Pilot Design for Large-Scale Multi-Cell Multiuser MIMO Systems

Anzhong Hu*, Tiejun Lv*, Hui Gao†, Yueming Lu*, and Enjie Liu‡
*Key Laboratory of Trustworthy Distributed Computing and Service, Ministry of Education
School of Information and Communication Engineering
Beijing University of Posts and Telecommunications, Beijing, China 100876
†Singapore University of Technology and Design, 20 Dover Drive, Singapore 138682
‡Department of Computer Science and Technology, University of Bedfordshire, Luton, United Kingdom
Email: {huanzhong, lvtiejun, ymlu}@bupt.edu.cn, hui_gao@sutd.edu.sg, e.liu@beds.ac.uk

***Abstract*—Large-scale multi-cell multiuser multiple-input multiple-output (LS-MIMO) systems can greatly increase the spectral efficiency. But the performance of these systems is deteriorated by pilot contamination. In this paper, first, a pilot design criterion is proposed by exploiting the orthogonality of channel vectors of LS-MIMO systems. Second, following this criterion, Chu sequences based pilots are designed. Because of the proposed pilots, the channel estimate of most terminals of a cell is only interfered by the partial cells rather than all the other cells, where the latter is caused by traditional pilots. As a result, pilot contamination is mitigated. Numerical results verify the effectiveness of the proposed pilots.**

I. Introduction

Multiple-input multiple-output (MIMO) technology can provide diversity gain and spatial multiplexing gain [1]. Moreover, large-scale multi-cell multiuser MIMO (LS-MIMO) systems have received much attention recently, in which the base station (BS) comprises a hundred, or a few hundred antennas, and serves tens of users simultaneously [2]. Because the channel vectors between the BS and terminals are approximately orthogonal, if the BS has perfect channel state information (CSI), the terminals can share the same band of frequencies simultaneously, and the effects of noise and small scale fading vanish. As a result, the spectral efficiency can be increased and the transmit power can be reduced [3], [4].

However, pilot contamination deteriorates the performance of LS-MIMO systems [5], [6]. Pilots are known to the receiver and transmitted with the data for channel estimation. But the pilots can not be orthogonal since they are limited in length. As a result, for LS-MIMO systems that use nonorthogonal pilots, the channel estimate at the BS is interfered by terminals of other cells, and this effect is called pilot contamination [7]. Although this is the only remaining impairment for LS-MIMO systems [5], little attention has been paid to this problem up to now. On the other hand, Chu sequences based pilots have been employed to mitigate the inter-cell interference (ICI) for MIMO orthogonal frequency division multiplexing (OFDM) systems [8], [9]. These pilots are orthogonal to their cyclic shifted ones, and new pilots can be generated by shifting the phase or changing the unique sequence parameter [10]. However, shadow fading coefficients are not considered [8] [9]. These pilots result in the same cross-correlation values for different terminals, thus are not optimal when the shadow fading coefficients are different [8]. Meanwhile, the pilots in [9] correspond to the reused orthogonal pilots in different cells when doppler spread is not considered. Hence, these pilots are not good enough to mitigate pilot contamination.

In this paper, pilot contamination is mitigated by the pilots designed for LS-MIMO systems. To be specific, the main contributions of this paper are two-fold. First, a pilot design criterion is proposed. By exploiting the orthogonality of the channel vectors of LS-MIMO systems, the signal-to-interference ratio (SIR) of the equalized signal is derived to measure the pilot contamination. Hence, according to the expression of SIR, the pilot design criterion should be minimizing the inner product of the correlation vectors of the pilots of different cells. Second, Chu sequences based pilots are designed according to the proposed criterion. Especially, when the length of the pilots is not less than K, where K is the number of simultaneously active terminals of each cell, the pilots inside a cell are orthogonal because of different cyclic shifts. Moreover, these pilots are reused among the L cells after being multiplied by phase shifts, where L is the number of cells that share the same band of frequencies. Because the phase shifts affect the pilot contamination, the suboptimal phase shifts for pilots of length N_p are derived according to the proposed criterion, where N_p should satisfy $K + L - 1 \leq N_p \leq 2K$. Then the cross-correlation values of pilots are different for different terminals, which fit scenarios with large shadow fading variance. Moreover, the channel estimate of most terminals is only interfered by the partial cells. As a result, pilot contamination is mitigated.

