CSE 211: Discrete Mathematics
Homework 1

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1. (10 points) Let p and q be the propositions.
   p : She speaks English. q: She speaks Turkish.
   Give a simple verbal sentence which describes each of the following:

   (a) $(p \land q)$. : *She speaks English and Turkish.*

   (b) $(p \lor q)$. : *She speaks English or Turkish.*

   (c) $(p \land \neg q)$. : *She speaks English and doesn't speak Turkish.*

   (d) $(p \leftrightarrow q)$. : *She speaks English if and only if she speaks Turkish.*

   (e) $(p \lor q) \land (p \to \neg q)$. : *She speaks English or Turkish, and if she speaks English then she doesn't speak Turkish*

2. (10 points) Show whether the following propositions are logically equivalent to $p \to q$.

   (a) $q \to p$.

   One counter example will be enough to show that $p \to q$ and $q \to p$ are not logically equivalent to each other:

p	q	$p \to q$	$q \to p$
F	T	T	F

   (b) $\neg p \to \neg q$.

   Again, a counter example:

p	q	$p \to q$	$\neg p \to \neg q$
T	F	F	T

   (c) $\neg q \to \neg p$. (1)

   Truth Table:

p	q	$p \to q$	$\neg q \to \neg p$
T	T	T	T
F	F	T	T
T	F	F	F
F	T	T	T

   Hence, $p \to q \equiv \neg q \to p$.

(d) $\neg p \vee q$.

Truth Table:

p	q	$p \to q$	$\neg p \vee q$
T	T	T	T
F	F	T	T
T	F	F	F
F	T	T	T

Hence, $p \to q \equiv \neg p \vee q$. **(1)**

(e) $\neg(p \wedge \neg q)$.

$\neg(p \wedge \neg q) \equiv \neg p \vee q$. *(De Morgan's Law)*
$p \to q \equiv \neg p \vee q$. **(1)**

Hence, $\neg(p \wedge \neg q) \equiv p \to q$

3. (10 points) Construct truth tables for the following and determine whether each of the following is a tautology or neither.

(a) $[p \to (q \wedge r)] \leftrightarrow [(p \to q) \wedge (p \to r)]$.

Truth Table:

p	q	r	$[p \to (q \wedge r)]$	$[(p \to q) \wedge (p \to r)]$	$[p \to (q \wedge r)] \leftrightarrow [(p \to q) \wedge (p \to r)]$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	F	T	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T
F	T	T	T	T	T

Tautology.

(b) $(a \vee (b \bigoplus c)) \vee (c \to b)$.

Truth Table:

a	b	c	$(a \vee (b \bigoplus c))$	$(c \rightarrow b)$	$(a \vee (b \bigoplus c)) \vee (c \rightarrow b)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	F	T
T	F	F	T	T	T
F	F	T	T	F	T
F	T	F	T	T	T
F	F	F	F	T	T
F	T	T	F	T	T

Tautology.

4. (10 points) Write these propositions using p, q and r and logical connectives. Test the validity of the following argument using truth table.
   p: Tom is a singer. q: Tom is a footballer. r: Tom has good voice.
   Tom is either a singer or a footballer. If he is a singer then he has good voice. Tom does not have good voice so he is a footballer.

   Proposition: $(p \bigoplus q) \wedge (p \rightarrow r) \wedge (\neg r \rightarrow q)$

   Truth Table:

p	q	r	$(p \bigoplus q)$	$(p \rightarrow r)$	$(\neg r \rightarrow q)$	$(p \bigoplus q) \wedge (p \rightarrow r) \wedge (\neg r \rightarrow q)$
T	T	T	F	T	T	F
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	F	F
F	F	T	F	T	F	F
F	T	F	T	T	T	T
F	F	F	F	T	F	F
F	T	T	T	T	T	T

5. (10 points) Show, without using truth table, if $(d \vee (a \wedge c \wedge d)) \wedge ((a \wedge b \wedge \neg c) \vee a \vee (a \wedge b))$ is logically equivalent to $(a \wedge d)$. Explain why (simplify and use the laws of logic).

   $(d \vee (a \wedge c \wedge d)) \equiv$d Absorption law
   $(a \wedge b \wedge \neg c) \vee a \equiv$a Absorption Law
   $(a \vee (a \wedge b)) \equiv (d \wedge a)$ Absorption law

6. (6 points) $A = \{1, 2, 3, 4\}, B = \{3, 4, 5\}, X = \{a, b\}, Y = \{b, c, d\}$. List the elements of each of the following sets.

   (a) $(A \times X) \cap (B \times Y) = \{(3, b), (4, b)\}$

(b) $(A \cap X) \times Y. = \{b, c, d\}$

(c) $(A \times X) \cup (B \times Y) = \{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b),(4,a),(4,b),(3,c),(3,d),(4,c)$ $,(4,d),(5,b),(5,c),(5,d)\}$

7. (4 points) Find the power set $P(E)$ of $E = [\{a,b,c\},\{b,c\},\{1,2\}]$.

$E = \{a,b,c\} \cup \{b,c\} \cup \{1,2\}$
$E = \{a,b,c,1,2\}$

$P(E) = [\{a\},\{b\},\{c\},\{1\},\{2\},\{a,b\},\{a,c\},\{c,b\},\{a,1\},\{a,2\},\{b,1\},\{b,2\},$
$\{c,1\},\{c,2\},\{1,2\},\{a,b,c,1\},\{a,b,c,2\},\{2,b,c,1\},\{a,2,c,1\},\{a,b,2,1\}$
$\{a,b,c,1,2\},\{\}]$

8. (15 points) Prove by mathematical induction that for all positive integers $n \geq 1$, $4^{2n+1} + 3^{n+2}$ is divisible by 13.

$a_1 \equiv \ 0 \ mod(13)$

$a_n = 4^{2n+1} + 3^{n+2} \rightarrow a_{n+1} = 4^{2n+3} + 3^{n+3}$

We assume the proposition is true, hence:

$m, n \in \mathbf{N}$

$a_n = 13.m \Rightarrow 13.m = 4.4^n + 9.3^n \Rightarrow 3.13.m = 12.4^n + 27.3^n$

$a_{n+1} = 13.n = 64.4^n + 27.3^n$

$a_{n+1} - a_n \Rightarrow 13(n-m) = 52.2^n \Rightarrow 13(n-m) - 51.2^n \equiv 0 \ mod(13)$

9. (15 points) Use mathematical induction to prove that for $n \geq 1$

$$\sum_{1 \leq x \leq n} \frac{1}{x(x+1)} = 1 - \frac{1}{(n+1)}$$

$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$

$$\sum_{1 \leq x \leq n} \frac{1}{x} - \frac{1}{x+1} = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + ... - \frac{1}{n+1}$$

$$\sum_{1 \leq x \leq n} \frac{1}{x} - \frac{1}{x+1} = 1 - \frac{1}{n+1}$$

$$\sum_{1 \leq x \leq n} \frac{1}{x(x+1)} = 1 - \frac{1}{(n+1)}$$