Logistic Regression

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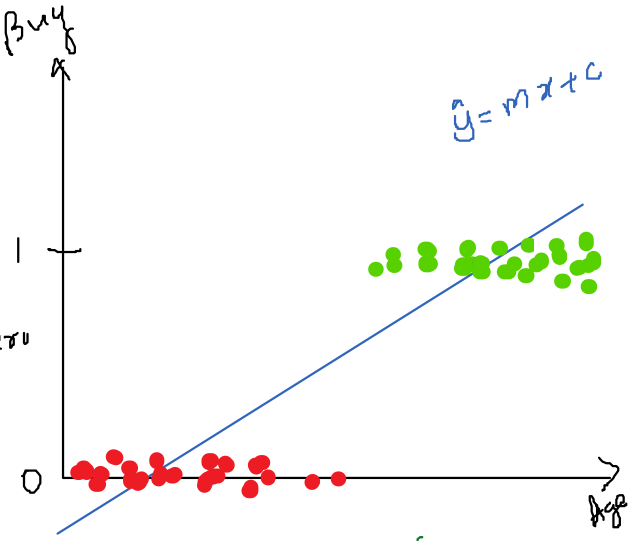
y = 1 / 0 prediction ⇒ 1 / 0

Sigmoid function / logistic function $f(x) = \frac{1}{1+e^{-x}}$ { 1 / 0



$$\frac{1}{1+e^{-\hat{y}}} = P = \text{probability of observ to be in class 1}$$

If $P \geq 0.5$ ⇒ class 1

If $P < 0.5$ ⇒ class 0

$\hat{y} = mx + c$

$$\frac{1}{1+e^{-(mx+c)}} = P \Rightarrow \frac{1-P}{P} = e^{-(mx+c)} \Rightarrow \log\left(\frac{P}{1-P}\right) = mx + c$$

(Error) $E = \frac{1}{n} \sum (y - \hat{y})^2$ MSE

Regression

(Error) $E = \frac{1}{n} \sum \left[-y \log \hat{y} + (1-y) \log(1-\hat{y})\right]$

binary cross entropy

A = zero

P = 999 / 1002

y = 0 NB

y = 1 YB

M1 C1 A = 0 P = 0.45 ⇒ P = 0

C2 A = 1 P = 0.55 ⇒ P = 1

M2 C1 A = 0 P = 0.01 ⇒ P = 0

C2 A = 1 P = 0.95 ⇒ P = 1

A = 0 ⇒ y = 0

$1 - \hat{y}$ = Prob of NB

$\hat{y}$ = Prob of YB

y = 1 (−x)

Linear Reg. — Regression	Log. Reg. — Classification
$\hat{y} = mx + c$	$\log\left(\frac{P}{1-P}\right) = mx + c$
Error (MSE) $E = \frac{1}{n} \sum (y - \hat{y})^2$	Binary cross entropy $E = \frac{1}{n} \sum \left(-y \log \hat{y} + (1-y) \log(1-\hat{y})\right)$
Gradient Descent	Gradient Descent