Two dimensional Kinematics in Rectangular Co-ordinates – Motion of Projectiles

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Contents

Introduction

Projectile:

- When an object or particle is thrown with an initial velocity in the air, in the direction other than vertical and further motion is exclusively under the action of gravity. We observe that the object moves in a curved path(Parabolic), such an object or particle is called projectile and it's motion is called motion of a projectile.

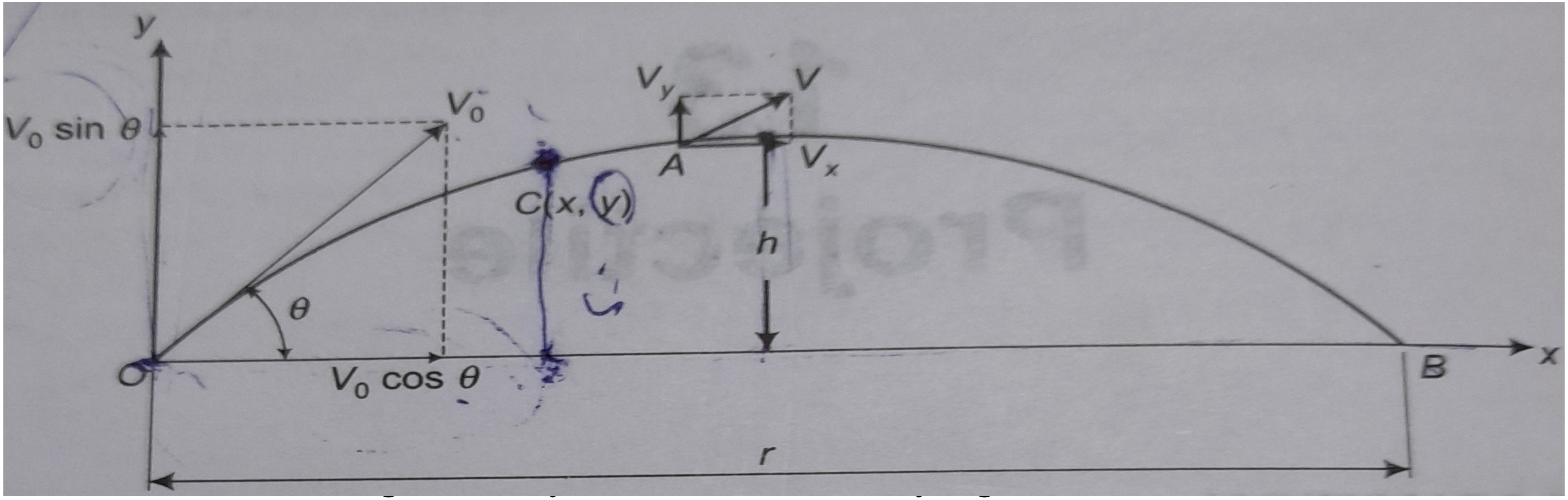


Figure-1

Consideration :

- The resistance of air against the motion is neglected.

Examples

- Throwing a stone other than vertical direction.
- Firing a bullet from a gun.
- Release of a bomb from a fighter jet.
- Throwing a Javelin.
- Fountains

Terminology associated with projectile motions

Velocity of Projection

- The initial velocity with which a projectile is thrown is called velocity of projection.

Angle of Projection

- The angle at which the projectile is thrown with the horizontal is called the angle of projection of the projectile.

Range of Projectile

- Range of projectile is the horizontal distance it covers from the point of origin and the point on the ground, where it touches.

Trajectory of a Projectile

- The path a projectile travels through space is called it’s trajectory. The trajectory of a projectile is Parabolic.

Schematic diagram Representing Projectile motions

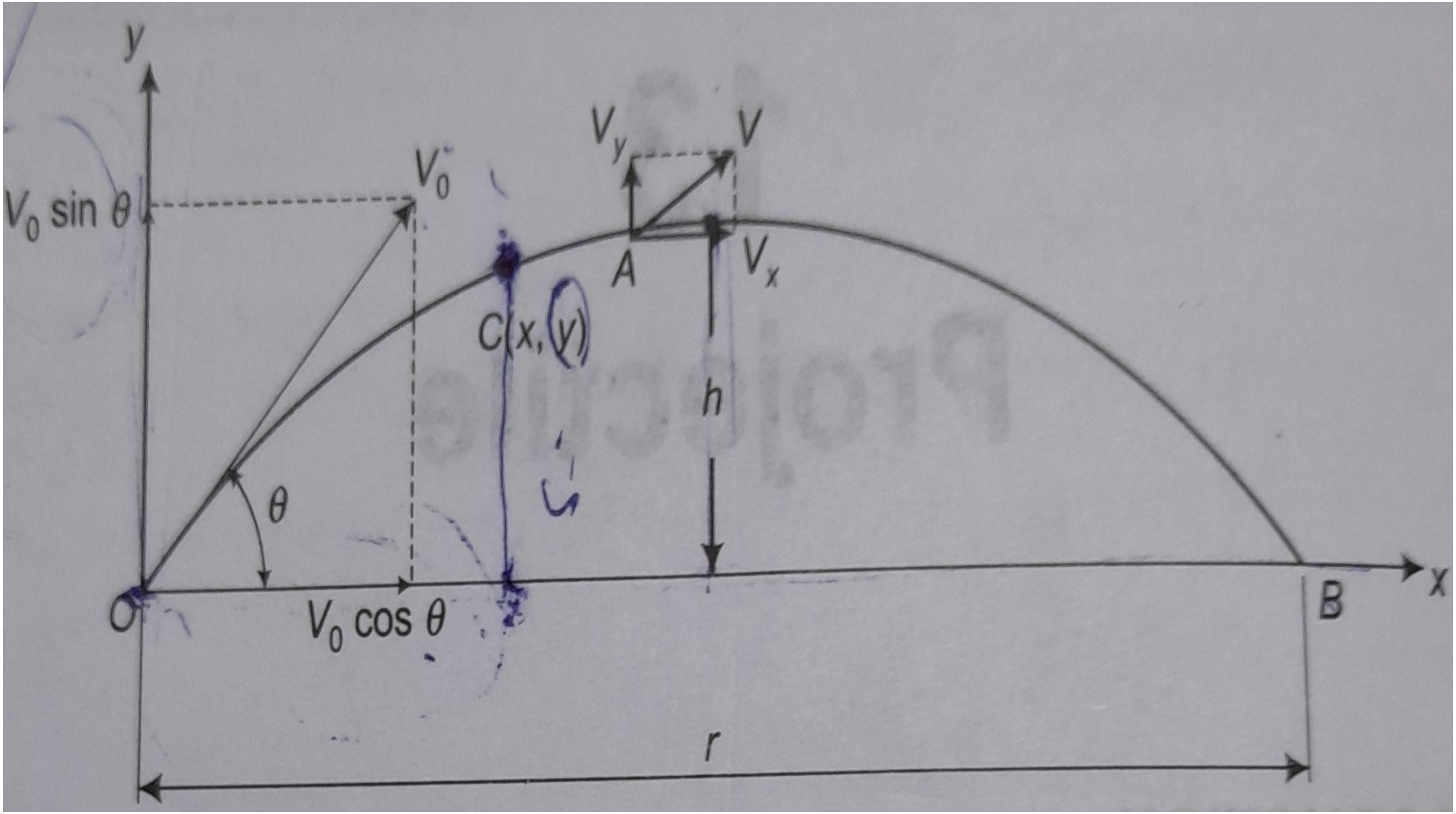


Figure-1

Equation of the Path of Projectile

Let us consider a particle that is thrown from the point O with an initial velocity V_0 that makes an angleθ with the horizontal as shown in figure.

