CSC420 – Assignment 3

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In solving the questions in this assignment, I worked together with my classmate Uttkarsh Berwal & 1005355018. I confirm that I have written the solutions/code/report in my own words.

1.1) The equation of a circle centered at the origin with radius r can be represented by $x^2+y^2=r^2$ in cartesian coordinates.

→ For the given feature, the scale that best matches the size of the feature being detected produces maximum or minimum response.

→ When there is a transition ~~from~~ of an edge going from dark to bright, the intensity of the image is increasing. This increase corresponds to a positive second order derivative in the Laplacian of Gaussian (LOG) response.

→ Let $I(x,y)$ be our image, let $I(x,y)$ be equal to 1 outside the circle and 0 inside the circle.

→ We need to convolve the image with LOG Kernel to get response of LOG.

→ $\iint \nabla^2 G(x,y,\sigma) * I(x,y) . dr\, d\theta$

$$= \int_r^{\infty} \cdot \int_0^{2\pi} \frac{1}{\pi\sigma^4}\left(\frac{r^2}{2\sigma^2}-1\right)e^{-\frac{r^2}{2\sigma^2}} \cdot r \cdot d\theta \cdot dr$$

$$= \left[x^2+y^2=r^2\right]$$

Now we normalize [using hint from Ed discussion]

$$= \int_r^{\infty} \int_0^{2\pi} \frac{1}{\pi\sigma^4}\left(\frac{r^2}{2\sigma^2}-1\right)e^{-\frac{r^2}{2\sigma^2}} . \sigma^2 \cdot r \cdot d\theta\, d$$

$$= \frac{2}{\sigma^2} \int_r^{\infty} \left(\frac{r^2}{2\sigma^2} - 1\right) e^{-r^2/2\sigma^2} \cdot r\,dr$$

$$= \frac{2}{\sigma^2}\left(\int_r^{\infty} \frac{r^3}{2\sigma^2} \cdot e^{-r^2/\sigma^2 \cdot 2} \cdot dr\right) - \left(\int_r^{\infty} r \cdot e^{-r^2/2\sigma^2} \cdot dr\right)$$

$$= \frac{2}{\sigma^2}\left[-\frac{r^2}{2} \cdot e^{\frac{-r^2}{2\sigma^2}} - \sigma^2 \cdot e^{-r^2/2\sigma^2} + \sigma^2 \cdot e^{-r^2/2\sigma}\right]_r^{\infty}$$

$$= \frac{2}{\sigma^2}\left[\frac{1}{2} r^2 e^{-\frac{r^2}{2\sigma^2}}\right] = \boxed{\frac{r^2}{\sigma^2} e^{-r^2/2\sigma^2}} \quad (1)$$

$$= I * \nabla^2 G(x, y, \sigma)$$

~~* In order to differentiate the results above~~

* In order to get the value of σ that gives max. response we need to differentiate the result above (1).

$$\rightarrow \frac{d}{d\sigma}\left(\frac{r^2}{\sigma^2} \cdot e^{-r^2/2\sigma^2}\right) = 0$$

$$= \frac{r^4 \cancel{e^{-r^2/2\sigma^2}}}{\sigma^5} - \frac{2r^2 \cancel{e^{-r^2/2\sigma^2}}}{\cancel{\sigma^3}} = 0$$

$$= \frac{r^2}{\sigma^2} - 2 = 0 \Rightarrow \boxed{\frac{r^2}{2} = \sigma^2}$$

$$= \frac{D^2}{8} = \sigma^2 \quad \left[r = \frac{D}{2}\right]$$

Therefore, $\boxed{\sigma = \frac{D}{2\sqrt{2}}}$ maximizes the magnitude

1.2) The intensity decreases in the cases when an edge goes from bright to dark which corresponds to negative second order derivative

→ This time, $I(x,y)$ is equal to 1 inside of circle and 0 outside of circle.

→ ~~Same as~~ Similar to (1.1) part we need

$$I * \nabla^2 G(x,y,\sigma)$$

→ This time we will get:

