Refractive index

Chp. 23- J&W; Chp.7-T.B.

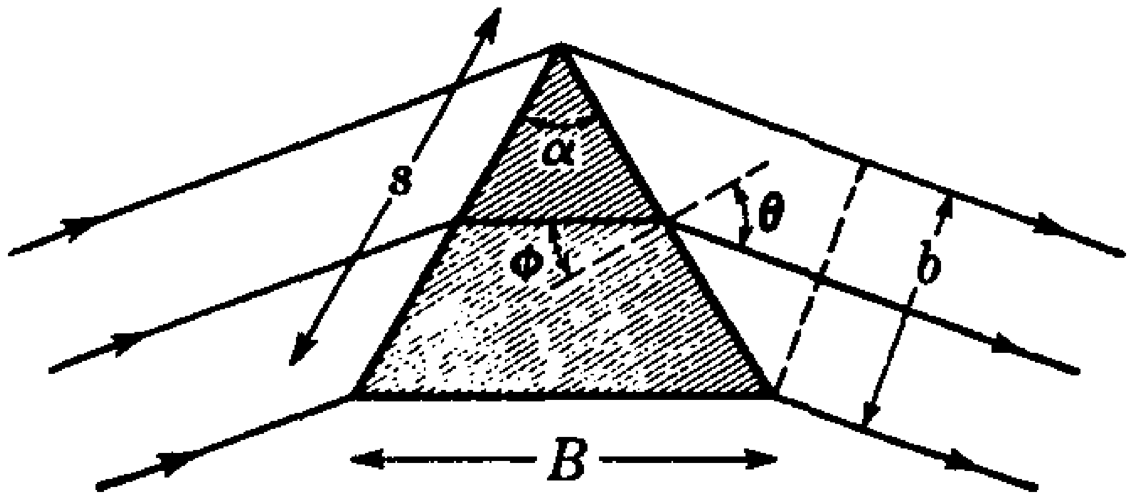


FIGURE 23A
Refraction by a prism at minimum deviation.

