Deep Learning - Convolutional Neural Networks

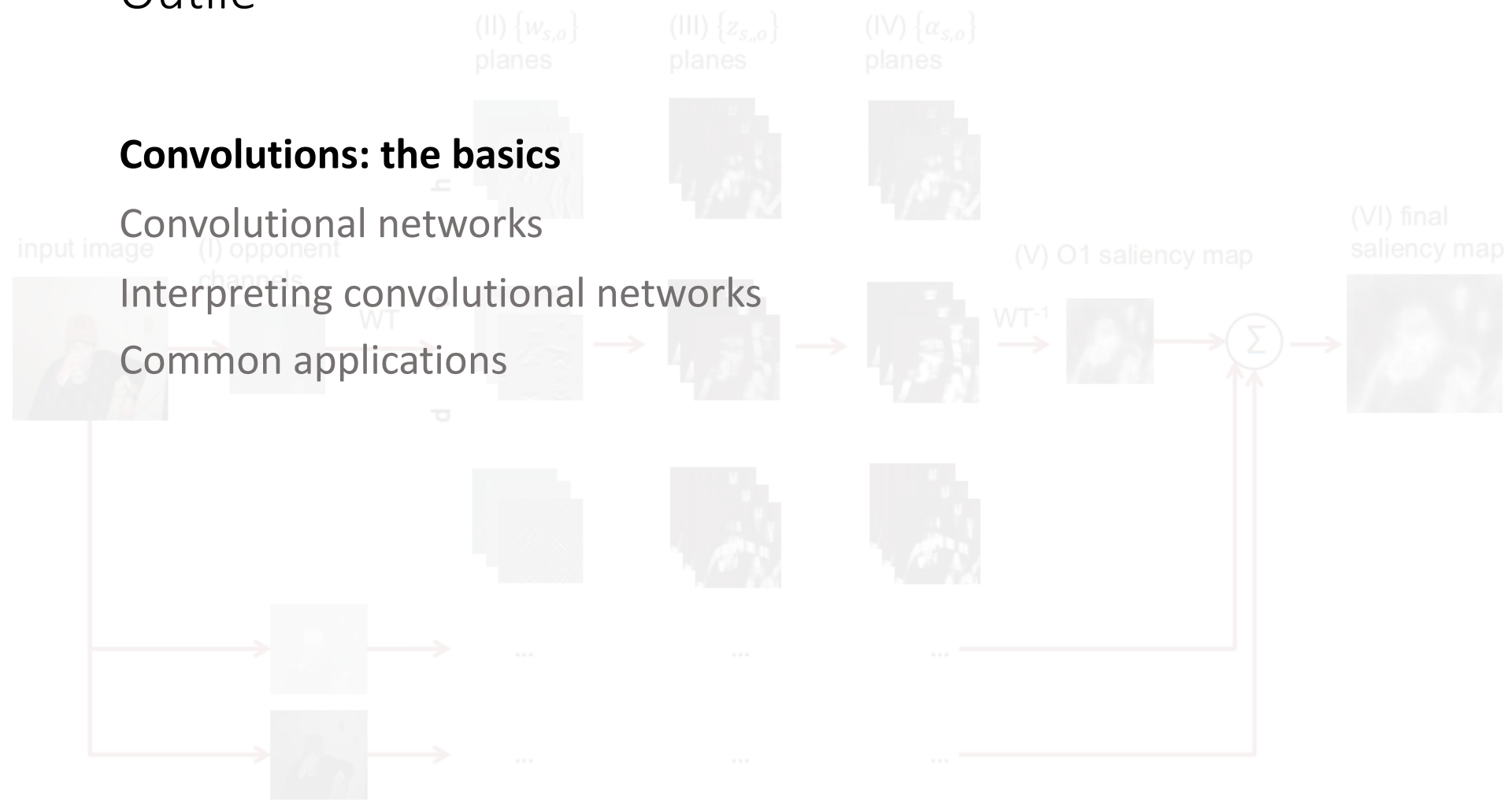
Outile

Convolutions: the basics

Convolutional networks

Interpreting convolutional networks

Common applications



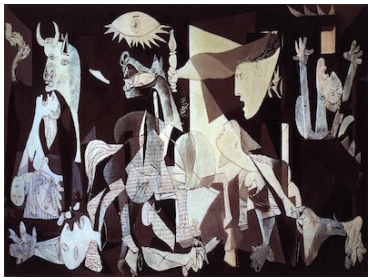
Computer Vision Problems

Image Classification

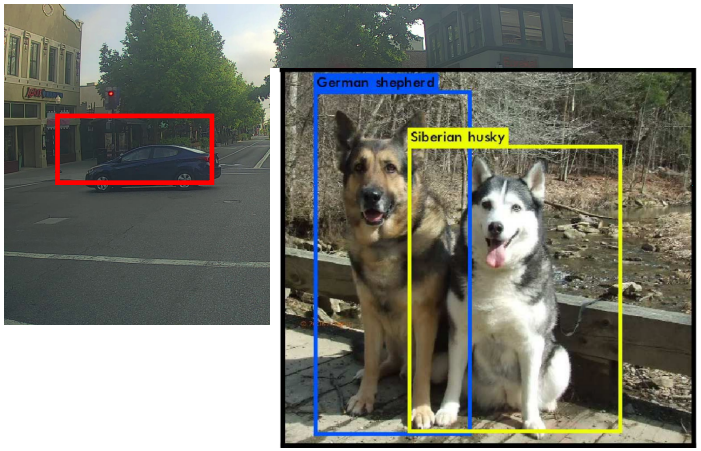
64x64

Cat? (0/1)

Neural Style Transfer



Object detection

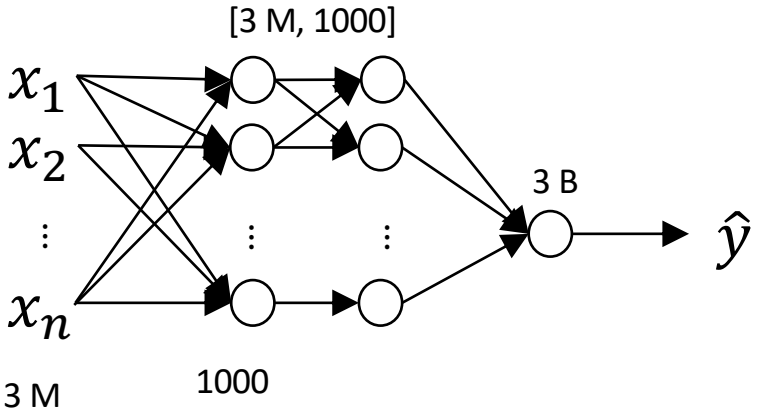


Deep Learning on large images

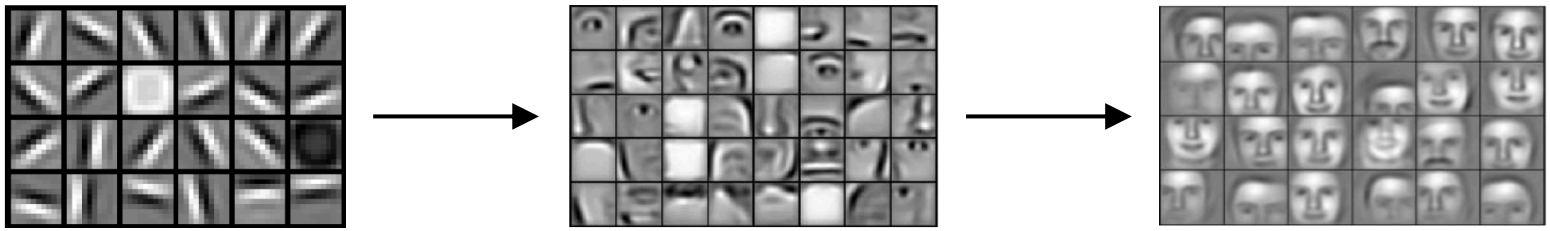
64x64x3 = 12.288 pixels

1000 x 1000 x 3 = 3 M pixels

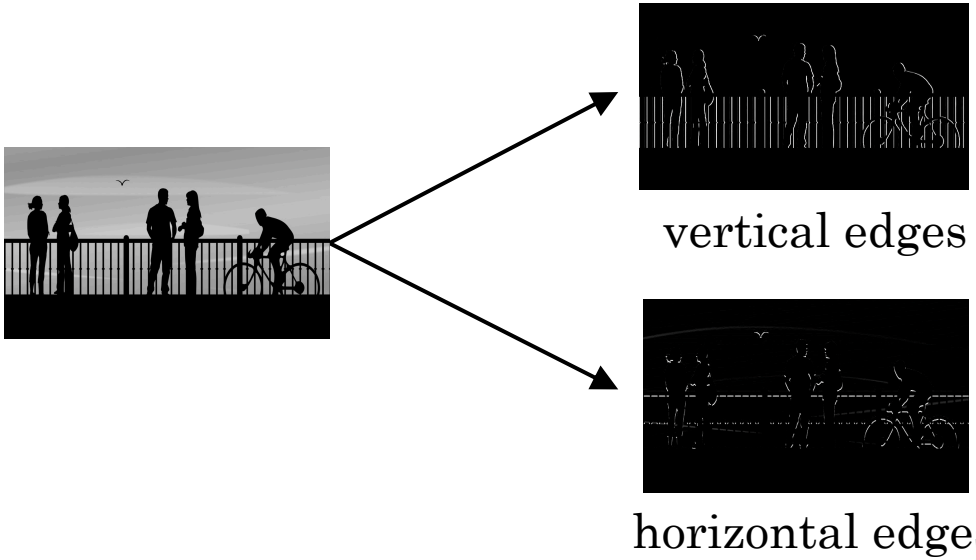
Cat? (0/1)



Computer Vision Problem



Detection of layers of Neural Networks

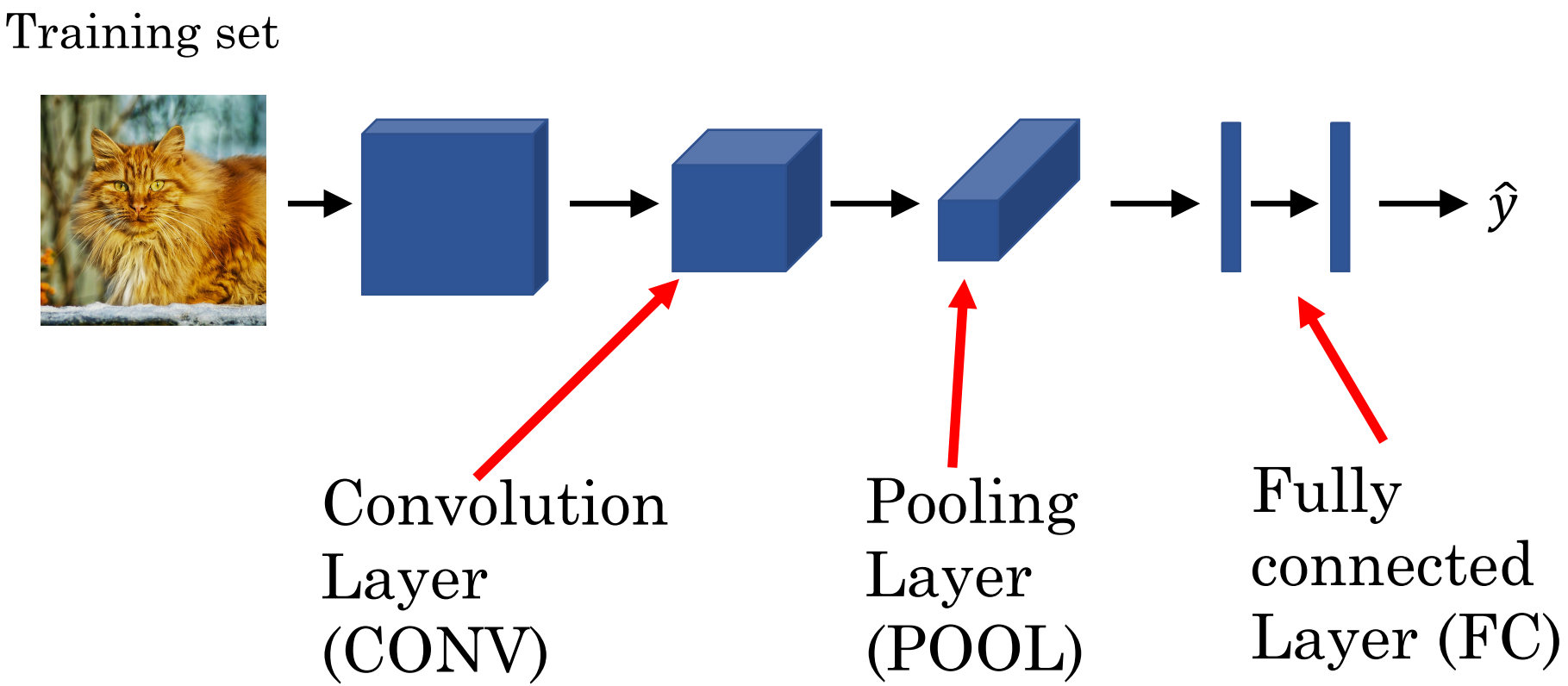


vertical edges

horizontal edges

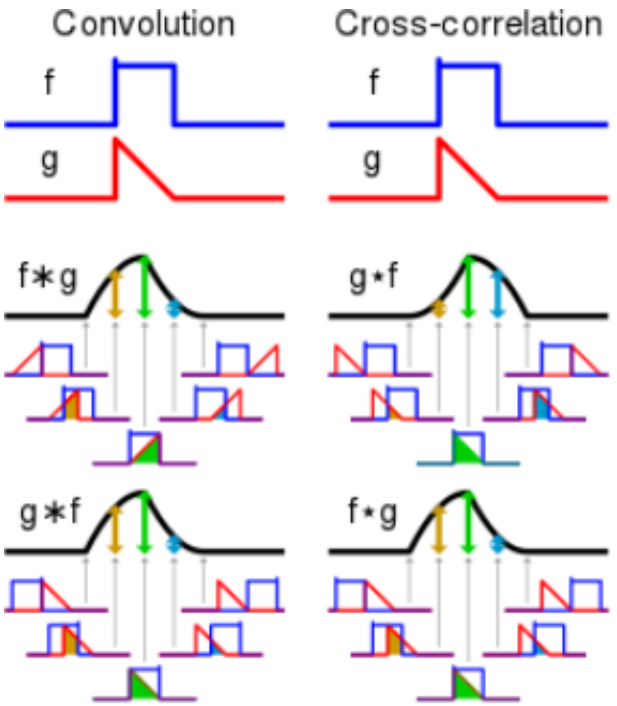
Convolutional Neural Network

Neural networks that include convolution operations



Convolution Layer

- A network layer that convolves its receptive input before passing it to the next layer.
- Most ML libraries implement convolutional layers as **cross-correlation layers**.



$$(I \star K)(i,j) == \sum_m \sum_n I(m,n)K(i+m, j+n)$$

Discrete cross-correlation: 2-D example

$$(I \star K)(0,0) = I(0,0)K(0,0) + I(0,1)K(0,1) + I(0,2)K(0,2) + I(1,0)K(1,0) + I(1,1)K(1,1) + I(1,2)K(1,2) + I(2,0)K(2,0) + I(2,1)K(2,1) + I(2,2)K(2,2)$$

I

1	0	0	1	2
0	0	0	3	0
0	1	2	1	1
1	1	3	0	0
3	0	0	0	1

$\star$

K

0	0	1
0	2	0
1	1	0

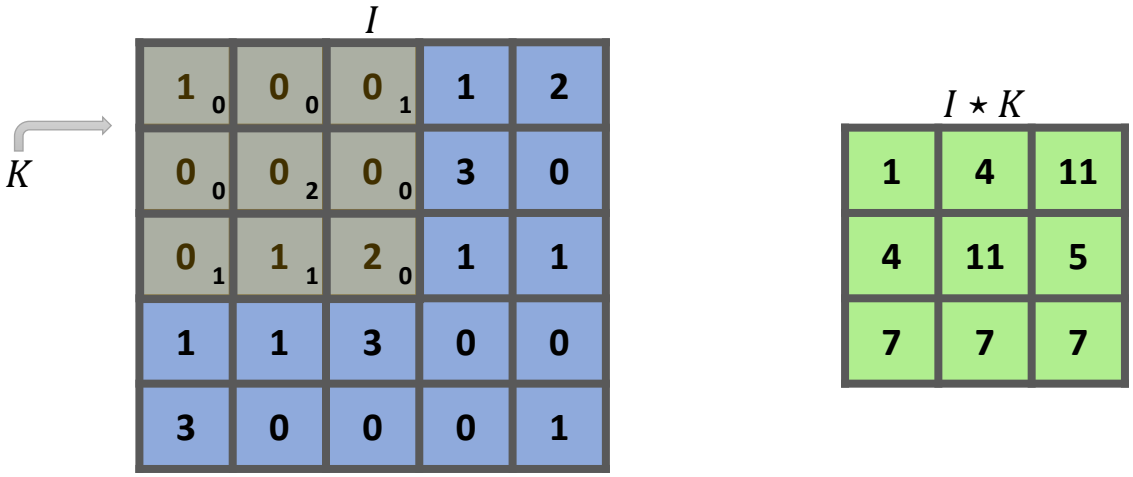
$$= 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 + 0 \cdot 2 + 0 \cdot 0 + 0 \cdot 1 + 1 \cdot 1 + 2 \cdot 0$$

$$= 1$$

$$(I \star K)(i,j) == \sum_m \sum_n I(m,n)K(i+m, j+n)$$

Discrete cross-correlation: 2-D example

Can be viewed as a “sliding window” operation:



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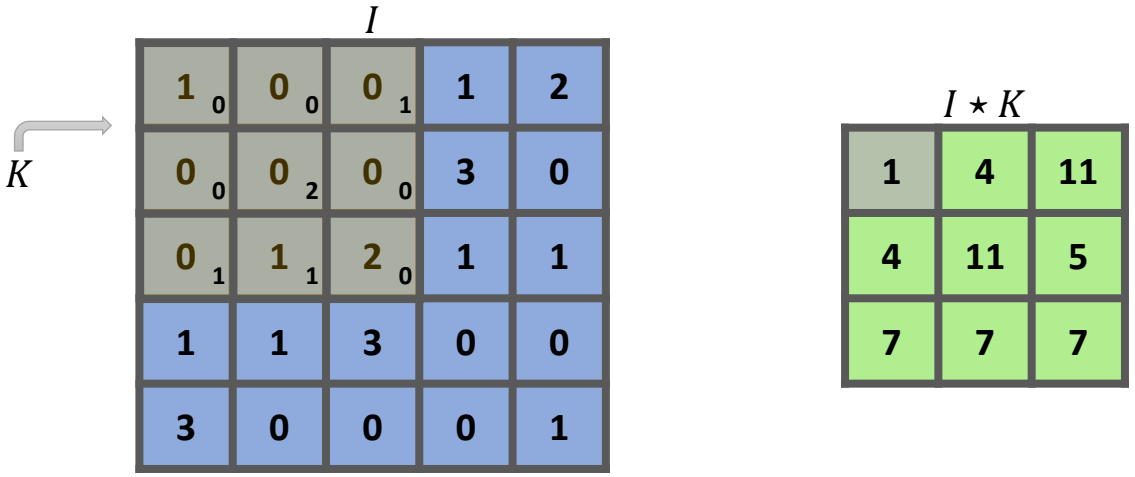

