

Fuzzy Logic | Set 2 (Classical and Fuzzy Sets)

Difficulty Level: Medium • Last Updated: 24 Jan, 2022

Prerequisite: Fuzzy Logic | Introduction

In this post, we will discuss classical sets and fuzzy sets, their properties and operations that can be applied on them.

Set: A set is defined as a collection of objects, which share certain characteristics.

Classical set

- 1. Classical set is a collection of **distinct** objects. For example, a set of students passing grades.
- 2. Each individual entity in a set is called a **member** or an **element** of the set.
- 3. The classical set is defined in such a way that the universe of discourse is splitted into two groups **members** and **non-members**. Hence, In case classical sets, **no partial membership exists**.
- 4. Let A is a given set. The membership function can be use to define a set A is given by:

$$\mu A(x)=\{1 \text{ if } x \in A$$

0 if x ∉ A }

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

- This operation is also called **logical OR**.
- Intersection:

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

- This operation is also called logical AND.
- Complement:

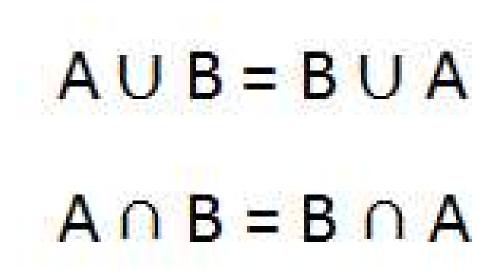
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• Difference:

$$A \mid B = \{x \mid x \in A \text{ and } x \notin B\}$$

- 1. Properties of classical sets: For two sets A and B and Universe X:
 - Commutativity:



• Associativity:

$$A \cup (B \cup C) = (A \cup B) \cup C$$

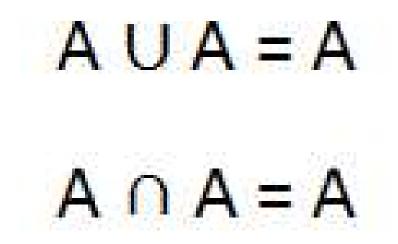
 $A \cap (B \cap C) = (A \cap B) \cap C$

Distributivity:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

• Idempotency:



• Identity:

- $A \cup \emptyset = A$
- $A \cap X = A$
- $A \cap \emptyset = \emptyset$
- $A \cup X = X$

If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

Fuzzy set:

- 1. **Fuzzy set** is a set having **degrees of membership** between 1 and 0. Fuzzy sets are represented with tilde character(~). For example, Number of cars following traffic signals at a particular time out of all cars present will have membership value between [0,1].
- 2. Partial membership exists when member of one fuzzy set can also be a part of other fuzzy sets in the same universe.
- 3. The degree of membership or truth is not same as probability, fuzzy truth represents membership in vaguely defined sets.
- 4. A fuzzy set A~ in the universe of discourse, U, can be defined as a set of ordered pairs and it is given by

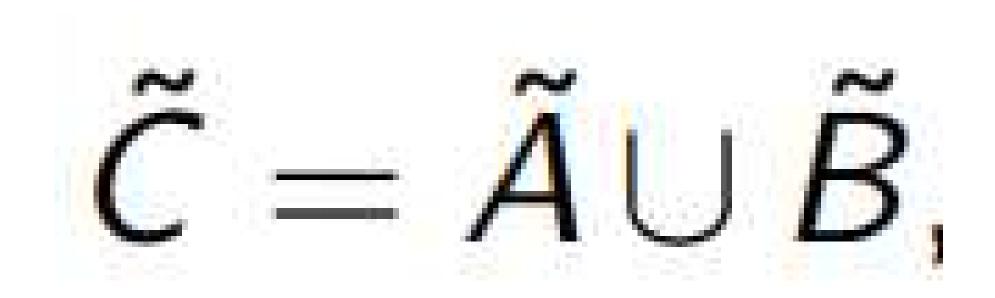
$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$$