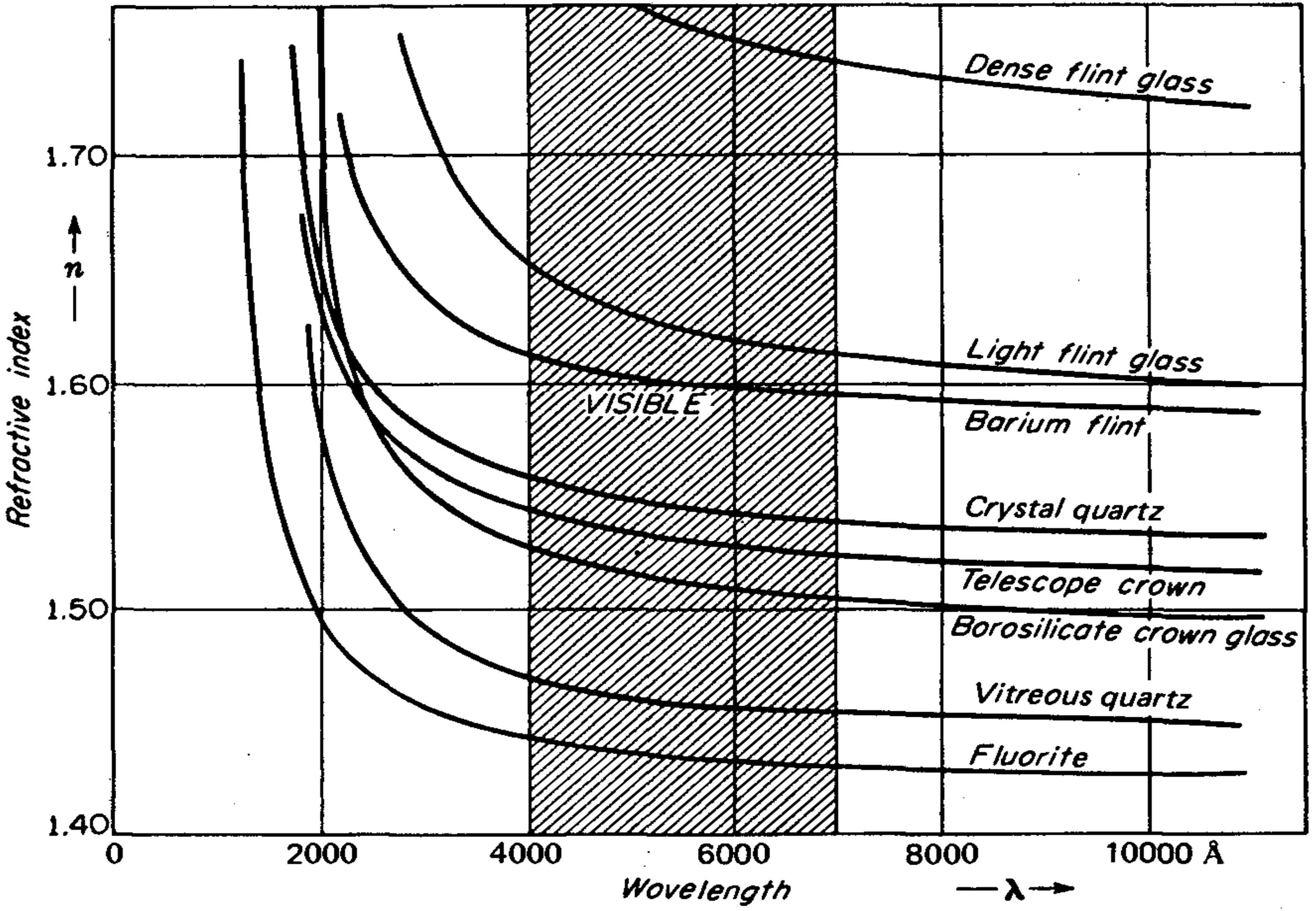


FIGURE 23B
Dispersion curves for several different materials commonly used for lenses and prisms.