Notations: $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ are the conjugate, transpose, and conjugate transpose respectively; $[\cdot]_i$ is the ith column of a matrix, and $[\cdot]_{i,j}$ is the (i, j)th entry of a matrix; $\mathbb{E}\{\cdot\}$ is the expectation; $||\cdot||$ is the norm of a vector; $\mathbf{I}_K$ is the size K identity matrix; $||\cdot||_\mathrm{F}$ is the Frobenius norm of a matrix; $\delta(\cdot)$ is the Kronecker delta function; $\langle\cdot\rangle_n$ is the left circular shift of a vector with shift length n; $x \bmod N$ is the modulo-N operation for x.

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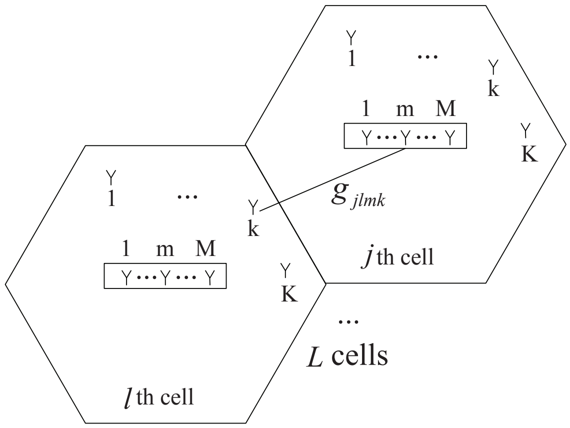

Fig. 1. System model. The channel from the kth terminal in the lth cell to the mth antenna of the BS in the jth cell is g_{jlmk}.

II. System Model

We consider a system model as shown in Fig. 1. The system consists of L hexagonal cells that share the same band of frequencies. In each cell, one BS with M antennas in the center serves K single-antenna terminals simultaneously. Because LS-MIMO is considered here, it is required that $M \gg K$. In addition, the system employs OFDM, works in time-division duplex (TDD). The reverse-link channels are estimated by the BSs with the received signals, and the forward-link channels are acquired by utilizing channel reciprocity.

In one coherence interval, the channel responses of N_a symbols are assumed to be static, which means that the channel estimate is only effective in this interval. Consequently, a total of N_{c} symbols are used for pilots from all the terminals to the BS, and the rest $N_a - N_{\mathrm{c}}$ symbols are used for two-way data transmission. Apart from that, follow the model in [5], the channel responses are assumed to be constant in the frequency smooth interval, which composes N_{s} consecutive subcarriers. Therefore, to estimate the channel response of one subcarrier, every terminal can transmit pilots over N_{s} subcarriers. In the following, channel responses of the N_{s} subcarriers will be estimated. As a result, the pilot length is $N_{\mathrm{p}} = N_{\mathrm{c}} N_{\mathrm{s}}$, and N_{p} terminals can transmit pilots to the BS without intra-cell interference.

As seen from Fig. 1, the channel response from the kth terminal in the lth cell to the mth antenna of the BS in the jth cell is

$$g_{jlmk} = h_{jlmk} \beta_{jlk}^{1/2}, \tag{1}$$

where h_{jlmk} is the small scale fading coefficient, and β_{jlk} is the large scale fading coefficient which is composed of path loss and shadow fading, and is assumed invariant for different antennas of the same BS. Then the channel matrix from terminals of the lth cell to the BS in the jth cell is

$$\mathbf{G}_{jl} = \mathbf{H}_{jl} \mathbf{D}_{jl} \in \mathbb{C}^{M \times K}, \tag{2}$$

