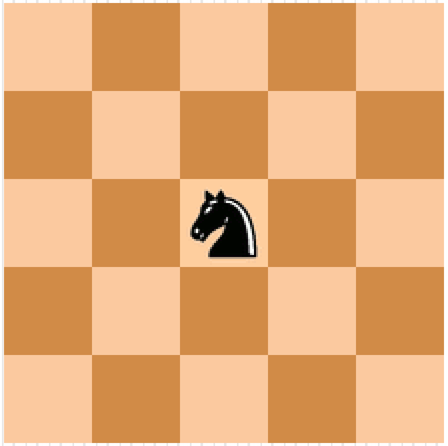
Knight’s Tour



Kelum Senanayake

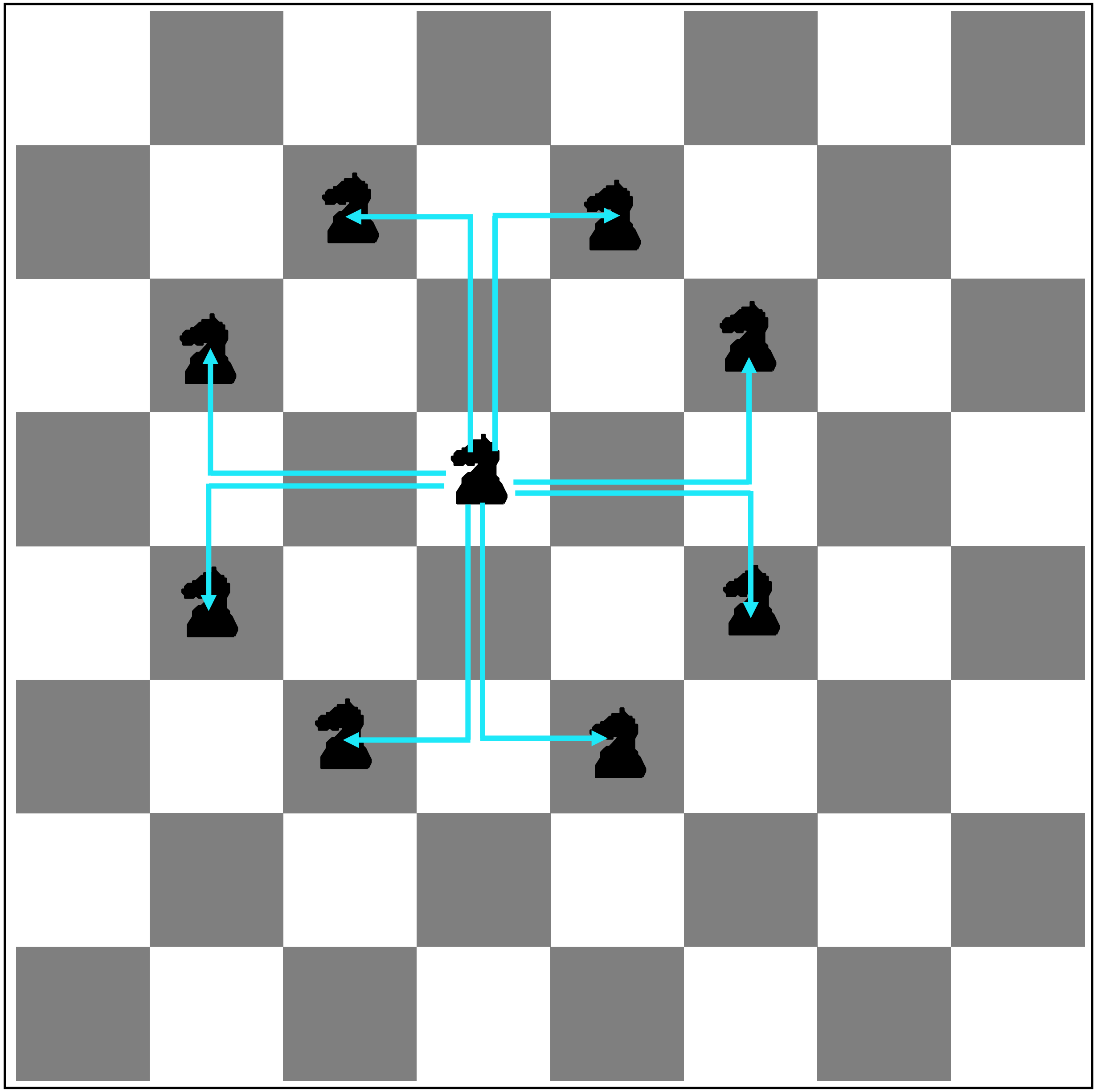
Outline

- What is ‘Knight’s Tour
- Theory
- Knight's graph
- Computer Algorithms
- Warnsdorff's Algorithm
- Magic Knight’s Tours
- Knight’s Tours and Cryptography
- Longest uncrossed knight's path

What is ‘Knight’s Tour?

- Chess problem involving a knight
- Start on a random square
- Visit each square exactly ONCE according to rules