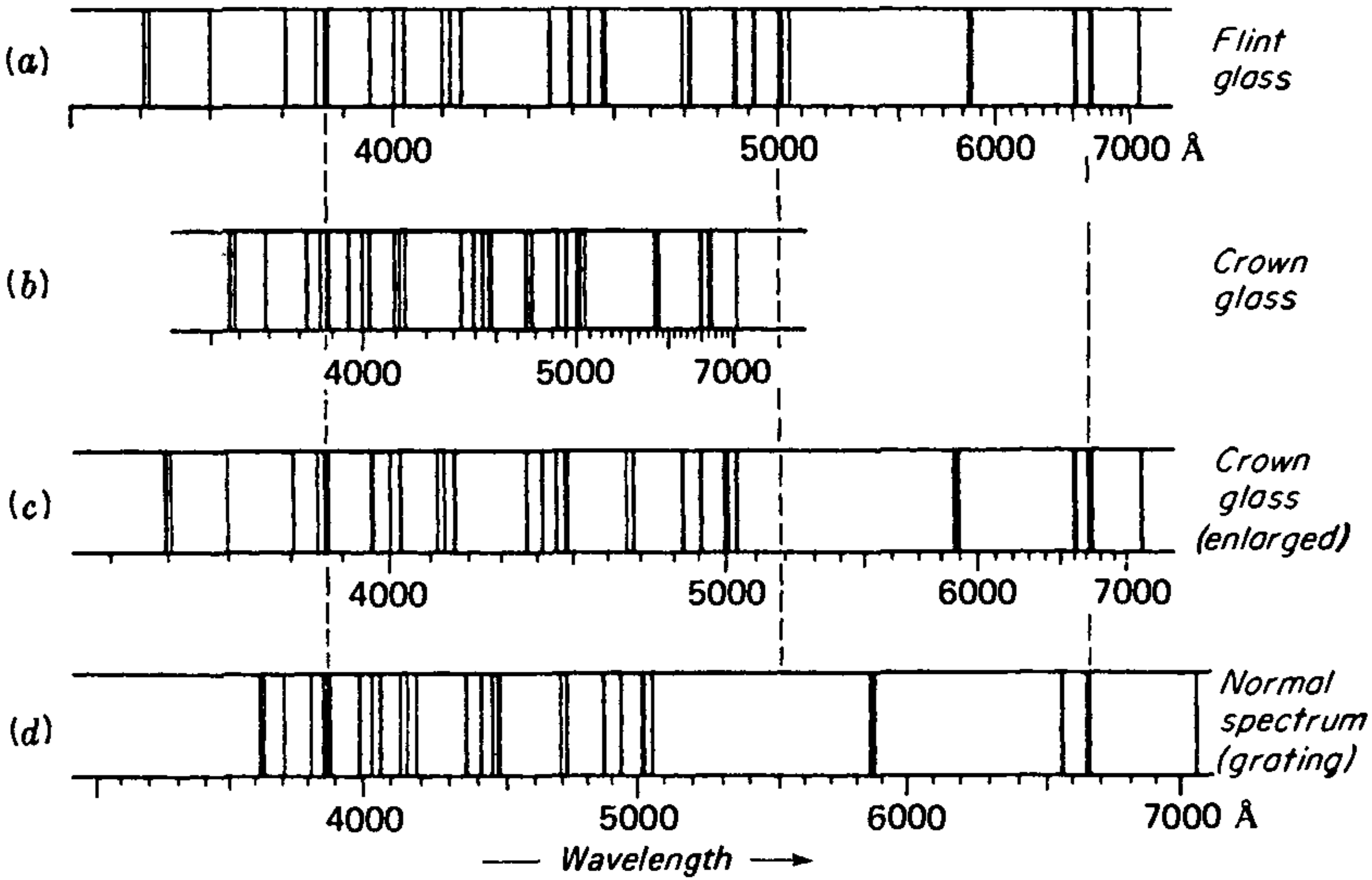


FIGURE 23C
Comparison of the helium spectrum produced by flint-glass and crown-glass prism spectrographs with a normal spectrum.