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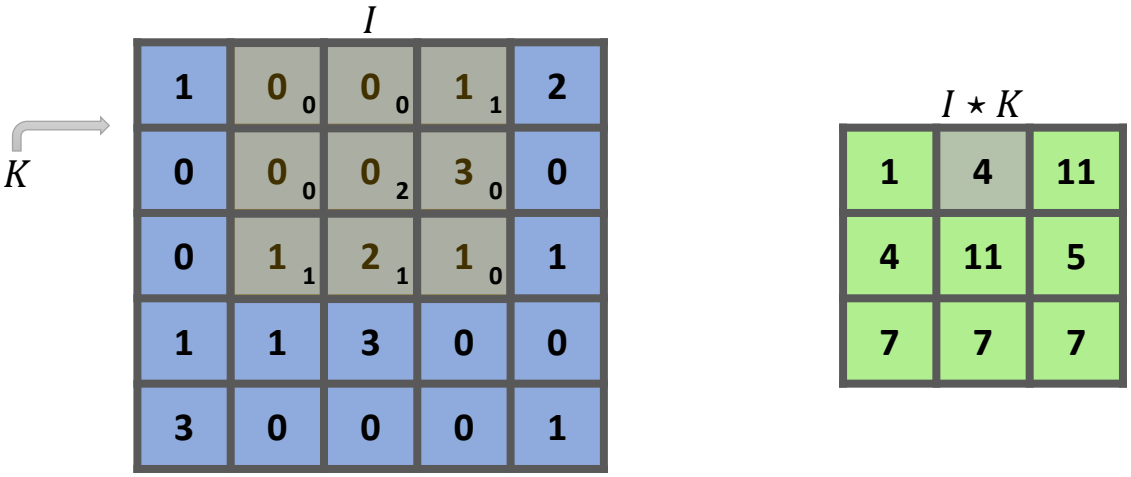

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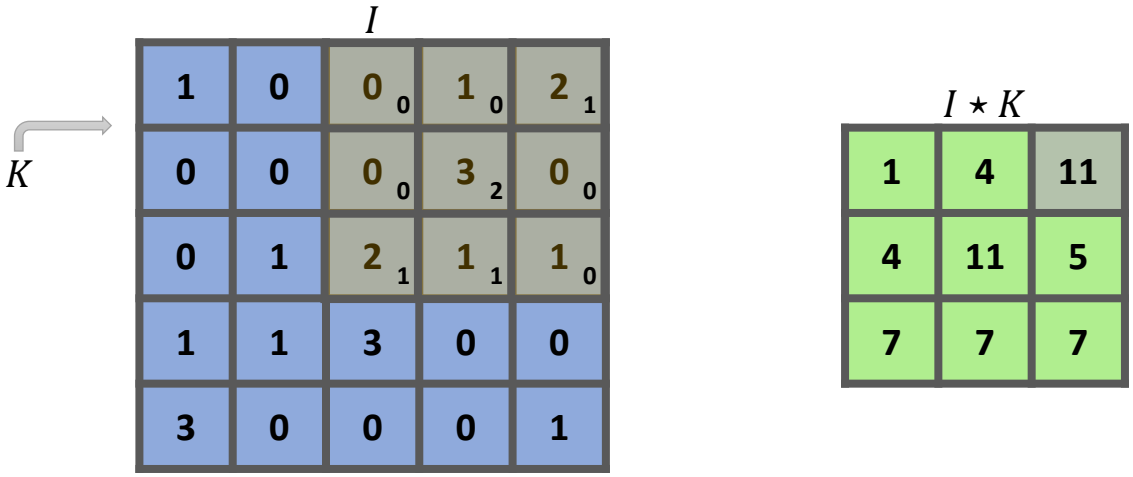

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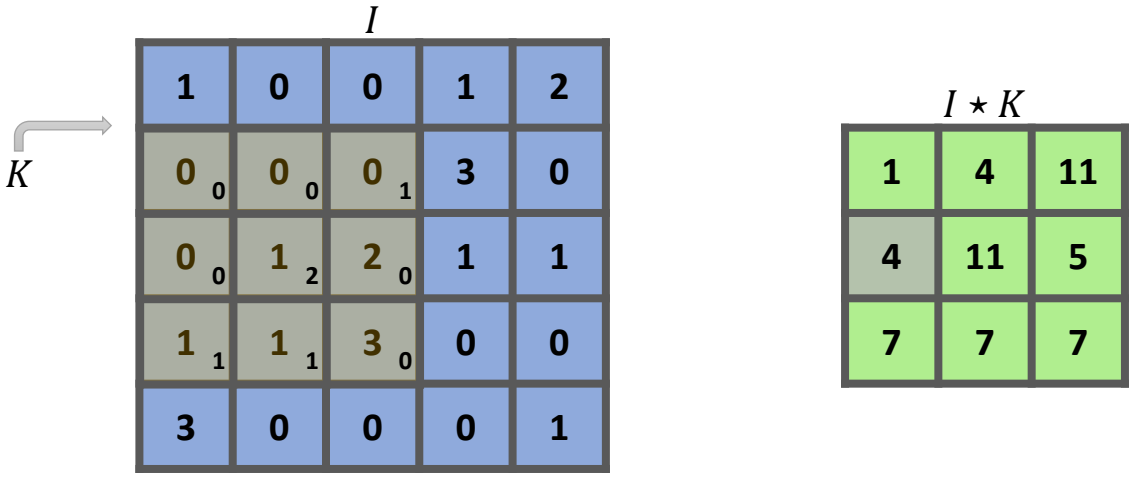

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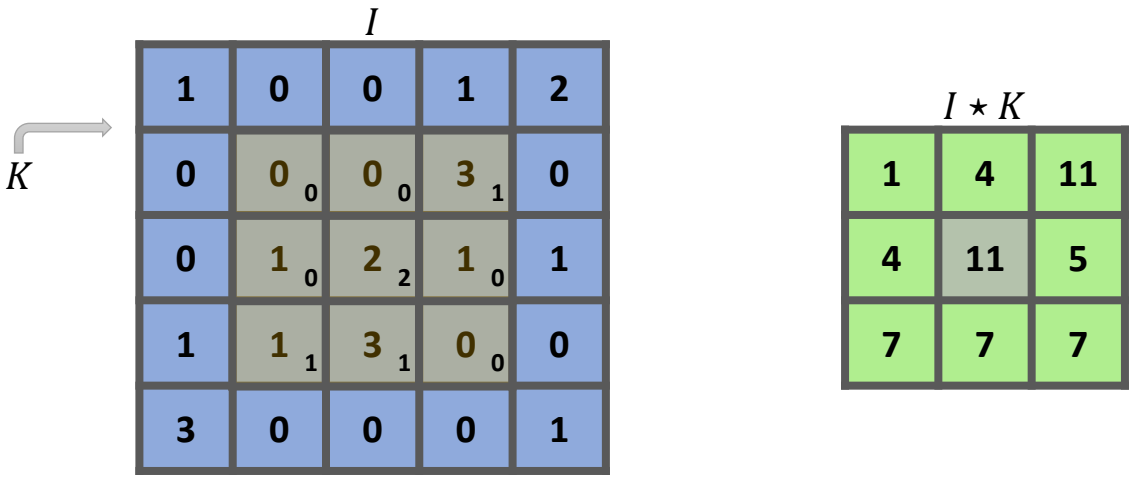

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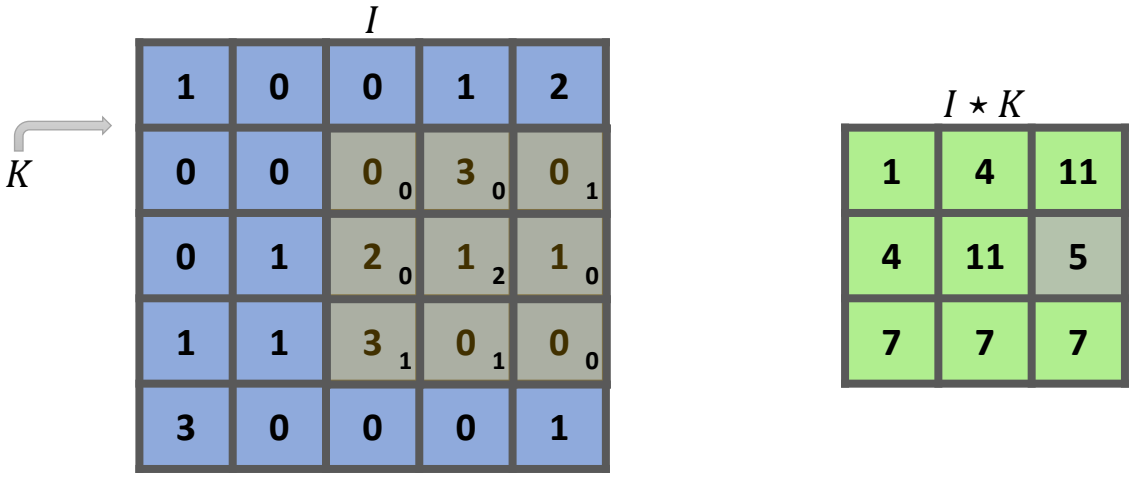

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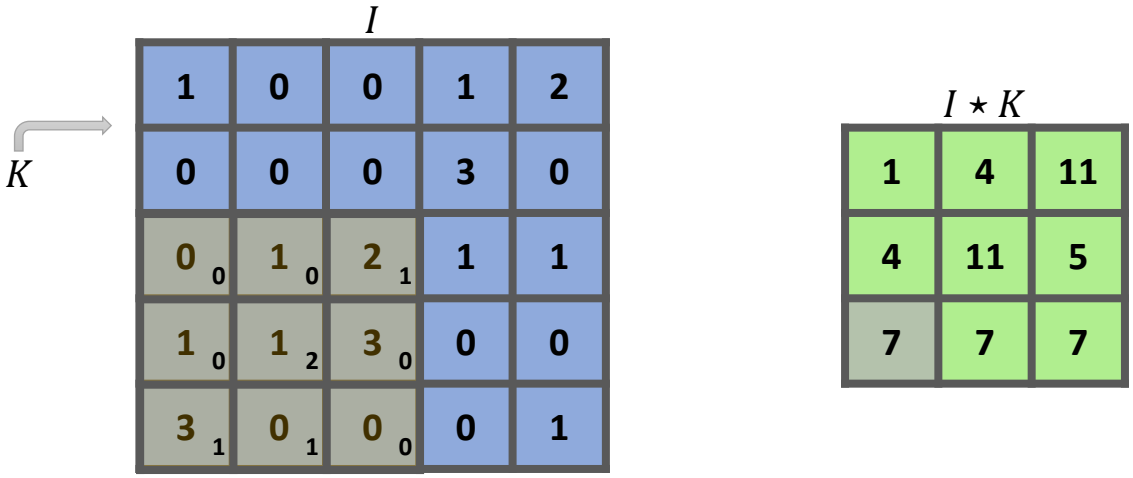

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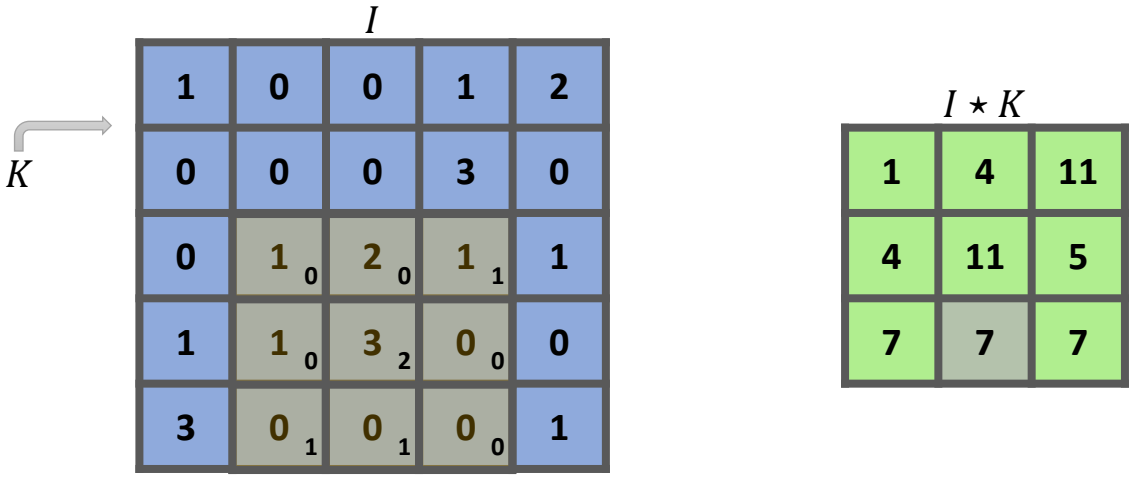

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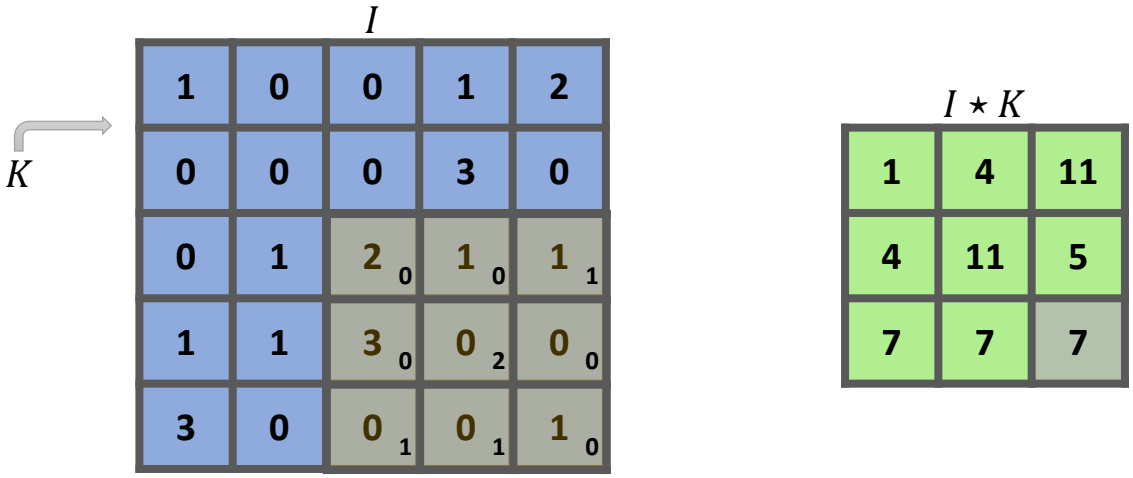

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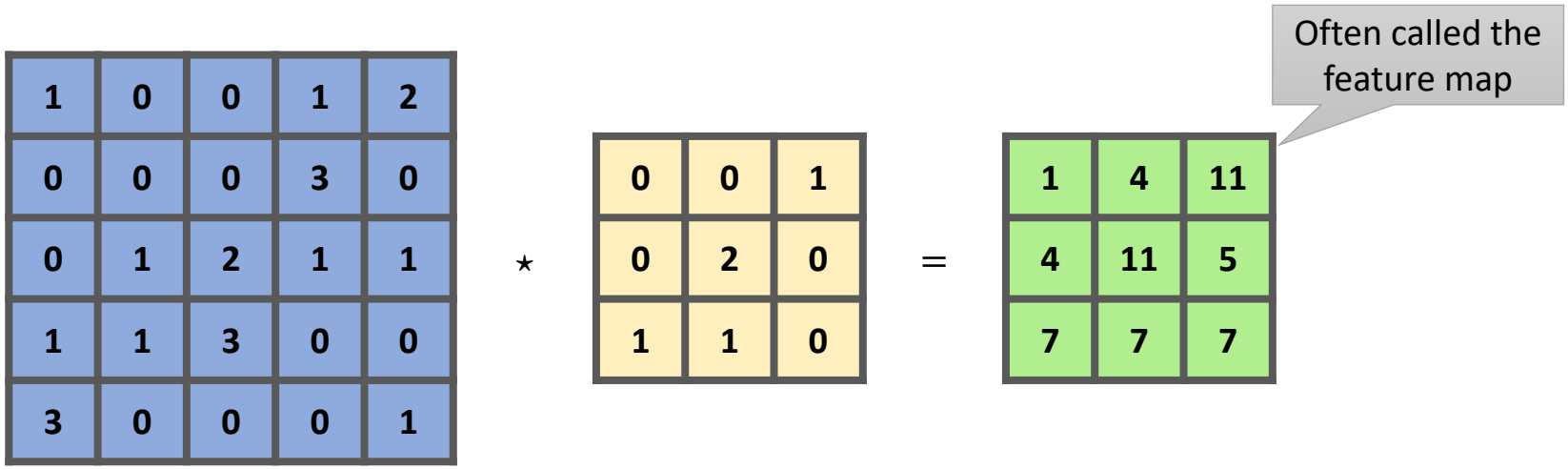
Discrete cross-correlation: 2-D example



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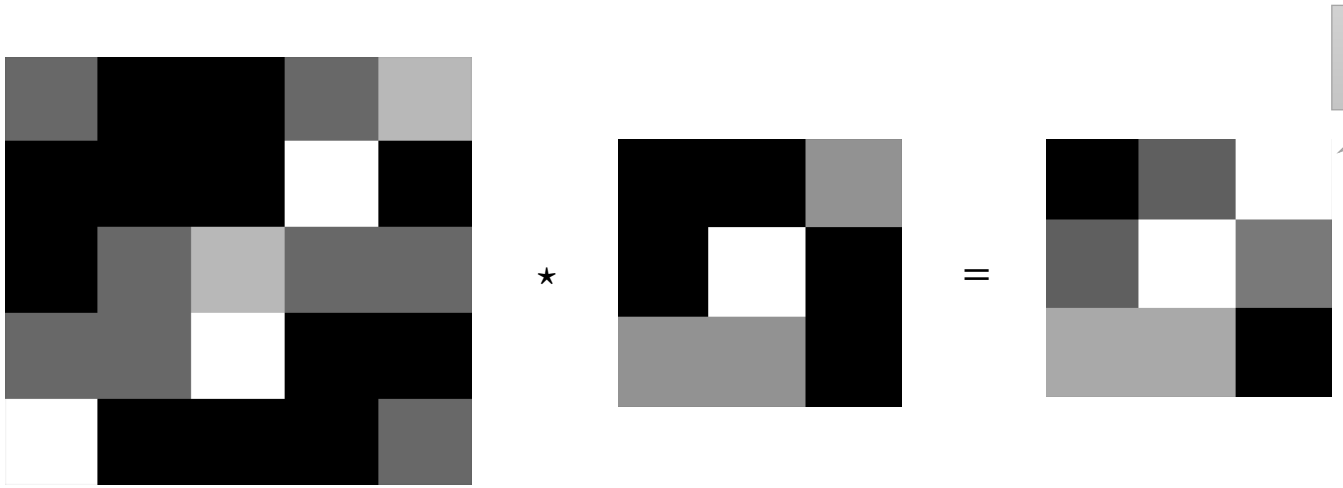
Discrete cross-correlation: 2-D example



$$(I \star K)(i,j) == \sum_m \sum_n I(m,n)K(i+m, j+n)$$

Vertical edge detection

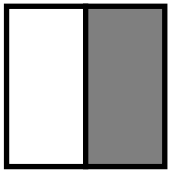
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