- Tour called closed, if ending square is same as the starting.
- Variations of the knight's tour problem.
  - Different sizes than the usual 8×8
  - Irregular (non-rectangular) boards.

A Knights Tour – Legal Moves

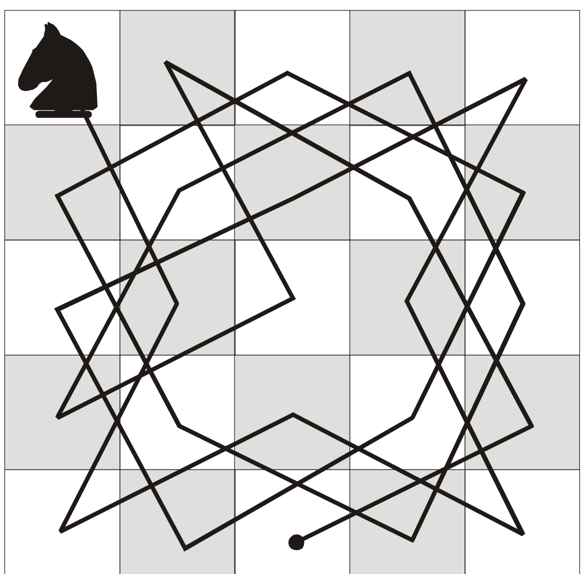


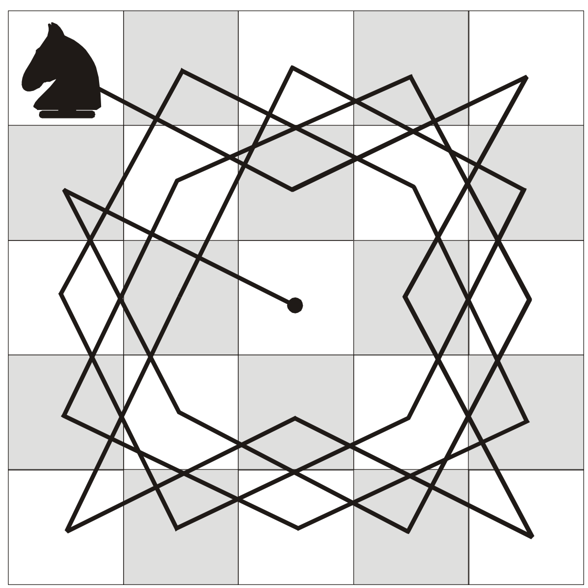
1	22	13	18	7
14	19	8	23	12
9	2	21	6	17
20	15	4	11	24
3	10	25	16	5

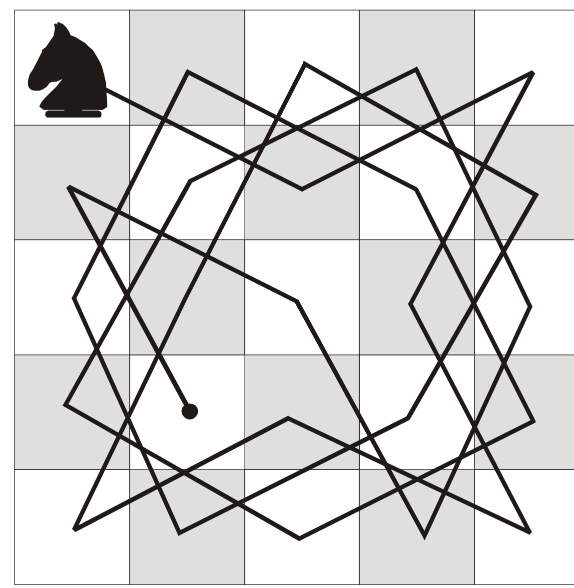
1	14	9	20	3
24	19	2	15	10
13	8	25	4	21
18	23	6	11	16
7	12	17	22	5

