# Using the Rebalancing Strategy to Construct Hedged CFMM LP Returns

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## 1. Introduction

The top panel of Figure 1 shows the dollar returns to liquidity provision in the Uniswap v2 ETH-USDC pair. We measure this simply as how the US dollar value of LP assets changes over time, accounting for mints and burns. Under this measure – the red line – LP returns appear very risky, with frequent gains or losses of tens of millions of dollars daily. However, this is mostly *market risk*: the Uni v2 pool holds a large long ETH position, and ETH prices are very volatile. In Milionis et al. [2022], we show that market risk can be removed simply by shorting the *rebalancing strategy*. The residual after hedging away market risk is a purer measure of LP returns. The result is the yellow line, also shown magnified on the bottom panel. **Hedging decreases the volatility of LP P&L by a factor of almost 100:** daily hedged P&L fluctuates between approximately 0 and $300,000 USD!

Studying delta-hedged LP returns thus eliminates a large amount of noise when analyzing LP returns, allowing empirical researchers to focus on analyzing economic drivers of LP returns with increased statistical power. This note is a simple practitioner's guide illustrating how to implement our delta-hedging procedure, moving from the red line to the yellow line. The procedure has extremely low data requirements: the researcher needs data on LP holdings, which essentially all researchers studying LP profitability have access to; and CEX prices of risky assets, which are widely available from various free and paid data providers. The data and code, in R and Stata, for these calculations is available here.

## 2. Data requirements

**CEX prices.** The researcher must have a time series of prices for the risky asset, generally from a centralized exchange (CEX). There are many sources for CEX price data; some free options include Coingecko for daily data, and Binance for higher-frequency data. Paid data providers for CEX data include Tardis, Kaiko, and Cryptotick.

**Pool holdings.** To construct the rebalancing strategy, the researcher needs a time series $x_t$ of how much risky assets a CFMM LP position holds. For example, for the Uniswap v2 ETH-USDC pair, the researcher must have a time series of ETH held in the LP pool. To measure LP profits, the researcher will generally need time series of LP holdings of all assets – in the Uni v2 example, ETH and USDC – as well as mints and burns. In our example, we gathered minute-level data on how much ETH and USDC is minted or burned in the ETH-USDC pool in each minute. Our method can also in principle be applied with data at more aggregated levels, such as hourly or daily data.

**LP P&L.** We measure the P&L on the LP position in a standard manner. Let $y_t$ and $x_t$ be respectively USDC and ETH held by the pool at time $t$, and let $P_t$ be the CEX price. We define $V_t$ as the monetary value of pool reserves at time $t$, if all sold at CEX prices:

