Received October 19, 2017, accepted November 9, 2017, date of publication November 14, 2017, date of current version December 5, 2017.

Digital Object Identifier 10.1109/ACCESS.2017.2773491

Concavity Approximation Based Power Allocation in Millimeter-Wave MIMO Systems

ANZHONG HU, (Member, IEEE) AND PENG PAN, (Member, IEEE)
School of Communication Engineering, Hangzhou Dianzi University, Hangzhou 310018, China

Corresponding author: Anzhong Hu (huaz@hdu.edu.cn)

This work was supported in part by the Zhejiang Provincial Natural Science Foundation of China under Grant LQ16F010007 and in part by the National Natural Science Foundation of China under Project 61601152, Project 61401130, and Project U1609216.

ABSTRACT This paper investigates the power allocation in millimeter-wave multiple-input multiple-output systems. The asymptotic concavity of the sum rate of the system is utilized to form the proposed power allocation approach. In contrast to the traditional approaches, the proposed approach utilizes a channel asymptotic orthogonality-based approximation which is more suitable, thus achieves higher spectral efficiency. The analysis demonstrates that the sum rate of the system increases as the number of antennas increases. Numerical results verify the analysis and show that the proposed approach can perform better than those traditional approaches.

INDEX TERMS Multiple-input multiple-output (MIMO), millimeter-wave (mm-wave), beamspace, power allocation.

I. INTRODUCTION

The traditional technologies cannot provide enough spectral efficiency and sum rate for future wireless systems [1], [2]. In order to tackle this problem, the millimeter-wave (mm-wave) multiple-input multiple-output (MIMO) technology [3] and the massive MIMO technology [4], [5] have been proposed. In mm-wave MIMO systems, wide bandwidth can be used to increase sum rate. Moreover, many antennas can be equipped in mm-wave MIMO systems, which also constitute massive MIMO systems. It can be seen that the spectral efficiency can be greatly improved in mm-wave MIMO systems [6].

The sum rate of mm-wave MIMO systems with line-of-sight (LOS) transmission is investigated in [7]. Due to the limited number of radio frequency chains, hybrid beamforming, which is constituted with analog beamforming and digital beamforming, is becoming a promising technology for outdoor mm-wave systems [9]. The architecture of hybrid beamforming in mm-wave MIMO systems has been investigated in many papers [10]–[12], and the capacity of these systems has also been derived in [13]–[15]. These hybrid beamforming approaches fully take use of the sparsity of the mm-wave channels to improve the energy efficiency and simplify the analog front-end. One of these hybrid beamforming approaches is the beamspace MIMO, which multiplexes the data symbols onto orthogonal spatial beams with the analog beamforming and achieves almost the optimal performance, i.e., the performance with conventional MIMO transceivers [16]–[19].

Power allocation is an efficient way for mitigating interference in mm-wave beamspace MIMO systems. The signals in mm-wave frequency usually undergo LOS propagation [17]. Thus, the channels of the co-located user terminals (UTs) may be highly correlated and cause severe interference. Besides, other channels in mm-wave MIMO systems are correlated in low degrees and cause slight interference. Since the interference varies over the UTs, power allocation at the access point (AP) is an efficient way to mitigate such interference. Power allocation schemes for interference channels have been extensively studied, such as the geometric programming (GP) [20]–[22], the fractional programming (FP) [23], [24], the successive convex approximation (SCA) [25]–[27], and the iterative water-filling (IWF) [28]. However, the existing approaches for traditional systems are not suitable for the considered systems. For example, the GP in [20] and the SCA in [26] approximate the power allocation problem as a convex optimization problem but cause approximation error, which means these approaches are not optimizing the power for the original problem, the FP in [23] and [24] maximizes the power efficiency, which means that the ratio of the spectral efficiency to the power rather than the spectral efficiency is maximized, the IWF in [28] allocates the power at each UT rather than at the AP. For mm-wave beamspace MIMO systems, there are also several existing power allocation approaches. For example, [29] employs waterfilling to allocate power in the frequency domain, [30] investigates the problem of allocating power for each link independently, [31] studies the power on/off mechanism for the base station, [32] solves the problem of power allocation in the presence of inter-cell interference, and [33] studies the power allocation for each user pair. However, these approaches do not tackle the inter-user interference for the transmission between the AP and multiple UTs.

In this paper, a concavity approximation based power allocation approach is proposed for mm-wave MIMO systems. The sum rate of the system with minimum mean square error (MMSE) precoding is presented. The MMSE is employed because it can mitigate the interference and is general for MIMO systems, as in [17] and [18]. Note that the zero-forcing (ZF) is not employed here as the noise will be largely amplified when the channels are highly correlated. The asymptotic orthogonality of the channels is used to reveal the asymptotic concavity and monotonicity of the sum rate. Note that the asymptotic orthogonality means that the channels tend to be orthogonal when the number of the AP antennas tends to infinity. The reason is that each channel is the combination of multiple array response vectors, and the array response vectors tend to be orthogonal when the number of the AP antennas tends to infinity, which has been shown in [17] and [34]. Then, power allocation that exploits the approximate concavity of the sum rate is proposed to mitigate interference and achieve the near maximal sum rate. As the number of the AP antennas tends to infinity, the approximation error tends to zero, which means the proposed approach tends to achieve the optimal performance. With typical system parameters, the simulation shows that more than 200 AP antennas are enough to show the merits of the proposed approach. Finally, the impact of the system parameters on the system performance is analyzed. The main contributions of this paper are two-fold.

1) The concavity and the monotonicity of the asymptotic sum rate is revealed. Since the approximation error decreases with the increase of the AP antennas, the proposed method is expected to cancel more interference and achieve higher spectral efficiency than these existing approaches.
2) The impact of the system parameters on the system performance is analyzed. The analysis shows that interference decreases with the increase if the number of the AP antennas.

This paper is organized as follows. In Section II, the system model and the assumptions are given. Section III first presents the sum rate, then shows the proposed power allocation scheme. In Section IV, the impact of the system parameters on the performance of the proposed approach is analyzed. Section V gives the simulation parameters and the numerical results. Finally, conclusions are drawn in Section VI.

Notations: Lower-case (upper-case) boldface symbols denote vectors (matrices); $\mathbf{I}_K$ represents the $K \times K$ identity matrix; $(\cdot)^H$ and $\mathbb{E}\{\cdot\}$ denote the conjugate transpose and the expectation, respectively; $[\cdot]_j$ is the j-th element of a vector or the j-th column of a matrix; $[\cdot]_{j,k}$ is the element in the j-th row and k-th column of a matrix; $\mathrm{tr}\{\cdot\}$ is the trace of a matrix; $|\cdot|$ is the cardinality of a set; and i is the imaginary unit.

II. SYSTEM MODEL

We considered the mm-wave MIMO system that employs the beamspace processing. The system is depicted in Fig. 1 and is similar to that in [16]. The system consists of one AP and K UTs. The AP is equipped with one discrete lens array (DLA), which can be analytically modeled as a uniform rectangular array (URA) with N antennas [16], [17]. Each UT has M antennas which form a linear array. The UTs randomly distribute in front of the array and communicate with the AP simultaneously. Since the propagation in mm-wave systems is highly directional, only partial beams (each beam corresponds to one direction in the space) need to be utilized for transmission. Thus, the beamspace principle, which takes use of only partial beams, can mitigate the interference and improve the power efficiency for mm-wave MIMO systems. Moreover, advanced analog front-end design can be employed to simplify the system. As can be seen, the number of analog-to-digital/digital-to-analog (A-D/D-A) converters or transceiver modules is N_b, rather than N for conventional MIMO systems, where $N_b \ll N$. The difference is that the beam selector and the DLA can form the beams and there are only N_b analog inputs rather than N digital inputs in the conventional MIMO systems.

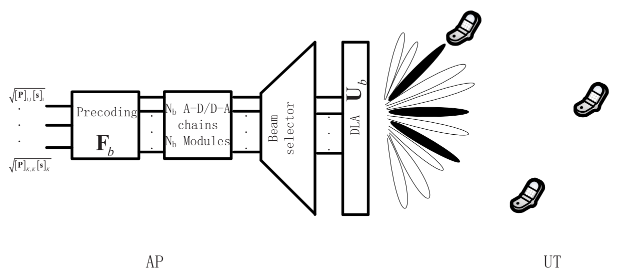


FIGURE 1. An example of the considered system model. The AP is equipped with a DLA and serves $K = 3$ UTs simultaneously.

