Linear Algebra - MTH 215 - Spring 2002

Section 1.3 - The proof that the transpose of a product is the product of the transposes in reverse order

Theorem: If A is m by n and B is m by n then

$$(AB)^T = B^T A^T$$

proof: We will show that for any row i and column j, the i,j-entry on the LHS is equal to the i,j-entry on the RHS.

The i,j-entry of the LHS is $(AB)^T_{i,j}$. It is the same as the j,i-entry of AB. That is,

$$(AB)^T_{i,j} = (AB)_{j,i}$$

The j,i-entry of AB is (row j of A) dot (column i of B).

On the other hand, the i,j-entry of the RHS is the i,j-entry of the product $B^T A^T$. This can be expressed as (row i of B^T) dot (column j of A^T). Which is the same as saying (column i of B) dot (row j of A).

So we see that we expressed the i,j-entries of each side as the dot product of the same two vectors. Therefore, the LHS = RHS. ♡