The velocity V_0 has got two components-the horizontal component $v_0 \cos\theta$, the vertical component $v_0 \sin\theta$, .

Since there is no force acting along the horizontal direction, the particle can not have any acceleration in this direction. However , along the vertical direction, It is under the action of gravity. But since the particle is moving against the gravity, It will decelerate with constant deceleration ***g*** **.**

- Let the time of travel of particle is ***t*** corresponds to position ***C***(x,y) .
- Thus the distance covered by the projectile along horizontal is

$$x = v_0 \cos\theta t ....[A]$$

and the distance along the vertical direction is

- $$y = v_0 \sin\theta t - \frac{1}{2} g t^2 ....[B]$$

Eliminating t from Eqs [A] and [B] yields.

$$y = v_0 \sin\theta \frac{x}{v_0 \cos\theta} - \frac{1}{2} g \left(\frac{x}{v_0 \cos\theta}\right)^2$$

Equation of Trajectory

$$y = \tan\theta x - \left(\frac{g}{2v_0^2\cos^2\theta}\right) x^2 \text{.....}[C]$$

➢This equation is in the form of $y = Ax + B\,x^2$ which the eqn of a parabola

Where $A = \tan\theta,$

$$B = -\left(\frac{g}{2v_0^2\cos^2\theta}\right)$$

➢**<u>CASE-I, Horizontal Projection</u>:**

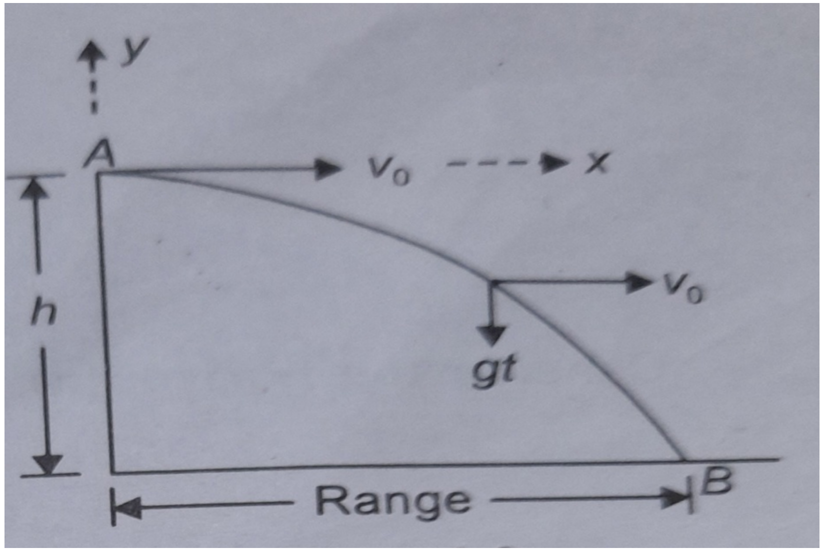
➢Consider a particle thrown horizontally from a point A with a velocity v_0 m/s as shown in figure-2. At any instant the particle is subjected to :

1. The horizontal motion with constant velocity v_0 m/s, i.e. , $v_x = v_{x0} = v_0$
2. The vertical motion with initial velocity zero and moving with acceleration due to gravity g , i.e., $v_y = v_{y0} + at = -gt$ since $v_{y0} = 0$ Let h be the height of A from the ground.

Considering vertical motion ,

$$y_B = -h \quad \Rightarrow \quad -h = 0 \times t - \frac{1}{2} g t^2 \Rightarrow h = \frac{1}{2} g t^2$$

➢From this expression the time of flight can be found . During this period, the particle moves horizontally with uniform velocity, v_0 m/s.



$$Range \;\; (R) = v_0 t$$

<u>Figure-2</u>

Case-II, Inclined Projection on Level ground

Consider the motion of a projectile, Projected from point A with velocity of projection v_0 and angle of projection α as shown in figure-3. Let the ground be a horizontal surface.

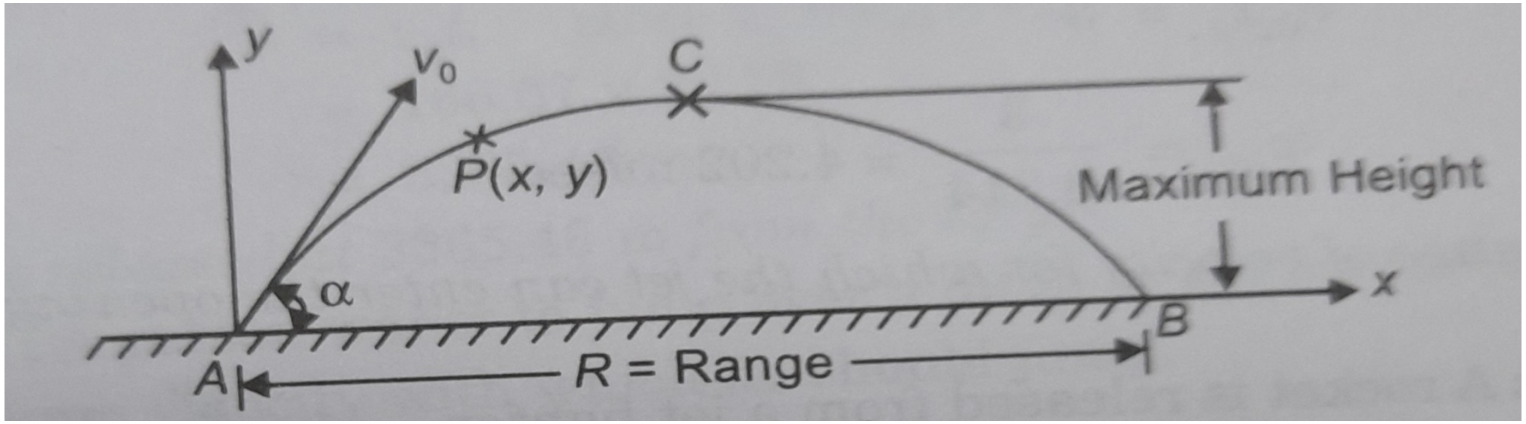


Figure-3

The particle has motion in vertical as well as horizontal directions.

Vertical motion

Initial velocity $v_{y0} = v_0 \sin\alpha$, upwards

Gravitational acceleration =g=9.81 m/s^2, downward .

$$i.e., \quad a = -g = -9.81 m/s^2 \Rightarrow \therefore \; v_y = v_{y0} - gt = v_0 \sin\alpha - gt$$

Hence initially the particle move upward with velocity

$$v_{y0} \sin\alpha \quad \text{and} \quad a = -9.81 m/s^2$$

Continuation from previous slide

Velocity becomes zero after sometime at (at C) and then the particle starts moving downward with g.