1	14	9	20	3
24	19	2	15	10
13	8	23	4	21
18	25	6	11	16
7	12	17	22	5

A Knights Tour on a 5x5 Chessboard





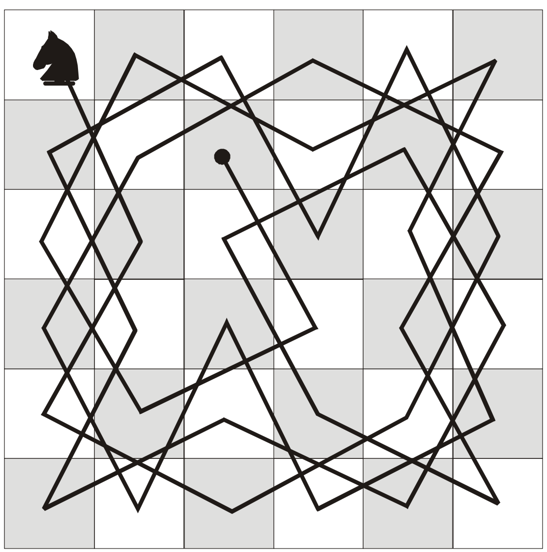


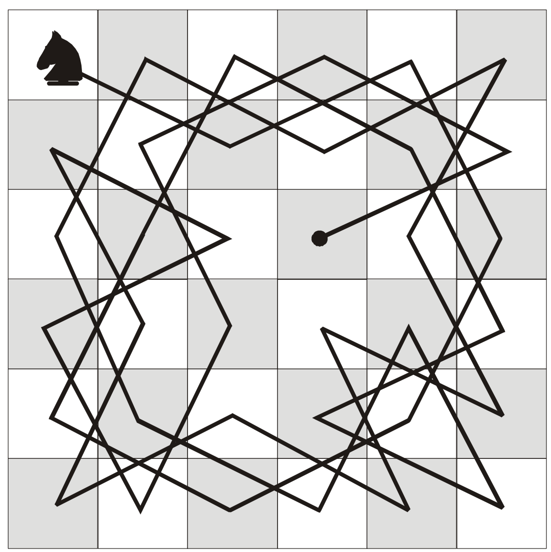
1	32	9	22	7	30
10	23	36	31	16	21
33	2	17	8	29	6
24	11	26	35	20	15
3	34	13	18	5	28
12	25	4	27	14	19

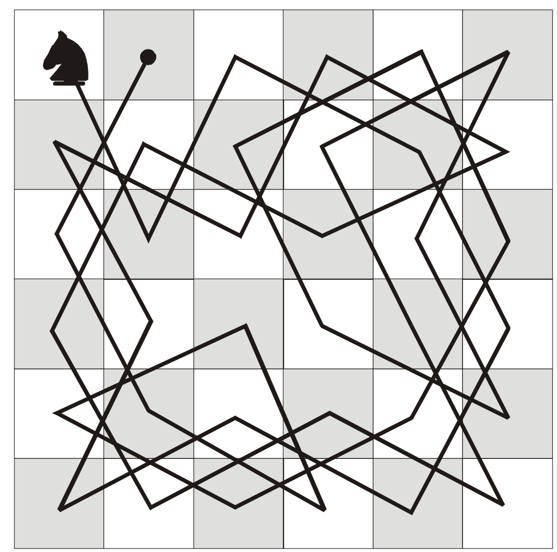
1	18	9	34	3	20
28	33	2	19	10	35
17	8	29	36	21	4
30	27	32	23	14	11
7	16	25	12	5	22
26	31	6	15	24	13

