Subspace-Based Semi-Blind Channel Estimation for Large-Scale Multi-Cell Multiuser MIMO Systems

Anzhong Hu, Tiejun Lv, and Yueming Lu
Key Laboratory of Trustworthy Distributed Computing and Service, Ministry of Education
School of Information and Communication Engineering
Beijing University of Posts and Telecommunications, Beijing, China 100876
Email: {huanzhong, lvtiejun, ymlu}@bupt.edu.cn

***Abstract*—Large-scale multi-cell multiuser multiple-input multiple-output (LS-MIMO) systems have received much attention recently. But the performance of these systems is deteriorated by imperfect channel state information (CSI). Hence, in this paper, a subspace-based semi-blind channel estimator is proposed. Based on the approximate orthogonality of the channel vectors of LS-MIMO systems, singular value decomposition (SVD) is employed on the received signals to determine the channel matrix up to an ambiguity matrix. Then matrix inversion is avoided in resolving the ambiguity matrix, which is essential to traditional subspace-based estimators and loses the partial CSI. The properties of the estimators are analyzed, and the analysis shows that the estimation accuracy of the proposed estimator is improved. Simulations comparing the proposed approach with others illustrate improvement of the performance of the proposed approach.**

I. Introduction

Because multiuser multiple-input multiple-output (MU-MIMO) systems can provide multiplexing gain and improve system reliability, they have been intensively investigated recently [1]. In large-scale multi-cell MU-MIMO (LS-MIMO) systems, the base station (BS) comprises a hundred or a few hundreds of antennas, and serves tens of users simultaneously [2]. Since the multiplexing gain of MIMO systems is proportional to the number of BS antennas, LS-MIMO systems can achieve higher spectral efficiency. Because the channel vectors between different terminals and the BS are orthogonal in rich scattering environments in LS-MIMO systems, linear signal processing is optimal [3].

However, the performance of LS-MIMO systems is deteriorated by imperfect channel state information (CSI). The imperfect CSI is caused by the channel estimation error [4], [5]. In the estimation process, the BS estimates the channels with the received pilot signals and the known pilots. Because the length of the pilot sequence is proportional to the transmitting interval, which is limited by the channel coherence interval, the pilot sequences of different terminals cannot be orthogonal. When the system is synchronous, the channel estimate of terminals of one cell is contaminated by terminals of other cells. This effect is called pilot contamination [6]. A semi-blind channel estimator has been proposed to improve the channel estimate in LS-MIMO systems [7]. The estimator employs eigenvalue decomposition (EVD) on the received signals to determine the channel matrix depending on an ambiguity matrix, and resolves the diagonal entries of the ambiguity matrix with short pilots. Because the channel vectors are not perfectly orthogonal, the ambiguity matrix is not a diagonal matrix, and is not completely resolved. For traditional systems, the subspace-based semi-blind channel estimators achieve good performance [8], [9]. But the left-null space of the channel matrix has to be utilized without the properties of LS-MIMO systems, thus these estimators either perform poor or are complicated when modified for LS-MIMO systems.

In this paper, a subspace-based semi-blind channel estimator is proposed for LS-MIMO systems, and the properties of the estimator are analyzed. The estimator employs singular value decomposition (SVD) on the received signals. Because of the approximate orthogonality of the channel vectors of LS-MIMO systems, the K left-singular vectors that correspond to the largest K singular values are approximately in the subspace spanned by the channel vectors, where K is the number of simultaneously active terminals of each cell. Then, the matrix that is consisted of these left-singular vectors is approximately expressed as the product of the normalized channel matrix and an ambiguity matrix. The ambiguity matrix is approximately unitary and can be estimated by means of pilots. To be specific, the main contributions of this paper are two-fold. First, the subspace-based semi-blind channel estimator for LS-MIMO systems is presented. The inversion of the estimated ambiguity matrix, which is used in [8], is replaced by its conjugate transpose, thus the CSI is not lost. Meanwhile, the ambiguity matrix is completely resolved, which is only partially resolved in [7]. Second, the properties of the proposed estimator are analyzed. The analysis shows that the proposed estimator is unbiased in the absence of inter-cell interference (ICI), and is consistent when there is no ICI and noise, while the modified estimator in [8] is biased. It is also shown that the mean square error (MSE) of the proposed estimator decreases with the increasing of the number of BS antennas. Hence, the proposed estimator improves the channel estimation accuracy for LS-MIMO systems.

Notations: $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ are the conjugate, transpose, and conjugate transpose respectively; $[\cdot]_i$ is the ith column of

This work is financially supported by the National Natural Science Foundation of China (NSFC) (Grant No. 61271188) and 973 program (No. 2011CB302702).

978-1-4673-6337-2/13/$31.00 ©2013 IEEE

a matrix, and $[\cdot]_{i,j}$ is the (i,j)th entry of a matrix; $[\mathbf{A};\mathbf{B}]$ is the concatenation of the columns of $\mathbf{A}$ and $\mathbf{B}$; $\mathbb{E}\{\cdot\}$ is the expectation; $||\cdot||$ is the norm of a vector; $\mathbf{I}_K$ is the size K identity matrix, and $\mathbf{0}_{M\times K}$ is a zero matrix of $M\times K$; $\delta(\cdot)$ is the Kronecker delta function.

