Logistic regression

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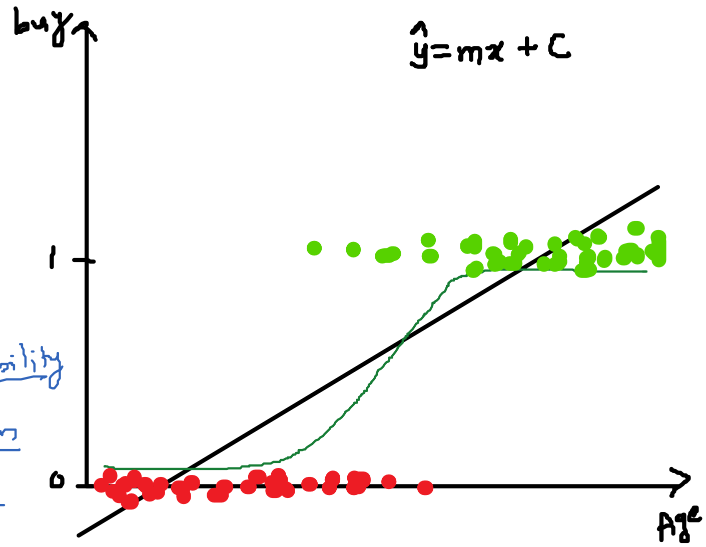
Logistic Regression
- A linear model, used to solve classification problems, two class classification problems

Linear Regression $\xrightarrow{\text{Sigmoid}}$ logistic Regression



Sigmoid

$$f(x) = \frac{1}{1+e^{-x}} \qquad f(x) = \begin{cases} 0 \\ 1 \end{cases}$$

$$\frac{1}{1+e^{-\hat{y}}} = \frac{1}{1+e^{-(mx+c)}} = P = \text{Probability to be in class 1}$$

If $P \geq 0.5 \Rightarrow$ class 1

$P < 0.5 \Rightarrow$ class 0

$$P = \frac{1}{1+e^{-(mx+c)}} \Rightarrow e^{-(mx+c)} = \frac{1}{P} - 1 = \frac{1-P}{P}$$

$$e^{mx+c} = \frac{P}{1-P}$$

$$\rightarrow \boxed{\log\left(\frac{P}{1-P}\right) = mx + c}$$

Linear Regression	Logistic Regression
Regression	Classification
$\hat{y} = mx + c$	$\log\left(\frac{P}{1-P}\right) = mx + c$
$E = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \rightarrow$ MSE	binary Crossentropy $E = -\frac{1}{n}\sum_{i=1}^{n}\left(y \log \hat{y} - (1-y)\log(1-\hat{y})\right)$ MLE

Actual	Model 1
$\rightarrow$ C1 $\rightarrow$ 1	C1 $\rightarrow$ 0.55 $\rightarrow$ 1 ✓ } high entropy
C1 $\rightarrow$ 0	C2 $\rightarrow$ 0.40 $\rightarrow$ 0 ✓

$\hat{y} =$

$\frac{F = 20}{P =}$ 0.8

−0.8

−0.9

→ C1 → 1

→ C2 → 0

C1 → 0.5

C2 → 0.40 → 0 ✓

Model2

C1 → 0.95 → 1 ✓

C2 → 0.10 → 0 ✓ } low entropy

$$\boxed{E = \frac{-1}{n} \sum \left(y \log \hat{y} - (1-y) \log (1-\hat{y}) \right)}$$

C1 → $y = 1$ $E = -\log \hat{y}$ $-\log \hat{y}$

C2 → $y = 0$ $E = \log(1-\hat{y})$