Modelli Univariati

Modello lineare

$$\begin{cases} Y_i \mid \mu_i, \sigma^2 \overset{ind}{\sim} \mathcal{N}(\mu_i,\ \sigma^2) & i = 1:N \\ \mu_i = \beta_0 + X_i^t \boldsymbol{\beta} \\ \beta_0 \mid \hat{\beta}_0, \tau_0^2 \ \sim\ \mathcal{N}(\hat{\beta}_0,\ \tau_0^2) \\ \boldsymbol{\beta} \mid \hat{\boldsymbol{\beta}}, \tau_1^2, \ldots, \tau_p^2 \ \sim\ \mathcal{N}_p\big(\hat{\boldsymbol{\beta}},\ \begin{bmatrix} \tau_1^2 & & 0 \\ & \cdots & \\ 0 & & \tau_p^2 \end{bmatrix}\ \big) \\ \sigma \sim \mathcal{U}\text{nif}(0, \sigma_0) \end{cases} \tag{1}$$

SSVS

$$\begin{cases} Y_i \mid \mu_i, \sigma^2 \overset{ind}{\sim} \mathcal{N}(\mu_i,\ \sigma^2) & i = 1:N \\ \mu_i = \beta_0 + X_i^t \boldsymbol{\beta} \\ \beta_j \mid \tau_j \overset{ind}{\sim} \mathcal{N}(0,\ \tau_j) & j = 0:p \\ \tau_j = c_1(1 - \gamma_j) + c_2\gamma_j \\ \gamma_j \mid \lambda \overset{iid}{\sim} \mathcal{B}(\lambda) & j = 0:p \end{cases} \tag{2}$$

Elastic net

$$\begin{cases} Y_i \mid \mu_i, \sigma^2 \overset{ind}{\sim} \mathcal{N}(\mu_i,\ \sigma^2) & i = 1:N \\ \mu_i = \beta_0 + X_i^t \boldsymbol{\beta} \\ \beta_0 \mid \tau_0^2 \ \sim\ \mathcal{N}(0,\ \tau_0^2) \\ \beta_j \mid a_2, \tau_j \overset{ind}{\sim} \mathcal{N}\big(0,\ \frac{\tau_j - 1}{a_2 \tau_j}\big) & j = 1:p \\ \tau_j \mid a_1, a_2 \overset{iid}{\sim} \text{tr-}\mathcal{G}(0.5, \frac{a_1^2}{8\, a_2},\ 1, \infty) & j = 1:p \\ a_1, a_2 \overset{iid}{\sim} \mathcal{E}(\lambda) + 0.5 \end{cases} \tag{3}$$

Modello gerarchico

$$\begin{cases} Y_{ij} \mid \mu_{ij}, \sigma^2 \overset{ind}{\sim} \mathcal{N}(\mu_{ij},\ \sigma^2) & i = 1:N;\ j = 1:ng \\ \mu_{ij} = \gamma_{0j} + X_j^t \boldsymbol{\theta} + Z_{ij}^t \boldsymbol{\gamma}_j \\ \gamma_{0j} \mid \hat{Y}, \tau_0^2 \overset{iid}{\sim} \mathcal{N}(\hat{Y},\ \tau_0^2) & j = 1:ng \\ \boldsymbol{\theta} \mid \tau_\theta^2 \ \sim\ \mathcal{N}_p\big(\mathbf{0}, \tau_\theta^2\, \mathbb{I}_p\big) \\ \boldsymbol{\gamma}_j \mid \tau_1^2, \ldots, \tau_v^2 \overset{iid}{\sim} \mathcal{N}_v\big(\mathbf{0},\ \begin{bmatrix} \tau_1^2 & & 0 \\ & \cdots & \\ 0 & & \tau_v^2 \end{bmatrix}\big) & j = 1:ng \\ \sigma \mid a_1, a_2 \ \sim\ \text{Inv-}\mathcal{G}(a_1,\ a_2) \\ \tau_0^2, \tau_l^2 \overset{iid}{\sim} \text{Inv-}\mathcal{G}(b_1,\ b_2) & l = 1:v; \\ 1/\tau_\theta^2 \ \sim\ \mathcal{E}(\lambda) + 0.5 \end{cases} \tag{4}$$