In this paper, we focus on the downlink transmission. It is assumed that time division duplex mode is adopted, which means the reciprocity of the two-way channels holds. The received signal at the UTs can be expressed as

$$\mathbf{r} = \mathbf{H}^H \mathbf{x} + \mathbf{n} \in \mathbb{C}^{KM \times 1}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N \times KM}$ is the channel matrix from the UTs to the AP, $\mathbf{x} \in \mathbb{C}^{N \times 1}$ is the transmitted signal from the AP, $\mathbf{n} \in \mathbb{C}^{KM \times 1}$ is the noise received at the UTs, and is composed of independent and identically distributed (i.i.d.) complex random variables with zero mean and variance one. According to the beamspace principle, the transmitted signal from the AP can be expressed as [17]

$$\mathbf{x} = \mathbf{U}_{\mathrm{b}}\mathbf{F}_{\mathrm{b}}\mathbf{T}\sqrt{\mathbf{P}}\mathbf{s}, \tag{2}$$

where $\mathbf{U}_{\mathrm{b}} \in \mathbb{C}^{N\times N_{\mathrm{b}}}$ is the beamforming matrix formed by the DLA and satisfies $\mathbf{U}_{\mathrm{b}}^H\mathbf{U}_{\mathrm{b}} = \mathbf{I}_{N_{\mathrm{b}}}$, N_{b} is the number of beams selected for all the UTs; $\mathbf{F}_{\mathrm{b}} \in \mathbb{C}^{N_{\mathrm{b}}\times KM}$ is the precoding matrix employed for the K digital streams; $\mathbf{s} \in \mathbb{C}^{K\times 1}$ is the transmitted data symbols for the UTs, and are i.i.d. complex random variables with zero mean and variance one; $\mathbf{P} \in \mathbb{R}^{K\times K}$ is a diagonal matrix of the transmission powers of the data symbols; $\mathbf{T} \in \mathbb{R}^{KM\times K}$ is the transforming matrix with $[\mathbf{T}]_{(k-1)M+m,k} = 1, k = 1, 2, \ldots, K, m = 1, 2, \cdots, M$ and $[\mathbf{T}]_{m_1,k} = 0$ for $m_1 < (k-1)M + 1$ or $m_1 > kM$. Here, the matrix $\mathbf{T}$ is used to allocate the symbols for each UT. As an example, the k-th UT has M antennas, the $(k-1)M+1$-th to te kM-th columns of $\mathbf{F}_{\mathrm{b}}$ are designed for this UT, and $\mathbf{T}$ makes the corresponding rows of $\mathbf{T}\sqrt{\mathbf{P}}\mathbf{s}$ being only related to the symbol $[\mathbf{s}]_k$. In (2), the beamforming matrix $\mathbf{U}_{\mathrm{b}}$ is constructed according to beam selection result. There are some beam selection approaches in the papers, such as [17], [19], and [34]. The direct way is to select beams for each UT that are of the highest correlation with the channel of this UT. Since the beams are in the form of the array manifold vector, and can be represented by two parameters, usually named spatial frequencies, the correlation value can be replaced with the differences between the spatial frequencies and the corresponding channel parameters of each UT. This can further simplify the beam selection process. Here, we take the channel correlation as the metric for beam selection.

As known to all, the LOS propagation is predominant in mm-wave transmissions, and there are also some multipaths. Then, the channel from the m-th antenna of the k-th UT to the AP can be expressed as $[\mathbf{H}]_{(k-1)M+m} \in \mathbb{C}^{N\times 1}$ and

$$[\mathbf{H}]_{(k-1)M+m} \triangleq \sum_{p=0}^{P} e^{i\pi(m-1)\cos\varphi_{k,p}}\alpha_{k,p}\mathbf{a}(\theta_{k,p},\phi_{k,p}), \tag{3}$$

where $\alpha_{k,0}$ and $\alpha_{k,p}, p \neq 0$ are the complex path losses of the LOS path and the multipath, $\theta_{k,p}$ and $\phi_{k,p}$ are the azimuth direction-of-arrival (DOA) and elevation DOA, respectively, $\varphi_{k,p}$ is the direction-of-departure, P is the number of multipaths. Note that $\alpha_{k,0}$ can be zero for the blockage of the LOS path. In addition, $\mathbf{a}(\theta_{k,p},\phi_{k,p}) \in \mathbb{C}^{N\times 1}$ is the array manifold defined as $[\mathbf{a}(\theta_{k,p},\phi_{k,p})]_n = e^{i\pi(n_{\mathrm{x}}-0.5(N_{\mathrm{x}}-1))\cos\phi_{k,p}\sin\theta_{k,p}} \times e^{i\pi(n_{\mathrm{y}}-0.5(N_{\mathrm{y}}-1))\sin\phi_{k,p}}$, where N_{x} and N_{y} are the numbers of antennas in the horizontal and vertical directions of the URA. This channel model is the analytical model of the DLA. In addition, the indices are $n = n_{\mathrm{y}}N_{\mathrm{x}} + n_{\mathrm{x}} + 1$, $n_{\mathrm{x}} = 0, 1, \cdots, N_{\mathrm{x}} - 1$, $n_{\mathrm{y}} = 0, 1, \cdots, N_{\mathrm{y}} - 1$.

The other assumptions and the corresponding results used in this paper are as follows.

1) The AP has perfect knowledge of the channel $\mathbf{H}$. Then, the beamforming matrix $\mathbf{U}_{\mathrm{b}}$ in (2) can be constructed according to the channel matrix $\mathbf{H}$, such as those shown in [17] and [34].

2) The transmission power of the AP is limited by ρ, i.e., $\mathrm{tr}\{\mathbb{E}\{\mathbf{x}\mathbf{x}^H\}\} = \rho$. According to the expression of $\mathbf{x}$ in (2), we have $\mathrm{tr}\{\mathbb{E}\{\mathbf{x}\mathbf{x}^H\}\} = \mathrm{tr}\{\mathbf{F}_{\mathrm{b}}\mathbf{T}\mathbf{P}\mathbf{T}^H\mathbf{F}_{\mathrm{b}}^H\}$. Thus, the power constraint can be re-expressed as

$$\mathrm{tr}\{\mathbf{F}_{\mathrm{b}}\mathbf{T}\mathbf{P}\mathbf{T}^H\mathbf{F}_{\mathrm{b}}^H\} = \rho. \tag{4}$$

With the system model and the assumptions, power allocation will be investigated in the following section.

III. POWER ALLOCATION

As shown earlier, power allocation is important to mm-wave beamspace MIMO systems but is rarely investigated. Meanwhile, the existing power allocation approaches are not suitable for mm-wave beamspace MIMO systems due to the approximation error. In this section, the sum rate of the considered system is presented, and one scheme is proposed to optimize the power allocation.

Substituting (2) into (1) results into

$$\mathbf{r} = \mathbf{H}_{\mathrm{b}}^H\mathbf{F}_{\mathrm{b}}\mathbf{T}\sqrt{\mathbf{P}}\mathbf{s} + \mathbf{n}, \tag{5}$$

where $\mathbf{H}_{\mathrm{b}} = \mathbf{U}_{\mathrm{b}}^H\mathbf{H} \in \mathbb{C}^{N_{\mathrm{b}}\times KM}$ is the beamspace channel matrix. With traditional linear precoding strategies, the sum rate of each UT is affected by the channels of other UTs. In the following, we will analyze the sum rate of the system with one linear precoding strategy, the MMSE precoding.

