Theorem: Let A and B be matrices. $(AB)^T = B^T A^T$.

Proof:

First observe that the ij entry of AB can be written as

$$(AB)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}.$$

Furthermore, if we transpose a matrix we switch the rows and the columns. These facts together mean that we can write

$$\left((AB)^T\right)_{ij} = (AB)_{ji} = \sum_{k=1}^{n} a_{jk} b_{ki}$$

and

$$(B^T A^T)_{ij} = \sum_{k=1}^{n} (B^T)_{ik} (A^T)_{kj} = \sum_{k=1}^{n} b_{ki} a_{jk}.$$

From here is is clear that the ij entry of the left and right sides are equal. Therefore the matrices are equal.