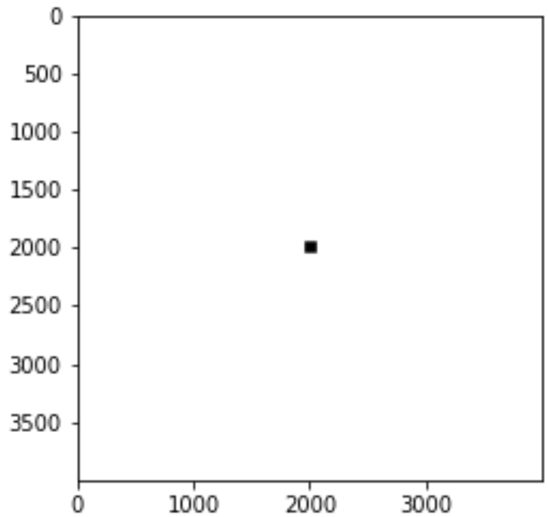
$$I * \nabla^2 G(x,y,\sigma) = \frac{-r^2}{\sigma^2} e^{-r^2/2\sigma^2} \text{ (negative)}$$

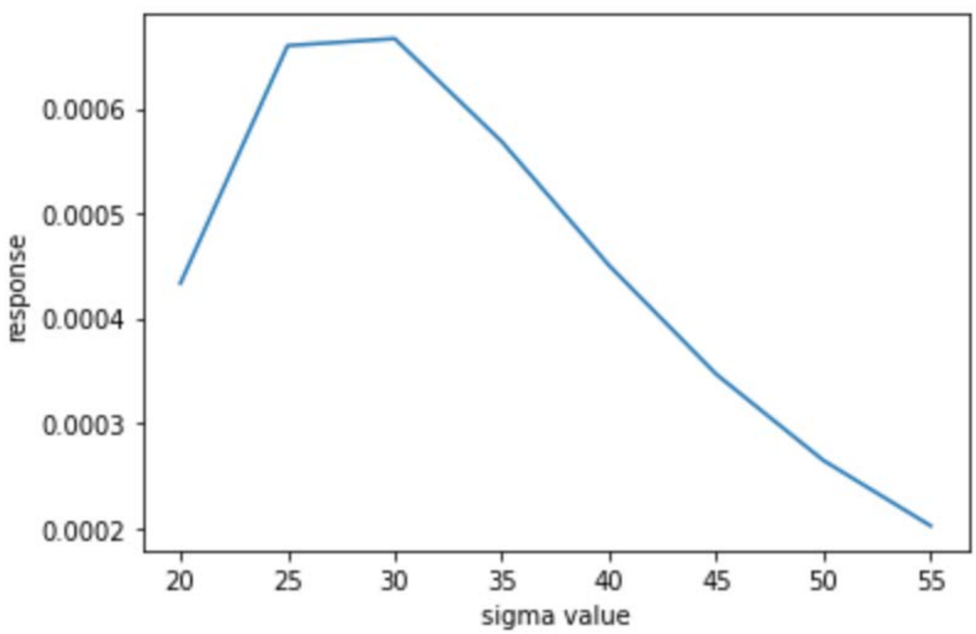
→ Now taking the derivative w.r.t σ,

$$\Rightarrow \frac{2r^2 e^{-r^2/2\sigma^2}}{\sigma^3} - \frac{r^4 e^{-r^2/2\sigma^2}}{\sigma^5} = 0$$

Again we get $\boxed{\sigma = \frac{D}{2\sqrt{2}}}$

Question 1. Part 3





Sigma value = 30

2) Given $N = \begin{bmatrix} Ix^2 & IxIy \\ IxIy & Iy^2 \end{bmatrix}$

→ Now to find eigenvalues of N

$$|N - \lambda I| = 0$$

$$\left| \begin{bmatrix} Ix^2 & IxIy \\ IxIy & Iy^2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} Ix^2 - \lambda & IxIy \\ IxIy & Iy^2 - \lambda \end{bmatrix} \right| = 0$$

$$\rightarrow (I^2x - \lambda)(I^2y - \lambda) - (IxIy)(IxIy) = 0$$

$$\rightarrow (I^2x I^2y - \lambda(Ix^2 + Iy^2) + \lambda^2 - I^2_x I^2_y) = 0$$

$$\rightarrow \lambda^2 = \lambda(Ix^2 + Iy^2)$$

$$\rightarrow 0 = \lambda(\lambda - (Ix^2 + Iy^2))$$

So, $\lambda_1 = 0$ & $\lambda_2 = Ix^2 + Iy^2$

eigenvalues of N

2.2) ~~From the answer of 2) part~~ $\Rightarrow$ we Know that a matrix is positive semi-definite if all its eigenvalues are non negative.

$\rightarrow$ Now, N from part 2) has non-negative eigenvalues Therefore, N is positive semi-definite. which implies. $\Rightarrow$ $VNV^T \geq 0, \ \forall$ vectors V

$\rightarrow$ We Know that window function is 1 in window and 0 outside which is never negative.

