University Roll No. - T91/ECE/204058

Subject - PHYSICS

Semester - 2nd

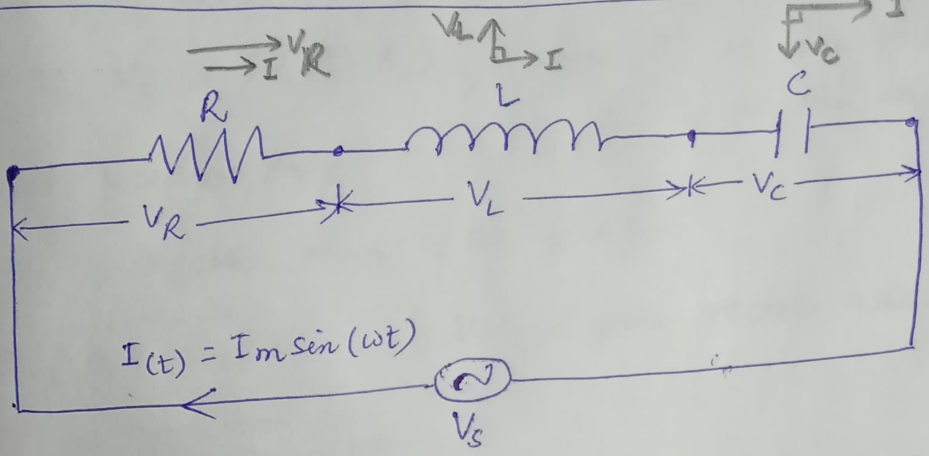
Paper Code - PH201

Date-of-examination - 10.08.2021

Signature - Archane Kri.

Group - A :

(1.) a) Derivation for obtaining the condition for flowing maximum current in a series LCR circuit biased with an ac power source :-



Circuit diagram

Let, a resistance, R ; an inductance, L & a capacitor, C are connected in series with an a.c. supply source, V_s.

Let, their respective potential across them be V_R, V_L and V_C.

Then, $V_s = V_0 \sin \omega t$. (where, V_0 is the peak voltage)

Now, let,

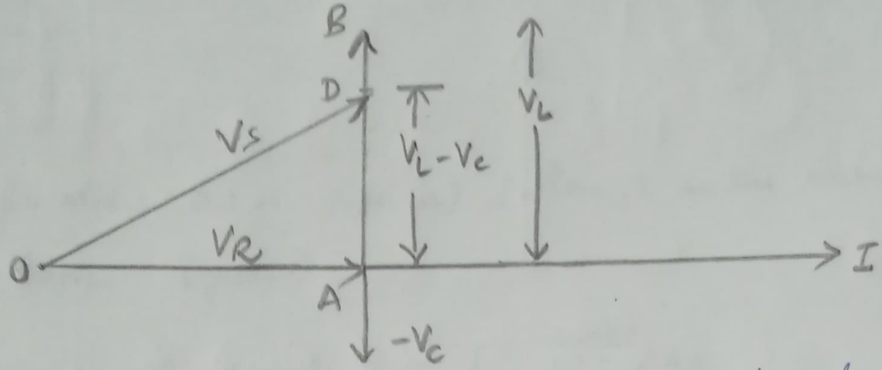
$V_R = IR$, drop across R inphase with I

$V_L = I X_L$, drop across L, leads I by 90°

$V_C = I X_C$, drop across C, lags I by 90°

~~QA~~

The corresponding phasor diagram for series LCR is :-



~~Note~~, This the vector diagram (also called as, impedance triangle)

Now, $OA = V_R$, $AB = V_L$, & $AC = V_c$

& $AD = V_L - V_c = I(X_L - X_c)$, $\left\{\begin{array}{l}\text{where, } X_L \text{ is the inductive reactance,} \\ \quad \& \ X_C \text{ is the capacitive reactance.}\end{array}\right.$

$\Rightarrow AD =$ net reactive drop.

$\rightarrow OD = V_s =$ vector sum of OA & AD.

$\Rightarrow OD = \sqrt{OA^2 + AD^2}$ (Using ~~pata~~ vector addition)

Or, $V_S = \sqrt{V_R^2 + (V_L - V_c)^2}$

$= \sqrt{(IR)^2 + (IX_L - IX_c)^2}$

$= I\sqrt{R^2 + (X_L - X_c)^2}$

$\Rightarrow I = \dfrac{V_s}{\sqrt{R^2 + (X_L - X_c)^2}}$, where, $\sqrt{R^2 + (X_L - X_c)^2}$ is called the impedance of the circuit, Z.