*

1	0	-1
1	0	-1
1	0	-1

=

0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0



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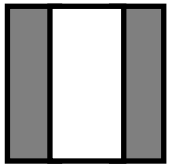
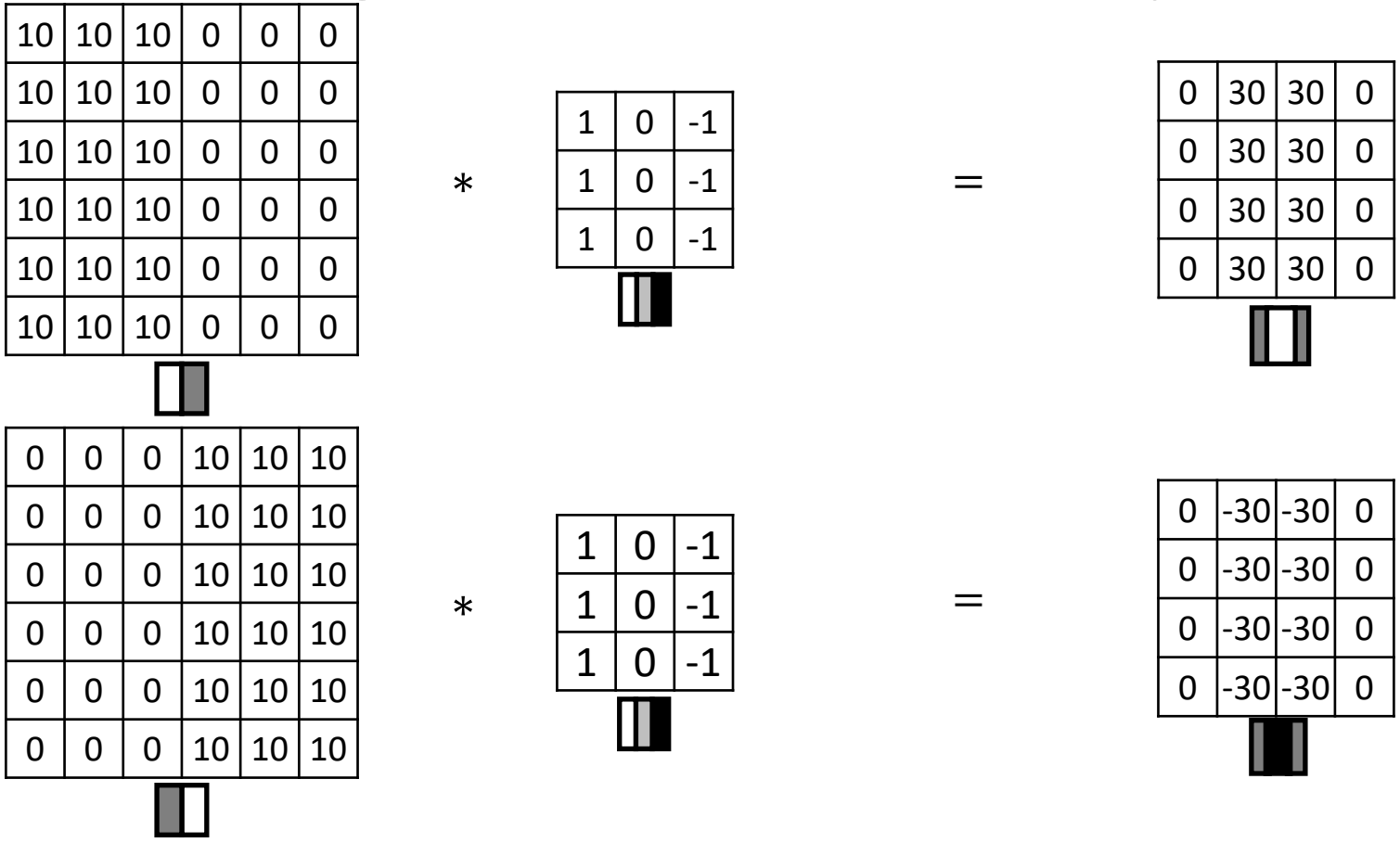


Vertical edge detection examples



Vertical and Horizontal Edge Detection

1	0	-1
1	0	-1
1	0	-1

Vertical

1	1	1
0	0	0
-1	-1	-1

Horizontal

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
0	0	0	10	10	10
0	0	0	10	10	10
0	0	0	10	10	10

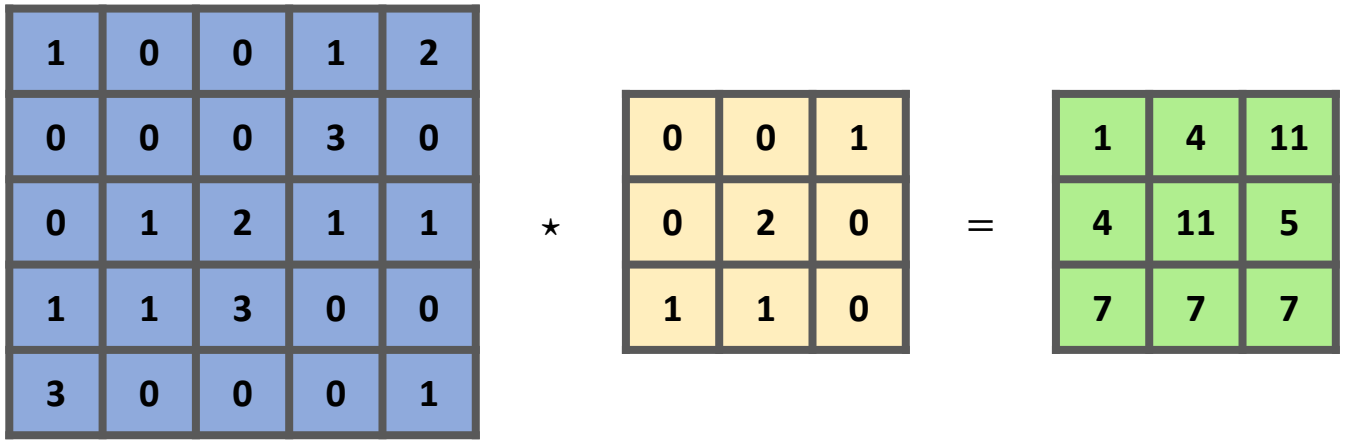
*

1	1	1
0	0	0
-1	-1	-1

=

0	0	0	0
30	10	-10	-30
30	10	-10	-30
0	0	0	0

Discrete cross-correlation: 2-D example



Something's not quite right…

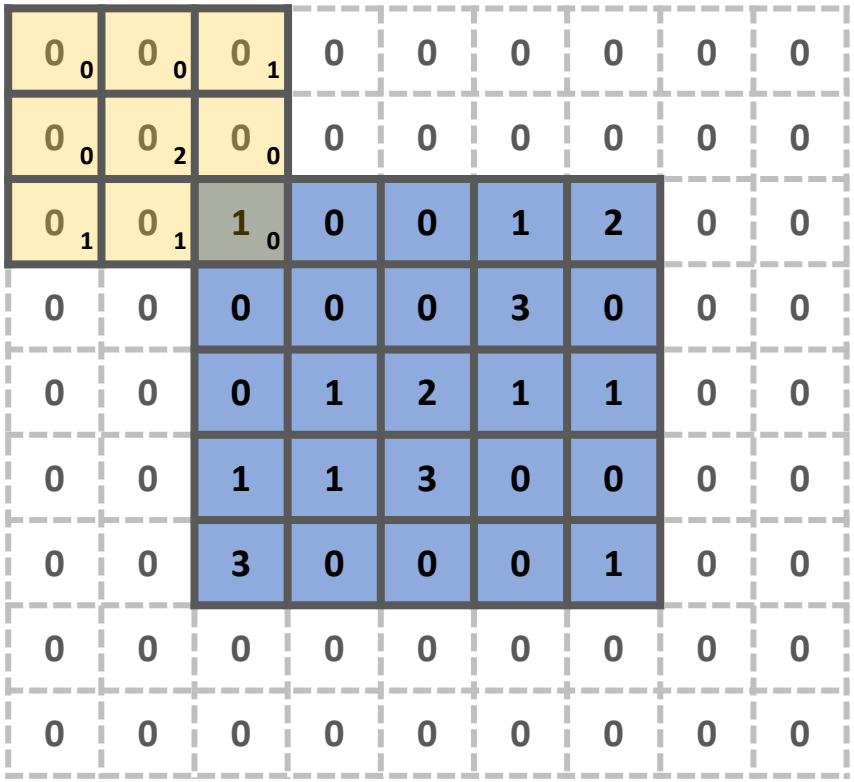
$$(I \star K)(i,j) == \sum_m \sum_n I(m,n)K(i+m, j+n)$$

