Assignment 3 Part 2: Anthony, William

1. Consider a relation V with attributes LMNOPQRST and functional dependencies W.
W = { LPR → Q, LR → ST, M → LO, MR → N }

(a) State which of the given FDs violate BCNF.

LPR^+ = LPRQST
is not a superkey thus violates BCNF

LR^+ = LRST
also violates BCNF

M^+ = LMO
also violates BCNF

MR^+ = LMNORST
does not have P and Q thus violates BCNF

(b) Employ the BCNF decomposition algorithm to obtain a lossless and redundancy-preventing decomposition of relation R into a collection of relations that are in BCNF. Make sure it is clear which relations are in the final decomposition, and don't forget to project the dependencies onto each relation in that final decomposition. Because there are choice points in the algorithm, there may be more than one correct answer. List the final relations in alphabetical order (order the attributes alphabetically within a relation, and order the relations alphabetically).

Decompose R using FD LPR → Q
R_1 = LPRQST
R_2 = LMNOPR

Project onto R1 = LPRQST
L^+ = L nothing
P^+ = P nothing
R^+ “” nothing
O^+ “” nothing
S^+ “” nothing
T^+ “” nothing
LP^+ “” nothing
LR^+ = LRST LR → ST violates BCNF since it is not a superkey
Decompose R_1 using FD LR → ST

R_3 = LRST and R_4 = LPRQ

Project onto R_3

Single attributes all contribute nothing

LR^+ = LRST LR → ST this is a superkeyt

LS^+ = LS nothing

LT^+ = LT nothing

LRS^+ superset of LR^+

LRT^+ superset LR^+

LST^+ = LST nothing

$LRST^+$ superset of LR^+

This relation is in BCNF

Project onto R_4 = LPRQ

Single attributes all contribute nothing

LP^+ = LP nothing

LR^+ = LR nothing

LQ^+ = LQ nothing useful

LPR^+ = LPRQ this is a superset LPR → Q

LRQ^+ = LRQ nothing useful

All other three attribute combinations are weaker supersets of LPR^+

$LPRQ^+$ is the superset of LPR^+

This relation is in BCNF

Now return to R_2 = LMNOPR

Projecting on R_2

M^+ = MLO M → LO is not a superkey

Decompose to

R_5 = MLO

R_6 = MNPR

Project onto R_5 = MLO

M^+ = MLO M → LO is a superkey of R_5

L^+ = L nothing

O^+ = O nothin

LO^+ = LO nothing

Everything else is a superset of M^+

This relation is in BCNF

Project onto R_6

R_6 = MNPR

M^+ = M nothing

N^+ = N nothing

P^+ = P nothing

R^+ = R nothing

MN^+ = MN nothing
MP^+ = MP nothing
MR^+ = MRN MR $\rightarrow$ N is not a superkey of this relation

Decompose to
R_7 = MRN
R_8 = MPR

Project onto R_7 = MRN
M^+ = M nothing
R^+ = R nothing
N^+ = N nothing
MR^+ = MRN MR $\rightarrow$ N is a superkey
RN^+ = RN nothing
Everything else is a superset of MR^+
Thus this relation is in BCNF

Project onto R_8 = MPR
M^+ = M nothing
P^+ = P nothing
N^+ = N nothing
Two attributes all contribute nothing
All three together trivial superkey

Final decomposition
R_3 = LRST with LR $\rightarrow$ ST
R_4 = LPQR with LPR $\rightarrow$ Q
R_5 = LMO with M $\rightarrow$ LO
R_7 = MNR with MR $\rightarrow$ N
R_8 = MPR with no FDs

2 (a) Compute a minimal basis for T. In your final answer, put the FDs into alphabetical order.

- We'll simplify to singleton right-hand sides and call this set S1:

  1 AB $\rightarrow$ C
  2 AB $\rightarrow$ D
  3 ACDE $\rightarrow$ B
  4 ACDE $\rightarrow$ F
  5 B $\rightarrow$ A
  6 B $\rightarrow$ C
  7 B $\rightarrow$ D
  8 CD $\rightarrow$ A
  9 CD $\rightarrow$ F
  10 CDE $\rightarrow$ F
  11 CDE $\rightarrow$ G
  12 EB $\rightarrow$ D

- We'll look for redundant FDs to eliminate.

