6. $p(x_1 \ldots x_n | \sigma, \mu) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)$

tentukan μ_{map}.

$\cancel{p(x_1 \ldots x_n|\sigma,\mu) \Rightarrow}$ $\mu_{map} = \arg\max\, p(x_1\ldots x_n|\mu,\sigma)\cdot p(\mu,\sigma)$.

$$p(\mu) = \frac{1}{\sigma_p\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\mu-\mu_p}{\sigma_p}\right)^2\right)$$

$$p(x_1 \ldots x_n|\mu) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right).$$

$$p(\mu,\sigma|x_1\ldots x_n) = \frac{p(x_1\ldots x_n|\mu,\sigma)\cdot p(\mu,\sigma)}{p(x_1\ldots x_n)} \approx p(x_1\ldots x_n|\mu,\sigma)\cdot p(\mu,\sigma)$$

$= \frac{1}{\sigma_p\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\mu-\mu_p}{\sigma_p}\right)^2\right)\cdot \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)$; untuk mencari μ_{max}, persamaan diatas diturunkan dan hasil turunannya sama dengan nol.

$$\ln(p(\mu,\sigma|x_1\ldots x_n)) = \ln\left(\frac{1}{\sigma_p\sqrt{2\pi}}\right) + \ln \exp\left(-\frac{1}{2}\left(\frac{\mu-\mu_p}{\sigma_p}\right)^2\right) + \ln\left[\prod_{i=1}^{n}\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)\right]$$

$$= \ln\left(\frac{1}{\sigma_p\sqrt{2\pi}}\right) + -\frac{1}{2}\left(\frac{\mu-\mu_p}{\sigma_p}\right)^2 + \ln\left[\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n\right] + \ln\left[\prod_{i=1}^{n}\left(-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)\right].$$

$$= \ln\left(\frac{1}{\sigma_p\sqrt{2\pi}}\right) + -\frac{1}{2}\left(\frac{\mu-\mu_p}{\sigma_p}\right)^2 + n\ln\left(\frac{1}{\sigma\sqrt{2\pi}}\right) + \sum_{i=1}^{n} -\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2;$$

$\frac{d}{d\mu}\left[\ln p(\mu,\sigma|x_1\ldots x_n)\right] = 0$, sehingga.

$$\frac{d}{d\mu}\ln\left(\frac{1}{\sigma_p\sqrt{2\pi}}\right) + \frac{d}{d\mu} -\frac{1}{2}\left(\frac{\mu-\mu_p}{\sigma_p}\right)^2 + \frac{d}{d\mu} n\ln\left(\frac{1}{\sigma\sqrt{2\pi}}\right) + \frac{d}{d\mu}\sum_{i=1}^{n} -\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2 = 0$$

$$\Leftrightarrow 0 + -\frac{(\mu-\mu_p)}{\sigma_p^2} + 0 + \sum_{i=1}^{n}\frac{d}{d\mu}\left(-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right) = 0$$

$$\Leftrightarrow -\left(\frac{\mu-\mu_p}{\sigma_p^2}\right) + \sum_{i=1}^{n} -\frac{(x_i-\mu)}{\sigma^2} = 0 \Leftrightarrow \frac{\sum_{i=1}^{n}\mu - \sum_{i=1}^{n}x_i}{\sigma^2} = \frac{\mu-\mu_p}{\sigma_p^2} \Leftrightarrow \frac{n\mu - \sum_{i=1}^{n}x_i}{\sigma^2} = \frac{\mu-\mu_p}{\sigma_p^2}.$$

$$\Leftrightarrow \frac{\mu_p}{\sigma_p^2} + \frac{\sum_{i=1}^{n}x_i}{\sigma^2} = \frac{\mu}{\sigma_p^2} + \frac{n\mu}{\sigma^2} \Leftrightarrow \frac{\sigma^2\mu_p + \left(\sum_{i=1}^{n}x_i\right)\sigma_p^2}{\cancel{\sigma_p^2\sigma^2}} = \frac{\sigma^2\mu + \sigma_p^2 n\mu}{\cancel{\sigma_p^2\sigma^2}} \Leftrightarrow$$

$$\sigma^2\mu_p + \sigma_p^2\sum_{i=1}^{n}x_i = \mu\left(\sigma^2 + \sigma_p^2 n\right). \Leftrightarrow \mu = \frac{\sigma^2\mu_p + \sigma_p^2\sum_{i=1}^{n}x_i}{\sigma^2+\sigma_p^2 n}$$