$$V_t = y_t + P_t x_t \tag{1}$$

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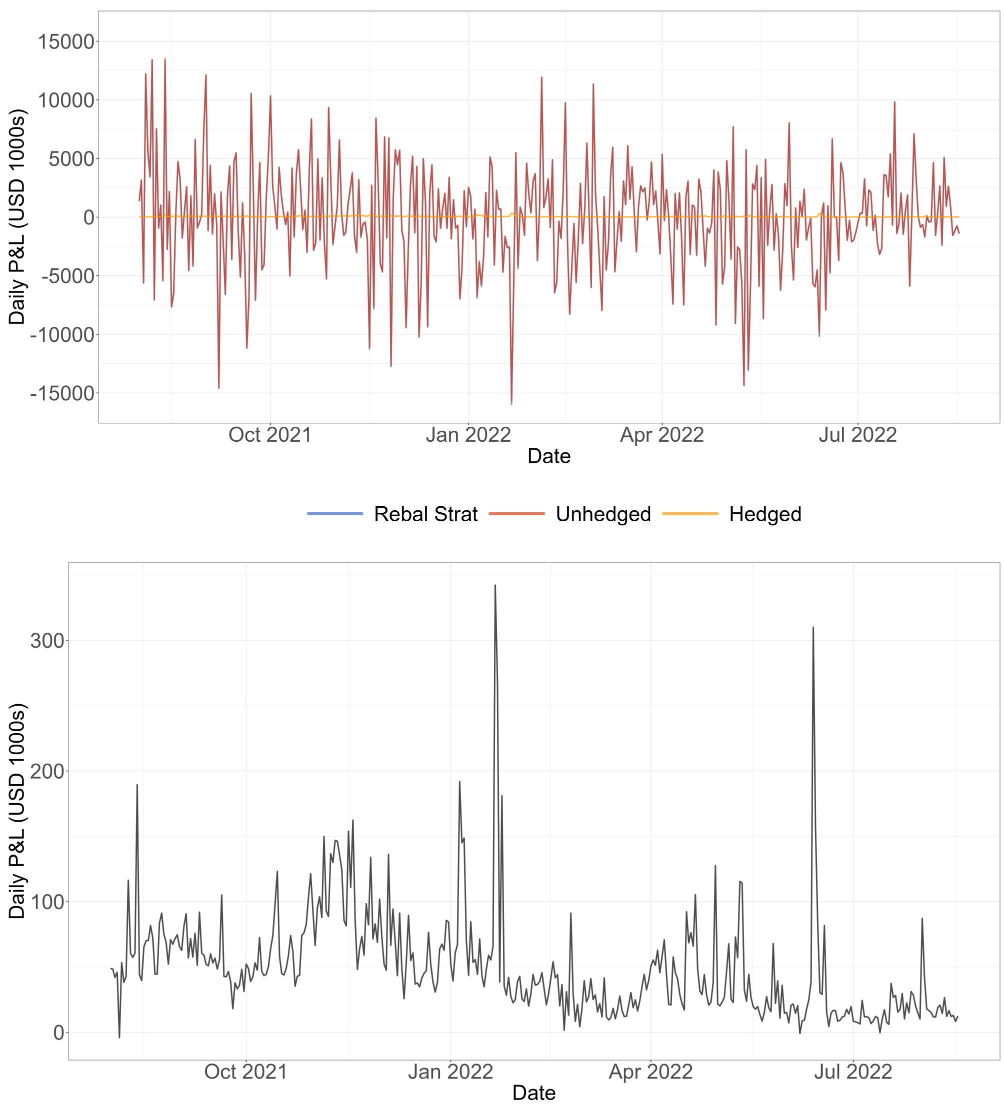
**Figure 1:** Unhedged LP P&L, Rebalancing Strategy P&L, and Hedged LP P&L




Notes. The top panel shows unhedged pool P&L, defined by (2), in red; the P&L from the one-minute-updated rebalancing strategy, defined by (3), in blue; and hedged LP P&L, the difference between unhedged P&L and the rebalancing strategy P&L, in yellow. The blue line and red line are very close to overlaid with each other. The bottom panel shows hedged P&L on a renormalized axis for clarity. All units are USD thousands.

If there were no minting and burning, LP P&L in period $t$ would simply be the change in the monetary value of the pool's holdings, $V_t - V_{t-1}$. To account for mints and burns, we define LP P&L in period $t$ as:

$$\Delta\text{LP }\mathsf{P\&L}_t \triangleq V_t + \Pi_t^{\text{burn}} - \Pi_t^{\text{mint}} - V_{t-1}. \tag{2}$$

where $\Pi_t^{\text{burn}}$ and $\Pi_t^{\text{mint}}$ are the monetary value of burns and mints. In our analysis of the Uni v2 ETH-USDC pair, we will set:

$$\Pi_t^{\text{mint}} = y_t^{\text{mint}} + P_t x_t^{\text{mint}}$$
$$\Pi_t^{\text{burn}} = y_t^{\text{burn}} + P_t x_t^{\text{burn}}$$

Essentially, a mint can be thought of as a "buy in", and a burn as a "cash out". That is, if in period $t$ $y_t$ USDC and $x_t$ ETH are "minted" into the LP pool, we value the ETH as a "buy in" at the CEX closing price $P_t$ in the minute the mint occured; if $x_t$ ETH is burned, we value it as a "cash out" at the CEX price $P_t$.

