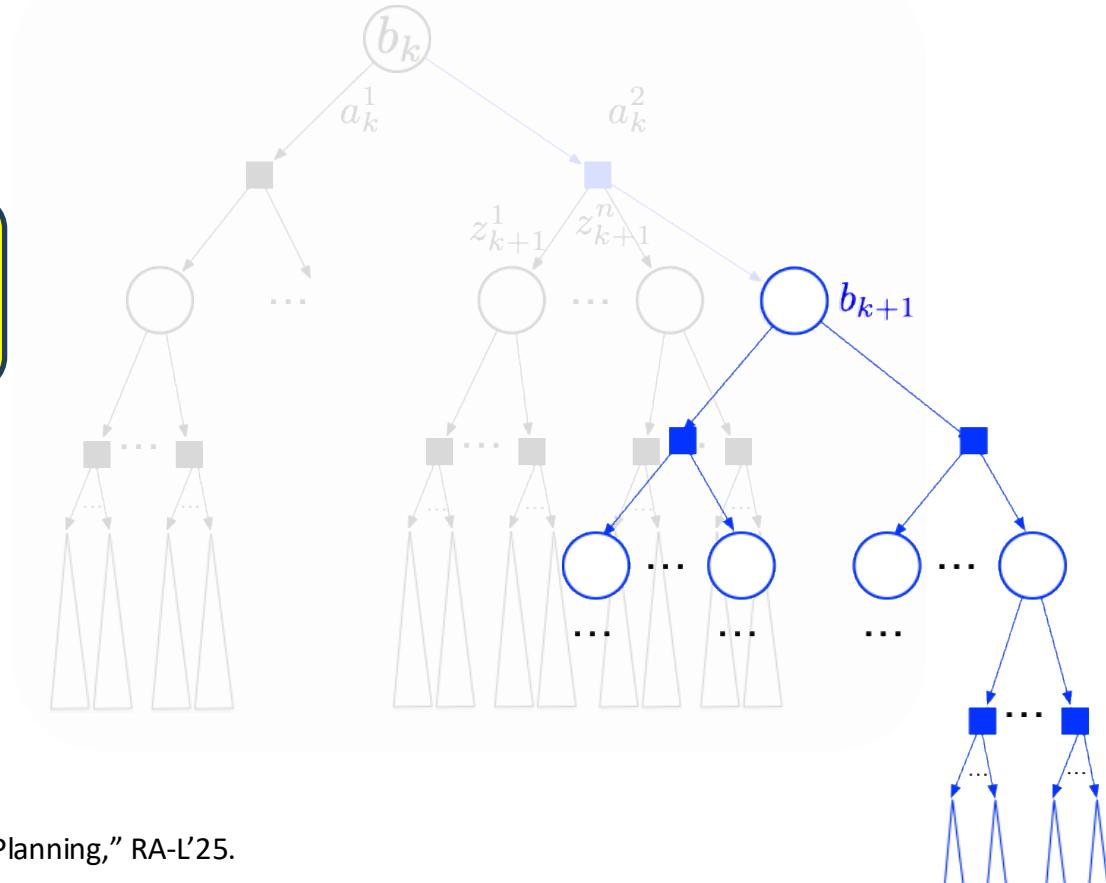
- Consider POMDPs with continuous state, action, and observation spaces
- The probability of sampling the same belief/observation twice is zero



Experience Reuse in POMDP Planning

- Consider POMDPs with continuous state, action, and observation spaces
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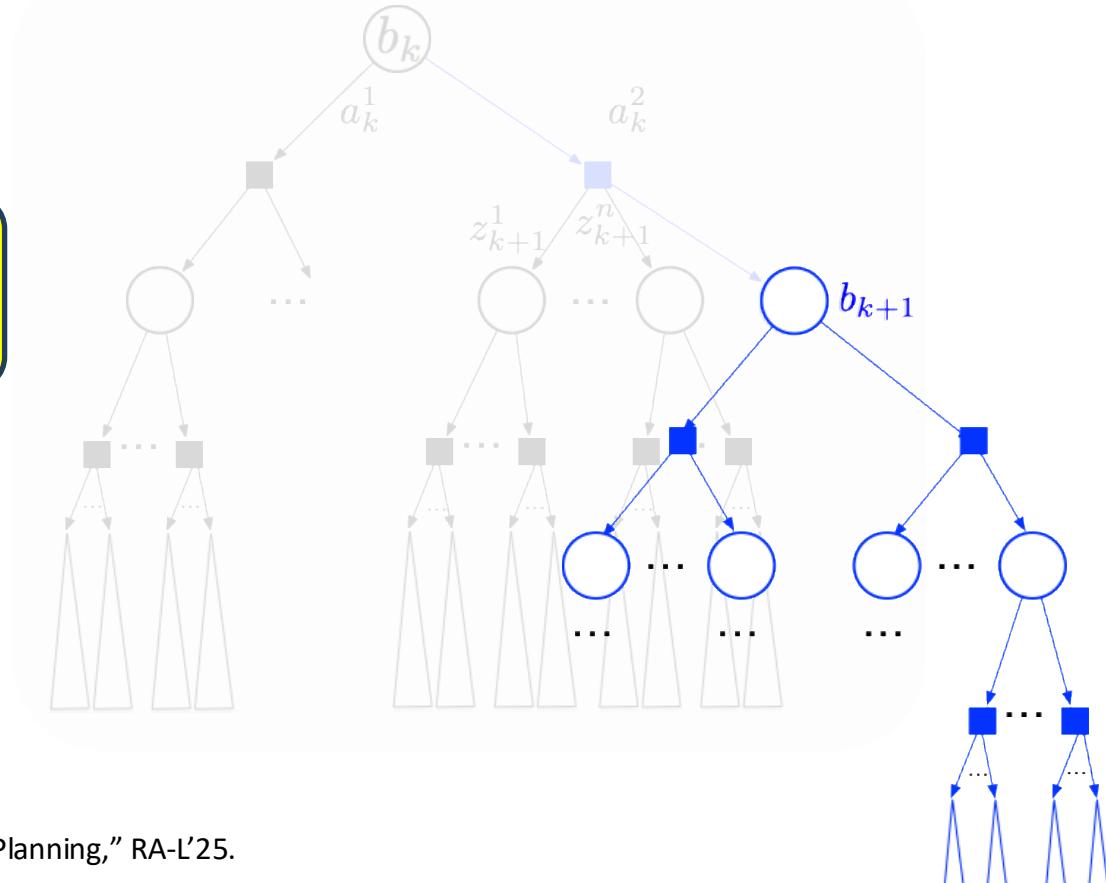
Online SOTA POMDP solvers typically perform calculations from **scratch at each planning session**



Experience Reuse in POMDP Planning

- Consider POMDPs with continuous state, action, and observation spaces
- The probability of sampling the same belief/observation twice is zero
- Previous planning sessions (experience) can provide useful information in the current planning session

Online SOTA POMDP solvers typically perform calculations from **scratch at each planning session**



Experience Reuse in POMDP Planning

- Consider POMDPs with continuous state, action, and observation spaces
- The probability of sampling the same belief/observation twice is zero
- Previous planning sessions (experience) can provide useful information in the current planning session

Key idea: Reuse previous planning session(s) to get an efficient estimation of

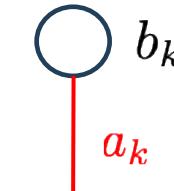
$$Q^\pi(b, a) = \mathbb{E}_\pi \left[\sum_{i=k}^{k+L-1} \gamma^{i-k} r(b_i, \pi_i(b_i), b_{i+1}) \mid b_k = b, a_k = a \right] \triangleq \mathbb{E}_\pi[G \mid b_k = b, a_k = a]$$

- Instead of calculating each planning session from scratch (state of the art)

Experience Reuse in POMDP Planning

- Consider a planning session at time instant k

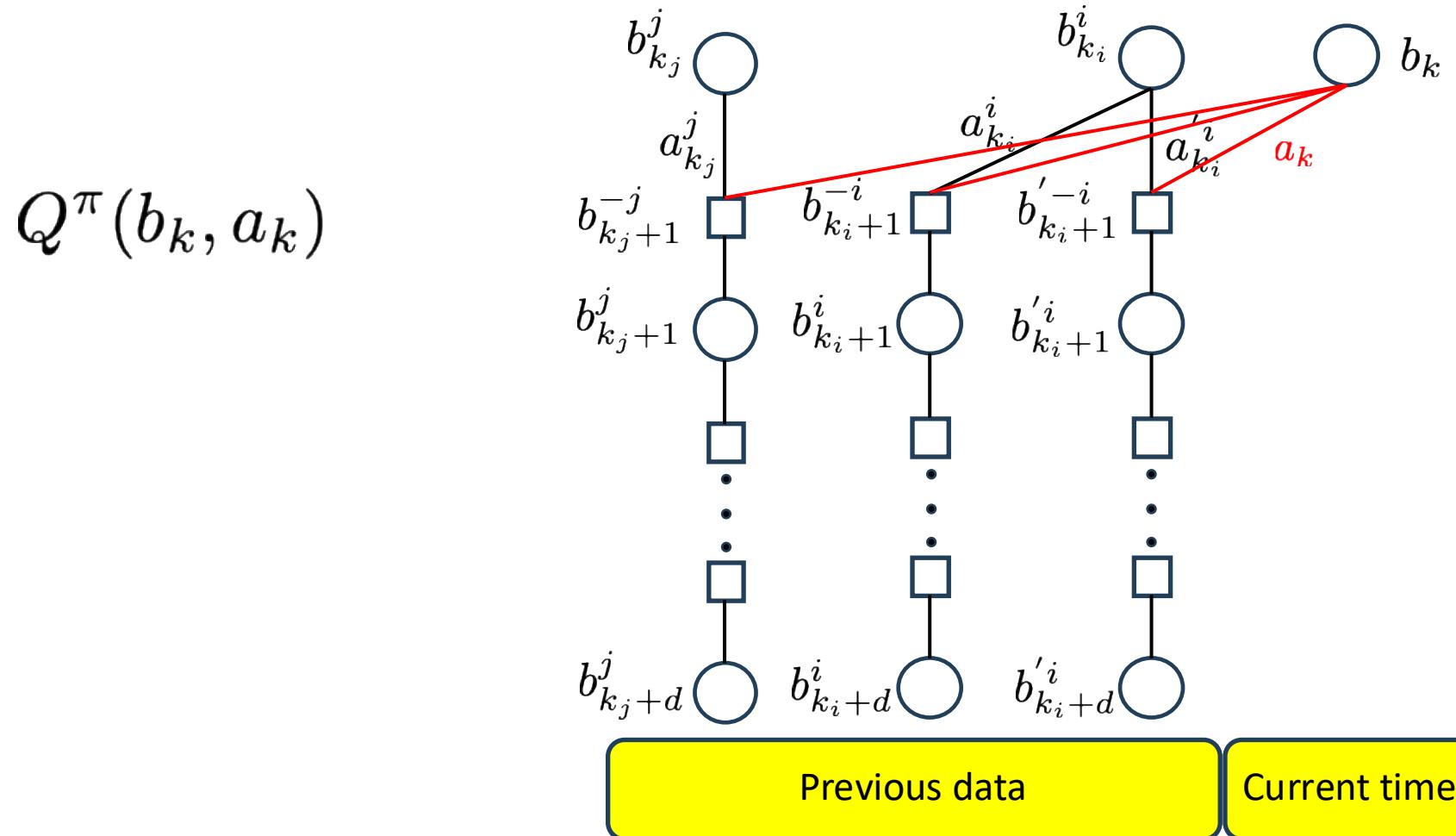
$$Q^\pi(b_k, a_k)$$



Current time

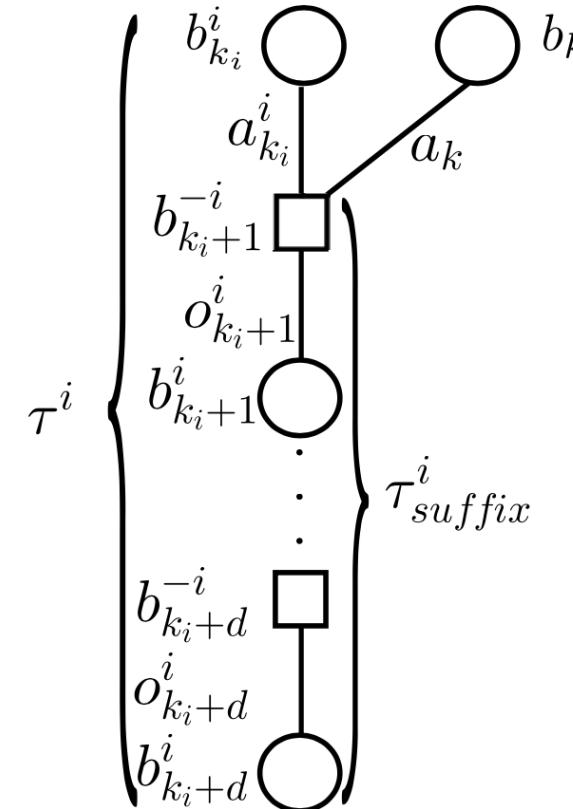
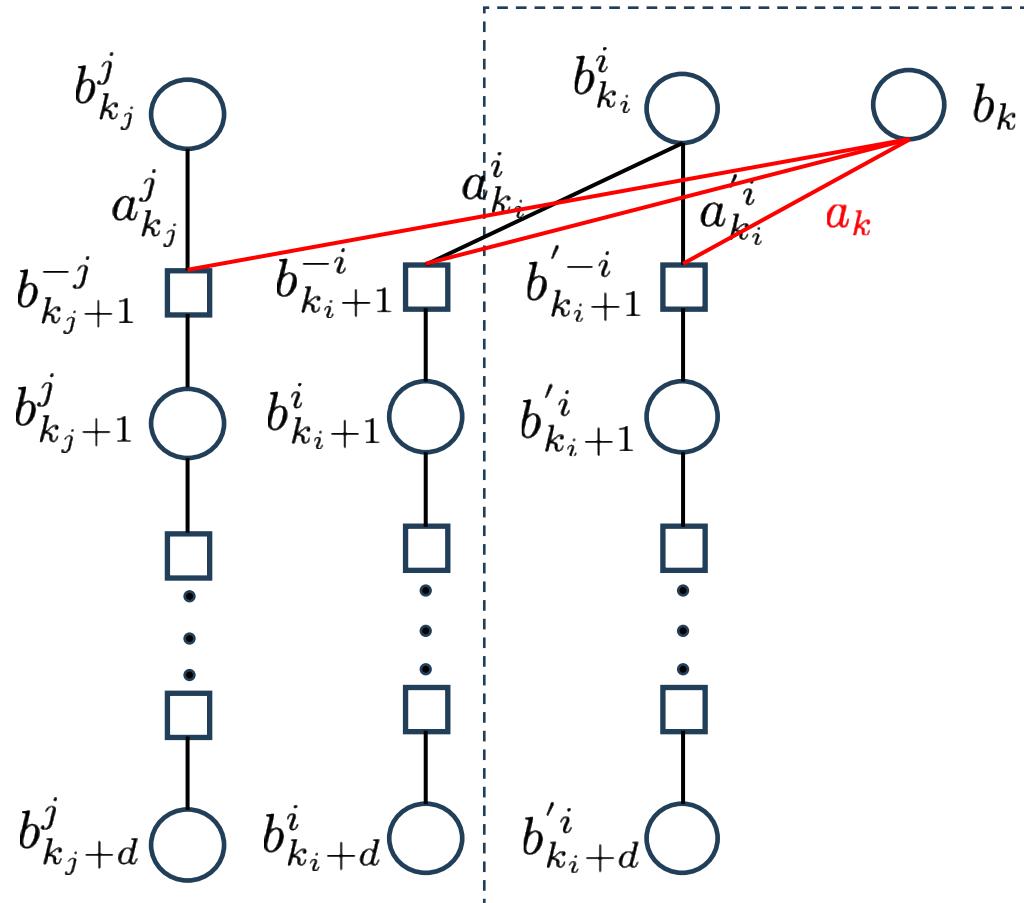
Experience Reuse in POMDP Planning

- Consider a planning session at time instant k



Experience Reuse in POMDP Planning

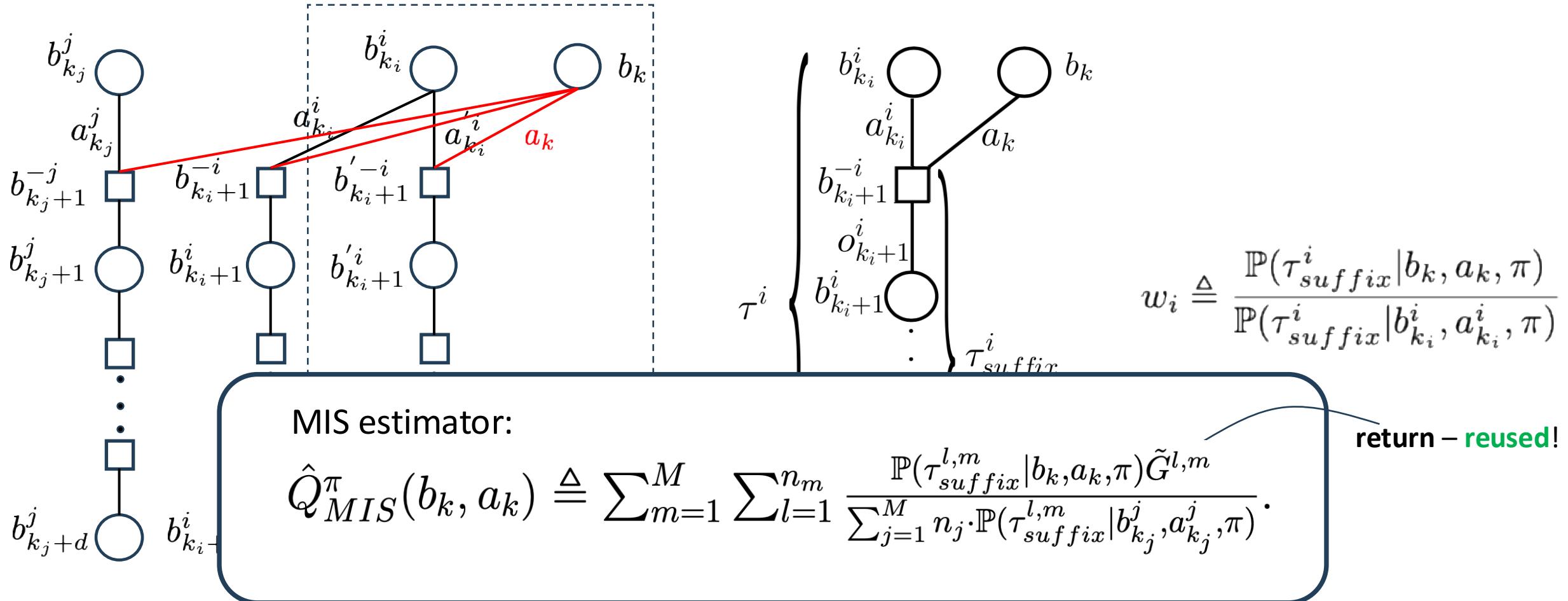
- Key idea: multiple importance sampling (MIS) estimator



$$w_i \triangleq \frac{\mathbb{P}(\tau_{suffix}^i | b_k, a_k, \pi)}{\mathbb{P}(\tau_{suffix}^i | b_{k_i}^i, a_{k_i}^i, \pi)}$$