Horizontal Motion:

Horizontal component of initial velocity $v_{x0} = v\cos\alpha$ Neglecting air friction , We can say that the projectile is having uniform velocity $v\cos\alpha$ during it's entire flight, i.e. ,

$$v_x = v\cos\alpha$$

Equation of the Trajectory

Let *P(x,y)* represented the position of projectile after *t* seconds. Considering the vertical motion,

$$y = (v_0 \sin \alpha)t - \frac{1}{2} g t^2 \text{.......[1]}$$

Considering horizontal motion,

$$x = (v_0 \cos \alpha)t \text{.......... ..[2]}$$

$$\therefore \quad t = \frac{x}{v_0 \cos \alpha}$$

Substituting this value in eqn[1],we get

$$y = v_0 \sin \alpha \frac{x}{v_0 \cos \alpha} - \frac{1}{2} g \left(\frac{x}{v_0 \cos \alpha}\right)^2$$

i.e.,

$$y = x \tan \alpha - \frac{1}{2} \frac{g x^2}{v_0^2 \cos^2 \alpha} \text{.......[3]}$$

But
$$\frac{1}{\cos^2 \alpha} = \sec^2 \alpha = (1 + \tan^2 \alpha)$$

Hence, eqn [3] reduces to the form

$$\Rightarrow y = x \tan \alpha - \frac{1}{2} \frac{x^2}{v_0^2}(1 + \tan^2 \alpha) \text{...[4]}$$

This is an Equation of a parabola .Hence, The equation of trajectory is a parabola.

Expression for Maximum Height

When the particle reaches maximum height, the vertical component of the velocity will be zero. Considering vertical motion,

Initial velocity $= (v_0 \sin \alpha)$ and Final velocity $= 0$

Acceleration= -g ,Using equation of rectilinear motion

$$v^2 - u^2 = 2as \text{, we get}$$

$$0 - (v_0 \sin \alpha)^2 = -2gh$$

$$h = \frac{v_0^2 \sin^2 \alpha}{2g} \text{........[5]}$$

Time Required to Reach Maximum Height

Using first equation of motion ($v=u+at$), When projectile reaches maximum height,

$$0 = v_0 \sin \alpha - gt$$

$$t = \frac{v_0 \sin \alpha}{g} \text{.........[6]}$$

Time of flight of projectile

Motion of the projectile in vertical direction is given in Eqn [1]as

$$y = (v_0 \sin\alpha)t - \frac{1}{2} g t^2$$

At the end of flight , $y = 0$

$$\therefore \qquad 0 = (v_0 \sin\alpha t) - \frac{1}{2} g t^2$$

$$= t(v_0 \sin\alpha - \frac{1}{2} gt)$$

$$\therefore \quad t = 0$$

Or
$$t = \frac{2 v_0 \sin\alpha}{g}$$

$t=0$, gives initial position`0' of the projectile motion. Hence time of flight is given

$$t = \frac{2 v_0 \sin\alpha}{g} \text{.......[7]}$$

Horizontal Range

During the time of the flight, projectile moves in horizontal direction with uniform velocity $v_0 \cos\alpha$. Hence, the horizontal distance traced by the projectile in this time is given by:

$$R = (v_0 \cos\alpha)t$$

$$= v_0 \cos\alpha \frac{2v_0 \sin\alpha}{g}$$

$$R = \frac{v_0^2 \sin 2\alpha}{g} \text{......[8]}$$

Maximum range

In the Eqn [8],sin2α can have maximum value of 1.

Hence the maximum range $\dfrac{v_0^2}{g}$.......... ..[9]

And the angle of projection for this is when

$$\sin 2\alpha = 1$$

or $2\alpha = 90^o$ i.e, $\alpha = 45^o$

Angle of projection for the required Range

In Eqn[8],we have the expression for range as $R = \frac{v_0^2 \sin 2\alpha}{g}$

$$\therefore \quad \sin 2\alpha = \frac{gR}{v_0^2}$$

Since

$$\sin 2\alpha = \sin(180 - 2\alpha)$$

There are two values of α which give the same result

$$2\alpha_1 = 2\alpha \quad \text{i.e,} \alpha_1 = \alpha$$

And another is

$$2\alpha_2 = 180 - 2\alpha \qquad \text{i, e,} \alpha_2 = 90^o - \alpha$$

$$\alpha_1 + \alpha_2 = 90^o$$

$$\textit{Hence} \quad \text{If} \quad \alpha_1 = 45^\circ + \theta \text{ and } \alpha_2 = 45^\circ - \theta$$

Thus there are two angles of projections for the required range as shown in figure-4

Angle of Projection for the Required Range

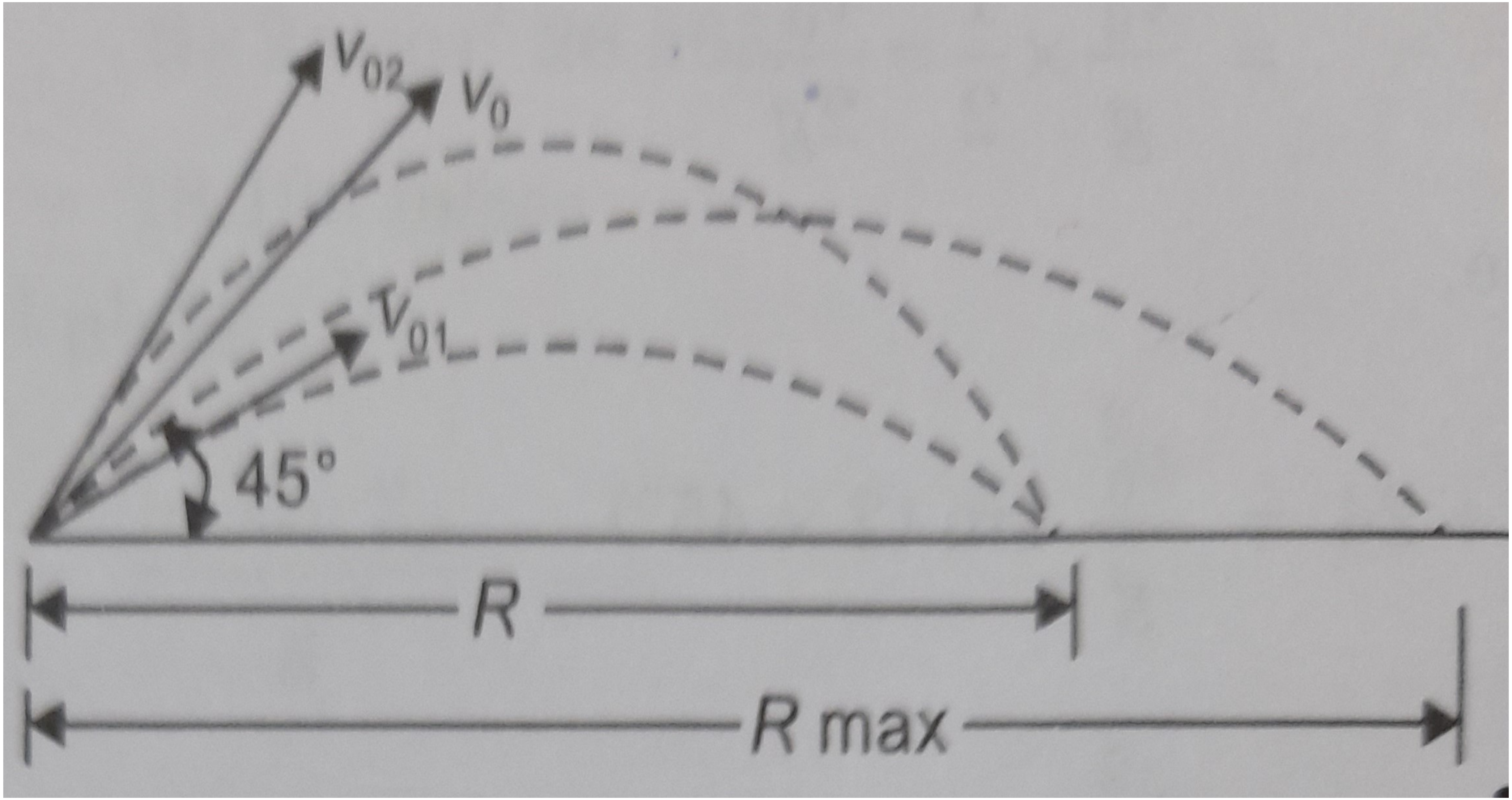


Figure-4

Case-III, Inclined Projection with point of Projection and point of strike at different Levels

