Bayes Net

Graphical Models

- Key Idea:
  - Conditional independence assumptions useful
  - but Naïve Bayes is extreme!
  - Graphical models express sets of conditional independence assumptions via graph structure
  - Graph structure plus associated parameters define *joint probability distribution over set of variables/ nodes*

- Two types of graphical models:
  - Directed graphs (aka Bayesian Networks)
  - Undirected graphs (aka Markov Random Fields)

today

Graphical Models – Why Care?

- Among most important ML developments of the decade
- Graphical models allow combining:
  - Prior knowledge in form of dependencies/independencies
  - Observed data to estimate parameters
- Principled and ~general methods for
  - Probabilistic inference
  - Learning
- Useful in practice
  - Diagnosis, help systems, text analysis, time series models, ...

Conditional Independence

Definition: X is conditionally independent of Y given Z, if the probability distribution governing X is independent of the value of Y, given the value of Z

$$(\forall i, j, k) P(X = x_i | Y = y_j, Z = z_k) = P(X = x_i | Z = z_k)$$

Which we often write $P(X|Y, Z) = P(X|Z)$

E.g., $P(Thunder|Rain, Lightning) = P(Thunder|Lightning)$

Marginal Independence

Definition: X is marginally independent of Y if

$$(\forall i, j) P(X = x_i, Y = y_j) = P(X = x_i) P(Y = y_j)$$

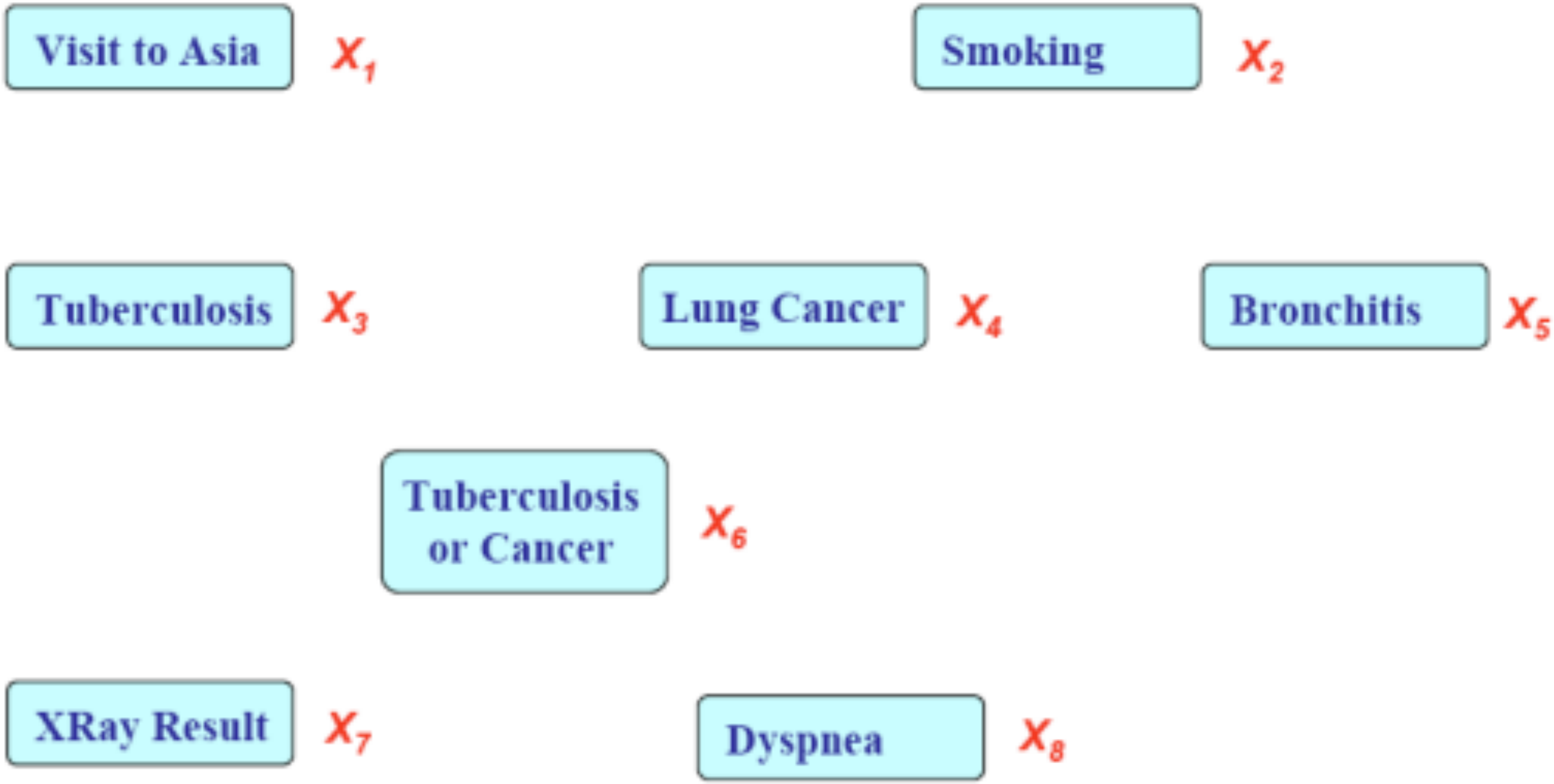
Equivalently, if

$$(\forall i, j) P(X = x_i | Y = y_j) = P(X = x_i)$$