When MMSE precoding is employed, the precoding matrix is

$$\mathbf{F}_{\mathrm{b}} = \mathbf{H}_{\mathrm{b}}\left(\mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}} + \frac{KM}{\rho}\mathbf{I}_{KM}\right)^{-1}. \tag{6}$$

Then, substituting (6) into (5) yields

$$\begin{aligned}\mathbf{r} &= \mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}}\left(\mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}} + \frac{KM}{\rho}\mathbf{I}_{KM}\right)^{-1}\mathbf{T}\sqrt{\mathbf{P}}\mathbf{s} + \mathbf{n}\\ &= \mathbf{V}\tilde{\mathbf{D}}\mathbf{V}^H\mathbf{T}\sqrt{\mathbf{P}}\mathbf{s} + \mathbf{n},\end{aligned} \tag{7}$$

where (7) is derived by substituting the singular value decomposition of $\mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}}$, i.e.,

$$\mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}} = \mathbf{V}\mathbf{D}\mathbf{V}^H, \tag{8}$$

into the first equation, where $\mathbf{V} \in \mathbb{C}^{KM\times KM}$ is the matrix of the left singular vectors, $\mathbf{D} \in \mathbb{R}^{KM\times KM}$ is the matrix of singular values, and

$$\tilde{\mathbf{D}} = \mathbf{I}_{KM} - \left(\mathbf{I}_{KM} + \frac{\rho}{KM}\mathbf{D}\right)^{-1} \in \mathbb{R}^{KM\times KM}. \tag{9}$$

On the other hand, the power constraint has a simpler expression. Substituting (8) into (6) results into

$$\mathbf{F}_{\mathrm{b}} = \mathbf{H}_{\mathrm{b}}\mathbf{V}\bar{\mathbf{D}}\mathbf{V}^H, \tag{10}$$

where

$$\bar{\mathbf{D}} = \left(\frac{KM}{\rho}\mathbf{I}_{KM} + \mathbf{D}\right)^{-1} \in \mathbb{R}^{KM\times KM}. \tag{11}$$

Then, substituting (10) into (4) yields

$$\begin{aligned}\mathrm{tr}\{\mathbf{F}_\mathrm{b}\mathbf{T}\mathbf{P}\mathbf{T}^H\mathbf{F}_\mathrm{b}^H\} &= \mathrm{tr}\{\mathbf{H}_\mathrm{b}\mathbf{V}\bar{\mathbf{D}}\mathbf{V}^H\mathbf{T}\mathbf{P}\mathbf{T}^H\mathbf{V}\bar{\mathbf{D}}\mathbf{V}^H\mathbf{H}_\mathrm{b}^H\} \\ &= \mathrm{tr}\{\mathbf{T}^H\mathbf{V}\bar{\mathbf{D}}\mathbf{V}^H\mathbf{H}_\mathrm{b}^H\mathbf{H}_\mathrm{b}\mathbf{V}\bar{\mathbf{D}}\mathbf{V}^H\mathbf{T}\mathbf{P}\} \\ &= \mathrm{tr}\{\mathbf{T}^H\mathbf{V}\bar{\mathbf{D}}^2\mathbf{D}\mathbf{V}^H\mathbf{T}\mathbf{P}\} = \rho, \end{aligned} \tag{12}$$

where the third equation is derived by substituting (8) into the second equation.

According to the expression of the received signal in (7), the sum rate of the system is

$$R_\mathrm{MMSE} = \sum_{k=1}^{K} \log_2\left(1 + \frac{a_{k,k}p_k}{\sum_{\substack{k'=1\\k'\neq k}}^{K} a_{k,k'}p_{k'} + 1}\right), \tag{13}$$

where

$$a_{k,k'} \triangleq |\sum_{m_1=1}^{M}\sum_{m_2=1}^{M}[\mathbf{V}\tilde{\mathbf{D}}\mathbf{V}^H]_{(k-1)M+m_1,(k'-1)M+m_2}|^2, \tag{14}$$

in which $\tilde{\mathbf{D}}$ is defined in (9).

As a result, the maximization of the sum rate can be denoted as

$$\begin{aligned}&\max_{p_k, k=1,2,\cdots,K} R_\mathrm{MMSE} \\ &\text{s.t. } p_k \geq 0 \\ &\qquad \sum_{k=1}^{K} b_k p_k = \rho, \end{aligned} \tag{15}$$

where

$$b_k \triangleq \sum_{m_1=1}^{M}\sum_{m_2=1}^{M}[\mathbf{V}\bar{\mathbf{D}}^2\mathbf{D}\mathbf{V}^H]_{(k-1)*M+m_1,(k-1)*M+m_2},$$

$\bar{\mathbf{D}} = \left(\frac{KM}{\rho}\mathbf{I}_{KM} + \mathbf{D}\right)^{-1} \in \mathbb{R}^{KM\times KM}$. Note that the optimization arguments, $p_k, k = 1, 2, \cdots, K$ are the powers of the data symbols of the UTs. From this object, we have the following observation. The object is to optimize the powers of the UTs subject to power constraint, which means that the object is an optimization of the power allocation.

With the optimization problems presented, we will continue to solve this problem and present the proposed scheme. It is known that the sum rate in (13) is a standard form of the sum rate of systems with interference channels, and the optimization problem in (15) is not a convex optimization problem. In order to solve this problem, various optimization approaches have been proposed, among which the GP approach [20]–[22] and the SCA approach [25]–[27] are mostly used. These approaches approximate the original problem as a convex optimization problem, but the high signal-to-noise ratio (SNR) approximation [20] or the first-degree Taylor polynomial approximation [26] causes error. Thus, we will try to form an optimization approach by taking use of the asymptotic orthogonality of the mm-wave beamspace MIMO channels. First, we will approximate the initial optimization problem in (13) as a convex optimization problem with the aid of the asymptotic orthogonality of the mm-wave beamspace MIMO channels.

Denote the summation item in (13) as

$$f_k \triangleq h(g_k), \tag{16}$$

where $h(x) \triangleq \log_2(1 + x)$,

$$g_k \triangleq \frac{a_{k,k}p_k}{\sum_{\substack{k'=1\\k'\neq k}}^{K} a_{k,k'}p_{k'} + 1}. \tag{17}$$

Then, we have

$$R_\mathrm{MMSE} = \sum_{k=1}^{K} f_k. \tag{18}$$

When $a_{k,k'} = 0, \forall k' \neq k$, we have $f_k = h(\tilde{g}_k)$, where $\tilde{g}_k = a_{k,k}p_k$. According to the concavity of the log functions, it can be seen that R_MMSE is a concave function of the vector $[p_1, p_2, \cdots, p_K]$. When $a_{k,k'} \approx 0, \forall k' \neq k$, R_MMSE may still be a concave function of the vector $[p_1, p_2, \cdots, p_K]$. Then, the standard convex optimization approaches can be employed to solve the problem. Here, one proposition is presented to show that the approximation $a_{k,k'} \approx 0, \forall k' \neq k$ is reasonable for mm-wave beamspace MIMO systems.

Proposition 1: As the number of the AP antennas, N_x and N_y, tend to infinity, $a_{k,k'} \to 0, \forall k' \neq k$.

It is shown in [13] and [14] that $\mathbf{a}^H(\theta_{k,p}, \phi_{k,p})\mathbf{a}(\theta_{k',p_{k'}}, \phi_{k',p_{k'}})/N \to \delta(k - k')\delta(p_k - p_{k'})$. This asymptotic orthogonality can be extended to the asymptotic orthogonality in the proposition.

It is known that mm-wave beamspace MIMO system is likely to equip a large number of antennas at the AP, which means the conclusion in Proposition 1 holds for the considered system. Thus, we can take the sum rate R_MMSE as a concave function of the vector $[p_1, p_2, \cdots, p_K]$. Then, various convex optimization algorithms, such as the gradient descent method and the Newton's method, can be employed to search for the optimum power allocation. However, these algorithms update the power vector $[p_1, p_2, \cdots, p_K]$ in each iteration. Also, the water filling approach can be employed. However, $a_{k,k'} \neq 0, \forall k' \neq k$, which makes the water filling solution not the optimal solution. Hence, an approach that utilizes the approximate concavity, which means that the sum rate may still be a concave function with $a_{k,k'} \neq 0$, $\forall k' \neq k$, will yield a better solution than the water filling approach.