$\rightarrow$ To prove that M is positive semi-definite we need to show $VMV^T \geq 0$

$$\Rightarrow \quad vMv^T \geq 0 = V\left(\sum\sum w(x,y)N\right)V^T$$

$$= \sum\sum w(x,y)\, vNv^T$$

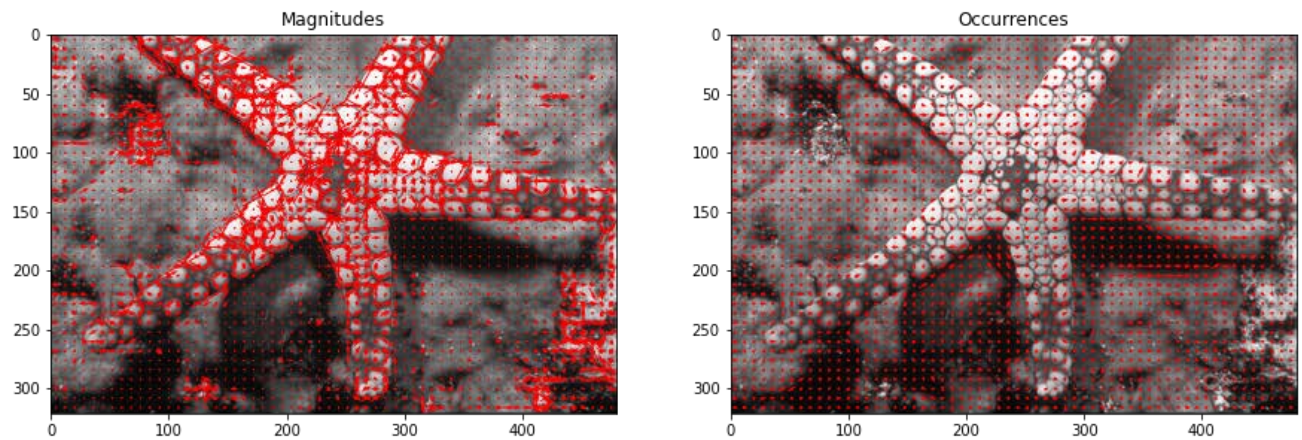
[since v is not part of summation (constant)]

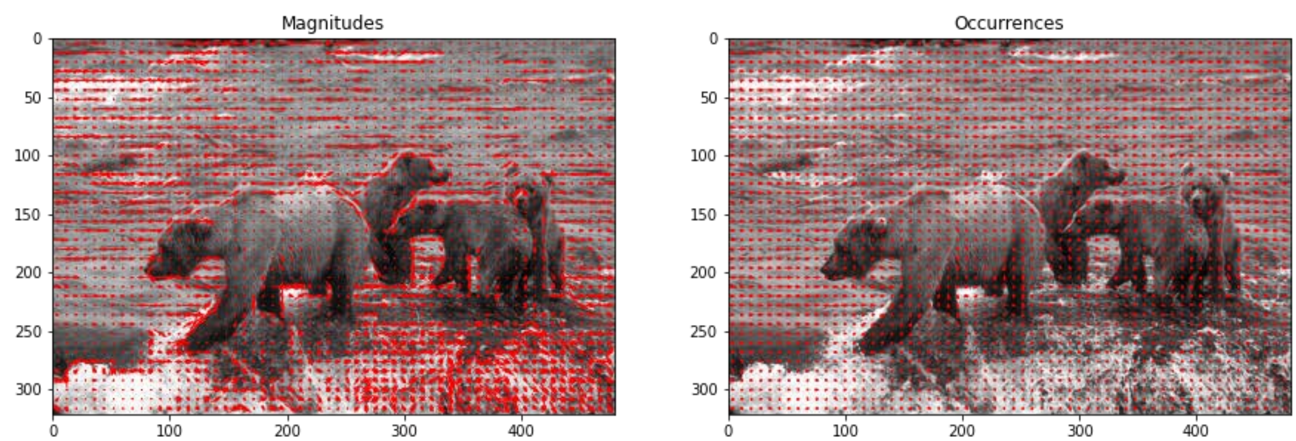
Now, we proved $vNv^T \geq 0$ & $w(x,y) \geq 0$

$$\therefore \ V\sum\sum w(x,y)Nv^T = VMV^T \geq 0$$

Therefore, M is positive semi-definite.