II. System Model

Consider a system with L hexagonal cells that share the same band of frequencies. Every cell adjoins the other $L-1$ cells. The system works in time-division duplex (TDD), the BSs estimate the reverse-link channels by received signals, and acquire the forward-link channels by utilizing the reciprocity of the channels. The BS in each cell is equipped with M antenna elements, and K single antenna terminals are served simultaneously. In accordance with the definition of LS-MIMO systems, $M \gg K$ is assumed in this paper. The channels from terminals of the lth cell to the BS of the jth cell constitute

$$\mathbf{G}_{jl} = \mathbf{H}_{jl}\mathbf{D}_{jl} \in \mathbb{C}^{M\times K}, \tag{1}$$

where $\mathbf{H}_{jl} \in \mathbb{C}^{M\times K}$ is of small scale fading coefficients, and $\mathbf{D}_{jl} \in \mathbb{R}^{K\times K}$ is a diagonal matrix of square roots of large scale fading coefficients. More specifically, the channel from the kth terminal in the lth cell to the mth BS antenna of the jth cell is

$$g_{jlmk} \triangleq [\mathbf{G}_{jl}]_{m,k} = h_{jlmk}\beta_{jlk}^{1/2}, \tag{2}$$

where $h_{jlmk} = [\mathbf{H}_{jl}]_{m,k}$, $\beta_{jlk}^{1/2} = [\mathbf{D}_{jl}]_{k,k}$, and β_{jlk} is the large scale fading coefficient that is composed of path loss and shadow fading. It is assumed that the large scale fading coefficient is constant, independent of the receiving antenna, and known *a priori*. Because the path loss decreases exponentially with distance, (i) $\beta_{jjk_1} \gg \beta_{jlk_2}, \forall l \neq j$, is assumed when terminal k_1 or k_2 is far away from the cell edge, (ii) $\beta_{jjk_1} \approx \beta_{jlk_2}, \forall l \neq j$, is assumed when terminals k_1 and k_2 are close to the cell edge. As the terminals move randomly, $\mu_\beta = \mathbb{E}\{\beta_{jlk}\}, \forall l \neq j, k$, is assumed. The entries of $\mathbf{H}_{jl}$ are independent and identically distributed (i.i.d.) and circularly-symmetric Gaussian random variables with mean zero and variance one ($\mathcal{CN}(0,1)$).

The received signal matrix $\mathbf{Y}_j \in \mathbb{C}^{M\times(N_\mathrm{p}+N_\mathrm{d})}$ can be expressed as

$$\mathbf{Y}_j = \sum_{l=1}^{L}\mathbf{G}_{jl}\mathbf{S}_l^T + \mathbf{N}_j, \tag{3}$$

where $\mathbf{Y}_j = \left[\mathbf{Y}_j^\mathrm{p}, \mathbf{Y}_j^\mathrm{d}\right]$, $\mathbf{Y}_j^\mathrm{p} \in \mathbb{C}^{M\times N_\mathrm{p}}$ and $\mathbf{Y}_j^\mathrm{d} \in \mathbb{C}^{M\times N_\mathrm{d}}$ are matrices of received pilots and data symbols respectively; $\mathbf{S}_l = \sqrt{p_\mathrm{u}}\left[\sqrt{N_\mathrm{p}}\mathbf{\Phi}_l; \mathbf{A}_l\right] \in \mathbb{C}^{(N_\mathrm{p}+N_\mathrm{d})\times K}$ is the matrix of transmitted symbols of the lth cell, where p_u the average transmitted power per symbol, $\mathbf{\Phi}_l \in \mathbb{C}^{N_\mathrm{p}\times K}$ and $\mathbf{A}_l \in \mathbb{C}^{N_\mathrm{d}\times K}$ are matrices of pilots and data symbols of the lth cell; $\mathbf{N}_j = \left[\mathbf{N}_j^\mathrm{p}, \mathbf{N}_j^\mathrm{d}\right] \in \mathbb{C}^{M\times(N_\mathrm{p}+N_\mathrm{d})}$ is the noise matrix, where $\mathbf{N}_j^\mathrm{p} \in \mathbb{C}^{M\times N_\mathrm{p}}$ and $\mathbf{N}_j^\mathrm{d} \in \mathbb{C}^{M\times N_\mathrm{d}}$ are of i.i.d. $\mathcal{CN}(0,1)$ entries. Here, it is assumed that the channel response is constant in transmitting the $N_\mathrm{p}+N_\mathrm{d}$ symbols, and the target is to estimate the channel matrix $\mathbf{H}_{jj}$ with the received signal matrix $\mathbf{Y}_j$. More explicitly, the two parts of $\mathbf{Y}_j$ can be expressed as

$$\mathbf{Y}_j^\mathrm{p} = \sqrt{p_\mathrm{u}N_\mathrm{p}}\sum_{l=1}^{L}\mathbf{G}_{jl}\mathbf{\Phi}_l^T + \mathbf{N}_j^\mathrm{p}, \tag{4}$$

$$\mathbf{Y}_j^\mathrm{d} = \sqrt{p_\mathrm{u}}\sum_{l=1}^{L}\mathbf{G}_{jl}\mathbf{A}_l^T + \mathbf{N}_j^\mathrm{d}. \tag{5}$$

