

Algorithmic Thinking

for

Adventurous Minds



Written By
William Xu, Raymond Xu, and Claire Xu

Algorithmic Thinking for Adventurous Minds

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Mr. Sanjit Singh, MIT, helped review the code and technical explanation.

We feel tremendously grateful to students in our community who have actively participated in our CS and algorithm classes and online seminars over the last few years. Your valuable feedback has been incorporated in this book.

ENDORSEMENT

“I would recommend this book to anyone interested in a fun and engaging look at the fundamentals of algorithms. It will change your perspective.” - Jen Lee, Harvard University

“It is one of a kind to bring the abstraction process to life. Fascinating.” - Bryan Ng, Stanford University

“I’m a visual learner. CS abstraction logic used to scare me. Sitting in William, Raymond and Claire’s classes changed my belief. Algorithms have a lovely side. In fact, they solve your problems. This book is a compilation of part of their class content. Hope you enjoy it like I did.” - Jeff Thomas, 9th grader at MVHS

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INTRODUCTION

This book is about how to work smart to avoid unnecessary work. Algorithmic thinking is about identifying the most efficient steps to solve a seemingly complex problem without detouring. It is a necessary skill for future jobs.

This book introduces many computer science terms like Fibonacci, graph, recursion, queue, stack, Greedy, Dynamic Programming, Prim, Kruskal, Dijkstra, BFS, DFS, etc. While these concepts all have rigid definitions and are not easy to grasp at a glance, this book describes them in visualizations, graphs, miniature poems, and fun facts. What a unique approach to engaging with both visual learners and reading/writing learners!

Through a magical lens, CalliLens, you will observe abstraction and recognize patterns in the Land of Apple Pi. You will adventure a series of challenges and naturally come up with the solution steps. Without forcefully knowing it, you will master the skill of algorithmic thinking.

Computer World Is an Animal Kingdom

Coming up with the right names for the characters in this book is equally important as making the algorithms and programs right. The names spark the inspiration. Spending excessive brain power gazing at the cosmos without a clue, but with a flash on the screen, the characters jump out of the computer! Computer world is an Animal Kingdom, our characters live in this Kingdom —

The sky is blue; the grass is green. Deep in the forest lives the BestFour:



Dark Knight

A gallant hero and adventurer, he is admired by all who meet him.
He was born to lead!



Banana Split

Intelligent and enthusiastic, he brings energy to the crowd. This social butterfly will make friends with everyone!



Bubble Gum

Creative and cheerful, she brings balance to the group. This peacekeeper is a natural problem solver with an unparalleled memory.



Mighty Python

An embodiment of computer science, he brings a plethora of knowledge to the group. When it comes to Algorithms and Code, he's the one to call.

Computer World is an Animal Kingdom



From as big as Elk (analytics software) to as tiny as Ant (build automation tool)



From as friendly and social as Llama (heterogeneous system development) to as deadly and evil as Cobra (OOP language)



From as sleek as Penguin (graphical user interfaces development) to as sharp as Porcupine (web app server)



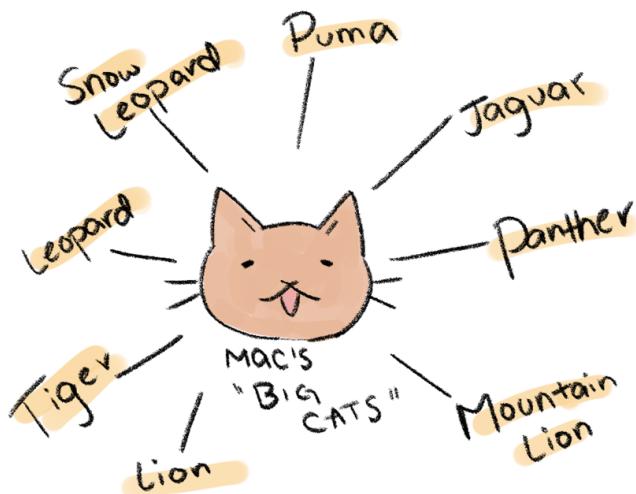
From as fast as Cheetah (template engine) to as slow as Sloth (slowest language by Larry Page)



From as elegant as Orca (parallel application development) to as gross as Cockroach (SQL Database)



Programmers just cannot have enough animals. They added a WHOLE pack to the macOS big cat family: 10.0 “Cheetah,” 10.1 “Puma,” 10.2 “Jaguar” ...



You get the point!

Intended Audience

This book is designed to introduce the essential framework that computer science is built upon: algorithmic thinking. No matter if you are a child or an adult, we invite you to journey through the visual illustrations of abstraction, decomposition, pattern recognition and algorithm design steps. The only prerequisite is to keep an open mind and open eyes. If you don't know coding yet, we offer flowcharts as your friendly starting point. What? Some of you say that you want to dive into coding? Alright, we offer Python code and Pygame as a bonus for you to craft your programming skills.

