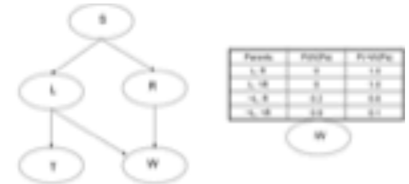
# Bayes Net – Inferences

# Bayes Network Definition



A Bayes network represents the joint probability distribution over a collection of random variables

A Bayes network is a directed acyclic graph and a set of CPD's

- Each node denotes a random variable
- Edges denote dependencies
- CPD for each node $X_i$ defines $P(X_i \mid Pa(X_i))$
- The joint distribution over all variables is defined as

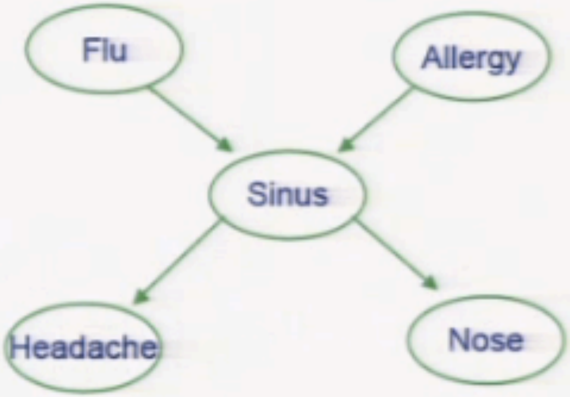
$$P(X_1 \ldots X_n) = \prod_i P(X_i | Pa(X_i))$$

Pa(X) = immediate parents of X in the graph

# Inference in Bayes Nets

- In general, intractable (NP-complete)
- For certain cases, tractable
  - Assigning probability to fully observed set of variables
  - Or if just one variable unobserved
  - Or for singly connected graphs (ie., no undirected loops)
    - Belief propagation
- For multiply connected graphs
    - Junction tree
- Sometimes use Monte Carlo methods
  - Generate many samples according to the Bayes Net distribution, then count up the results
- Variational methods for tractable approximate solutions

# Prob. of joint assignment: easy

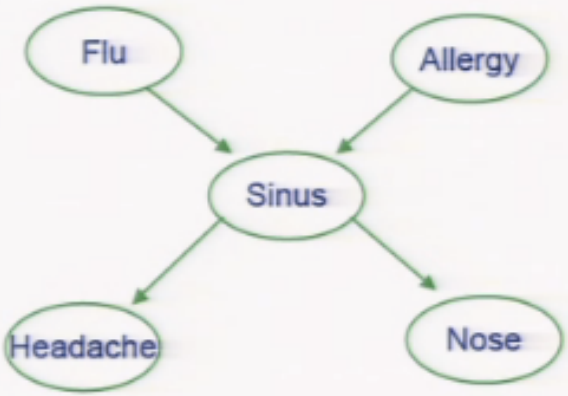



- Suppose we are interested in joint assignment <F=f,A=a,S=s,H=h,N=n>

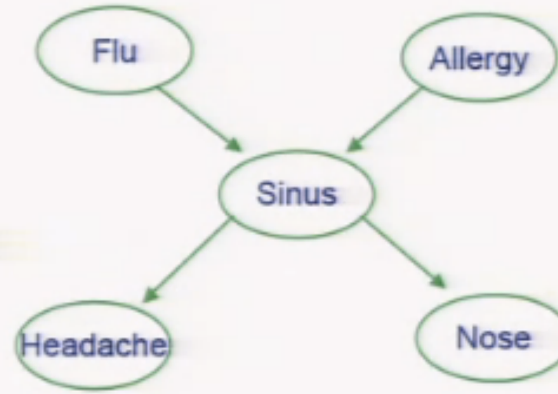
What is P(f,a,s,h,n)?

# Prob. of marginals: not so easy

- How do we calculate P(N=n) ?

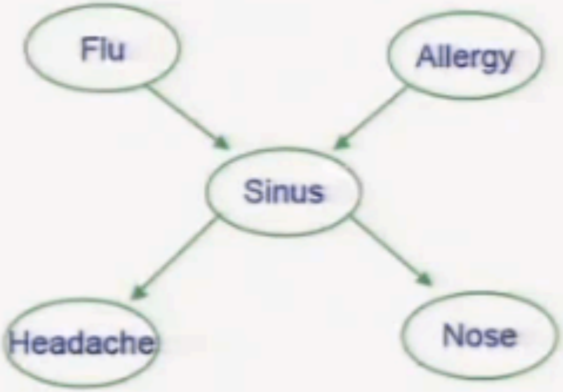


# Generating a sample from joint distribution: easy




How can we generate random samples drawn according to P(F,A,S,H,N)?