Cross-correlation: padding

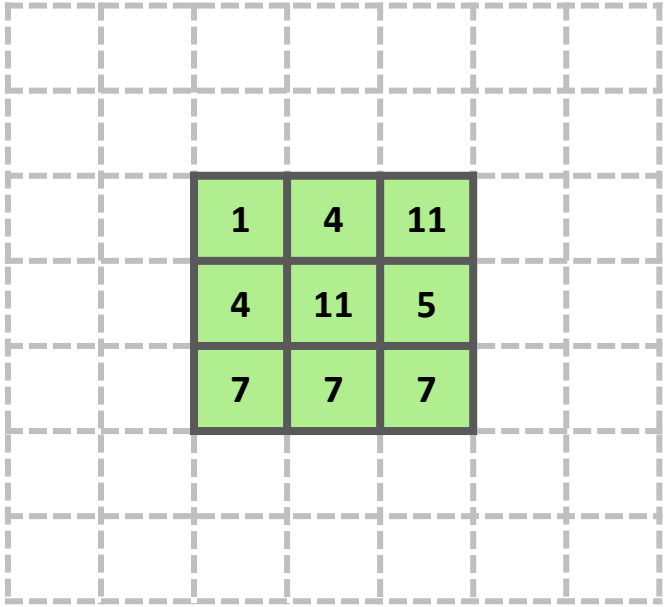
Adding extra pixels outside the image

0	0	0	0	0	0
0	35	19	25	6	0
0	13	22	16	53	0
0	4	3	7	10	0
0	9	8	1	3	0
0	0	0	0	0	0

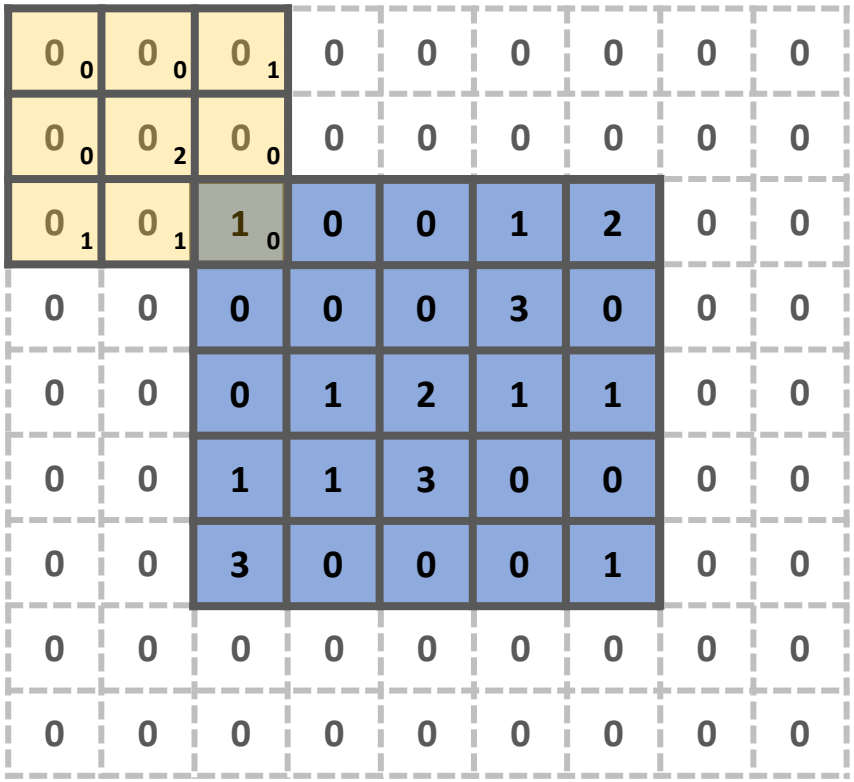
Discrete cross-correlation: padding

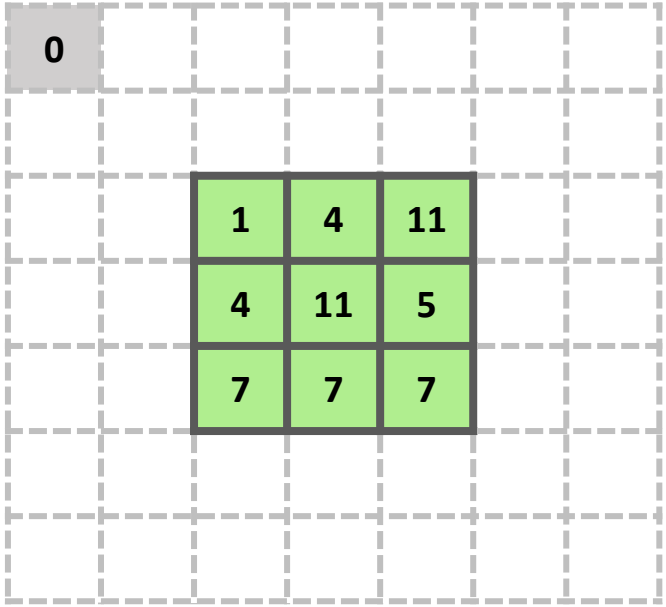


Technically, our signals have infinite extent…
we solve by padding with zeros

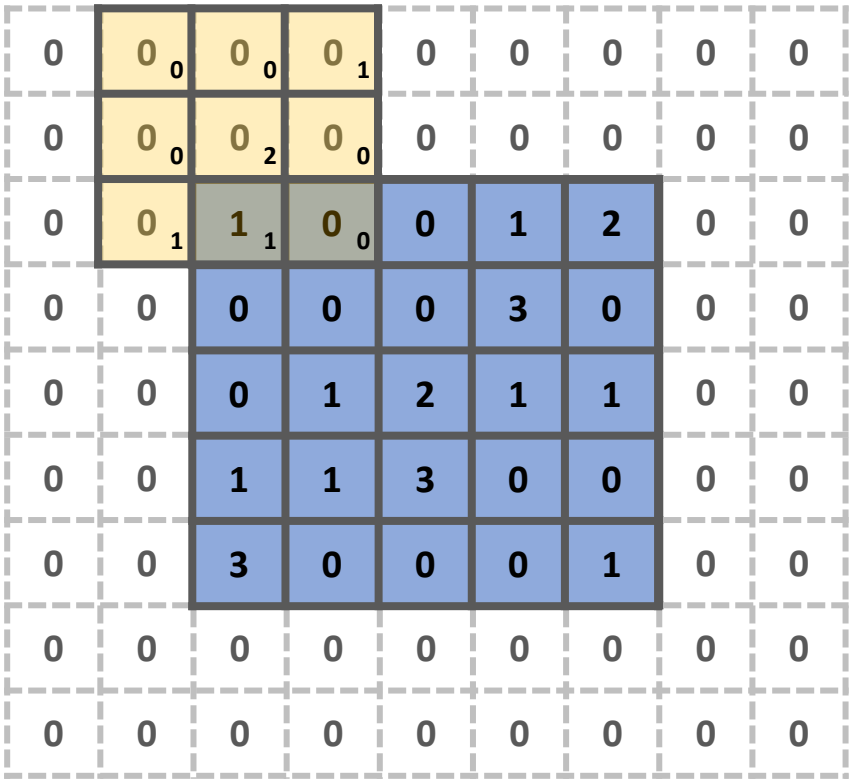


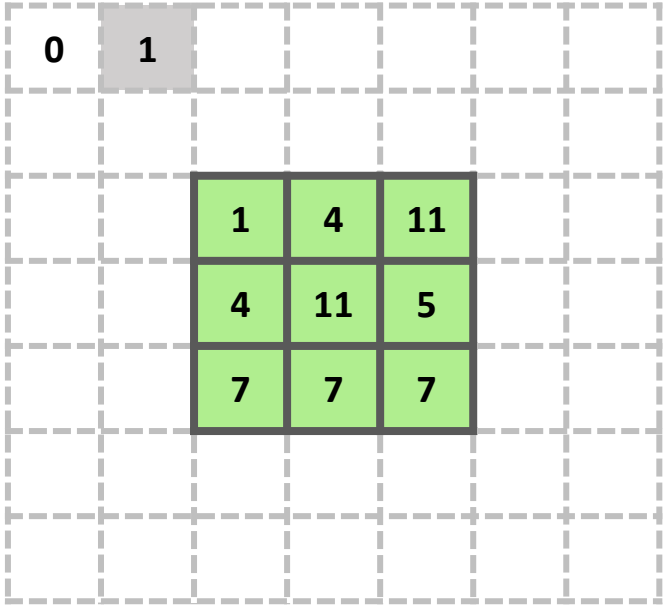
Discrete cross-correlation: padding



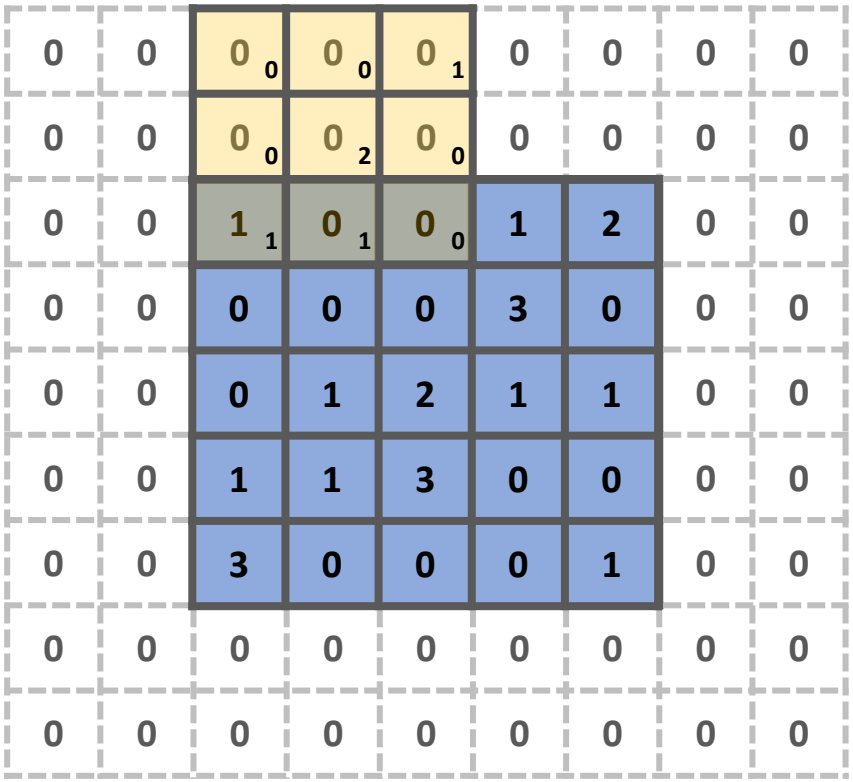


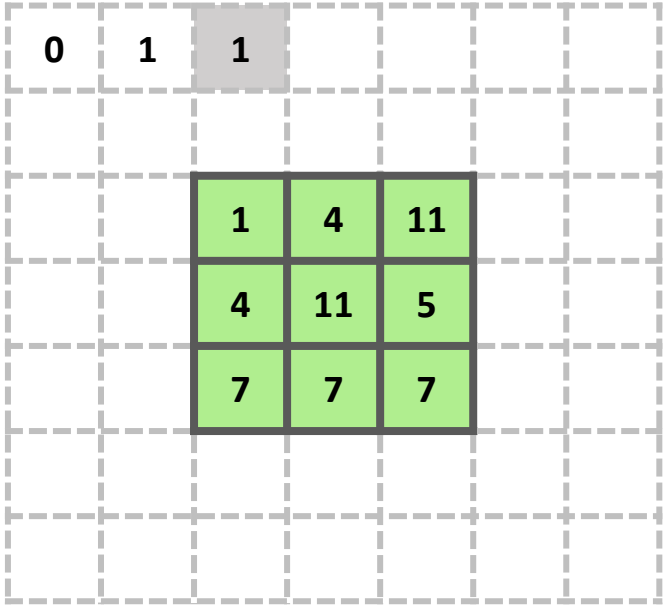
Discrete cross-correlation: padding



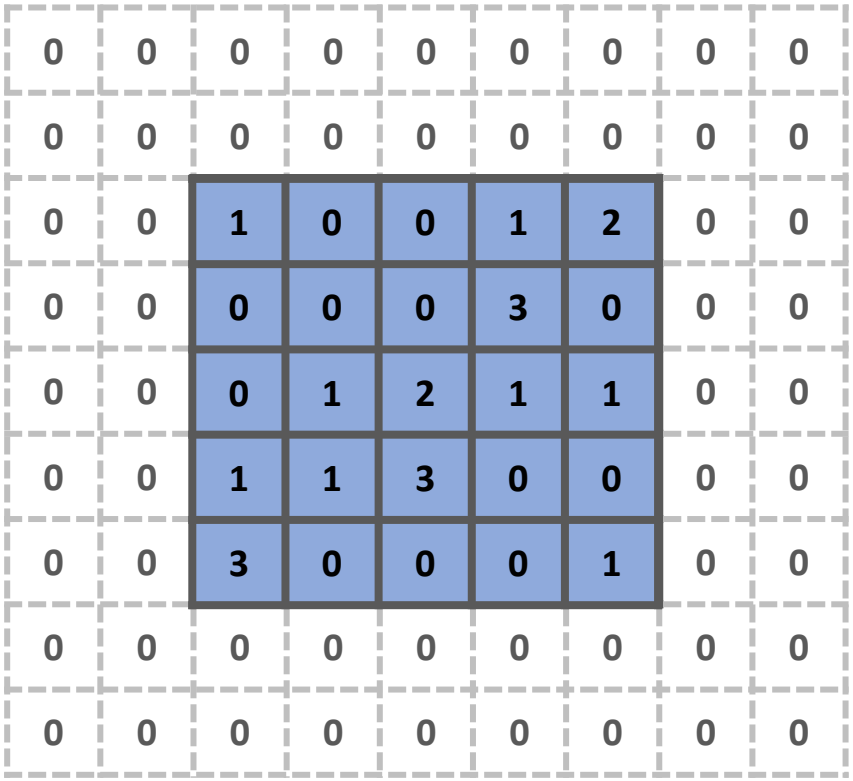


Discrete cross-correlation: padding

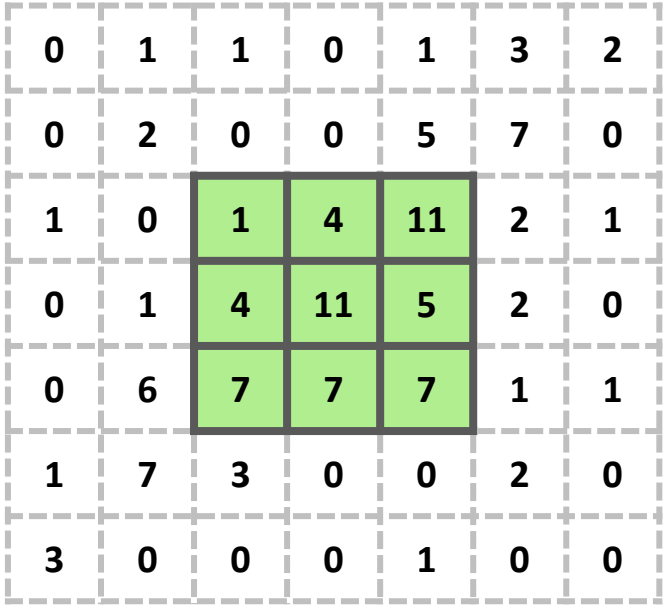




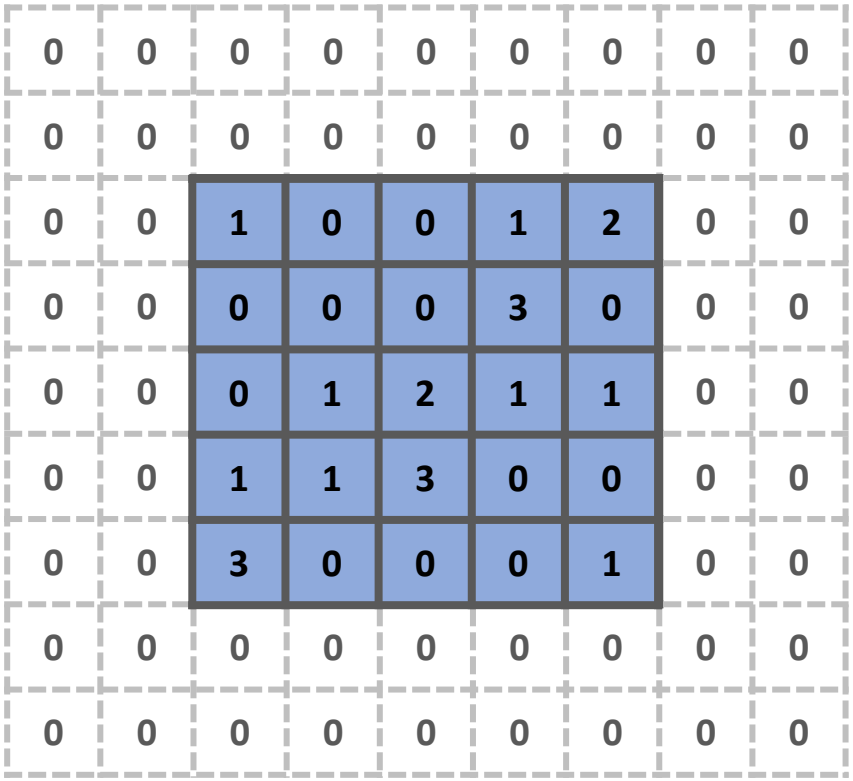
Discrete cross-correlation: padding



Normally we want the output to maintain the input size



Discrete cross-correlation: padding



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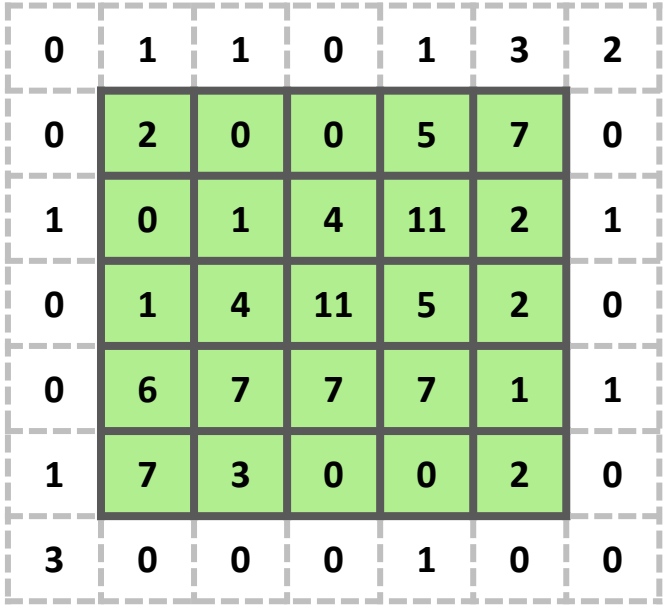
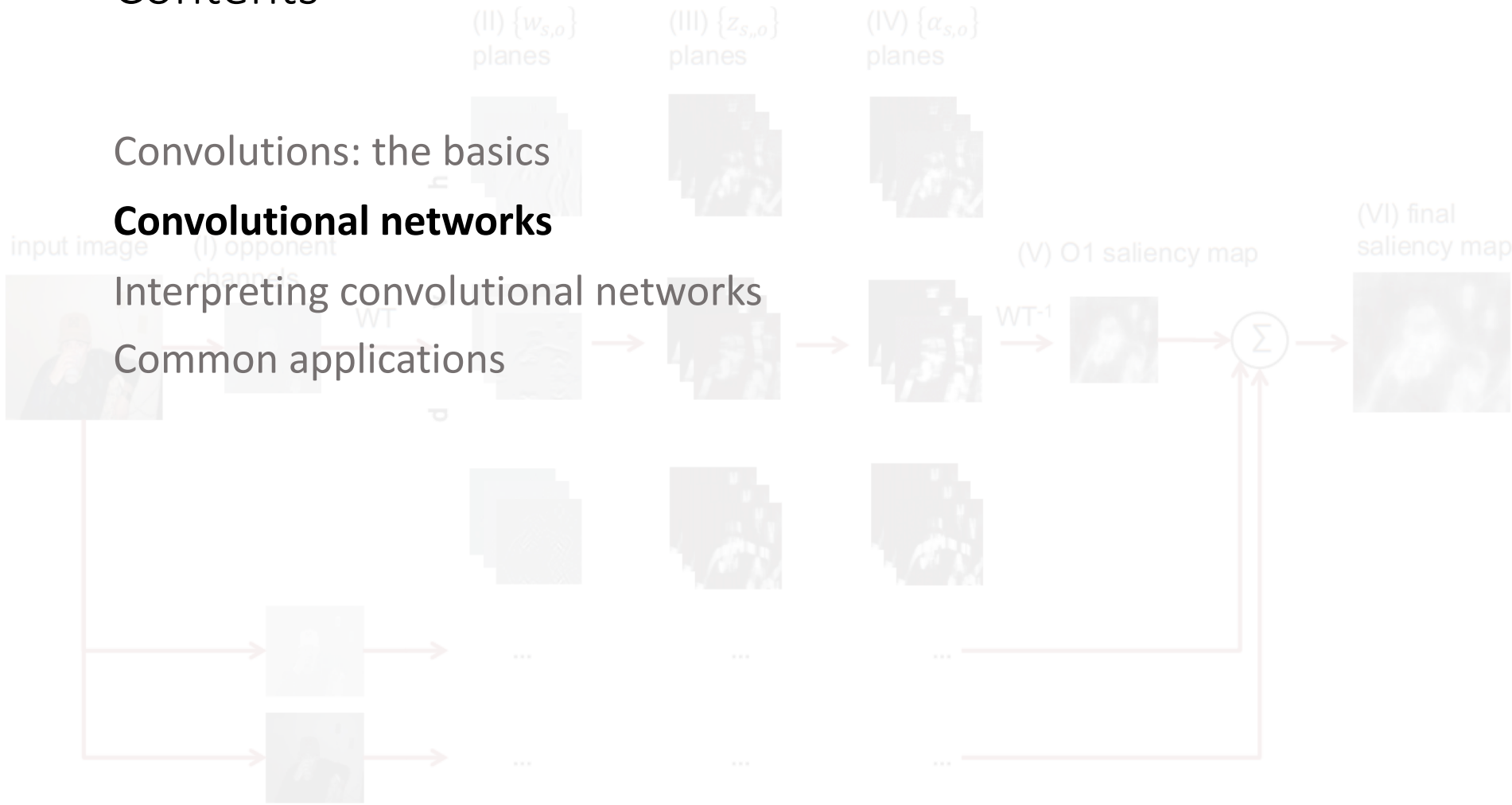


Contents



Convolutional networks

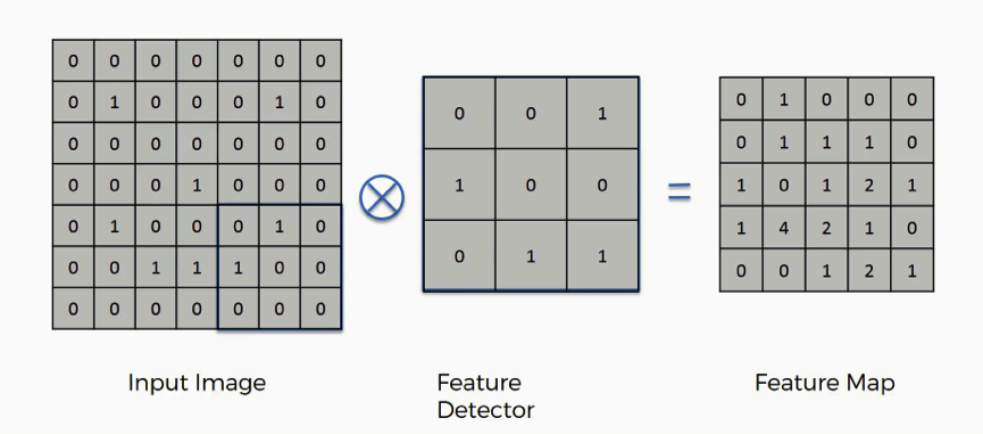
Reminder: Neural networks that include convolution operations

Can be used in place of dense matrix multiplication (i.e. fully-connected layers)

Motivations:

- Sparse connectivity
- Parameter sharing
- Translation equivariance
- Arbitrary input sizes

Why convolutions: Motivation



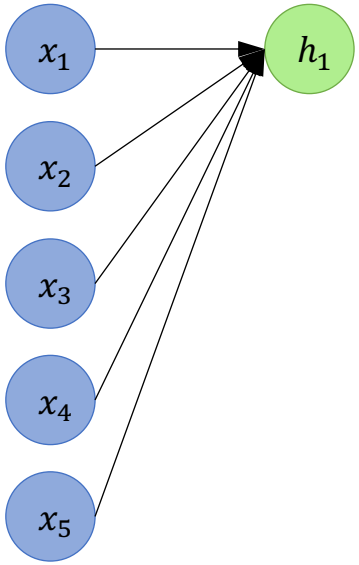
Sparsity of connections: In each layer, each output value depends only on a small number of inputs.

Parameter sharing: A feature detector (such as a vertical edge detector) that's useful in one part of the image is probably useful in another part of the image.

Translation Equivariance

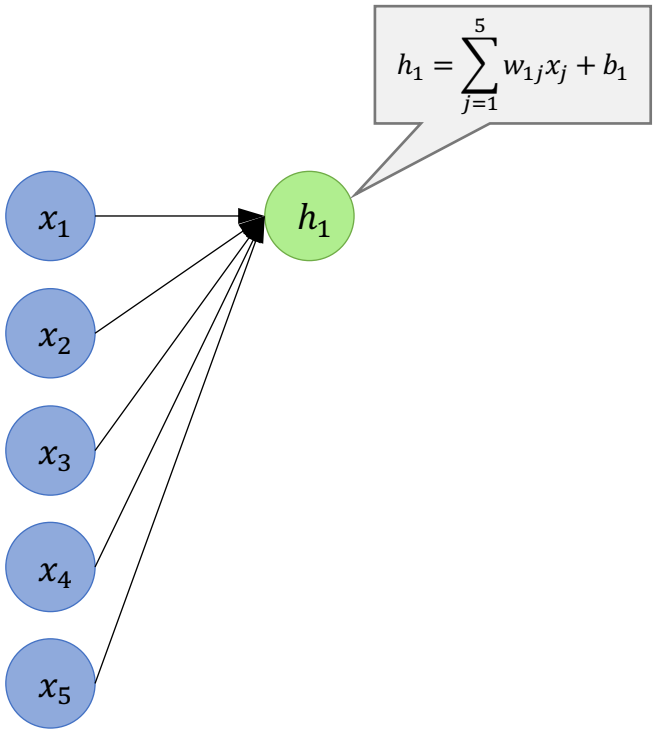
Arbitrary Input Sizes

Neural networks ⇒ Convolutional networks



$$\boldsymbol{h} = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{b};\ h_i = \sum_j w_{ij} x_j + b_i$$

Neural networks ⇒ Convolutional networks



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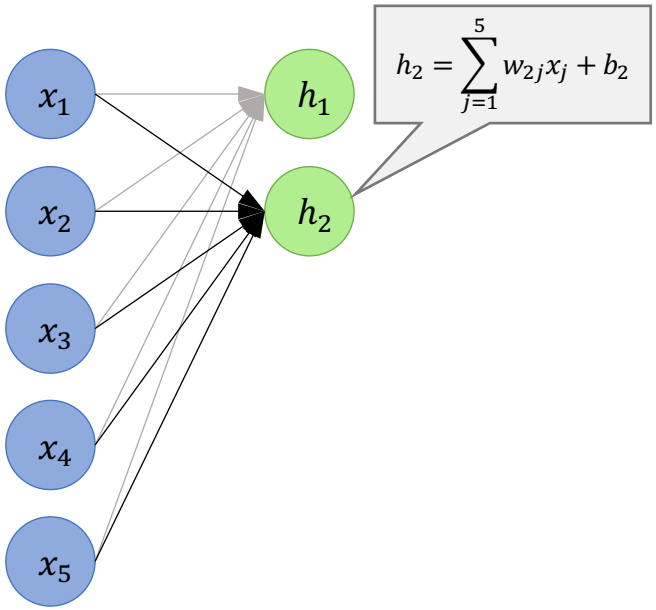
Neural networks ⇒ Convolutional networks



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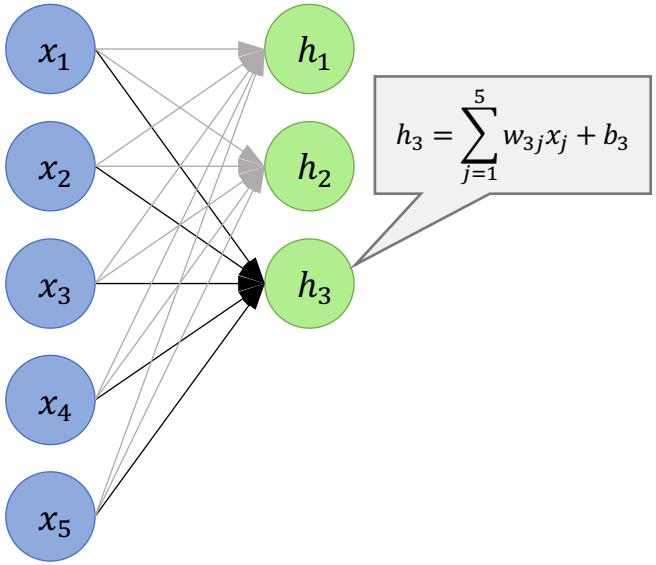