We have identified a set of fundamental algorithms after pre-work analyzing the questions in the USA Computing Olympiad (USACO) and CS job interviews. We weave the algorithms into the adventures.

Python Code

Why Python?

Python is an easy-going friend to make. If you know the English language, you know a portion of Python! It resembles the English language and focuses on what you want to achieve but not the programming language itself.

Python is a celebrity on all computer continents: Windows, macOS, Linux. Your Python code will work no matter where it runs.

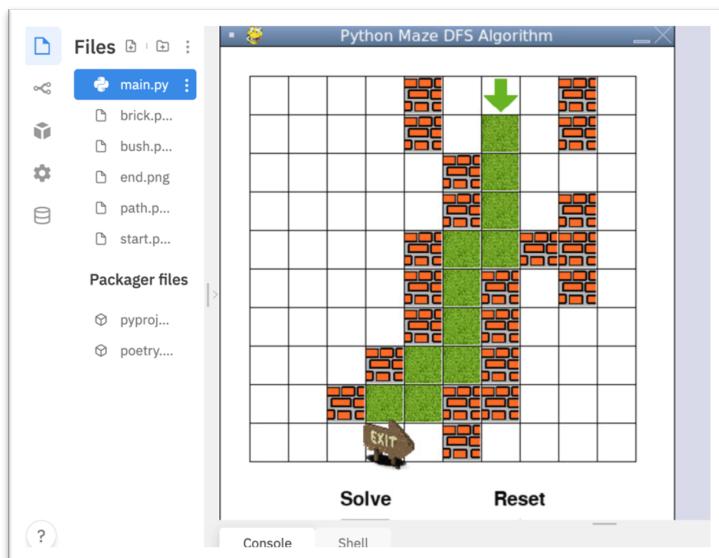
Python is a big brain for you to pick, the artificial intelligence brain. With the support of Python clans (libraries) like ‘NumPy,’ ‘Pandas’ and ‘Matplotlib’, Python handles statistics, matrix data, and visualization very well. Furthermore, some of the strong family members, “Keras”, “TensorFlow” and “OpenCV” enable artificial intelligence. It’s like you have two brains now.

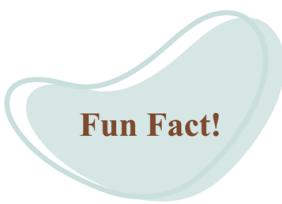
Python Code and Development Environment

You can download all the Python code used in this book from
<https://github.com/applepiinc/arithmaticthinking>

To practice Python and Pygame code, you can either setup development environment on your laptop or go to online environment:

- To setup development environment on your laptop, please install
 - Python and IDLE (Integrated DeveLopment Environment) <https://www.python.org/downloads/>
 - Pygame <https://www.pygame.org/download.shtml>
- To use online Python interpreter, you can go to
 - <https://repl.it/>
Create account, select language Python or Pygame (for Pygame code) and upload necessary code and images.





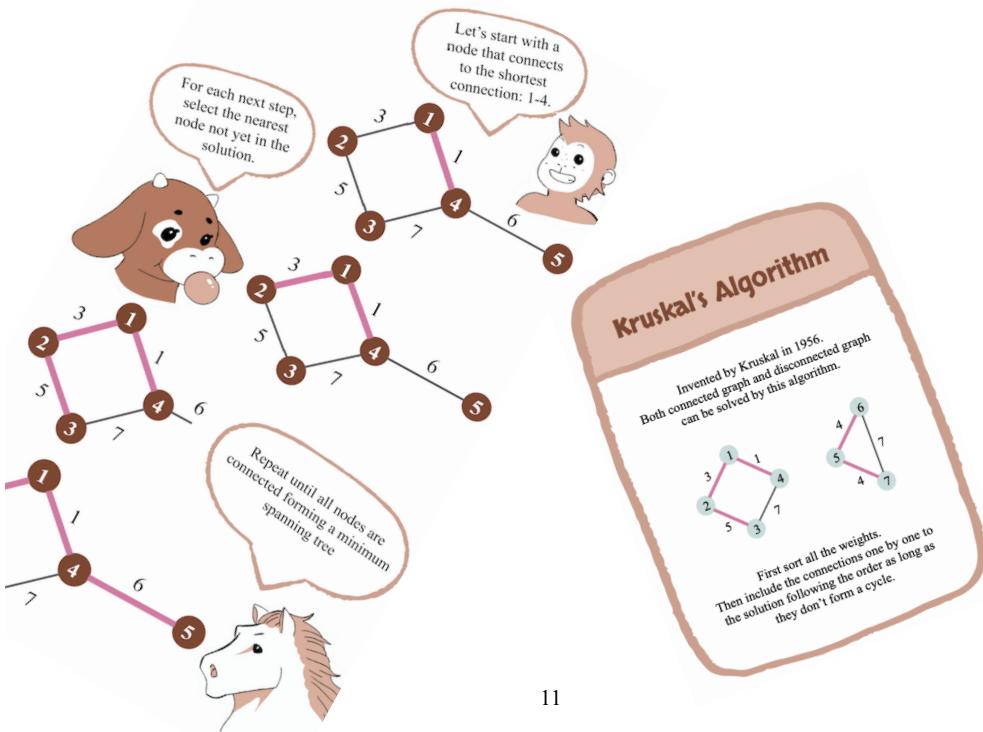
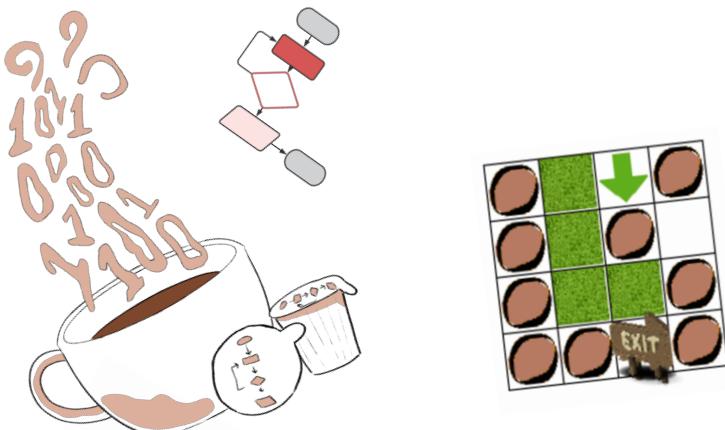
Fun Fact!