Here, an optimization algorithm that only updates the powers of partial UTs is proposed. According to the convex optimization theory, the near optimal power allocation should satisfy $\frac{\partial R_\mathrm{MMSE}}{\partial p_k} = 0, \forall k$. According to (16) and (18), the above equation can be equivalently written as $\sum_{\tilde{k}=1}^{K} \frac{\partial f_{\tilde{k}}}{\partial p_k} = 0, \forall k$. Note that the satisfaction of the above equation is approximately equivalent to the object in (15). Then, the optimization problem in (15) is changed into

$$\begin{aligned} &\min_{p_k, k=1,2,\cdots,K} \left| \sum_{\tilde{k}=1}^{K} \frac{\partial f_{\tilde{k}}}{\partial p_k} \right|, \quad \forall k \\ &\text{s.t. } p_k \geq 0 \\ &\quad \sum_{k=1}^{K} b_k p_k = \rho. \end{aligned} \tag{19}$$

Meanwhile, the power constraint in (15) can be equivalently expressed as

$$p_{k_0} = \tilde{\rho} - \sum_{\substack{k'=1 \\ k' \neq k_0}}^{K} \tilde{b}_{k'} p_{k'}, \quad \forall k_0 \neq k, \tag{20}$$

where $\tilde{b}_{k'} = b_{k'}/b_{k_0}$, $\tilde{\rho} = \rho/b_{k_0}$. Substituting (20) into (17) can eliminate the equality constraint in (15), which yields

$$\bar{g}_k = \begin{cases} \dfrac{a_{k,k} p_k}{x_k}, & k \neq k_0 \\ \dfrac{a_{k_0,k_0} \left(\tilde{\rho} - \sum_{\substack{k'=1 \\ k' \neq k_0}}^{K} \tilde{b}_{k'} p_{k'} \right)}{\sum_{\substack{k'=1 \\ k' \neq k_0}}^{K} a_{k_0,k'} p_{k'} + 1}, & k = k_0, \end{cases} \tag{21}$$

where $x_k = \sum_{\substack{k'=1 \\ k' \neq k,k_0}}^{K} a_{k,k'} p_{k'} + a_{k,k_0} (\tilde{\rho} - \sum_{\substack{k'=1 \\ k' \neq k_0}}^{K} \tilde{b}_{k'} p_{k'}) + 1$. Correspondingly, we have

$$\bar{f}_k \triangleq h(\bar{g}_k), \tag{22}$$

and the optimization problem in (19) is changed into

$$\begin{aligned} &\min_{p_k, k \neq k_0} \left| \sum_{\tilde{k}=1}^{K} \frac{\partial \bar{f}_{\tilde{k}}}{\partial p_k} \right|, \quad \forall k \neq k_0 \\ &\text{s.t. } p_k \geq 0 \\ &\quad \sum_{\substack{k=1 \\ k \neq k_0}}^{K} b_k p_k \leq \rho. \end{aligned} \tag{23}$$

Then, another proposition should be presented to simplify the optimization object in (23).

Proposition 2: As the number of the AP antennas, N_{x} and N_{y}, tend to infinity, the optimization object in (23) is equivalent to

$$\min_{p_k, k \neq k_0} \left| \frac{1}{b_k} \frac{\partial \bar{f}_k}{\partial p_k} - \frac{1}{b_{k_0}} \frac{\partial f_{k_0}}{\partial p_{k_0}} \right|. \tag{24}$$

Proof: According to Proposition 1, as N_{x} and N_{y}, tend to infinity, $a_{k,k'} \to 0, \forall k' \neq k$. In this case, only the derivative to the numerator of $\bar{g}_k$ needs to be taken into consideration in the derivative of $\bar{g}_k$. Thus, the derivative of $\bar{g}_k$ can be written as

$$\frac{\partial \bar{g}_{\tilde{k}}}{\partial p_k} \to \begin{cases} \dfrac{a_{\tilde{k},\tilde{k}}}{x_{\tilde{k}}}, & \tilde{k} \neq k_0, \ \tilde{k} = k \\ \dfrac{-a_{k_0,k_0} \tilde{b}_k}{\sum_{\substack{k'=1 \\ k' \neq k_0}}^{K} a_{k_0,k'} p_{k'} + 1}, & \tilde{k} = k_0, \ \tilde{k} \neq k \\ 0, & \text{otherwise.} \end{cases} \tag{25}$$

From (21), (22), and (25), it can be found that

$$\frac{\partial \bar{f}_{\tilde{k}}}{\partial p_k} \to \begin{cases} \dfrac{1}{\ln 2(1+\bar{g}_k)} \dfrac{\partial \bar{g}_k}{\partial p_k} = \dfrac{\partial \bar{f}_k}{\partial p_k}, & \tilde{k} \neq k_0, \ \tilde{k} = k \\ \dfrac{1}{\ln 2(1+\bar{g}_{k_0})} \dfrac{\partial \bar{g}_{k_0}}{\partial p_k} = \dfrac{\partial \bar{f}_{k_0}}{\partial p_k} & \tilde{k} = k_0, \ \tilde{k} \neq k \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

Substituting the above equation into (19) yields

$$\left| \sum_{\tilde{k}=1}^{K} \frac{\partial \bar{f}_{\tilde{k}}}{\partial p_k} \right| \to \left| \frac{\partial \bar{f}_k}{\partial p_k} + \frac{\partial \bar{f}_{k_0}}{\partial p_k} \right|. \tag{27}$$

From (25) and (26), we have

$$\frac{\partial \bar{f}_{k_0}}{\partial p_k} = -\frac{b_k}{b_{k_0}} \frac{\partial f_{k_0}}{\partial p_{k_0}}, \tag{28}$$

where

$$\frac{\partial f_{k_0}}{\partial p_{k_0}} = \frac{1}{\ln 2(1+\bar{g}_{k_0})} \frac{a_{k_0,k_0}}{\sum_{\substack{k'=1 \\ k' \neq k_0}}^{K} a_{k_0,k'} p_{k'} + 1}.$$

Substituting (28) into (27) yields

$$\left| \sum_{\tilde{k}=1}^{K} \frac{\partial \bar{f}_{\tilde{k}}}{\partial p_k} \right| \to \left| \frac{\partial \bar{f}_k}{\partial p_k} - \frac{b_k}{b_{k_0}} \frac{\partial f_{k_0}}{\partial p_{k_0}} \right|.$$

Therefore, the optimization object in (23) is equivalent to the optimization object in (24). ■

According to Proposition 2, we can replace the optimization problem in (23) with

$$\begin{aligned} &\min_{p_k, k \neq k_0} \left| \frac{1}{b_k} \frac{\partial \tilde{f}_k}{\partial p_k} - \frac{1}{b_{k_0}} \frac{\partial \tilde{f}_{k_0}}{\partial p_{k_0}} \right|, \quad \forall k \neq k_0 \\ &\text{s.t. } p_k \geq 0 \\ &\quad \sum_{\substack{k=1 \\ k \neq k_0}}^{K} b_k p_k \leq \rho, \end{aligned} \tag{29}$$

where

$$\tilde{f}_k = \begin{cases} \bar{f}_k, & k \neq k_0 \\ f_{k_0}, & k = k_0 \end{cases}$$

is defined for the convenience of writing the expressions. Note here we also uses the asymptotic property to simplify the problem, but we do not take $a_{k,k'}$ as zero. In other words, only the approximation of $a_{k,k'} \approx 0$ is used, but it is not taken as zero here.

According to the optimization problem in (29) and the monotonicity and the concavity of $\tilde{f}_k$, the power allocation should obey the following principles.

Principle 1: In order to increase the sum rate of the system, the power of the UT with the largest $\partial \tilde{f}_k / \partial p_k / b_k$ should be increased, and the power of the UT with the smallest $\partial \tilde{f}_k / \partial p_k / b_k$ should be decreased.

Principle 2: If we sort $\partial \tilde{f}_k / \partial p_k / b_k, k \in \tilde{\mathcal{C}}_{\mathrm{MMSE}}$ in decreasing order, this order may change with the change of the power $p_k, k \in \tilde{\mathcal{C}}_{\text{MMSE}}$. Note that $\tilde{\mathcal{C}}_{\text{MMSE}}$ is the set of UTs with positive power, and $\mathcal{C}_{\text{MMSE}}$ is the set of UTs with power of zero. Thus, the UTs that should change power are not fixed when the power changes, and the changes of the powers and UTs will not stop until $\partial\tilde{f}_k/\partial p_k/b_k$ is the same for $k \in \tilde{\mathcal{C}}_{\text{MMSE}}$.