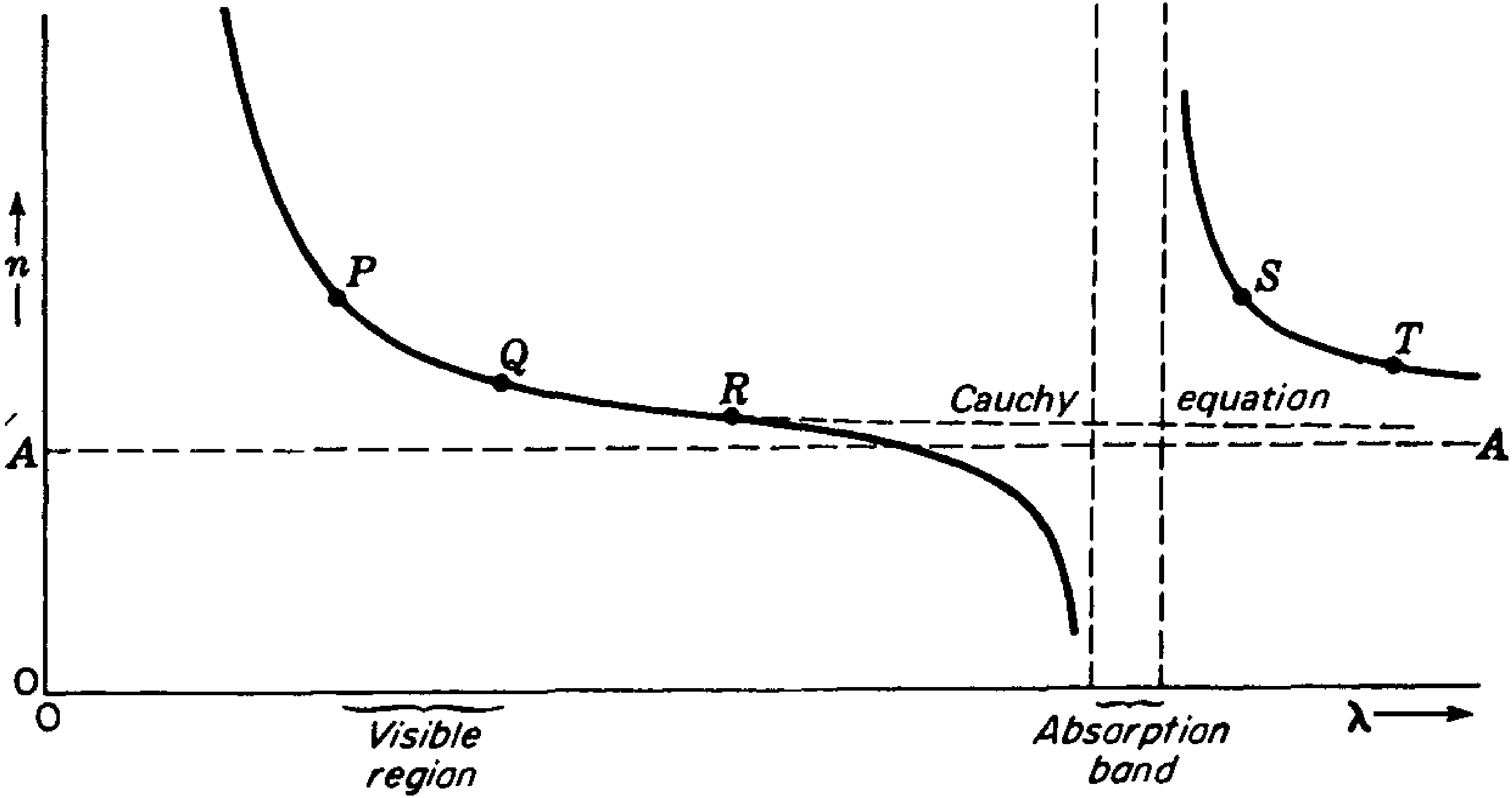


FIGURE 23D
Anomalous dispersion of a transparent substance like quartz in the infrared.

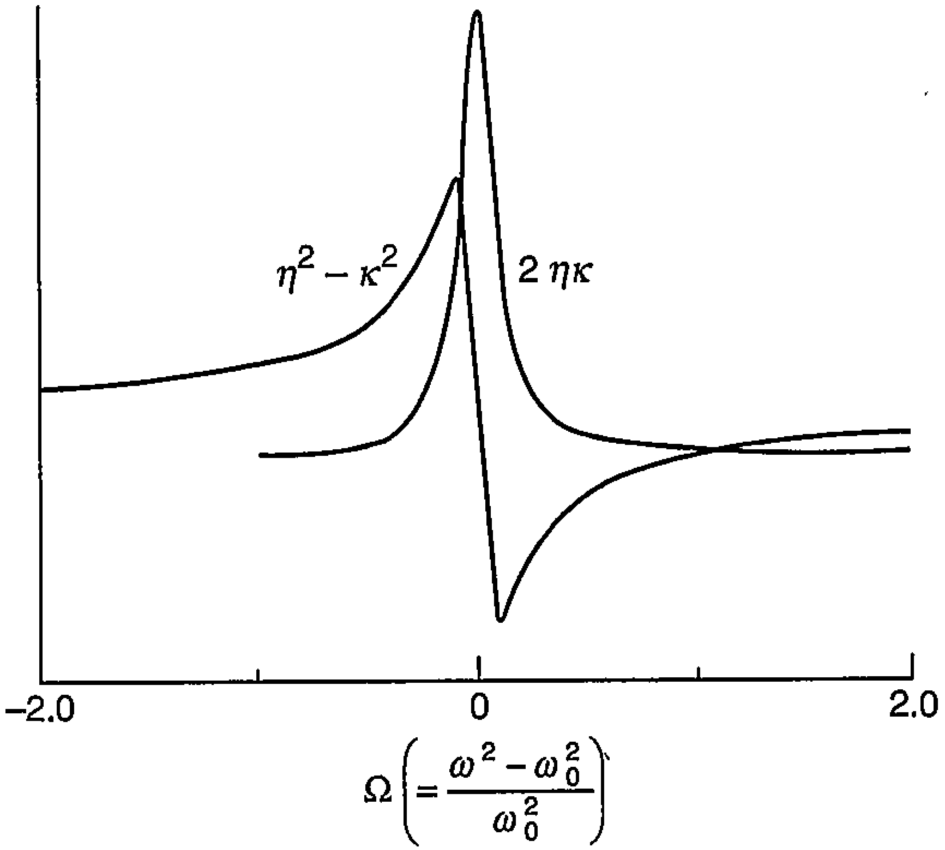


Fig. 7.13 Qualitative variation of $(\eta^2 - \kappa^2)$ and $2\eta\kappa$ with Ω.