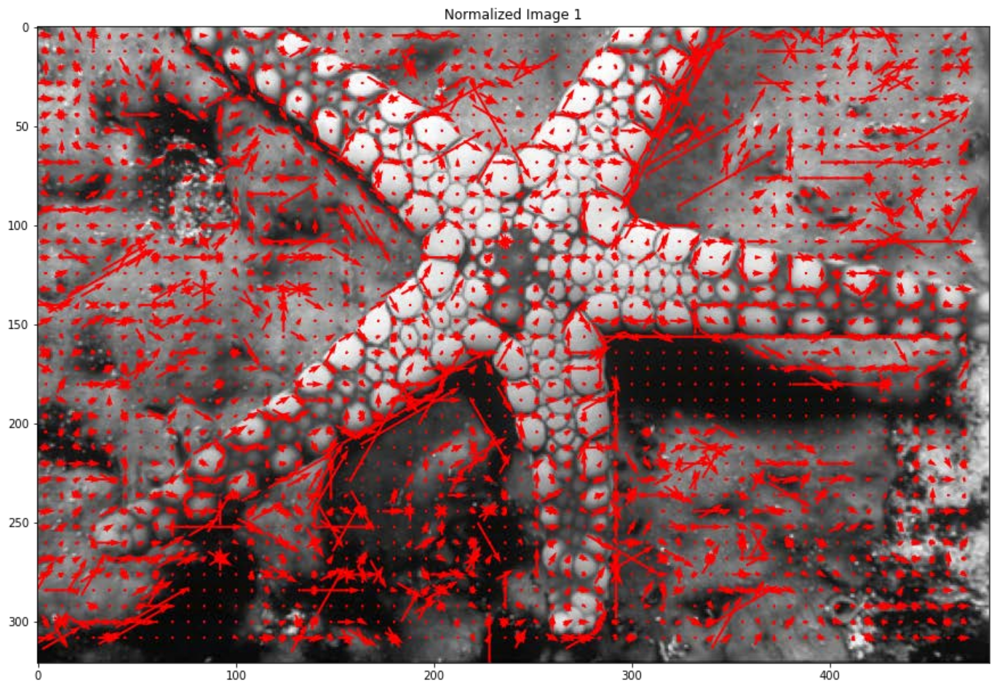
Question 3.