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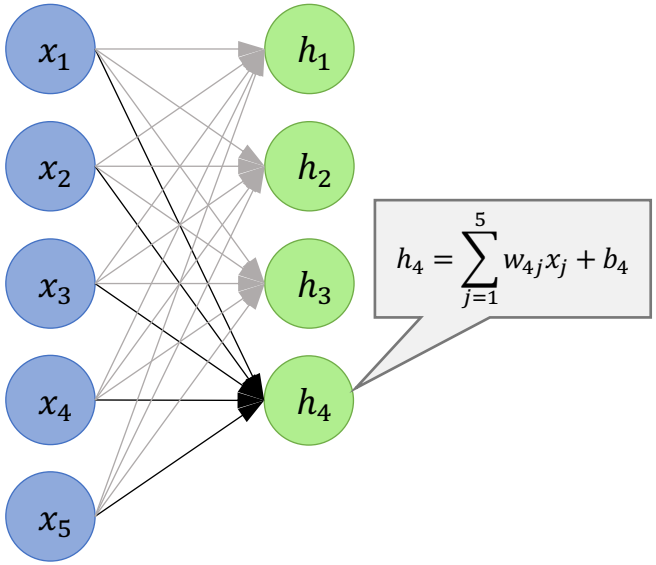

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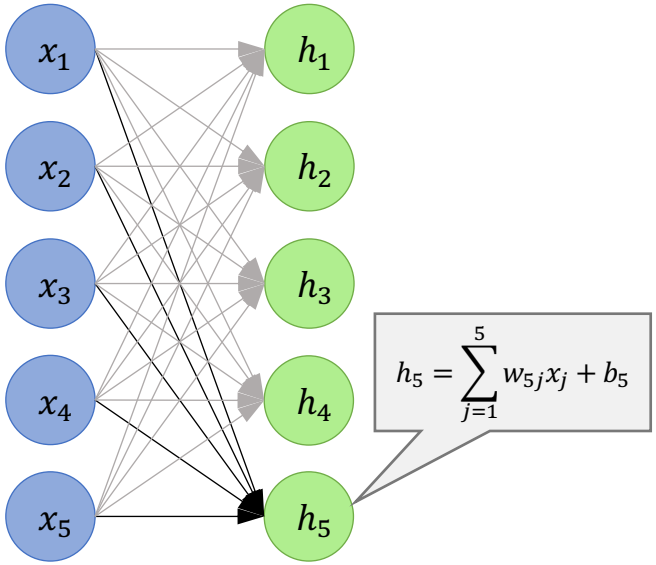

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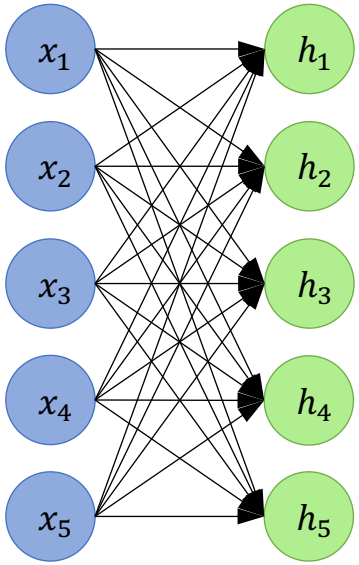

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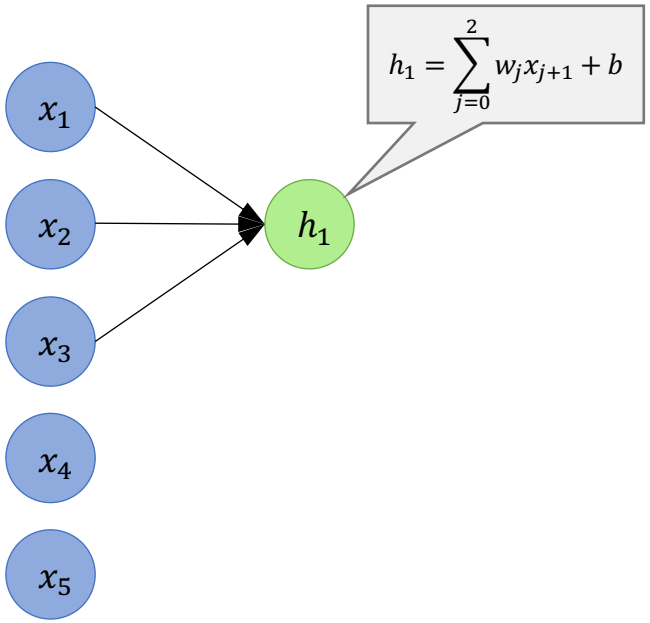
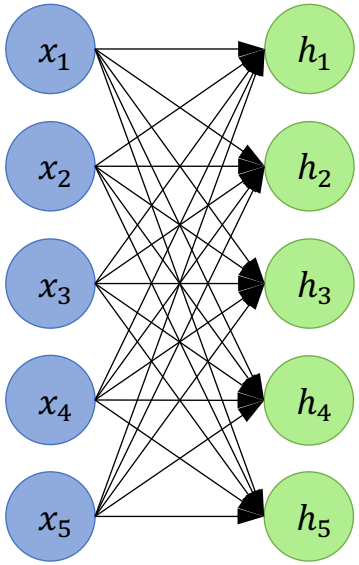


$$h_i = \sum_j w_j x_{j+i} + b$$

Neural networks ⇒ Convolutional networks



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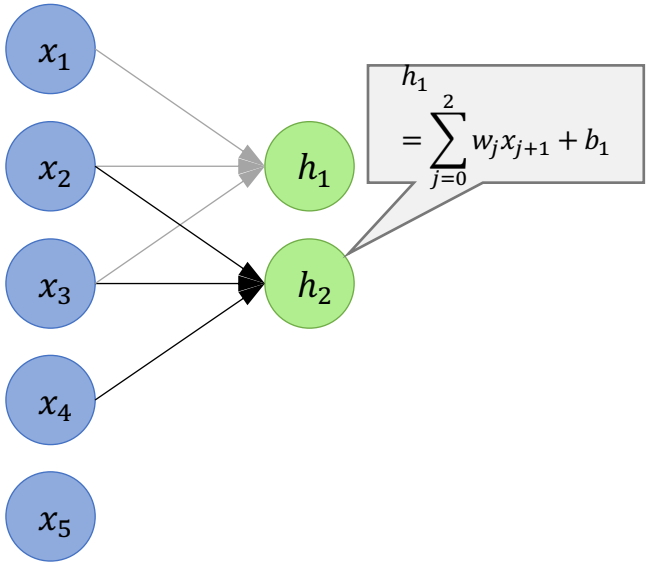

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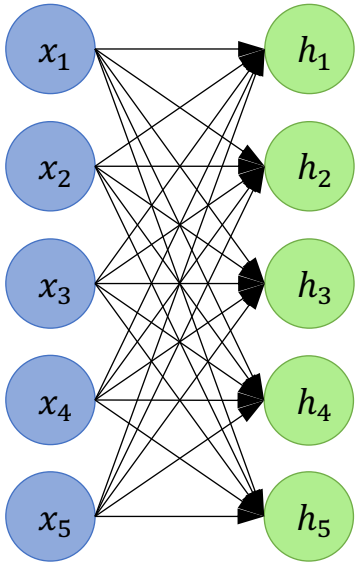

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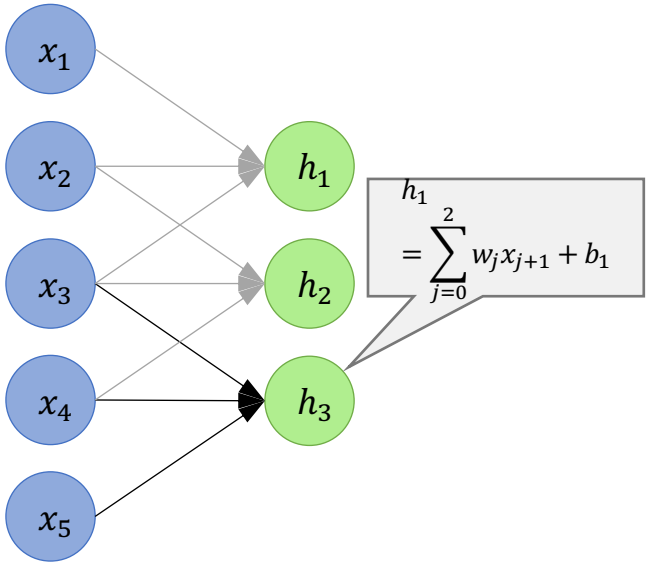

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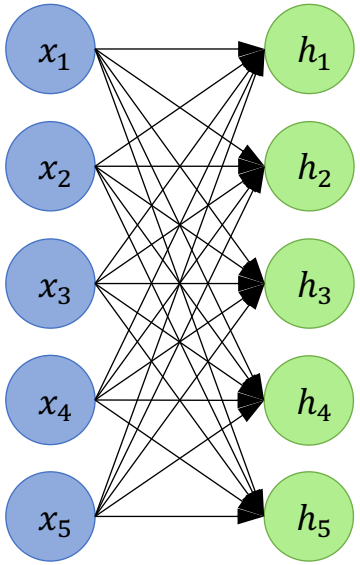
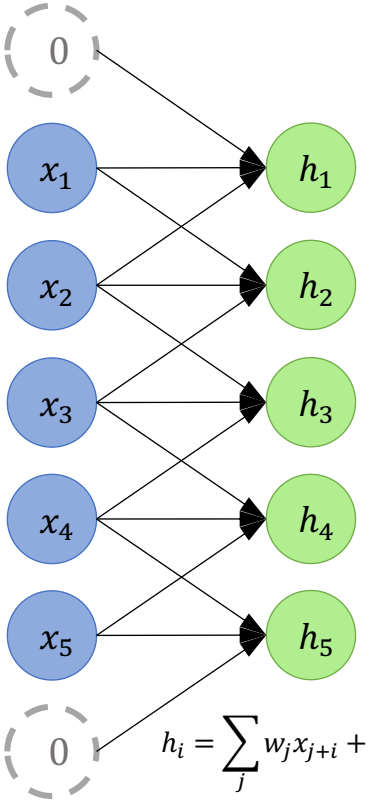


$$\boldsymbol{h} = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{b};\ h_i = \sum_j w_{ij} x_j + b_i$$

dense connectivity vs sparse connectivity

In our example:
5x5 multiplications vs 5x3 multiplications

Sparse connectivity scales better
e.g. 25x25 vs 25x3



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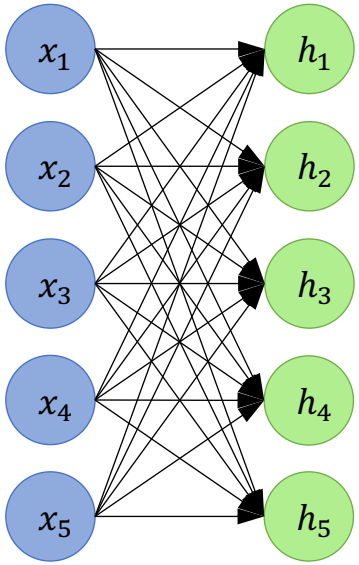
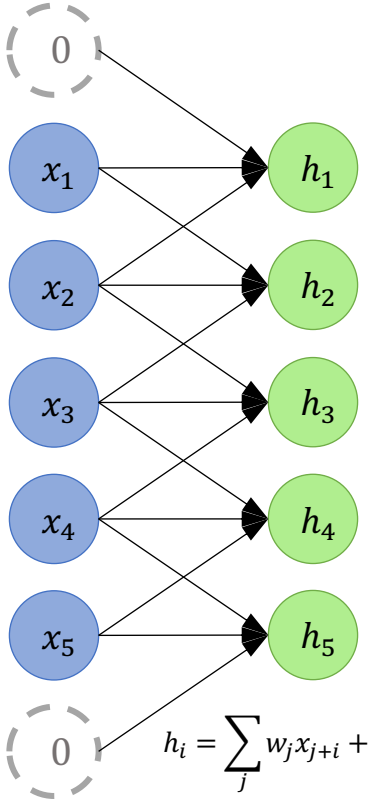


$$\boldsymbol{h} = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{b};\ h_i = \sum_j w_{ij} x_j + b_i$$

unshared vs shared weights

In our example:
5x5 vs 3 weights

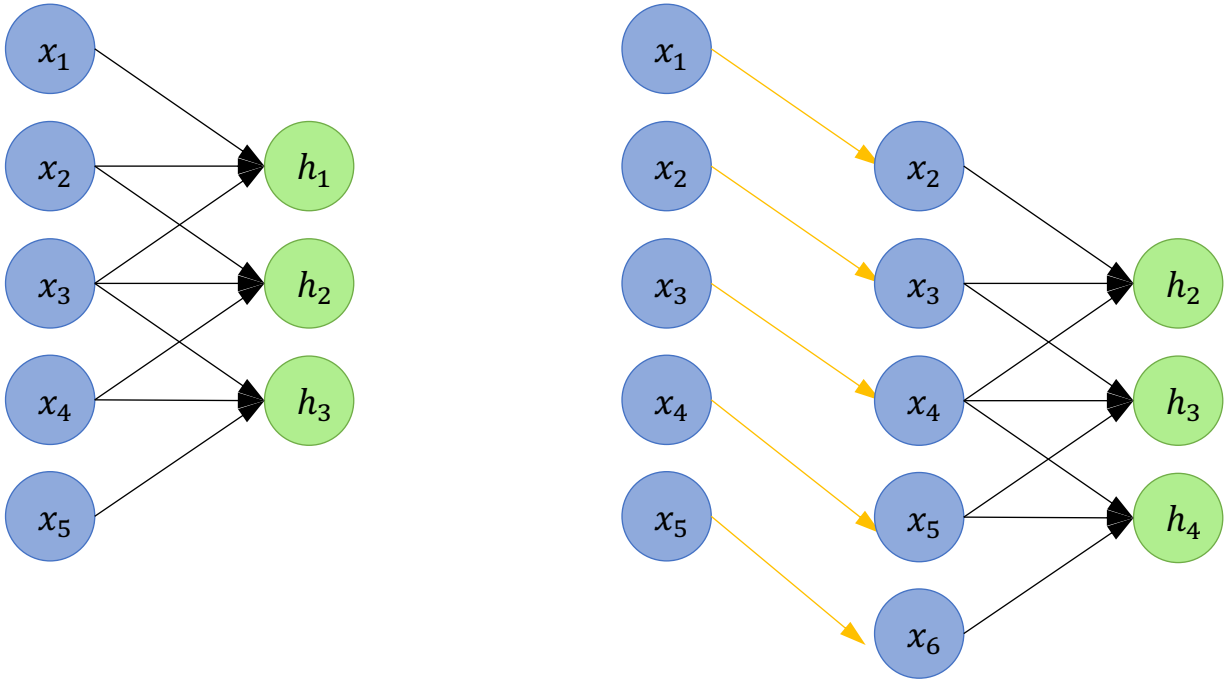
shared weights scale way better:
e.g. 25x25 vs 3



$$h_i = \sum_j w_j x_{j+i} + b$$

Translation equivariance

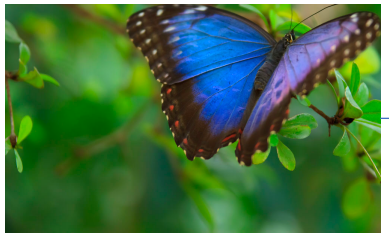
x_i $\quad x_i' = x_{i-1}$ $\quad h_i' = h_{i-1}$



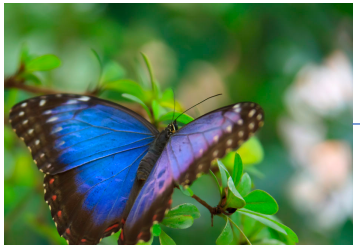
Translation equivariance

Why is translation equivariance useful?

Helps make predictions **translation invariant**



CNN → “butterfly”



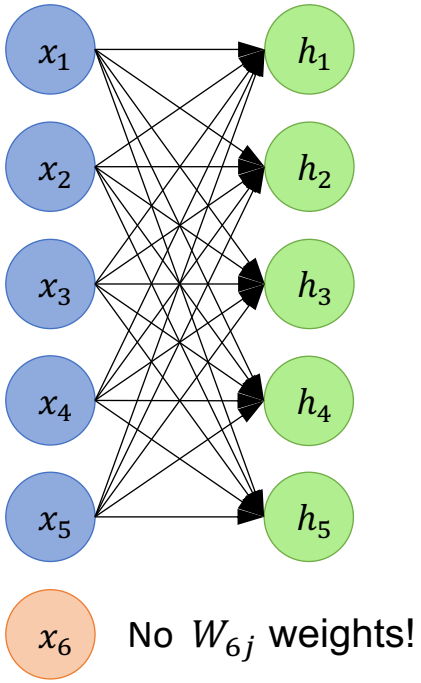
CNN → “butterfly”

CNN → “butterfly”

What other types of invariance could be useful?