**Rebalancing strategy profits.** The rebalancing strategy aims to holds as much of the risky asset as the CFMM LP position does at any point in time, thus having the same exposure to market risk. Let $x_t^{RB}$ denote the ETH held by the rebalancing strategy. In the theoretical analysis of Milionis et al. [2022], we set $x_t^{RB}$ exactly equal to the pool's holdings $x_t$, so the rebalancing strategy updates holdings whenever the pool's holdings update. However, when implementing the rebalancing strategy in practice, due to data limitations, we may update $x_t^{RB}$ less frequently. For example, if we observe the CFMM LP's holdings periodically – at the end of every minute, or hour, or day – we may construct the rebalancing strategy to update, say, at the end of every hour, so that $x_t^{RB}$ is constant within each hour.[1] In Section 5 below, we show that hedging frequency does not appear to substantially affect results, in the setting we analyze.

Regardless of how $x_t^{RB}$ is constructed, the dollar return on the rebalancing strategy in period $t$ is simply:

$$\Delta\text{RB }\mathsf{P\&L}_t = x_t^{RB}\left(P_{t+1} - P_t\right). \tag{3}$$

For example, if the rebalancing holds $x_t^{RB}$ ETH in period $t$, and ETH prices increase by $P_{t+1} - P_t = \$2$, then the rebalancing strategy makes $\$2x_t$ dollars.

**Hedged P&L.** Hedged P&L is simply the return from a long position in the LP, and a short position in the rebalancing strategy – that is, the difference between LP P&L and the rebalancing strategy's P&L:

$$\Delta\text{Hedged }\mathsf{P\&L}_t = \Delta\text{LP }\mathsf{P\&L}_t - \Delta\text{RB }\mathsf{P\&L}_t$$

As Milionis et al. [2022] discuss, this essentially "delta-hedges" the LP position: the trader invests in the LP position, and then maintains a short position on a CEX which shorts as much of the risky asset as the LP position is long at any point in time, thus removing the market risk component of $\Delta\text{LP }\mathsf{P\&L}_t$. As a result, the trader collects fees, and is exposed to slippage losses from $\mathsf{LVR}$, but has no directional exposure to prices of the risky asset.

## 3. Numerical Example

Table 1 presents a simple 4-period numerical example of our calculations.[2] We illustrate a hypothetical world in which, from January 1st to 4th, ETH prices move from \$2000 to \$2200, back to \$2000, and then down to \$1750. Columns 3 and 4 show the pool's hypothetical ETH and USD holdings.

We calculate $V_t$ (column 5), the monetary value of pool holdings each day, just as the cash value of pool holdings. For example, on January 2nd, the pool holds 0.954 ETH and 2098 USD, and the ETH price is \$2200, hence:

$$V_{Jan\ 2} = (0.954)(2200) + 2098.67 = 4197.33$$

[1] Using the hedging interpretation of the rebalancing strategy, we can think of $x_t^{RB}$ with infrequent updates like a delta-hedged LP strategy, which updates the delta-hedge only infrequently, and thus incurs some additional market risk within each update period, when the CFMM LP's holdings diverge slightly from the rebalancing strategy's holdings.

[2] For simplicity we assume there are no mints and burns. This example is loosely structured around the CPMM with fees, with a few approximations: we generate holdings using the $xy = k$ function each period, then scale up holdings by a factor 1.0005 as if there is a 5bps fee. This is not exactly how LP holdings would evolve with fees in practice, but we use this simply to illustrate our results.