FD	Exclude these from S1 when computing closure	Closure	Decision
1	1	AB+ = ABDCF	discard
2	1, 2	AB+ = ABCDF	discard
3	1, 2, 3	There's no way to get B without this FD	keep
4	1, 2, 4	ACDE+ = ACDEFG	discard
5	1, 2, 4, 5	B+ = BCDAF	discard
6	1, 2, 4, 5, 6	There's no way to get C without this FD	keep
7	1, 2, 4, 5, 7	B+ = BC	keep
8	1, 2, 4, 5, 8	There's no way to get A without this FD	keep
9	1, 2, 4, 5, 9	CD+ = CDA	keep
10	1, 2, 4, 5, 10	CDE+ = CDEGAFB	discard
11	1, 2, 4, 5, 10, 11	There's no way to get G without this FD	keep
12	1, 2, 4, 5, 10, 12	EB+ = EBCDAFG	discard

- Let's call the remaining FDs S2:

  3 ACDE $\rightarrow$ B
  6 B $\rightarrow$ C
  7 B $\rightarrow$ D
  8 CD $\rightarrow$ A
  9 CD $\rightarrow$ F
  11 CDE $\rightarrow$ G

- Let's try reducing the LHS

  3 ACDE $\rightarrow$ B
  A+ = A so we can't reduce the LHS to A.
  C+ = C so we can't reduce the LHS to C.
  D+ = D so we can't reduce the LHS to D.
  E+ = E so we can't reduce the LHS to E.
  AC+ = AC so we can't reduce the LHS to AC.
  AD+ = AD so we can't reduce the LHS to AD.
  AE+ = AE so we can't reduce the LHS to AE.
  CD+ = CDA . . . so we can reduce the LHS to CDE.

8 CD $\rightarrow$ A
C+ = C so we can't reduce the LHS to C.
D+ = D so we can't reduce the LHS to D.
So this FD remains as it is

9 CD $\rightarrow$ F
C+ = C so we can't reduce the LHS to C.
D+ = D so we can't reduce the LHS to D.
So this FD remains as it is

11 CDE $\rightarrow$ G
C+ = C so we can't reduce the LHS to C.
D+ = D so we can't reduce the LHS to D.
E+ = E so we can't reduce the LHS to E.
CD+ = CDAF so we can't reduce the LHS to CD.
CE+ =CE so we can't reduce the LHS to CE.
DE+ = DE so we can't reduce the LHS to DE.
So this FD remains as it is

- Let's call the set of FDs that we have after reducing left-hand sides S3:

3' CDE $\rightarrow$ B
6 B $\rightarrow$ C
7 B $\rightarrow$ D
8 CD $\rightarrow$ A
9 CD $\rightarrow$ F
11 CDE $\rightarrow$ G

- Look again in case any of the changes we made allow further simplification.

FD	Exclude these from S1 when computing closure	Closure	Decision
3'	3'	There's no way to get B without this FD	keep
6	6	There's no way to get C without this FD	keep
7	7	There's no way to get D without this FD	keep
8	8	There's no way to get A without this FD	keep
9	9	There's no way to get F without this FD	keep
11	11	There's no way to get G without this FD	keep

- No further simplifications are possible.
- So the following set S4 is a minimal basis:

1 B $\rightarrow$ C
2 B $\rightarrow$ D
3 CD $\rightarrow$ A
4 CD $\rightarrow$ F
5 CDE $\rightarrow$ B
6 CDE $\rightarrow$ G

(b) Using your minimal basis from the last subquestion, compute all keys for P.

- To summarize, each attrtibutes in P:

Attribute	Appears on		Conclusion
	LHS	RHS	
H	-	-	must be in every key
E	$\surd$	-	must be in every key
A, F, G	-	$\surd$	is not in any key
B, C, D	$\surd$	$\surd$	must chek

- Therefore, we only need to consider the combination of B, C, D
  BEH+ = BEHCDAFG. So BEH is a key. CEH+ = CEH. This is not a key DEH+ = DEH. This is not a key
- The keys for P are BEH

(c) Employ the 3NF synthesis algorithm to obtain a lossless and dependency-preserving decomposition of relation P into a collection of relations that are in 3NF.

- Let's call the revised FDs S5:
  1 B $\rightarrow$ CD
  2 CD $\rightarrow$ AF
  3 CDE $\rightarrow$ BG
- The set of relations that would result would have these attributes:
  R1(BCD), R2(ACDF), R3(BCDEG)
- Since the attributes BCD occur within R3, we don't need to keep the relation R1.
- BEH is a key of P which is not in the set of relations so we need to add another relation that includes a key.
- So the final set of relations is:
  R2(ACDF), R3(BCDEG), R4(BEH)

(d) Does your schema allow redundancy?

- We need to find out the projection of the FDs onto each relation to ensure the FDs do not violate BCNF and therefore redundancy is allowed.
- Clearly, CD $\rightarrow$ AF will project onto the relation R4 and it is not a superkey of this relation.
- So yes, these schema allows redundancy.