Arbitrary input sizes

$$h_i = \sum_j W_{ij} x_j + b_i$$



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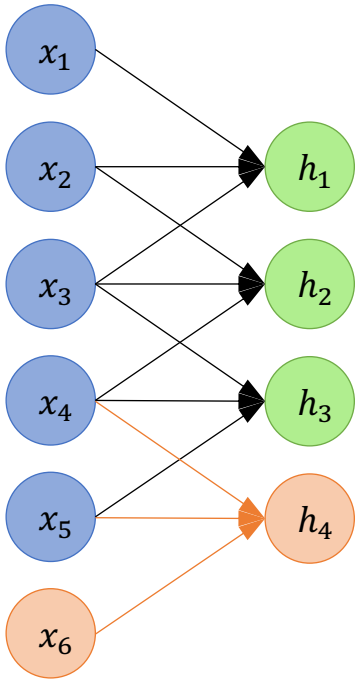

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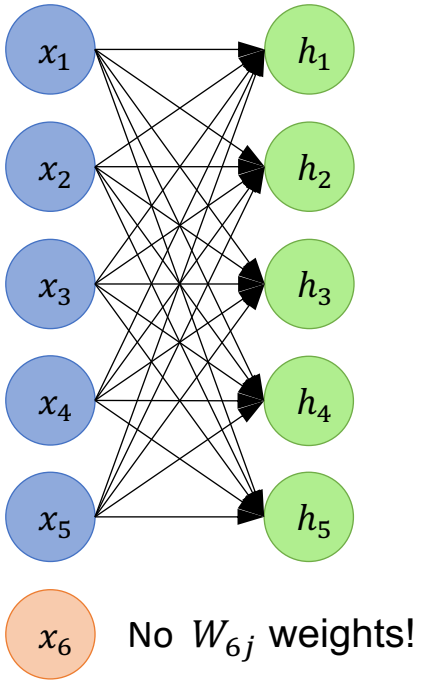

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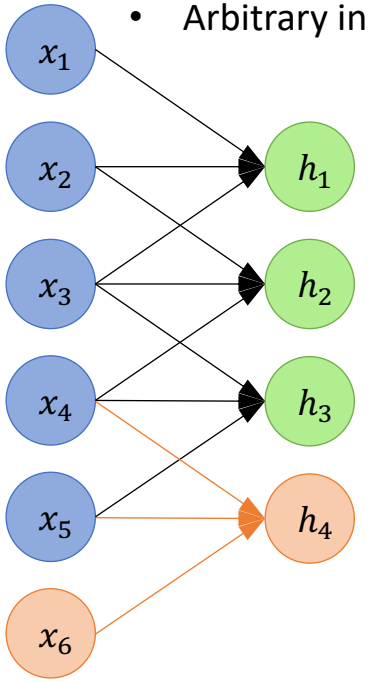
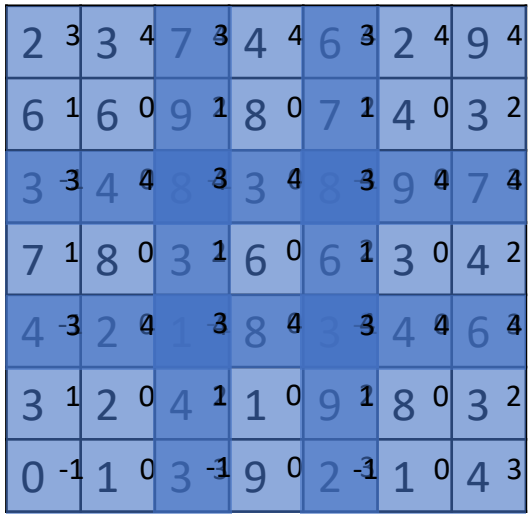


- Sparse connectivity
- Parameter sharing
- Translation equivariance
- Arbitrary input sizes

Strided convolution



*

3	4	4
1	0	2
-1	0	3

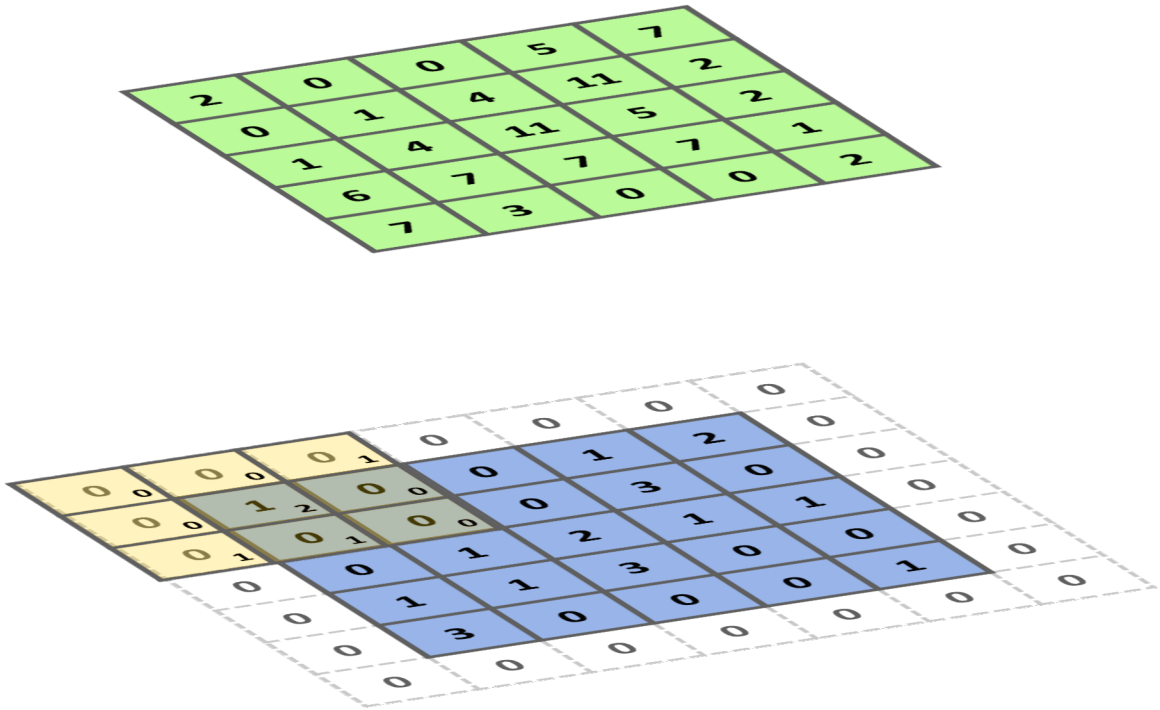
=

91	100	83
69	91	127
44	72	74

$\text{Stride} = 2$

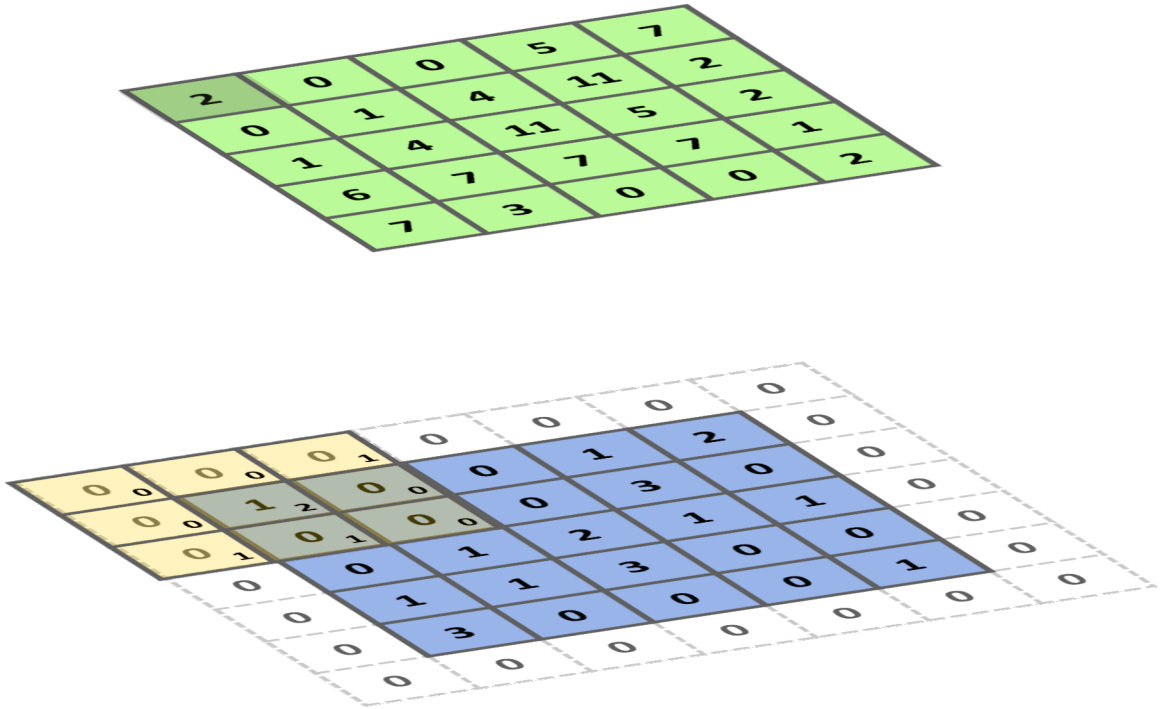
Stride is the number of pixels shifts over the input matrix (sliding step).

The effect of different strides



The effect of different strides

Stride: 1x1



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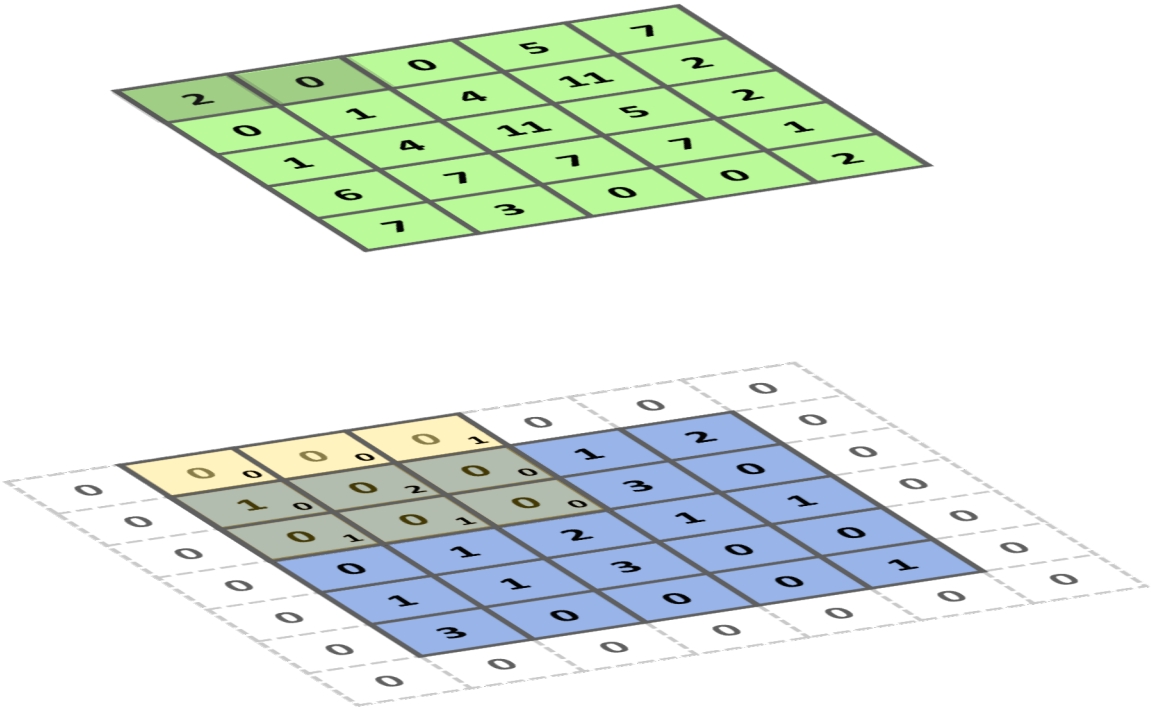

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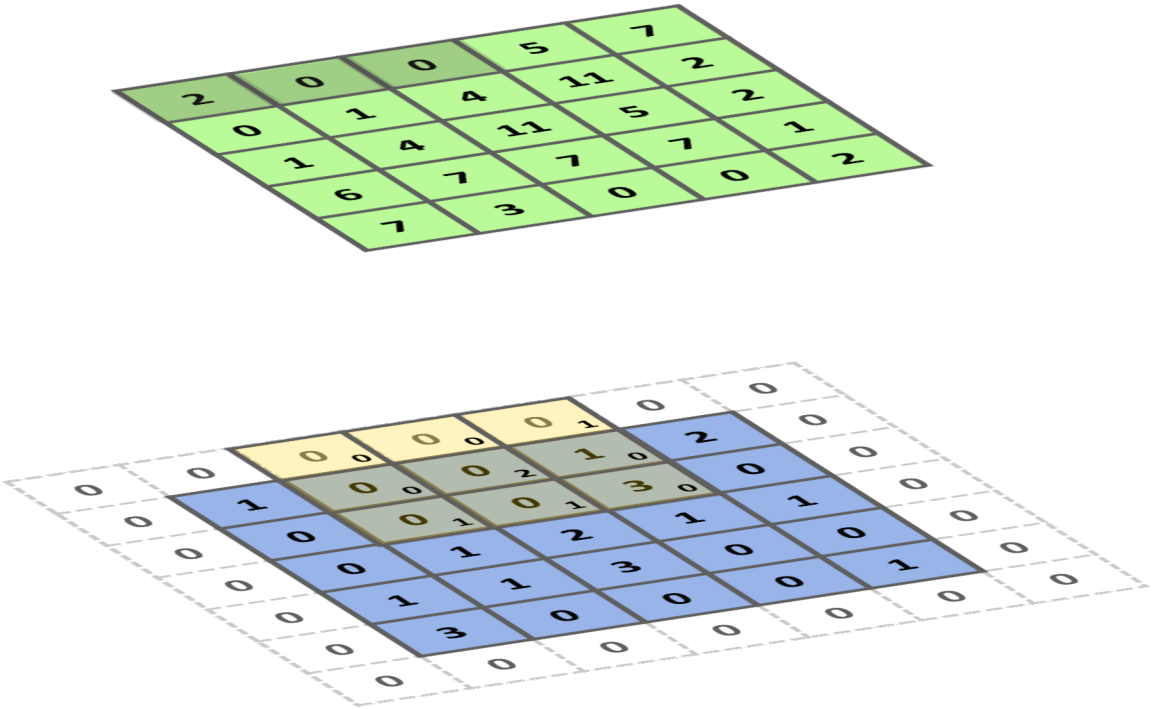

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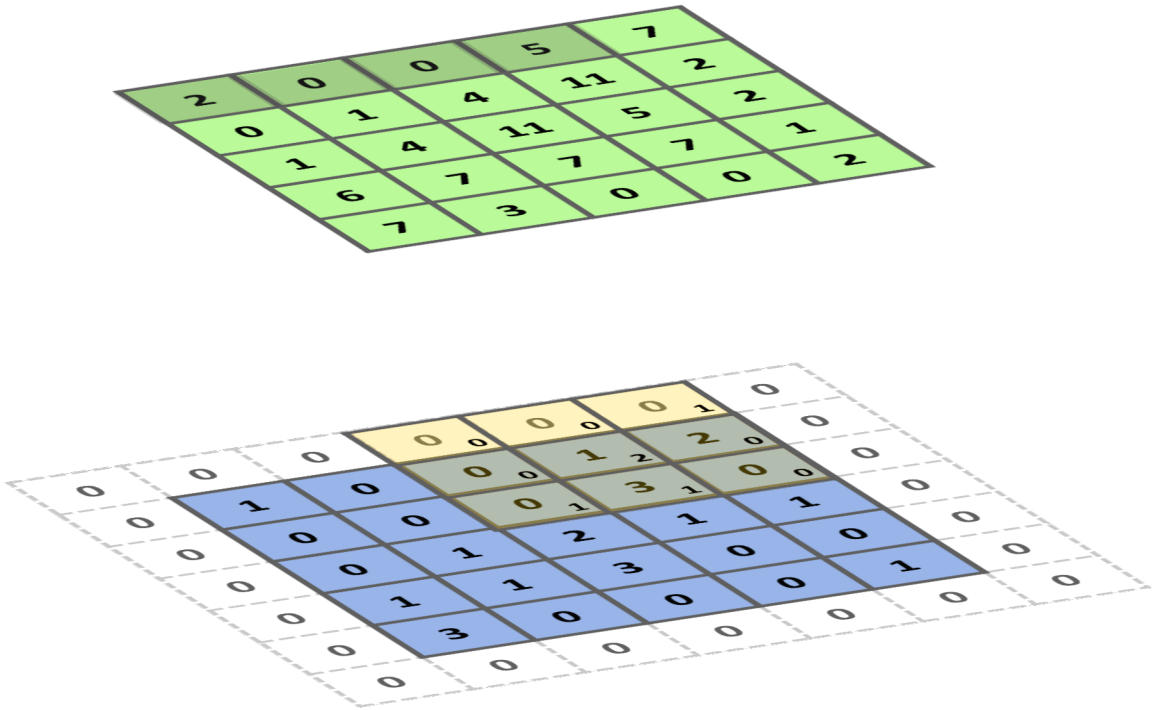
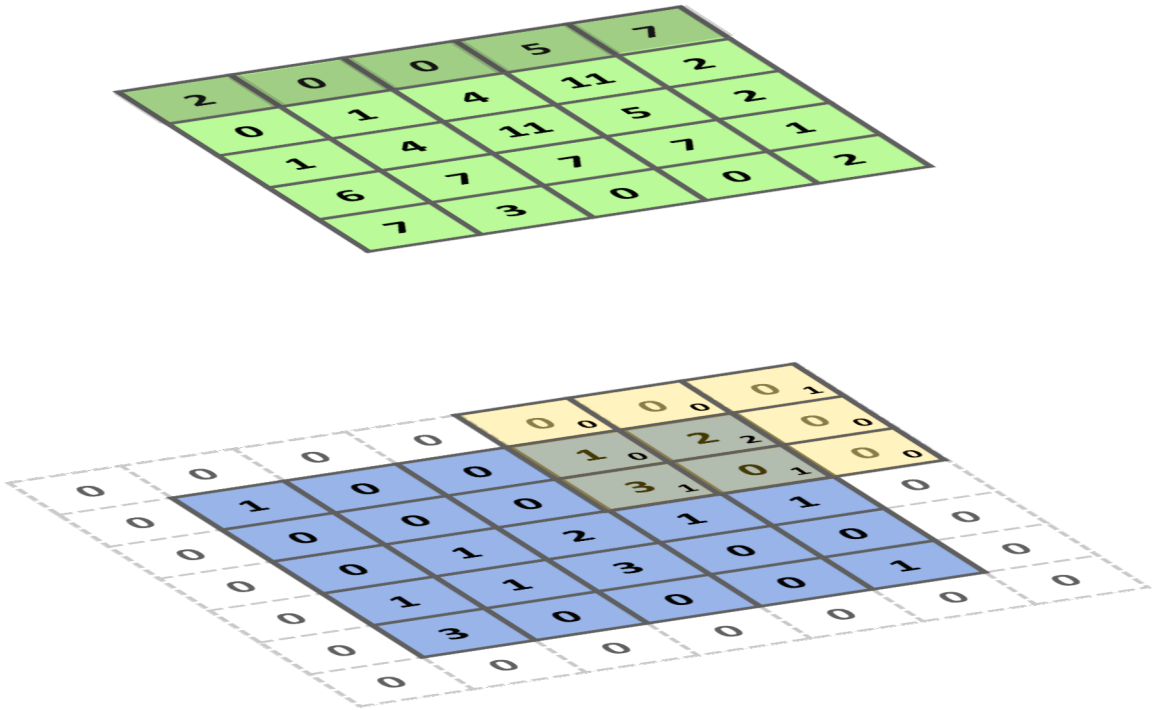


