

CS224N: NATURAL LANGUAGE PROCESSING WITH DEEP LEARNING
ASSIGNMENT #1

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1. (a) For any input vector \mathbf{x} and any constant c ,

$$\begin{aligned}\text{softmax}(\mathbf{x} + c)_i &= \frac{e^{x_i + c}}{\sum_j e^{x_j + c}} \\ &= \frac{e^c e^{x_i}}{\sum_j e^c e^{x_j}} \\ &= \frac{e^c e^{x_i}}{e^c \sum_j e^{x_j}} \\ &= \frac{e^{x_i}}{\sum_j e^{x_j}} \\ &= \text{softmax}(\mathbf{x})_i\end{aligned}\tag{1}$$

Since (1) is true for any arbitrary element i , we can conclude that:

$$\text{softmax}(\mathbf{x}) = \text{softmax}(\mathbf{x} + c)$$

- (b) Please see the coding portion of the assignment.

2. (a) First, we can rearrange the definition of the sigmoid function to obtain:

$$e^{-x} = \frac{1}{\sigma(x)} - 1$$

Now we can derive the gradient of the sigmoid function w.r.t. x , assuming x is a scalar.

$$\begin{aligned}\frac{\partial \sigma(x)}{\partial x} &= \frac{\partial}{\partial x} \frac{1}{1 + e^{-x}} \\ &= \frac{-1}{(1 + e^{-x})^2} (-e^{-x}) \\ &= \frac{1}{(1 + e^{-x})^2} (e^{-x}) \\ &= (\sigma(x))^2 \left(\frac{1}{\sigma(x)} - 1 \right) \\ &= \sigma(x) (1 - \sigma(x))\end{aligned}$$

- (b) First, let's consider the fact that \mathbf{y} is the one-hot label vector, i.e.

$$y_i = \begin{cases} 1, & \text{if } i = k \\ 0, & \text{otherwise} \end{cases}$$

where k is the index of the true label.

Therefore, we can simplify the cross entropy function as:

$$\text{CE}(\mathbf{y}, \hat{\mathbf{y}}) = - \sum_i y_i \log(\hat{y}_i) = - \log(\hat{y}_k)$$

To derive the gradient w.r.t the inputs of a softmax function when cross entropy loss is used for evaluation, let's consider its individual elements:

$$\begin{aligned}
\frac{\partial}{\partial \theta_i} \text{CE}(\mathbf{y}, \hat{\mathbf{y}}) &= \frac{\partial}{\partial \theta_i} [-\log(\hat{y}_k)] \\
&= \frac{\partial}{\partial \theta_i} \left[-\log\left(\frac{e^{\theta_k}}{\sum_j e^{\theta_j}}\right) \right] \\
&= \frac{\partial}{\partial \theta_i} \left[-\theta_k + \log \sum_j e^{\theta_j} \right] \\
&= -\frac{\partial \theta_k}{\partial \theta_i} + \frac{\sum_j e^{\theta_j} \frac{\partial \theta_j}{\partial \theta_i}}{\sum_j e^{\theta_j}}
\end{aligned} \tag{2}$$

By noting that:

$$\frac{\partial \theta_j}{\partial \theta_i} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

We can simplify (2) as:

$$\frac{\partial}{\partial \theta_i} \text{CE}(\mathbf{y}, \hat{\mathbf{y}}) = -y_i + \frac{e^{\theta_i}}{\sum_j e^{\theta_j}} = -y_i + \hat{y}_i$$

Thus, the gradient w.r.t the inputs of a softmax function when cross entropy loss is used for evaluation is:

$$\frac{\partial}{\partial \boldsymbol{\theta}} \text{CE}(\mathbf{y}, \hat{\mathbf{y}}) = \hat{\mathbf{y}} - \mathbf{y}$$

(c) Let's denote:

$$\begin{aligned}
\boldsymbol{\theta}_1 &= \mathbf{x} \mathbf{W}_1 + \mathbf{b}_1 \\
\boldsymbol{\theta}_2 &= \mathbf{h} \mathbf{W}_2 + \mathbf{b}_2
\end{aligned}$$