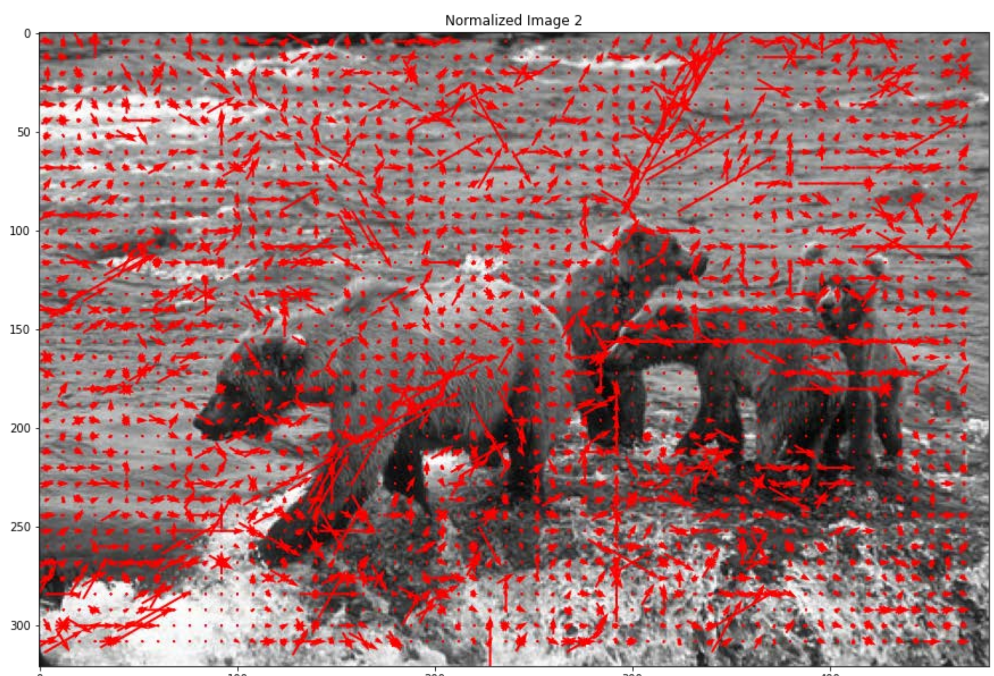


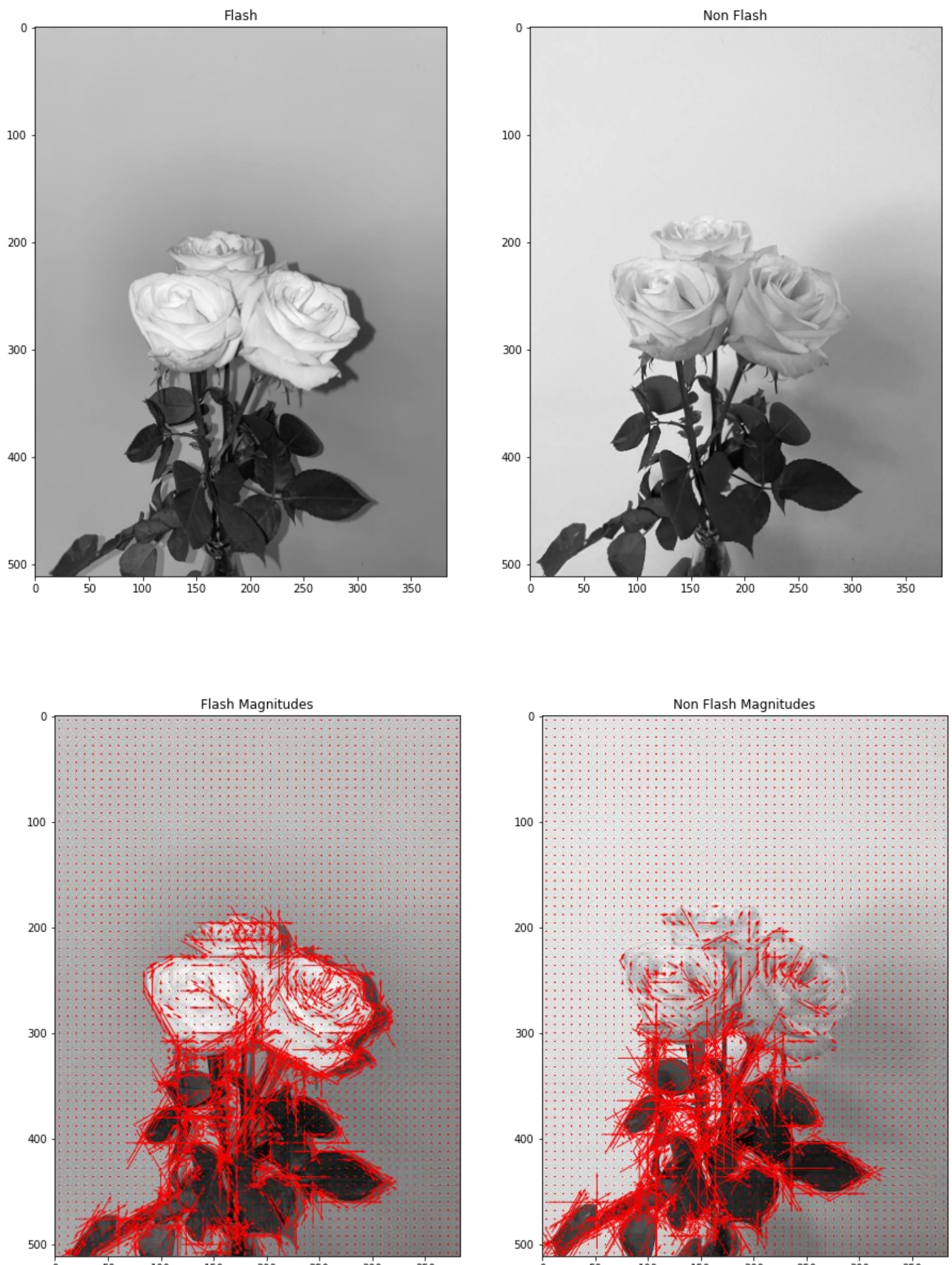


- Compared to counting occurrences approach, the method of accumulating gradient magnitudes tends to produce more accurate and visually informative quiver plots around the jellyfish and bears.
- Decided to go ahead with the approach of Gradient Magnitudes for the rest of the task as the reason stated above.