Elastic net gerarchica

$$\begin{cases} Y_{ij} \mid \mu_{ij}, \sigma^2 \overset{ind}{\sim} \mathcal{N}(\mu_{ij},\ \sigma^2) & i = 1:N;\ j = 1:ng \\ \mu_{ij} = \gamma_{0j} + X_j^t \boldsymbol{\theta} + Z_{ij}^t \boldsymbol{\gamma}_j & i = 1:N;\ j = 1:ng \\ \gamma_{0j} \mid s_0^2 \overset{iid}{\sim} \mathcal{N}(0,\ s_0^2) & j = 1:ng \\ \boldsymbol{\gamma}_{kj} \mid s_k^2 \overset{ind}{\sim} \mathcal{N}(0,\ s_k^2) & k = 1:v;\ j = 1:ng \\ s_k \overset{iid}{\sim} \mathcal{U}\text{nif}(0,\ 100) & k = 1:v \\ \theta_k \mid \tau_k, a_2 \overset{ind}{\sim} \mathcal{N}\big(0,\ \frac{\tau_k - 1}{\tau_k a_2}\big) & k = 1:p \\ \tau_k \mid a_1, a_2 \overset{iid}{\sim} \text{tr-}\mathcal{G}\big(0.5, \frac{a_1^2}{8\, a_2^2},\ 1, \infty\big) & k = 1:p \\ a_1, a_2 \overset{iid}{\sim} \mathcal{E}(0.1) + 0.5 \\ \sigma^2 \ \sim\ \text{inv-}\mathcal{G}(\alpha_1, \alpha_2) \end{cases} \tag{5}$$

Modelli Bivariati

Modello lineare

$$\begin{cases} \boldsymbol{Y}_i \,|\, \boldsymbol{\mu}_i, \Sigma \overset{ind}{\sim} \mathcal{N}_2(\boldsymbol{\mu}_i,\, \Sigma) \qquad i = 1:N \\ \boldsymbol{\mu}_i = \begin{bmatrix} \beta_0[1] + \boldsymbol{X}_i^t \boldsymbol{\beta}_1 \\ \beta_0[2] + \boldsymbol{X}_i^t \boldsymbol{\beta}_2 \end{bmatrix} \\ \boldsymbol{\beta}_0 \,|\, \tau_0^2 \sim \mathcal{N}_2(\mathbf{0},\, \tau_0^2\, \mathbb{I}_2) \\ \boldsymbol{\beta}_1 \,|\, \tau_1^2 \sim \mathcal{N}_p(\mathbf{0},\, \tau_1^2\, \mathbb{I}_p) \\ \boldsymbol{\beta}_2 \,|\, \tau_2^2 \sim \mathcal{N}_p(\mathbf{0},\, \tau_2^2\, \mathbb{I}_p) \\ \Sigma = \left[\begin{smallmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{smallmatrix}\right] \\ \sigma_1, \sigma_2 \overset{iid}{\sim} \mathcal{U}\text{nif}(0, 120) \\ \rho \sim \mathcal{U}\text{nif}(-1, 1) \end{cases} \tag{6}$$