Equation 6 & 7 are to be used only when the point of projection and point of striking the ground are at same level. Now let us consider the case , when the point of projection is at a height y0 above the point of strike as shown in Figure-5

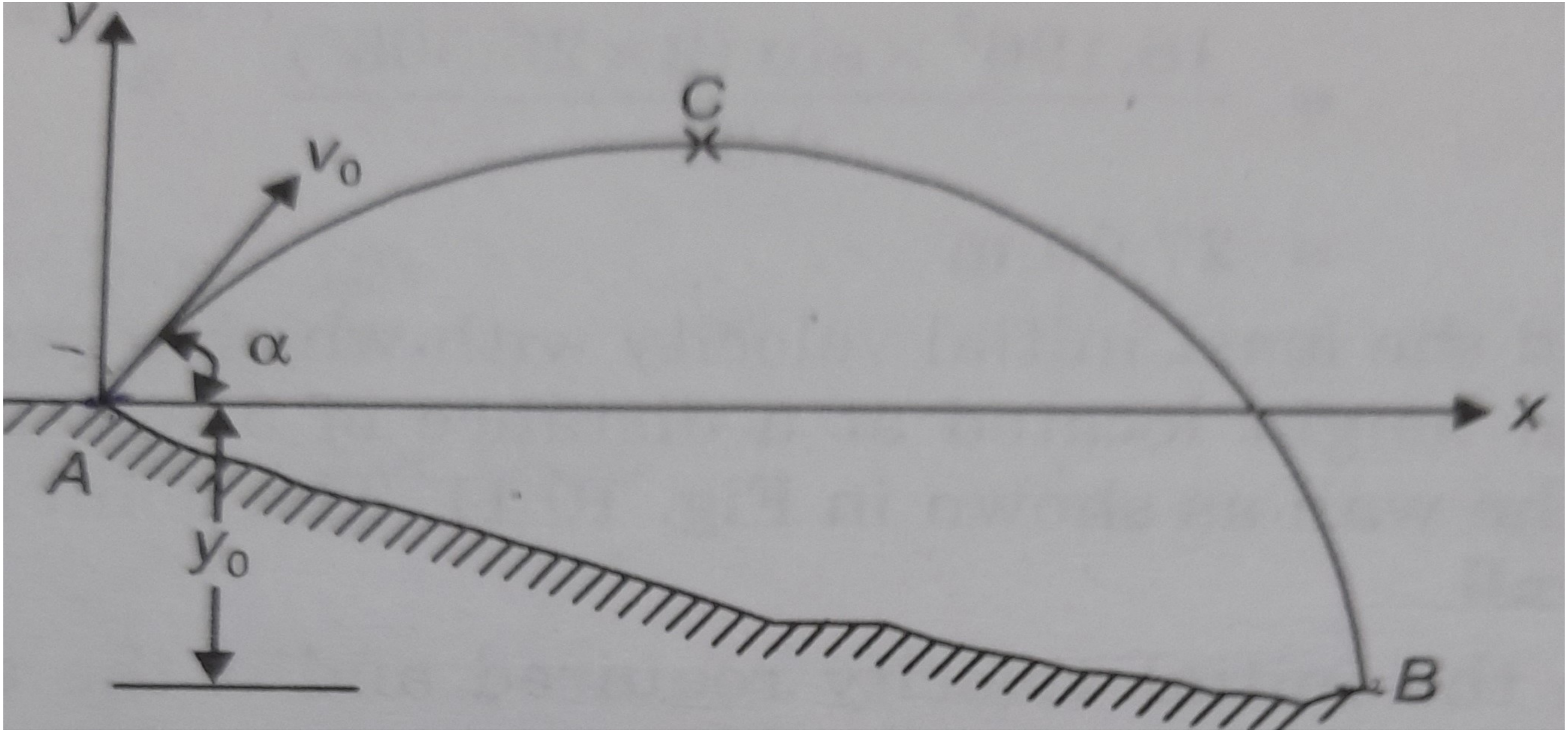


Figure-5

From equation of motion in vertical direction, $y = (v_0 \sin \alpha) \times t \quad \frac{1}{2} g t^2$ By putting $y = -y_0$ in the above equation , the time required to reach B(Time of flight) is

$$-y_0 = (v_0 \sin \alpha) \times t - \frac{1}{2} g t 2$$

Once the time of flight is known , horizontal range can be found from the relation:

$$R = (v_0 \cos \alpha) \times t$$

Maximum height above the point of projection and time required to reach it can be found as usual from equation[5] &[6] .

Case-IV, Projection on Inclined Plane

Let AB be a plane inclined at an angle β to the horizontal as shown in the Figure-6. A projectile is fired up the plane from point A with initial velocity v_0 m/s and an angle α . Now the range on the inclined plane AB and the time of flight are to be determined.