Experience Reuse in POMDP Planning

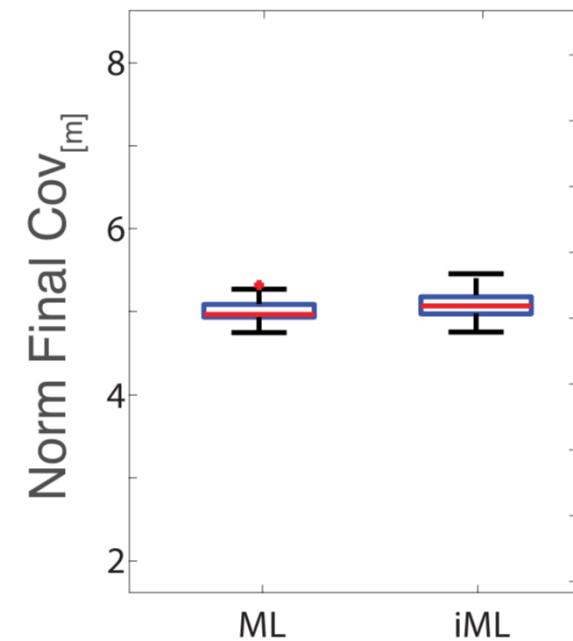
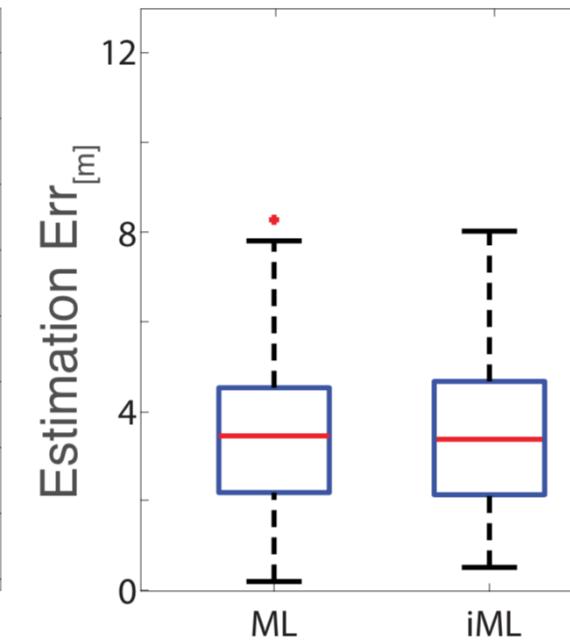
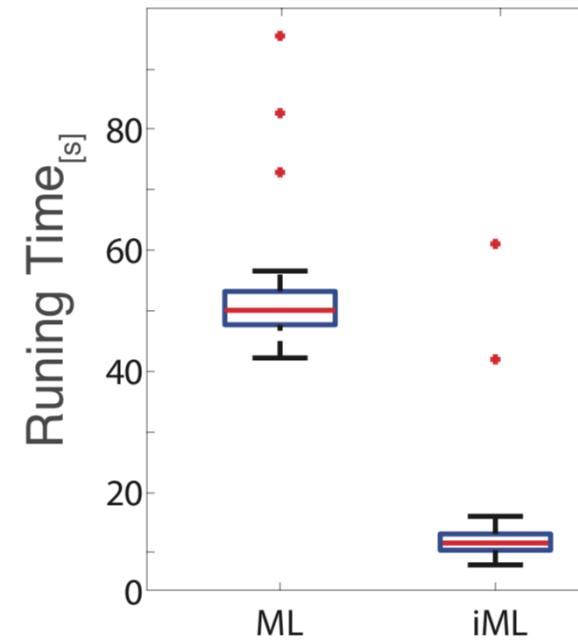
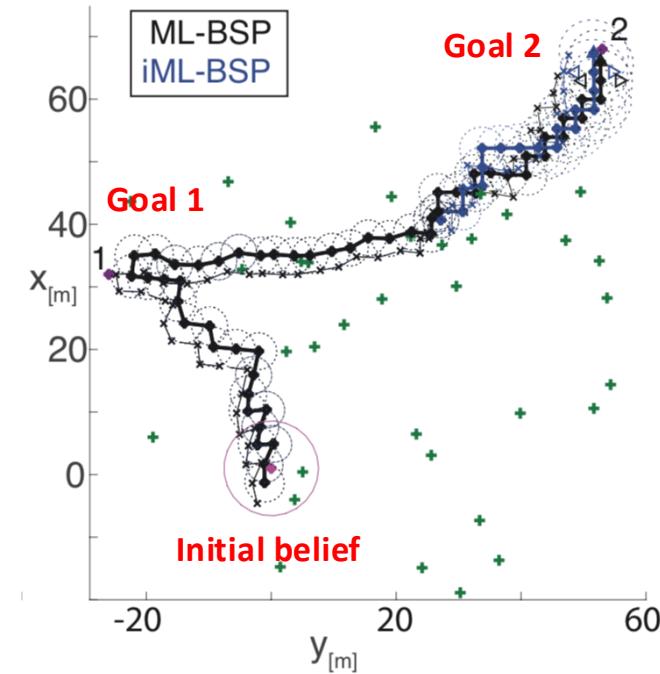
- Key idea: multiple importance sampling (MIS) estimator



Incremental Belief Space Planning

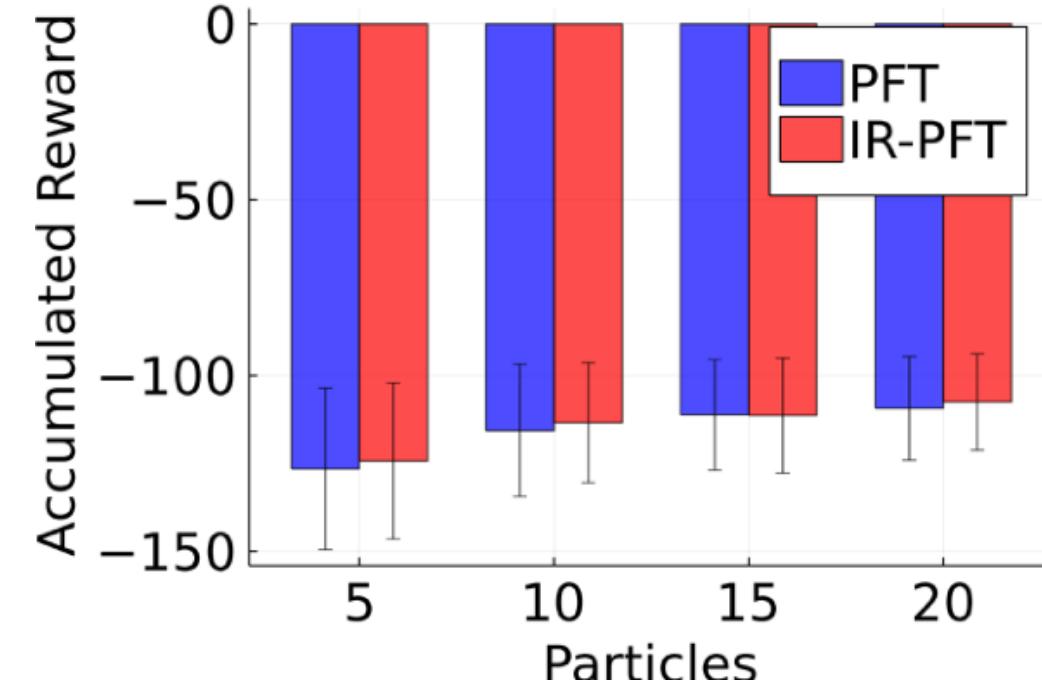
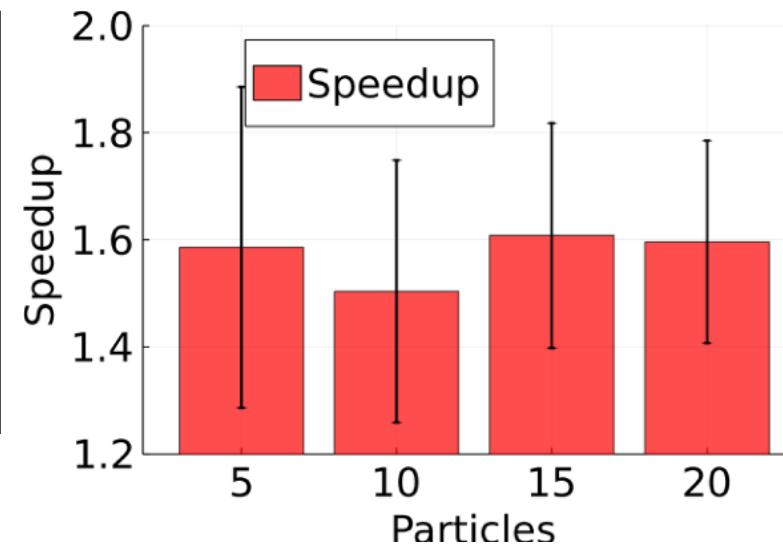
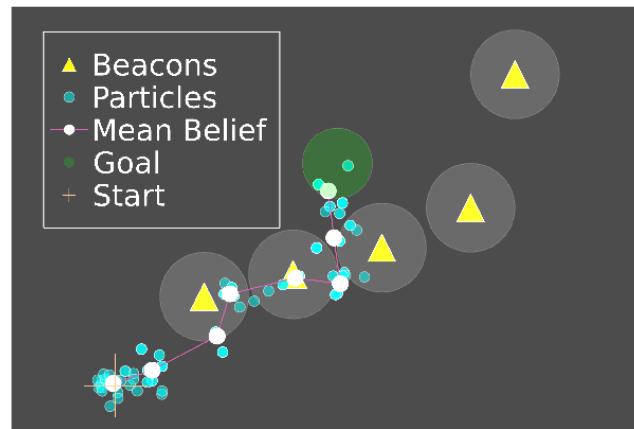
Basic simulation – autonomous navigation in unknown environments:

ML-BSP: BSP with ML observations
(one sample per look ahead step)



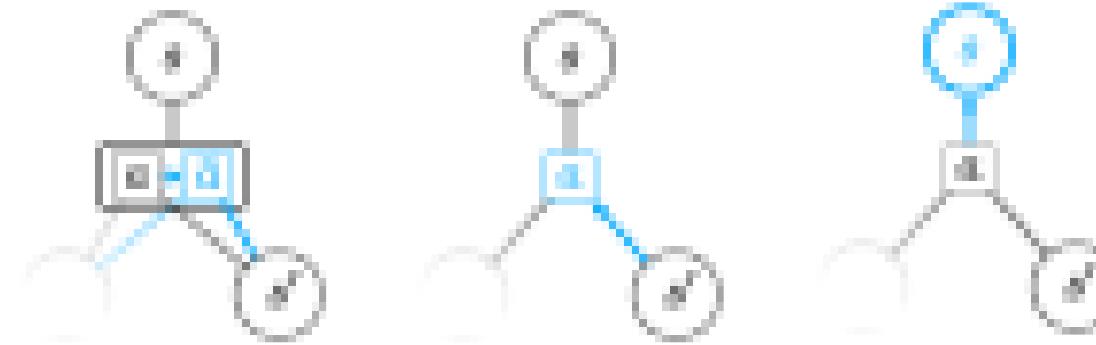
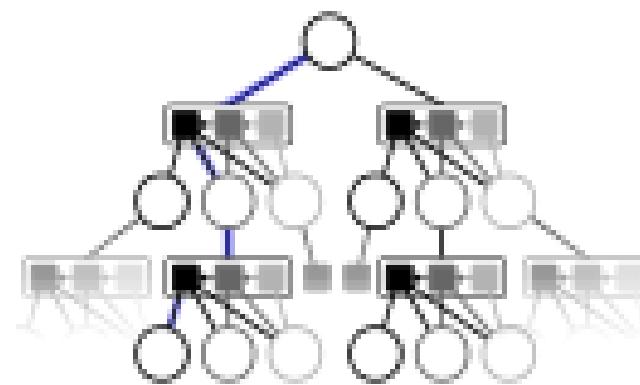
Incremental Reuse Particle Filter Tree (IR-PFT)

- Extend PFT-DPW¹, incorporating trajectories from previous planning sessions for fast estimation of $Q(b_k, a_k)$



¹Z. Sunberg and M. Kochenderfer. "Online algorithms for POMDPs with continuous state, action, and observation spaces." ICAPS, 2018.

Action-Gradient Monte Carlo Tree Search for Non-Parametric Continuous (PO)MDPs



Action update

Action backpropagation

State backpropagation

Agenda

Experience Reuse in POMDP Planning

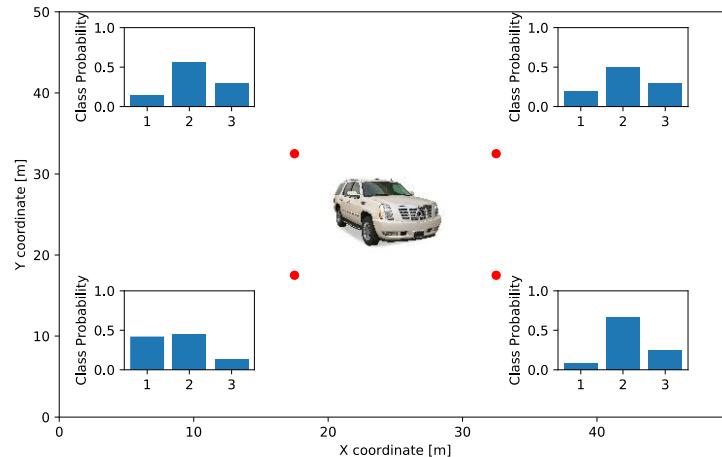
POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

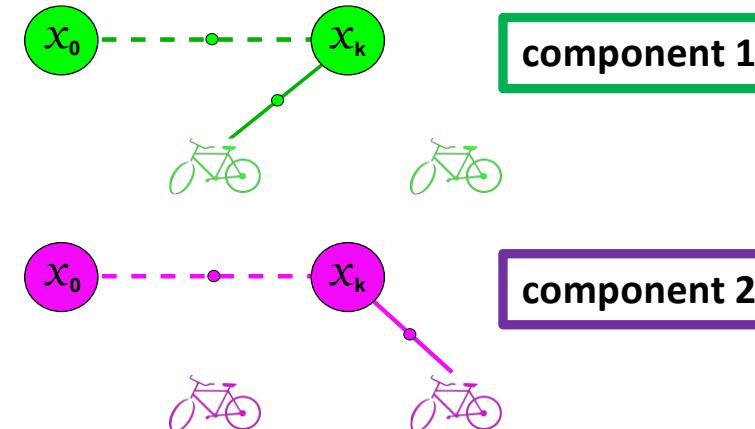
POMDP Planning with Hybrid Beliefs

Autonomous Semantic Perception



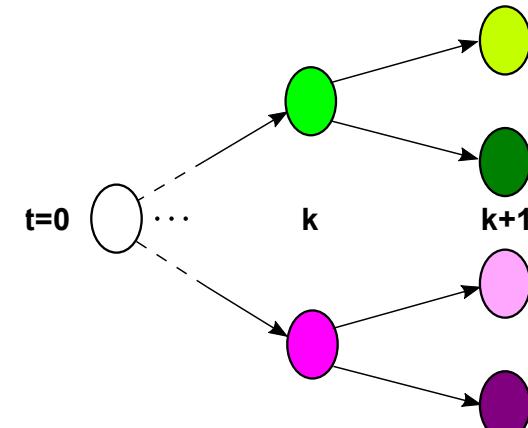
Viewpoint-dependent semantic models

Ambiguous Environments



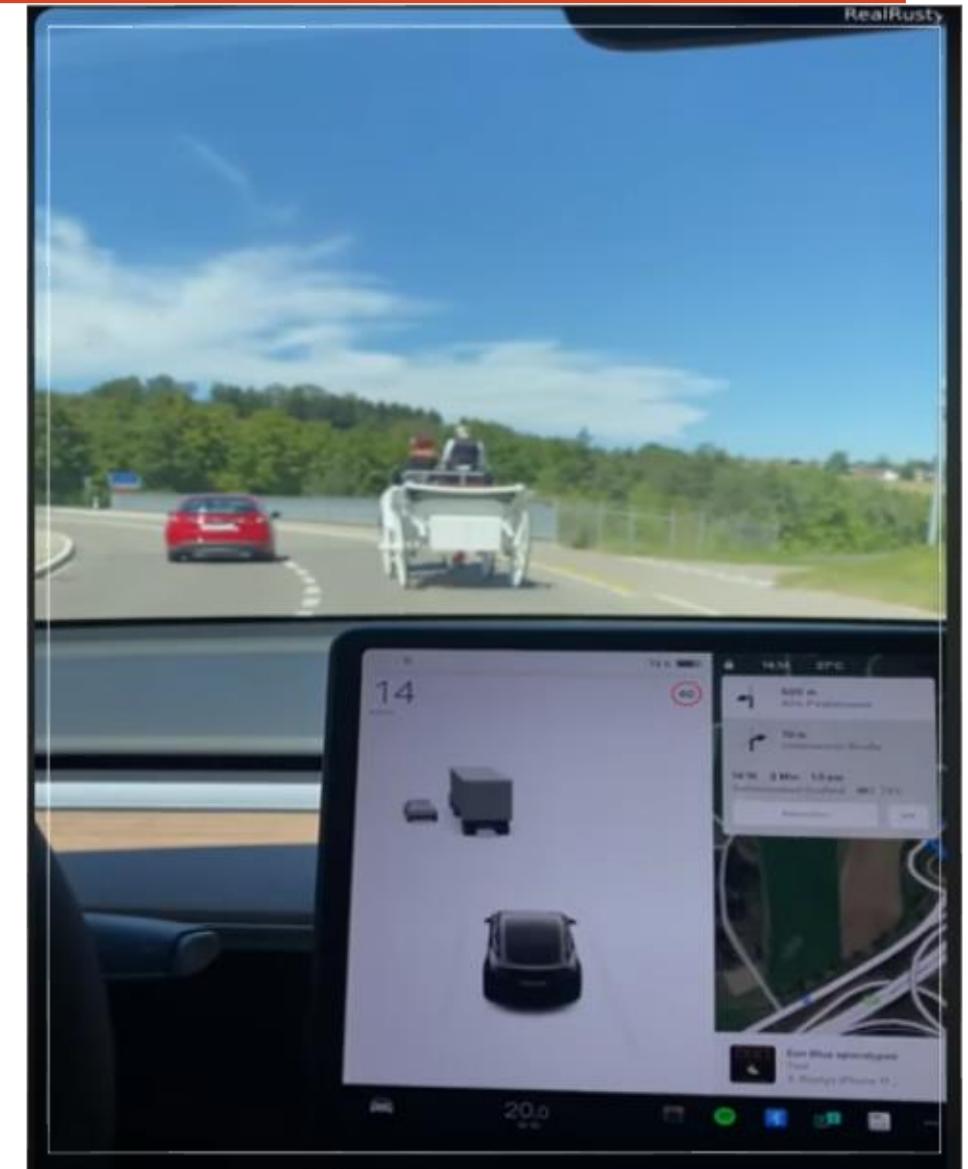
Data association hypotheses

- Hybrid beliefs (over continuous and discrete random variables)
- The number of hypotheses can grow exponentially
- How do we do probabilistic inference and POMDP planning?