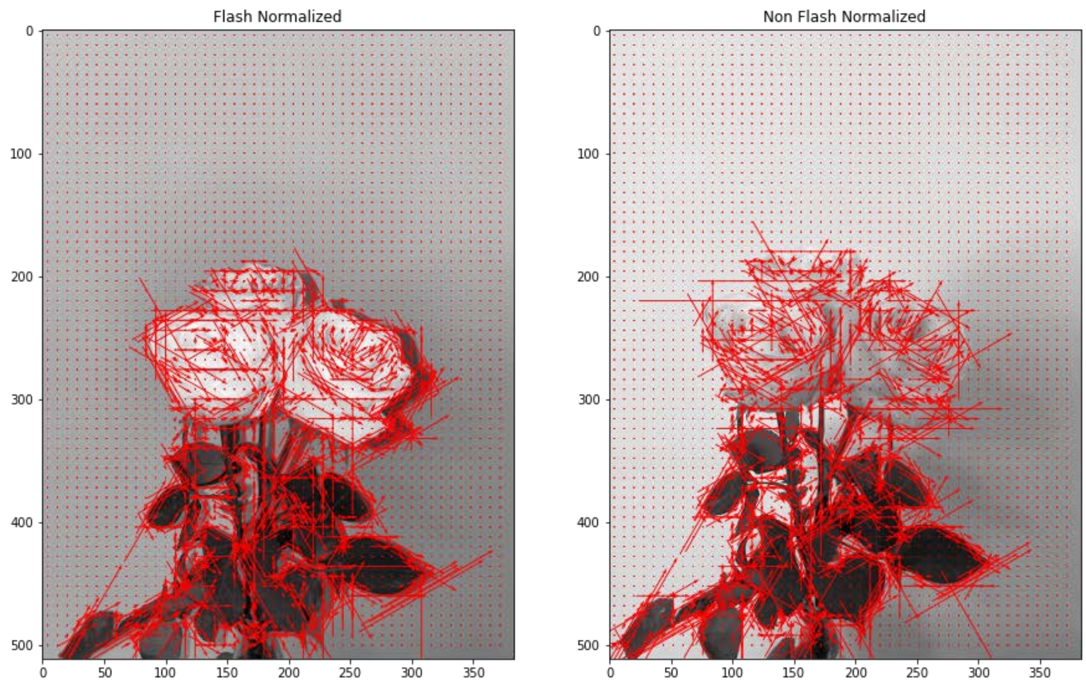
Question 3, Part 4.







Flash
Non Flash
Flash Magnitudes
Non Flash Magnitudes



The gradient magnitude approach is more effective than the normalized HOG approach in detecting the edges of the rose in the flash photo. This is because the flash photo produces well-defined edges that the magnitude approach can accurately detect. On the other hand, I found that the normalized HOG approach was not beneficial in my case. This method tends to decrease the effect of large magnitudes, leading to noise in the shadow on the white wall in the non-flash photo. Additionally, the normalized HOG approach picks up on small details in the flash photo, which can be irrelevant for the analysis. I have Visually compared the outcomes of both methods on several other flash and non-flash photos too and all had similar results as noted above.

Question 4.

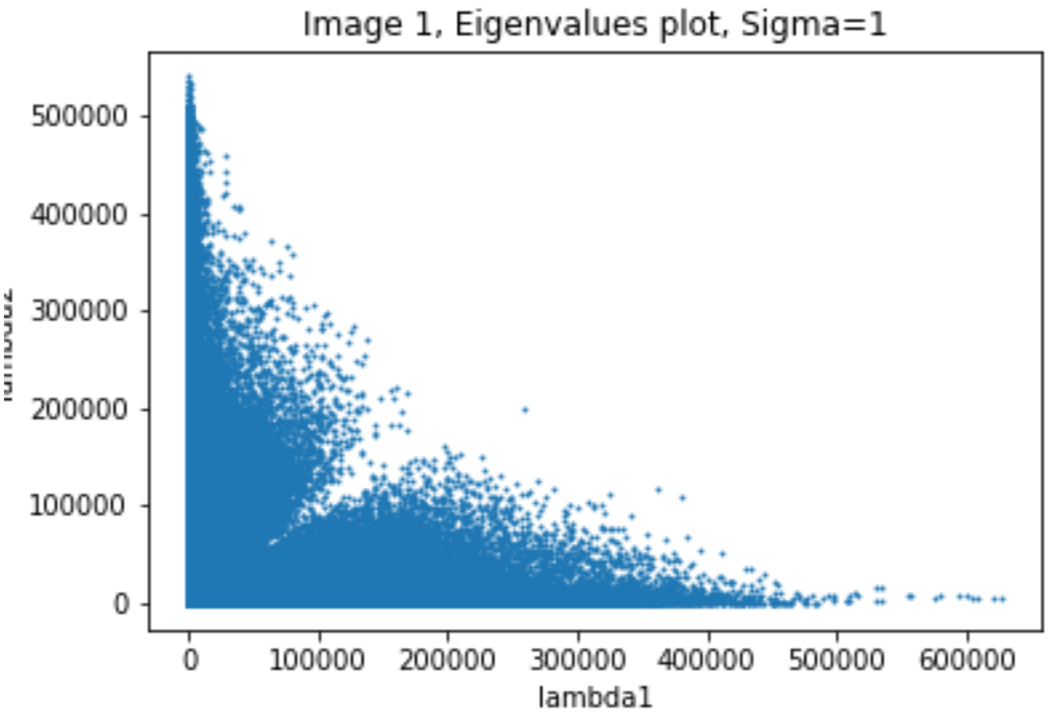


Image 1, Eigenvalues plot, Sigma=1
500000
400000
300000
200000
100000
0
0
100000
200000
300000
400000
500000
600000
lambda1