Is Python language named after the snake?

When Guido van Rossum began implementing Python in the 1980s, he was watching the comedy series “Monty Python’s Flying Circus”. Van Rossum thought he needed a name that was short, unique, and slightly mysterious, so he decided to call the language Python. The language was first released in 1991.

A Picture's Worth a Thousand Words

We weave in lots of illustrations and fun facts throughout the book to help with the learning process.



The Magical Lens and The Land of Apple Pi

Do algorithms sound like a terrifying monster topic? Don't worry. Algorithm, in simple words, is the sequence of steps to solve a type of problem. We will grant our audience a magical lens called CalliLens.



The CalliLens symbolizes a set of problem-solving methods that are used to abstract the problems and identify algorithmic solutions in a computer-friendly way. When we use the lens, we will see a crystal-clear world out of the chaotic reality.

Our story will take place in the Land of Apple Pi. The "Apple" is a symbol of realization, health and innovation (e.g.

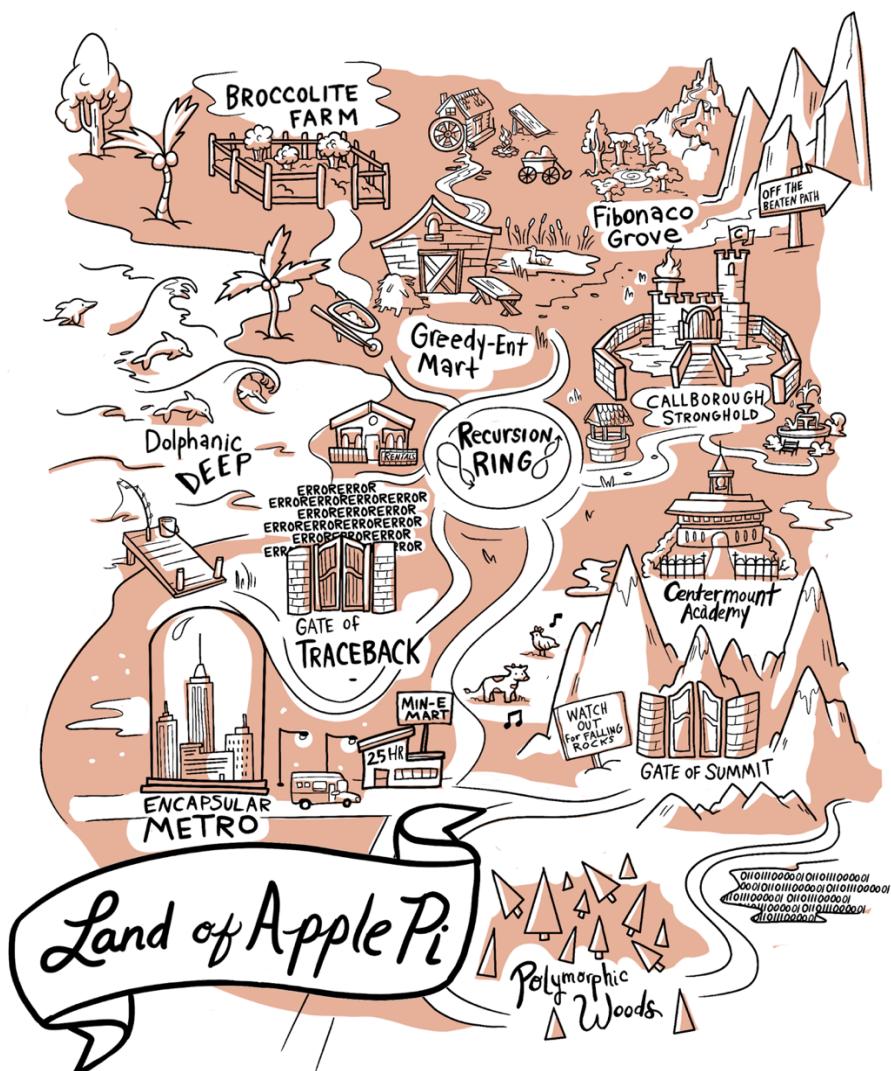
Newton's apple, an apple a day keeps doctor away, Apple laptop).
Pi is for Math. Welcome!