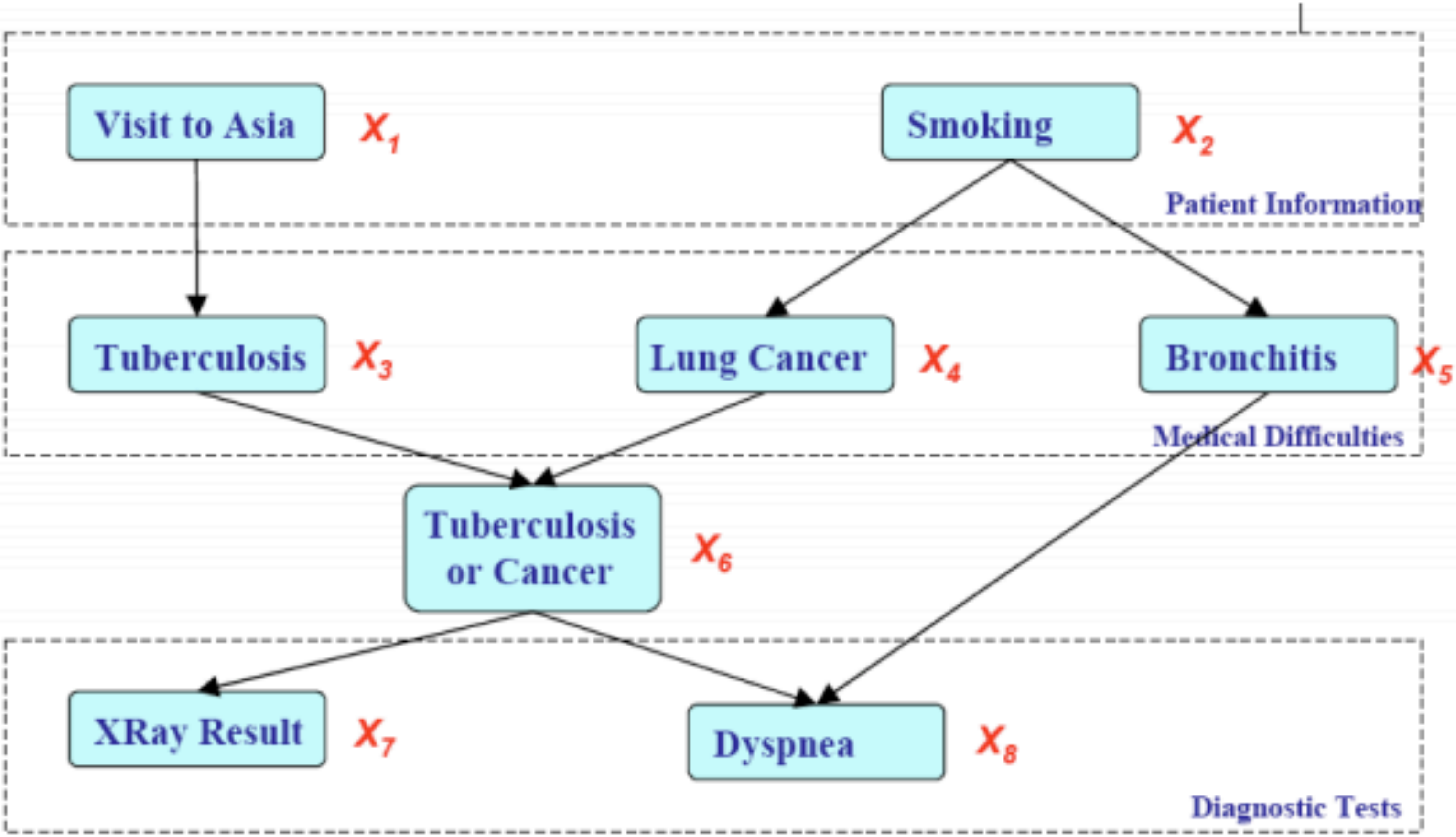
Equivalently, if

$$(\forall i, j) P(Y = y_i | X = x_j) = P(Y = y_i)$$

Represent Joint Probability Distribution over Variables

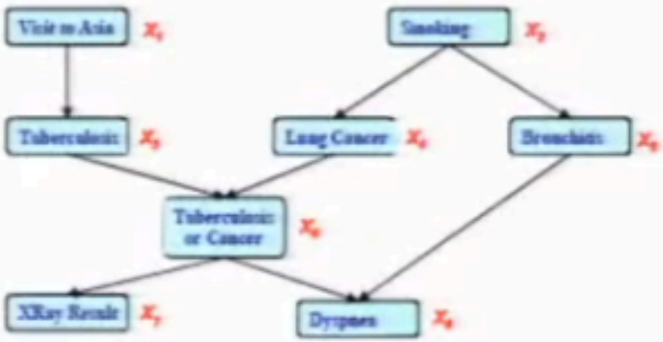


Describe network of dependencies



Bayesian Networks define Joint Distribution in terms of this graph, plus parameters

- If X_i's are **conditionally independent** (as described by a **PGM**), the joint can be factored to a product of simpler terms, e.g.,



$$P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8)$$

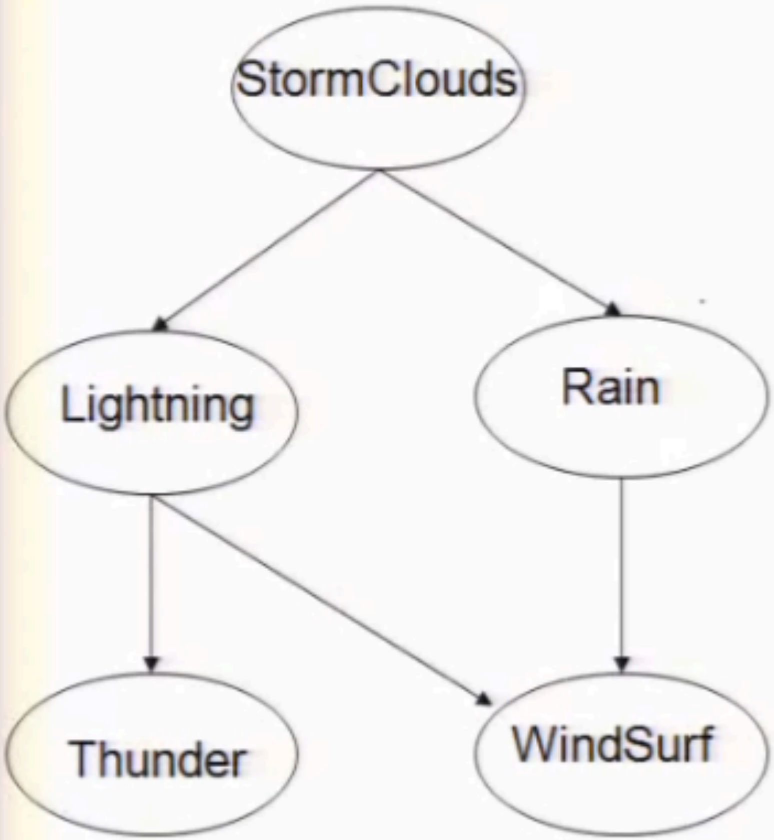
$$= P(X_1)\, P(X_2)\, P(X_3 | X_1)\, P(X_4 | X_2)\, P(X_5 | X_2)\, P(X_6 | X_3, X_4)\, P(X_7 | X_6)\, P(X_8 | X_5, X_6)$$