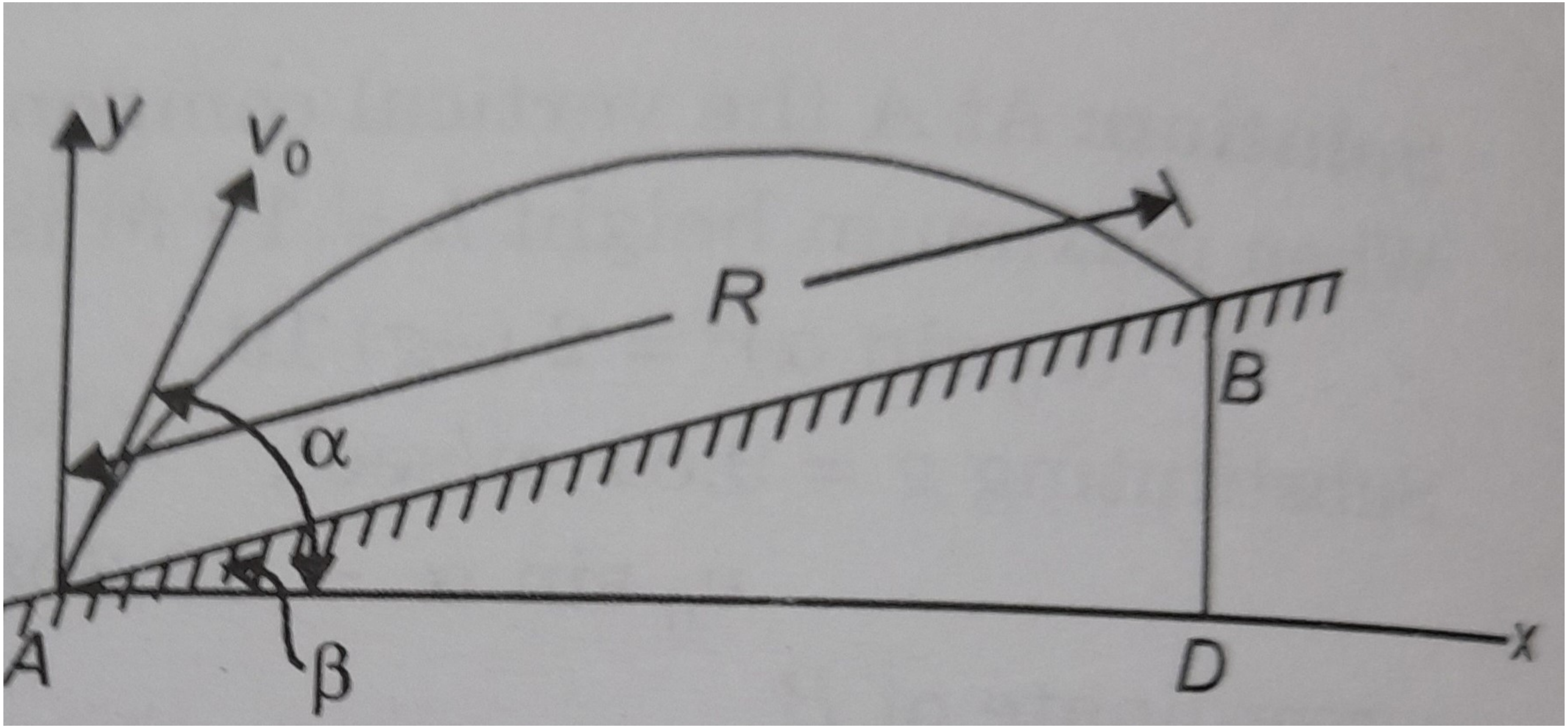


Figure-6

Let the inclined range **AB** be denoted by **R.** **AD** be the corresponding horizontal range.

$$\therefore AD = R\cos\beta \text{ and } \mathrm{DB} = \mathrm{R}\sin\beta$$

The equation of trajectory of the projectile is given by

$$y = x\tan\alpha - \frac{1}{2}\frac{gx^2}{v_0^2\cos^2\alpha}$$

Applying this equation to point B, we get

$$R\sin\beta = R\cos\beta\tan\alpha - \frac{1}{2}\frac{gR^2\cos^2\beta}{v_0^2\cos^2\alpha}$$

i.e.,

$$R\frac{1}{2}\left(\frac{g\cos^2\beta}{v_0^2\cos^2\alpha}\right) = \cos\beta\tan\alpha - \sin\beta$$

$$\therefore \mathrm{R} = \frac{2v_0^2\cos^2\alpha}{g\cos^2\beta}(\cos\beta\tan\alpha - \sin\beta)$$

$$\therefore \mathrm{R} = \frac{2v_0^2\cos\alpha}{g\cos^2\beta}(\cos\beta.\sin\alpha - \sin\beta.\cos\alpha)$$

$$\Rightarrow R = \frac{2v_0^2 \cos\alpha}{g\cos^2\beta}\sin(\alpha - \beta)....[11a]$$

$$\therefore \mathrm{R} = \frac{v_0^2}{g\cos^2\beta}[\sin(2\alpha - \beta) - \sin\beta]....[11b]$$

[Since , 2cosA sin B=sin(A+B)-Sin(A-B)]

Time of Flight:

Let t be the time of flight. The horizontal distance covered during the flight AD as shown in figure-6

$$AD = (v_0\cos\alpha)\times t \Rightarrow t = \frac{AD}{v_0\cos\alpha} = \frac{R\cos\beta}{v_0\cos\alpha}$$

$$\Rightarrow t = \frac{2v_0^2\cos\alpha}{g\cos^2\beta}\times\frac{\sin(\alpha-\beta)}{v_0\cos\alpha}\cos\beta$$

$$\Rightarrow t = \frac{2v_0\sin(\alpha - \beta)}{g\cos\beta}......[12]$$

For the given values of v and β, the range is maximum when :

$\sin(2\alpha-\beta)=1$

$(2\alpha-\beta)=\pi/2$

Or, $\alpha=\pi/4 + \beta/2$.....[13]

Reffering to Figure-7

$\theta_1= \pi/4 + \beta/2- \beta$

$=\pi/4 - \beta/2$

And

$\theta_2= \pi/2 - \beta$

Thus, $\theta_2=2\ \theta_1$

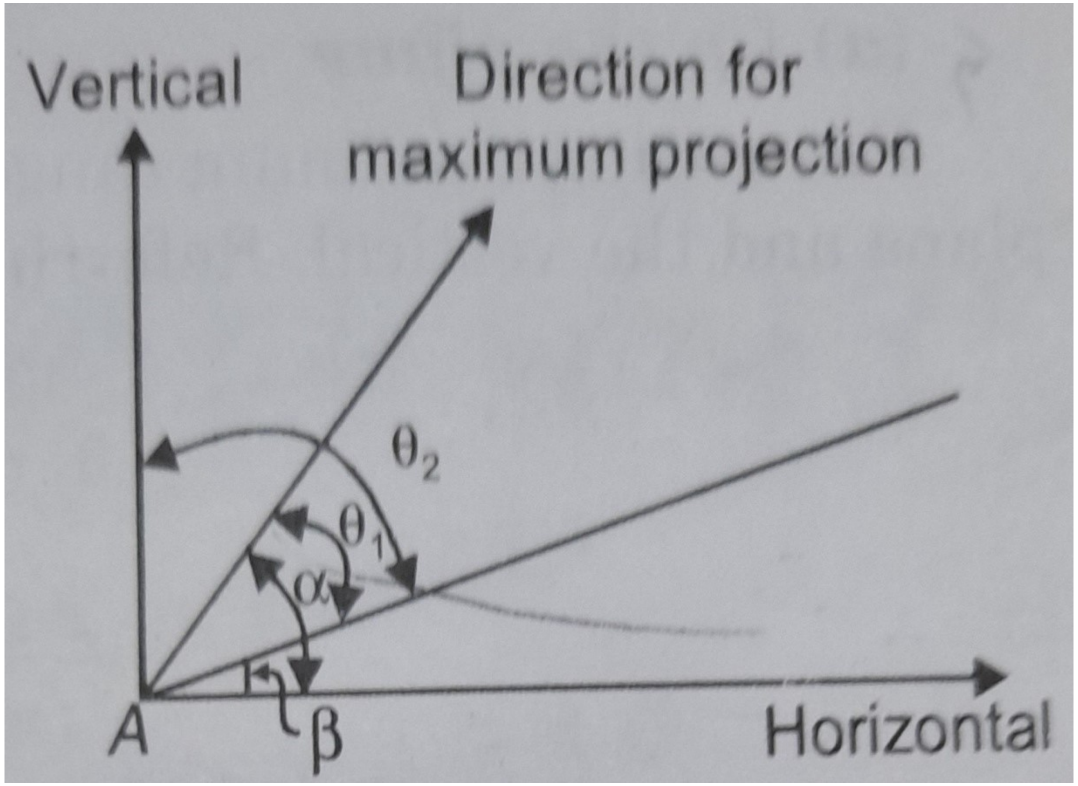


Figure-7

i.e The Range on the given plane is maximum , when the angle of projection bisects the angle between the vertical and inclined planes. If the projection is down the plane , the equation 11 to 13 can be still used but the value of β should be taken negative.