The entries of $\mathbf{A}_l, l=1,2,\cdots,L$, are assumed to be i.i.d. random variables with mean zero and variance one. In addition, it is assumed that the same pilot matrix $\mathbf{\Phi} \in \mathbb{C}^{N_\mathrm{p}\times K}$, which satisfies $\mathbf{\Phi}^H\mathbf{\Phi} = \mathbf{I}_K$, is used in all L cells, thus $N_\mathrm{p} \geq K$ is required. With these assumptions, $p_\mathrm{u}\beta_{jjk}$ has the interpretation of the average of the received signal-to-noise ratio (SNR) for the kth terminal in the jth cell.

III. Subspace-Based Semi-Blind Channel Estimation

The ambiguity is not completely resolved with the semi-blind estimator in [7]. While the subspace-based semi-blind channel estimators in [8] and [9] resolve the ambiguity matrix, they utilize the left-null space of the channel matrix, and either perform poor or are complicated when modified for LS-MIMO systems. Hence, a subspace-based semi-blind channel estimator for LS-MIMO systems is proposed here.

A. Theoretical Investigation

From the asymptotic property of LS-MIMO systems, it is known that

$$\frac{1}{M}\mathbf{H}_{jl_1}^H\mathbf{H}_{jl_2} \to \delta(l_1-l_2)\mathbf{I}_K, \text{ as } M\to\infty. \tag{6}$$

Because the number of BS antennas M is finite, the channel vectors are approximately orthogonal, and can be expressed as

$$\mathbf{H}_{jj} = \tilde{\mathbf{H}}_{jj}\mathbf{\Xi}_j, \tag{7}$$

where $\tilde{\mathbf{H}}_{jj} \in \mathbb{C}^{M\times K}$ satisfies

$$\frac{1}{M}\tilde{\mathbf{H}}_{jj}^H\tilde{\mathbf{H}}_{jj} = \mathbf{I}_K. \tag{8}$$

$\mathbf{\Xi}_j \in \mathbb{C}^{K\times K}$ can be expressed as

$$\mathbf{\Xi}_j = \mathbf{I}_K + \mathbf{\Gamma}_j, \tag{9}$$

where the absolute values of the entries of $\mathbf{\Gamma}_j \in \mathbb{C}^{K\times K}$ are far less than one.

Like other subspace-based semi-blind estimators, the second-order statistics (SOS) of the received symbols are utilized to obtain the channel estimate. The covariance matrix of $\left[\mathbf{Y}_j^\mathrm{d}\right]_n$ is

$$\begin{aligned}\mathbf{R}_{\mathbf{Y}_j^\mathrm{d}} &= \mathbb{E}\left\{\left[\mathbf{Y}_j^\mathrm{d}\right]_n\left(\left[\mathbf{Y}_j^\mathrm{d}\right]_n\right)^H\right\}\\ &= p_\mathrm{u}\sum_{l=1}^{L}\mathbf{H}_{jl}\mathbf{D}_{jl}^2\mathbf{H}_{jl}^H + \mathbf{I}_M\\ &= p_\mathrm{u}\sum_{l=1}^{L}\tilde{\mathbf{H}}_{jl}\mathbf{X}_{jl}^2\tilde{\mathbf{H}}_{jl}^H + \mathbf{I}_M \in \mathbb{R}^{M\times M},\end{aligned} \tag{10}$$

where $\tilde{\mathbf{H}}_{jl} \in \mathbb{C}^{M\times K}$, satisfies

$$\frac{1}{M}\tilde{\mathbf{H}}_{jl}^H\tilde{\mathbf{H}}_{jl'} = \delta(l-l')\mathbf{I}_K, \forall l \neq j, l'. \tag{11}$$

Meanwhile, $\mathbf{X}_{jl} \approx \mathbf{D}_{jl}, \forall l$ can be derived from (6). From the structure of (10), $\mathbf{X}_{jl}^2$ is a normal matrix and its SVD can be expressed as

$$\mathbf{X}_{jl}^2 = \mathbf{V}_l\tilde{\mathbf{\Sigma}}_l\mathbf{V}_l^H, \tag{12}$$

where $\mathbf{V}_l \in \mathbb{C}^{K\times K}$ is composed the K left-singular vectors, and $\mathbf{\Sigma}_l \in \mathbb{R}^{K\times K}$ is a diagonal matrix that is composed of the K singular values. Then $\mathbf{R}_{\mathbf{Y}_j^{\rm d}}$ can be re-expressed as

$$\mathbf{R}_{\mathbf{Y}_j^{\rm d}} = p_{\rm u}\sum_{l=1}^{L}\tilde{\mathbf{H}}_{jl}\mathbf{V}_l\tilde{\mathbf{\Sigma}}_l\mathbf{V}_l^H\tilde{\mathbf{H}}_{jl}^H + \mathbf{I}_M. \tag{13}$$