# Generating a sample from joint distribution: easy




Note we can estimate marginals like $P(N=n)$ by generating many samples from joint distribution, by summing the probability mass for which $N=n$

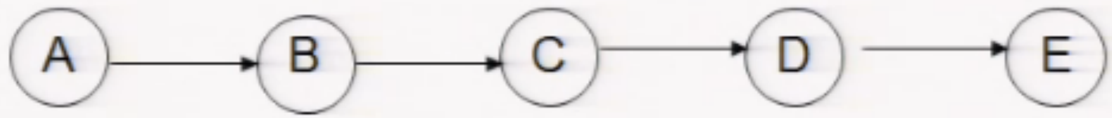
Similarly, for anything else we care about $P(F=1|H=1, N=0)$

→ weak but general method for estimating <u>any</u> probability term…

# Prob. of marginals: not so easy

But sometimes the structure of the network allows us to be clever → avoid exponential work

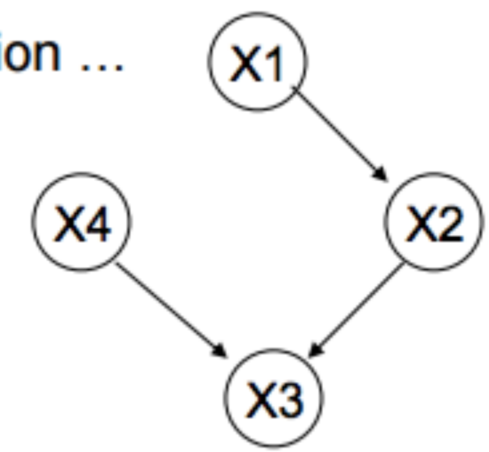
eg., chain



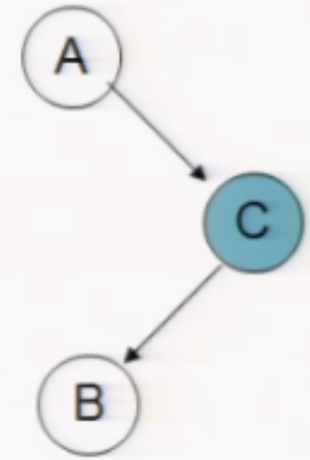
# Conditional Independence, Revisited

- We said:
  - Each node is conditionally independent of its non-descendents, given its immediate parents.

- Does this rule give us all of the conditional independence relations implied by the Bayes network?
  - No!
  - E.g., X1 and X4 are conditionally indep given {X2, X3}
  - But X1 and X4 not conditionally indep given X3
  - For this, we need to understand D-separation …



# Easy Network 1: Head to Tail

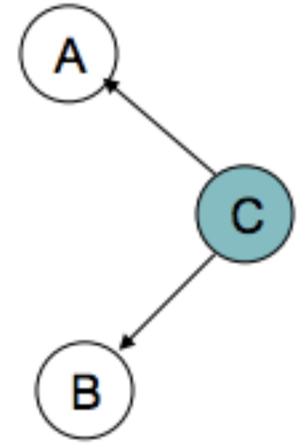



prove A cond indep of B given C?

ie., $p(a,b|c) = p(a|c)\, p(b|c)$

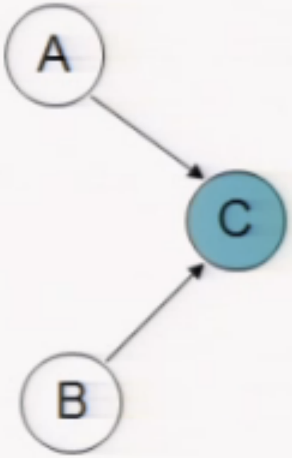
# Easy Network 2: Tail to Tail

prove A cond indep of B given C? ie., $p(a,b|c) = p(a|c)\, p(b|c)$

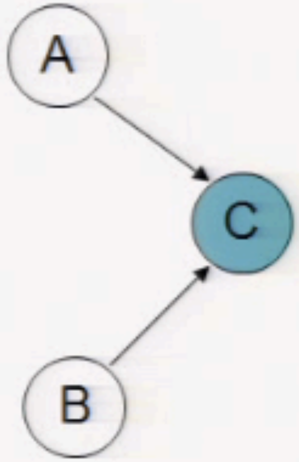


# Easy Network 3: Head to Head

prove A cond indep of B given C?     ie., $p(a,b|c) = p(a|c)\, p(b|c)$



# Easy Network 3: Head to Head




prove A cond indep of B given C?  NO!