By applying chain rule, we can rewrite the gradient as:

$$\frac{\partial J}{\partial \mathbf{x}} = \frac{\partial \text{CE}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \mathbf{x}} = \frac{\partial \text{CE}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \boldsymbol{\theta}_2} \frac{\partial \boldsymbol{\theta}_2}{\partial \mathbf{h}} \frac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}_1} \frac{\partial \boldsymbol{\theta}_1}{\partial \mathbf{x}}$$

The first component is simply the result of part (b):

$$\frac{\partial \text{CE}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \boldsymbol{\theta}_2} = \hat{\mathbf{y}} - \mathbf{y}$$

The second component is:

$$\frac{\partial \boldsymbol{\theta}_2}{\partial \mathbf{h}} = \frac{\partial}{\partial \mathbf{h}} (\mathbf{h} \mathbf{W}_2 + \mathbf{b}_2) = \mathbf{W}_2^\top$$

The third component is a $H \times H$ matrix and can be computed by examining its individual elements:

$$\left(\frac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}_1} \right)_{ij} = \left(\frac{\partial \sigma(\boldsymbol{\theta}_1)}{\partial \boldsymbol{\theta}_1} \right)_{ij} = \frac{\partial (\sigma(\boldsymbol{\theta}_1))_i}{\partial \theta_{1j}} = \frac{\partial \sigma(\theta_{1i})}{\partial \theta_{1j}} = \begin{cases} \sigma'(\theta_{1i}), & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

where $\sigma'(\theta_{1i}) = \sigma(\theta_{1i})(1 - \sigma(\theta_{1i}))$ is the sigmoid gradient as shown in part (a).

Thus, we can define:

$$\frac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}_1} = \mathbf{S}(\boldsymbol{\theta}_1)$$

where \mathbf{S} denotes a $H \times H$ diagonal matrix where the diagonal elements are $\sigma'(\theta_i)$ for $i = 1, \dots, H$.

The fourth component is similar to the second component:

$$\frac{\partial \boldsymbol{\theta}_1}{\partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{x}} (\mathbf{x} \mathbf{W}_1 + \mathbf{b}_1) = \mathbf{W}_1^\top$$

Therefore, the the gradient with respect to the inputs \mathbf{x} to an one-hidden-layer neural network is:

$$\begin{aligned}\frac{\partial J}{\partial \mathbf{x}} &= \frac{\partial \text{CE}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \boldsymbol{\theta}_2} \frac{\partial \boldsymbol{\theta}_2}{\partial \mathbf{h}} \frac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}_1} \frac{\partial \boldsymbol{\theta}_1}{\partial \mathbf{x}} \\ &= (\hat{\mathbf{y}} - \mathbf{y}) \mathbf{W}_2^\top \mathbf{S}(\boldsymbol{\theta}_1) \mathbf{W}_1^\top \\ &= (\hat{\mathbf{y}} - \mathbf{y}) \mathbf{W}_2^\top \mathbf{S}(\mathbf{x} \mathbf{W}_1 + \mathbf{b}_1) \mathbf{W}_1^\top\end{aligned}$$

Or, equivalently:

$$\frac{\partial J}{\partial \mathbf{x}} = (\hat{\mathbf{y}} - \mathbf{y}) \mathbf{W}_2^\top \circ \sigma(\mathbf{x} \mathbf{W}_1 + \mathbf{b}_1) \circ (1 - \sigma(\mathbf{x} \mathbf{W}_1 + \mathbf{b}_1)) \mathbf{W}_1^\top$$

where \circ denotes the Hadamard product of two vectors.

(d) The dimensions of the weights and biases are as follows:

| Parameter | Dimension |
|----------------|----------------|
| \mathbf{W}_1 | $D_x \times H$ |
| \mathbf{b}_1 | $1 \times H$ |
| \mathbf{W}_2 | $H \times D_y$ |
| \mathbf{b}_2 | $1 \times D_y$ |

Therefore, the number of parameters in this neural network is:

$$\# \text{ parameters} = D_x H + H + H D_y + D_y = (D_x + 1)H + D_y(H + 1)$$

(e) Please see the coding portion of the assignment.