- Why we may favor a PGM?
  - Representation cost: how many probability statements are needed?

    2+2+4+4+4+8+4+8=36, an 8-fold reduction from 2^8!

  - Algorithms for systematic and efficient inference/learning computation
    - Exploring the graph structure and probabilistic (e.g., Bayesian, Markovian) semantics
  - Incorporation of domain knowledge and causal (logical) structures

Bayesian Network

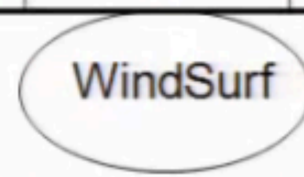


Bayes network: a directed acyclic graph defining a joint probability distribution over a set of variables

Each node denotes a random variable

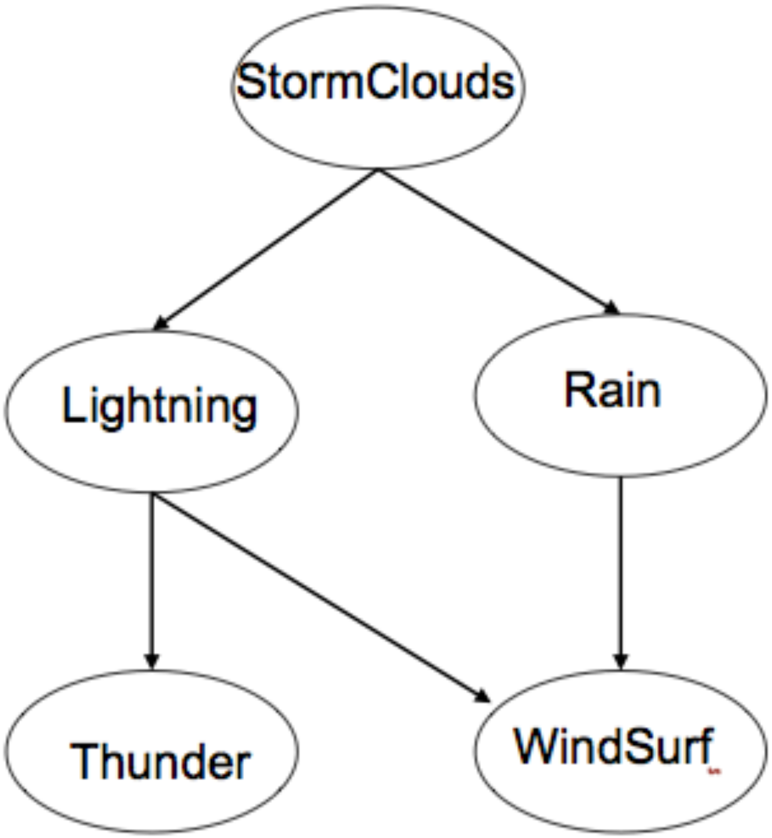
A conditional probability distribution (CPD) is associated with each node N, defining P(N | Parents(N))

Parents	P(W|Pa)	P(¬W|Pa)
L, R	0	1.0
L, ¬R	0	1.0
¬L, R	0.2	0.8
¬L, ¬R	0.9	0.1



The joint distribution over all variables in the network is defined in terms of these CPD's, plus the graph

Bayesian Network



What can we say about conditional independencies in a Bayes Net?