1. When the universe of discourse, U, is **discrete and finite**, fuzzy set A~ is given by

$$ilde{A} = \sum_{i=1}^n rac{\mu_{ ilde{A}}(x_i)}{x_i} = rac{\mu_{ ilde{A}}(x_1)}{x_1} + rac{\mu_{ ilde{A}}(x_2)}{x_2} + \ldots + rac{\mu_{ ilde{A}}(x_n)}{x_n}$$

$$ilde{A} = \int rac{\mu_{ ilde{A}}(x)}{x_n} dx$$

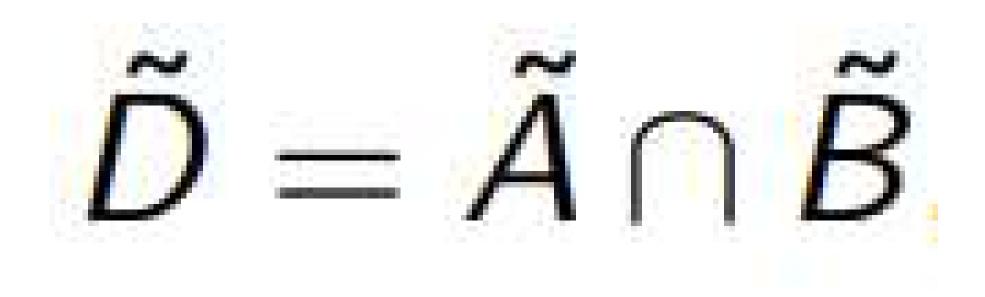
- 1. Fuzzy sets also satisfy every property of classical sets.
- 2. Common Operations on fuzzy sets: Given two Fuzzy sets A^{\sim} and B^{\sim}
 - **Union**: Fuzzy set C~ is union of Fuzzy sets A~ and B~:



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$$\mu_{\tilde{c}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

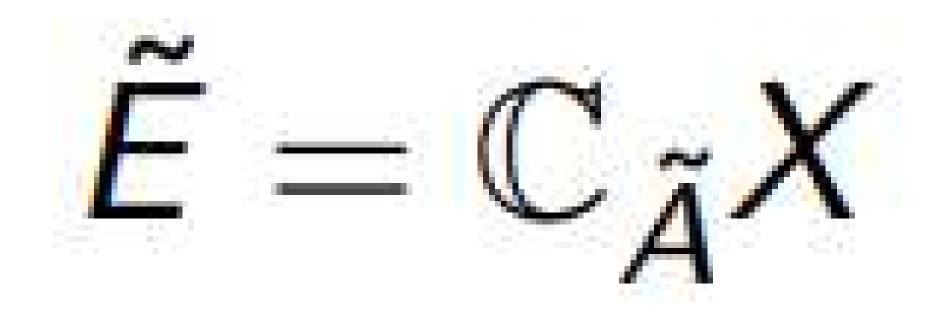
• **Intersection**: Fuzzy set D~ is intersection of Fuzzy sets A~ and B~:



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$$\mu_{\tilde{D}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

• Complement: Fuzzy set E~ is complement of Fuzzy set A~:



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$$\mu_{\tilde{E}}(x) = 1 - \mu_{\tilde{A}}(x)$$

- 1. Some other useful operations on Fuzzy set:
 - Algebraic sum:

$$\mu A + B(x) = \mu A(x) + \mu B(x) - \mu A(x)$$
. $\mu B(x)$

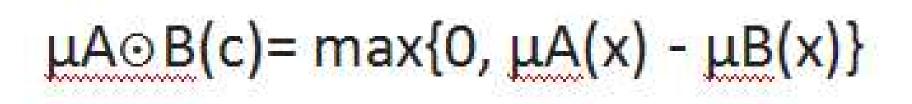
• Algebraic product:



• Bounded sum:

$$\mu A_{\oplus}B(x)=\min\{1, \mu A(x) + \mu B(x)\}$$

• Bounded difference:

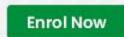


Sources:

- (1) http://staff.cs.upt.ro/~todinca/cad/Lectures/cad_fuzzysets.pdf
- (2) Principles of Soft Computing

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