(f) Please see the coding portion of the assignment.

(g) Please see the coding portion of the assignment.

3. (a) Let's denote:

$$\begin{aligned}\theta_w &= \mathbf{u}_w^\top \mathbf{v}_c \\ \boldsymbol{\theta} &= \mathbf{U}^\top \mathbf{v}_c\end{aligned}$$

where θ_w is a scalar, \mathbf{u}_w and \mathbf{v}_c are column vectors of dimensions $N \times 1$, $\boldsymbol{\theta}$ is a column vector of dimension $V \times 1$, and $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_V]$ is a matrix of dimension $N \times V$.

The softmax predictions for every word can then be written as:

$$\hat{\mathbf{y}} = \frac{\exp(\mathbf{U}^\top \mathbf{v}_c)}{\sum_{w=1}^V \exp(\mathbf{u}_w^\top \mathbf{v}_c)} = \frac{\exp(\boldsymbol{\theta})}{\sum_{w=1}^V \exp(\theta_w)}$$

where $\hat{\mathbf{y}}$ is a column vector of softmax predictions for every word of dimension $V \times 1$.

By using chain rule and the result of 2(b), the gradient of the cross entropy cost w.r.t. \mathbf{v}_c can be derived as:

$$\begin{aligned}\frac{\partial}{\partial \mathbf{v}_c} J_{\text{softmax-CE}} &= \frac{\partial \boldsymbol{\theta}}{\partial \mathbf{v}_c} \frac{\partial J}{\partial \boldsymbol{\theta}} \\ &= \frac{\partial \mathbf{U}^\top \mathbf{v}_c}{\partial \mathbf{v}_c} \frac{\partial \text{CE}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \boldsymbol{\theta}} \\ &= \mathbf{U} (\hat{\mathbf{y}} - \mathbf{y})\end{aligned}$$

where \mathbf{y} is a column vector of expected word of dimension $V \times 1$.

(b) As in the previous part, we can apply chain rule and the result of 2(b):

$$\begin{aligned}\frac{\partial}{\partial \mathbf{u}_k} J_{\text{softmax-CE}} &= \frac{\partial \boldsymbol{\theta}}{\partial \mathbf{u}_k} \frac{\partial J}{\partial \boldsymbol{\theta}} \\ &= \frac{\partial \mathbf{U}^\top \mathbf{v}_c}{\partial \mathbf{u}_k} \frac{\partial \text{CE}(\mathbf{y}, \hat{\mathbf{y}})}{\partial \boldsymbol{\theta}} \\ &= \frac{\partial \mathbf{U}^\top \mathbf{v}_c}{\partial \mathbf{u}_k} (\hat{\mathbf{y}} - \mathbf{y})\end{aligned} \tag{3}$$

Rewriting matrix multiplication in (3) explicitly:

$$\begin{aligned}\frac{\partial}{\partial \mathbf{u}_k} J_{\text{softmax-CE}} &= \sum_j^V (\hat{\mathbf{y}} - \mathbf{y})_j \left(\frac{\partial \mathbf{U}^\top \mathbf{v}_c}{\partial \mathbf{u}_k} \right)_{\cdot j} \\ &= \sum_j^V (\hat{y}_j - y_j) \left(\frac{\partial \mathbf{U}^\top \mathbf{v}_c}{\partial \mathbf{u}_k} \right)_{\cdot j}\end{aligned}$$

where $\left(\frac{\partial \mathbf{U}^\top \mathbf{v}_c}{\partial \mathbf{u}_k} \right)_{\cdot j}$ is the j -th column of $\frac{\partial \mathbf{U}^\top \mathbf{v}_c}{\partial \mathbf{u}_k}$ which is a $N \times V$ matrix. It can be simplified to:

$$\left(\frac{\partial \mathbf{U}^\top \mathbf{v}_c}{\partial \mathbf{u}_k} \right)_{\cdot j} = \begin{cases} \mathbf{v}_c, & \text{if } j = k \\ 0, & \text{otherwise} \end{cases}$$