1	36	3	12	27	22
10	15	26	21	4	13
35	2	11	14	23	28
16	9	32	25	20	5
31	34	7	18	29	24
8	17	30	33	6	19

A Knights Tour on a 6x6 Chessboard



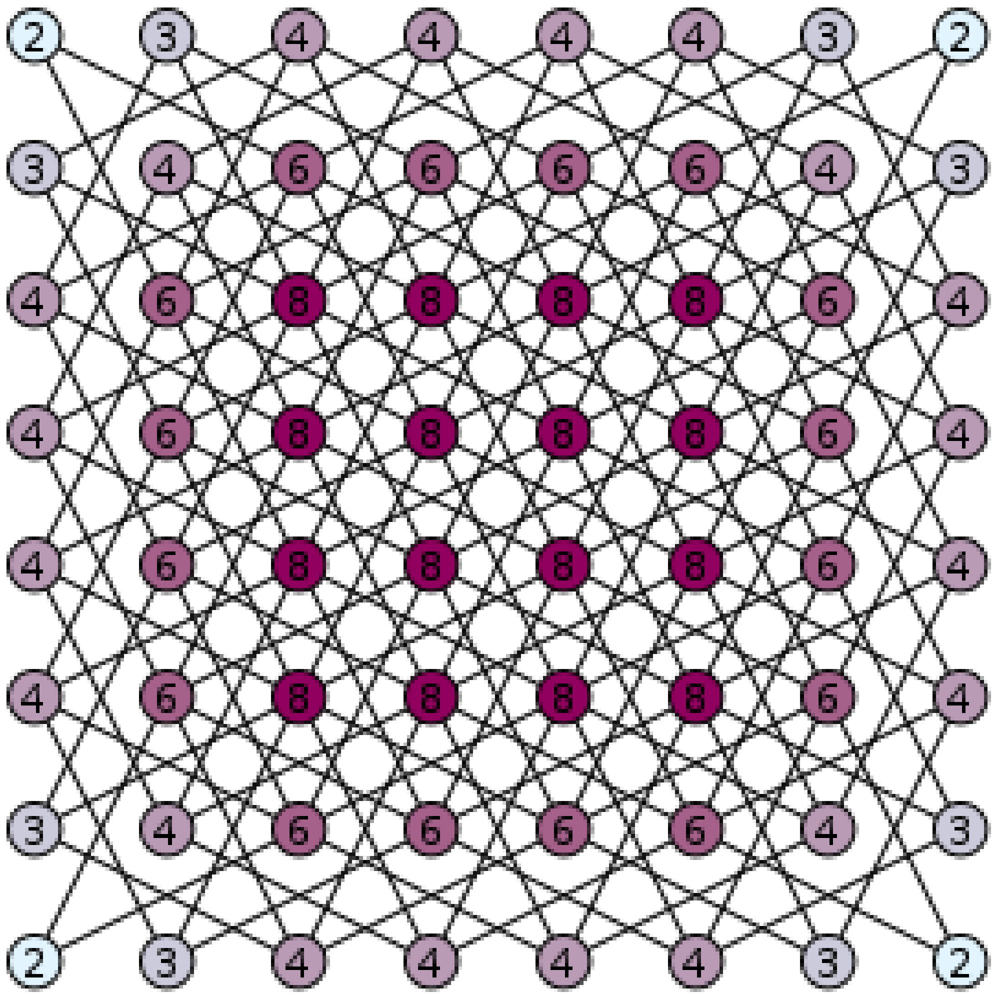




Theory

- Knight’s tour can be represented as a graph.
- The vertices - Represent the squares of the board.
- The edges - Represent a knight’s legal moves between squares.
- Knight’s tour is simply an instance of Hamiltonian path.
- A closed tour is a Hamiltonian cycle.
- Knight's tour problem can be solved in linear time.