where $\mathbf{H}_{jl} \in \mathbb{C}^{M \times K}$ is of small scale fading coefficients, i.e., $[\mathbf{H}_{jl}]_{m,k} = h_{jlmk}$. The entries of $\mathbf{H}_{jl}$ are assumed to be independent and identically distributed (i.i.d.) and circularly-symmetric Gaussian random variables ($\mathcal{CN}(0,1)$); $\mathbf{D}_{jl} \in \mathbb{R}^{K \times K}$ is a diagonal matrix, where $[\mathbf{D}_{jl}]_{k,k} = \beta_{jlk}^{1/2}$. Then the received signal matrix of pilots and the received signal vector of data for the BS in the jth cell are

$$\mathbf{x}_j = \sqrt{\rho_{\mathrm{p}}} \sum_{l=1}^{L} \mathbf{G}_{jl} \mathbf{\Phi}_l^T + \mathbf{w}_j \in \mathbb{C}^{M \times N_{\mathrm{p}}}, \tag{3}$$

$$\mathbf{y}_j = \sqrt{\rho_{\mathrm{d}}} \sum_{l=1}^{L} \mathbf{G}_{jl} \mathbf{d}_l + \mathbf{v}_j \in \mathbb{C}^{M \times 1} \tag{4}$$

respectively, where ρ_{p} is the transmit power for pilots, and ρ_{d} is the transmit power for data; $\mathbf{w}_j \in \mathbb{C}^{M \times N_{\mathrm{p}}}$ and $\mathbf{v}_j \in \mathbb{C}^{M \times 1}$ are of i.i.d. $\mathcal{CN}(0,1)$ noise variables, and $\mathbf{d}_l \in \mathbb{C}^{K \times 1}$ is of i.i.d. data symbols; $\mathbf{\Phi}_l \in \mathbb{C}^{N_{\mathrm{p}} \times K}$ is of pilots from all the terminals of the lth cell. It is known that the number of pilot symbols is limited by channel coherence interval, thus $N_{\mathrm{p}} < 2K$ is assumed here. In addition, $\mathbf{\Phi}_l$ must satisfy the requirement $\mathbf{\Psi}_{ll} = \mathbf{I}_K$, where $\mathbf{\Psi}_{jl} = \mathbf{\Phi}_l^T \mathbf{\Phi}_j^* \in \mathbb{C}^{K \times K}$ is the correlation matrix between pilots. Consequently, $N_{\mathrm{p}} \geq K$ is required, which means that the number of users per-cell K should be no larger than the pilot length N_{p}. Then, only pilots with length no less than K and less than $2K$ will be designed here. It should be noticed that the specific length of pilots employed is determined by the channel coherence interval and the frequency smoothness interval. Because the received signal vector of data in the forward-link is similar to $\mathbf{y}_j$, only the reverse-link data transmission is analyzed.

Here is the process of reverse-link data detection. First, the BS correlates the received signal vector of pilots with $\mathbf{\Phi}_j^*$ and gets the observation

$$\begin{aligned} \hat{\mathbf{G}}_{jj} &= \mathbf{x}_j \mathbf{\Phi}_j^* \\ &= \sqrt{\rho_{\mathrm{p}}} \sum_{l=1}^{L} \mathbf{G}_{jl} \mathbf{\Psi}_{jl} + \mathbf{w}_j \mathbf{\Phi}_j^*. \end{aligned} \tag{5}$$

Because $\mathbf{\Psi}_{jj} = \mathbf{I}_K$, the observation can be regarded as a channel estimate.

Then, the BS detects the reverse-link data with the matching filter, and the equalized signal vector is

$$\begin{aligned} \mathbf{z}_j &= \frac{1}{M\sqrt{\rho_{\mathrm{p}} \rho_{\mathrm{d}}}} \hat{\mathbf{G}}_{jj}^H \mathbf{y}_j \\ &\approx \mathbf{D}_{jj}^2 \mathbf{d}_j + \sum_{l \neq j} \mathbf{\Psi}_{jl}^H \mathbf{D}_{jl}^2 \mathbf{d}_l, \end{aligned} \tag{6}$$

where the approximation is derived according to the asymptotic results of random matrix theory

$$\frac{1}{M} \mathbf{H}_{jl_1}^H \mathbf{H}_{jl_2} \to \delta(l_1 - l_2) \mathbf{I}_K, \text{ as } M \to \infty. \tag{7}$$

It can be seen that the effects of noise and small scale fading vanish in (6).

