



Fuzzy Logic | Set 2 (Classical and Fuzzy Sets)

Difficulty Level : Medium • Last Updated : 24 Jan, 2022

Prerequisite : [Fuzzy Logic | Introduction](#)

In this post, we will discuss classical sets and fuzzy sets, their properties and operations that can be applied on them.

Set: A set is defined as a collection of objects, which share certain characteristics.

Classical set

1. Classical set is a collection of **distinct** objects. For example, a set of students passing grades.
2. Each individual entity in a set is called a **member** or an **element** of the set.
3. The classical set is defined in such a way that the universe of discourse is splitted into two groups **members** and **non-members**. Hence, In case classical sets, **no partial membership exists**.
4. Let A is a given set. The membership function can be use to define a set A is given by:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \end{cases}$$

$$\begin{cases} 0 & \text{if } x \notin A \end{cases}$$

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

- This operation is also called **logical OR**.
- **Intersection:**

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

- This operation is also called **logical AND**.
- **Complement:**

$$A' = \{x \mid x \in A, x \in X\}$$

- **Difference:**

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

1. **Properties of classical sets:** For two sets A and B and Universe X:

- **Commutativity:**

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

- **Associativity:**

$$A \cup (B \cap C) = (A \cup B) \cap C$$

$$A \cap (B \cup C) = (A \cap B) \cup C$$

- Distributivity:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



- **Idempotency:**

$$A \cup A = A$$

$$A \cap A = A$$

- **Identity:**

$$A \cup \emptyset = A$$

$$A \cap X = A$$

$$A \cap \emptyset = \emptyset$$

$$A \cup X = X$$

If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

Fuzzy set:

1. **Fuzzy set** is a set having **degrees of membership** between 1 and 0. Fuzzy sets are represented with tilde character(\sim). For example, Number of cars following traffic signals at a particular time out of all cars present will have membership value between $[0,1]$.
2. Partial membership exists when member of one fuzzy set can also be a part of other fuzzy sets in the same universe.
3. The degree of membership or truth is not same as probability, fuzzy truth represents membership in vaguely defined sets.
4. A fuzzy set A_{\sim} in the universe of discourse, U , can be defined as a set of ordered pairs and it is given by

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$$

1. When the universe of discourse, U , is **discrete and finite**, fuzzy set \tilde{A} is given by

$$\tilde{A} = \sum_{i=1}^n \frac{\mu_{\tilde{A}}(x_i)}{x_i} = \frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + \dots + \frac{\mu_{\tilde{A}}(x_n)}{x_n}$$

$$\tilde{A} = \int \frac{\mu_{\tilde{A}}(x)}{x}$$

1. Fuzzy sets also satisfy every property of classical sets.
2. **Common Operations on fuzzy sets:** Given two Fuzzy sets \tilde{A} and \tilde{B}
 - **Union** : Fuzzy set \tilde{C} is union of Fuzzy sets \tilde{A} and \tilde{B} :

$$\tilde{C} = \tilde{A} \cup \tilde{B}$$

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$$\mu_{\tilde{C}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

- **Intersection:** Fuzzy set D^{\sim} is intersection of Fuzzy sets A^{\sim} and B^{\sim} :

$$D^{\sim} = A^{\sim} \cap B^{\sim}$$

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$$\mu_{\tilde{D}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

- **Complement:** Fuzzy set E^{\sim} is complement of Fuzzy set A^{\sim} :

$$\tilde{E} = C_{\tilde{A}} X$$

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$$\mu_{\tilde{E}}(x) = 1 - \mu_{\tilde{A}}(x)$$

1. Some other useful operations on Fuzzy set:

- **Algebraic sum:**

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

- **Algebraic product:**

$$\underline{\mu_{A.B}(x)} = \underline{\mu_A(x)} \cdot \underline{\mu_B(x)}$$

- **Bounded sum:**

$$\mu_{A \oplus B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\}$$

- **Bounded difference:**

$$\mu_{A \ominus B}(x) = \max\{0, \mu_A(x) - \mu_B(x)\}$$

Sources:

- (1) http://staff.cs.upt.ro/~todinca/cad/Lectures/cad_fuzzysets.pdf
- (2) [Principles of Soft Computing](#)

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