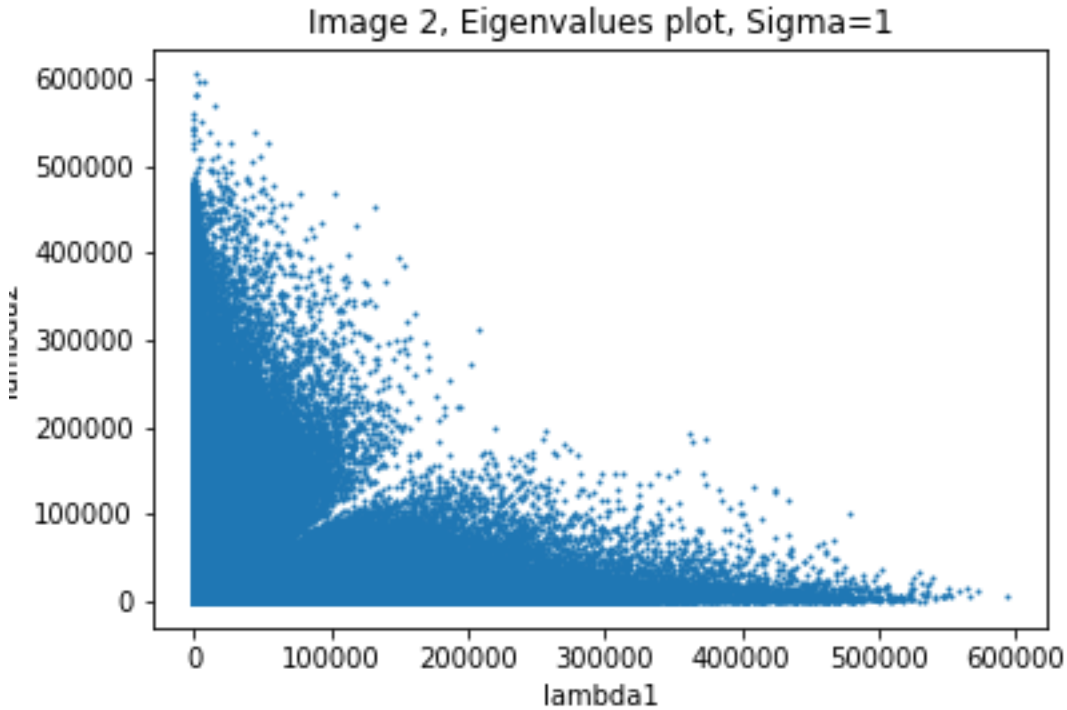


Image 2, Eigenvalues plot, Sigma=1
600000
500000
400000
300000
200000
100000
0
0
100000
200000
300000
400000
500000
600000
lambda1