Semantic Perception & SLAM

- Usually, semantics and geometry are considered **separately**
- Cannot use coupled observation models or priors
- Can lead to absurd results



Class- and Viewpoint-Dependency

- Is it a floor or a roof?
- Depending on the viewpoint of the viewer!
 - Looking on the people below - it's a floor
 - Looking on the people above - it's a roof
- How do we know the viewpoint?

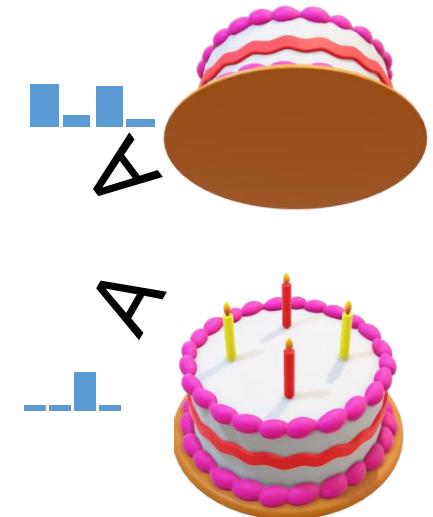
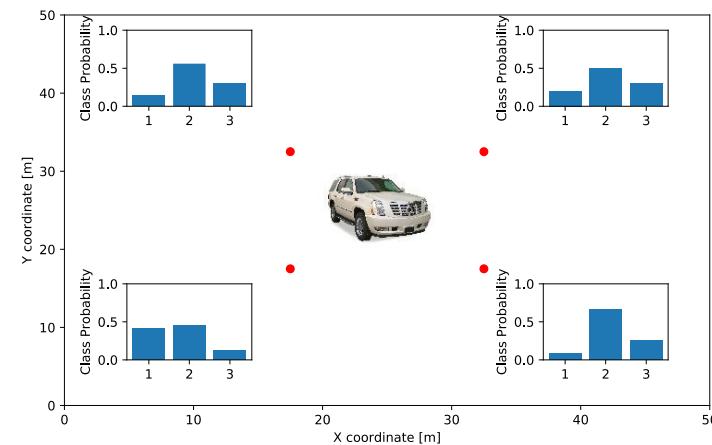


Class- and Viewpoint-Dependency

- View-dependent semantic observation model:

$$\mathbb{P}(z^s \mid c, \mathcal{X}^{rel})$$

Object class
 Semantic observation
 (from a classifier) Agent's viewpoint
 relative to object



- Classes and poses can be coupled via learned prior probabilities.
- Reward/constraint can depend on both classes and poses (e.g., object search, search & rescue)

Y. Feldman and V. Indelman, "Bayesian Viewpoint-Dependent Robust Classification under Model and Localization Uncertainty," ICRA'18, AURO'20.

V. Tchuiiev, Y. Feldman, and V. Indelman, "Data Association Aware Semantic Mapping and Localization via a Viewpoint Dependent Classifier Model," IROS'19.

V. Tchuiiev and V. Indelman, "Epistemic Uncertainty Aware Semantic Localization and Mapping for Inference and Belief Space Planning," Artificial Intelligence, 2023.

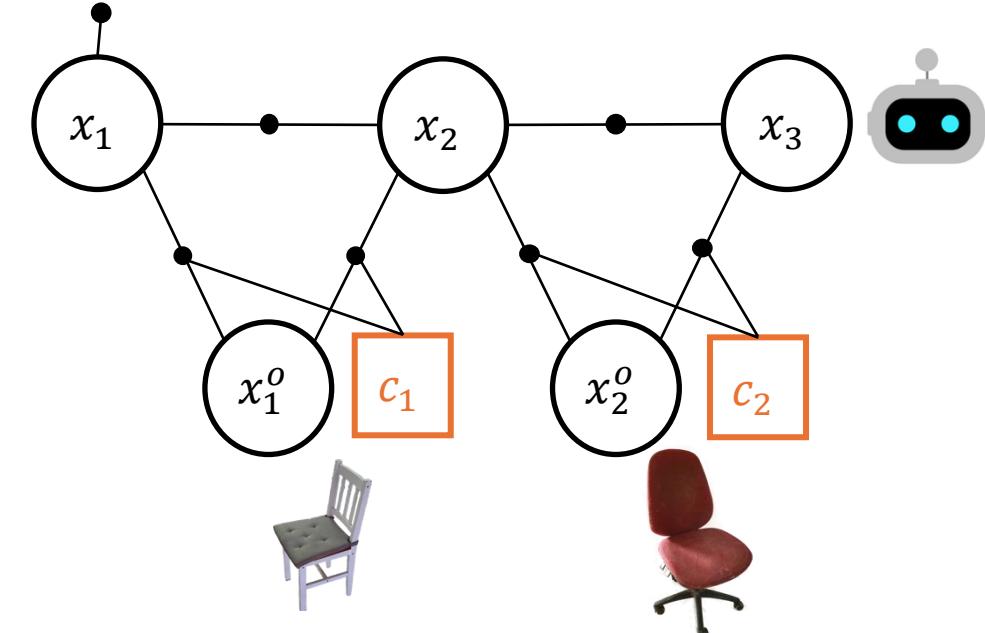
T. Lemberg and V. Indelman, "Online Hybrid-Belief POMDP with Coupled Semantic-Geometric Models and Semantic Safety Awareness", arXiv'25.

Hybrid Belief

- **Hybrid Belief** at time instant k:

$$b[X_k, C] = \mathbb{P}(X_k, C \mid \mathcal{H}_k)$$

Robot's and objects' poses Objects' classes History (actions, geometric & semantic observations)



- Classes and agent poses are dependent
- Classes of different objects are dependent

Y. Feldman and V. Indelman, "Bayesian Viewpoint-Dependent Robust Classification under Model and Localization Uncertainty," ICRA'18 , AURO'20.

V. Tchuiiev, Y. Feldman, and V. Indelman, "Data Association Aware Semantic Mapping and Localization via a Viewpoint Dependent Classifier Model," IROS'19.

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POMDP Planning with Hybrid Semantic-Geometric Beliefs

- Value function

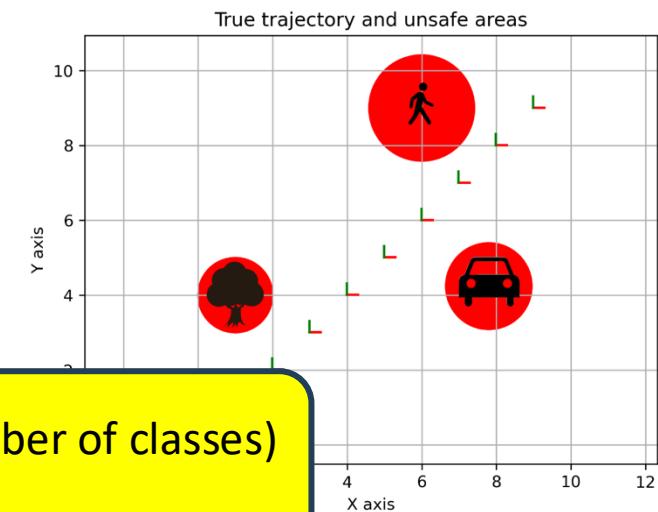
$$V^\pi(b_k) = \mathbb{E}_{z_{k+1:k+L}} \left[\sum_{l=k}^{k+L-1} \rho(b_l, \pi_l(b_l), b_{l+1}) \right]$$

- Semantic Risk Awareness

$$\mathbb{P}_{safe} \triangleq \mathbb{P}(\{\wedge_{t=k+1}^L x_t \notin \mathcal{X}_{unsafe}(C, X^o)\} \mid b_k[x_k, C, X^o], \pi)$$

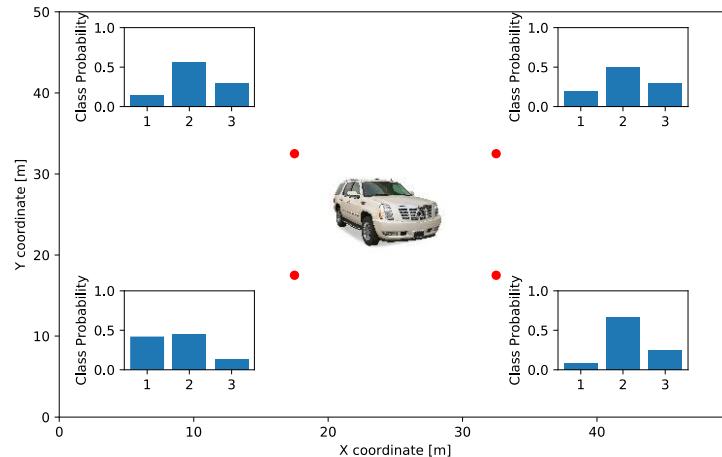
Objects' classes Objects' poses

The number of classification hypotheses is M^N (N: number of objects, M: number of classes)
 How to sample w/o pruning hypotheses? How to estimate \mathbb{P}_{safe} ?



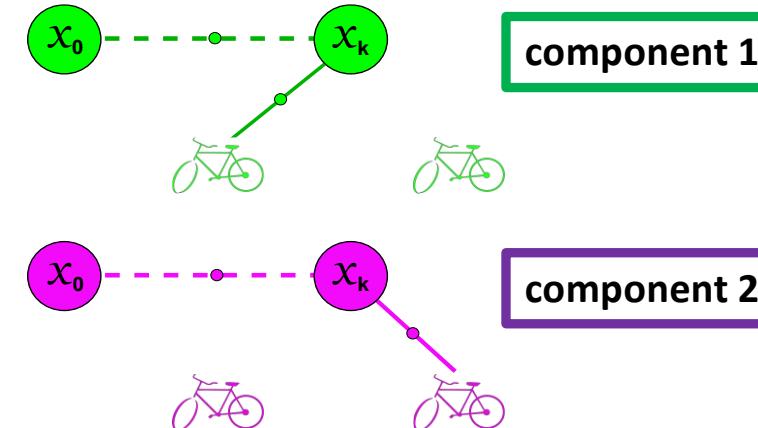
POMDP Planning with Hybrid Beliefs

Autonomous Semantic Perception



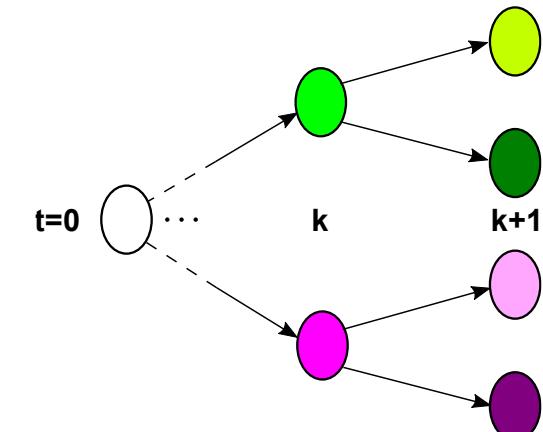
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- How do we do probabilistic inference and POMDP planning?

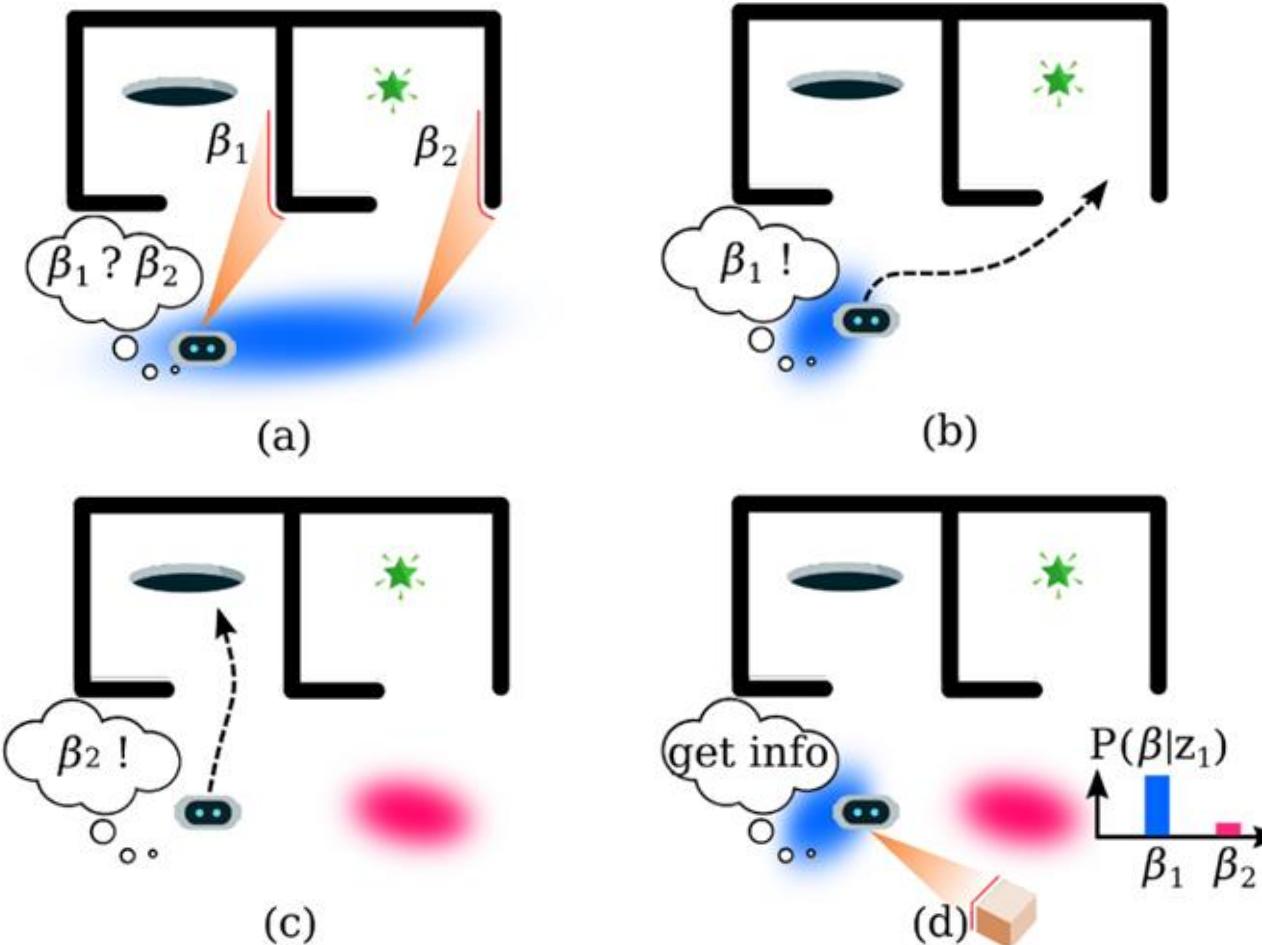


Ambiguous Scenarios

- Have to reason about data association hypotheses within inference and planning

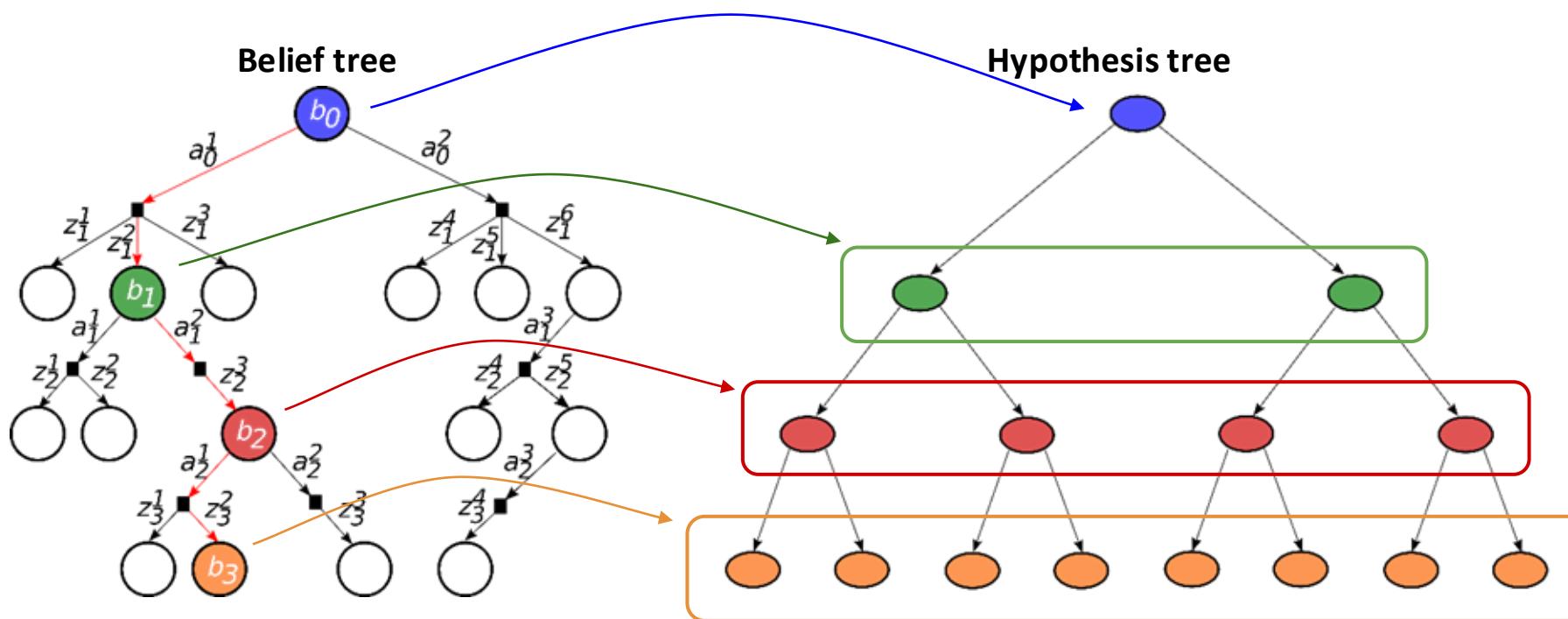
An observation:
(e.g. LIDAR)

How should the agent act?