It can be seen that $\mathbf{R}_{\mathbf{Y}_j^{\rm d}}$ is also a normal matrix and its SVD can be denoted as

$$\mathbf{R}_{\mathbf{Y}_j^{\rm d}} = \mathbf{U}_j\mathbf{\Sigma}_j\mathbf{U}_j^H, \tag{14}$$

where $\mathbf{U}_j \in \mathbb{C}^{M\times M}$ is composed the M left-singular vectors, $\mathbf{\Sigma}_j \in \mathbb{R}^{M\times M}$ is a diagonal matrix that is composed of the M singular values, and these singular values are in descending order from the upper left corner. More explicitly, $\mathbf{U}_j$ is denoted as $\mathbf{U}_j = \left[\mathbf{U}_j^{\rm s}, \mathbf{U}_j^{\rm n}\right]$, where $\mathbf{U}_j^{\rm s} \in \mathbb{C}^{M\times K}$, and $\mathbf{U}_j^{\rm n} \in \mathbb{C}^{M\times(M-K)}$. From (13), it can be seen that the columns of $(1/\sqrt{M})\tilde{\mathbf{H}}_{jl}\mathbf{V}_l, l = 1, 2, \cdots, L$, are the left-singular vectors that correspond to the largest KL singular values of $\mathbf{R}_{\mathbf{Y}_j^{\rm d}}$. Because $\beta_{jjk_1} \gg \beta_{jlk_2}, \forall l \neq j$, holds in most cases, and $\beta_{jjk_1} \approx \beta_{jlk_2}, \forall l \neq j$, holds in other cases, $\mathbf{U}_j^{\rm s}$ can be expressed as

$$\begin{aligned}\mathbf{U}_j^{\rm s} &= \left(\frac{1}{\sqrt{M}}\tilde{\mathbf{H}}_{jj}\mathbf{V}_j + \mathbf{B}_j\right)\mathbf{P}_j \\ &= \frac{1}{\sqrt{M}}\left(\tilde{\mathbf{H}}_{jj} + \mathbf{F}_j\right)\mathbf{V}_j\mathbf{P}_j, \end{aligned} \tag{15}$$

where $\mathbf{B}_j \in \mathbb{C}^{M\times K}$ is approximately a zero matrix, with each nonzero column be the linear combination of the columns of $\tilde{\mathbf{H}}_{jj}\mathbf{V}_j$ and $\tilde{\mathbf{H}}_{jl}\mathbf{V}_l$ for some $l \neq j$. $\mathbf{P}_j \in \mathbb{R}^{K\times K}$ is a permutation matrix, and $\mathbf{F}_j = \sqrt{M}\mathbf{B}_j\mathbf{V}_j^H \in \mathbb{C}^{M\times K}$ corresponds to the ICI in the received data symbols. Then, the columns of $\mathbf{U}_j^{\rm s}$ are approximately in the column space of $\mathbf{H}_{jj}$, and the columns of $\mathbf{U}_j^{\rm n}$ are approximately in the left null space of $\mathbf{H}_{jj}$. For simplicity, $\mathbf{U}_j^{\rm s}$ is re-expressed as

$$\mathbf{U}_j^{\rm s} = \frac{1}{\sqrt{M}}\left(\tilde{\mathbf{H}}_{jj} + \mathbf{F}_j\right)\mathbf{E}_j, \tag{16}$$

where $\mathbf{E}_j = \mathbf{V}_j\mathbf{P}_j \in \mathbb{C}^{K\times K}$ is a unitary matrix. This is an important property of LS-MIMO systems, and will be used in the proposed estimator. Despite $\mathbf{F}_j$, it can be seen that $\mathbf{U}_j^{\rm s}$ determines the channel matrix $\mathbf{H}_{jj}$ depending on the ambiguity matrix

$$\tilde{\mathbf{E}}_j = \mathbf{\Xi}_j^{-1}\mathbf{E}_j \in \mathbb{C}^{K\times K}. \tag{17}$$

Because $\tilde{\mathbf{E}}_j \approx \mathbf{E}_j$, the estimate of $\mathbf{E}_j$ can be used to resolve the ambiguity.

Here, comparisons with other semi-blind estimators are also presented. The semi-blind estimator in [7] employs EVD on $\mathbf{R}_{\mathbf{Y}_j^{\rm d}}$. Because the channel vectors are not perfectly orthogonal, the selected eigenvectors are linear combinations of some columns of $\mathbf{H}_{jl}, l = 1, 2, \cdots, L$. Hence, the ambiguity matrix is not a diagonal matrix, and is not completely resolved. When the semi-blind estimator in [9] is modified for LS-MIMO systems, $\mathbf{R}_{\mathbf{Y}_j^{\rm d}}$ is first decomposed with EVD, then SVD is employed on an $M \times M$ matrix that is composed of some of these eigenvectors to obtain $\mathbf{U}_j^{\rm s}$. Apparently, the modified estimator is more complicated because of the EVD step.