**Table 1:** Stylized Numerical Example of $\Delta$Hedged $\mathsf{P\&L}_t$ calculations

| Date | ETH price | ETH held | USD held | $V_t$ | $\Delta$LP $\mathsf{P\&L}_t$ | $\Delta$RB $\mathsf{P\&L}_t$ | $\Delta$Hedged $\mathsf{P\&L}_t$ |
|---|---|---|---|---|---|---|---|
| Jan 1 | \$2000.00 | 1.000 | \$2000.00 | \$4000.00 | | | |
| Jan 2 | \$2200.00 | 0.954 | \$2098.67 | \$4197.33 | \$197.33 | \$200.00 | \$-2.67 |
| Jan 3 | \$2000.00 | 1.001 | \$2002.00 | \$4004.00 | \$-193.33 | \$-190.79 | \$-2.54 |
| Jan 4 | \$1750.00 | 1.071 | \$1873.64 | \$3747.27 | \$-256.73 | \$-250.25 | \$-6.48 |

$\Delta$LP $\mathsf{P\&L}_t$ (column 6) is then just the increments of $V_{Jan\ 2}$: from Jan 1 to Jan 2, the LP pool makes \$197.33 in USD value.

We calculate $\Delta$RB $\mathsf{P\&L}_t$ (column 7) by simply taking the lagged ETH holdings and multiplying by the change in prices. For example, the rebalancing strategy holds 1 ETH from Jan 1 to Jan 2, hence $\Delta$RB $\mathsf{P\&L}_t$ is:

$$(ETH\ holdings) \times \left(p^{ETH}_{Jan\ 2} - p^{ETH}_{Jan\ 1}\right) = (1)\,(2200 - 2000) = 200$$

Similarly, for Jan 2 to Jan 3, the rebalancing strategy holds around 0.954 ETH – the amount held by the LP on Jan 2 – hence the rebalancing strategy has profits:

$$(ETH\ holdings) \times \left(p^{ETH}_{Jan\ 3} - p^{ETH}_{Jan\ 2}\right) = (0.954)\,(2000 - 2200) = -190.79$$

$\Delta$Hedged $\mathsf{P\&L}_t$ (column 8) is then just the difference between $\Delta$LP $\mathsf{P\&L}_t$. For example, from Jan 1 to 2, the rebalancing strategy makes \$200, whereas the LP pool value only gains \$197.33; hence, hedged LP P&L is -\$2.67 USD.

Analyzing Table 1, we see that the pool's value, in column 4, grows and shrinks with ETH prices, because the pool is systematically long ETH. Thus, $\Delta$LP $\mathsf{P\&L}_t$ in Column 5 is positive whenever prices increase, and negative whenever prices decrease. The goal of the rebalancing strategy is to hedge away this exposure: we see that $\Delta$RB $\mathsf{P\&L}_t$ similarly is positive or negative depending on whether prices increased or decreased. $\Delta$Hedged $\mathsf{P\&L}_t$ in column 8 – the difference between $\Delta$LP $\mathsf{P\&L}_t$ and $\Delta$RB $\mathsf{P\&L}_t$ – is much smaller in magnitude than $\Delta$LP $\mathsf{P\&L}_t$ and $\Delta$RB $\mathsf{P\&L}_t$, since "noise" from market risk exposure is eliminated. $\Delta$Hedged $\mathsf{P\&L}_t$ also tells a very different story from $\Delta$LP $\mathsf{P\&L}_t$. Hedged P&L is negative in all periods in our example: the CFMM does not collect enough in fees to offset what is lost in price slippage.

Note that it is an empirical question whether $\Delta$Hedged $\mathsf{P\&L}_t$ is positive or negative: we will now show a data-based example where $\Delta$Hedged $\mathsf{P\&L}_t$ turns out to be positive.

## 4. Data Example: The Uniswap v2 ETH-USDC Pair

We illustrate our methodology in the context of the Uniswap v2 ETH-USDC pair. We construct the rebalancing strategy $x_t$ at minutely, hourly, and daily frequencies, simply by setting holdings equal to the LP pool's total quantity of ETH held at the start of each minute, hour, or day respectively.