Summary:

- $p(a,b)=p(a)p(b)$
- $p(a,b|c)$ NotEqual $p(a|c)p(b|c)$

Explaining away.

e.g.,

- A=earthquake
- B=breakIn
- C=motionAlarm

## X and Y are conditionally independent given Z, **<u>if and only if</u>** X and Y are D-separated by Z.

[Bishop, 8.2.2]

Suppose we have three sets of random variables: X, Y and Z

X and Y are **<u>D-separated</u>** by Z (and therefore conditionally indep, given Z) iff every path from any variable in X to any variable in Y is **<u>blocked</u>**

A path from variable A to variable B is **blocked** if it includes a node such that either

1. arrows on the path meet either head-to-tail or tail-to-tail at the node and this node is in Z
2. the arrows meet head-to-head at the node, and neither the node, nor any of its descendants, is in Z

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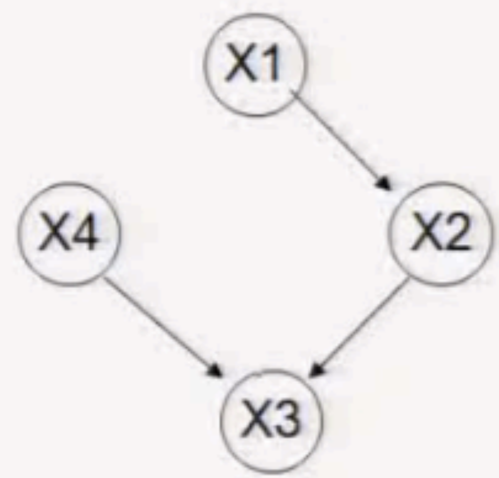
X4 indep of X1 given X3?

X4 indep of X1 given {X3, X2}?

X4 indep of X1 given {}?



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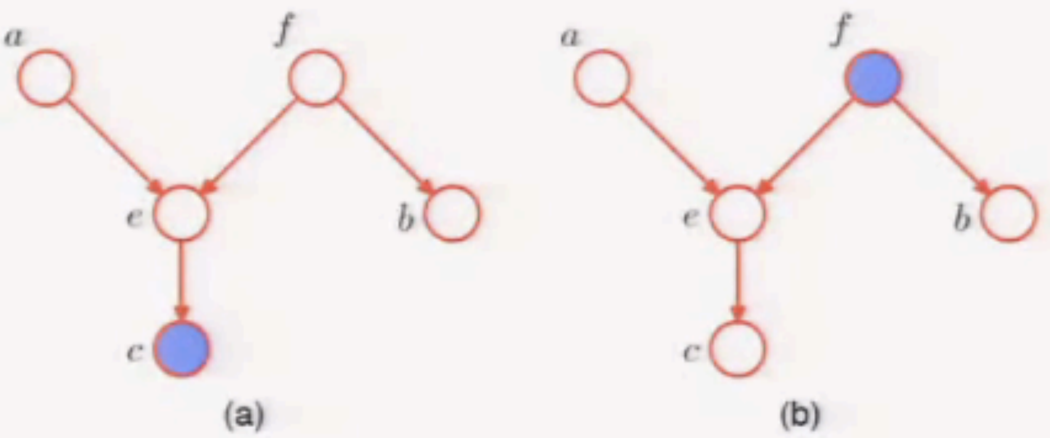
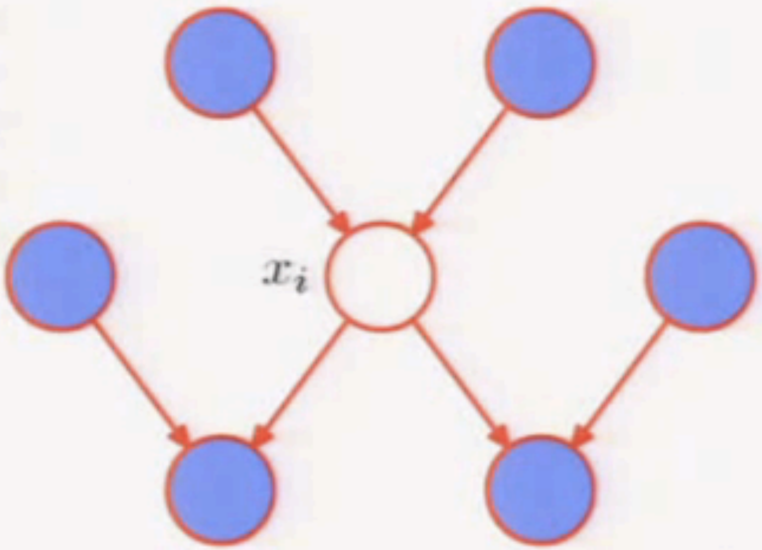
a indep of b given c?

a indep of b given f ?



# Markov Blanket

The Markov blanket of a node $\mathbf{x}_i$ comprises the set of parents, children and co-parents of the node. It has the property that the conditional distribution of $\mathbf{x}_i$, conditioned on all the remaining variables in the graph, is dependent only on the variables in the Markov blanket.




from [Bishop, 8.2]

# What You Should Know

- Bayes nets are convenient representation for encoding dependencies / conditional independence
- BN = Graph plus parameters of CPD's
  - Defines joint distribution over variables
  - Can calculate everything else from that
  - Though inference may be intractable
- Reading conditional independence relations from the graph
  - Each node is cond indep of non-descendents, given its immediate parents
  - D-separation
  - 'Explaining away'