Elastic net

$$\begin{cases} \boldsymbol{Y}_i \,|\, \boldsymbol{\mu}_i, \Sigma \overset{ind}{\sim} \mathcal{N}_2(\boldsymbol{\mu}_i,\, \Sigma) & i = 1:N \\ \boldsymbol{\mu_i} = \boldsymbol{\beta_0} + X_{i,1}\boldsymbol{\beta_1} + ... + X_{i,p}\boldsymbol{\beta_p} & \\ \boldsymbol{\beta}_0 \,|\, \tau_0^2, \sim \mathcal{N}_2(\mathbf{0},\, \tau_0^2 \mathbb{I}_2) & \\ \boldsymbol{\beta}_j \,|\, \tau_j, a_2 \overset{ind}{\sim} \mathcal{N}_2(\mathbf{0},\, \frac{\tau_j - 1}{\tau_j a_2} \mathbb{I}_2) & j = 1:p \\ \tau_j \,|\, a_1, a_2 \overset{iid}{\sim} \text{trunc-}\mathcal{G}(\frac{1}{2},\, \frac{a_1^2}{8a_2},\, 1, \infty) & j = 1:p \\ a_1, a_2 \overset{iid}{\sim} \mathcal{E}(0.1) + 0.5 & \\ \Sigma = \left[\begin{smallmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{smallmatrix}\right] & \\ \sigma_1, \sigma_2 \overset{iid}{\sim} \mathcal{U}\text{nif}(0, 100) & \\ \rho \sim \mathcal{U}\text{nif}(-1, 1) & \end{cases} \tag{7}$$

Modello gerarchico

$$\begin{cases} \boldsymbol{Y}_{ij} \,|\, \boldsymbol{\mu}_{ij}, \Sigma \overset{ind}{\sim} \mathcal{N}_2(\boldsymbol{\mu}_{ij},\, \Sigma) & i = 1:N;\ j = 1:ng \\ \boldsymbol{\mu}_{ij} = \begin{bmatrix} \boldsymbol{\gamma}_{0j,1} + \boldsymbol{Z}_{ij}^t \gamma_{1j} + \boldsymbol{X}_j^t \boldsymbol{\beta}_1 \\ \gamma_{0j,2} + \boldsymbol{Z}_{ij}^t \boldsymbol{\gamma}_{2j} + \boldsymbol{X}_j^t \boldsymbol{\beta}_2 \end{bmatrix} & \\ \boldsymbol{\gamma}_{0j} \,|\, \hat{\boldsymbol{Y}}, \tau_0^2 \overset{ind}{\sim} \mathcal{N}_2(\hat{\boldsymbol{Y}},\, \tau_0^2 \mathbb{I}_2) & j = 1:ng \\ \boldsymbol{\gamma}_{1j,k} \,|\, \tau_{1,k}^2 \overset{ind}{\sim} \mathcal{N}(0,\, \tau_{1,k}^2) & j = 1:ng; k = 1:v \\ \boldsymbol{\gamma}_{2j,k} \,|\, \tau_{2,k}^2 \overset{ind}{\sim} \mathcal{N}(0,\, \tau_{2,k}^2) & j = 1:ng; k = 1:v \\ \boldsymbol{\beta}_1 \,|\, \omega_1^2 \sim \mathcal{N}_p(\mathbf{0},\, \omega_1^2 \mathbb{I}_p) & \\ \boldsymbol{\beta}_2 \,|\, \omega_2^2 \sim \mathcal{N}_p(\mathbf{0},\, \omega_2^2 \mathbb{I}_p) & \\ \tau_0^2,\, \tau_{1,1}^2, \ldots, \tau_{1,v}^2,\, \tau_{2,1}^2, \ldots, \tau_{2,v}^2 \overset{iid}{\sim} \text{Inv-}\mathcal{G}(2, 50) & \\ 1/\omega_1^2,\, 1/\omega_2^2 \overset{iid}{\sim} \mathcal{E}(100) + 0.5 & \\ \Sigma = \left[\begin{smallmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{smallmatrix}\right] & \\ \sigma_1, \sigma_2 \overset{iid}{\sim} \mathcal{U}\text{nif}(0, 120) & \\ \rho \sim \mathcal{U}\text{nif}(-1, 1) & \end{cases} \tag{8}$$