NUMERICAL

[1]A bullet is fired from a rifle with an initial velocity of 50 m/s so as to just clear a vertical wall of 20 m heigh and located at a distance of 30 m measured horizontally from the point of projection. Find the two angles of projections with horizontal.

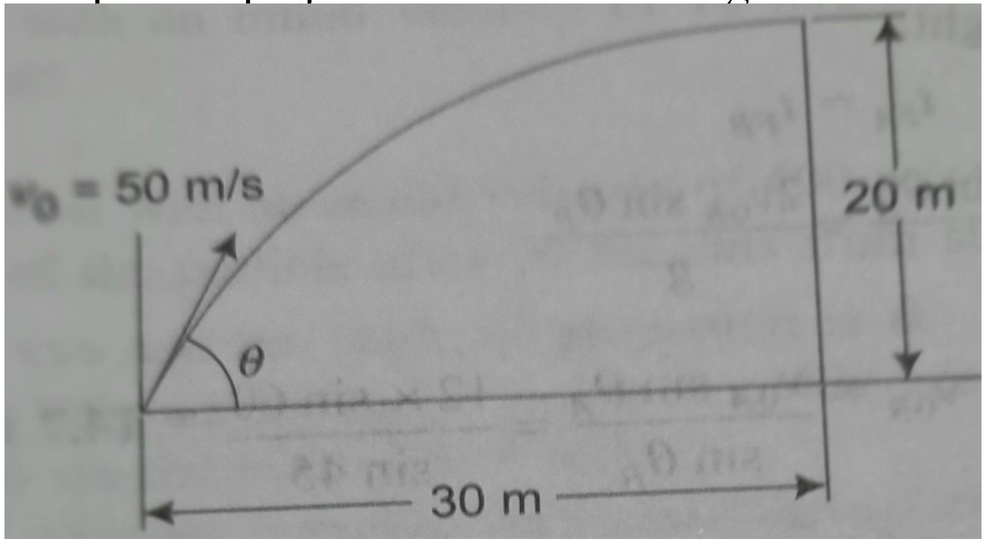


Figure-8

Solution The trajectory of the bullet can be expressed by Using the equation below

$$y = \tan\theta x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

From the given situation,the top of the wall will lie on the Trajectory, Thus the coordinate of the top of the wall(30 m, 20m) satisfies the equation .
Thus we get

$$20 = \tan\theta \times 30 - \left(\frac{g}{2\times 50^2\cos^2\theta}\right)30^2$$

$$1.7658\tan^2\theta + 30\tan\theta - 21.7658 = 0$$

Solving the given equation, we get θ=86.76º or 34.87º

[2]A person throws a stone so as to clear a wall of height 3.685 m located at a distance 5.25 m from the origin.The stone touches the ground at a distance of 3.58 m from the wall- away from the origin. Find the least initial velocity at which the stone to be thrown along with it's direction.

Solution:

The range of the stone R =5.25+3.58 m = 8.83 m

The trajectory of the stone can be expressed by

$$y = v_0 \tan\theta x - \left(\frac{g}{2v_0^2\cos^2\theta}\right) x^2$$

From the given situation, the top of the wall will lie on the trajectory. Thus the coordinate of the top of the wall (5.25 m, 3.685 m) will satisfy the equation.

$$Thus \qquad 3.685 = \tan\theta \times 5.25 - \left(\frac{g}{2v_0^2\cos^2\theta}\right) 5.25^2$$

Further range of the stone =

$$r = \frac{v_0^2 \sin 2\theta}{g} = 8.83 \Rightarrow \frac{v_0^2}{g} = \frac{8.83}{\sin 2\theta}$$

[Comparing two equation we get

$$Thus \qquad 3.685 = \tan\theta \times 5.25 - \left(\frac{\sin 2\theta}{2\times 8.83\times\cos^2\theta}\right) 5.25^2$$

$$\Rightarrow \quad 3.685 = \tan\theta \times 5.25 - \left(\frac{2\sin\theta\cos\theta}{2\times 8.83\times\cos^2\theta}\right) 5.25^2$$

$$3.685 = 5.25\tan\theta - 3.12\tan\theta = 2.13\tan\theta \Rightarrow \tan\theta = \frac{3.685}{2.13} = 1.73 \Rightarrow \theta = 60°$$

Thus,From the given situation, the top of the wall will lie on the trajectory. Thus the coordinate of the top of the wall (5.25 m, 3.685 m) will satisfy the equation.

$$\Rightarrow \frac{v_0^2}{g} = \frac{8.83}{\sin 120°} \qquad \Rightarrow \frac{8.83\times 9.81}{\sin 120°} = 100$$

v_0= 10 m/s **Ans**

[3] A bullet is fired at an angle of 30º up the horizontal with velocity 100 m/s from the top of a tower, 50 m high. Determine followings

(i) The time of Flight (ii) The horizontal range along the ground.(iii) The maximum height the bullet can attain from the ground.(iv) The velocity of the bullet after 6 s.

Assume horizontal ground at the foot of tower.

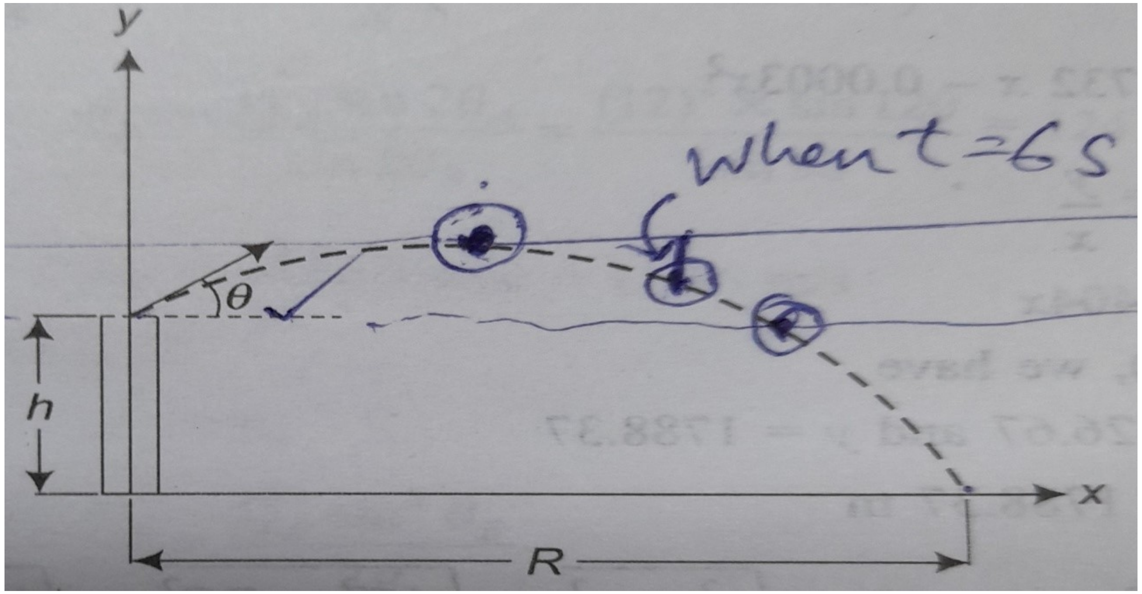


Figure-8

Solution:

The initial velocity v_o has got two components-the horizontal component being $v_0\cos\theta$,the vertical component is $v_0\sin\theta$.