Now, for current to be maximum, impedance is minimum.

$\Rightarrow \left(\sqrt{R^2 + (X_L - X_c)^2}\right)_{min}$, when $\boxed{X_L = X_C}$.

$\Rightarrow I = \dfrac{V_s}{Z} = \dfrac{V_s}{R}$

So, for maximum current in series LCR,
i) $X_L = X_C$
ii) $Z = R \rightarrow Z_{(min)} = R$
iii) $I_{max} = \dfrac{V_s}{R}$

b) For maximum current, $X_L = X_C$ (to get minimum impedance)

$\Rightarrow \omega L = \frac{1}{\omega C}$

$\Rightarrow \boxed{C = \frac{1}{\omega^2 L}}$

Now, $L = 100\,mH = 0.1\,H$ (given) (where, L is the inductance.)

frequency, $f = 50\,Hz$ (given)

Then, capacitance, $C = \frac{1}{(2\pi f)^2 \cdot L}$ $\{\because$ using $\underline{\omega = 2\pi f}\}$.

$\Rightarrow C = \frac{1}{(2 \cdot \pi \cdot 50)^2 (0.1)}$

$\Rightarrow C = \frac{1}{1000\pi^2}\,F = 0.00010132\,F$

$= 101.32 \times 10^{-6}\,F$

$= \underline{101.32\,\mu F}$.

~~c) Effect of humidity of air on velocity of sound :-~~

c) The maximum current through the circuit is obtained when, $X_L = X_C$. (where, X_C is capacitive reactance & X_L is ~~cap~~

$\Rightarrow$ Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

when $X_L = X_C$,

$\Rightarrow Z = \sqrt{R^2 + 0^2}$

$\Rightarrow Z = R \longrightarrow \boxed{Z_{min} = R}$

So, $I_{max} = \frac{V}{Z_{min}} = \frac{V}{R} = \frac{220}{10} = \underline{22\,A}$

$\{\because$ Max^m current $= \frac{\text{Voltage}}{\text{Impedance}}\}$.

② a) Doppler effect:

→ whenever there is a relative motion between the source of sound and the listener, the pitch (frequency) of the sound, heard by the listener differs from the true pitch.

→ This phenomenon of the apparent change in frequency of sound owing to the relative motion of the source and the listener is known as Doppler effect.

b)
$$\boxed{f_o = \frac{v \pm v_o}{v \pm v_s} f_s}$$

where, f_o = observer frequency of sound
v = speed of sound wave
v_o = observer velocity
v_s = source velocity
f_s = actual frequency of sound waves.

Now, A/c to question:
speed of sound, $v = 340$ m/s.
speed of source, $v_s = 36$ km/h $= 10$ m/s
frequency of siren, $f_s = 500$ Hz

Case I: Direct waves:

The, apparent frequency of direct waves, $f_o = \frac{v}{v + v_s} f_s$

$$\Rightarrow f_o = \frac{340}{340+10} \times 500 = 486 \text{ Hz}$$

Case II: Reflected waves:

~~frequency~~ $f_o' = \frac{v}{v - v_s} \cdot f_s = \frac{340}{340-10} \times 500 = 515$ Hz {where, f_o' is the frequency received by the wall.}

Thus, the wall reflects this sound without changing the frequency and as there is no relative motion between the observer & wall, the frequency of the reflected wave as head by observer is 515 Hz.

c) Effect of humidity of air on the velocity of sound :-

we know, moist air is less dense than dry air.

So, the velocity of sound is greater in moist air than in dry air.

Let, V_m be the velocity of sound wave in moist air of density, ρ_m.

& V_d be the velocity of sound wave at same pressure of density, ρ_d

Let, p be the pressure exerted by moisture.

Then, Proportion of dry air in each unit volume of moist air is $\left(\frac{P-p}{P}\right)$ and the moisture is (p/P).