Therefore, the gradient can be simplified to:

$$\frac{\partial}{\partial \mathbf{u}_k} J_{\text{softmax-CE}} = (\hat{y}_k - y_k) \mathbf{v}_c$$

Specifically,

$$\frac{\partial}{\partial \mathbf{u}_k} J_{\text{softmax-CE}} = \begin{cases} (\hat{y}_k - 1) \mathbf{v}_c, & \text{if } k = o \\ \hat{y}_k \mathbf{v}_c, & \text{otherwise} \end{cases}$$

(c) Let's denote:

$$\begin{aligned}\theta_o &= \mathbf{u}_o^\top \mathbf{v}_c \\ \theta_k &= -\mathbf{u}_k^\top \mathbf{v}_c\end{aligned}$$

The gradient of the negative sampling loss w.r.t. \mathbf{v}_c is:

$$\begin{aligned}\frac{\partial}{\partial \mathbf{v}_c} J_{\text{neg-sample}} &= \frac{\partial}{\partial \mathbf{v}_c} \left[-\log(\sigma(\mathbf{u}_o^\top \mathbf{v}_c)) - \sum_{k=1}^K \log(\sigma(-\mathbf{u}_k^\top \mathbf{v}_c)) \right] \\ &= \frac{\partial}{\partial \mathbf{v}_c} \left[-\log(\sigma(\theta_o)) - \sum_{k=1}^K \log(\sigma(\theta_k)) \right] \\ &= -\frac{1}{\sigma(\theta_o)} \frac{\partial \sigma(\theta_o)}{\partial \theta_o} \frac{\partial \theta_o}{\partial \mathbf{v}_c} - \sum_{k=1}^K \frac{1}{\sigma(\theta_k)} \frac{\partial \sigma(\theta_k)}{\partial \theta_k} \frac{\partial \theta_k}{\partial \mathbf{v}_c} \\ &= -\frac{1}{\sigma(\theta_o)} \sigma(\theta_o)(1 - \sigma(\theta_o)) \frac{\partial \theta_o}{\partial \mathbf{v}_c} - \sum_{k=1}^K \frac{1}{\sigma(\theta_k)} \sigma(\theta_k)(1 - \sigma(\theta_k)) \frac{\partial \theta_k}{\partial \mathbf{v}_c} \\ &= (\sigma(\mathbf{u}_o^\top \mathbf{v}_c) - 1) \frac{\partial \mathbf{u}_o^\top \mathbf{v}_c}{\partial \mathbf{v}_c} + \sum_{k=1}^K (\sigma(-\mathbf{u}_k^\top \mathbf{v}_c) - 1) \frac{\partial (-\mathbf{u}_k^\top \mathbf{v}_c)}{\partial \mathbf{v}_c} \\ &= (\sigma(\mathbf{u}_o^\top \mathbf{v}_c) - 1) \mathbf{u}_o - \sum_{k=1}^K (\sigma(-\mathbf{u}_k^\top \mathbf{v}_c) - 1) \mathbf{u}_k\end{aligned}$$

Similarly, the gradient of the negative sampling loss w.r.t. \mathbf{u}_o is:

$$\begin{aligned}\frac{\partial}{\partial \mathbf{u}_o} J_{\text{neg-sample}} &= (\sigma(\mathbf{u}_o^\top \mathbf{v}_c) - 1) \frac{\partial \mathbf{u}_o^\top \mathbf{v}_c}{\partial \mathbf{u}_o} + \sum_{k=1}^K (\sigma(-\mathbf{u}_k^\top \mathbf{v}_c) - 1) \frac{\partial (-\mathbf{u}_k^\top \mathbf{v}_c)}{\partial \mathbf{u}_o} \\ &= (\sigma(\mathbf{u}_o^\top \mathbf{v}_c) - 1) \mathbf{v}_c\end{aligned}$$