Let the total time of flight is T s. Also consider the projectile will cover h' m height above the tower to reach it's peak and it's corresponding time is t' and during the fall, it cover h m in t s.Thus T=2t'+t

$$v_0 \sin\theta - gt' = 0$$

$$2t' = \frac{2v_0 \sin\theta}{g} = \frac{2 \times 100 \sin 30^\circ}{9.81} = 10.194\ s$$

Further, $h = v_0(\sin\theta)t + \frac{1}{2}gt^2 \quad 50 = 100(\sin 30^\circ)t + \frac{1}{2} \times 9.81 \times t^2 \Rightarrow t = 0.92\ s$

Thus total time of flight becomes T=2t'+t=10.194+0.92=11.114 s.

Now, $0=(v_0\sin\theta)^2-2gh'$

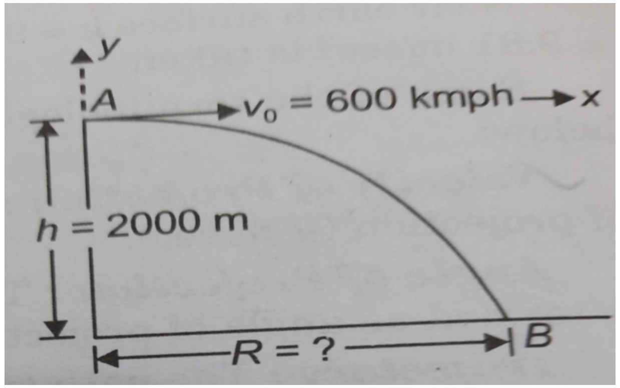
$$h' = \frac{(100 \sin 30^\circ)^2}{2 \times 9.81} = 127.42\ m$$

Total height above the ground becomes H=h'+h=127.42+50=177.42 m **Ans**

The range $R=(v_0\cos\theta)T=100\cos30^o \times 11.114=962.5$ m. Since t'=5.097 s , after 6s the projectile on it's downward movement from the peak height. Thus the downward component of velocity after 6 s if denoted by $v_{v,6}=g \times 0.903=8.858$ m/s. The horizontal component being unchanged at $V_0\cos\theta$ implying $v_{h,6}=100\cos30^o=86.6$ m/s. Thus resultant velocity after 6s is

$v_6=\sqrt{((v_{h,6})^2+(v_{v,6})^2)}= \sqrt{((86.6)^2+(8.858)^2)}=87.05$ m/s

[4] A pilot flying his bomber at a height of 2000 m with a uniform horizontal velocity of 600 km/hr wants to strike a target (ref Fig) . At what distance from the target , he should release the bomb ?



Solution : $y_B = -h = -2000m$

$v_{x0} = 600$ km/h

$= 600 \times 1000/60\times60$

$= 166.67$ m/sec

Initial velocity is vertical direction= $v_{y0} = 0$, and gravitational acceleration = 9.81 m/s^2

If t is the time of flight, considering vertical motion we get

$$-2000=0\times t- ½\times9.81t^2$$

$t = 20.19$ s

During this period horizontal distance travelled by the bomb

$= v_0 t$, since $v_x = v_{x0} = v_0$

$= 166.67 \times 20.19$

$= 3365.46$ m

Bomb should be released at 3365.46 m from the target.

[5] A person wants to jump over a ditch as shown in figure .Find the minimum velocity with which he should jump.

Solution; Taking A as origin

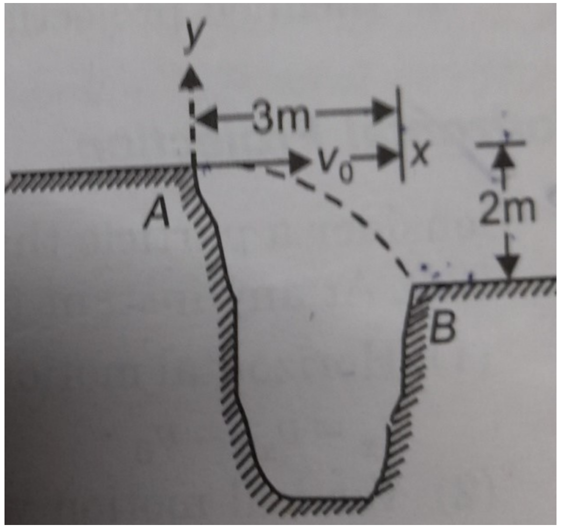
$$y_B = -h = -2 \text{ m and range} = 3 \text{ m}$$

Let t be the time of flight and v_{x0} *the* minimum horizontal velocity required. Considering the vertical motion $-h = -1/2gt^2$

$$2 = \tfrac{1}{2}\, 9.81\, t^2$$

$t = 0.6386\ s.$

Considering horizontal motion of uniform velocity, we get



$3 = v_0 \times 0.6386$ since $v_x = v_{x0} = v_0$

Therefore $v_0 = 4.698\ m/s$

[6] A projectile is aimed at a target on the horizontal plane and falls 12m short when the angle of projection is 15^o , while overshoots by 24 m when the angle is 45^o Find the angle of projection to hit the target.

Solution:

Let s be the distance of the target from the point of projection and v0 be the velocity of projection.

Range of projection is given by the expression

$$R = \frac{v_0^2 \sin 2\alpha}{g}$$

Applying it to first case ,

$$s - 12 = \frac{v_0^2 \sin(2 \times 15°)}{g} = \frac{v_0^2}{g} \times \frac{1}{2} = \frac{v_0^2}{2g} .....[1]$$

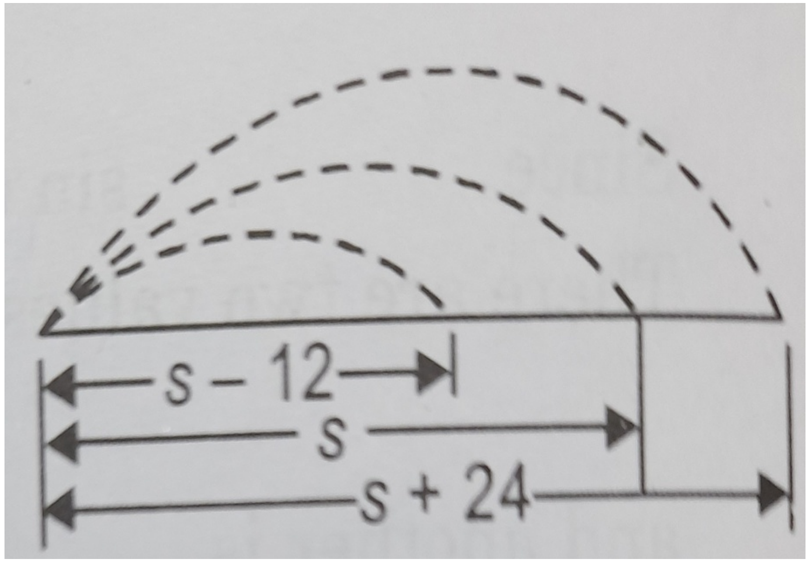
Applying it to second case

$$s + 24 = \frac{v_0^2 \sin(2 \times 45°)}{g} = \frac{v_0^2}{g} ....[2]$$

From eqn [1] & [2] we get $s + 24 = 2(s - 12) \Rightarrow s = 48m$

Let the correct angle of projection be α, then

$$48 = \frac{v_0^2 \sin 2\alpha}{g} \quad .....[3]$$



From eqn[2] ,

$$s + 24 = \frac{v_0^2}{g} = 48 + 24 = 72\ m$$

From equation [3] we get

$$48 = 72 \sin 2\alpha \quad \Rightarrow 2\alpha = 41.81° \Rightarrow \alpha = 20.905°$$

[7] A particle is thrown with an initial velocity of 12m/s at angle 60^o with the horizontal .If another particle is thrown from the same position at an angle 45^o with the horizontal , Find the velocity of latter for the following situations.