B. Resolving the Ambiguity Matrix

The ambiguity matrix $\tilde{\mathbf{E}}_j$ can only be resolved with pilots. From (4), the pilot-based channel estimate of the jth cell is

$$\begin{aligned}\hat{\mathbf{H}}_{jj}^{\rm p} &= \frac{1}{\sqrt{p_{\rm u}N_{\rm p}}}\mathbf{Y}_j^{\rm p}\mathbf{\Phi}^*\mathbf{D}_{jj}^{-1} &(18)\\ &= \mathbf{H}_{jj} + \mathbf{\Delta}_j, &(19)\end{aligned}$$

where

$$\mathbf{\Delta}_j = \sum_{l\neq j}\left(\mathbf{H}_{jl}\mathbf{D}_{jl} + \frac{1}{\sqrt{p_{\rm u}N_{\rm p}}}\mathbf{N}_j^{\rm p}\mathbf{\Phi}^*\right)\mathbf{D}_{jj}^{-1} \in \mathbb{C}^{M\times K} \tag{20}$$

corresponds to the sum of the ICI and the noise. From (16) and (19), the estimate of $\mathbf{E}_j$ is

$$\begin{aligned}\hat{\mathbf{E}}_j &= \frac{1}{\sqrt{M}}\left(\hat{\mathbf{H}}_{jj}^{\rm p}\right)^H\mathbf{U}_j^{\rm s} \\ &= \mathbf{E}_j + \left(\mathbf{\Gamma}_j^H + \mathbf{\Upsilon}_j\right)\mathbf{E}_j, \end{aligned} \tag{21}$$

where

$$\mathbf{\Upsilon}_j = \frac{1}{M}\mathbf{\Delta}_j^H\left(\tilde{\mathbf{H}}_{jj} + \mathbf{F}_j\right) + \frac{1}{M}\mathbf{H}_{jj}^H\mathbf{F}_j \in \mathbb{C}^{K\times K} \tag{22}$$

corresponds to the sum of the ICI in the received symbols and the noise in the received pilots. After that, $\hat{\mathbf{E}}_j^H$ is used to resolve the ambiguity matrix in the proposed semi-blind channel estimator, and the channel estimate of the jth cell is

$$\begin{aligned}\hat{\mathbf{H}}_{jj}^{\rm s} &= \sqrt{M}\mathbf{U}_j^{\rm s}\hat{\mathbf{E}}_j^H \\ &= \mathbf{U}_j^{\rm s}\left(\mathbf{U}_j^{\rm s}\right)^H\hat{\mathbf{H}}_{jj}^{\rm p} &(23)\\ &= \mathbf{H}_{jj} + \frac{1}{M}\tilde{\mathbf{H}}_{jj}\tilde{\mathbf{H}}_{jj}^H\mathbf{\Delta}_j + \tilde{\mathbf{F}}_j, &(24)\end{aligned}$$

where

$$\begin{aligned}\tilde{\mathbf{F}}_j &= \frac{1}{M}\left[\left(\tilde{\mathbf{H}}_{jj} + \mathbf{F}_j\right)\mathbf{F}_j^H\left(\mathbf{H}_{jj} + \mathbf{\Delta}_j\right)\right] \\ &+ \mathbf{F}_j\left(\frac{1}{M}\tilde{\mathbf{H}}_{jj}^H\mathbf{\Delta}_j + \mathbf{\Xi}_j\right) \in \mathbb{C}^{M\times K} \end{aligned} \tag{25}$$

is caused by the ICI in the received data symbols.

On the other hand, it can be noticed that $\hat{\mathbf{E}}_j^{-1}$ can also be used to resolve the ambiguity matrix, and the corresponding channel estimate is

$$\begin{aligned}\hat{\mathbf{H}}_{jj}^{\rm n} &= \sqrt{M}\mathbf{U}_j^{\rm s}\hat{\mathbf{E}}_j^{-1} \\ &= \left(\mathbf{H}_{jj} + \mathbf{F}_j\mathbf{\Xi}_j\right)\left(\mathbf{\Xi}_j + \mathbf{\Gamma}_j^H\mathbf{\Xi}_j + \mathbf{\Upsilon}_j\mathbf{\Xi}_j\right)^{-1}. \end{aligned} \tag{26}$$

It can be proved (the proof is omitted due to space constraints) that $\hat{\mathbf{H}}_{jj}^{\mathrm{n}}$ is obtained when the subspace-based semi-blind estimators in [8] is modified for LS-MIMO systems. With the modified estimator in [8], the channel estimate is obtained with $\mathbf{U}_j^{\mathrm{n}}$, which is approximately in the left null space of $\mathbf{H}_{jj}$.