So, mass of unit volume of moist air at a given temperature, density;

$$\rho_m = (P-p)\frac{\rho_d}{P} + \frac{p}{P} \times 0.62\,\rho_d \quad \left\{ \because \frac{\rho_d}{\rho_m} = \frac{1}{0.62} \right\}$$

$$= (P - 0.38p)\frac{\rho_d}{P}$$

Therefore, we get from the above relation;

$$\frac{\rho_m}{\rho_d} = \frac{P - 0.38p}{P}, \quad \& \quad \frac{V_d}{V_m} = \sqrt{\frac{\rho_m}{\rho_d}}$$

$$\therefore \frac{V_d}{V_m} = \sqrt{\frac{P - 0.38p}{P}}$$

Thus, the velocity of sound in moist air is greater than in dry air.

---

(3) a) Given, mass of body suspended = 1 kg

spring's stiffness constant, k = 25 N/m

Also, the suspended frequency = $\frac{2}{\sqrt{3}}$ × damped frequency —— (1)

To find : the damping factor.

Calculation: we know, suspended frequency $= \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

and damped frequency $= \frac{1}{2\pi}\sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$

where, b is the damping factor.

Using the above eqns. and eqn ①, we get;

$$\frac{\cancel{1}}{2\pi}\sqrt{\frac{K}{m}} = \frac{2}{\sqrt{3}} \cdot \frac{\cancel{1}}{2\pi}\sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$$

On squaring both sides, we get,

$$\Rightarrow \frac{K}{m} = \frac{4}{3}\left(\frac{K}{m} - \frac{b^2}{4m^2}\right)$$

$$\Rightarrow \frac{4}{3} \cdot \frac{b^2}{4m^2} = \left(\frac{4}{3} - 1\right)\frac{K}{m}$$ (On rearranging the terms)

$$\Rightarrow \frac{b^2}{3m^2} = \frac{1}{3} \cdot \frac{K}{m}$$

$$\Rightarrow b^2 = Km$$

$$\Rightarrow \boxed{b = \sqrt{Km}}$$

On putting the values of K and m given in the question, we get,

$$b = \sqrt{25 \times 1 \, N kg/m}$$

$$b = \sqrt{25 \; kg^2/s^2}$$

$$b = 5 \; kg/s = 5 \; Ns/m$$

Thus, the damping factor is 5 kg/s. or 5 Ns/m

b) Critically damping oscillation is defined as the condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position, without oscillating back and forth about its mean position.

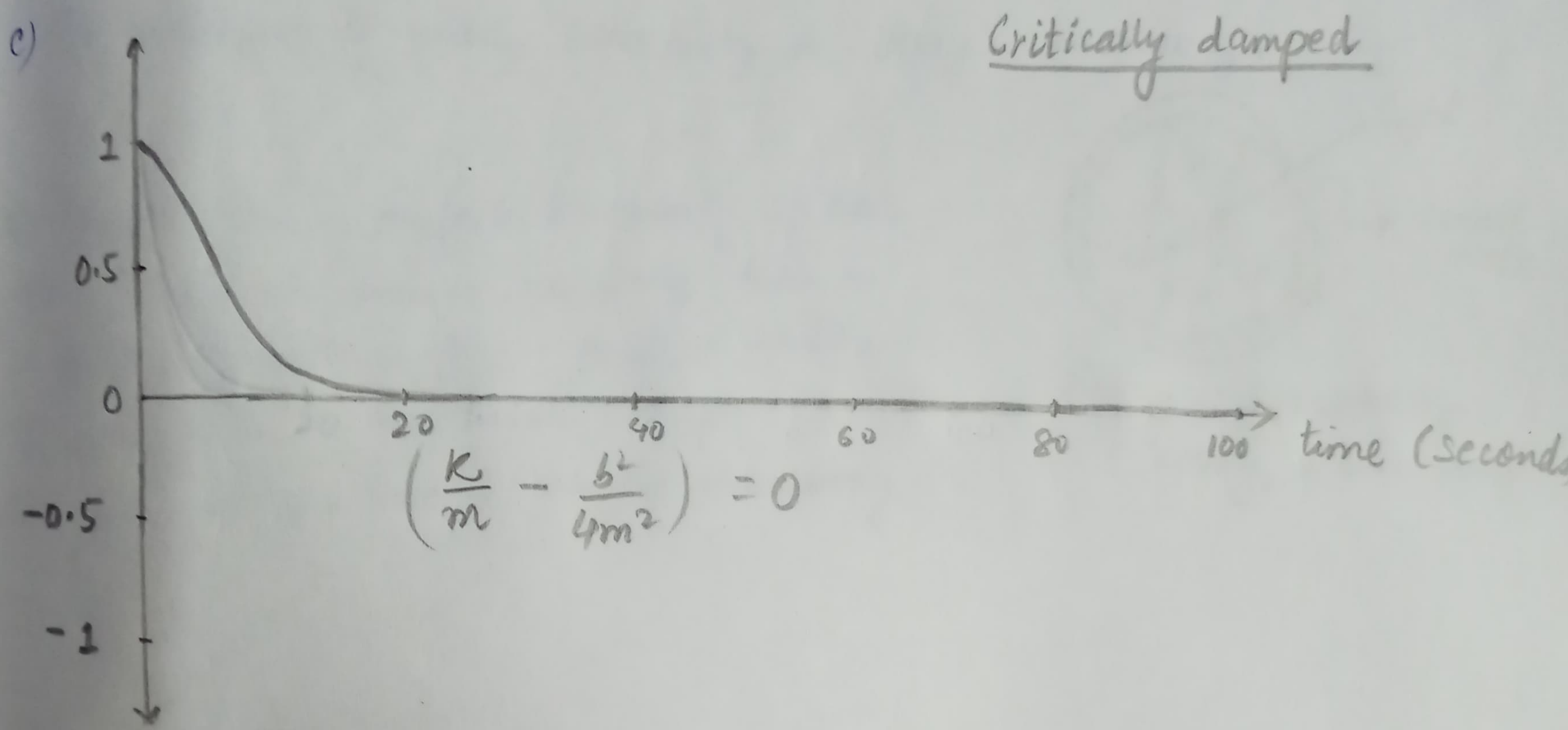
→ The automobile shock absorber is an example of a critically damped device.