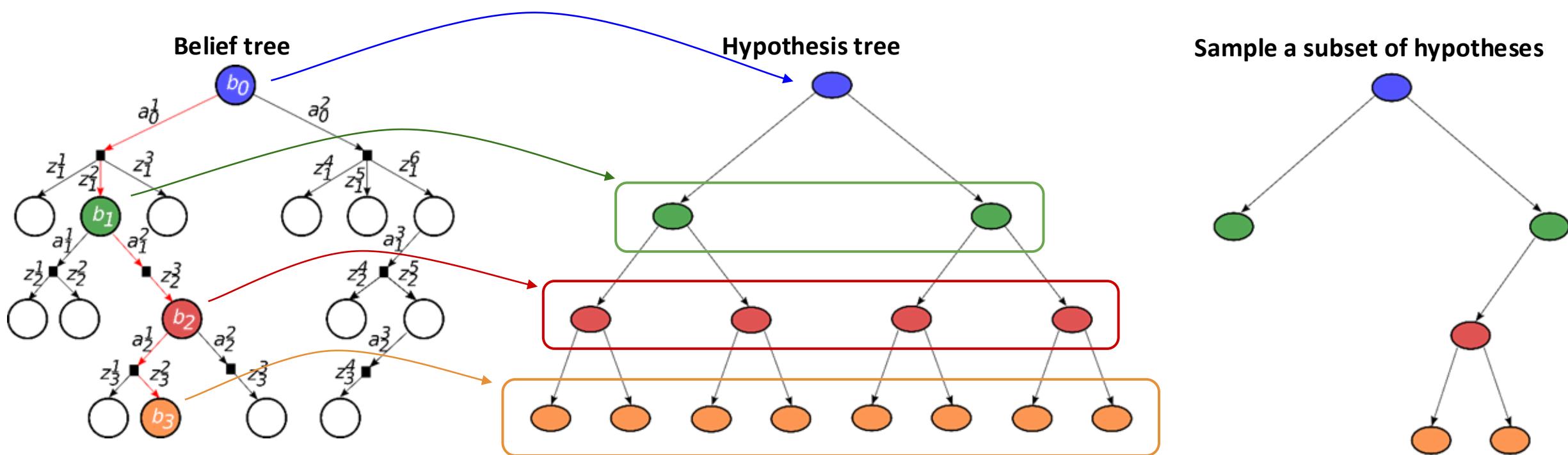
Continuous-Discrete State Spaces

- The number of hypotheses may grow **exponentially** with the planning horizon!



Continuous-Discrete State Spaces

- The number of hypotheses may grow **exponentially** with the planning horizon!



Impact on decision making?

Agenda

Experience Reuse in POMDP Planning

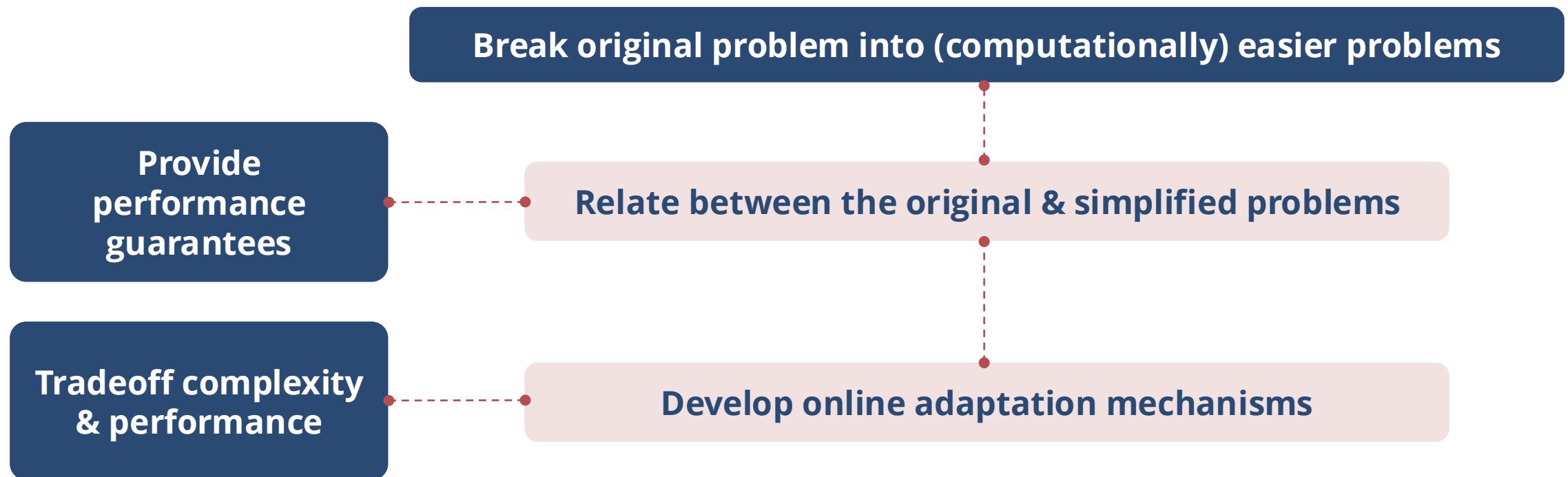
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Simplification Framework

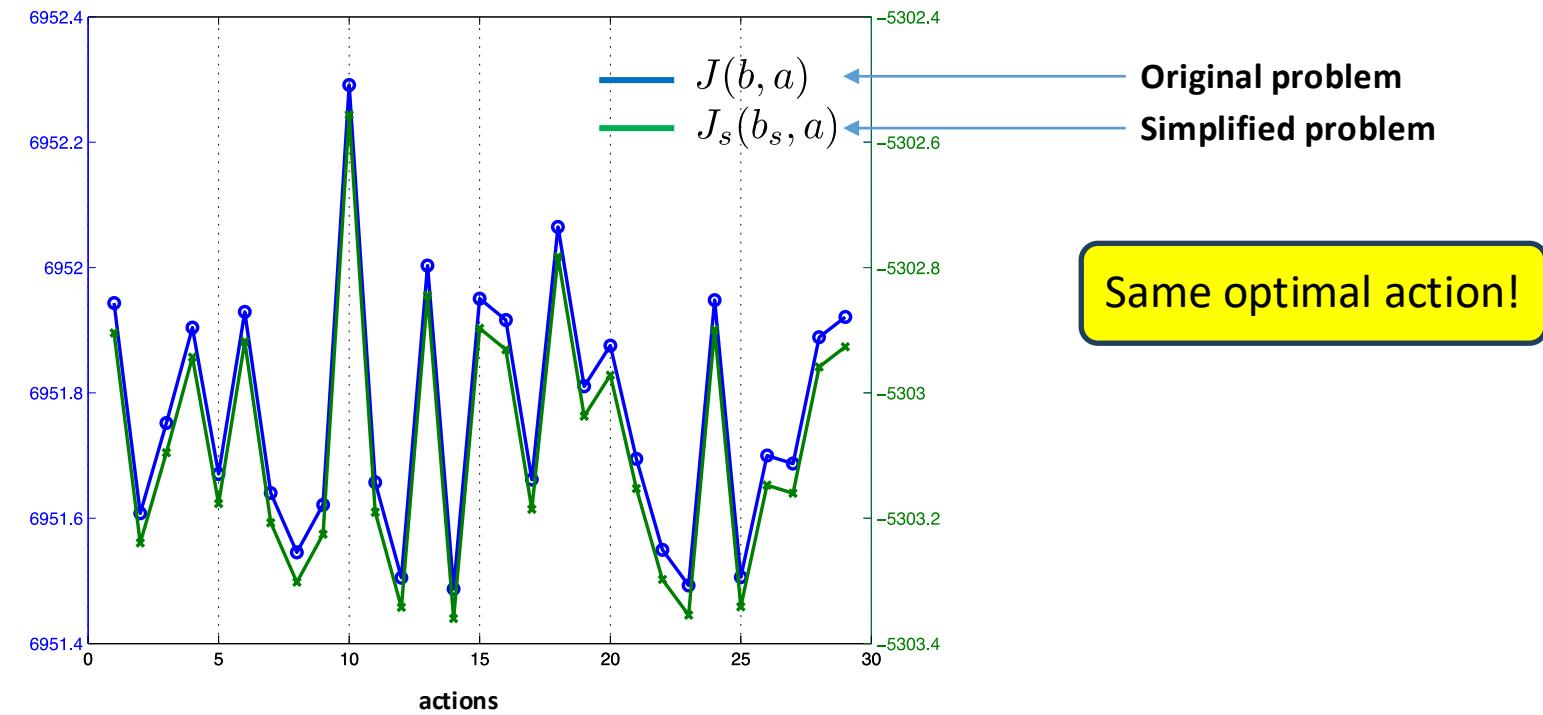
Accelerate decision making by adaptive simplification while providing performance guarantees



[Indelman RAL'16; Elimelech & Indelman IJRR'22; Szyglic & Indelman IROS'22; Zhitnikov & Indelman AIJ'22, TRO'24; Shienman & Indelman ICRA'22; Barenboim & Indelman NIPS'23; Kitanov & Indelman IJRR'24; Zhitnikov et al. IJRR'24; Lev-Yehudi, Barenboim & Indelman AAAI'24, Yotam & Indelman TRO'24, Da & Indelman ISRR'24]

Simplification of Decision-Making Problems

- Each element of the decision-making problem can be simplified
- **Action-consistent simplification preserves order** between actions w.r.t. original problem



Simplification of Decision-Making Problems

$$\mathcal{LB}(b, a) \leq Q(b, a) \leq \mathcal{UB}(b, a)$$

Computationally cheap(er)
bounds

Simplification of Decision-Making Problems

Concept:

- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

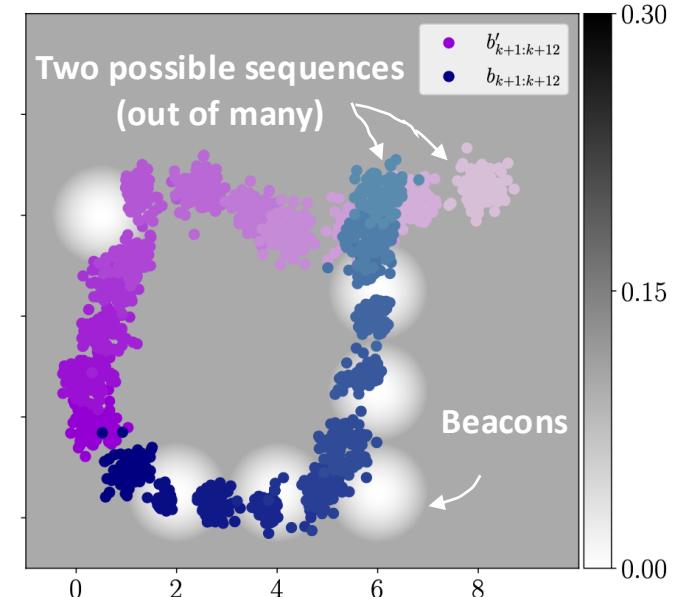
Specific simplifications include:

- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of policy space
- Simplification of Risk-Averse & Robust Planning
- Simplification in a multi-agent setting

Simplification of POMDPs with Nonparametric Beliefs

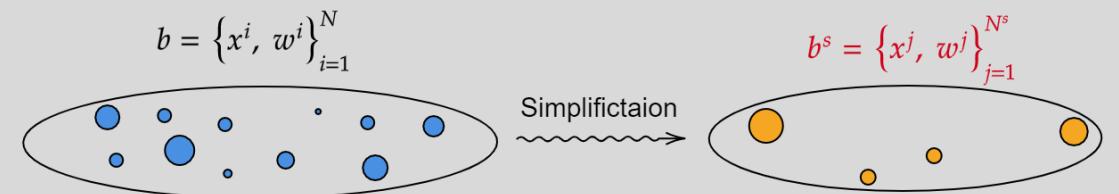
- Value function

$$V^\pi(b_k) = \mathbb{E}_{z_{k+1:k+L}} \left[\sum_{\ell=0}^L \rho(b_{k+\ell}, \pi_{k+\ell}(b_{k+\ell})) \right]$$



Simplification:

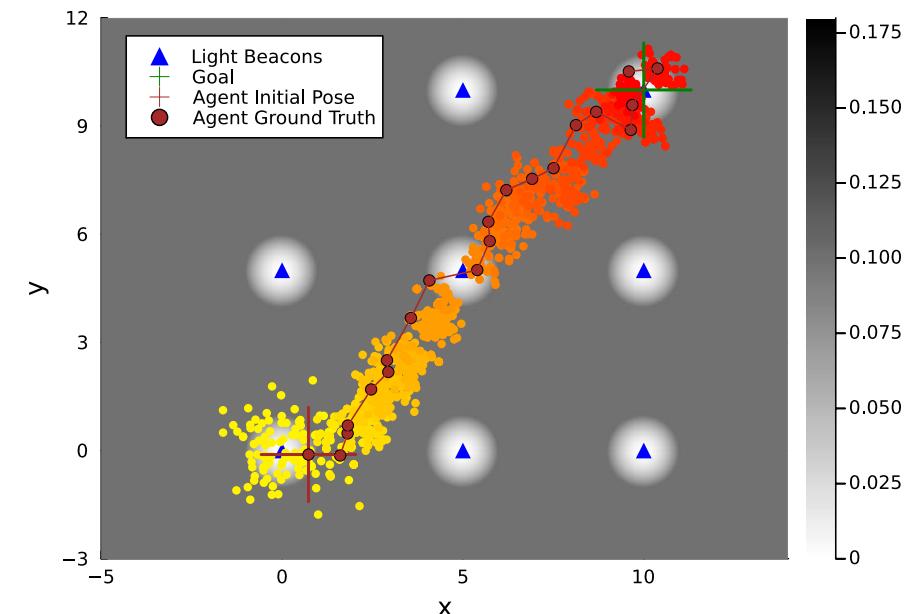
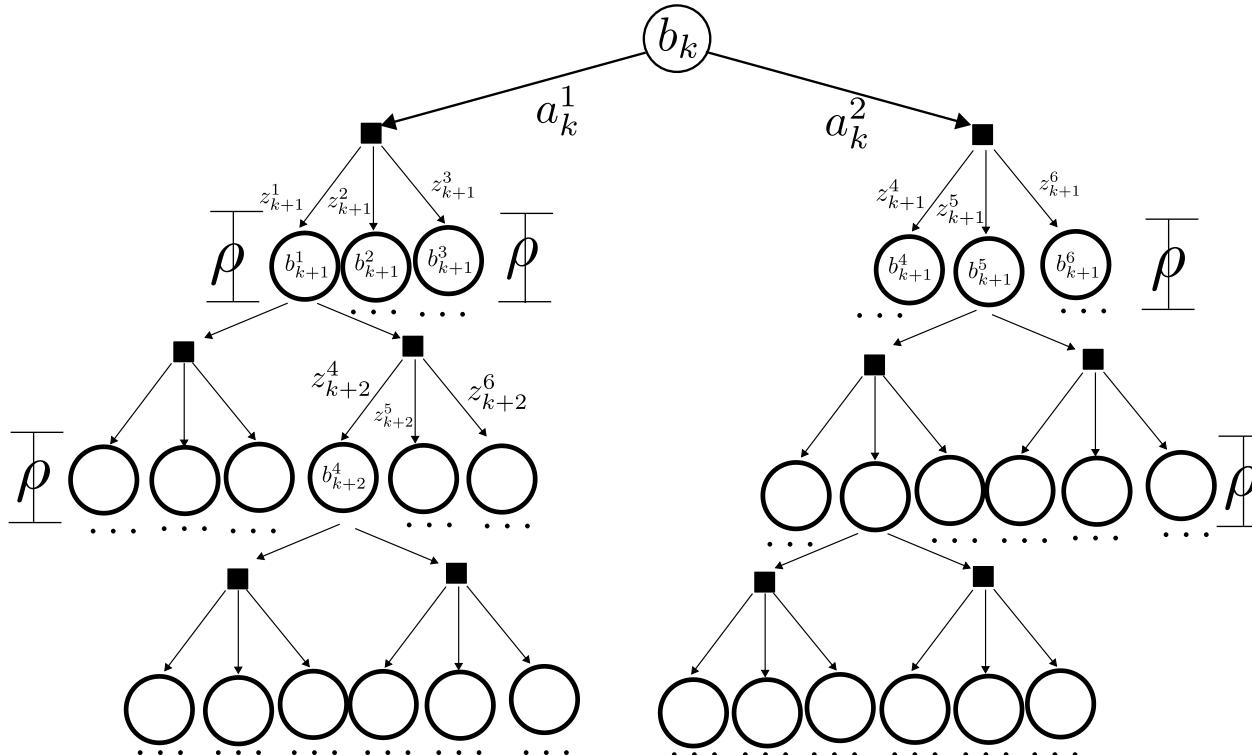
- Utilize a **subset** of samples for planning
- Information-theoretic reward (entropy)
- Analytical (**cheaper**) bounds over the reward



$$lb(b, b^s, a) \leq \rho(b, a) \leq ub(b, b^s, a)$$

Simplification of POMDPs with Nonparametric Beliefs

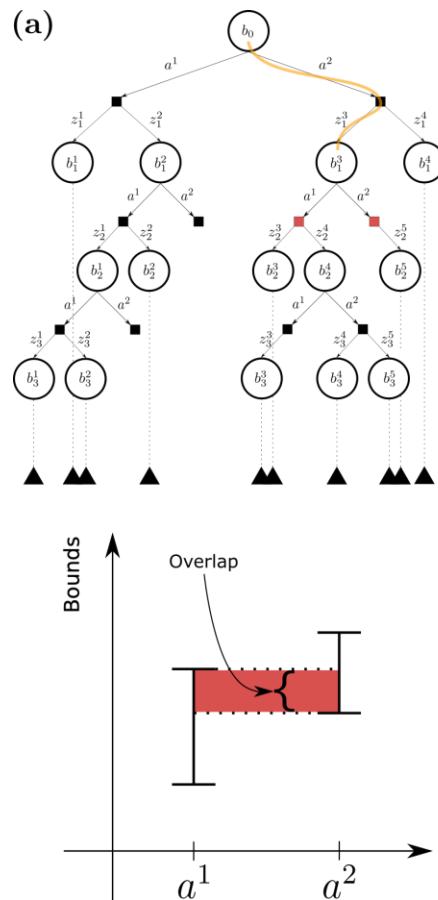
- Adaptive multi-level simplification in a Sparse Sampling setting:



Typical speedup of 20% - 50%,
Same performance!