By the way, do you have to love animals to continue reading this book? No, but if you start now, together we can make the world a better place.

Oh, one more thing: if any of you geniuses in the audience figure out a better solution than the one presented in the book, please feel free to share the joy with us!

Now let the adventure begin!

MISSION TO THE LAND OF APPLE PI





“Look!” Dark Knight shouts as he gallops to his clan, “I found this in Grandpa’s treasure box.”

“What is it?” asks the BestFour.

Shining beneath the warmth of the sun, Dark Knight holds out a glistening and

immaculate lens. However, right before the rest decide to lay a hand on it, a line of fine print catches Banana Split’s eyes:
CalliLens welcomes you to the crystal world.

“CalliLens?” Bubble Gum murmurs, “I’ve only heard of HoloLens by Microsoft. Their transparent lens shows you the augmented reality. If I’m hungry, a heap of alfalfa hay in front of my nose will display the number of calories it has; and when I’m thirsty, a small stream of river along my hooves will display the mineral ingredients.”



“Alright,” Banana Split chimes in, “if *holo* means a three-dimensional image, then what does *Calli* mean?”

Everyone turns to look at each other blankly before falling silent again.

“Hmm...” Banana Split continues, “if calligraphy is a style of beautiful writing, and *Calli* means beautiful in Greek, then CalliLens must be—”

“It must mean that we’re going to see a beautiful world through these CalliLens!” Bubble Gum exclaims. “That’s wonderful.”

Crystal Clear and Abstract World



The roads are straight and the
stones are smooth.

The cookies are crispy and
cakes are fluffy.



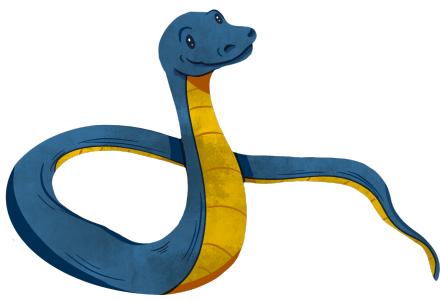
The dreams are vast and
our friendship is warm.



“*We get it!*” the BestFour shout together,
“*Beautiful. We can see an abstract world out of the
unnecessarily chaotic reality!*”

Everyone gasps and turns towards Mighty Python. “What is this mysterious CalliLens?”

Lifting his head and gazing towards the horizon, Mighty Python begins to explain:



“The CalliLens has a remarkable ability. When you are agitated by a monster problem, wear this gadget, and you will see.

- 1) The monster problem will shrink and decompose into several miniature sub-problems that are easier to manage.
- 2) The patterns among the closely related problems will jump out, and the irrelevant details will fade away.
- 3) The instructions on how to attack the monster will emerge in front of your eyes.”

“Who here has a monster problem?” Mighty Python asks while waving the CalliLens in the air.

“Me,” Banana Split replies, “Mom asked me to clean my room today. Where should I start—”

“Remember...the CalliLens will tell you.”

“Oh right...it shows that I should gather my clean clothes in a pile, toss the dirty ones into the laundry bin, and place my books onto the shelf. This is easier than I anticipated!”

Reverse Earth Aging

“Yep, now let me tell you something.” Mighty Python continues, “long, long ago, the Land of Apple Pi was thriving and transforming with humans’ inventions generation after generation. They saved people’s lives, lightened them in the darkness, and elevated the power of controlling their own destiny.