C. Implementation of the Proposed Semi-Blind Estimator

In practice, $\mathbf{R}_{\mathbf{Y}_j^{\mathrm{d}}}$ is not available, and is replaced by the sample covariance matrix

$$\hat{\mathbf{R}}_{\mathbf{Y}_j^{\mathrm{d}}} = \frac{1}{N_{\mathrm{d}}} \sum_{n=1}^{N_{\mathrm{d}}} \left[\mathbf{Y}_j^{\mathrm{d}}\right]_n \left(\left[\mathbf{Y}_j^{\mathrm{d}}\right]_n\right)^H. \tag{27}$$

According to the strong law of large numbers, when the number of the data symbols N_{d} increases, the average matrix $\hat{\mathbf{R}}_{\mathbf{Y}_j^{\mathrm{d}}}$ converges almost surely to the expectation $\mathbf{R}_{\mathbf{Y}_j^{\mathrm{d}}}$. In this paper, it is assumed that N_{d} is large enough so that the average matrix $\hat{\mathbf{R}}_{\mathbf{Y}_j^{\mathrm{d}}}$ approximates the expectation matrix $\mathbf{R}_{\mathbf{Y}_j^{\mathrm{d}}}$. Then the proposed estimation algorithm is presented as follows.

Algorithm 1: Subspace-Based Semi-Blind Channel Estimation for LS-MIMO Systems

Step 1) Compute the average covariance matrix $\hat{\mathbf{R}}_{\mathbf{Y}_j^{\mathrm{d}}}$ as (27).

Step 2) Perform SVD decomposition of $\hat{\mathbf{R}}_{\mathbf{Y}_j^{\mathrm{d}}}$ as (14).

Step 3) Find the K left-singular vectors that correspond to the largest K singular values of $\hat{\mathbf{R}}_{\mathbf{Y}_j^{\mathrm{d}}}$, which are further used to form the columns of $\mathbf{U}_j^{\mathrm{s}}$.

Step 4) Compute the pilot-based channel estimate $\hat{\mathbf{H}}_{jj}^{\mathrm{p}}$ as (18).

Step 5) Obtain the channel estimate $\hat{\mathbf{H}}_{jj}^{\mathrm{s}}$ as (23).

IV. Properties of the Subspace-Based Semi-Blind Estimators

The unbiasedness and the consistency are two important properties of an estimator and are analyzed here. Meanwhile, the MSE of the channel estimate is analyzed.

A. Unbiasedness and Consistency of the Estimator

From (20) and (24), it is known that the bias matrix of the proposed estimator is

$$\begin{aligned} \mathbb{B}\left\{\hat{\mathbf{H}}_{jj}^{\mathrm{s}}\right\} &= \mathbb{E}\left\{\hat{\mathbf{H}}_{jj}^{\mathrm{s}}\right\} - \mathbf{H}_{jj} \\ &= \frac{1}{M\sqrt{p_{\mathrm{u}} N_{\mathrm{p}}}} \tilde{\mathbf{H}}_{jj} \tilde{\mathbf{H}}_{jj}^H \mathbb{E}\left\{\mathbf{N}_j^{\mathrm{p}}\right\} \mathbf{\Phi}_j^* \mathbf{D}_{jj}^{-1} \\ &= \mathbf{0}_{M\times K} \end{aligned}$$

when the ICI is eliminated. Hence, the proposed estimator is unbiased in an ICI-free environment. When the number of the data symbols N_{d} tends to infinity, $\hat{\mathbf{R}}_{\mathbf{Y}_j^{\mathrm{d}}}$ converges to $\mathbf{R}_{\mathbf{Y}_j^{\mathrm{d}}}$, and $\hat{\mathbf{H}}_{jj}^{\mathrm{s}} = \mathbf{H}_{jj}$ in the absence ICI and noise. Hence, the proposed estimator is consistent in a noiseless and ICI-free environment.

On the other hand, the bias matrix of the modified estimator in [8] is

$$\begin{aligned} \mathbb{B}\left\{\hat{\mathbf{H}}_{jj}^{\mathrm{n}}\right\} &= \mathbb{E}\left\{\hat{\mathbf{H}}_{jj}^{\mathrm{n}}\right\} - \mathbf{H}_{jj} \\ &= \mathbf{H}_{jj}\left(\mathbf{\Xi}_j + \mathbf{\Gamma}_j^H \mathbf{\Xi}_j\right)^{-1} - \mathbf{H}_{jj} \end{aligned}$$

in the absence ICI and noise. Because $\left(\mathbf{\Xi}_j + \mathbf{\Gamma}_j^H \mathbf{\Xi}_j\right)$ is not an identity matrix, and the entries are not random variables, this estimator is biased in a noiseless and ICI-free environment. The conclusion is that the matrix inversion in (26) loses the partial CSI, and the modified estimator in [8] is biased. When the number of BS antennas M increases, the channel vectors tends to be orthogonal, and the bias of this estimator decreases.