Principle 3: If $\partial\tilde{f}_{k_x}/\partial p_{k_x}/b_{k_x}$ and $\partial\tilde{f}_{k_y}/\partial p_{k_y}/b_{k_y}$ are the maximum and minimum in $\partial\tilde{f}_k/\partial p_k/b_k$, $k \in \tilde{\mathcal{C}}_{\text{MMSE}}$, then the increase of p_{k_x} should make $\partial\tilde{f}_{k_x}/\partial p_{k_x}/b_{k_x}$ and $\partial\tilde{f}_{k_y}/\partial p_{k_y}/b_{k_y}$ close. Hence, reasonable changes of p_{k_x} and p_{k_y}, which are denoted as $\delta_{p_{k_x}}$ and $\delta_{p_{k_y}}$, should satisfy the power constraint $b_{k_x}\delta_{p_{k_x}} + b_{k_y}\delta_{p_{k_y}} = 0$ and the approximate equality of the increasing rate $(\partial\tilde{f}_{k_x}/\partial p_{k_x} + \partial^2\tilde{f}_{k_x}/\partial p_{k_x}^2\delta_{p_{k_x}} + \partial^2\tilde{f}_{k_x}/\partial p_{k_x}/\partial p_{k_y}\delta_{p_{k_y}})/b_{k_x} = (\partial\tilde{f}_{k_y}/\partial p_{k_y} + \partial^2\tilde{f}_{k_y}/\partial p_{k_y}^2\delta_{p_{k_y}} + \partial^2\tilde{f}_{k_y}/\partial p_{k_x}/\partial p_{k_y}\delta_{p_{k_x}})/b_{k_y}$. Solving the above two equations results into

$$\delta_{p_{k_x}} = \left(\frac{1}{b_{k_x}}\frac{\partial\tilde{f}_{k_x}}{\partial p_{k_x}} - \frac{1}{b_{k_y}}\frac{\partial\tilde{f}_{k_y}}{\partial p_{k_y}}\right)\Big/\left(\frac{1}{b_{k_y}}\frac{\partial^2\tilde{f}_{k_x}}{\partial p_{k_x}\partial p_{k_y}} + \frac{1}{b_{k_y}}\frac{\partial^2\tilde{f}_{k_y}}{\partial p_{k_x}\partial p_{k_y}} - \frac{b_{k_x}}{b_{k_y}^2}\frac{\partial^2\tilde{f}_{k_y}}{\partial p_{k_y}^2} - \frac{1}{b_{k_x}}\frac{\partial^2\tilde{f}_{k_x}}{\partial p_{k_x}^2}\right) \tag{30}$$

and

$$\delta_{p_{k_y}} = -\delta_{p_{k_x}}\frac{b_{k_x}}{b_{k_y}}. \tag{31}$$

Based on these principles, a concavity approximation based power allocation algorithm, i.e., Algorithm 1, is proposed, where the parameter σ can be set to be $10^{-3}|\partial\tilde{f}_k/\partial p_k/b_k|^2, \forall k \in \tilde{\mathcal{C}}_{\text{MMSE}}$. Other details are not shown in the algorithm for brevity. For example, the algorithm will break if the variance of the derivatives in the current loop is much larger than that in the previous loop, and this is not shown in the algorithm.

The convergence of the proposed algorithm is easy to verify. As $\tilde{f}_k$ monotonically increases with p_k and is a concavity function of p_k, the powers of UTs with smaller/larger values of $\partial\tilde{f}_k/\partial p_k/b_k$ can be decreased/increased. Moreover, UTs with extremely small $\partial\tilde{f}_k/\partial p_k/b_k$ and p_k can be allocated with no power. In the extreme case, only one UT is allocated with power, as can be seen, this definitely satisfies the stop criterion in the algorithm. Therefore, it is guaranteed that the iteration will converge.

For the case that some of the paths are blocked, which means some beams may also be blocked, the proposed approach can still work with supplementary strategies. If all the paths of a UT are blocked, then this UT will not be scheduled and the UT will not be included in the power allocation process. If the paths of a UT are not completely blocked, then the effective paths can be utilized for transmission. Thus, the beams for this UT can be selected and the UT will be included in the power allocation process in the proposed approach.

From the above analysis, it can be found that the proposed approach updates the powers toward the point of equal first order derivative, which is also the point of the maximal sum rate for mm-wave beamspace MIMO systems with

Algorithm 1 Proposed Power Allocation

Initialize: Set $\mathcal{C}_{\text{MMSE}}$ as an empty set, set $\tilde{\mathcal{C}}_{\text{MMSE}}$ as the set of the indices of all the UTs, set the powers of the UTs as $p_k = \rho/\sum_{k'=1}^{K} b_{k'}$, $\forall k \in \tilde{\mathcal{C}}_{\text{MMSE}}$, take an arbitrary k as k_0

1: **while** $\sum_{k\in\tilde{\mathcal{C}}_{\text{MMSE}}}(\partial\tilde{f}_k/\partial p_k/b_k - \sum_{k'\in\tilde{\mathcal{C}}_{\text{MMSE}}}\partial\tilde{f}_{k'}/\partial p_{k'}/b_{k'}/\tilde{K}_{\text{MMSE}})^2 > \sigma$ **do**
2: Find the maximum and minimum in $\partial\tilde{f}_k/\partial p_k/b_k$, $k \in \tilde{\mathcal{C}}_{\text{MMSE}}$, and the indices of the corresponding UTs are denoted as k_x, k_y
3: Calculate $\delta_{p_{k_x}}$ and $\delta_{p_{k_y}}$ according to (30) and (31)
4: **if** $p_{k_y} + \delta_{p_{k_y}} \le 0$
5: **then** Update p_{k_x} with $p_{k_x} + p_{k_y}b_{k_y}/b_{k_x}$, set $p_{k_y} = 0$, pick k_y out of $\tilde{\mathcal{C}}_{\text{MMSE}}$ and put it into $\mathcal{C}_{\text{MMSE}}$,
6: **else** Update p_{k_x}, p_{k_y} with $p_{k_x} + \delta_{p_{k_x}}, p_{k_y} + \delta_{p_{k_y}}$
7: **end if**
8: **end while**

N_x and N_y tending to infinity. Therefore, it is expected that the proposed approach can achieve near optimal performance in the considered mm-wave beamspace MIMO system.

Moreover, the proposed approach approximates the original optimization problem as a convex optimization problem by using the asymptotic orthogonality of the mm-wave channels. In contrast, the existing SCA approach [25]–[27] and the GP approach [20]–[22] approximate the original problem as a convex optimization problem in other ways, such as the high signal-to-interference-plus-noise ratio (SINR) approximation [20] or the first-degree Taylor polynomial approximation [26]. Thus, the approximation error deteriorates the SCA and the GP. The channel asymptotic orthogonality is evident in mm-wave systems and the approximation error is expected to be smaller than those in the SCA and the GP. Therefore, the proposed approach is expected to perform better than the SCA and the GP.

Till now, the optimization problem of power allocation has been presented and solved. In the following section, we will analyze the impact of system parameters on the performance of the proposed scheme and compare it with the existing approaches in more detail.

IV. PERFORMANCE ANALYSIS

According to the previous derivation, it can be seen that the proposed approach is suited for mm-wave beamspace MIMO system with MMSE precoding. In this section, the relation between the performance of the MMSE precoding and the system parameters will be analyzed, which also indicates the relation between the proposed scheme and the system parameters. Moreover, the proposed approach will be compared with the existing approaches.

A. IMPACT OF SYSTEM PARAMETERS

The impact of the beamspace channel correlation on the performance of the system can be summarized in the following theorem.

Theorem 1: As the beamspace channel correlation, i.e.,

$$\frac{\left|([\mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}}]_j)^H[\mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}}]_l\right|^2}{\left|\left|[\mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}}]_j\right|\right|^2\left|\left|[\mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}}]_l\right|\right|^2}, \quad \forall j \neq l,$$

decreases, the interference between the j-th UT and the l-th UT decreases.