Knight's graph



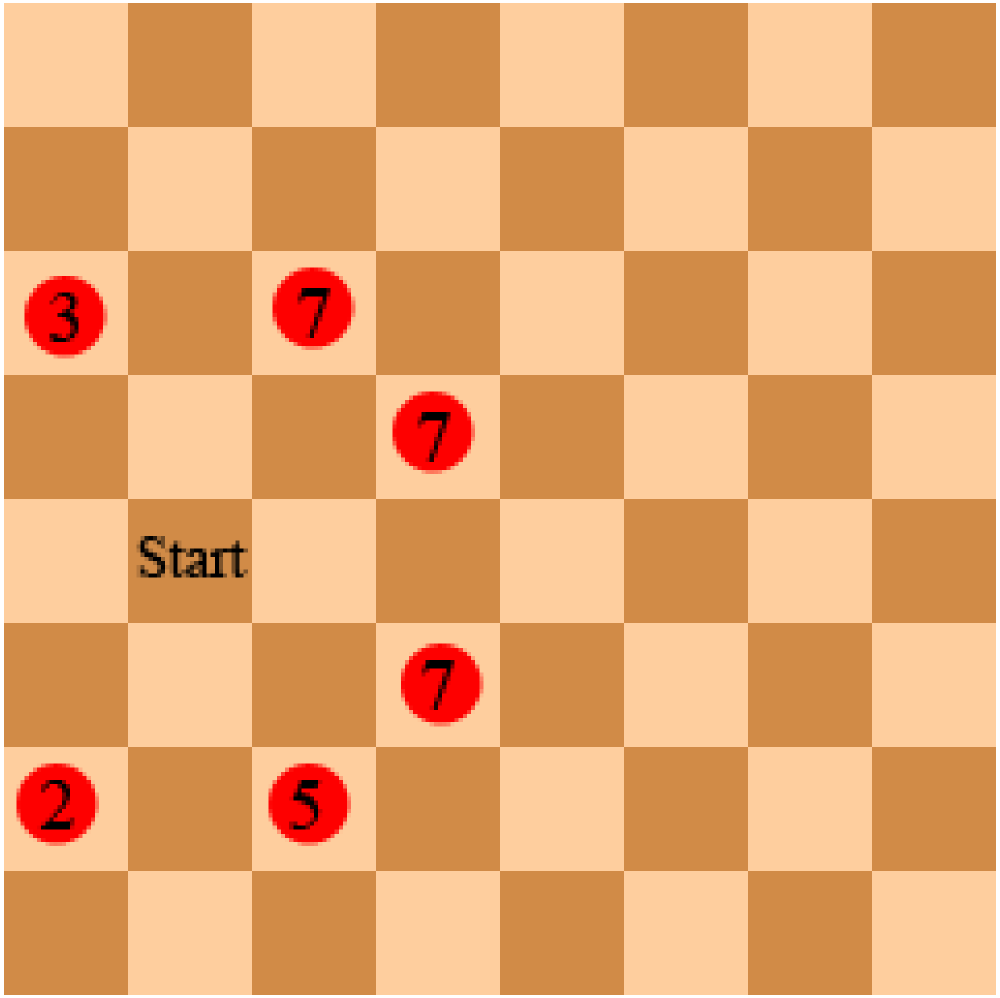
Computer Algorithms

- **Brute-force search ???**
  - Iterates through all possible move sequences.
  - For a regular **8x8** chess board, there are approximately $\mathbf{4\times10^{51}}$ possible move sequences.
  - Brute force approach can ***never*** be applied to the Knight's Tour problem.
- **Neural network solutions.**
  - Can be solved by a neural network implementation.
  - Every legal knight's move is represented by a neuron.
- **Warnsdorff's algorithm**

Warnsdorff's Algorithm

- Heuristic Method.
- Introduced by H. C. Warnsdorff in 1823.
- Any initial position of the knight on the board.
- Each move is made to the adjacent vertex with the least degree.
- May also more generally be applied to any graph.
- On many graphs that occur in practice this heuristic is able to successfully locate a solution in linear time.