B. MSE Analysis

A sensible measure of the channel estimation error is the MSE, which is no less than the Cramér-Rao Bound (CRB). Hence, the MSE of the proposed estimator is analyzed here. The estimation error matrix of the proposed estimator is

$$\begin{aligned} \tilde{\mathbf{H}}_{jj}^{\mathrm{s}} &= \hat{\mathbf{H}}_{jj}^{\mathrm{s}} - \mathbf{H}_{jj} \\ &= \frac{1}{M} \tilde{\mathbf{H}}_{jj} \tilde{\mathbf{H}}_{jj}^H \mathbf{\Delta}_j + \tilde{\mathbf{F}}_j. \end{aligned} \tag{28}$$

By substituting (20) into (28), it can be derived that the MSE of the estimate $\left[\hat{\mathbf{H}}_{jj}^{\mathrm{s}}\right]_{m,n}$ is

$$\begin{aligned} \mathbb{E}\left\{\left|\left[\tilde{\mathbf{H}}_{jj}^{\mathrm{s}}\right]_{m,n}\right|^2\right\} = \left[\mathbf{R}_{\tilde{\mathbf{F}}_j}\right]_{m,n} + \frac{\left|\left|\left[\tilde{\mathbf{H}}_{jj}^T\right]_m\right|\right|^2}{M} \\ \times\left(p_{\mathrm{u}}^{-1}\beta_{jjn}^{-1}N_{\mathrm{p}}^{-1} + \beta_{jjn}^{-1}\mu_\beta \sum_{l\neq j}\left|\left|\left[\mathbf{\Psi}_{jl}\right]_n\right|\right|^2\right), \end{aligned}$$

where $\mathbf{R}_{\tilde{\mathbf{F}}_j} \in \mathbb{R}^{M\times K}$ can be expressed as

$$\mathbf{R}_{\tilde{\mathbf{F}}_j} = \mathbb{E}\left\{\tilde{\mathbf{F}}_j \odot \tilde{\mathbf{F}}_j^* + 2\ \mathrm{Re}\left(\left((1/M)\tilde{\mathbf{H}}_{jj}\tilde{\mathbf{H}}_{jj}^H \mathbf{\Delta}_j\right) \odot \tilde{\mathbf{F}}_j^*\right)\right\},$$

where $\mathrm{Re}(\cdot)$ is the real part of a matrix, and $\odot$ is the Hadamard product.

The ICI in the received data symbols corresponds to $\mathbf{R}_{\tilde{\mathbf{F}}_j}$, and the ICI in the received pilots corresponds to $\mathbf{\Psi}_{jl}$. It can be seen that the MSE decreases with the increasing of the number of BS antennas M, while not in the pilot-based estimator. Hence, when the entries of $\mathbf{R}_{\tilde{\mathbf{F}}_j}$ are small enough, the remaining ICI in the estimate $\hat{\mathbf{H}}_{jj}^{\mathrm{s}}$ is less than that in the pilot-based estimator in LS-MIMO systems.

V. Numerical Results

A system with 3 neighboring hexagonal cells is considered in the simulation. Every cell adjoins the other two cells. The cell radius (from center to edge) is 800 meters, the decay exponent is 3.8, and the shadow fading is not considered. The BS of each cell serves 3 terminals simultaneously, the terminals are distributed uniformly at random in the cell, and the distance from any terminal to the serving BS is no less than 100 meters and no larger than 800 meters. The number of pilots per terminal is 3, and the BPSK modulation is used. The

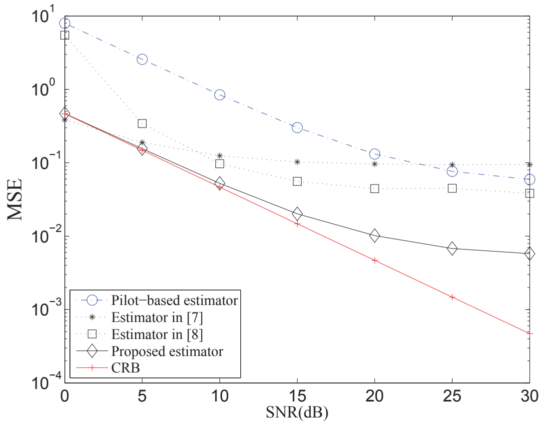

Fig. 1. MSE versus γ for $M = 100$, $K = 3$, and $N_{\mathrm{d}} = 100$.

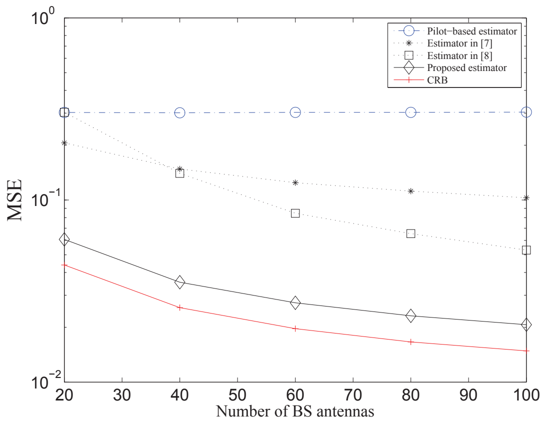

Fig. 2. MSE versus M for $K = 3$, $N_{\mathrm{d}} = 100$, and $\tilde{\gamma} = 15$ dB.

