

$$1. P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}.$$

Buktikan pernyataan diatas.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ dan } P(A \cap B) = P(B|A) \cdot P(A) = P(A|B) \cdot P(B)$$

$$\text{Sehingga } P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}.$$

2. Tentukan rumus $P(A|-B)$ dalam $P(A)$, $P(B)$, dan $P(A \cap B)$.

$$P(A|-B) = \frac{P(-B|A) P(A)}{P(-B)}; P(-B) = 1 - P(B) \text{ dan } P(-B|A) = 1 - P(B|A)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow 1 - P(B|A) = 1 - \frac{P(A \cap B)}{P(A)} = \frac{P(A) - P(A \cap B)}{P(A)}$$

$$P(A|-B) = \frac{\frac{P(A) - P(A \cap B)}{P(A)} \cdot P(A)}{1 - P(B)} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

3. Jika A dan B independen, buktikan

a) A dan $\neg B$ independen

b) $\neg A$ dan $\neg B$ independen

A dan B independen jika $P(A|B) = P(A)$ dan $P(B|A) = P(B)$.

a. A dan $\neg B$ independen jika $P(A|\neg B) = P(A)$ dan $P(\neg B|A) = P(\neg B)$.

$$P(A|\neg B) = \frac{P(\neg B|A) \cdot P(A)}{P(\neg B)} = \frac{[1 - P(B|A)] \cdot P(A)}{1 - P(B)}; \text{ karena } P(B|A) = P(B),$$

$$\text{maka } \frac{[1 - P(B|A)] \cdot P(A)}{1 - P(B)} = \frac{[1 - P(B)] P(A)}{1 - P(B)} = P(A)$$

$$\begin{aligned} P(\neg B|A) &= 1 - P(B|A) = 1 - \frac{P(B \cap A)}{P(A)} = \frac{P(A) - P(B \cap A) \cdot P(A)}{P(A)} \\ &= \frac{\cancel{P(A)} [1 - P(B|A)]}{\cancel{P(A)}} = 1 - P(B|A) = 1 - P(B) = P(\neg B). \end{aligned}$$

Karena $P(A|\neg B) = P(A)$ dan $P(\neg B|A) = P(\neg B)$ maka A dan $\neg B$ independen.

b. $\neg A$ dan $\neg B$ ~~xxx~~ independen jika $P(\neg A|\neg B) = P(\neg A)$ dan $P(\neg B|\neg A) = P(\neg B)$

$$P(\neg A|\neg B) = 1 - P(A|\neg B); \text{ dari 3a } P(A|\neg B) = P(A) \text{ sehingga}$$

$$1 - P(A|\neg B) = 1 - P(A) = P(\neg A).$$

$$P(\neg B|\neg A) = \frac{P(\neg A|\neg B) \cdot P(\neg B)}{P(\neg A)} = \frac{[1 - P(A|\neg B)] \cdot P(\neg B)}{P(\neg A)}; \text{ karena } P(A|\neg B) = P(A)$$

$$\text{maka } \frac{[1 - P(A|\neg B)] \cdot P(\neg B)}{P(\neg A)} = \frac{[1 - P(A)] P(\neg B)}{P(\neg A)} = \frac{P(\neg A)}{P(\neg A)} P(\neg B) = P(\neg B)$$

Karena $P(\neg A|\neg B) = P(\neg A)$ dan $P(\neg B|\neg A) = P(\neg B)$ maka $\neg A$ dan $\neg B$ adalah independen.

4.

	$x = \alpha$	$x = \beta$
$Y = \delta$	15	20
$Y = \eta$	75	30

$$a. p(x = \alpha) = \frac{15 + 75}{140} = \frac{90}{140}$$

$$b. p(x \neq \beta) = 1 - p(x = \beta) = p(x = \alpha) = \frac{90}{140}$$

$$c. p(x = \beta \text{ atau } Y = \delta) = p(x = \beta) + p(Y = \delta) - p(x = \beta, Y = \delta) \\ = \frac{50}{140} + \frac{35}{140} - \frac{20}{140} = \frac{65}{140}$$

$$d. p(x = \alpha \text{ atau } Y = \eta) = p(x = \alpha) + p(Y = \eta) - p(x = \alpha, Y = \eta) \\ = \frac{90}{140} + \frac{105}{140} - \frac{75}{140} = \frac{120}{140}$$

$$e. p(x = \beta | Y = \eta) = \frac{p(x = \beta, Y = \eta)}{p(Y = \eta)} = \frac{\frac{30}{140}}{\frac{105}{140}} = \frac{30}{105}$$

5. Peluang orang terkena kanker = 0,01, Peluang
 Tes positif jika orang terkena kanker = 0,98.
 Peluang tes negatif jika orang tidak kanker = 0,97.

$$P(K) = 0,01 \rightarrow P(-K) = 0,99; \text{ :}$$

$$P(P|K) = 0,98 \rightarrow P(-P|K) = 0,02$$

$$P(-P|-K) = 0,97 \rightarrow P(P|-K) = 0,03$$

Peluang orang terkena kanker jika hasil tes positif
 adalah $P(K|P)$.

$$P(K|P) = \frac{P(P|K) \cdot P(K)}{P(P)} = \frac{0,98 \cdot 0,01}{P(P)}$$

	K	-K
P	$P \cap K$	$P \cap -K$
-P	$K \cap -P$	$-P \cap -K$

$$\begin{aligned} P(P) &= P(P \cap K) + P(P \cap -K) \\ &= P(P|K) \cdot P(K) + P(P|-K) \cdot P(-K) \\ &= 0,98 \cdot 0,01 + 0,03 \cdot 0,99 \end{aligned}$$

$$\text{Sehingga } P(K|P) = \frac{0,98 \cdot 0,01}{0,98 \cdot 0,01 + 0,03 \cdot 0,99} = 0,24$$