One thing is this:

Each node is conditionally independent of its non-descendents, given only its immediate parents.

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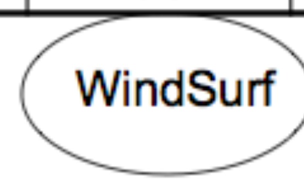
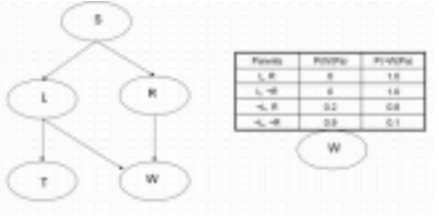


Bayesian Networks Definition



A Bayes network represents the joint probability distribution over a collection of random variables

A Bayes network is a directed acyclic graph and a set of CPD's

- Each node denotes a random variable
- Edges denote dependencies
- CPD for each node X_i defines $P(X_i \mid Pa(X_i))$
- The joint distribution over all variables is defined as

$$P(X_1 \ldots X_n) = \prod_i P(X_i|Pa(X_i))$$

Pa(X) = immediate parents of X in the graph

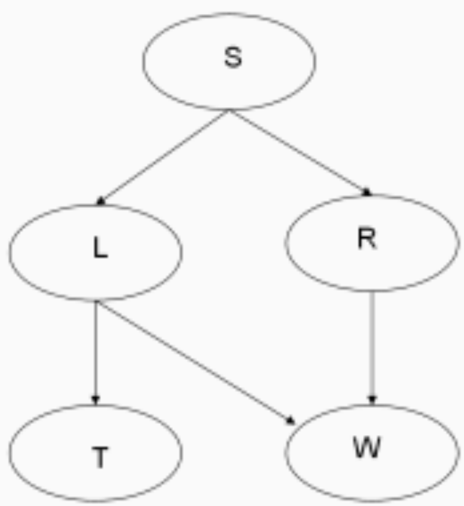
Some helpful terminology

Parents = Pa(X) = immediate parents

Antecedents = parents, parents of parents, ...

Children = immediate children

Descendents = children, children of children, ...



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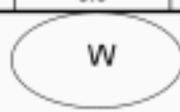
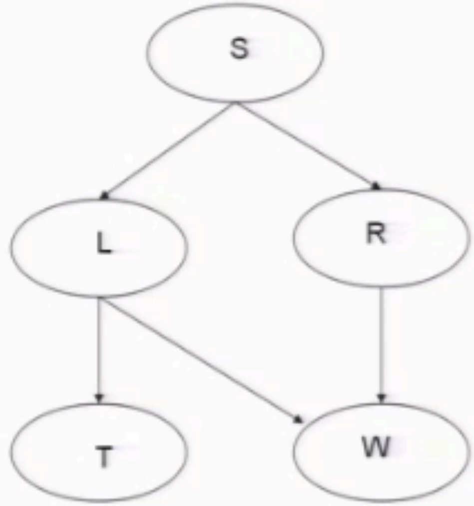