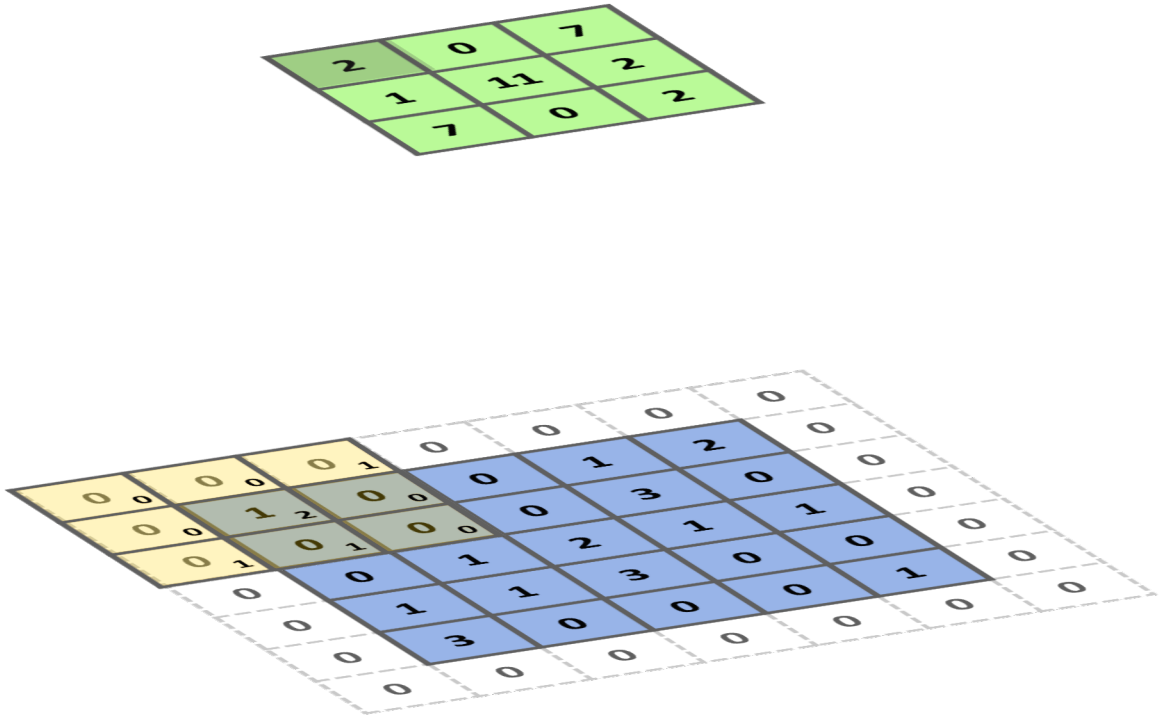
The effect of different strides

Stride: 1x1



The effect of different strides

Stride: 2x2



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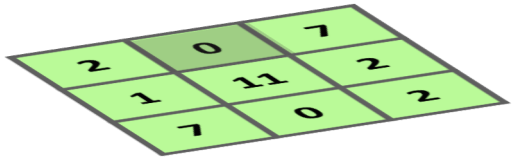
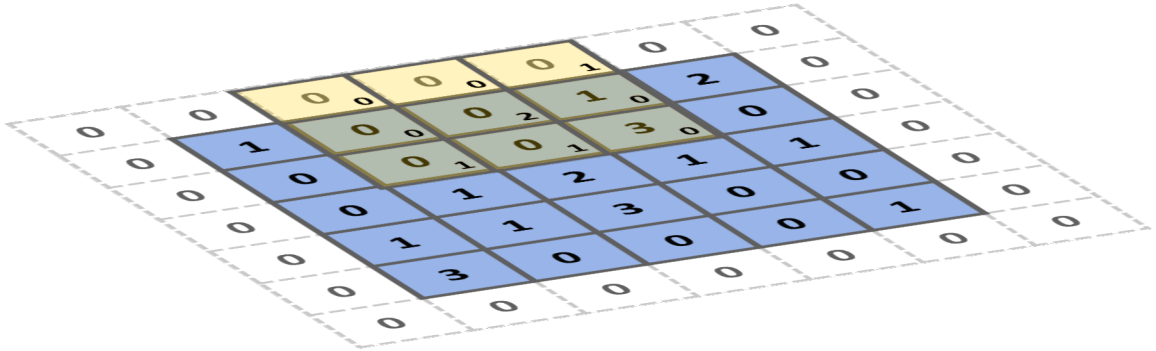
Stride: 2x2





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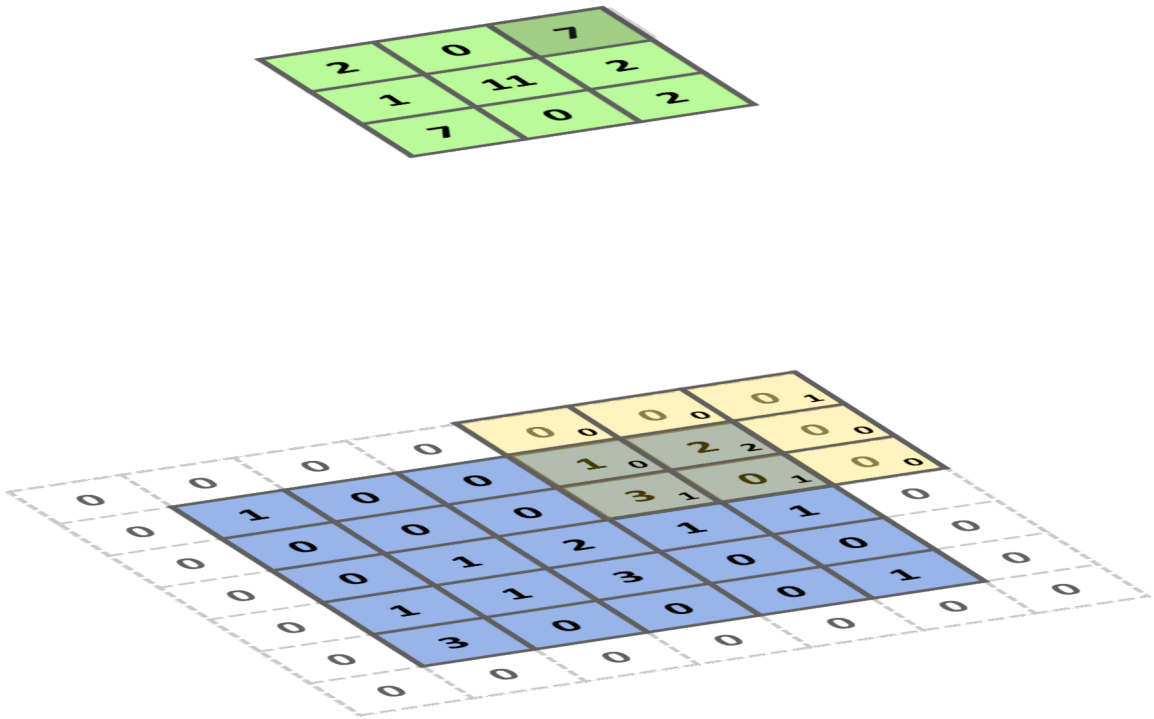
Stride: 2x2



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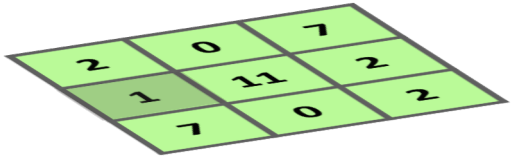
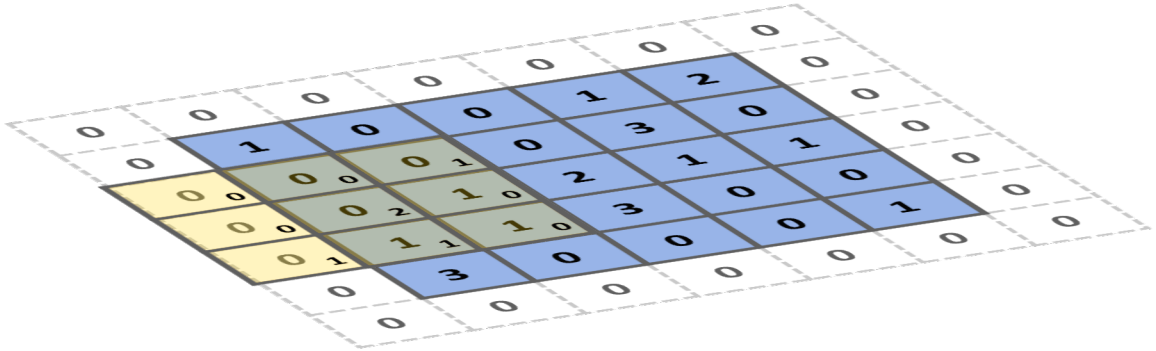
Stride: 2x2





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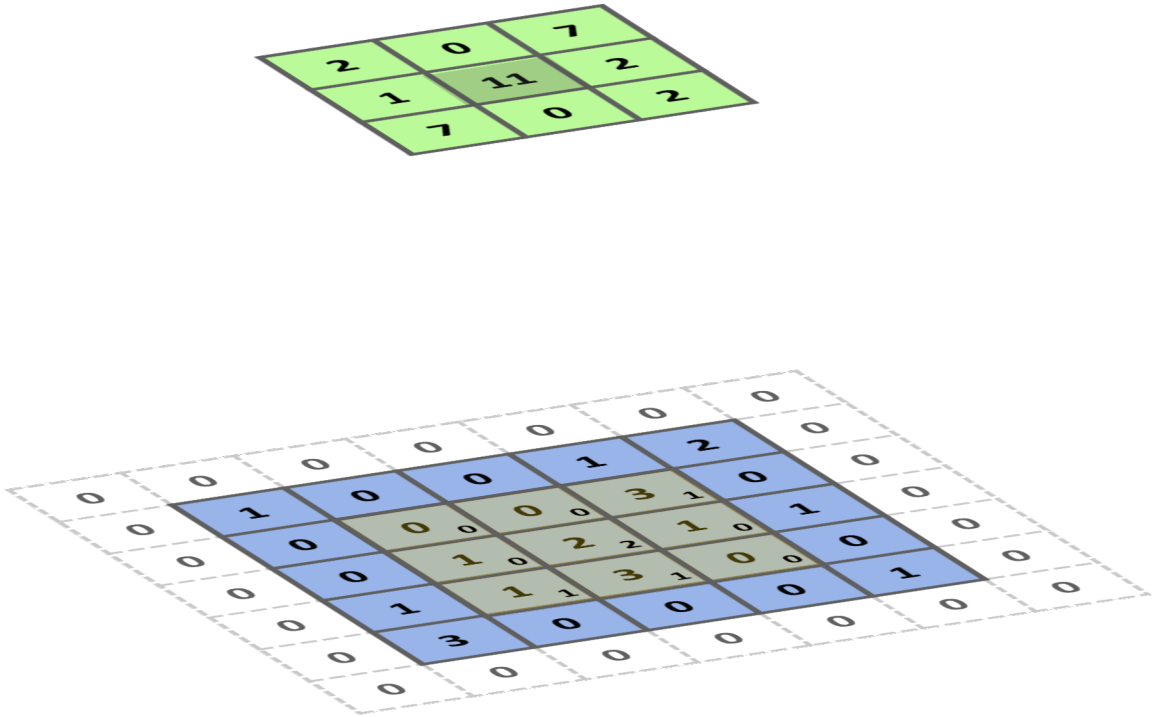
Stride: 2x2



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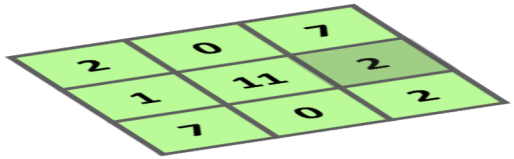
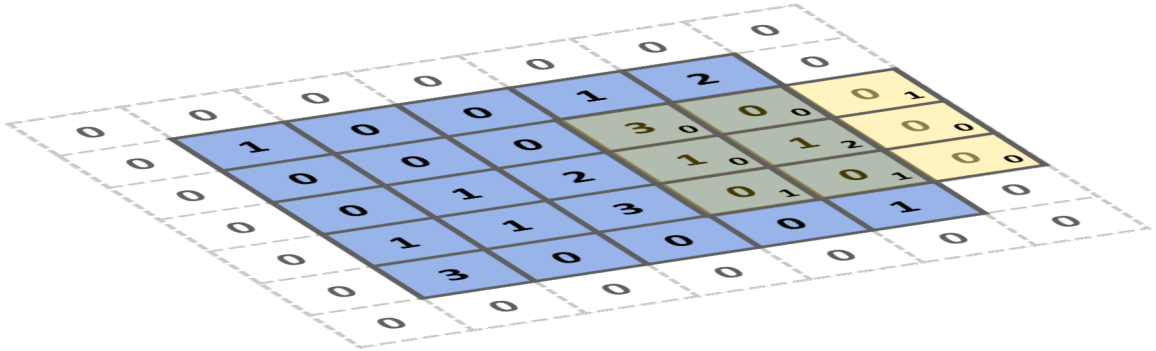
Stride: 2x2





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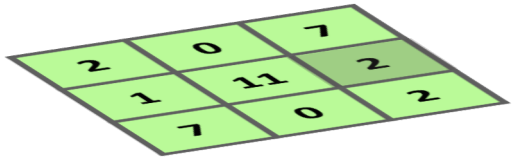
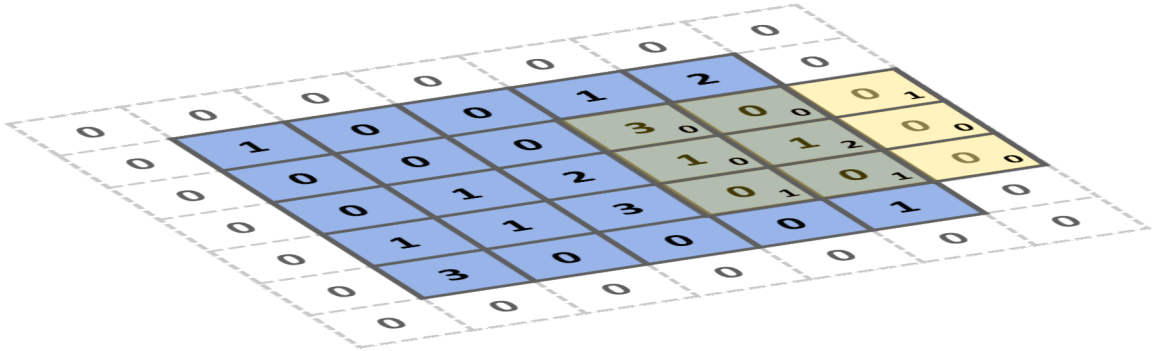


Why use stride > 1?

- Reduce redundancy
- Compress feature map



Summary of convolutions

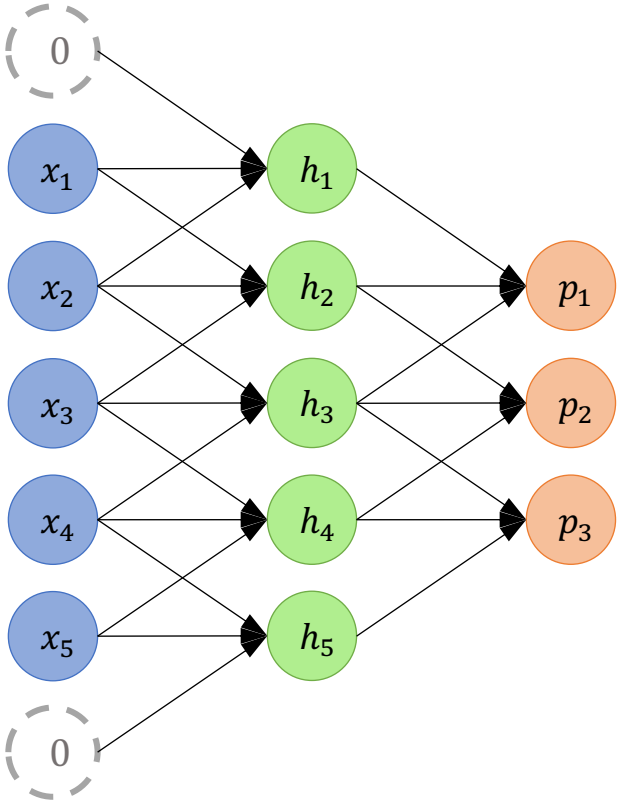
$n \times n$ image $\quad$ $f \times f$ filter

padding p $\quad$ stride s

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

Pooling

Operation to aggregate or "summarize" sub-region of input.



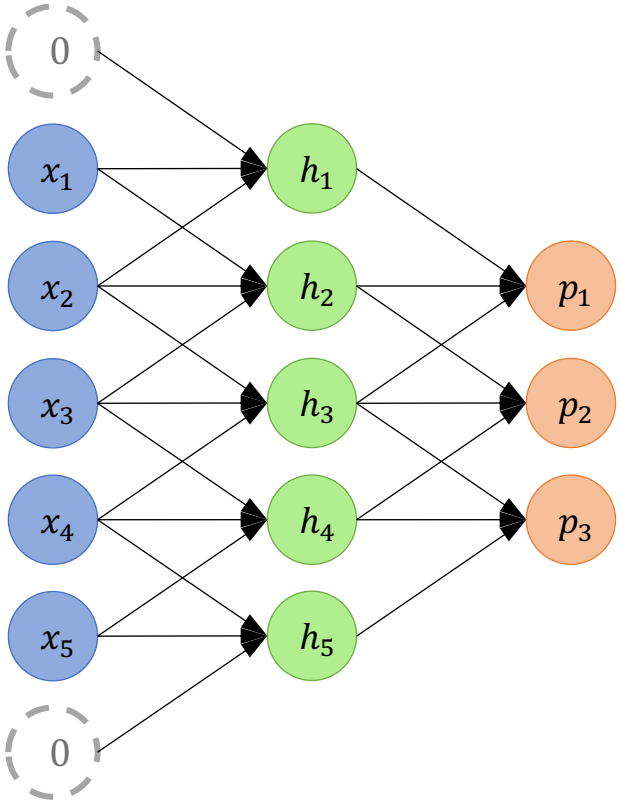
Pooling

Operation to aggregate or "summarize" sub-region of input.

$$p = f(\boldsymbol{a})$$

$$f_{max}(\boldsymbol{a}) = \max_i(a_i)$$

$$f_{avg}(\boldsymbol{a}) = \frac{1}{N}\sum_{i=1}^{N} a_i$$



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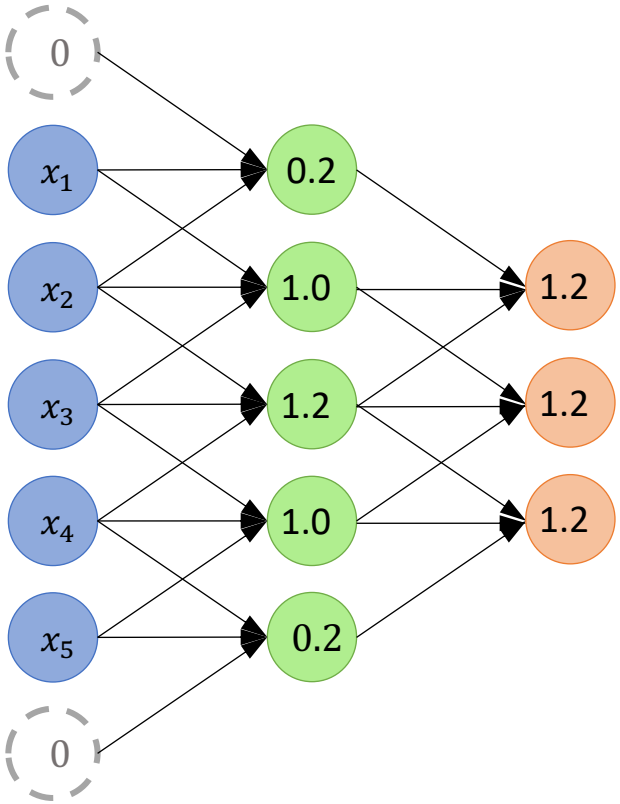

Pooling

Operation to aggregate or “summarize” sub-region of input.