In (6), the former part is composed of the desired data and the latter part is composed of ICI. Because the kth element of $\mathbf{z}_j$ is the signal of the kth terminal in the jth cell, the effective

SIR for the reverse-link transmission of the kth terminal in the jth cell is

$$\begin{aligned}\mathrm{SIR}_{jk} &= \frac{E\left\{\left|\beta_{jjk}[\mathbf{d}_j^T]_k\right|^2\right\}}{E\left\{\left|\sum_{l\neq j}\left([\boldsymbol{\Psi}_{jl}]_k\right)^H\mathbf{D}_{jl}^2\mathbf{d}_l\right|^2\right\}} \\ &= \frac{\beta_{jjk}^2}{\sum_{l\neq j}\sum_{m=1}^{K}\left|[\boldsymbol{\Psi}_{jl}]_{m,k}\right|^2\beta_{jlm}^2}. \qquad (8)\end{aligned}$$

It is assumed that the BSs are far apart, which is usually true because the radiuses of most cells are long. Hence, the large scale fading coefficient from the mth terminal in the lth cell to the BS in the jth cell β_{jlm} can be approximated by the mean value $\bar{\beta}_{jl} = (1/K)\sum_{m=1}^{K}\beta_{jlm}$. Then the effective SIR is given by

$$\mathrm{SIR}_{jk} = \frac{\beta_{jjk}^2}{\sum_{l\neq j}\bar{\beta}_{jl}^2\,||[\boldsymbol{\Psi}_{jl}]_k||^2}. \qquad (9)$$

Because $\boldsymbol{\Psi}_{jl}$ is not related with $\bar{\beta}_{jl}$, minimizing $||[\boldsymbol{\Psi}_{jl}]_k||^2$ is necessary for maximizing SIR_{jk}. Because the coefficients $\bar{\beta}_{jl}, l = 1, 2, \cdots, L$ are different and vary with different propagation environments, they cannot be considered in designing pilots. Therefore, the proposed pilot design criterion is

$$\begin{aligned}\min \quad & \sum_{l\neq j}||[\boldsymbol{\Psi}_{jl}]_k||^2 \\ \text{s.t.} \quad & \boldsymbol{\Psi}_{jj} = \mathbf{I}_K, \\ & \forall N_\mathrm{p},\ K \leq N_\mathrm{p} < 2K, \\ & \forall j = 1, 2, \cdots, L,\ k = 1, 2, \cdots, K. \qquad (10)\end{aligned}$$

III. Chu Sequences Based Pilot Design

Chu sequences are employed in designing pilots here for the correlation property of these sequences. As discussed in the introduction, the Chu sequences are designed with different criterions in [8] and [9]. Here, the proposed pilots are constructed in different ways from the pilots in [8] and [9], which will be explained in the following.

A. Chu Sequences for Different Cells

The nth entry of Chu sequence $\mathbf{a} = [a_0, a_1, \cdots, a_{N_\mathrm{p}-1}]$ can be expressed as [10]

$$a_n = \exp\, i\frac{N\pi}{N_\mathrm{p}}n\left(n + (N_\mathrm{p} \bmod 2)\right), n = 0, 1, \cdots, N_\mathrm{p} - 1, \qquad (11)$$

where N_p is the length of the sequence, and N is the unique sequence parameter, which should be an integer that is relatively prime to N_p. The circular autocorrelation function of the sequence $\mathbf{a}$ is defined as

$$r_j = \sum_{n=0}^{N_\mathrm{p}-1} a_n a^*_{(n+j) \bmod N_\mathrm{p}}, j = 0, 1, \cdots, N_\mathrm{p} - 1. \qquad (12)$$

Then the autocorrelation values are $r_0 = N_\mathrm{p}$, and $r_j = 0$ when $j \neq 0$. In addition, a new sequence $\mathbf{b} = [b_0, b_1, \cdots, b_{N_\mathrm{p}-1}]$ with

$$b_n = a_n \exp\, i\frac{2\pi qn}{N_\mathrm{p}}, n = 0, 1, \cdots, N_\mathrm{p} - 1, \qquad (13)$$

is obtained when the entries of sequence $\mathbf{a}$ are shifted with linear phases, where q is an integer that is not exactly divisible by N_p. From (12), it can be seen that sequence $\mathbf{b}$ results in the same autocorrelation values as $\mathbf{a}$.