The condition for critically damped harmonic oscillator is:

$$\left(\frac{k}{m} - \frac{b^2}{4m^2}\right) = 0$$

$$\Rightarrow (b^2 - 4mk) = 0$$

c)



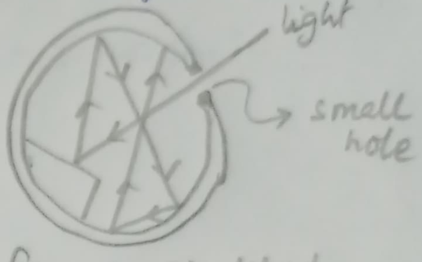
Critically damped harmonic oscillator with $k = 30$ N/m, $m = 20$ kg, $b = 50$ N s/m

Group B (4 ques.)

(6) a) Blackbody is a material object that absorbs all the radiation falling on it and hence, appears as black under reflection when illuminated from outside.

→ A blackbody is a perfect absorber & a perfect emitter of radiation.

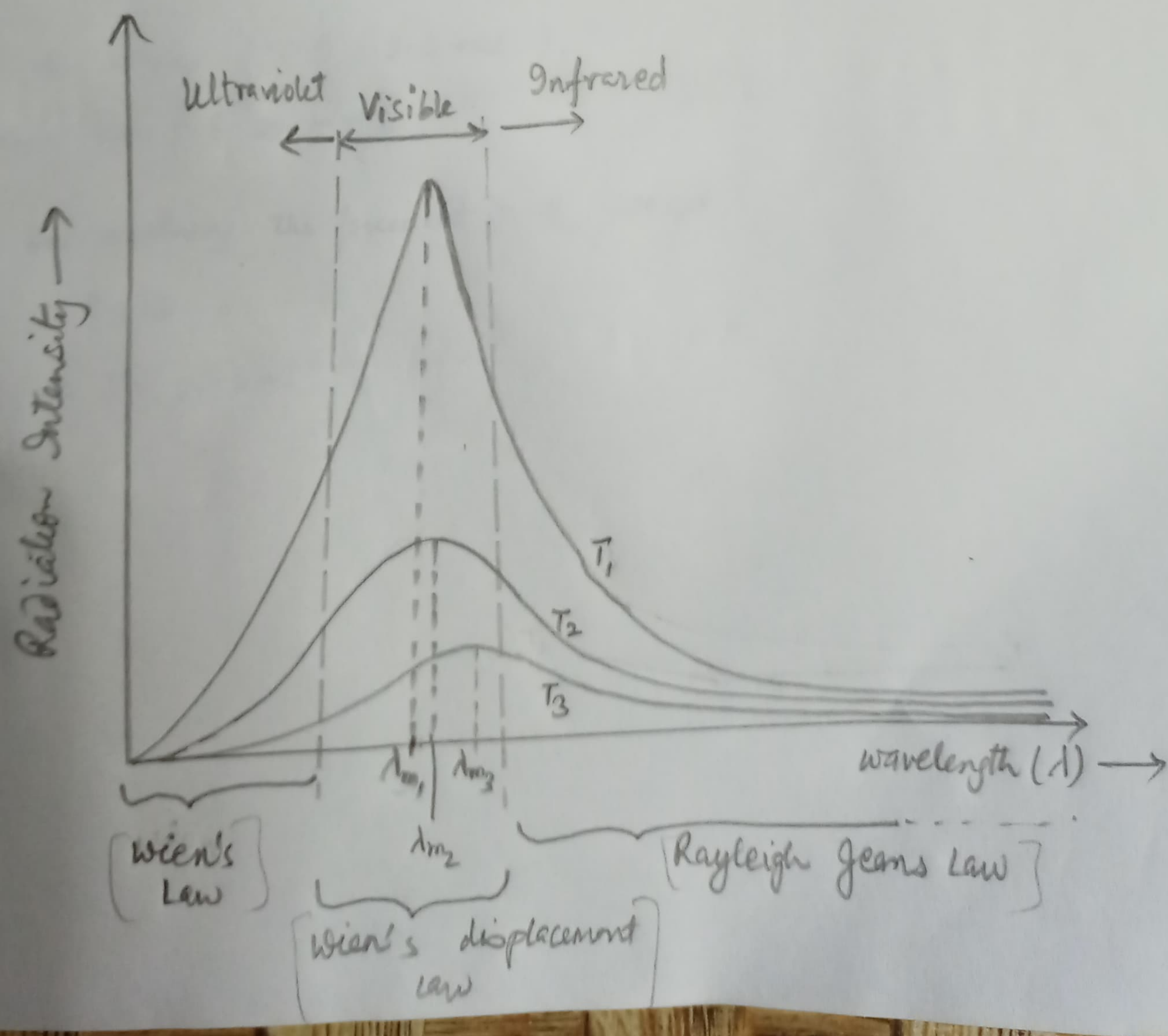
An example of nearly blackbody is Ferry's blackbody.



Ferry's blackbody
(nearly, a blackbody)

We cannot have a perfect Blackbody in real life, because, there is no body which is a perfect absorber or a perfect emitter of light. It is an ideal concept. However, we can have a (nearly) black body.

b)



(c) <u>ultraviolet catastrophe</u> :

→ In the limit of low frequencies, the classical spectrum approaches the experimental results, but as the frequency becomes large, the theoretical prediction goes to infinity.

But, experiments show that the energy remains finite and goes to zero at very high frequencies.

→ This unrealistic behaviour of the prediction of classical theory at high frequencies is called as "ultraviolet catastrophe".

→ So, this is the error at short wavelength.

(8) a) ~~Normalle~~ Normal scattering of light is the phenomenon in which light rays get deviated from its straight path on striking an obstacle like dust, or gas molecules, water vapours etc. and the wavelength remains unchanged ($\lambda = \lambda'$)

whereas, Crompton Scattering, is the scattering of X-rays from electrons in a target where wavelength of scattered light is different from that of incident ~~ligh~~ radiation (due to recoil e^-).

There both the wavelengths are observed and the new wavelength is greater than the original one, unlike normel scattering.