Bayesian Networks

- CPD for each node X_i describes $P(X_i \mid Pa(X_i))$



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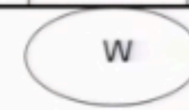
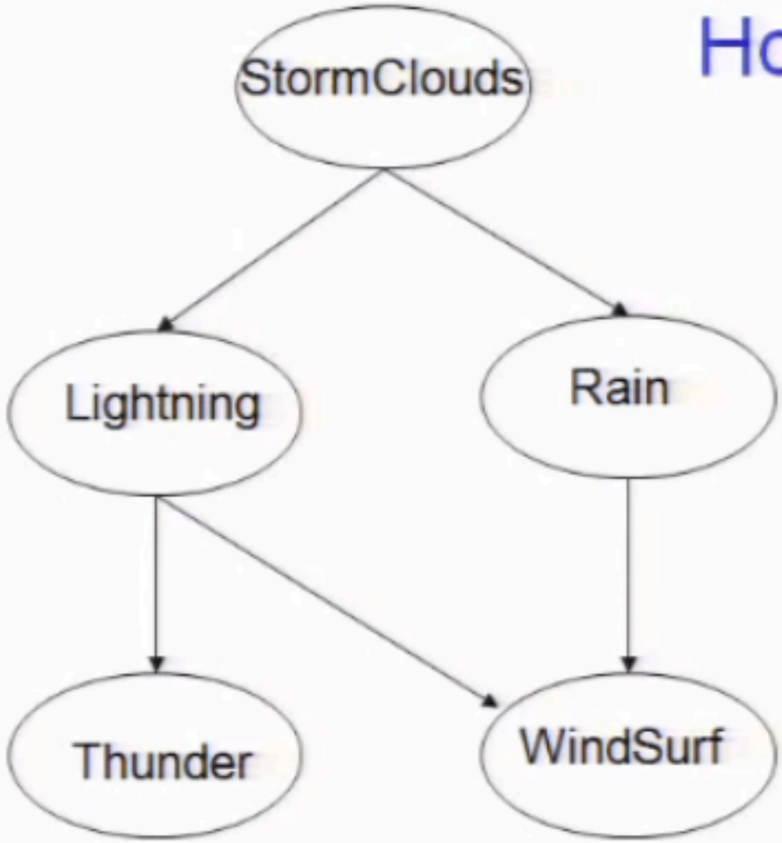


Chain rule of probability:

$$P(S,L,R,T,W) = P(S)P(L|S)P(R|S,L)P(T|S,L,R)P(W|S,L,R,T)$$

But in a Bayes net: $$P(X_1 \ldots X_n) = \prod_i P(X_i|Pa(X_i))$$

How Many Parameters?



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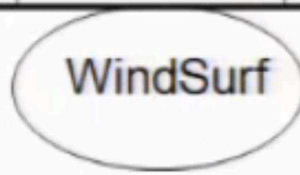
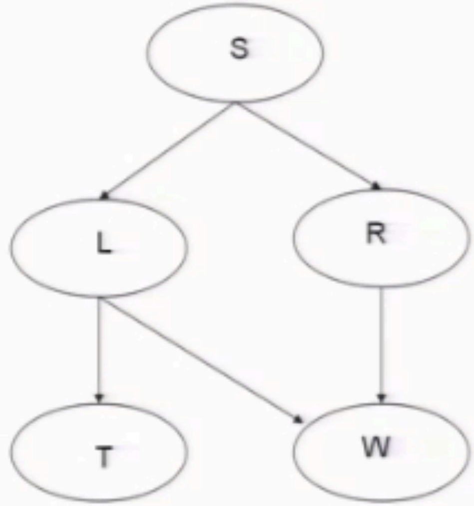


In full joint distribution?

Given this Bayes Net?

Bayesian Networks

- CPD for each node X_i describes $P(X_i \mid Pa(X_i))$



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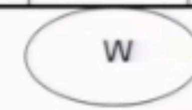