The bottom panel of Figure 1 shows the series for $\Delta$LP $\mathsf{P\&L}_t$ and $\Delta$RB $\mathsf{P\&L}_t$. The red line shows the pool P&L, defined by (2). The pool's value is very volatile, frequently gaining or losing over 10 million dollars within a day. However, the vast majority of this volatility is simply market risk: the pool has a long position in ETH, whose price is very volatile. The rebalancing strategy's return, shown in blue, is almost identical to pool P&L: the blue line is slightly below the red line at the troughs, but is otherwise almost exactly overlaid. Shorting the rebalancing strategy thus hedges the vast majority of risk in the LP pool.

The yellow line shows hedged P&L – the difference between pool P&L and the rebalancing strategy's P&L, which can be thought of as the returns from taking a long position in the CFMM LP, and a short position in the rebalancing strategy. The yellow line is much less risky and the red line. The bottom panel of Figure 1 zooms in to hedged P&L: the hedged LP position returns roughly \$0 to \$300,000 USD each day, with volatility almost 100x lower than the unhedged P&L line.

This exercise illustrates that the majority of risk in LP P&L is simply market risk, and that this risk can be accounted for simply by shorting the rebalancing strategy. Studies focusing on analyzing the determinants of

LP returns can thus dramatically reduce noise by simply accounting for market risk through the rebalancing strategy.

## 5. The Effect of Hedging Frequency

The frequency at which the rebalancing strategy $x_t^{RB}$ is adjusted affects the results, but the effect is quantitatively relatively small. Figure 2, which is essentially Figure 5 in Milionis et al. [2022], compares the results from constructing $x_t^{RB}$ at 5 different frequencies: 1 minute, 4 minutes, 1 hour, 4 hours, and daily. The top panel shows cumulative P&L, calculated by summing $\Delta\text{LP }\mathsf{P\&L}_t$ and $\Delta\text{RB }\mathsf{P\&L}_t$ from the start of our time period. All hedging frequencies generate substantially smoother cumulative P&L, relative to the unhedged pool P&L. Moreover, all frequencies besides daily rebalancing are visually very similar.

The bottom panel shows non-cumulative, daily P&L, for all hedging frequencies other than daily. For all hedging frequencies, the pool P&L series appears fairly similar, suggesting that hedging frequencies should not have a very large effect on any empirical findings based on hedged P&L. In practice, we suggest trying a few different hedging frequencies to ensure that results are robust, though it is unlikely that different frequencies will produce substantially different results.

## 6. Extensions

**Fees not paid in kind.** In the setting of Uniswap v2, fees do not need to be explicitly accounted for, because they are collected in-kind, increasing total pool reserves: if fees are high and prices do not change, then pool reserves $x_t, y_t$ will increase, and thus the value $V_t$ of the pool will increase through (1). For a hypothetical CFMM which paid fees out to LPs, rather than directly accruing trading fees into pool reserves, we would simply add a term $Fee_t$, representing the monetary value of trading fees paid in period $t$.

**Multiple risky assets.** If the LP pool has two risky assets, and no numeraire, we simply construct the rebalancing strategy by buying and holding both of the risky assets. Letting $x_{1,t}^{RB}$ and $x_{2,t}^{RB}$ denote the holdings of the two risky assets, and $P_{1,t}$ and $P_{2,t}$ denote prices, we then have:

$$\Delta\text{RB }\mathsf{P\&L}_t = x_{1,t}^{RB}\left(P_{1,t+1} - P_{1,t}\right) + x_{2,t}^{RB}\left(P_{2,t+1} - P_{2,t}\right)$$