According to the correlation property, Chu sequences are employed as pilots here. First, the designed pilots are phase shifted between cells, while the pilots in [8] are changed by the unique sequence parameter N between cells, and the pilots in [9] are cyclic shifted between cells. The pilot vector of one terminal in the lth cell is denoted as $\mathbf{s}_l \in \mathbb{C}^{1\times N_\mathrm{p}}$, and the nth entry of $\mathbf{s}_l$ is

$$[\mathbf{s}_l]_n = a_n \exp\, i\frac{2\pi q_l n}{N_\mathrm{p}}, n = 1, 2, \cdots, N_\mathrm{p} - 1, \qquad (14)$$

where $\exp\, i\left(2\pi q_l n/N_\mathrm{p}\right)$ is the phase shift, q_l is an integer that is not exactly divisible by N_p, and differs with l, $l = 1, 2, \cdots, L$. Second, the designed pilots are cyclic shifted for terminals inside a cell. The cyclic shifted vector of pilots with shift length n is

$$\langle\mathbf{s}_l\rangle_n = \left[[\mathbf{s}_l]_n, [\mathbf{s}_l]_{n+1}, \cdots, [\mathbf{s}_l]_{N_\mathrm{p}-1}, [\mathbf{s}_l]_0, [\mathbf{s}_l]_1, \cdots, [\mathbf{s}_l]_{n-1}\right].$$

Then, the pilot matrix of the lth cell can be constructed as

$$\boldsymbol{\Phi}_l = \frac{1}{\sqrt{N_\mathrm{p}}}\left[\left(\langle\mathbf{s}_l\rangle_0\right)^T, \left(\langle\mathbf{s}_l\rangle_1\right)^T, \cdots, \left(\langle\mathbf{s}_l\rangle_{K-1}\right)^T\right]. \qquad (15)$$

Because $N_\mathrm{p} \geq K$, the autocorrelation properties of $\mathbf{a}$ and $\mathbf{b}$ result in $\boldsymbol{\Psi}_{ll} = \mathbf{I}, \forall l$. Hence, the designed pilots satisfy the requirement of the pilots, which is stated below (4). Then the pilots are designed to meet the criterion in (10). With the designed pilot matrix, there is

$$\begin{aligned}[\boldsymbol{\Psi}_{jl}]_{m,k} &= \frac{1}{N_\mathrm{p}}\langle\mathbf{s}_l\rangle_{m-1}\left(\langle\mathbf{s}_j\rangle_{k-1}\right)^H \\ &= \frac{\zeta_{jlmk}}{N_\mathrm{p}}\sum_{n=0}^{N_\mathrm{p}-1}\exp\, i\frac{2\pi}{N_\mathrm{p}}\left(N(m-k) + q_l - q_j\right)^n,\end{aligned}$$

where ζ_{jlmk} is a complex number of unit norm. According to the property of N_pth root of unity, the module of $[\boldsymbol{\Psi}_{jl}]_{m,k}$ is

$$|[\boldsymbol{\Psi}_{jl}]_{m,k}| = \delta\left(\left(N(m-k) + q_l - q_j\right) \bmod N_\mathrm{p}\right). \qquad (16)$$

From the criterion in (10), it is known that the sequence $\mathbf{q} = [q_1, q_2, \cdots, q_L]$ should be designed such that $|[\boldsymbol{\Psi}_{jl}]_{m,k}|$ equals one as less times as possible when m changes from 1 to K, i.e., the best phase shifts should be selected to meet the criterion.

B. Selection of Phase Shifts

From (14), it can be found that there are only N_p different phase shifted sequences. When the entries of $\mathbf{q}$ are in the set $\mathcal{A} = \{0, 1, \cdots, N_\mathrm{p} - 1\}$, all these different sequences can be obtained. Because it is difficult to select $\mathbf{q}$ in $\mathcal{A}$ for the best phase shifts, a proposition is presented below.

