Fuzzy Logic | Set 2 (Classical and Fuzzy Sets)

Difficulty Level : Medium • Last Updated : 24 Jan, 2022

Prerequisite : Fuzzy Logic | Introduction

In this post, we will discuss classical sets and fuzzy sets, their properties and operations that can be applied on them.

Set: A set is defined as a collection of objects, which share certain characteristics.

Classical set

1. Classical set is a collection of **distinct** objects. For example, a set of students passing grades.
2. Each individual entity in a set is called a **member** or an **element** of the set.
3. The classical set is defined in such a way that the universe of discourse is splitted into two groups **members** and **non-members**. Hence, In case classical sets, **no partial membership exists**.
4. Let A is a given set. The membership function can be use to define a set A is given by:

$$\mu A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases}$$

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

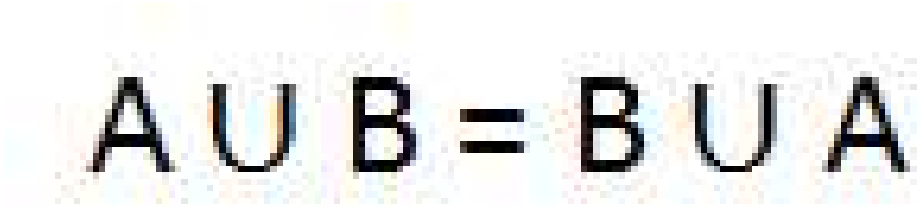
- This operation is also called **logical OR**.
- **Intersection**:

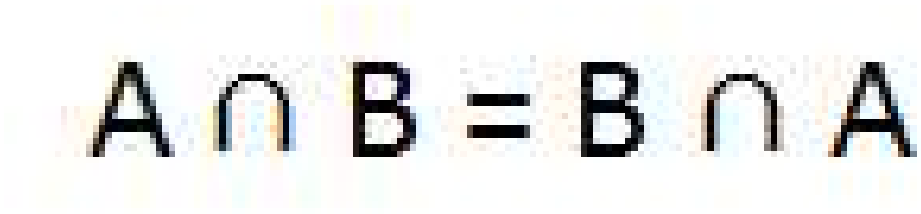
- This operation is also called **logical AND**.
- **Complement**:

- **Difference**:

$$A \mid B = \{x \mid x \in A \text{ and } x \notin B\}$$

1. **Properties of classical sets**: For two sets A and B and Universe X:
   - **Commutativity**:





- **Associativity**:

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

- **Distributivity**:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

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- **Idempotency**:

- **Identity**:

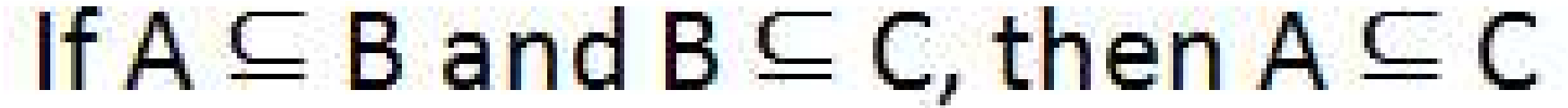
$$A \cup \emptyset = A$$

$$A \cap X = A$$

$$A \cap \emptyset = \emptyset$$

$$A \cup X = X$$

- **Transitivity**



Fuzzy set:

1. **Fuzzy set** is a set having **degrees of membership** between 1 and 0. Fuzzy sets are represented with tilde character(~). For example, Number of cars following traffic signals at a particular time out of all cars present will have membership value between [0,1].
2. Partial membership exists when member of one fuzzy set can also be a part of other fuzzy sets in the same universe.
3. The degree of membership or truth is not same as probability, fuzzy truth represents membership in vaguely defined sets.
4. A fuzzy set A~ in the universe of discourse, U, can be defined as a set of ordered pairs and it is given by

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$$

1. When the universe of discourse, U, is **discrete and finite**, fuzzy set A~ is given by

$$\tilde{A} = \sum_{i=1}^{n} \frac{\mu_{\tilde{A}}(x_i)}{x_i} = \frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + \ldots + \frac{\mu_{\tilde{A}}(x_n)}{x_n}$$

