invertible, then AB is invertible and (AB)^-1 =B^-1 A^-1?

This maths homework is hard! How to prove? Thanks :)

Update: If AB is invertible, is it true that both A and B are invertible?

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Answers

Relevance

Best Answer: Suppose that A and B are both invertible.

Then, since A is invertible, then there exists a matrix A^-1 such that AA^-1 = I and A^-1A = I, where I is the identity matrix.

Similarly, since B is invertible, then there exists a matrix B^-1 such that BB^-1 = I and B^-1B = I.

To show that AB is invertible, all that one has to do is to demonstrate that it has an inverse; that is, we must exhibit a matrix C such that (AB)C = I, and C(AB) = I.

Selecting B^-1A^-1 to be the matrix C works, because

(AB)(B^-1A^-1) = A(BB^-1)A^-1 = A(I)A^-1 = (AI)A^-1 = AA^-1 = I; and
(B^-1A^-1)AB = B^-1(A^-1A)B = B^-1(I)B = (B^-1I)B = B^-1B = I.

(Here, I've used the facts that matrix multiplication is associative; that a matrix multiplied by its inverse yields the identity matrix; and that the identity matrix times any matrix is the other matrix.)

So AB is invertible and B^-1A^-1 is the inverse of AB; in other words, B^-1A^-1 = (AB)^-1.

If AB is invertible, then yes, it will be true that both A and B are invertible.

Suppose that AB is invertible. Then there exists a matrix C such that (AB)C = I and C(AB) = I.

Since (AB)C = I and matrix multiplication is associative, then A(BC) = I. Thus, BC is an inverse for the matrix A. (To show that a square matrix has an inverse, it is enough to show that the multiplication works on one side only--if it does, then it will always work the other way also. That is to say, in this case, knowing that A(BC) = I, we can also conclude that (BC)A = I.)

Also, since C(AB) = I, then (CA)B = I. Thus, CA is an inverse for the matrix B.

Since A and B both have inverses, then both are invertible.

TheMathemagician · 6 years ago

1 0 Comment

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The proof that if A and B are invertible, then AB is invertible can be done more elegantly if you know these two results:

I. Det (AB) = (det (A))*(det(B)).
II. A matrix B is invertible if and only if det(B) =/ 0.

Proof. Suppose that both A and B are invertible. Then det(A) =/ 0 and det(B) =/ 0. Now by I, det(AB) =/ 0, so by II AB is invertible.

Tony · 6 years ago

0 0 Comment

(AB)(B^-!A^-1)=
A(BB^-1)A^-1=
AIA^-1=AA^-1=I

Reza · 6 years ago

0 0 Comment

Matrices: Prove that A and B are both invertible, then AB is invertible and (AB)^-1 =B^-1 A^-1?

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Prove that matrix B is invertible?

Proof: Let A, B be mxn and nxm matrices. Prove that I_m-AB is invertible iff I_n-BA is invertible?

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