(a) Both have same horizontal range

(b) Both have same maximum height

(c) Both have same time of flight.

Solution:

Let first particle be A and second particle be B . We have given data as V_{0A}=12 m/s and $\theta_A=60^o$ and $\theta_B=45^o$

(a) Both have same horizontal range

$$r_A = r_B$$

$$\frac{V_{0A}^2 \sin 2\theta_A}{g} = \frac{V_{0B}^2 \sin 2\theta_B}{g}$$

$$V_{0B}^2 = \frac{V_{0A}^2 \sin 2\theta_A}{\sin 2\theta_B}$$

$$V_{0B}^2 = \frac{(12)^2 \cdot \sin 120^\circ}{\sin 90^\circ}$$

$$V_{0B} = \sqrt{\frac{12^2 \sin(90^\circ + 30^\circ)}{1}}$$

$$V_{0B} = \sqrt{144 \times \cos 30^\circ}$$

$$V_{0B} = \sqrt{144 \times \left(\frac{\sqrt{3}}{2}\right)}$$

$$V_{0B} = \sqrt{124.707} \quad m/s$$

$$= 11.167 \ m/s$$

(b) both have same maximum height.

$$\frac{V_{oA}^2 \sin^2 \theta_A}{2g} = \frac{V_{oB}^2 \sin^2 \theta_B}{2g}$$

$$V_{oB}^2 = \frac{V_{oA}^2 \sin^2 \theta_A}{\sin^2 \theta_B}$$

$$= \frac{12^2 \sin^2 60^\circ}{\sin^2 45^\circ}$$

$$= \frac{144 \times \left(\frac{\sqrt{3}}{2}\right)^2}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$V_{oB}^2 = \frac{144 \times 3 \times \cancel{2}}{\cancel{4}\ 2}$$

$$V_{oB}^2 = 72 \times 3$$

$$V_{oB}^2 = 216$$

$$V_{oB} = \sqrt{216}\ m/s$$

$$V_{oB} = 14.7\ m/s$$

(c) Both have same time of flight

$$t_{FA} = t_{FB}$$

$$\frac{\cancel{2} V_{0A} \sin\theta_A}{\cancel{g}} = \frac{\cancel{2} V_{0B} \sin\theta_B}{\cancel{g}}$$

$$V_{0B} = \frac{V_{0A} \sin\theta_A}{\sin\theta_B}$$

$$= \frac{12 \times \sin 60^\circ}{\sin 45^\circ}$$

$$= \frac{12 \times \sqrt{3} \times \sqrt{2}}{2 \times}$$

$$= 6\sqrt{6}$$

$$= 14.69 \text{ m/s}$$

$$\boxed{V_{0B} = 14.7 \text{ m/s}}$$

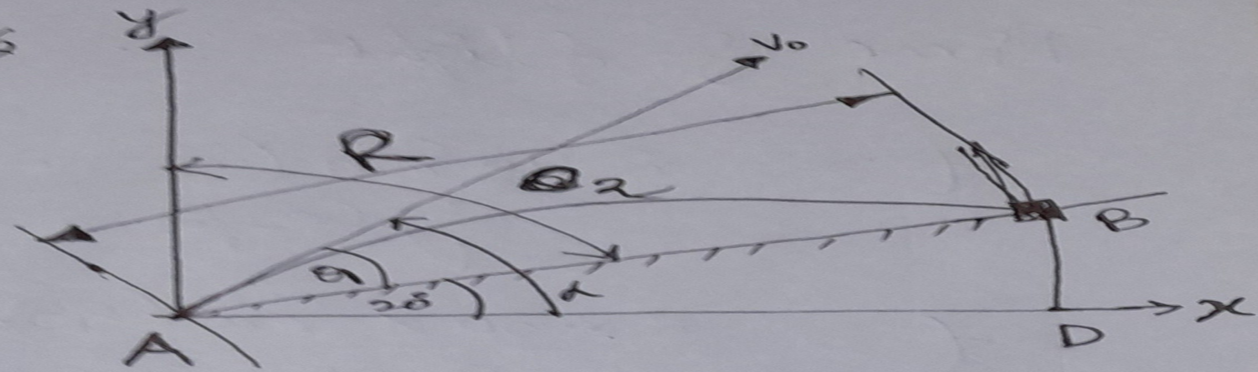
(12) A particle is projected from a point on an inclined with a velocity of 30 m/s. The angle of projection and angle of the plane ~~are~~ 55° and 20° to the horizontal respectively. Find the range and time of flight of particle and show the range is maximum

Inputs

$v_0 = 30\ m/s$

$\alpha = 55^\circ$

$\beta = 20^\circ$



$$R = \frac{v_0^2}{g\cos^2\beta}\left[\sin(2\alpha-\beta) - \sin\beta\right]$$

$$R = \frac{30^2}{9.81\cos^2 20^\circ}\left[\sin(2\times55^\circ - 20^\circ) - \sin 20^\circ\right]$$

$$= \frac{30\times30}{9.81\cos^2 20^\circ}\left[\sin 90^\circ - \sin 20^\circ\right]$$

$$= \frac{30\times30}{9.81\cos^2 20^\circ}\left[1 - \sin 20^\circ\right]$$

$$= \frac{900}{9.81\times\cos^2 20^\circ}\left[1 - \sin 20^\circ\right]$$

$$= 68.36\ m$$

$$\theta_1 = \alpha - \beta = \frac{\pi}{4} + \frac{\beta}{2} - \beta$$

$$\theta_1 = \frac{\pi}{4} - \beta/2 = 35^\circ$$

$$\theta_2 = \frac{\pi}{2} - \beta = 70^\circ$$

$\theta_2 = 2\theta_1$ so Range R is Maximu

Time of flight

$$t = \frac{2u_0 \sin(\alpha - \beta)}{g \cos\beta}$$

$$t = \frac{2 \times 30 \times \sin(55^\circ - 20^\circ)}{9.81 \times \cos 20^\circ}$$

$$t = \frac{60 \times \sin 35^\circ}{9.81 \times \cos 20^\circ}$$

$$t = \frac{60 \times 0.574}{9.81 \times 0.940}$$

$$\boxed{t = 3.735\ s}$$ Ans.

$R = 68.36 m$ $t = 3.735 s$

Ans

Thank You…

Q&A

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