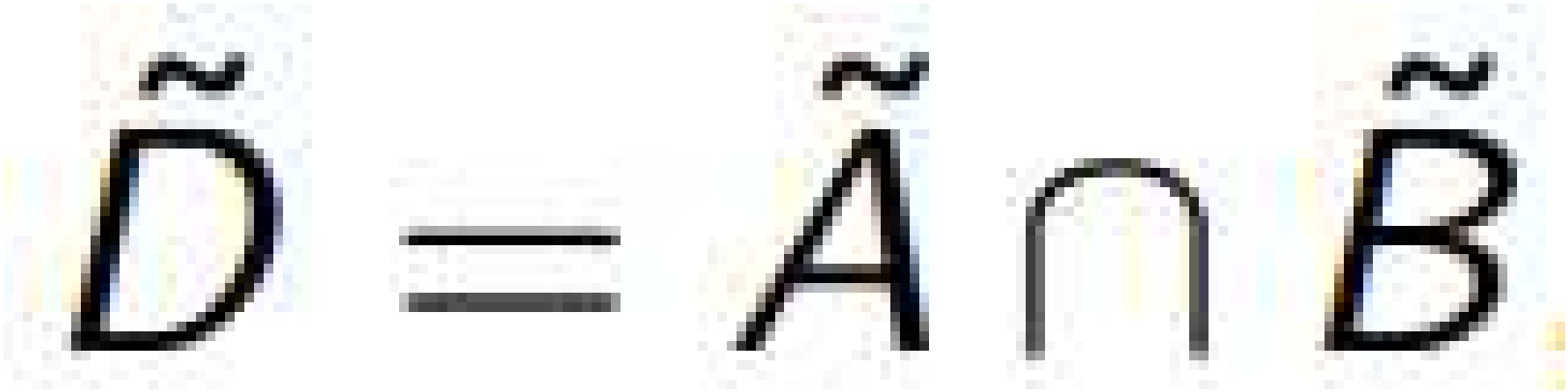
$$\tilde{A} = \int \frac{\mu_{\tilde{A}}(x)}{x}$$

1. Fuzzy sets also satisfy every property of classical sets.
2. **Common Operations on fuzzy sets**: Given two Fuzzy sets A~ and B~
   - **Union** : Fuzzy set C~ is union of Fuzzy sets A~ and B~ :

$$\tilde{C} = \tilde{A} \cup \tilde{B},$$

- $$\mu_{\tilde{C}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

- **Intersection**: Fuzzy set D~ is intersection of Fuzzy sets A~ and B~ :



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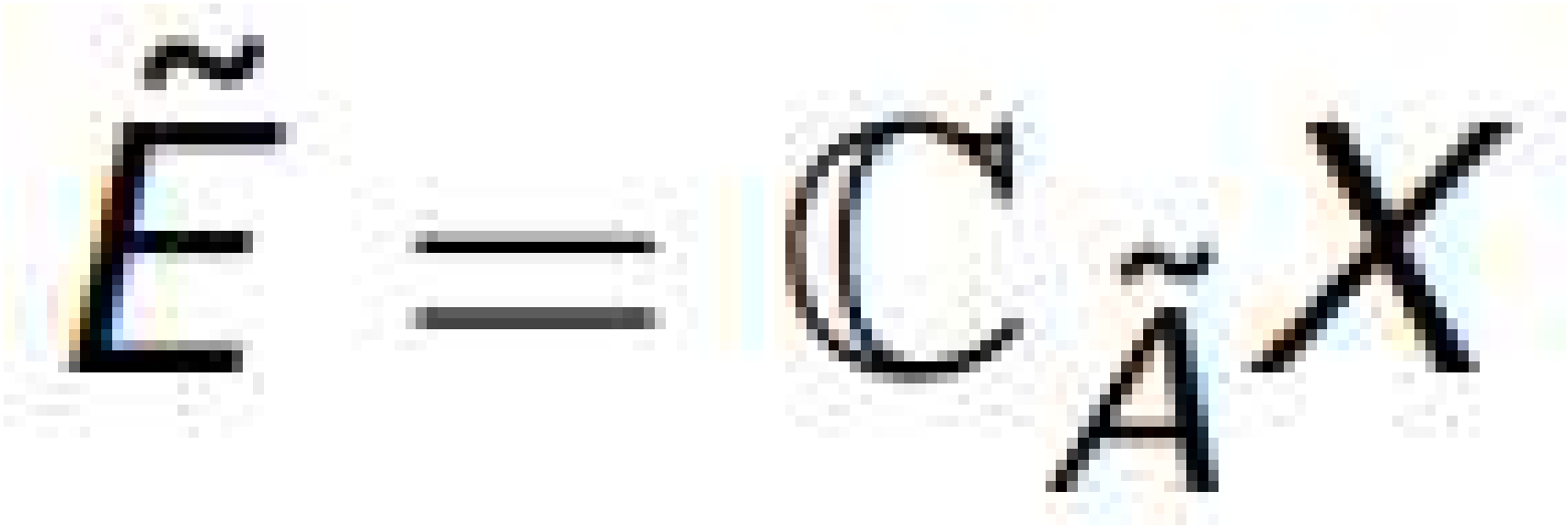
$$\mu_{\tilde{D}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

- **Complement**: Fuzzy set E~ is complement of Fuzzy set A~ :



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$$\mu_{\tilde{E}}(x) = 1 - \mu_{\tilde{A}}(x)$$

1. Some other useful operations on Fuzzy set:
   - **Algebraic sum**:

$$\mu A+B(x)= \mu A(x) + \mu B(x) - \mu A(x).\ \mu B(x)$$

   - **Algebraic product**:

- **Bounded sum**:

$$\mu A \oplus B(x) = \min\{1, \mu A(x) + \mu B(x)\}$$

- **Bounded difference**:

$$\mu A \odot B(c) = \max\{0, \mu A(x) - \mu B(x)\}$$

Sources:

(1) http://staff.cs.upt.ro/~todinca/cad/Lectures/cad_fuzzysets.pdf

(2) Principles of Soft Computing

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