Warnsdorff's Algorithm



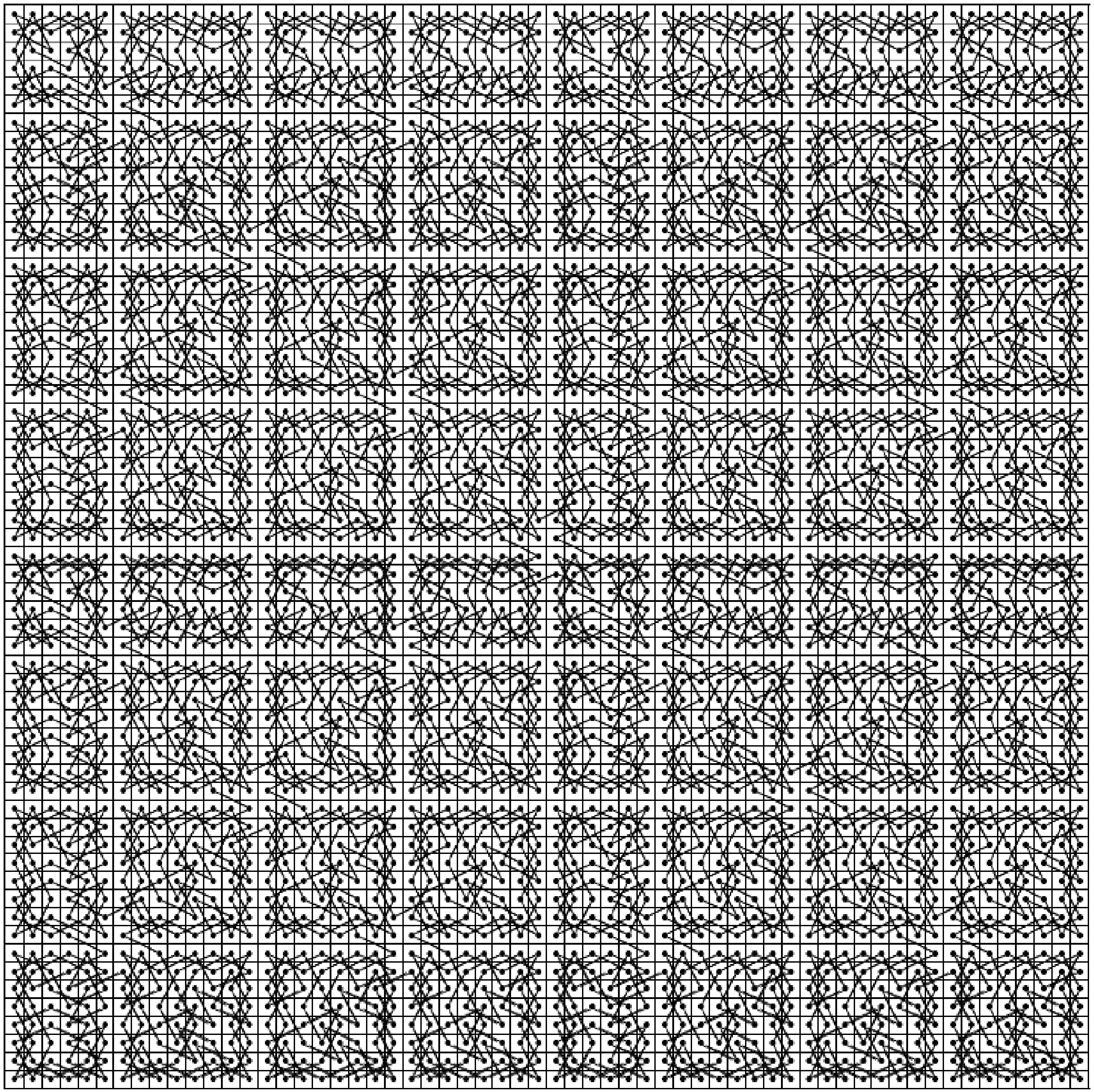
Warnsdorff's Algorithm

- Some definitions:
  - A position Q is accessible from a position P if P can move to Q by a single knight's move, and Q has not yet been visited.
  - The accessibility of a position P is the number of positions accessible from P.
- Algorithm:
  1. set P to be a random initial position on the board
  2. mark the board at P with the move number "1"
  3. for each move number from 2 to the number of squares on the board:
     1. let S be the set of positions accessible from the input position
     2. set P to be the position in S with minimum accessibility
     3. mark the board at P with the current move number
  4. return the marked board – each square will be marked with the move number on which it is visited.