Simplification of POMDPs with Nonparametric Beliefs

- Adaptive multi-level simplification in an MCTS setting:



Simplification of Decision-Making Problems

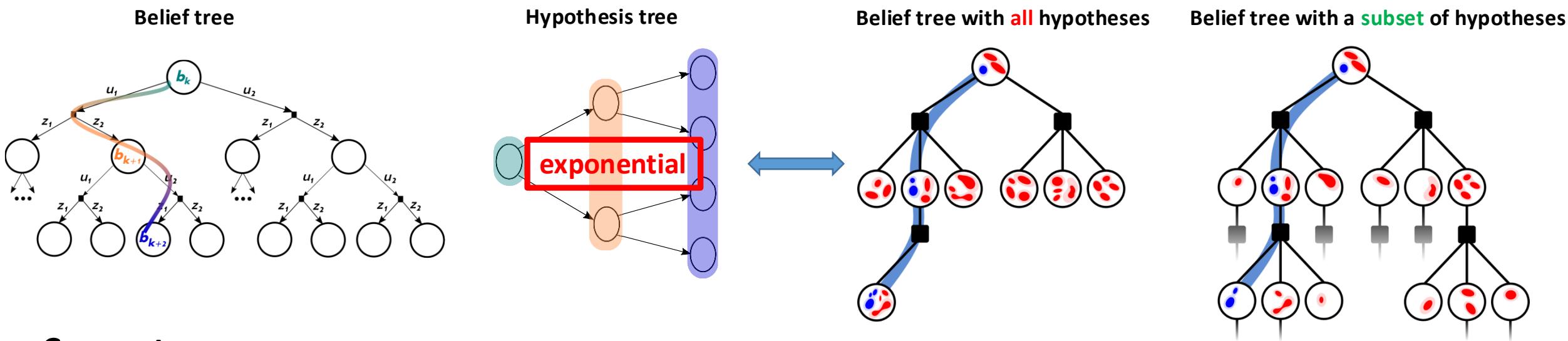
Concept:

- Identify and solve a **simplified (computationally) easier** decision-making problem
- Provide performance guarantees

Specific simplifications include:

- Sparsification of Gaussian beliefs (high dim. state)
- Topological metric for Gaussian beliefs (high dim. state)
- Utilize a subset of samples (nonparametric beliefs)
- Utilize a subset of hypotheses (hybrid beliefs)
- Simplified models and spaces
- Simplification of policy space
- Simplification of Risk-Averse & Robust Planning
- Simplification in a multi-agent setting

Simplification of BSP/POMDP with Hybrid Beliefs



Concept:

- Instead, utilize only a **subset** of hypotheses
- Derive reward bounds, given planning task (reward)
 - Disambiguate between hypotheses
 - Navigate to a goal
 - ..

$$\mathcal{LB}(b_k, \pi) \leq V^\pi(b_k) \leq \mathcal{UB}(b_k, \pi)$$

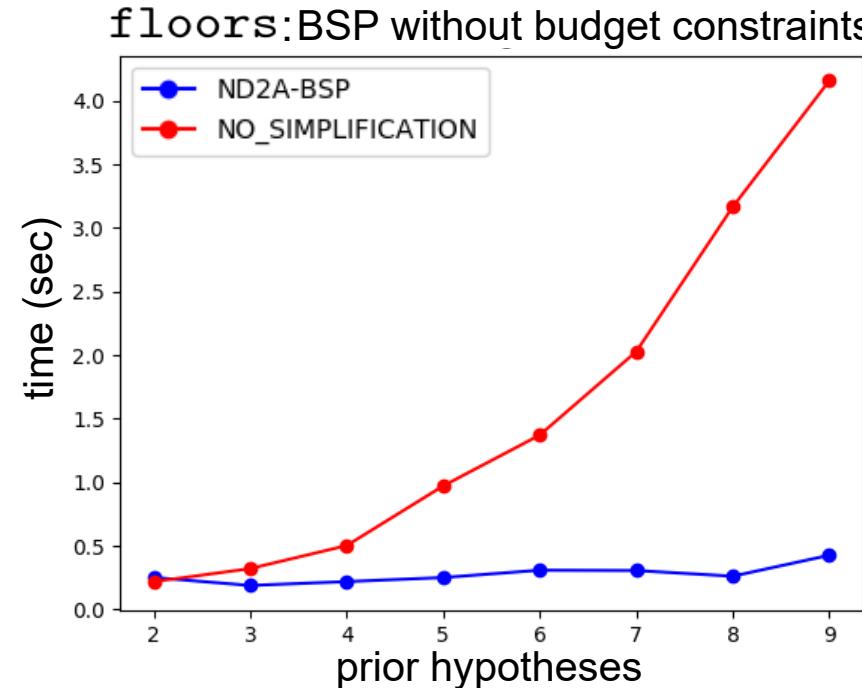
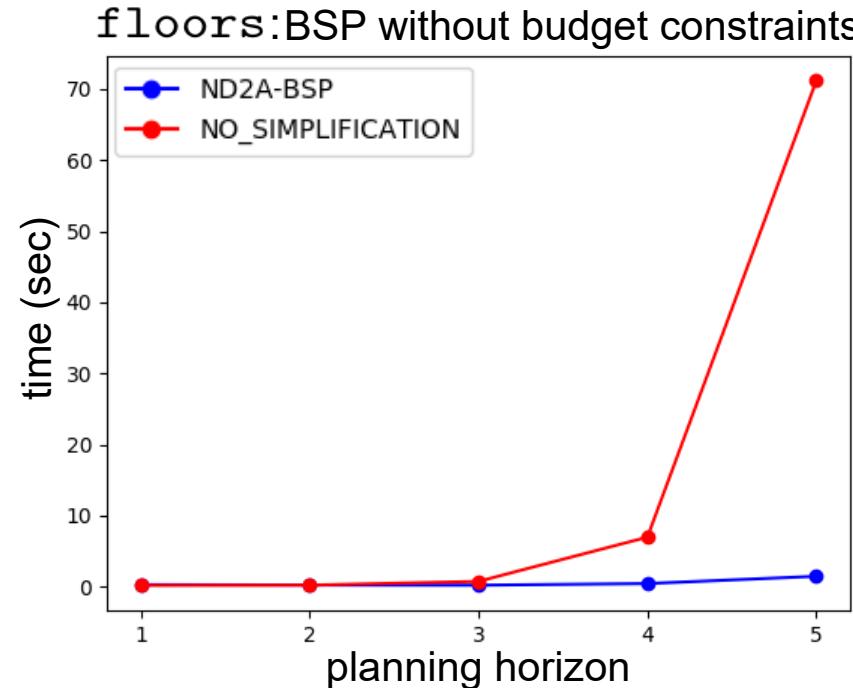
M. Shienman and V. Indelman, "D2A-BSP: Distilled Data Association Belief Space Planning with Performance Guarantees Under Budget Constraints," ICRA'22, **Outstanding Paper Award Finalist**.

M. Shienman and V. Indelman, "Nonmyopic Distilled Data Association Belief Space Planning Under Budget Constraints," ISRR'22.

M. Barenboim, M. Shienman, and V. Indelman, "Monte Carlo Planning in Hybrid Belief POMDPs," IEEE RA-L'23.

M. Barenboim, I. Lev-Yehudi, and V. Indelman, "Data Association Aware POMDP Planning with Hypothesis Pruning Performance Guarantees," IEEE RA-L'23.

Simplification of BSP/POMDP with Hybrid Beliefs



- Significant speed-up in planning
- Same planning performance is **guaranteed** (no overlap between bounds)

Simplification of BSP/POMDP with Hybrid Beliefs

- Derived a deterministic bound to relate the full set of hypotheses to a subset thereof,

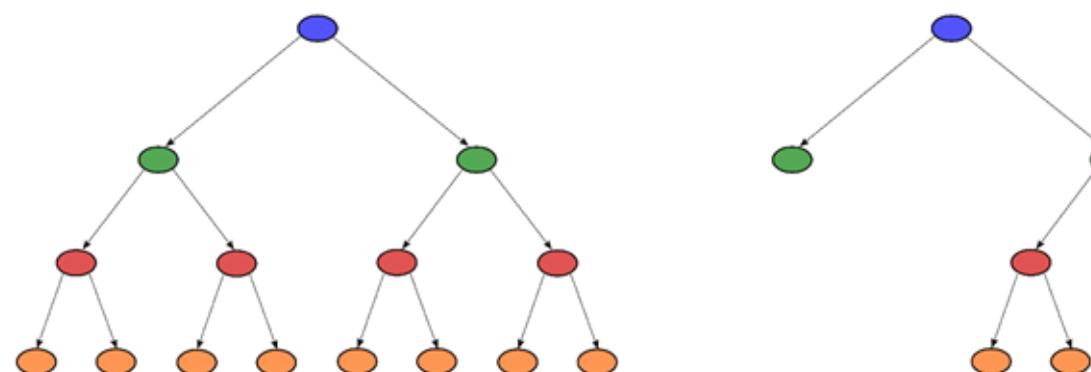
Corollary

For any policy π , and selection of hypotheses set $\{\beta_{0:\tau}^i\}_{i=0}^{|\mathcal{B}|}$ the following holds,

$$|V^\pi(b_0) - \bar{V}^\pi(\bar{b}_0)| \leq \mathcal{R}_{max} \left[\mathcal{T} \delta_0^\beta + \sum_{k=1}^{\mathcal{T}} \sum_{\tau=1}^k \mathbb{E}_{z_{1:\tau}} [\delta_\tau^\beta] \right].$$

Full tree

Any subset



Importantly, the bound relies on the available hypotheses

Can bound the theoretical value with access only to the simplified tree

Bounds can be evaluated online

Simplification of Decision-Making Problems

Concept:

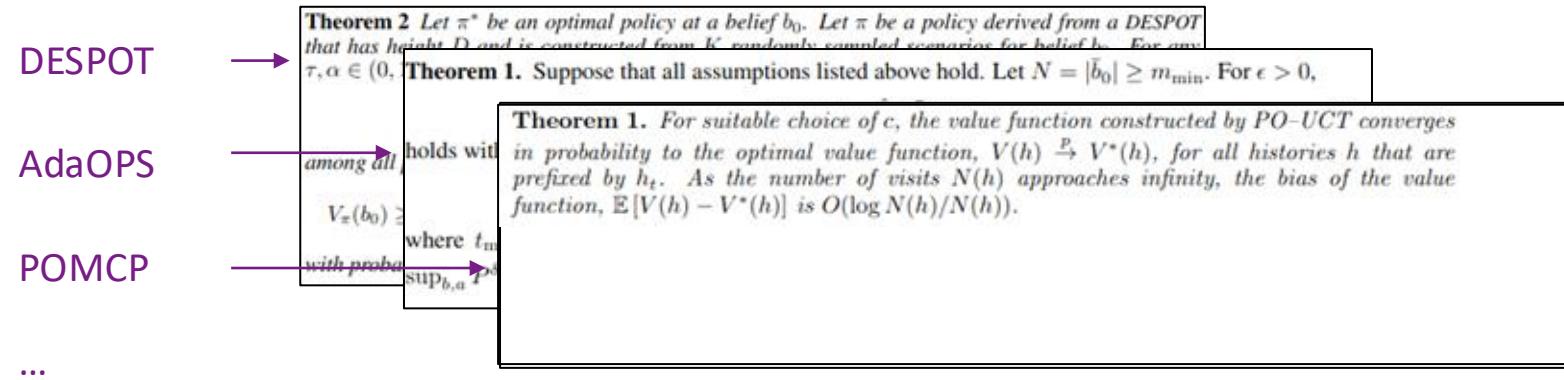
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POMDPs with Deterministic Guarantees

SOTA sampling based approaches come with probabilistic theoretical guarantees



Can we get deterministic guarantees?

We show that deterministic guarantees are indeed possible!

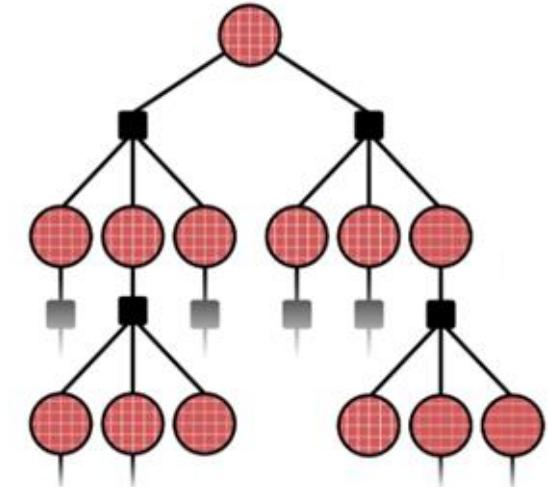
Online POMDP Planning with Anytime Deterministic Guarantees

Concept:

Instead of solving the original POMDP, consider a simplified version of that POMDP.

$$\mathcal{M} \xrightarrow{\text{wavy arrow}} \bar{\mathcal{M}}$$

Derive a mathematical relationship between the solution of the simplified, and the theoretical POMDP.

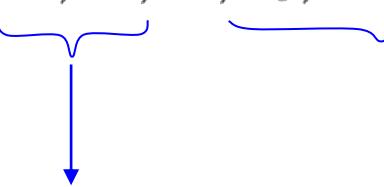


$$V(b) \xleftarrow{\text{wavy arrow}} ? \xrightarrow{\text{wavy arrow}} \bar{V}(b)$$

Online POMDP Planning with Anytime Deterministic Guarantees

- Given a POMDP: $\mathcal{M} = \langle \mathcal{X}, \mathcal{Z}, \mathcal{A}, b_0, \mathcal{P}_T, \mathcal{P}_Z, \rho, \gamma \rangle$
- Define a **simplified** POMDP,

$$\bar{\mathcal{M}} = \langle \bar{\mathcal{X}}, \bar{\mathcal{Z}}, \mathcal{A}, \bar{b}_0, \bar{\mathcal{P}}_T, \bar{\mathcal{P}}_Z, \rho, \gamma \rangle$$



 $\bar{\mathcal{X}}(H_t) \subset \mathcal{X}$ $\bar{b}_0(x) \triangleq \begin{cases} b_0(x) & , x \in \bar{\mathcal{X}}_0 \\ 0 & , otherwise \end{cases}$

 $\bar{\mathcal{Z}}(H_t) \subset \mathcal{Z}$ $\bar{\mathbb{P}}(x_{t+1} | x_t, a_t) \triangleq \begin{cases} \mathbb{P}(x_{t+1} | x_t, a_t) & , x_{t+1} \in \bar{\mathcal{X}}(H_{t+1}^-) \\ 0 & , otherwise \end{cases}$