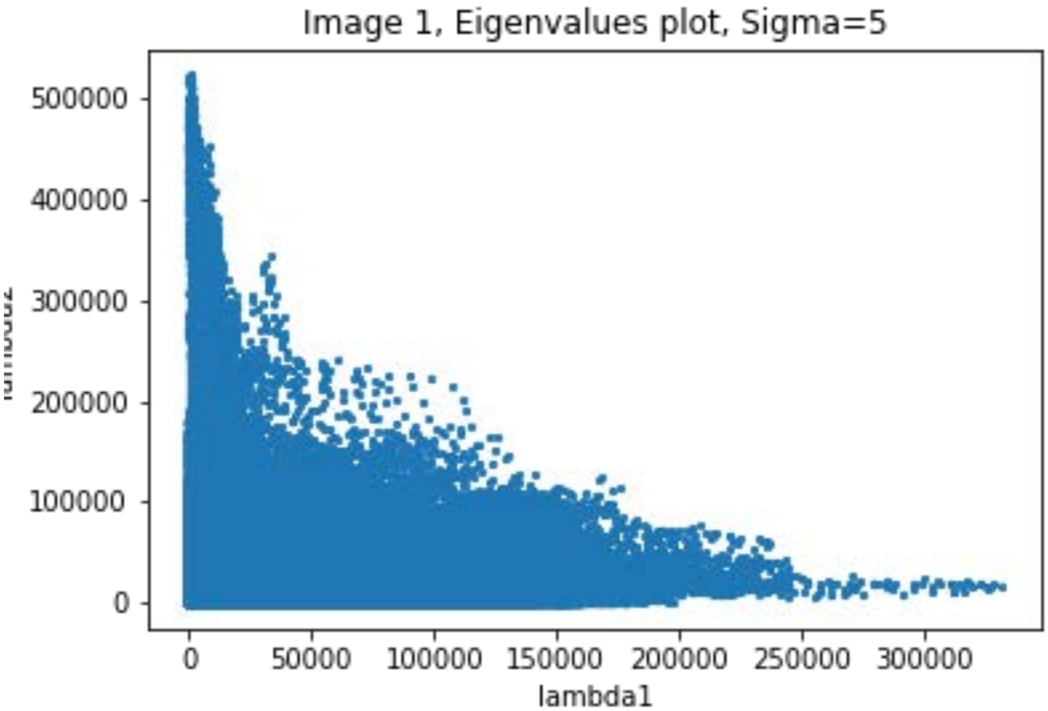


Image 1, Eigenvalues plot, Sigma=5
500000
400000
300000
200000
100000
0
0
50000
100000
150000
200000
250000
300000
lambda1

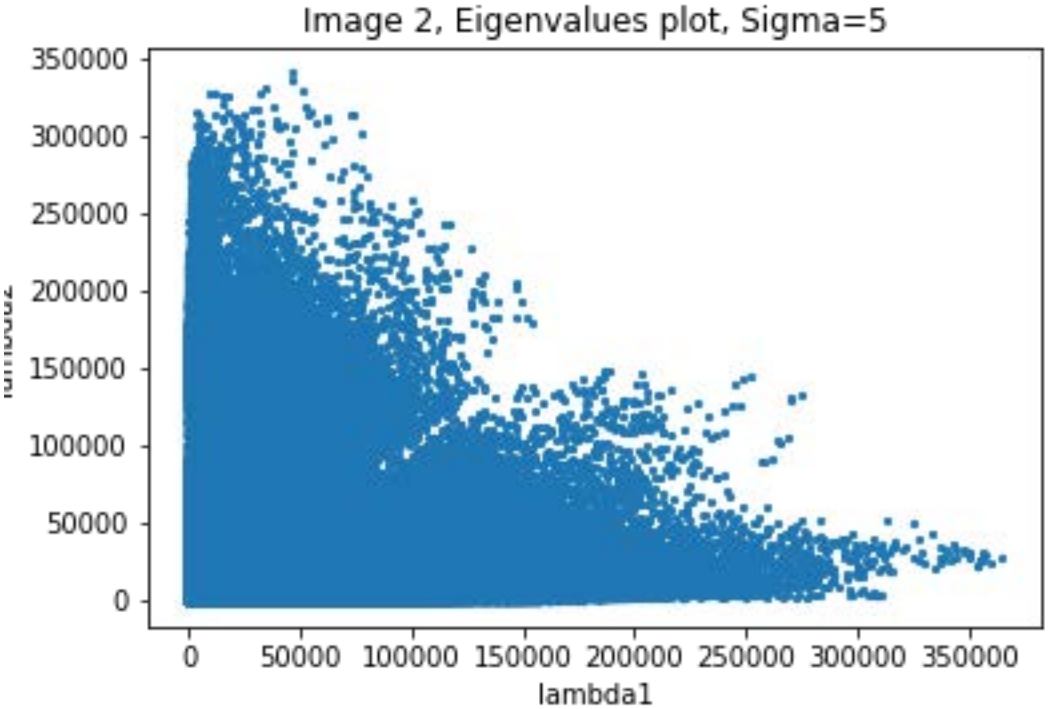


Image 2, Eigenvalues plot, Sigma=5
lambda1
0
50000
100000
150000
200000
250000
300000
350000

A smaller sigma produces a smaller filter, which leads to less smoothing, while a larger sigma produces a larger filter and more smoothing. A smaller sigma is more susceptible to noise, whereas a larger sigma reduces the impact of noise.

I have compared the corner detection above using different sigma values, a larger sigma causes corners to appear less distinct and more rounded due to blurring as you notice on the image above, while a smaller sigma results in sharper and more angular corners. It is crucial to select the appropriate sigma value to balance the trade-off between smoothing and preserving essential image characteristics.