2.4 Prove the following proposition:

If square matrix A can be divided into four parts:

$A = [IZXY]$

, where I is the identity matrix, Z is all zero and the first column of Y is all zero, **then A is singular.**

Hint: There are multiple ways to prove this problem.

- consider the rank of Y and A
- consider the determinate of Y and A
- consider certain column is the linear combination of other columns

TODO Please use latex （referring to the latex in problem may help）

TODO Proof：

Let I be a square of m * m, since I is a unit matrix, the first m rows of the first column of Y can be obtained by linear combinations of columns of I, and since Z is an all-zero matrix and the first column of Y is Zero, then the first column of Y is the linear combination of the preceding m columns, so A is not full rank, A is the singular matrix

3 Linear Regressions:

3.1 Compute the gradient of loss function with respect to parameters

(Choose one between two 3.1 questions)

We define loss function E as

$$E(m,b)=\sum_{i=1}^{n}(y_i-mx_i-b)^2$$

and we define vertex Y, matrix X and vertex h :

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{bmatrix}, X = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \cdots & \cdots \\ x_n & 1 \end{bmatrix}, h = \begin{bmatrix} m \\ b \end{bmatrix}$$

Proves that

$$E = Y^T Y - 2(Xh)^T Y + (Xh)^T Xh$$

$$\partial E / \partial h = 2X^T Xh - 2X^T Y$$

TODO Please use latex （refering to the latex in problem may help）

TODO Proof：

f(x)=mx + b

Y = $[y_1\ y_2\ \ldots\ y_n]$

, X = $[x_1\ 1\ x_2\ 1\ \ldots\ldots\ x_n\ 1\ ]$

, h = $[m\ b\ ]$

f(X)=Xh

$E(h) = (Y - f(X))^2 = (Y - Xh)^2 = (Y - Xh)^T \times (Y - Xh) = Y^T Y - 2(Xh)^T Y + (Xh)^T Xh$

$[\partial E/\partial m \quad \partial E/\partial b] = \partial E/\partial h = \partial (Y - Xh)^2 / \partial h = -2X^T (Y - Xh) = 2X^T Xh - 2X\, Y$