Warnsdorff's Algorithm

- Warnsdorff's rule is heuristic
  - It is not guaranteed to find a solution.
  - It can fail for boards larger than 76x76.
- The reason for using these heuristics instead of an algorithm guaranteed to work is speed.
- Improvements by decomposing;
  1. Decompose a large board into smaller rectangles.
  2. Solve those sub rectangles.
  3. Smaller solutions are then joined to form a knight's tour.

A 60x60 Knight's Tour generated by decomposition



Magic Knight’s Tours

- The squares of the chess board are numbered in the order of the knight’s moves.
- Each column, row, and diagonal must sum to the same number.
- The first magic knight’s tour (with sum 260) by William Beverley (1848).

Beverley's tour

1	48	31	50	33	16	63	18	260
30	51	46	3	62	19	14	35	260
47	2	49	32	15	34	17	64	260
52	29	4	45	20	61	36	13	260
5	44	25	56	9	40	21	60	260
28	53	8	41	24	57	12	37	260
43	6	55	26	39	10	59	22	260
54	27	42	7	58	23	38	11	260
260	260	260	260	260	260	260	260	

What's the magic number?

$$\frac{n(n+1)}{16}$$

$$\frac{64x65}{16}$$

1	48	31	50	33	16	63	18
30	51	46	3	62	19	14	35
47	2	49	32	15	34	17	64
52	29	4	45	20	61	36	13
5	44	25	56	9	40	21	60
28	53	8	41	24	57	12	37
43	6	55	26	39	10	59	22
54	27	42	7	58	23	38	11

(a)

1	48	31	50	33	16	63	18
30	51	46	3	62	19	14	35
47	2	49	32	15	34	17	64
52	29	4	45	20	61	36	13
5	44	25	56	9	40	21	60
28	53	8	41	24	57	12	37
43	6	55	26	39	10	59	22
54	27	42	7	58	23	38	11

(b)

1	48	31	50	33	16	63	18
30	51	46	3	62	19	14	35
47	2	49	32	15	34	17	64
52	29	4	45	20	61	36	13
5	44	25	56	9	40	21	60
28	53	8	41	24	57	12	37
43	6	55	26	39	10	59	22
54	27	42	7	58	23	38	11

(c)

a) First semi-magic knight's tour
b) In each quadrant, the sum of the numbers equals 520 and each of the rows and columns adds to 130
c) The sum of the numbers in each 2x2 section is 130

Existence of full magic knight's tour on 8x8 was a 150-year-old unsolved problem.

In August 5, 2003, after nearly 62 computation-days, a search showed that no 8x8 fully magic knight's tour is possible.

http://mathworld.wolfram.com/news/2003-08-06/magictours/

Knight’s Tours and Cryptography

- A cryptotour is a puzzle in which the 64 words or syllables of a verse are printed on the squares of a chessboard and are to be read in the sequence of a knight’s tour.
- The earliest known examples of a cryptotour were printed in the mid 1800s in a French magazine.
- Published before the invention of crossword puzzles (1890).

Example of a cryptotour from 1870

PUZZLE

sor	to	king	good	say	luck	loy	eth
and	moth	a	soon	dis	our	to	bad
place	ry	church	his	force	is	hat	al
er	queen	him	wight	he	to	may	truth
man	his	and	and	chess	es	knight	op's
a	sneer	the	and	un	lawn	of	tates
cas	that	at	less	pawn	no	bish	lant
eth	faith	tles	hath	the	gal	in	love

