

A detailed wireframe model of a particle accelerator complex. It features a large, roughly oval-shaped main ring with a complex internal structure. Several smaller rings and connecting sections are visible, particularly at the top of the main ring. The entire structure is rendered in a light gray wireframe style, showing the intricate geometry of the accelerator.

Lecture 13: Wrap-up Numerical Methods of Accelerator Physics

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Learning objects for lecture series:

- basic models of accelerator physics
- suitable methods for their implementation

Topics covered:

1. **reduced models:** steering, focusing, acceleration
2. **maps** of linear periodic systems + **stability analysis**
3. beam **transport models:** particles, beam distributions (self-consistent modelling)
4. **control-room** applications and **diagnostics:**
 - tune reconstruction
 - tomographic reconstruction of phase space
 - closed-orbit control

- **30 min** oral exam in TEMF library on **14 April 2023**
(check your TUCaN for the time slot or ask me)
- format = conceptual discussion on models of accelerator physics and numerical implementations
- exam material = Σ summary slides
- ⚠ you do **not** need to know how to write python code –
focus on the ideas and concepts we discussed!

- I. basic concepts: [lectures 1-3](#)
- II. longitudinal dynamics: [lectures 4-6](#)
- III. transverse dynamics: [lectures 7-8](#)
- IV. applications:

I. basic concepts: lectures 1-3

- ⇒ using the simple pendulum as example
- ⇒ time scales in a synchrotron
(transverse / longitudinal motion period, storage times)
- ⇒ phase space (system state), Hamiltonian (equations of motion)
- ⇒ discrete integrators: Euler, Euler-Cromer, leapfrog
- ⇒ statistical moments, emittance
- ⇒ non-linearities, Liouville theorem vs. filamentation (emittance growth)
- ⇒ discrete frequency analysis, NAFF algorithm (vs. FFT)
- ⇒ control of simulation error sources:
 1. discretisation error (symplecticity!)
 2. modelling error
 3. numerical artefacts
 4. (input error)
- ⇒ deterministic chaos,
early indicators: (max.) Lyapunov exponent, frequency map analysis

I. basic concepts: lectures 1-3

II. longitudinal dynamics: lectures 4-6

- Lorentz force, electric longitudinal field E_z to accelerate
- beam rigidity, paraxial approximation
- momentum compaction, phase slippage, transition energy
- phase focusing and stability (classical vs. relativistic regime)
- longitudinal tracking equations (discrete one-turn map)
- synchrotron Hamiltonian, rf bucket
- Monte-Carlo sampling (random number generation)
- equilibrium distributions (thermal PDF),
small-amplitude approximation vs. nonlinear matching
- emittance growth mechanisms (filamentation ↔ bucket non-linearity)
from dipole and quadrupole moment oscillations

- I. basic concepts: **lectures 1-3**
- II. longitudinal dynamics: **lectures 4-6**
- III. transverse dynamics: **lectures 7-8**
 - magnetic fields for bending (steering) and focusing
 - multipole representation, dipole / quadrupole / sextupole magnets
 - Hill differential equation, quasi-harmonic oscillation
 - betatron transport matrices
 - FODO cell, alternate-gradient focusing
 - optics / Twiss functions, β -function as beam envelope, dispersion function
 - stability of periodic transport maps
 - betatron tune, chromaticity

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- II. longitudinal dynamics: [lectures 4-6](#)
- III. transverse dynamics: [lectures 7-8](#)
- IV. applications:
 - longitudinal phase-space tomography ([lecture 9](#))
 - Radon transform, sinogram, Fourier Slice Theorem
 - filtered back projection vs. algebraic reconstruction technique
 - closed orbit distortion ([lecture 10](#))
 - local orbit correction (bumps)
 - global orbit correction (orbit response matrix, SVD)
 - self-consistent modelling / collective effects ([lecture 12](#))
 - categories of beam interactions (space charge, ...)
 - longitudinal space charge, line density derivative λ' model
 - microwave instability
 - machine learning ([lectures 11, 14](#))
 - reinforcement learning (discrete vs. continuous state/action spaces)
 - Bayesian optimisation (Gaussian processes, uncertainty modelling)

**You have now solid fundamental knowledge
on numerical modelling of periodic physics +
have seen in action a few dynamical
examples from accelerator physics!**

... and perhaps became a happy python user.

Well done! :))