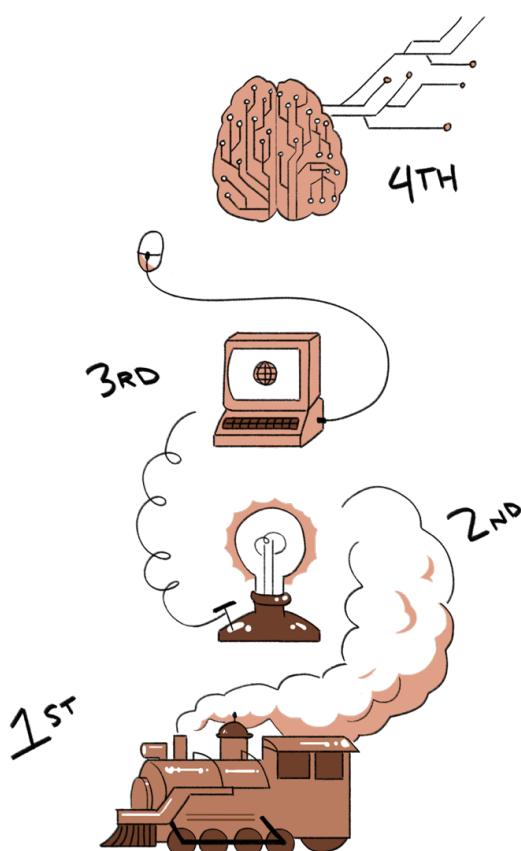
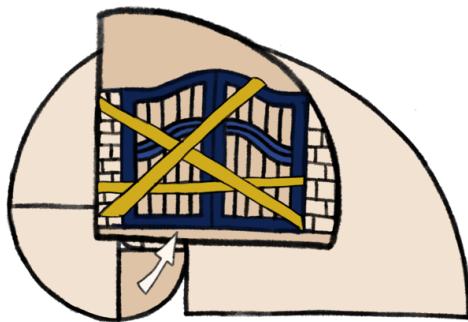


Figure 1.1 The Four Industrial Revolutions: inventions that help humans to thrive.

However, some of the inventions showed two sides of the coin: nuclear reaction as a power source but also as a weapon, dynamite to blast rocks in mining but also to blow people up! Some inventions were even purely driven by people's greed for possession and greed to win. This led the world to start twisting and wrinkling."



Figure 1.2 Toxic inventions that age the Earth: atomic bombs, deadly gasses, spam email, etc.



“Now what?” Everyone asks while looking towards Mighty Python.

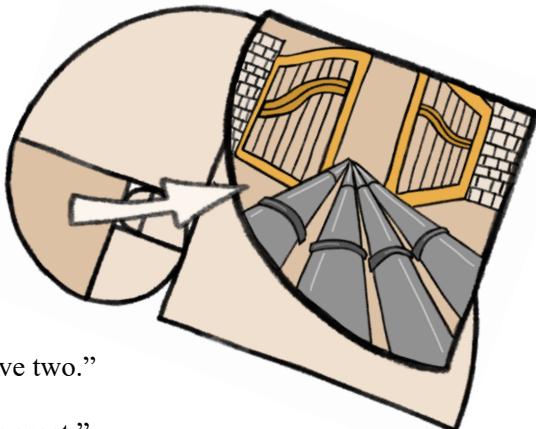
“There is only one chance to reverse the Earth’s aging: we first have to take the CalliLens back to the Land of Apple Pi. It will help us navigate through the chaos. Then we need

to build pipelines to flow the righteous inventions from the Gate of Summit to every city and town. Lastly, the Gate of Traceback must be sealed, for it is the source of the toxic inventions.”

“Next week is a travel back to the Land Earth’s aging!” Dark

holiday. Let’s secretly of Apple Pi and reverse the Knight proposes.

“CalliLens lens is the magical lens,” recites Bubble Gum, “if you wear it often, it’ll solve your question. I’ll just borrow the other one from Grandma, so we have two.”



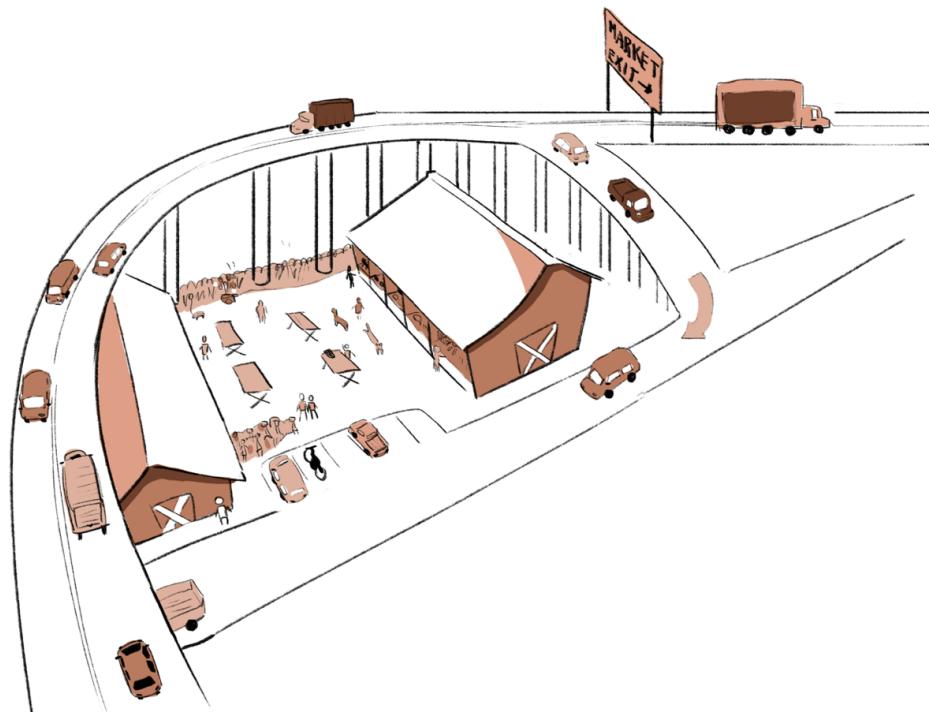
“Ok, sounds great.”