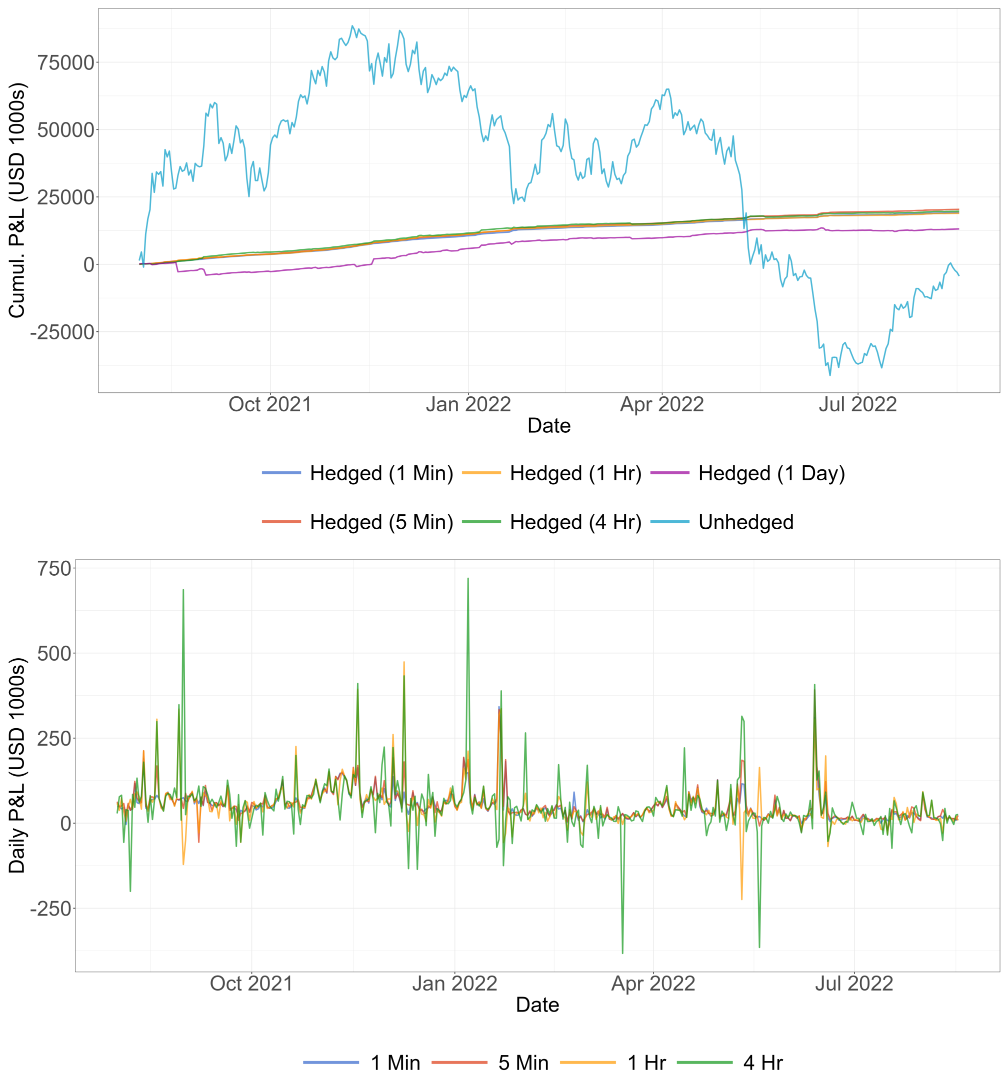
Similarly, in pools with $N > 2$ assets $i = 1 \ldots N$, the rebalancing strategy simply holds some quantity of each, and has profits:

$$\Delta\text{RB }\mathsf{P\&L}_t = \sum_{i=1}^{N} x_{i,t}^{RB}\left(P_{i,t+1} - P_{i,t}\right)$$

## References


Jason Milionis, Ciamac C Moallemi, Tim Roughgarden, and Anthony Lee Zhang. Automated market making and loss-versus-rebalancing. *arXiv preprint arXiv:2208.06046*, 2022.

**Figure 2:** Cumulative unhedged and hedged P&L, for different hedging frequencies




The top panel, which is essentially Figure 5 in Milionis et al. [2022], shows cumulative unhedged pool P&L, and hedged P&L for different hedging frequencies, calculated by summing daily P&L from the start of our sample. The bottom panel shows daily hedged P&L for different hedging frequencies.