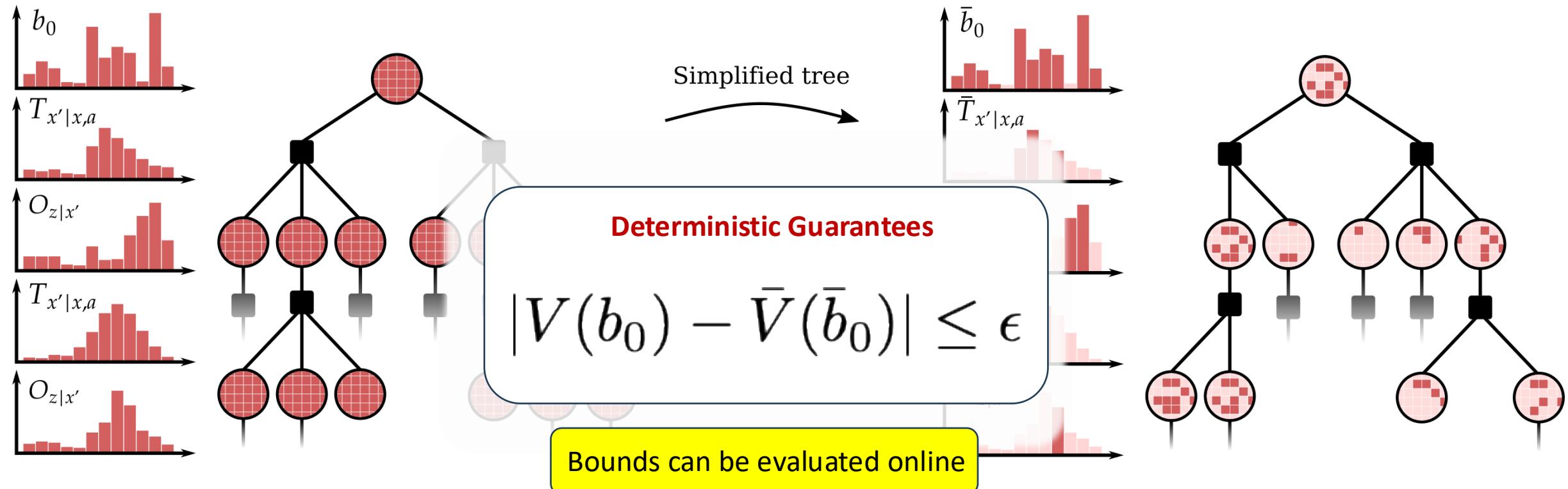
 $\bar{\mathbb{P}}(z_t | x_t) \triangleq \begin{cases} \mathbb{P}(z_t | x_t) & , z_t \in \bar{\mathcal{Z}}(H_t) \\ 0 & , otherwise \end{cases}$

- Simplified value function

$$\bar{V}^\pi(\bar{b}_t) \triangleq r(\bar{b}_t, \pi_t) + \bar{\mathbb{E}}_{z_{t+1:T}} [\bar{V}^\pi(\bar{b}_{t+1})]$$

Online POMDP Planning with Anytime Deterministic Guarantees

- Deterministic guarantees (assuming discrete spaces)



Online POMDP Planning with Anytime Deterministic Guarantees

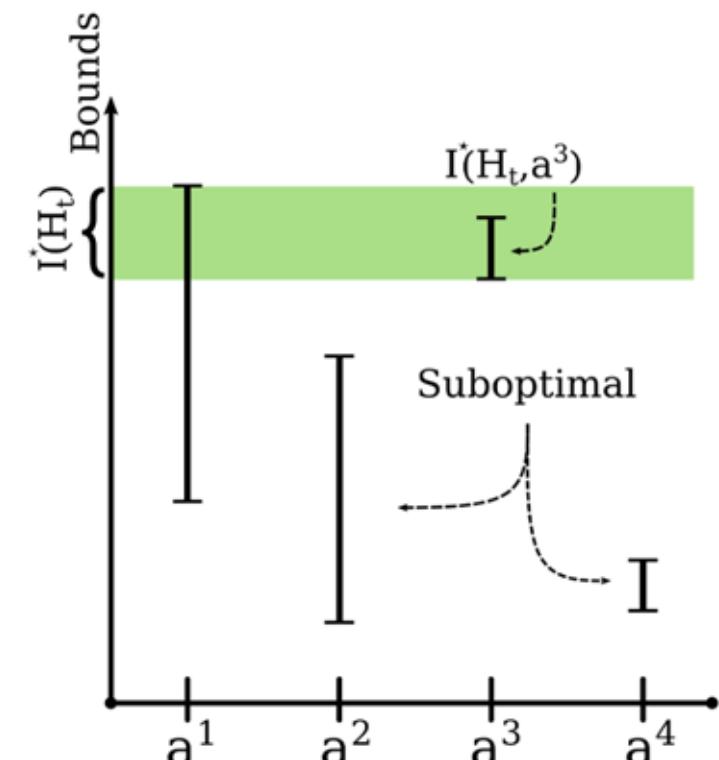
Importantly, the bounds can be calculated during planning.

How can we use them?

- **Pruning of sub-optimal branches**
 - Made possible by the deterministic guarantees
- **Stopping criteria for the planning phase**
 - Made possible by the deterministic guarantees
- **Finding the optimal solution in finite time**
 - Without recovering the theoretical tree

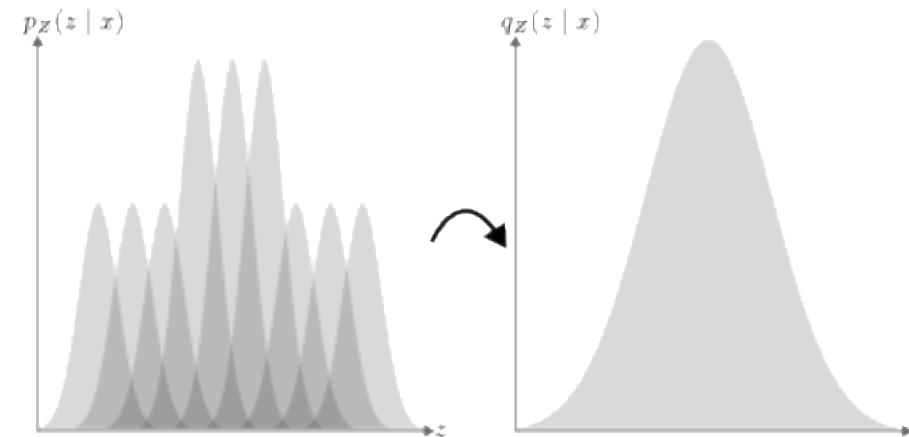
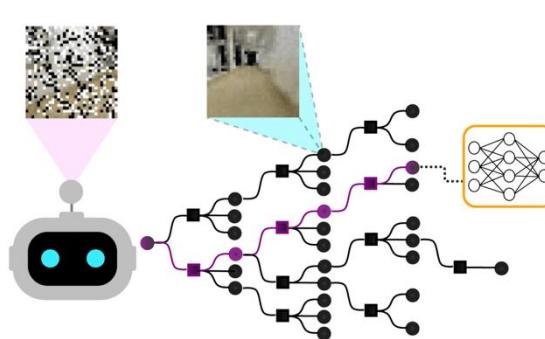
Deterministic Guarantees

$$|V(b_0) - \bar{V}(\bar{b}_0)| \leq \epsilon$$



Simplifying Complex Observation Models with Probabilistic Guarantees

- We replace the (learned) observation model p_Z with a cheaper model q_Z
 - Simpler GMM, Shallower Neural Network, etc.
 - Example:



Simplified models

$$p_\theta(z | x)$$

Original, **expensive**

$$q_\phi(z | x)$$

Simplified, **cheap**

Can we simplify the learned models?
What is the impact on planning performance?

Simplifying Complex Observation Models with Probabilistic Guarantees

- We replace the (learned) observation model p_Z with a cheaper model q_Z
- Simplified action-value function: $\hat{Q}_P^{q_Z}$

Corollary 3

For arbitrary $\varepsilon, \delta > 0$ there exists a number of particles for which

$$|Q_P^{p_Z}(b_t, a) - \hat{Q}_{M_P}^{q_Z}(\bar{b}_t, a)| \leq \hat{\Phi}_{M_P}(\bar{b}_t, a) + \varepsilon$$

with probability of at least $1 - \delta$ for any guaranteed planner

Theoretical Q function
of the POMDP, with
original models

Estimator of the Q function of a
particle-belief POMDP, with
simplified models

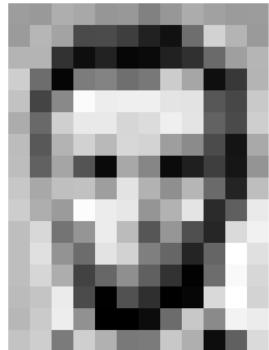
Partitioning of a Multivariate Observation Space

- Consider a **multivariate** random variable $Z \in \mathcal{Z}$, that represents future observations:

$$Z = (Z^1, Z^2, \dots, Z^m)$$

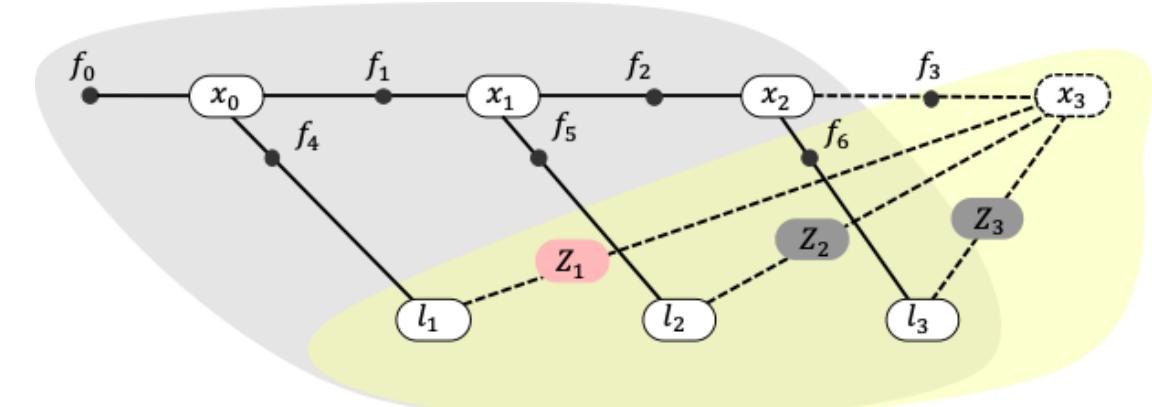
- Examples:

Raw measurement of an image sensor



157	153	174	168	150	152	129	151	172	161	155	156
155	182	163	74	55	62	93	17	110	210	180	154
180	180	50	14	54	6	10	33	48	106	159	181
204	109	5	124	131	111	120	204	168	19	66	180
194	68	157	251	237	239	228	228	227	67	71	201
172	106	207	233	238	214	220	239	228	98	74	206
188	68	179	209	185	215	211	158	139	75	20	169
180	97	164	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	171	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	85	150	79	38	218	241
196	224	147	108	227	210	127	103	36	101	255	224
190	214	173	66	103	143	95	50	2	109	249	216
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	198	243	236
195	206	123	207	177	121	123	200	175	13	95	218

Factor graph



Partitioning of a Multivariate Observation Space

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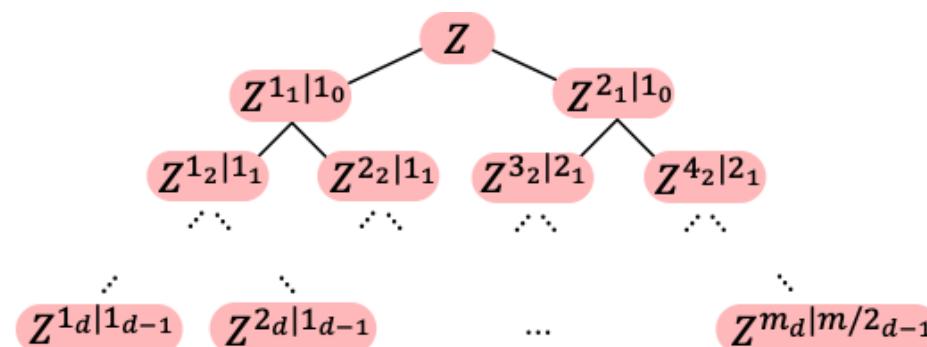
- We can partition $Z \in \mathcal{Z}$ into different subsets/components, e.g.

$$Z^s = \{Z^1, Z^2, \dots, Z^n\}$$

$$Z^{\bar{s}} = \{Z^{n+1}, Z^{n+2}, \dots, Z^m\}$$

$$Z = Z^s \cup Z^{\bar{s}}$$

- Hierarchical Partitioning:



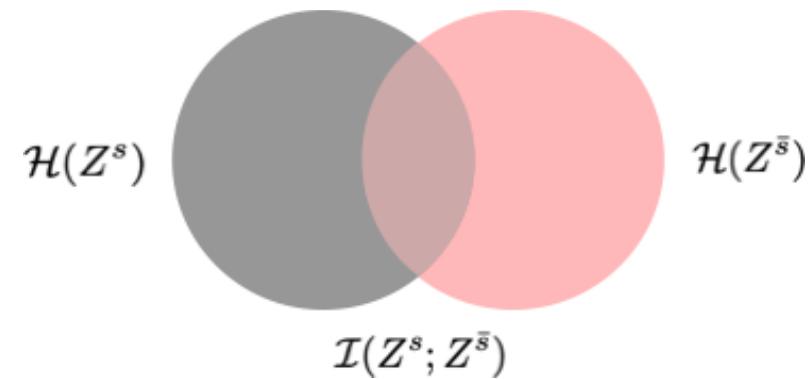
Partitioning of a Multivariate Observation Space

$$\mathcal{LB} \leq \mathcal{H}(X|Z) \leq \mathcal{UB}$$

$$\mathcal{LB} \triangleq \mathcal{H}(Z^s | X) + \mathcal{H}(Z^{\bar{s}}|X) - \mathcal{H}(Z^s) - \mathcal{H}(Z^{\bar{s}}) + \mathcal{H}(X)$$

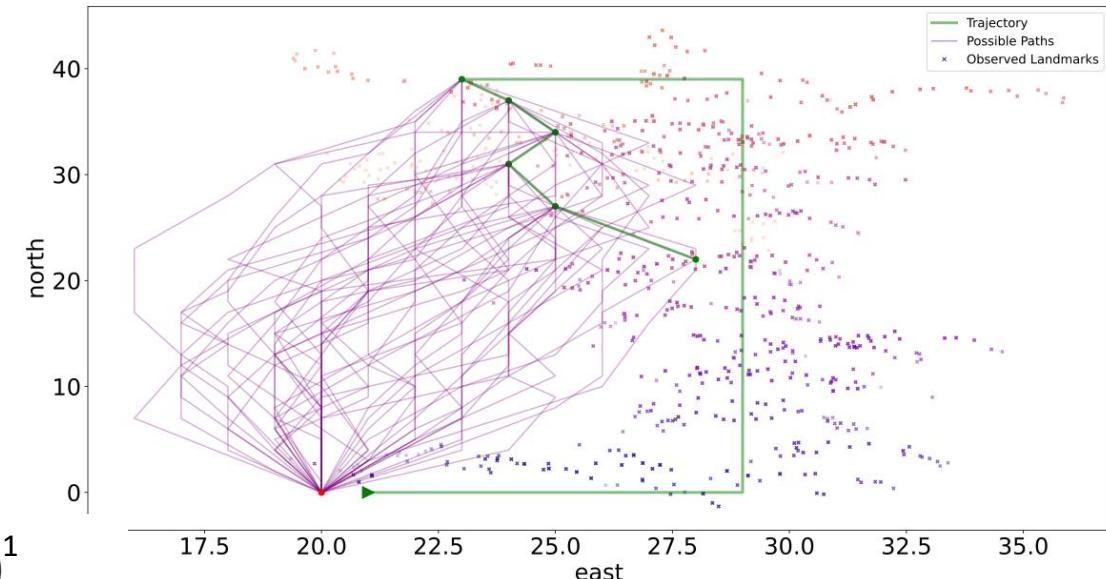
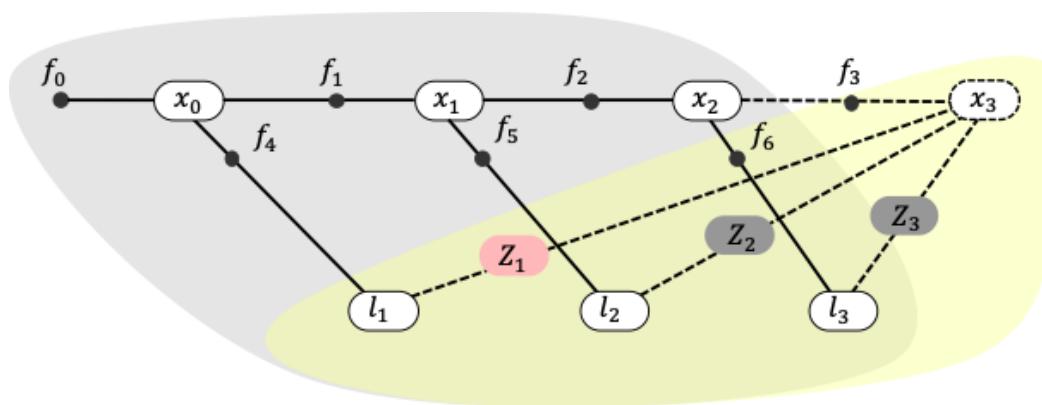
$$\mathcal{UB} \triangleq \mathcal{H}(Z^s|X) + \mathcal{H}(X) - \mathcal{H}(Z^s)$$

$$\mathcal{H}(Z^s, Z^{\bar{s}}) = \mathcal{H}(Z^s) + \mathcal{H}(Z^{\bar{s}}) - \mathcal{I}(Z^s; Z^{\bar{s}})$$



Partitioning of a Multivariate Observation Space

Application to Active SLAM



# Paths	# Factors	RP	rAMDL ²	MP (ours) ¹
100	2956	No	11.521 ± 0.537	6.888 ± 0.155
100	2956	Yes	24.636 ± 1.381	11.758 ± 0.372
100	5904	Yes	84.376 ± 14.458	32.069 ± 4.913