Proposition 1: For the phase shifted Chu sequence in (15), when the entries of $\mathbf{q}$ are in the set $\mathcal{B} = \{0, N, 2N, \cdots, (N_\mathrm{p} - 1)N\}$, all the N_p different sequences can be obtained.

Proof: Assume two different values in $\mathcal{B}$ are $q_1 = \lambda_1 N$ and $q_2 = \lambda_2 N$, where $\lambda_1, \lambda_2 \in \mathcal{A}$, and the corresponding Chu sequences are $\mathbf{s}_1$ and $\mathbf{s}_2$. Then the ratio of $[\mathbf{s}_1]_n$ to $[\mathbf{s}_2]_n$ is

$$\frac{[\mathbf{s}_1]_n}{[\mathbf{s}_2]_n} = \exp i\frac{2\pi N}{N_\mathrm{p}}(\lambda_1 - \lambda_2) \neq 1, \tag{17}$$

which is because $\lambda_1 - \lambda_2$ is in the set $\mathcal{C} = \{1 - N_\mathrm{p}, 2 - N_\mathrm{p}, \cdots, N_\mathrm{p} - 1\}\backslash\{0\}$. Therefore, all the sequences can be obtained when the entries of $\mathbf{q}$ are in the set $\mathcal{B}$. ■

Proposition 1 reveals that only the set $\mathcal{B}$ needs to be considered. Hence, q_l can be expressed as

$$q_l = c_l N,\ l = 1, 2, \cdots, L, \tag{18}$$

where $c_l \in \mathcal{A}$ and c_l differs with l. Because N is relatively prime to N_p, (16) can be re-expressed as

$$|[\mathbf{\Psi}_{jl}]_{m,k}| = \delta\left((m - k + c_l - c_j) \bmod N_\mathrm{p}\right). \tag{19}$$

Now the aim is to select the entries of $\mathbf{c} = [c_1, c_2, \cdots, c_L]$ so that (19) equals one with the least times when m changes from 1 to K. A suboptimal sequence $\mathbf{c}$ is derived in the following proposition.

Proposition 2: The best sequence $\mathbf{c}$ is the sequence that makes (19) equal one with the least times when m changes from 1 to K. A suboptimal sequence is $\mathbf{c} = [0, 1, \cdots, \tilde{L}, \tilde{L} + N_\mathrm{p} - K - L + 2, \tilde{L} + N_\mathrm{p} - K - L + 3, \cdots, N_\mathrm{p} - K]$, where $\tilde{L} = \lfloor L/2 \rfloor$.

Proof: From (10), it is known that only the cross-correlation values of the pilots of different cells need to be considered. It is denoted that $\tilde{c}_l = c_{l+1} - c_l \geq 1, l = 1, 2, \cdots, L-1$, and $\tilde{c}_{\max} = c_L - c_1$, then $\tilde{c}_{\max} \geq L-1$ holds. To simplify the analysis, $N_\mathrm{p} > K - 1 + \tilde{c}_{\max}$ is assumed, and the criterion in (10) is changed into minimizing the sum of all the ICI, which is

$$\begin{aligned} \min \quad & \sum_{l\neq j}\sum_{j=1}^{L} ||\mathbf{\Psi}_{jl}||_\mathrm{F}^2 \\ \text{s.t.} \quad & \forall N_\mathrm{p},\ K + L - 1 \leq N_\mathrm{p} < 2K. \end{aligned} \tag{20}$$

Moreover, $||[\mathbf{\Psi}_{jl}]_k||^2$ should not be uniform because the shadow fading coefficients of the terminals are different. Because $N_\mathrm{p} > K - 1 + \tilde{c}_{\max}$ and $N_\mathrm{p} < 2K$ must be satisfied, $\tilde{c}_{\max} \leq K$ holds. Hence, the sum of all the ICI to the channel estimate of the jth cell is

$$\sum_{l\neq j} ||\mathbf{\Psi}_{jl}||_\mathrm{F}^2 = K(L-1) - \sum_{n=1}^{L-1}\sum_{i=1}^{n} \tilde{c}_i. \tag{21}$$