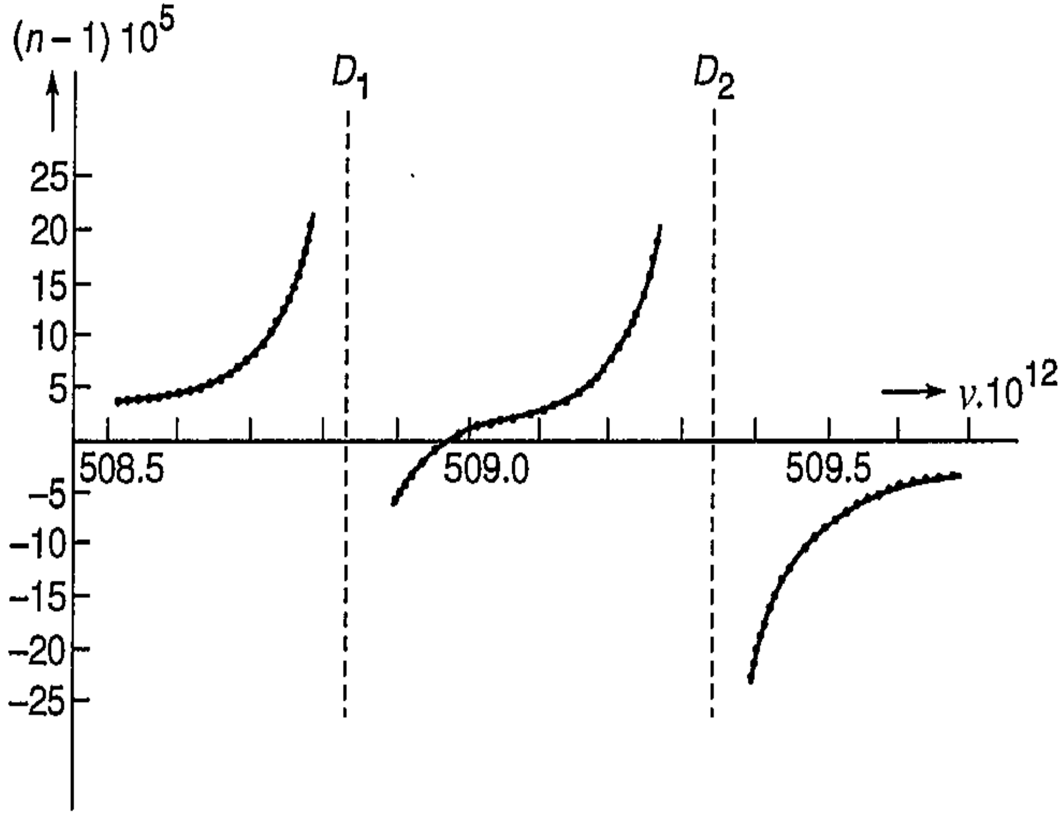
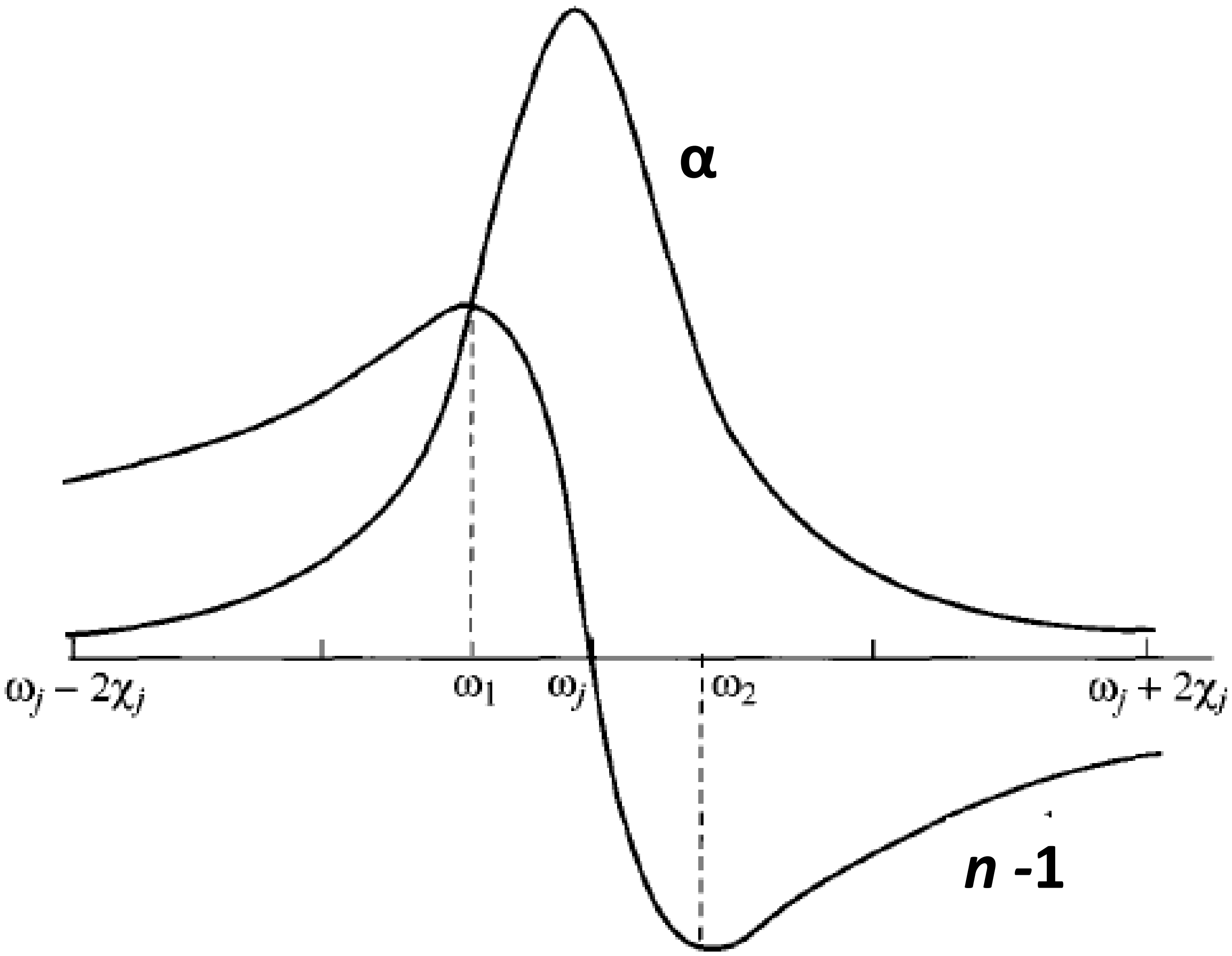