Shift input by 1 position:

5/5 inputs change but only 1/3 pooled outputs

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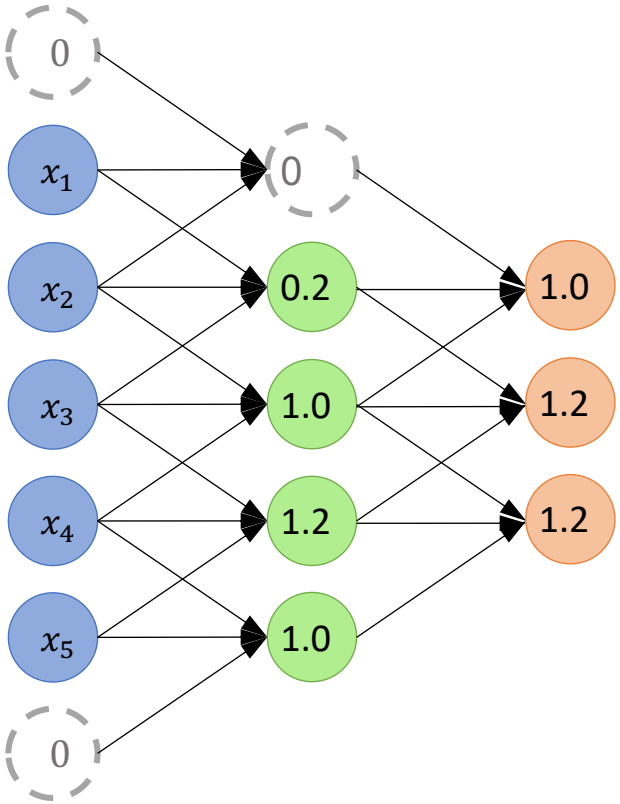

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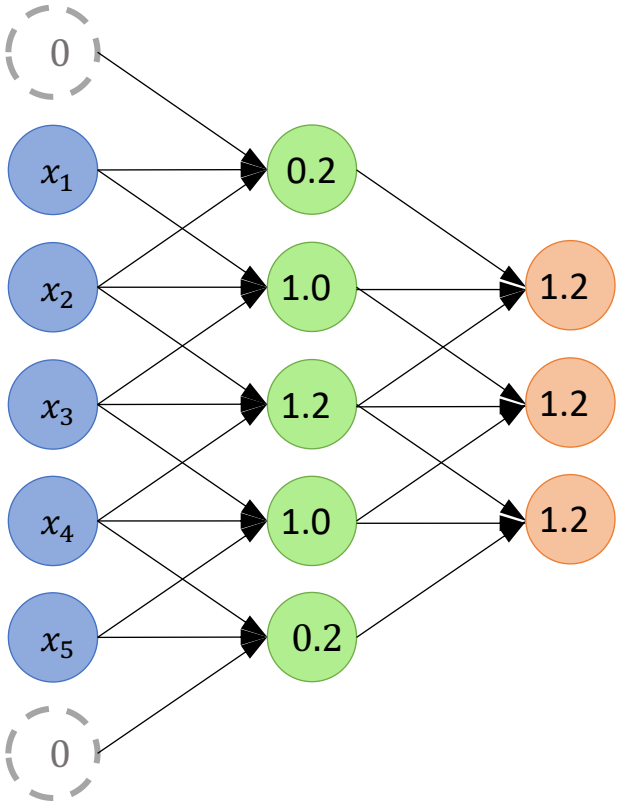

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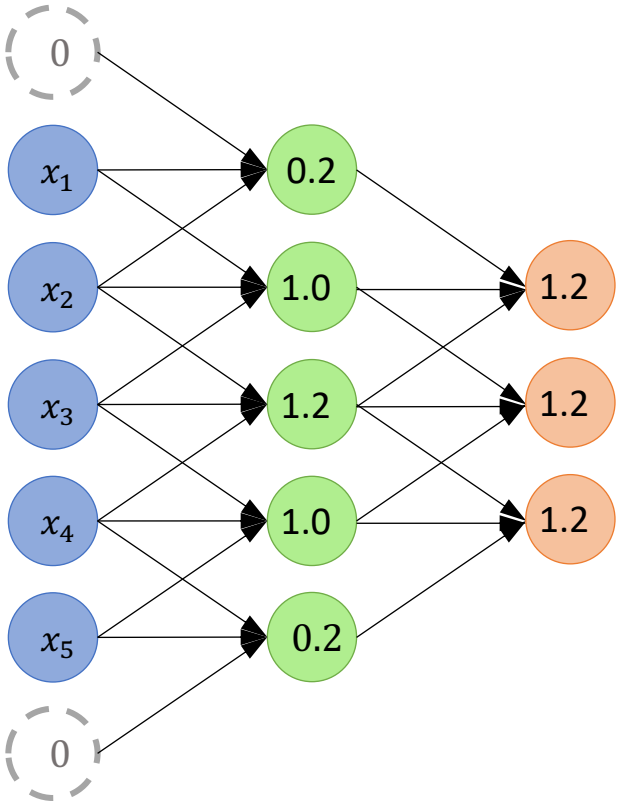
Operation to aggregate or “summarize” sub-region of input.

Shift input by 1 position:

5/5 inputs change but only 1/3 pooled outputs

Why is pooling useful?

- Adds **translation invariance** (useful when we care about “what” more than “where”)
- Can be used to **compress signal** (useful to improve computational efficiency)



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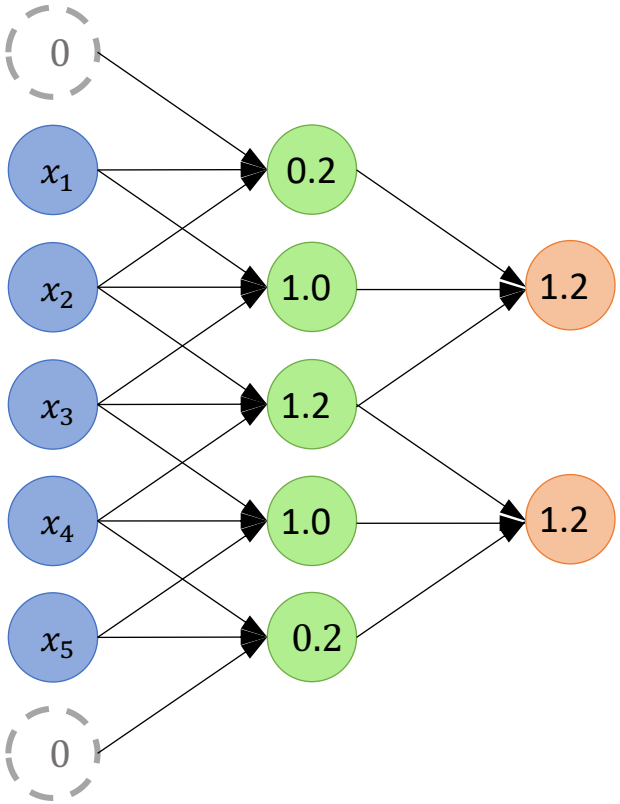


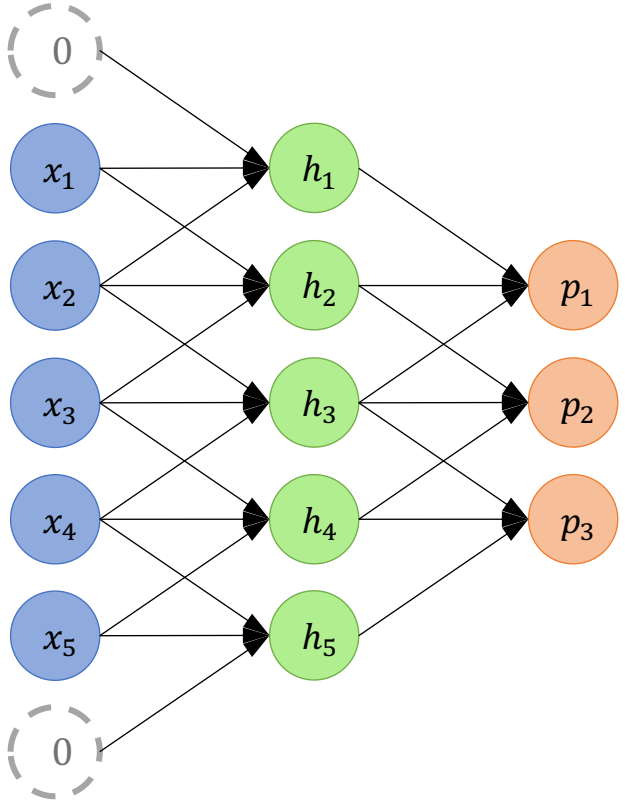
Receptive fields

Borrowed terminology from neuroscience:

⇒ stimulus region that impacts a neuron's firing

For neural networks:

⇒ Region of input signal that impacts node output



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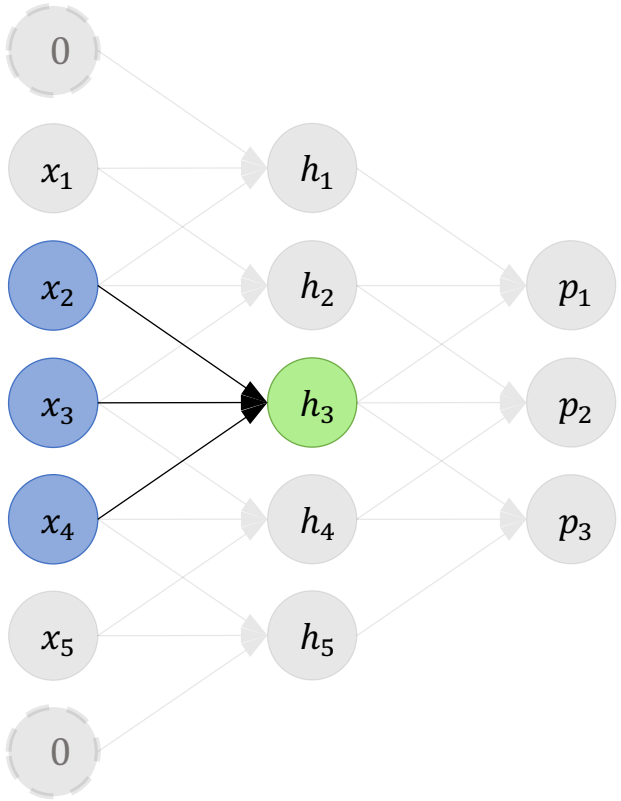

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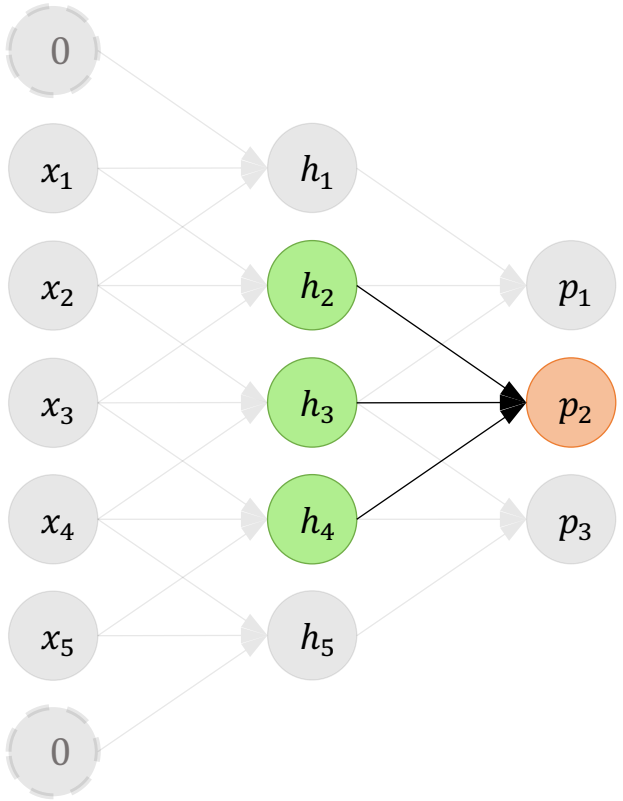


Receptive fields

Borrowed terminology from neuroscience:

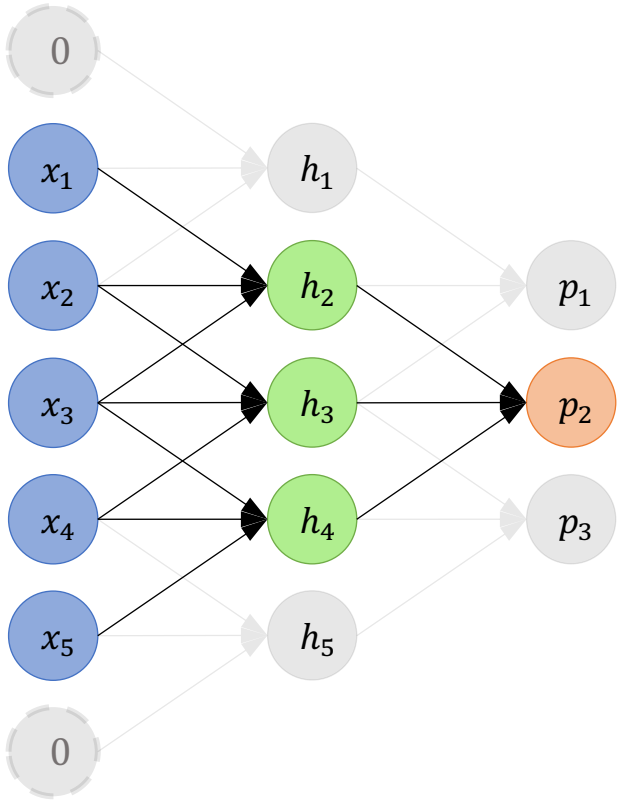
⇒ stimulus region that impacts neuronal firing

For neural networks:

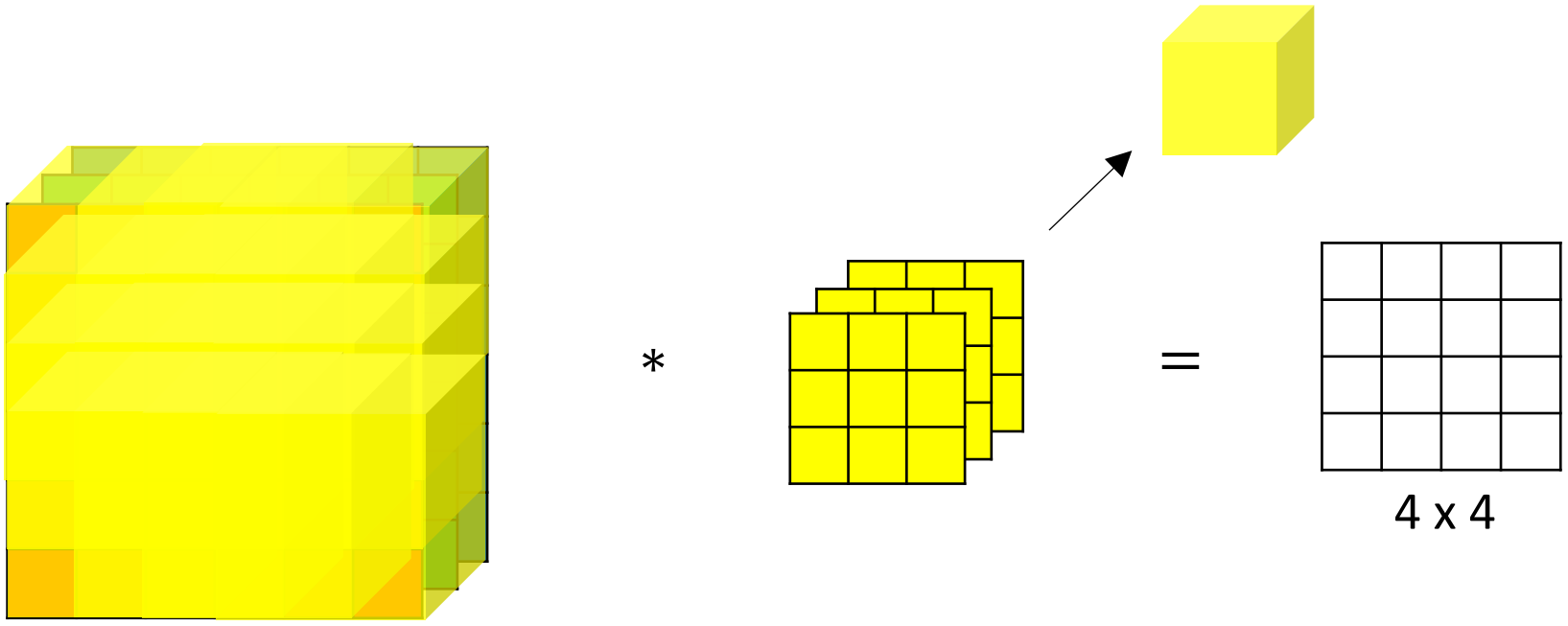
⇒ Region of input signal that impacts node output

Effective field size:

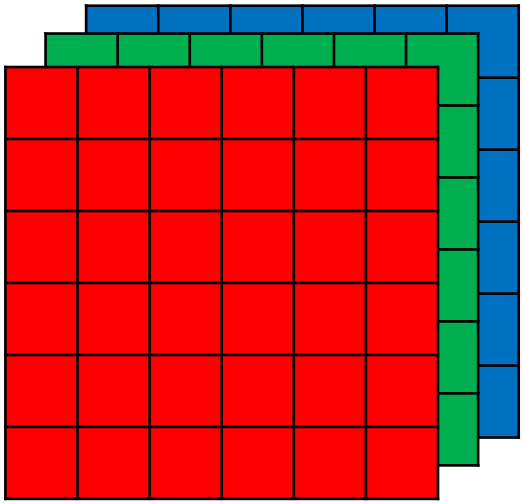
⇒ Region of **network** input signal that impacts node output



Convolutions on RGB image



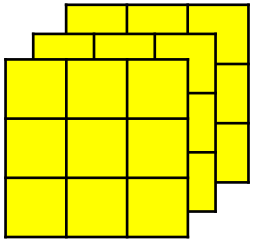
Multiple filters



6 x 6 x 3

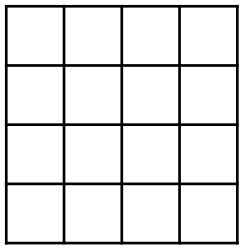
Vertical Edge

*



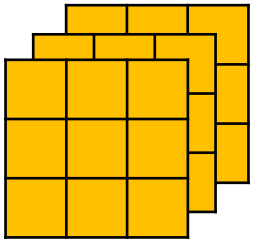
3 x 3 x 3

=



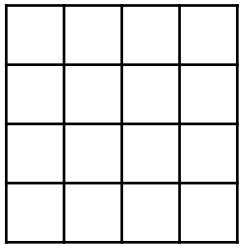
4 x 4

*



3 x 3 x 3

=



4 x 4

Horizontal Edge