As a result, the sum in (20) is

$$\sum_{l\neq j}\sum_{j=1}^{L} ||\mathbf{\Psi}_{jl}||_\mathrm{F}^2 = KL(L-1) - 2\sum_{i=1}^{L-1} i(L-i)\tilde{c}_i. \tag{22}$$

Because $\tilde{c}_l \geq 1$, the sum is minimized when

$$\tilde{c}_l = \begin{cases} 1, & \text{if } l \neq \tilde{L} \\ \tilde{c}_{\max} - L + 2, & \text{if } l = \tilde{L}, \end{cases} \tag{23}$$

where $\tilde{L} = \lfloor L/2 \rfloor$. Because $N_\mathrm{p} > K - 1 + \tilde{c}_{\max}$ must be satisfied, the minimal number of the pilot symbols is $N_\mathrm{p} = K + \tilde{c}_{\max}$. Hence, the selected sequence is $\mathbf{c} = [0, 1, \cdots, \tilde{L}, \tilde{L} + N_\mathrm{p} - K - L + 2, \tilde{L} + N_\mathrm{p} - K - L + 3, \cdots, N_\mathrm{p} - K]$. ■

Proposition 2 presents the suboptimal sequence $\mathbf{c}$, which corresponds to the phase shifts of the proposed pilots. With the proposed pilots, some of the terminals will not be interfered by terminals of other cells. While for [9], the same orthogonal pilots are reused in all cells, and every terminal will be interfered by other cells. From (22), it can be seen that the sum is much less than $KL(L-1)$, which is the sum of ICI if the pilots in [9] are used. Hence, the pilot contamination is mitigated with the proposed pilots. From (19) and (23), it can also be seen that the cross-correlation values of the pilots are not uniform for different terminals. While the cross-correlation values are uniform with the pilots in [8] and [9], and these pilots are optimal only when the large scale fading coefficients are the same for different terminals. Hence, the proposed pilots fit scenarios of large shadow fading variance.

TABLE I
SYSTEM PARAMETERS

Cell radius (from center to edge)	1600 meters
Decay exponent ν	3.8
Shadow fading standard deviation σ_{shad}	8 dB
Minimal terminal to the BS distance $r_{\min}$	1500 meters
Maximal terminal to the BS distance $r_{\max}$	1600 meters
Number of constant subcarriers N_s	14
Number of symbols for pilots N_c	3
Length of pilot sequence N_p	42
Number of cells L	7
Number of terminals per-cell K	27
Number of BS antennas M	400

IV. NUMERICAL RESULTS

In order to evaluate the proposed pilots, simulations of a LS-MIMO system have been performed. The system consists of seven cells sharing the same band of frequencies, with six cells surround one cell and the cell edges of the six cells overlap the edges of the central cell. In addition, the parameters of the system are given in Table. I. The terminals are distributed uniformly and are static, and are near the cell edges, thus the effect of ICI is more apparent. The shadow fading is modeled with log-normal distribution and is the same between each terminal and all the BSs. More explicitly, the large scale fading coefficient between each terminal and the BSs can be expressed as $\beta_{jlk} = 10^{\tilde{\beta}_{\text{shad}}/10}/(r_{jlk}/100)^\nu$ when the power is measured in watt, where r_{jlk} is the distance between the kth terminal in the jth cell to the BS in the lth cell, $\tilde{\beta}_{\text{shad}}$ is the corresponding coefficient in dB, and is a gaussian random variable with mean zero and variance σ_{shad}^2 ($\mathcal{N}(0, \sigma_{\text{shad}}^2)$). Following [5], the simulations are taken in a noiseless environment, the transmit power for pilots ρ_p and the transmit power for data ρ_d are the same for each terminal, and are set to be one. The data symbols are BPSK modulated. For the pilots in [8], the sequence parameter N in (11) is set to be $1, 5, 11, 17, 23, 25, 31$ respectively for the seven cells, and