Fig. 7.14 The measured variation of refractive index of sodium with frequency around the D_1 and D_2 lines. The measurements are of Roschdestwensky; the figure has been adapted from Ref. 1.



Plots of *α* (the absorption coefficient) and *n*-1 with frequency *ω*

Prob. 7.8 (a) In a metal, the electrons can be assumed to be essentially free. The drift velocity of the electron satisfies the following equation

$$m\frac{d\mathbf{v}}{dt} + m\nu\mathbf{v} = \mathbf{F} = -q\,\mathbf{E}_0\, e^{-i\omega t}$$

where ν represents the collision frequency. Calculate the steady state current density ($\mathbf{J} = -Nq\mathbf{v}$) and show that the conductivity is given by

$$\sigma(\omega) = \frac{Nq^2}{m}\frac{1}{\nu - i\omega}$$

Prob. 7.8 (b) If $\mathbf{r}$ represents the displacement of the electron, show that

$$\mathbf{P} = -Nq\mathbf{r} = -\frac{Nq^2}{m(\omega^2 + i\omega\nu)}\mathbf{E}$$

which represents the polarization. Using the above equation show that

$$\kappa(\omega) = 1 - \frac{Nq^2}{m\varepsilon_0(\omega^2 + i\omega\nu)}$$

which represents the dielectric constant variation for a free-electron gas.

Prob. J & W

23.1 The refractive indices of a piece of optical glass for the blue and green lines of the mercury spectrum, $\lambda = 4358$ Å and $\lambda = 5461$ Å, are 1.65250 and 1.62450, respectively. Using the two-constant Cauchy equation, calculate values for (*a*) the constants A and B, (*b*) the refractive index for the sodium yellow lines at $\lambda = 5893$ Å, and (*c*) the dispersion at this same wavelength.