And the gradient of the negative sampling loss w.r.t. \mathbf{u}_k is (changing summation indices from k to j to avoid confusion):

$$\begin{aligned}\frac{\partial}{\partial \mathbf{u}_k} J_{\text{neg-sample}} &= (\sigma(\mathbf{u}_o^\top \mathbf{v}_c) - 1) \frac{\partial \mathbf{u}_o^\top \mathbf{v}_c}{\partial \mathbf{u}_k} + \sum_{j=1}^K (\sigma(-\mathbf{u}_j^\top \mathbf{v}_c) - 1) \frac{\partial (-\mathbf{u}_j^\top \mathbf{v}_c)}{\partial \mathbf{u}_k} \\ &= (1 - \sigma(-\mathbf{u}_k^\top \mathbf{v}_c)) \mathbf{v}_c\end{aligned}$$

This cost function is much more efficient to compute than the softmax-CE loss because the computation of $\frac{\partial J}{\partial \mathbf{v}_c}$ for softmax-CE loss scales as V while the computation of $\frac{\partial J}{\partial \mathbf{v}_c}$ for negative sampling loss scales as K , resulting in a speed-up ratio of K/V , which could make a huge difference if one has a big vocabulary.

- (d) For skip-gram, the cost for a context centered around c is:

$$J_{\text{skip-gram}}(w_{t-m}, \dots, w_{t+m}) = \sum_{-m \leq j \leq m, j \neq 0} F(w_{t+j}, \mathbf{v}_c)$$

where

$$F(o, \mathbf{v}_c) = \begin{cases} J_{\text{softmax-CE}}(o, \mathbf{v}_c, \dots) = \text{CE}(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_i y_i \log(\hat{y}_i), & \text{for softmax-CE loss} \\ J_{\text{neg-sample}}(o, \mathbf{v}_c, \dots) = -\log(\sigma(\mathbf{u}_o^\top \mathbf{v}_c)) - \sum_{k=1}^K \log(\sigma(-\mathbf{u}_k^\top \mathbf{v}_c)), & \text{for negative sampling loss} \end{cases}$$

Therefore, the gradients w.r.t. the word vectors for the skip-gram model are:

$$\begin{aligned} \frac{\partial}{\partial \mathbf{v}_k} J_{\text{skip-gram}}(w_{t-m}, \dots, w_{t+m}) &= \begin{cases} \sum_{-m \leq j \leq m, j \neq 0} \frac{\partial F(w_{t+j}, \mathbf{v}_c)}{\partial \mathbf{v}_c}, & \text{if } k = c \\ 0, & \text{otherwise} \end{cases} \\ \frac{\partial}{\partial \mathbf{u}_k} J_{\text{skip-gram}}(w_{t-m}, \dots, w_{t+m}) &= \sum_{-m \leq j \leq m, j \neq 0} \frac{\partial F(w_{t+j}, \mathbf{v}_c)}{\partial \mathbf{u}_k} \end{aligned}$$

For CBOW, the cost is:

$$J_{\text{CBOW}}(w_{t-m}, \dots, w_{t+m}) = F(w_t, \hat{\mathbf{v}})$$

where

$$\hat{\mathbf{v}} = \sum_{-m \leq j \leq m, j \neq 0} \mathbf{v}_{w_{t+j}}$$

Therefore, the gradients w.r.t. the word vectors for the CBOW model are:

$$\begin{aligned} \frac{\partial}{\partial \mathbf{v}_k} J_{\text{CBOW}}(w_{t-m}, \dots, w_{t+m}) &= \begin{cases} \frac{\partial F(w_t, \hat{\mathbf{v}})}{\partial \hat{\mathbf{v}}} \frac{\partial \hat{\mathbf{v}}}{\partial \mathbf{v}_k} = \frac{\partial F(w_t, \hat{\mathbf{v}})}{\partial \hat{\mathbf{v}}}, & \text{if } t-m \leq k \leq t+m \text{ and } k \neq t \\ 0, & \text{otherwise} \end{cases} \\ \frac{\partial}{\partial \mathbf{u}_k} J_{\text{CBOW}}(w_{t-m}, \dots, w_{t+m}) &= \frac{\partial F(w_t, \hat{\mathbf{v}})}{\partial \mathbf{u}_k} \end{aligned}$$