“Nonni, we’re going camping, just typical camping in the woods—” The BestFour waves and calls out as they are off to the road.

DAY 1: BESTFOUR LANDING AT GREEDY-ENT MART



Landing at Greedy-ent Mart



Hovering on top of a peculiar and restless land, the BestFour notices tiny, little ants crawling along several intertwined lines.

They land at a marketplace after following a hazy cloud.

Banana Split cannot help but reach out for a tempting plate of nuts.



“Wait, young man, don’t you know that you should pay first?” The cashier growls as he points to a box of coins and bills collected from the customers.

Banana Split peers at the box for a split second before getting interrupted.

“Once,” Bubble Gum recalls, “Grandpa showed us the coins and bills in his treasure box. They were used as physical currency for goods exchange on the Land of Apple Pi.”

“Oh, this must be it!” Banana Split zooms in close to the cashier’s box, “well, can I pay with my fingerprint, facial recognition, retina recognition, or with any other sort of biometric identification? We don’t carry physical currency anymore.”

“W-What?!?” The cashier’s jaw drops. “Who are you guys? Aliens?”

“Ah no, unfortunately... but I have an idea: we can sell what we have in order to exchange the physical currency here.” Dark Knight announces.

“Glad we brought two CalliLenses. What about selling one?” Bubble Gum asks as she rummages through her pocket.



Decoding Nature: Fibonacci and Recursion

“Allow me to do the honors of trying out this masterpiece, if you insist.” A nearby farmer proudly states. “Oh my me! What happened to my

Romanesco broccoli? What are these 5 green spiral lines? Are they formed by the pinnacles as it curls to one side? What are these 8 red spiral lines curling to the other side?”

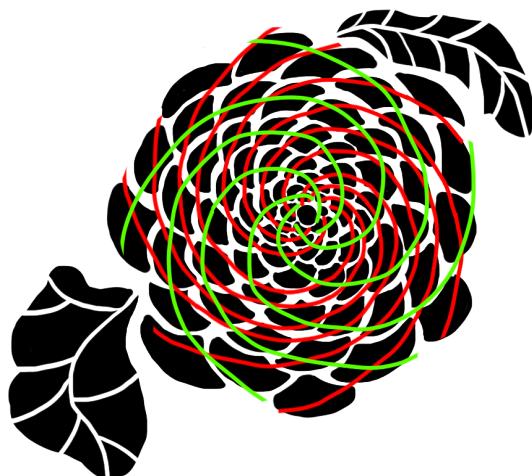
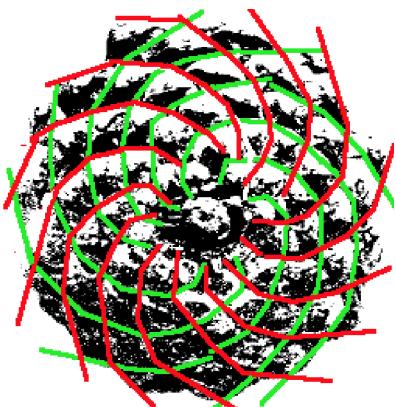


Figure 1.3 Spirals on romanesco

“What do you mean?” Another farmer asks after overhearing the conversation. “I...my dear pinecone. It has 8 green spirals and 13 red spirals drawn along it!”