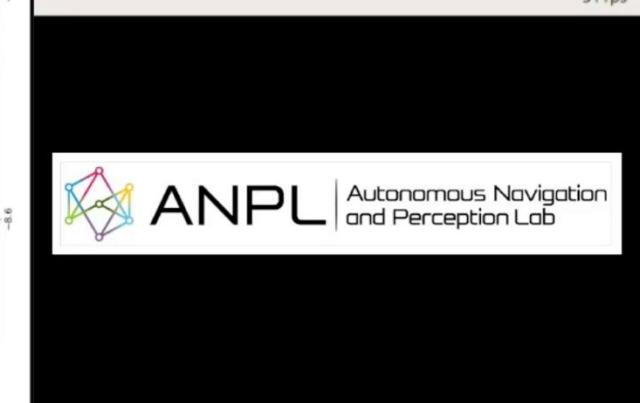
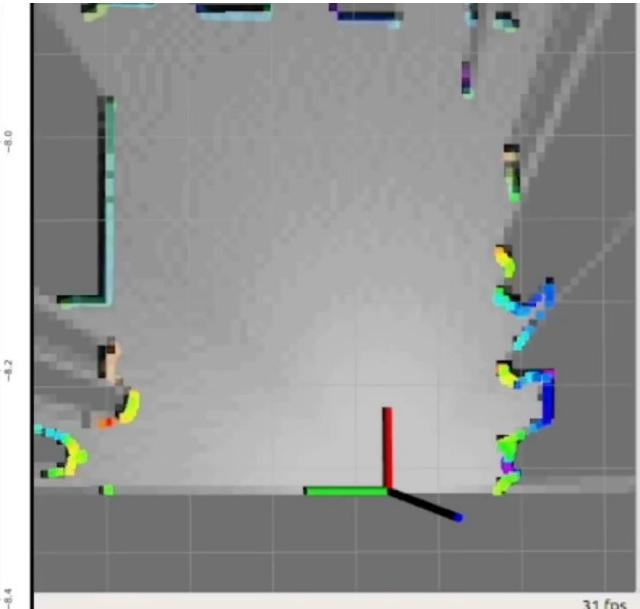
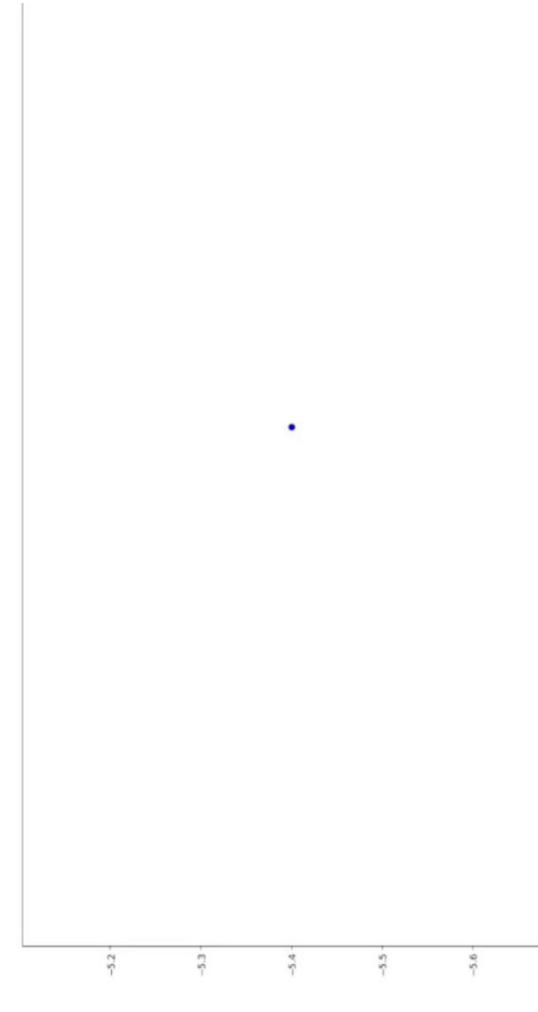
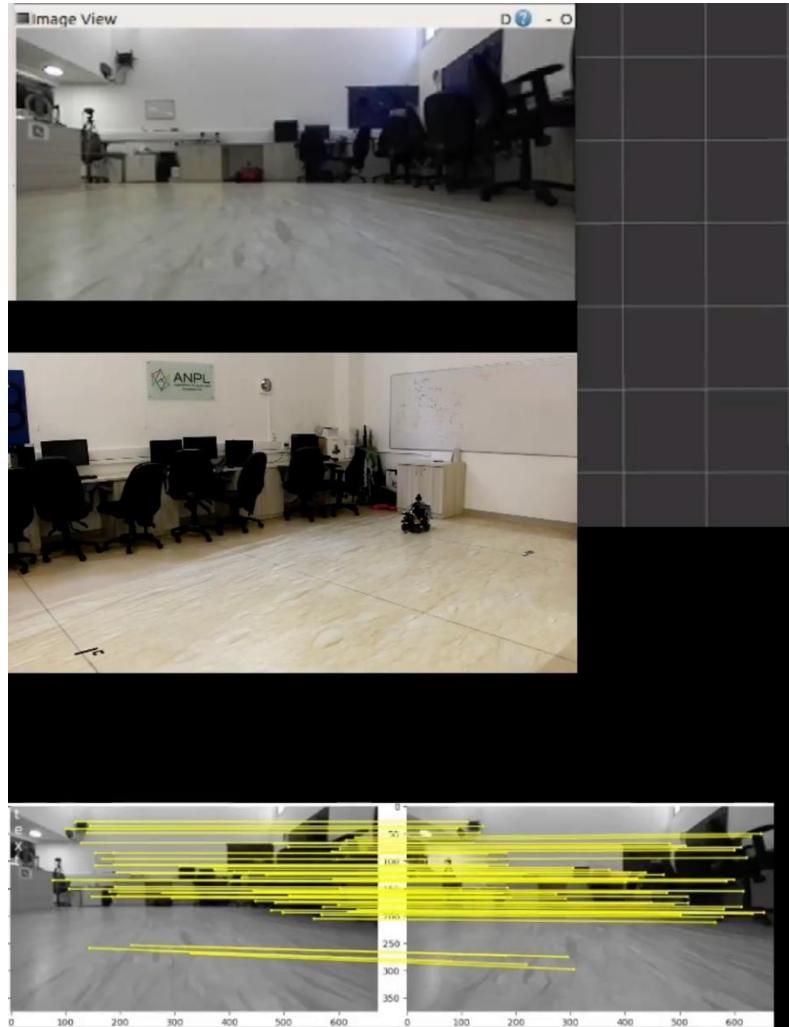
Table: Total planning time in seconds (lower is better)

¹T. Yotam and V. Indelman, "Measurement Simplification in p-POMDP with Performance Guarantees," IEEE T-RO'24.

²D. Kopitkov and V. Indelman, "No Belief Propagation Required: Belief Space Planning in High-Dimensional State Spaces via Factor Graphs, Matrix Determinant Lemma and Re-use of Calculation," IJRR'17.

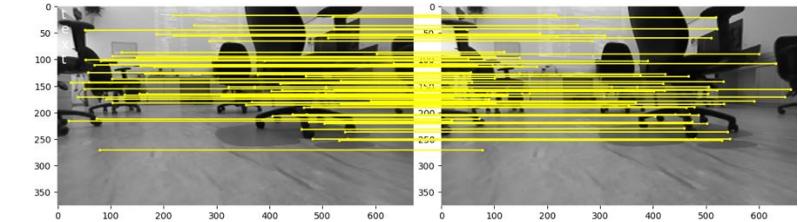
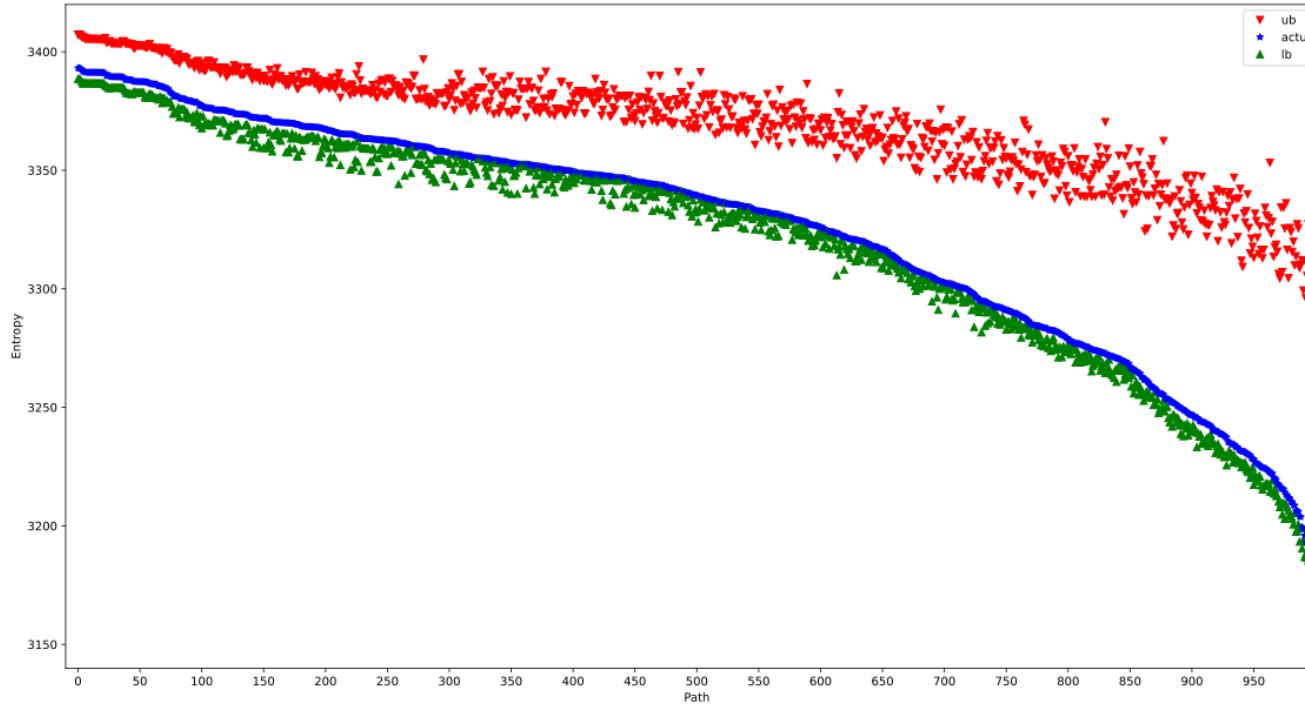
²D. Kopitkov and V. Indelman, "General-purpose incremental covariance update and efficient belief space planning via a factor-graph propagation action tree", IJRR'19.

Real Worl Experiment - Visual Active SLAM



Partitioning of a Multivariate Observation Space

Application to Active SLAM



Method	time [sec]
MP (ours) ¹	585.507 ± 27.153
rAMDL ²	802.545 ± 25.651
iSAM2 ³	1764.835 ± 26.521

Table: Total planning time in seconds (lower is better)

¹T. Yotam and V. Indelman, "Measurement Simplification in ρ -POMDP with Performance Guarantees," IEEE T-RO'24.

²D. Kopitkov and V. Indelman, "No Belief Propagation Required: Belief Space Planning in High-Dimensional State Spaces via Factor Graphs, Matrix Determinant Lemma and Re-use of Calculation," IJRR'17.

²D. Kopitkov and V. Indelman, "General-purpose incremental covariance update and efficient belief space planning via a factor-graph propagation action tree", IJRR'19.

³M. Kaess, et al., "iSAM2: Incremental smoothing and mapping using the Bayes tree," IJRR'12.

Simplification of Decision-Making Problems

Concept:

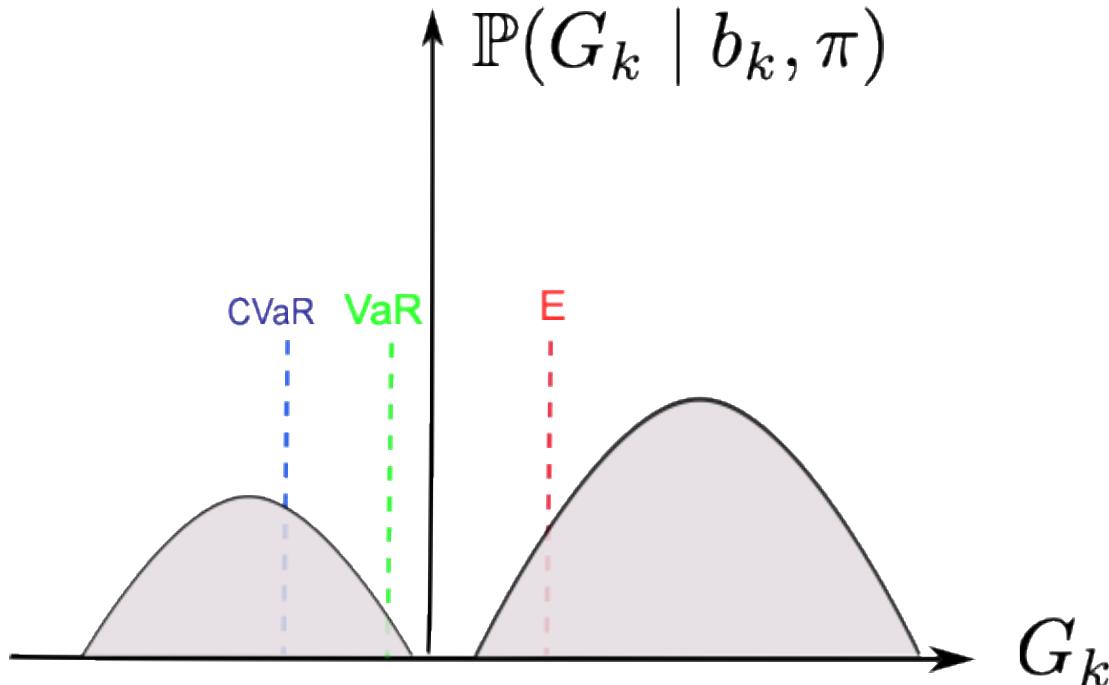
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Simplification of Risk Averse POMDP Planning

- **Distribution** over returns/rewards



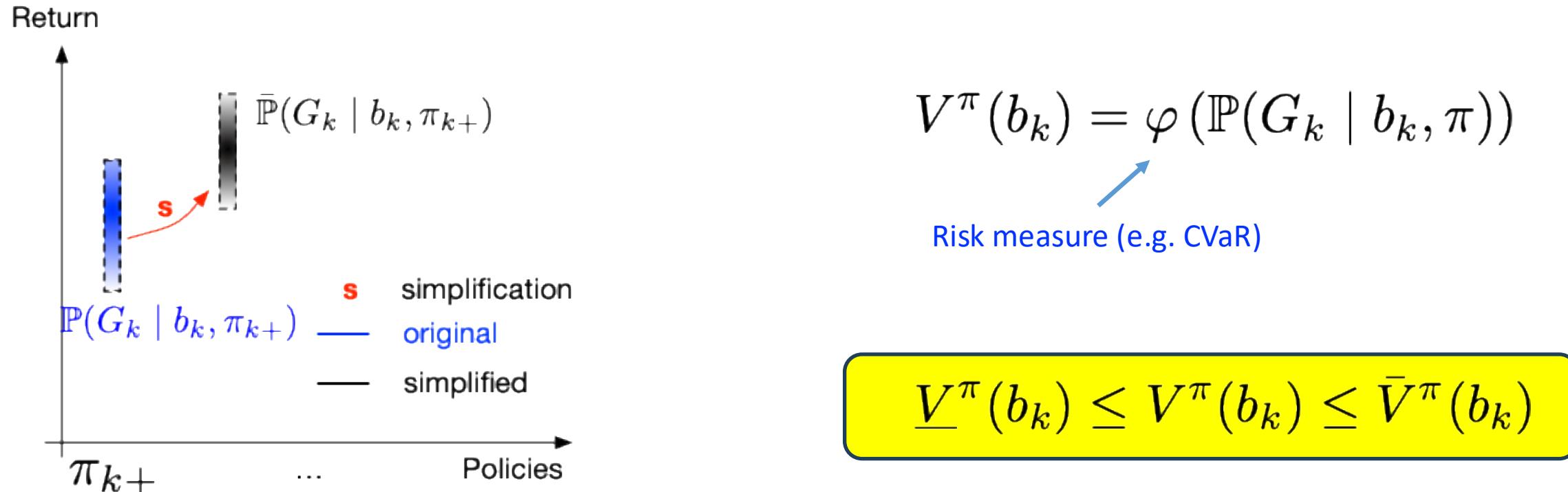
$$G_k = \sum_{\text{return}}_{t=k}^L r(b_t, a_t)$$

$$V^\pi(b_k) = \varphi(\mathbb{P}(G_k \mid b_k, \pi))$$

Risk measure (e.g. CVaR)

Simplification of Risk Averse POMDP Planning

- Impact of simplification on **distribution** over returns/rewards
- Simplified **risk-aware** decision-making with formal guarantees



A. Zhitnikov and V. Indelman, "Simplified Risk Aware Decision Making with Belief Dependent Rewards in Partially Observable Domains," Artificial Intelligence, 2022.

Y. Pariente and V. Indelman, "Simplification of Risk Averse POMDPs with Performance Guarantees," arXiv'24.

I. Nutov and V. Indelman, "Simplified Risk Aware CVaR-based POMDP With Performance Guarantees: a Risk Envelope Perspective", TR'24.

Y. Pariente and V. Indelman, "Bounding Conditional Value-at-Risk via Auxiliary Distributions with Bounded Discrepancies," arXiv'25.

Probabilistically Constrained Belief Space Planning

$$\max_{\pi_{k+}} \mathbb{E} \left[\sum_{\ell=k}^{k+L-1} \rho_{\ell+1} \mid b_k, \pi_{k+} \right]$$

subject to $P(c(b_{k:k+L}; \phi, \delta) = 1 \mid b_k, \pi_{k+}) \geq 1 - \epsilon$

Information gain¹:

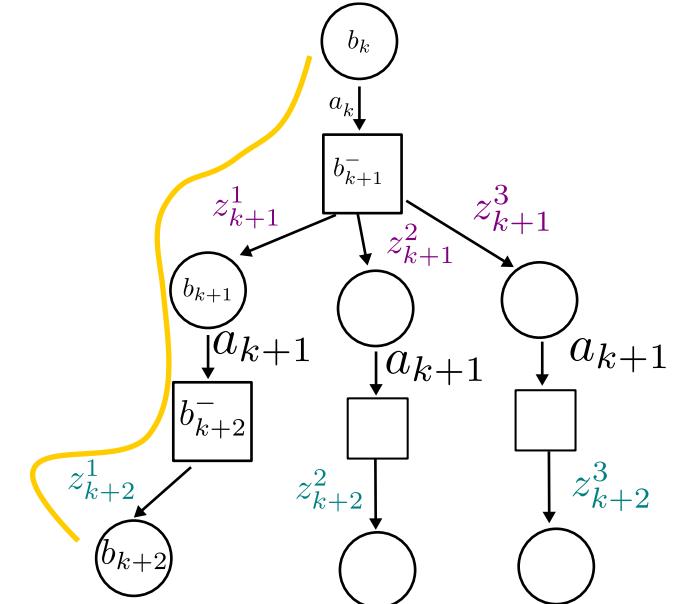
$$c(b_{k:k+L}; \phi, \delta) \triangleq \mathbf{1}_{\left\{ \left(\sum_{\ell=k}^{k+L-1} \phi(b_\ell, b_{\ell+1}) \right) \geq \delta \right\}} (b_{k:k+L})$$

↑
Information gain

Safety²:

$$c(b_{k:k+L}; \phi, \delta) \triangleq \prod_{\ell=k}^{k+L} \mathbf{1}_{\{b_\ell: \phi(b_\ell) \geq \delta\}} (b_\ell)$$

↑
Safety



¹A. Zhitnikov and V. Indelman, "Simplified Continuous High Dimensional Belief Space Planning with Adaptive Probabilistic Belief-dependent Constraints," T-RO'24.