- (e) Please see the coding portion of the assignment.
- (f) Please see the coding portion of the assignment.
- (g) Figure 1 shows the visualization for the word vectors. First we can immediately notice it that all the adjectives cluster together, while articles and punctuations scatter around. Also, we observe that many pairs of adjectives that have opposite meanings form vectors that point to approximately the same direction, such as bad \rightarrow good, boring \rightarrow enjoyable, dumb \rightarrow brilliant, and waste \rightarrow worth.
- (h)

4. (a) Please see the coding portion of the assignment.
- (b) Regularization helps prevent overfitting and reduce model complexity, which would help increase prediction and generalization power to new datasets.
- (c) Please see the coding portion of the assignment.

The code for `chooseBestModel` is as follow:

```
# Pick the result with the best validation accuracy
bestResult = max(results, key=lambda result: result['dev'])
```

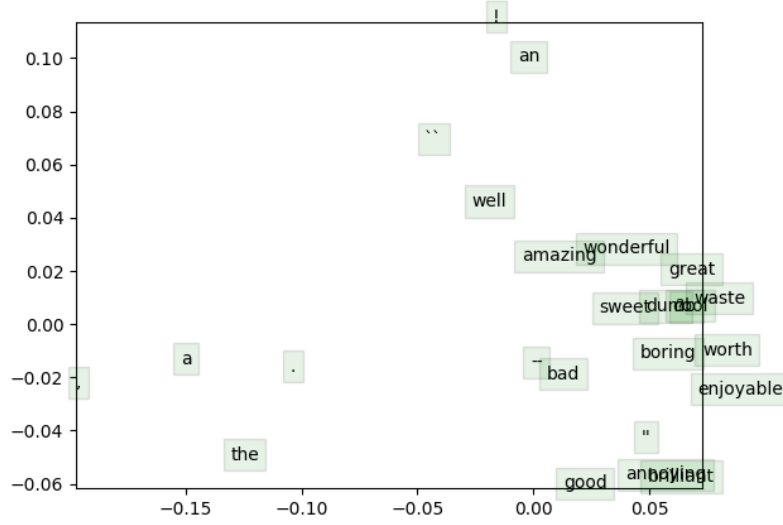


FIGURE 1. Visualization for the word vectors.

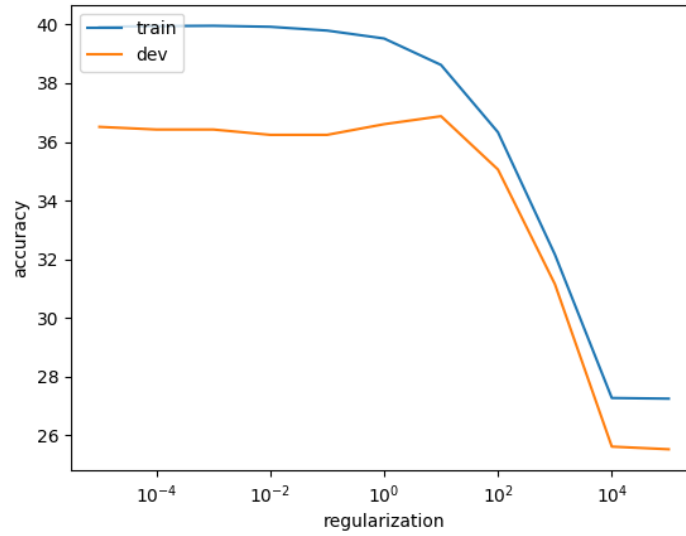


FIGURE 2. Classification accuracy on the train and dev set with respect to the regularization value for the pretrained GloVe vectors.