TOUR SOLUTION

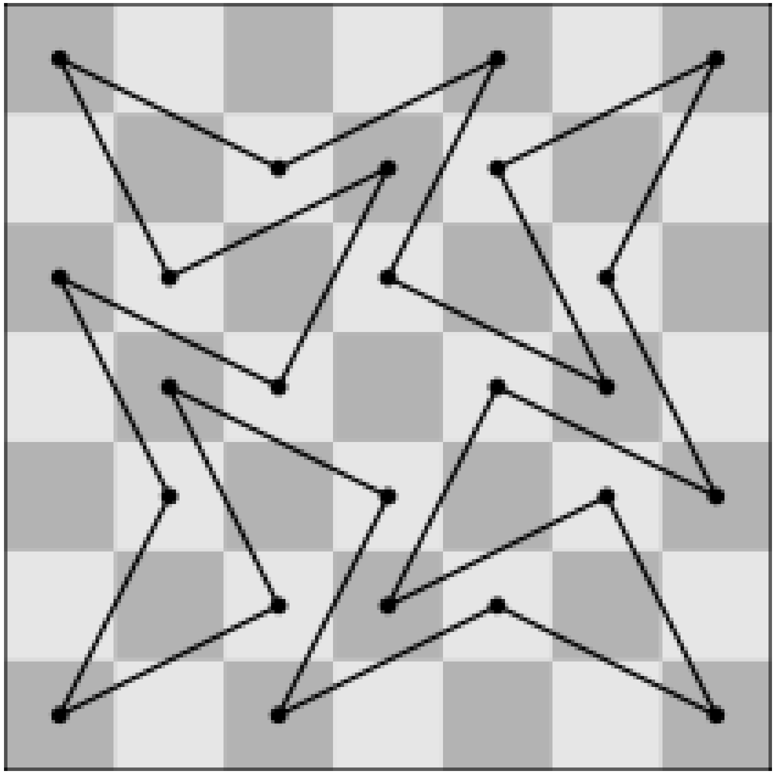
14	55	22	37	12	51	18	35
23	38	13	54	17	36	11	50
56	15	40	21	52	9	34	19
39	24	53	16	33	20	49	10
2	57	28	41	8	61	32	47
25	42	1	60	29	48	7	62
58	3	44	27	64	5	46	31
43	26	59	4	45	30	63	6

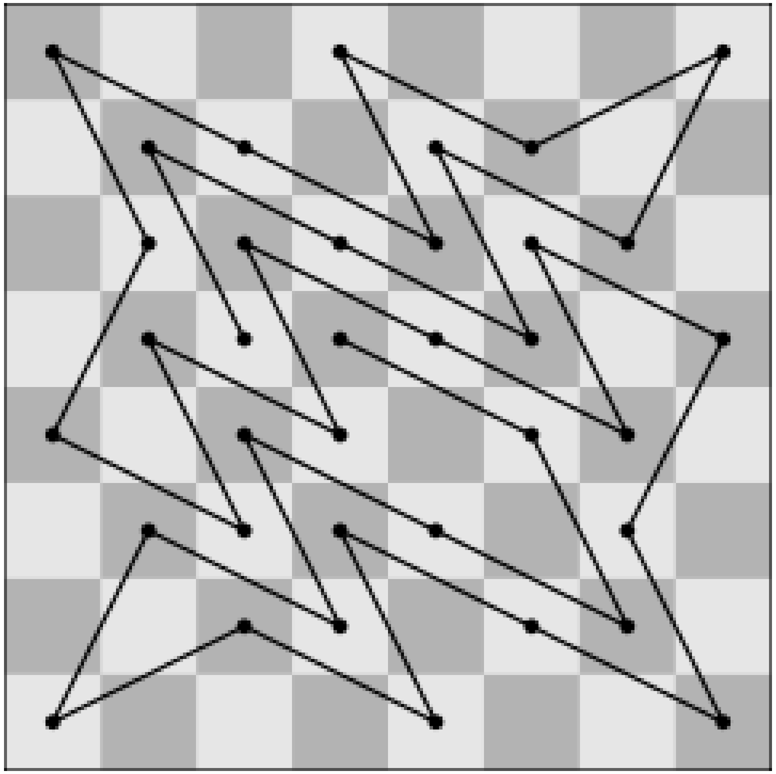
VERSE SOLUTION

The man that have no love of chess
Is, truth to say, a sorry wight,
Disloyal to his king and queen.
A faithless and ungallant knight;
He hateth our good mother church,
And sneereth at the bishop's lawn;
May bad luck force him soon to place
His castles and estates in pawn!

Longest uncrossed knight's path

- The problem is to find the longest path the knight can take on nxn board, such that the path does not intersect itself.





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- "*Longest uncrossed knight's path*" [Online]. Available: http://en.wikipedia.org/wiki/Longest_uncrossed_knight%27s_path
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Thank You