²A. Zhitnikov and V. Indelman, "Anytime Probabilistically Constrained Provably Convergent Online Belief Space Planning," T-RO'25.

Robust Online Planning Under Uncertainty

- So far, models were assumed to be given and perfect
- In practice, models are learned from data
- What happens when the models are **uncertain**?

How to do **online robust** planning?

Uncertainty set:

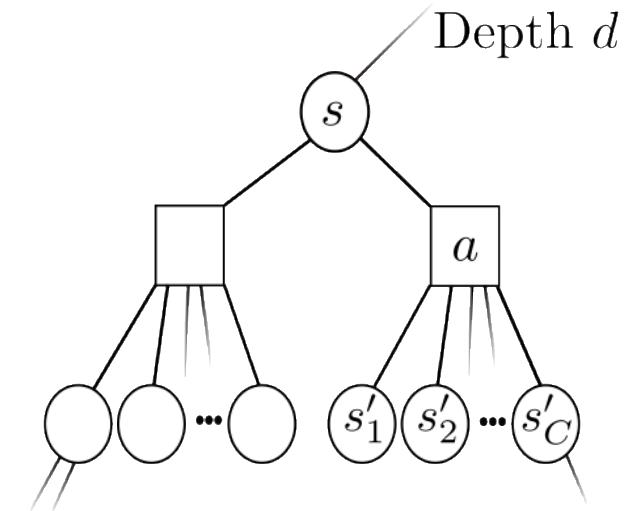
$$P_t(S_{t+1} \mid S_t = s, A_t = a) \in \mathcal{P}_t^{s,a}$$

Robust value function:

$$V^\pi(s) = \min_{P \in \mathcal{P}} V^{\pi,P}(s)$$

Robust Sparse Sampling (RSS) Algorithm:

- A sample-based online robust planner
- Applicable to infinite or continuous state spaces
- Finite-sample performance guarantees



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Depth d

Robust Sparse Sampling (RSS) Algorithm:

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- Finite-sample performance guarantees

Guarantees

$$|V^{\hat{\pi}^*}(s) - V^{\pi^*}(s)| \leq \epsilon$$

Agenda

Experience Reuse in POMDP Planning

POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

Simplification of Decision-Making Problems

Concept:

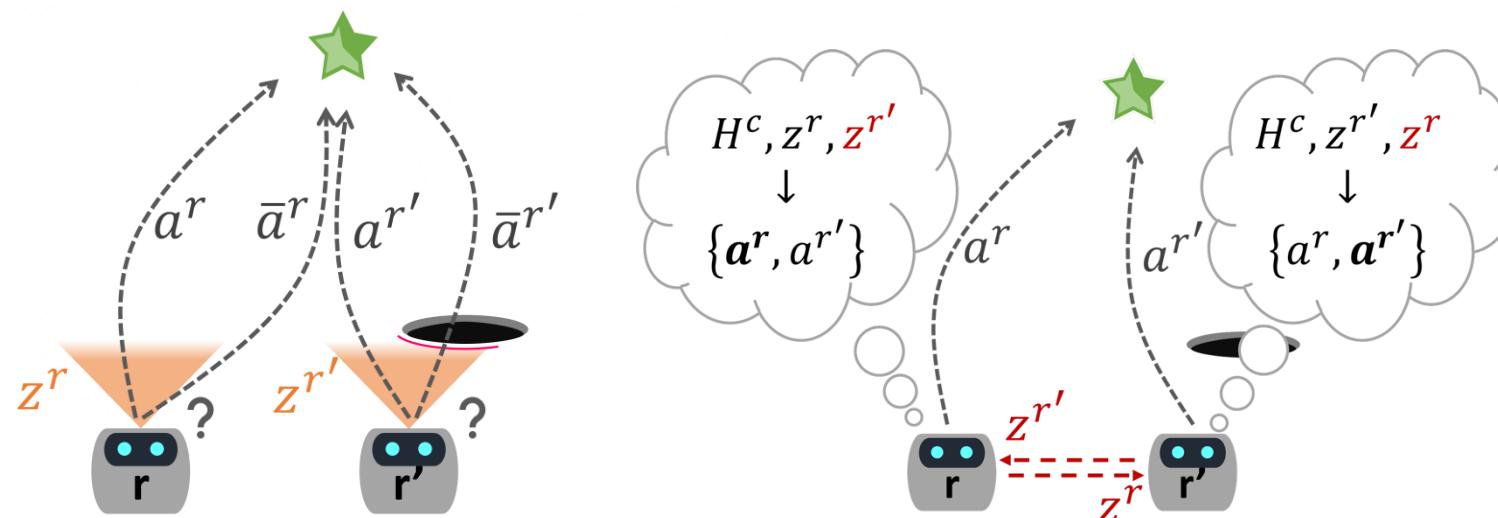
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Multi-Robot Belief Space Planning

- **A common assumption:** Beliefs of different robots are consistent at planning time
- Requires prohibitively frequent data-sharing capabilities!

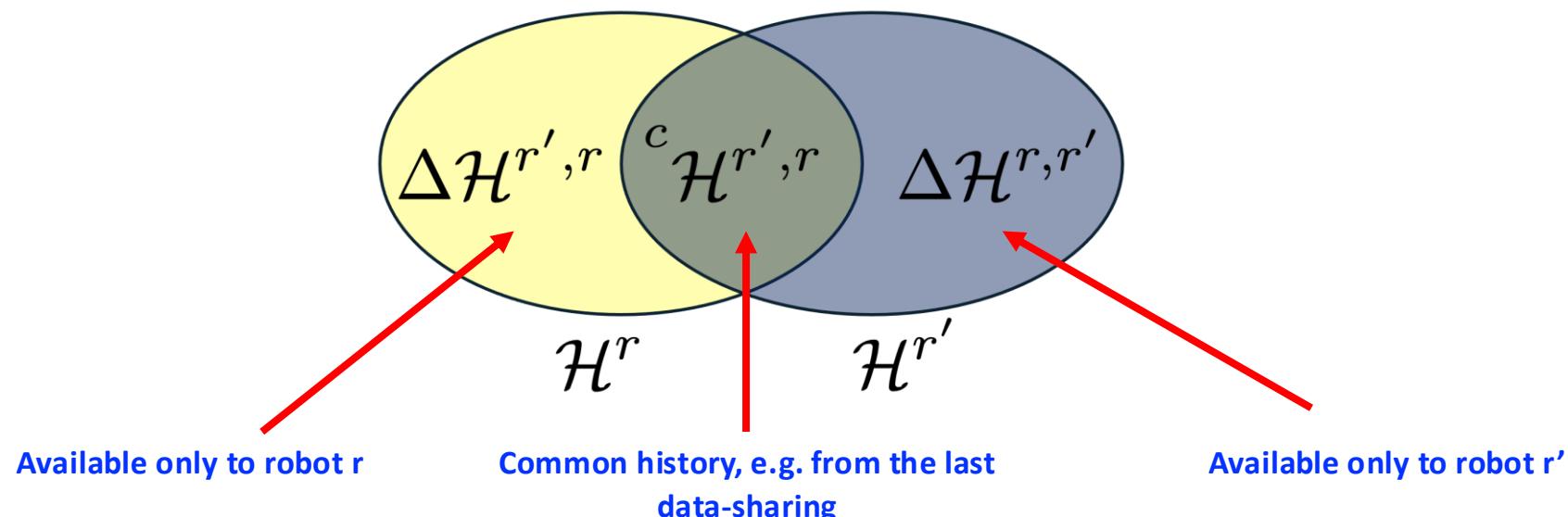


Multi-Robot Cooperative BSP with Inconsistent Beliefs

What happens when data-sharing capabilities between the robots are limited?

- Histories & beliefs of the robots may **differ** due to limited data-sharing capabilities

$$b_k^r = \mathbb{P}(x_k \mid \mathcal{H}_k^r) \quad b_k^{r'} = \mathbb{P}(x_k \mid \mathcal{H}_k^{r'}) \quad \mathcal{H}_k^r \neq \mathcal{H}_k^{r'}$$



T. Kundu, M. Rafaeli, and V. Indelman, "Multi-Robot Communication-Aware Cooperative Belief Space Planning with Inconsistent Beliefs: An Action-Consistent Approach," IROS'24.

T. Kundu, M. Rafaeli, A. Gulyaev, and V. Indelman, "Action-Consistent Decentralized Belief Space Planning with Inconsistent Beliefs and Limited Data Sharing: Framework and Simplification Algorithms with Formal Guarantees," Submitted, 2025.

M. Rafaeli, and V. Indelman, "Towards Optimal Performance and Action Consistency Guarantees in Dec-POMDPs with Inconsistent Beliefs and Limited Communication," Submitted, 2025.

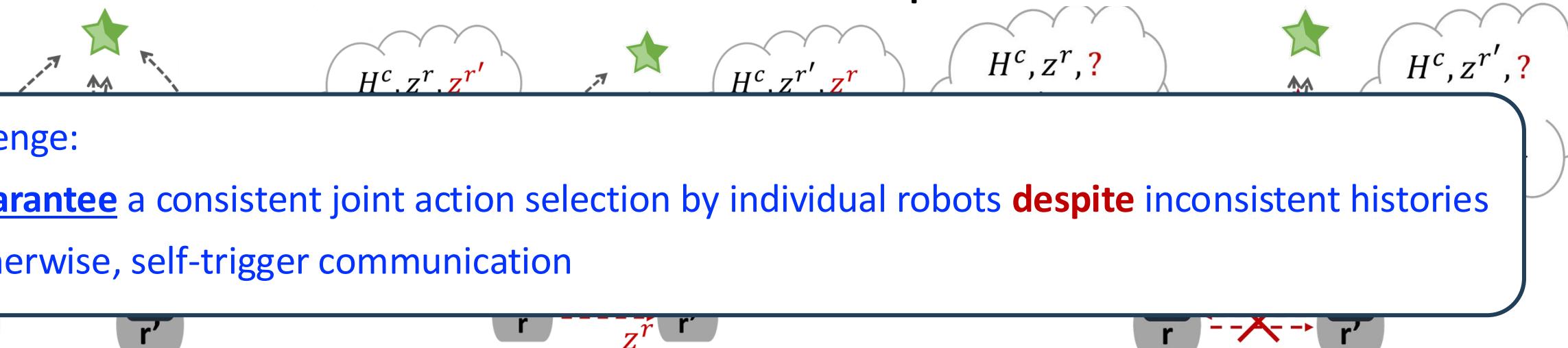
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- **Can lead to a lack of coordination and unsafe and sub-optimal actions**

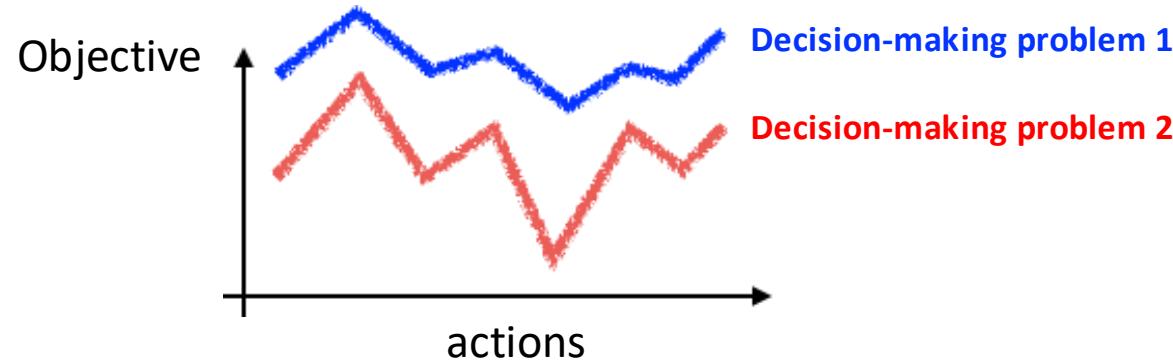


Challenge:

- **Guarantee** a consistent joint action selection by individual robots **despite** inconsistent histories
- Otherwise, self-trigger communication

Action Consistency

- If two decision-making problems have the **same action preference**, this implies both have the **same best action regardless of the actual objective/value function values**



- **Key idea:** to guarantee consistent multi-robot decision-making, each robot
 - reasons about its own and other robots' action preferences while accounting for the **missing information** between the robots
 - checks if for all these realizations, we get the same best joint action

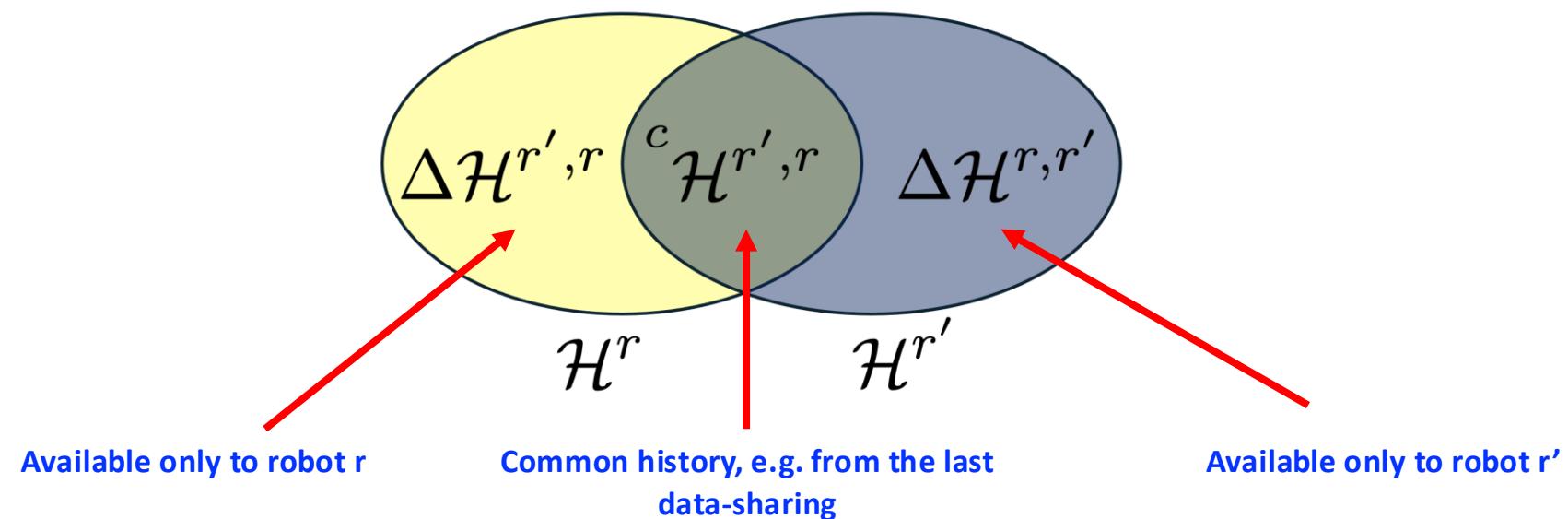
V. Indelman, "No Correlations Involved: Decision Making Under Uncertainty in a Conservative Sparse Information Space," IEEE RA-L'16.

K. Elimelech and V. Indelman, "Simplified decision making in the belief space using belief sparsification," IJRR'22.

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Decentralized Verification of Multi-Robot Action Consistency (MR-AC)

- From the perspective of robot r , MR-AC holds if the selected joint actions are the same based on:
 - Its local information
 - What it perceives about the reasoning of the other robot r'
 - What it perceives about the reasoning of itself perceived by the other robot r'



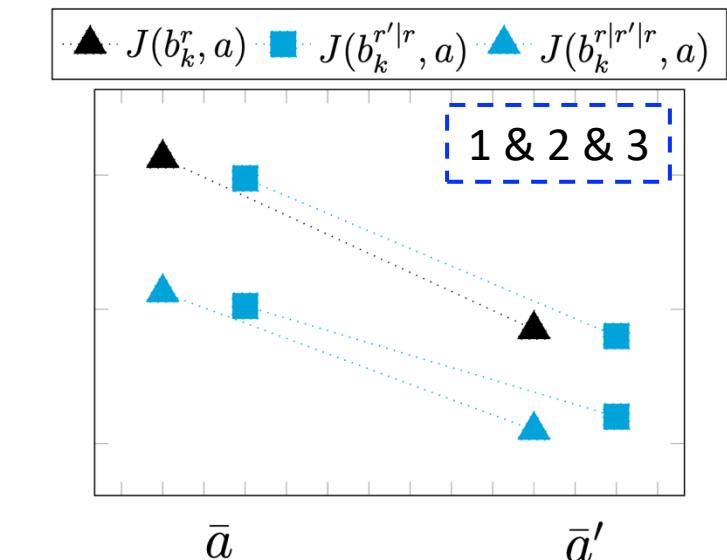
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Decentralized Verification of Multi-Robot Action Consistency (MR-AC)

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 1. Its local information
 2. What it perceives about the reasoning of the other robot r'
 3. What it perceives about the reasoning of itself perceived by the other robot r'
- Same best action in all cases?
 - Yes:** MR-AC is guaranteed to be satisfied
 - Robots are **guaranteed** to choose the same joint action
 - No further data sharing is needed!**
 - No:** self-trigger communication, share some data, repeat Steps 1-3



Agenda

Experience Reuse in POMDP Planning

POMDP Planning with Hybrid Beliefs

Simplification of POMDP with Formal Guarantees

Multi-agent POMDP Planning with Inconsistent Beliefs

Semantic Risk Awareness

Ambiguous Environments

Thank You





10 Year ANPL Anniversary Social Event, November 2024



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Autonomous Navigation
and Perception Lab