“Oh, 5, 8, and 13! All these spiral numbers are from the Fibonacci sequence.” Banana Split jumps in, “In fact, based on a statistical study in Norway, 95% of pinecones demonstrate Fibonacci spirals.” [1]

Figure 1.4 Spirals on pinecone

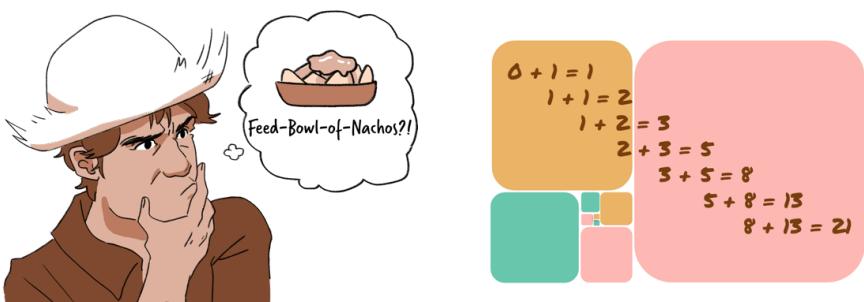


Figure 1.5 Fibonacci computation

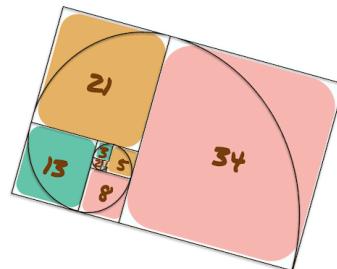
Fibonacci

A series of numbers in which each number is the sum of the previous two

Re-discovered by Leonardo Fibonacci, an Italian mathematician born somewhere around 1170

Golden Ratio

The ratios of Fibonacci numbers F_n/F_{n-1} approaches the factor 1.618 as n approaches infinity. This magic factor is called ϕ (Phi) or Golden Ratio, which was first proved by Scottish mathematician Robert Simson in 1753.



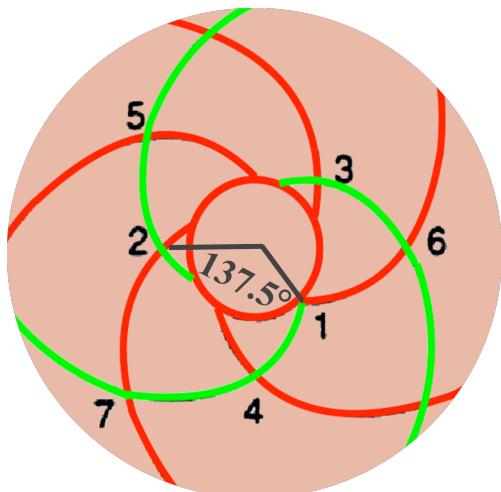
Golden Spiral

Therefore, this special spiral is called Golden Spiral. It gets wider by a factor of 1.618 for every quarter turn it makes.

Figure 1.6 Fibonacci number sequence, golden ratio and golden spiral

“Fibonacci numbers are very common in nature,” Bubble Gum adds.

“OH MY GOSH, is this even my flower?” Asks a nearby gardener.



In this image, the smaller the numbers, the younger the petals.
3 green clockwise spirals and 5 counterclockwise red spirals are shown.

Figure 1.7 Spirals on flower

Again, 3 and 5 are Fibonacci numbers! These spirals are called golden spirals.

The adjacent petals are always apart by a constant angle, 137.5°! 137.5° between petal 1 and 2, same 137.5° between petal 2 and 3, and so forth.

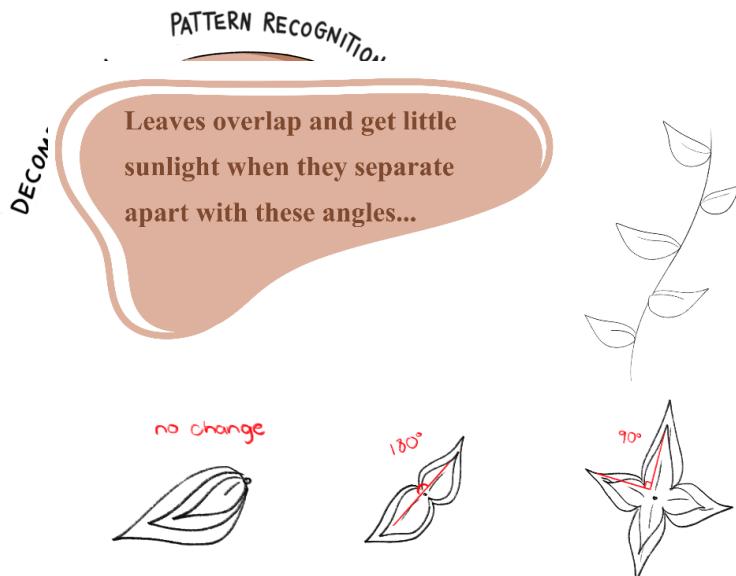
A crowd has formed around the BestFour by now.

“Look at the branches on the tree!” someone shouts, “The branches follow the same 137.5° angle to diverge from each other!” [4]

You might ask: why do plants follow this magical angle and hence form Fibonacci number of spirals (3, 5, 8, 13, etc.) for their growth? In fact, this was the question raised many times in history.

To decode nature's mystery, let's pose a hypothesis:

Ha, nature loves Fibonacci and the Golden Angle 137.5° !



Or 60? 30? 10? $360/n$ degrees?

Is there a magic angle so that leaves will never overlap?