The result is easy to see and the proof is omitted. It is known that the channel correlation in mm-wave MIMO systems decreases with the increase of the antennas. With beamspace processing, the channel is transformed into the beamspace with beams generated by the DLA. In fact, the analog beamforming beams are utilized to transform the MIMO channels into another orthogonal space with only the dominant dimensions left. Thus, the beamspace channel correlation also decreases with the increase of the antennas. Then, the impact of the distance between the UTs on the performance of the system can be illustrated.

Corollary 1: As the distance between two UTs decreases, the interference between the two UTs increases.

Proof: When the distance between two UTs decreases, the differences between the DOAs of two UTs tend to zero. According to the definitions of $\mathbf{H}$ in (3) and $\mathbf{H}_{\mathrm{b}}$, two rows of $\mathbf{H}_{\mathrm{b}}^H\mathbf{H}_{\mathrm{b}}$ tend to be the same, i.e., tend to be highly correlated. According to Theorem 1, the interference between the two UTs increases. ∎

Meanwhile, the impact of the number of the AP antennas on the performance of the system can be summarized in the following corollary.

Corollary 2: As the number of the AP antennas, N_{x} and N_{y}, tend to infinity, the interference among the UTs decreases.

Proof: According to Proposition 1, as N_{x} and N_{y} tend to infinity, $(1/N)\mathbf{H}_{\mathrm{b}}\mathbf{H}_{\mathrm{b}}^H$ tends to be a diagonal matrix. According to Theorem 1, the interference among the UTs decreases. ∎

According to these two corollaries, it can be seen that the increase of the number of the UTs results into the increase of the interference, while the increase of the number of the AP antennas results into the decrease of the interference. In mm-wave MIMO systems considered, the AP is equipped with a large number of antennas. Thus, the system can serve a large number of UTs.

B. COMPARISON WITH THE EXISTING APPROACHES

For the GP approach in [20], R_{MMSE} in (13) is approximated as $R_{\mathrm{MMSE}} \approx \sum_{k=1}^{K}\log_2\left(\frac{a_{k,k}e^{y_k}}{e^{z_k}+1}\right)$, where $y_k = \ln p_k$, $z_k = \ln\sum_{k'=1, k'\neq k}^{K} a_{k,k'}p_{k'}$, and the high SINR is necessary for the approximation. Then, the optimization in (15) is changed into

$$\begin{aligned}\min_{y_k, k=1,2,\cdots,K} &\sum_{k=1}^{K}\log_2\left(\frac{e^{z_k}+1}{a_{k,k}e^{y_k}}\right)\\ \text{s.t. } e^{z_k} &= \sum_{\substack{k'=1\\k'\neq k}}^{K} a_{k,k'}e^{y_{k'}}\\ &\sum_{k=1}^{K} b_k e^{y_k} = \rho.\end{aligned}$$

This is a convex optimization with equality constraints and is solved in [20].

For the SCA approach in [26], R_{MMSE} in (13) is written as $R_{\mathrm{MMSE}} = -u(\mathbf{p}) + v(\mathbf{p})$, where $u(\mathbf{p}) = -\sum_{k=1}^{K}\log_2\left(\sum_{k'=1}^{K}a_{k,k'}p_{k'}+1\right)$, $v(\mathbf{p}) = -\sum_{k=1}^{K}\log_2\left(\sum_{k'=1, k'\neq k}^{K}a_{k,k'}p_{k'}+1\right)$, and $\mathbf{p} = [p_1, p_2, \cdots, p_K]^T$. By approximating $v(\mathbf{p})$ with the first-degree Taylor polynomial, we have $-R_{\mathrm{MMSE}} \approx u(\mathbf{p}) - (v(\mathbf{p}^{(t)}) + \mathbf{r}_{\mathbf{p}^{(t)}}^T(\mathbf{p}-\mathbf{p}^{(t)}))$, where $\mathbf{p}^{(t)} \in \mathbb{C}^{K\times 1}$ is the derivative point in the t-th iteration, $\mathbf{r}_{\mathbf{p}^{(t)}} = \partial v(\mathbf{p}^{(t)})/\partial\mathbf{p}^{(t)} \in \mathbb{C}^{K\times 1}$. This approximated $-R_{\mathrm{MMSE}}$ is a convex function of $\mathbf{p}$. Then, the optimized value of $\mathbf{p}$ is taken as $\mathbf{p}^{(t+1)}$ and is used in the $t+1$-th iteration. However, this approximation also causes error, which is not negligible when the interference power, $\sum_{k'=1, k'\neq k}^{K}a_{k,k'}p_{k'}$, is close to zero. Thus, this approach is suitable for cases of large interference power. For clarity, the comparison is shown in Table 1.

In comparison with a system with no power allocation, the computational complexity of the proposed approach will also be analyzed here. By taking the division the same as the multiplication, the number of multiplications costed by step 2 of Algorithm 2 is on the order of $3K|\tilde{\mathcal{C}}_{\mathrm{MMSE}}|$, the number of multiplications costed by step 3 of Algorithm 2 is on the order of $13K$. Thus, the number of multiplications costed by the proposed approach is on the order of $3K|\tilde{\mathcal{C}}_{\mathrm{MMSE}}| + 13K$. As can be seen, the computational complexity of the proposed approach is not high. For the GP approach, in each iteration, there are K^2 multiplications, which means the number of multiplications costed by the GP is on the order of K^2. For the SCA approach, in each iteration, there are K^3 multiplications, which means the number of multiplications costed by the SCA is on the order of K^3. Because $|\tilde{\mathcal{C}}_{\mathrm{MMSE}}| \leq K$, when the GP and the SCA are compared with the proposed approach, it can be seen that the computational complexity of the proposed approach is lower than or similar to that of the other two approaches.

With the above analysis, we know that the MMSE precoding as well as the proposed scheme are suited to the mm-wave beamspace MIMO system considered. In the next section, we will evaluate the system performance with varying system parameters in simulations.

TABLE 1. Comparison of the power allocation approaches.

Approaches	Approximation	Application scenario
GP in [20]	$R_{\mathrm{MMSE}} \approx \sum_{k=1}^{K} \log_2\left(\frac{a_{k,k}e^{y_k}}{e^{z_k}+1}\right)$	high SINR
SCA in [26]	$R_{\mathrm{MMSE}} \approx v(\mathbf{p}^{(t)}) + \mathbf{r}_{\mathbf{p}^{(t)}}^T(\mathbf{p}-\mathbf{p}^{(t)}) - u(\mathbf{p})$	high interference power
Proposed	$R_{\mathrm{MMSE}} \approx \sum_{k=1}^{K} h(\tilde{g}_k(p_k))$	a large number of AP antennas

V. NUMERICAL RESULTS

In this section, the spectral efficiencies of the mm-wave beamspace MIMO system with the MMSE precoding and the considered approaches are compared by computer simulations. The simulation parameters are set by referring to [17]. The number of UTs is $K = 100$, the number of beams selected for each UT is $N_{\mathrm{b}}/K = 2$, the numbers of antennas are $N_{\mathrm{x}} = 32$ and $N_{\mathrm{y}} = 151$, the height of the AP is 10 m, the distance from the UT to the foot of the AP ranges from 10 m to 100 m, the signal-to-noise ratio (SNR) is -5 dB, the transmission frequency is 80 GHz. The UTs distribute randomly in a sector of 120°, the center of which is the foot of the AP. The number of multipaths for each UT is $P = 2$, and the probability of the blockage of the LOS path is 0.1. The number of antennas in each UT is $M = 4$. Since the expectation of the square of the complex path loss $\mathbb{E}\{\alpha_k^2\}$ is on the order of 10^{-12} for two points with the minimal and the maximal distances to the foot of the AP, and there are many antennas at the UT, the SNR is taken as $10^{-7}\rho$ in the simulations. Moreover, the MMSE precoding for comparison allocates the powers equally to all the UTs and is denoted as "Equal allocation". The GP approach in [20] and the SCA approach in [26] are modified for the considered system and the modified approaches are denoted as "GP" and "SCA", respectively. The proposed approach is denoted with the label "Proposed allocation".