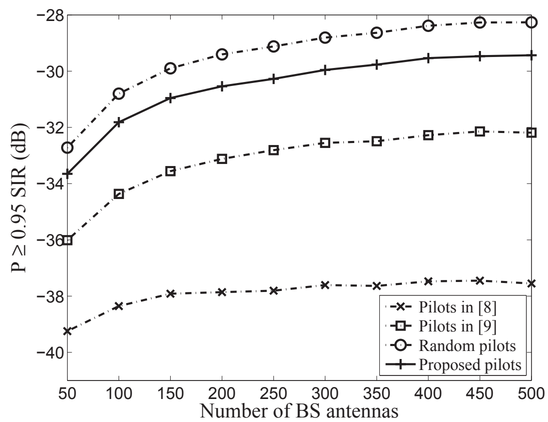

Fig. 2. P ≥ 0.95 SIR versus BS antenna number.

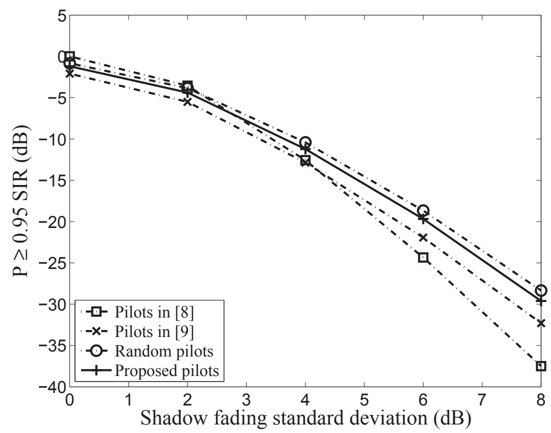

Fig. 3. P ≥ 0.95 SIR versus shadow fading standard deviation.

the same sequence is cyclic shifted for terminals in the same cell. The pilots in [9] for terminals in one cell are the cyclic shifted sequences of the sequence in (11), and are reused in different cells, which are the same as the orthogonal pilots in [5]. For the proposed pilots in (15), the sequence parameter N is set to be one, and the parameters $q_1, q_2, \cdots, q_7$ in (14) are set to be $0, 1, 2, 12, 13, 14, 15$ for the seven cells. The random pilots are similar to the proposed pilots, the only difference is that the parameters $q_1, q_2, \cdots, q_7$ are random variables and are uniformly distributed in the range from 0 to 41.

Fig. 2 shows the .95-likely SIRs of these pilots versus the number of BS antennas. The .95-likely SIRs of these pilots are compared following [5]. Denote the .95-likely SIR as ξ, then it satisfies the equation $\mathrm{P}(\mathrm{SIR} \geq \xi) = 0.95$, where P denotes the probability function. In this figure, the SIRs of all the pilots increase with the number of BS antennas. This is because the channel vectors tend to be orthogonal as the number of antennas increases, and the intra-cell interference decreases. It can be seen that the pilots in [8] perform worst. This is because these pilots result in uniform cross-correlation values, thus only minimize the ICI when the shadow fading coefficients are the same. The proposed pilots perform better than the pilots in [9], which verifies that the proposed pilots eliminates the partial ICI. Moreover, random pilots are orthogonal for terminals in the same cell and are independent between cell. It can be seen that the randomly chosen pilots perform better than the proposed pilots, which means that the phase shifts chosen in the proposed pilots are not optimal.

Fig. 3 shows the .95-likely SIRs of these pilots versus the shadow fading standard deviation. It can be seen that the proposed pilots perform close to the random pilots, which is because these pilots are constructed with the same manner. The performance of the pilots in [8] is best in low standard deviation and worst in high standard deviation. This is because the pilots in [8] result in uniform correlation values, thus the SIRs are uniform with low standard deviation of shadow fading, and the .95 likely SIR is high. The performance of all the pilots degrade with the increasing of the shadow fading standard deviation. This is because the number of terminals with severe fading increases as the shadow fading standard deviation increases.

V. Conclusion

The performance of LS-MIMO systems is mainly affected by pilot contamination. To eliminate this effect, pilots with small cross-correlation values are designed, and the pilot contamination is mitigated. Moreover, the designed pilots fit scenarios with large shadow fading variance. Hence, the proposed pilots increase the SIRs of the received signals, and can increase the spectral efficiency of LS-MIMO systems.

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