(d) These regularization parameters were used:

$$\lambda = [10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 10^0, 10^1, 10^2, 10^3, 10^4, 10^5]$$

and the best train, dev, and test accuracies are as follows:

| Best accuracy | --yourvectors | --pretrained |
|---------------|---------------|--------------|
| train | 31.156 | 39.958 |
| dev | 32.698 | 36.876 |
| test | 30.271 | 37.692 |

The pretrained vectors did better, because (1) the pretrained vectors has a much higher dimension ($N = 50$) than the word vector in q3 ($N = 10$), which presumably would capture more information; (2) the pretrained vectors were trained on many more samples (the whole Wikipedia dataset), versus

the much smaller Stanford Sentiment Treebank dataset (only 19539 tokens); (3) the pretrained vectors were trained with GloVe, which is probably better than skip-gram.

- (e) Figure 2 shows the classification accuracy on the train and dev set with respect to the regularization value for the pretrained GloVe vectors. We can see that the train accuracy is always higher than the dev accuracy, which makes sense. We can also observe that the accuracies change relatively little when the regularization value is smaller than 1 but experience sharp dropoffs when it increases.

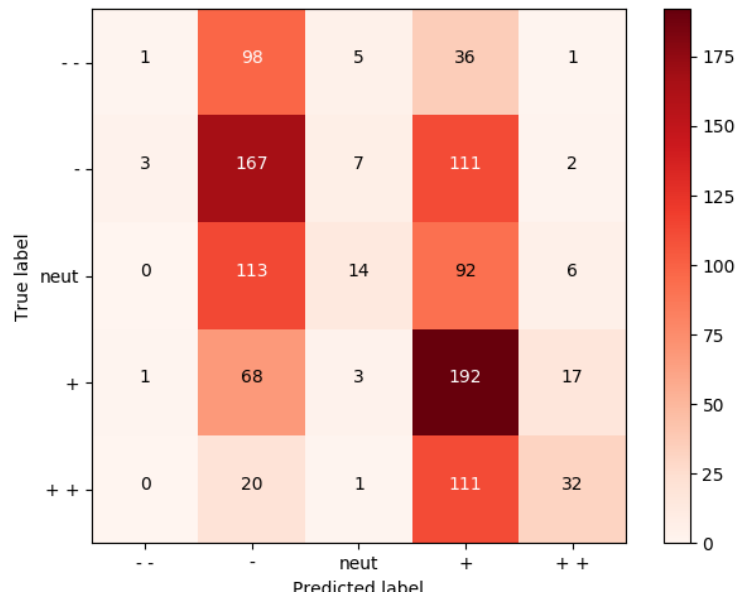


FIGURE 3. Confusion matrix for the pretrained GloVe vectors.

- (f) The confusion matrix is shown in figure 3. It looks like the classifier did a reasonably good job determining the general sentiment of sentence, i.e. whether a sentence is positive or not. It means that the classification accuracy would probably improve if this was implemented with only 3 classes (+, neutral, -) instead of 5 classes. Also, it looks like the classifier did not do a great job at classifying neutral samples, i.e. most neutral samples got classified as either positive or negative.

- (g) Here are 3 examples:

- (i) “despite its title , punch-drunk love is never heavy-handed”

Its true label is + but it got classified as -, probably because of the word “heavy-handed”. The problem may have been stemmed from the fact that averaging all word vectors within a sentence removes word order, and thus cannot process negation (the word “never”). The classifier may benefit from features that take the nearby words (e.g. preceding “not” or “never”) into account.

- (ii) “instead , he shows them the respect they are due .”

Its true label is ++ but it got classified as -. It may have been classified as - because of the word “due”, but in reality it doesn’t have too many strong sounding words. The classifier may benefit from learning phrases that have specific meanings “show respect that are due”.

- (iii) “a movie that successfully crushes a best selling novel into a timeframe that mandates that you avoid the godzilla sized soda .”

Its true label is + but it got classified as -. It may have been classified as - because of negative sounding words such as “crushes” and “avoid”. To understand this sentence correctly, the classifier will need to be able to parse different parts of the sentence (e.g. “a movie”, “successfully crushes”, “a best selling novel”) and perform comparisons.