Golden angle $\Psi \approx 137.5077^\circ$

A large orange speech bubble contains the text: "This is the exact angle we observed on pine cones, flowers and tree branches!"

“Then, how do we compute the Fibonacci number sequence?” asked one of the farmers.

Bubble Gum points to the lens. “It will tell.”

“Wow...”

- Say function $\text{fib}(n)$ is used to compute the n th Fibonacci number.
- Per Fibonacci definition, the function calls itself to add the previous two numbers together $\text{fib}(n-1) + \text{fib}(n-2)$.
- When we reach the first two numbers in the Fibonacci sequence (0 and 1), the function will stop the calling chain and return 0 and 1 respectively.

The Codephile Flowchart

Before getting down to coding, we'll draw a picture of the algorithm. Remember from this book's Introduction session that if code is coffee, then Flowchart is coffee mate! This picture is called **Flowchart**. It helps us sort through and visualize the thinking process. We may realize some logic defects during the drawing process, and hence have a chance to fix them before coding.

Flowchart Illustration [6][7]:

Rectangle: activity or operation of the algorithm

Diamond: decision, conditional operation to show which path to take next

Rhomboid: input/output of data, e.g. take user input, display result on screen

Lines and arrows: order and relationship of the steps

Rounded Rectangle: start and end of the process

We read flowcharts from top to bottom and left to right.

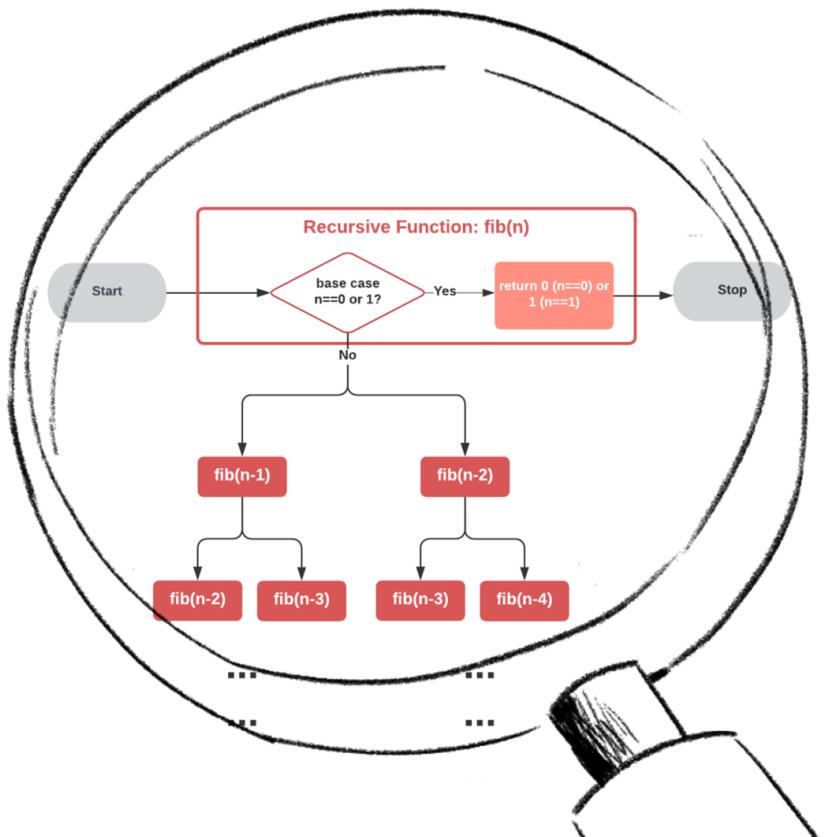
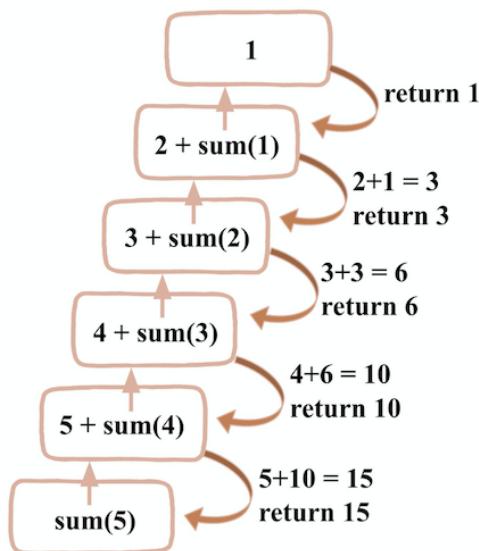


Figure 1.9 Flowchart: recursive function to solve Fibonacci number sequence

Recursion

A function that calls itself and stops the chain at the base case



sum() starts from the bottom of the stack, winding up to decompose until base case, and unwinding down to compute the result.

Figure 1.10 Recursive function definition, recursive approach to solve $\text{sum}()$