Ans. (*a*) $A = 1.57540$ and $B = 1.46431 \times 10^6$ Å^2, (*b*) $n = 1.61757$, (*c*) -1.43104×10^{-5} Å^{-1}

Prob. J & W

23.4 A 50° prism is made of glass for which the constants in the two-term Cauchy equation are $A = 1.53974$ and $B = 4.6528 \times 10^5$ Å^2. Find the angular dispersion in radians per angstrom when the prism is set for minimum deviation for a wavelength of 5500 Å.

Ans. $dn/d\lambda = -5.5932 \times 10^{-6}$ Å^{-1}, $d\theta/dn = 1.12145$,
$d\theta/d\lambda = -6.2725 \times 10^{-6}$ rad/Å

Prob.

The following data are given for the two materials:
For borosilicate glass: $n(\lambda_1)$ = 1.50883; $n(\lambda_2)$ = 1.51690.
For vitreous quartz: $n(\lambda_1)$ = 1.45640; $n(\lambda_2)$ = 1.46318 where λ_1 = 0.6563 μm and λ_2 = 0.4861 μm.
Calculate the values of A & B in the Cauchy's two constant formula.

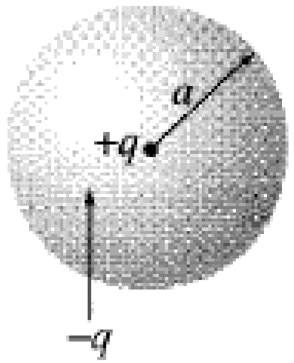
Using the Cauchy formula calculate the refractive index at 0.5890 μm and 0.3988 μm and compare with the corresponding experimental values:
(i) 1.51124 and 1.42546 for borosilicate glass
(ii) 1.45845 and 1.47 for vitreous quartz.

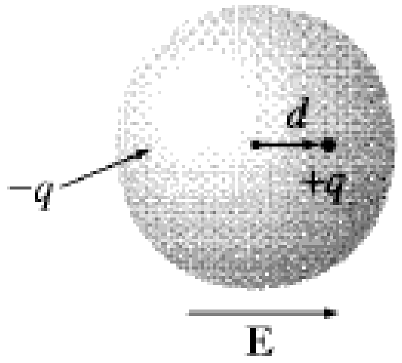
Ans. For borosilicate glass:
$A = 1.499$; $B \approx 4.22 \times 10^{-15}$ m^2.
At λ = 0.5890 μm, n = 1.51120 and at λ = 0.3988 μm, n = 1.52557.

For vitreous quartz:
$A = 1.44817$; $B \approx 3.546 \times 10^{-15}$ m^2.
At λ = 0.5890 μm, n = 1.45839 and at λ = 0.3988 μm, n = 1.47047.

Problem 9.24 Find the width of the anomalous dispersion region for the case of a single resonance at frequency ω_0. Assume $\gamma \ll \omega_0$. Show that the index of refraction assumes its maximum and minimum values at points where the absorption coefficient is at half-maximum.

A primitive model of an atom consists of a point nucleus *(+q)* surrounded by a uniformly charged spherical cloud *(-q)* of radius *a*. What is the natural frequency? Where does this lie in the electromagnetic spectrum, assuming the radius of the atom is 0.5 Å? Find the constants A & B in the Cauchy's formula and compare them with those for hydrogen (H_2) at 0˚C and atmospheric pressure: $A = 1.36\times 10^{-4}$, $B = 7.7\times10^{-15}$ m^2.





Ans. $\nu = 7.16\times10^{15}$ Hz; $A = 4.2\times 10^{-5}$, $B = 1.8 \times 10^{-15}$ m^2.

Prob. Fuchsin is a strong (aniline) dye, which in solution with alcohol has a deep red color. It appears red because it absorbs the green component of the spectrum. (As you might expect, the surfaces of crystals of fuchsin reflect green light rather strongly.) Imagine that you have a thin-walled hollow prism filled with this solution. What will the spectrum look like for incident white light?

Sol.