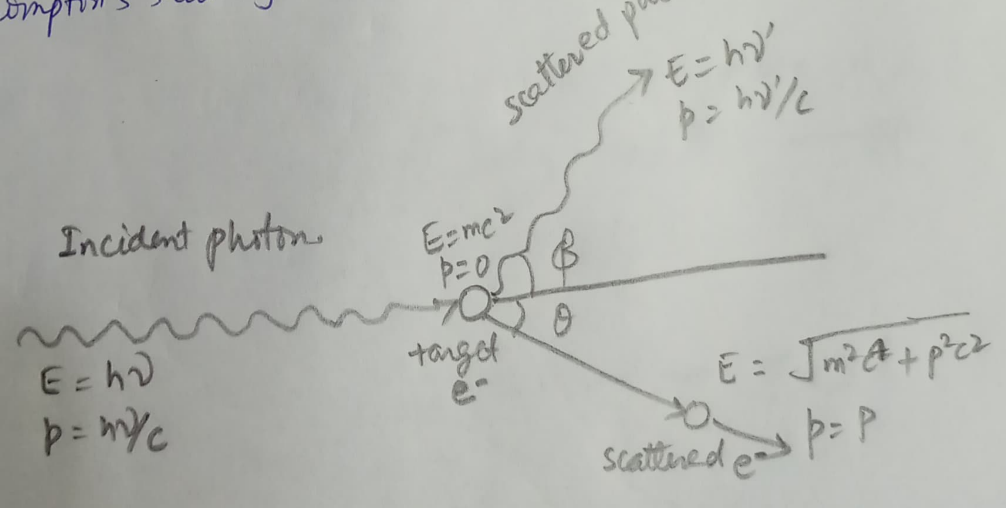
b) The intensity (Photon count) of new wavelength is higher than that of the original one because :-

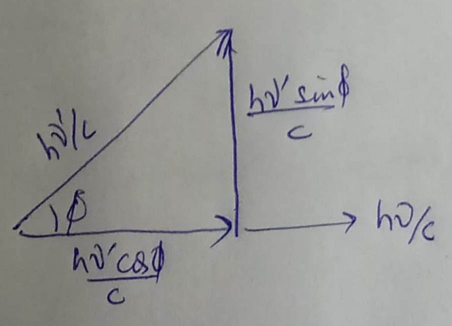
The collision of the ~~outerside~~ photon with outermost ($2p^{3/2}$ electrons) is more than the inner tightly bound electrons.

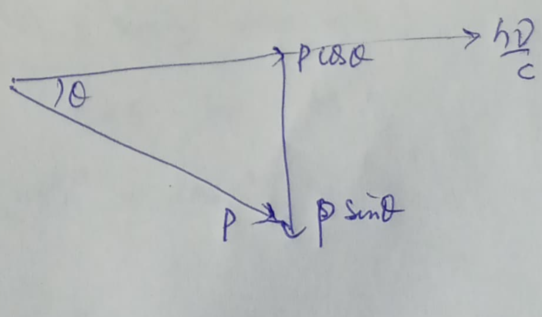
So, the probability of the Crompton's scattering is more than the normell scattering to occur.

So, the no. of scattered photon, hence the intensity is greater in Crompton's scattering than normel one.

c)







For photon, $m_0 = 0$

$$\Rightarrow E = \sqrt{m_0^2c^4 + p^2c^2} = pc = h\nu$$

$$\Rightarrow \boxed{p = \frac{E}{c} = \frac{h}{\lambda}}$$

Here, the initial photon momentum is $h\nu/c$ & the scattered photon momentum is $h\nu'/c$ and the initial & final e^- momenta are 0 & p resp.

In the original photon dir^n :-

Initial momentum = final momentum (elastic collision)

$$\Rightarrow \text{(along incident dir}^n\text{)} : \frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta \quad \text{— (i)}$$

$$(\perp r \text{ dir}^n) : 0 - \frac{h\nu'\sin\phi}{c} = p\sin\theta \quad \text{— (ii)}$$

where, ϕ is the angle betⁿ dirⁿ of initial & scattered photons & θ is the angle betⁿ dirⁿ of initial photon & recoil electron.

$\Rightarrow$ from (i), $pc\cos\theta = h\nu - h\nu'\cos\phi$

From (ii), $pc\sin\theta = h\nu'\sin\phi$

On squaring and adding the above eqⁿs., we get,

$$p^2c^2 = h^2\nu^2 - 2h\nu(h\nu')(\cos\phi) + (h\nu')^2 \quad \text{— (iii)}$$

Now, from energy conservation, we get,

$$h\nu = h\nu' + (KE)_{\text{recoil } e^-}$$

Also, $E = KE + m_0c^2$ (m_0 = rest mass of e^-)

& $E = \sqrt{m_0^2c^4 + p^2c^2}$

$$\Rightarrow (KE + m_0c^2)^2 = m_0^2c^4 + p^2c^2 \quad \text{(using above eq}^n\text{s.)}$$

$$\Rightarrow p^2c^2 = KE^2 + 2m_0c^2 \cdot KE \quad (KE = h\nu - h\nu')$$

$$\Rightarrow p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2m_0c^2(h\nu - h\nu') \quad \text{— (iv)}$$