In Fig. 2, the spectral efficiencies versus the SNR is depicted. It can be seen that the spectral efficiencies of all the schemes increase with the increase of the SNR. Additionally, the spectral efficiency of the proposed approach is higher than that of other approaches when the SNR is relatively low. This verifies that the approximation of concavity holds in mm-wave beamspace MIMO system, which is more suitable than the approximations employed in the GP approach and the SCA approach. When the SNR is high, the derivative of the sum rate is almost invariant, and the differences between the power allocations are small.

Fig. 3 shows the relation between the spectral efficiencies and the number of the UTs. It can be seen that the spectral efficiencies of all the schemes increase with the increase of the number of the UTs. This is because the effect of power allocation becomes more evident as the number of the UTs increases.

Fig. 4 demonstrates the change of the spectral efficiencies in accordance with the change of the number of the AP antennas. It can be seen that the spectral efficiencies of all the approaches increase with the increase of the number of the AP antennas. As the number of the AP antennas increases, the channels tend to be orthogonal, the interference among the UTs decreases. Thus, the performances of all the approaches improve. This phenomenon verifies the performance analysis given in Corollary 2. Meanwhile, the performance of a system with equal power allocation is better than that of the GP approach when the number of the AP antennas is small. This is because the SNR is very low in this case, and the approximation error of the GP approach is high, i.e., the GP approach is in fact optimizing the power allocation for a problem that differs much with the original problem.

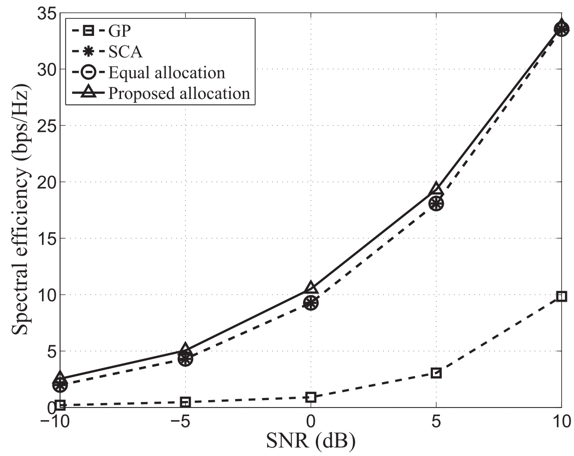


FIGURE 2. Spectral efficiency versus the SNR.

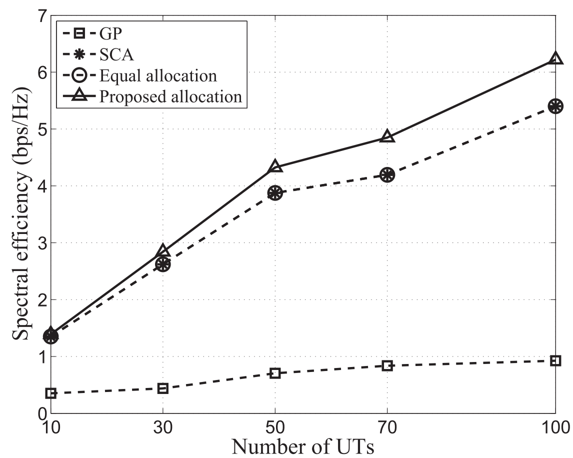


FIGURE 3. Spectral efficiency versus the number of the UTs.

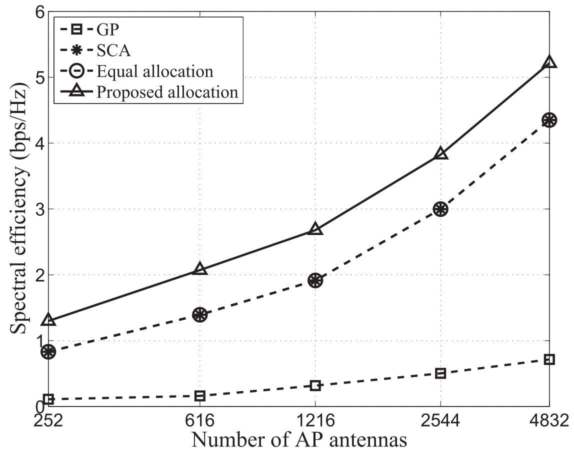


FIGURE 4. Spectral efficiency versus the number of the AP antennas.

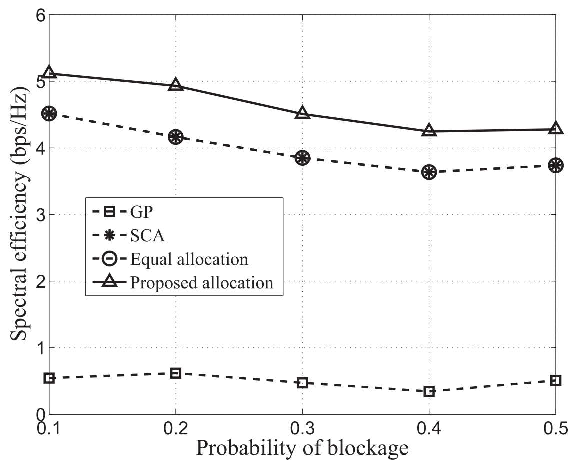


FIGURE 5. Spectral efficiency versus the probability of blockage of the LOS path.

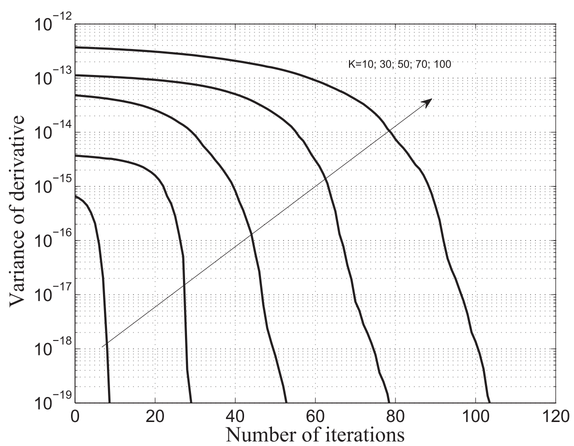


FIGURE 6. Variance of derivative versus the number of iterations.

Fig. 5 shows the spectral efficiency versus the probability of the blockage of the LOS path. As can be seen, the spectral efficiency decreases with the increase of the probability of the blockage. This is because the LOS path is of lower path loss than the multipath and the multipath should be used for transmission when the LOS is blocked. Meanwhile, the proposed approach still performs better than other approaches.

Fig. 6 shows the variance of derivative during the iterations of the proposed approach. From the illustration of Algorithm, it is known that the proposed approach converges when the variance of derivative decreases. Thus, Fig. 6 shows the speed of convergence of the proposed approach. In this figure, the larger the number of UTs in the system, the larger the variance of the initial derivative. As the iteration goes on, the variances of all the approaches decrease rapidly. This result verifies that the proposed approach can converge quickly and is suitable for implementation.

VI. CONCLUSIONS

In this paper, the power allocation is investigated for mm-wave beamspace MIMO system with the MMSE precoding. The sum rate of the system with the precoding method is presented first, which is then utilized to form the optimization problems. The asymptotic properties of the sum rate are employed to approximate the original optimization problem as a convex optimization problem and allocate the powers to maximize the sum rate. The analysis and the simulations verify that the proposed approach is suitable for the considered system.

ACKNOWLEDGMENT

The authors would like to thank the editor Dr. Jiayi Zhang and the three anonymous reviewers for their suggestions and comments.

REFERENCES

[1] Z. Pi and F. Khan, "An introduction to millimeter-wave mobile broadband systems," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 101–107, Jun. 2011.

[2] T. S. Rappaport *et al.*, "Millimeter wave mobile communications for 5G cellular: It will work!" *IEEE Access*, vol. 1, pp. 335–349, May 2013.

[3] A. L. Swindlehurst, E. Ayanoglu, P. Heydari, and F. Capolino, "Millimeter-wave massive MIMO: The next wireless revolution?" *IEEE Commun. Mag.*, vol. 52, no. 9, pp. 56–62, Sep. 2014.