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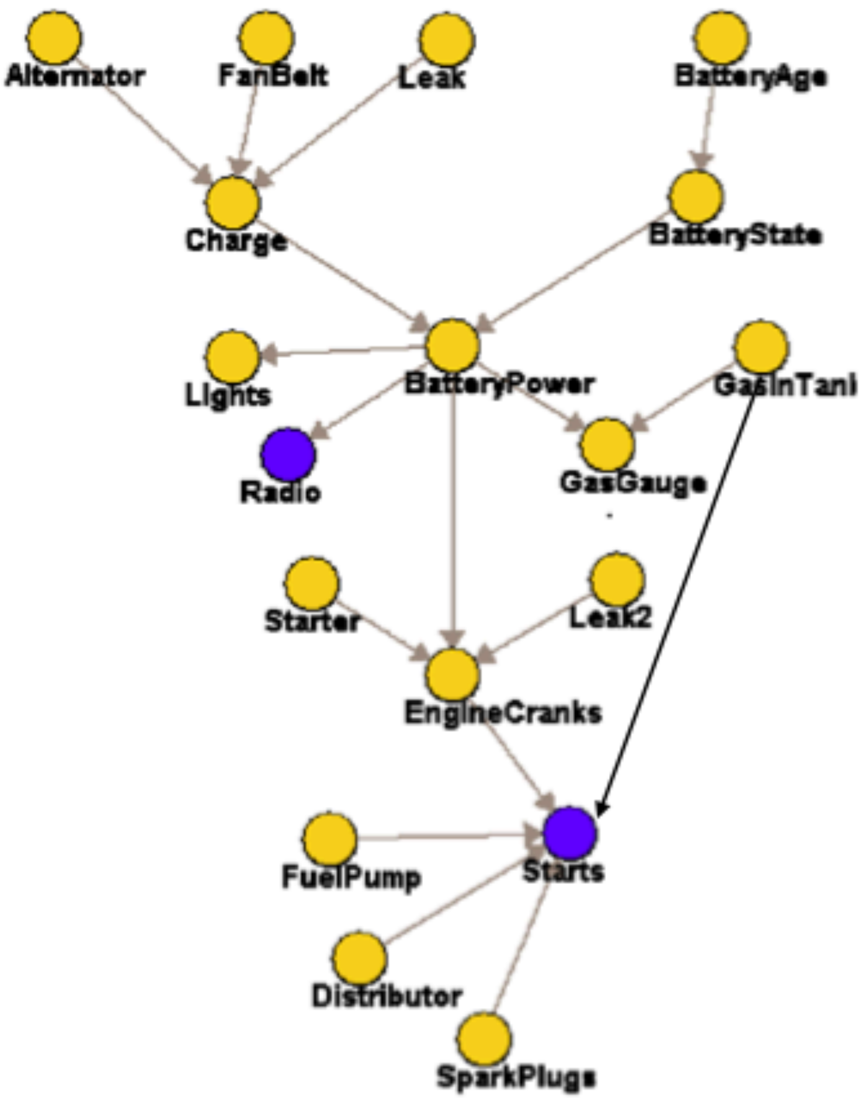
Bayes Net



<u>Inference</u>:

P(BattPower=t | Radio=t, Starts=f)

<u>Most probable explanation</u>:

What is most likely value of Leak, BatteryPower given Starts=f?

<u>Active data collection</u>:

What is most useful variable to observe next, to improve our knowledge of node X?

Algorithm for Constructing Bayes Network

- Choose an ordering over variables, e.g., $X_1, X_2, \ldots X_n$
- For i=1 to n
  - Add X_i to the network
  - Select parents $Pa(X_i)$ as minimal subset of $X_1 \ldots X_{i-1}$ such that

$$P(X_i|Pa(X_i)) = P(X_i|X_1, \ldots, X_{i-1})$$

Notice this choice of parents assures

$$P(X_1 \ldots X_n) = \prod_i P(X_i|X_1 \ldots X_{i-1}) \quad \text{(by chain rule)}$$

$$= \prod_i P(X_i|Pa(X_i)) \quad \text{(by construction)}$$

Example

- Bird flu and Allegies both cause Nasal problems
- Nasal problems cause Sneezes and Headaches

What is the Bayes Network for X1,...Xn with NO assumed conditional independencies?

What is the Bayes Network for Naïve Bayes?

What do we do if variables are mix of discrete and real valued?