MSE of each estimator is the mean of the estimation errors of all the entries of the channel matrices $\mathbf{H}_{jj}, j = 1, 2, \cdots, L$.

Fig. 1 shows the MSE versus the received SNR $\gamma = p_{\mathrm{u}}\beta M$ at the BS for different estimators and the CRB (the derivation is omitted due to space constraints), where β is the large scale fading coefficient from the cell edge to the corresponding BS. In this figure, the MSE of the proposed estimator is much less than the MSEs of other estimators, and is close to the CRB. The results verify the analysis, which shows that the proposed estimator outperforms the modified estimator in [8] and the estimator in [7].

Fig. 2 shows the MSE versus the number of BS antennas M for different estimators and the CRB. Here $\tilde{\gamma} = p_{\mathrm{u}}\beta M_{\max}$ is defined, where $M_{\max} = 100$ is the maximal value of M in the simulation. In this figure, the MSEs of the estimators in [7], [8] and the proposed estimator decreases when M increases, which verifies the MSE analysis of the proposed estimator. The performance of the estimator in [8] can be explained by the bias analysis, which shows that this estimator is biased because of the nonorthogonality part of the channel vectors.

Fig. 3 shows the MSE versus the number of the data symbols N_{d} for different estimators and the CRB. In this figure, the MSEs of these semi-blind estimators decrease with the increasing of N_{d}. The results verify the analysis of the implementation method. When the number of the data symbols N_{d} increases, the average of the sample covariance matrix $\hat{\mathbf{R}}_{\mathbf{Y}_j^{\mathrm{d}}}$ converges to the covariance matrix $\mathbf{R}_{\mathbf{Y}_j^{\mathrm{d}}}$, and the performance of these estimators is improved.

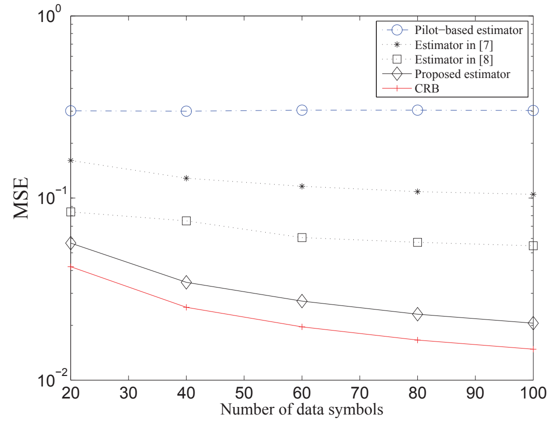

Fig. 3. MSE versus N_{d} for $K = 3$, $M = 100$, and $\gamma = 10$ dB.

VI. Conclusion

A subspace-based semi-blind channel estimator for LS-MIMO systems is proposed. The estimator takes advantage of the approximate orthogonality of the channel vectors to resolve the ambiguity completely without losing the CSI. The performance of the proposed estimator is much better than that of traditional semi-blind estimators, and is close to the CRB.

References

[1] D. Gesbert, M. Kountouris, R. W. Heath, C.-B. Chae, and T. Sälzer, "Shifting the MIMO paradigm," *IEEE Signal Processing Mag.*, vol. 24, no. 5, pp. 36–46, Sep. 2007.

[2] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser MIMO systems," submitted for publication.

[3] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Processing Mag.*, to be published.

[4] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 3590–3600, Nov. 2010.

[5] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, "Analysis of the pilot contamination effect in very large multicell multiuser MIMO systems for physical channel models," in *Proc. IEEE Int. Conf. Acoustics, Speech and Signal Processing (ICASSP 2011)*, Prague, Czech Repulic, 2011, pp. 3464–3467.

[6] J. Jose, A. Ashikhmin, T. L. Marzetta, and S. Vishwanath, "Pilot contamination problem in multi-cell TDD systems," in *Proc. IEEE Int. Symp. Information Theory (ISIT 2009)*, Seoul, South Korea, 2009, pp. 2184–2188.

[7] H. Q. Ngo and E. G. Larsson, "EVD-based channel estimation in multicell multiuser MIMO systems with very large antenna arrays," in *Proc. IEEE Int. Conf. Acoustics, Speech and Signal Processing (ICASSP 2012)*, Kyoto, Japan, 2012, pp. 3249–3252.

[8] B. Muquet, M. de Courville, and P. Duhamel, "Subspace-based blind and semi-blind channel estimation for OFDM systems," *IEEE Trans. Signal Process.*, vol. 50, pp. 1699–1712, Jul. 2002.

[9] Y. Zeng and T.-S. Ng, "A semi-blind channel estimation method for multiuser multiantenna OFDM systems," *IEEE Trans. Signal Process.*, vol. 52, pp. 1419–1429, May 2004.