[4] F. Rusek *et al.*, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.

[5] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.

[6] W. Hong, K.-H. Baek, Y. Lee, Y. Kim, and S.-T. Ko, "Study and prototyping of practically large-scale mmWave antenna systems for 5G cellular devices," *IEEE Commun. Mag.*, vol. 52, no. 9, pp. 63–69, Sep. 2014.

[7] W. Tan, S. Jin, and J. Wang, "Achievable rate of multiuser downlink for LOS-MIMO systems with uniform circular arrays," in *Proc. 6th Int. Conf. Wireless Commun. Signal Process.*, Hefei, China, Oct. 2014, pp. 1–6.

[8] S. Han, C.-L. I, Z. Xu, and C. Rowell, "Large-scale antenna systems with hybrid analog and digital beamforming for millimeter wave 5G," *IEEE Commun. Mag.*, vol. 53, no. 1, pp. 186–194, Jan. 2015.

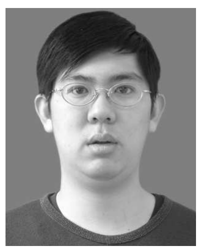
[9] S. Kutty and D. Sen, "Beamforming for millimeter wave communications: An inclusive survey," *IEEE Commun. Surveys Tuts.*, vol. 18, no. 2, pp. 949–973, 2nd Quart., 2016.

[10] J. A. Zhang, X. Huang, V. Dyadyuk, and Y. J. Guo, "Massive hybrid antenna array for millimeter-wave cellular communications," *IEEE Wireless Commun.*, vol. 22, no. 1, pp. 79–87, Feb. 2015.

[11] X. Huang, Y. J. Guo, and J. D. Bunton, "A hybrid adaptive antenna array," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1770–1779, May 2010.

[12] S.-H. Wu, L.-K. Chiu, K.-Y. Lin, and T.-H. Chang, "Robust hybrid beamforming with phased antenna arrays for downlink SDMA in indoor 60 GHz channels," *IEEE Trans. Wireless Commun.*, vol. 12, no. 9, pp. 4542–4557, Sep. 2013.
[13] O. El Ayach, R. W. Heath, Jr., S. Abu-Surra, S. Rajagopal, and Z. Pi, "The capacity optimality of beam steering in large millimeter wave MIMO systems," in *Proc. IEEE 13th Int. Workshop Signal Process. Adv. Wireless Commun.*, Cesme, Turkey, Jun. 2012, pp. 100–104.
[14] A. M. Sayeed, "Deconstructing multiantenna fading channels," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2563–2579, Oct. 2002.
[15] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, Jr., "Spatially sparse precoding in millimeter wave MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499–1513, Mar. 2014.
[16] J. Brady, N. Behdad, and A. M. Sayeed, "Beamspace MIMO for millimeter-wave communications: System architecture, modeling, analysis, and measurements," *IEEE Trans. Antennas Propag.*, vol. 61, no. 7, pp. 3814–3827, Jul. 2013.
[17] J. Brady and A. Sayeed, "Beamspace MU-MIMO for high-density gigabit small cell access at millimeter-wave frequencies," in *Proc. IEEE 15th Int. Workshop Signal Process. Adv. Wireless Commun.*, Toronto, ON, Canada, Jun. 2014, pp. 80–84.
[18] G. H. Song, J. Brady, and A. Sayeed, "Beamspace MIMO transceivers for low-complexity and near-optimal communication at mm-wave frequencies," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, Vancouver, BC, Canada, May 2013, pp. 4394–4398.
[19] P. V. Amadori and C. Masouros, "Low RF-complexity millimeter-wave beamspace-MIMO systems by beam selection," *IEEE Trans. Commun.*, vol. 63, no. 6, pp. 2212–2223, Jun. 2015.
[20] H. Liu, S. Xu, and K. S. Kwak, "Geometric programming based distributed resource allocation in ultra dense hetnets," in *Proc. IEEE 83rd Veh. Technol. Conf.*, Nanjing, China, May 2016, pp. 1–5.
[21] Y. Zhang and W.-P. Zhu, "Energy-efficient pilot and data power allocation in massive multi-user multiple-input multiple-output communication systems," *IET Commun.*, vol. 10, no. 14, pp. 1721–1729, Sep. 2016.
[22] K. Guo, Y. Guo, and G. Ascheid, "Security-constrained power allocation in MU-massive-MIMO with distributed antennas," *IEEE Trans. Wireless Commun.*, vol. 15, no. 12, pp. 8139–8153, Dec. 2016.
[23] W. Feng, Y. Chen, N. Ge, and J. Lu, "Optimal energy-efficient power allocation for distributed antenna systems with imperfect CSI," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 7759–7763, Sep. 2016.
[24] J. Zhang, Y. Jiang, P. Li, F. Zheng, and X. You, "Energy efficient power allocation in massive MIMO systems based on standard interference function," in *Proc. IEEE 83rd Veh. Technol. Conf.*, Nanjing, China, May 2016, pp. 1–6.
[25] P. Tsiaflakis, M. Diehl, and M. Moonen, "Distributed spectrum management algorithms for multiuser DSL networks," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 4825–4843, Oct. 2008.
[26] N. Vucic, S. Shi, and M. Schubert, "DC programming approach for resource allocation in wireless networks," in *Proc. Int. Symp. Modeling Optim. Mobile, Ad Hoc, Wireless Netw.*, Avignon, France, Jan. 2010, pp. 380–386.
[27] J. Papandriopoulos and J. S. Evans, "SCALE: A low-complexity distributed protocol for spectrum balancing in multiuser DSL networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 8, pp. 3711–3724, Aug. 2009.
[28] W. Yu, G. Ginis, and J. M. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 5, pp. 1105–1115, Jun. 2002.
[29] M. El Khaled, P. Fortier, M. L. Ammari, and S. A. M. Tariq, "Optimal power allocation for underground selective channel at 60 GHz," in *Proc. IEEE Int. Conf. Commun.*, Sydney, NSW, Australia, Jun. 2014, pp. 736–741.
[30] B. Ma, B. Niu, Z. Wang, and V. W. S. Wong, "Joint power and channel allocation for multimedia content delivery using millimeter wave in smart home networks," in *Proc. IEEE Global Commun. Conf.*, Austin, TX, USA, Dec. 2014, pp. 4745–4750.
[31] J. Kim, "Buffer-stable adaptive per-module power allocation for energy-efficient millimeter-wave modular antenna array (MAA) platforms," in *Proc. IEEE Conf. Comput. Commun.*, San Francisco, CA, USA, Apr. 2016, pp. 820–821.
[32] H. Zhang, S. Huang, C. Jiang, K. Long, V. C. M. Leung, and H. V. Poor, "Energy efficient user association and power allocation in millimeter-wave-based ultra dense networks with energy harvesting base stations," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 9, pp. 1936–1947, Sep. 2017.
[33] W. Hao, M. Zeng, Z. Chu, and S. Yang, "Energy-efficient power allocation in millimeter wave massive MIMO with non-orthogonal multiple access," *IEEE Wireless Commun. Lett.*, to be published.
[34] A. Hu, "Estimation of sparse channels in millimeter-wave MU-MIMO systems," *KSII Trans. Internet Inf. Syst.*, vol. 10, no. 5, pp. 2102–2123, May 2016.

ANZHONG HU (S'13–M'17) received the Ph.D. degree from the Beijing University of Posts and Telecommunications, Beijing, China, in 2014. He is currently a Lecturer with the School of Communication Engineering, Hangzhou Dianzi University, Hangzhou, China. His current research interests include channel estimation and array processing.



PENG PAN (M'13) received the B.Eng. and Ph.D. degrees in electronic engineering from Beihang University, Beijing, China, in 2005 and 2011, respectively. From 2007 to 2008, he was a Visiting Ph.D. Student with the School of Electronics and Computer Science, University of Southampton, Southampton, U.K. He is currently an Associate Professor with the School of Communication Engineering, Hangzhou Dianzi University, Hangzhou, China. His research interests include multicarrier systems, multiuser detection, and performance evaluation of wireless systems.