Substituting the value of p^2c^2 from (III) & (IV), we get,

$$2m_0c^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1-\cos\theta)$$

On simplifying the above eqn. in terms of wavelength, :-

$$\Rightarrow \nu - \nu' = \frac{h\nu\nu'}{m_0c^2}(1-\cos\theta) \quad \text{——— (V)}$$

Now, $\frac{1}{\lambda} = \frac{\nu}{c}$ & $\frac{1}{\lambda'} = \frac{\nu'}{c}$

Putting them in eqn (V), we get,

$$\frac{h}{m_0c}(1-\cos\phi) = \lambda' - \lambda$$

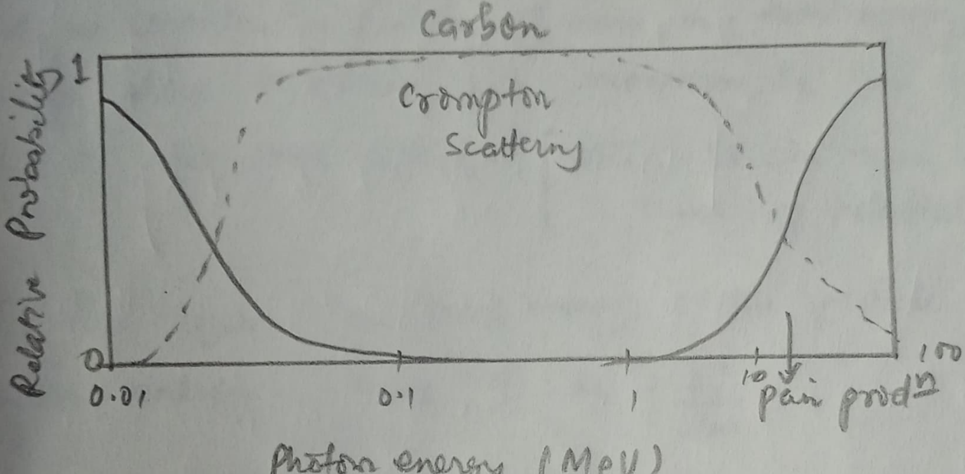
$$\Rightarrow \boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)}$$

This is the required expression for Crompton's shift.

d) We know, Crompton scattering becomes dominant at photon energies of a few keV.

~~For~~ pair production increasingly likely the more the photon energy exceeds the threshold of 1.02 MeV.

→ The greater the atomic no. of the absorber, the lower the energy at which pair production takes over as the principal mechanism of energy loss by gamma rays.



(10.) a) We know, from $\Delta p \cdot \Delta x \geq \hbar/2$ (from uncertainity principle) (where, $\hbar = h/2\pi$)

Now, we know, $L = mvr$ (from definition of angular momentum)

$\Rightarrow L = p \cdot r$

$\Rightarrow \Delta L = \Delta p \cdot r$ (from small changes).

$\Rightarrow \Delta p = \Delta L / r$ —(i)

Using, $\theta = \frac{x}{r}$ ($\because$ angle $= \frac{\text{arc length}}{\text{radius}}$) ~~from~~

$\Rightarrow \Delta\theta = \frac{\Delta x}{r}$

$\Rightarrow \Delta x = \Delta\theta \cdot r$ —(ii)

On putting ① & ⑪ in the main eqn we get,

$\Rightarrow \Delta p \cdot \Delta x \geq \hbar/2$

$\Rightarrow \frac{\Delta L}{r} \cdot \Delta\theta \cdot r \geq \frac{\hbar}{2}$

$\Rightarrow \boxed{\Delta L \cdot \Delta\theta \geq \hbar/2 .}$

This is the uncertainity principle

b) Let, us consider a particle of mass, m; total energy, E & moving along the x-axis with momentum, P_x in a free region

$\Rightarrow$ PE = 0 for free space. [NOTE: We can never find the absolute value of potential.]

In a free region, the total energy of the particle is equal to the kinetic energy. Hence, $E = KE = \frac{p_x^2}{2m}$.

In operator form, $\hat{E} = \frac{1}{2m}\hat{p}_x^2$. If $\psi(x,t)$ is the wave function corresponding to the particle then, it can be written as:

$$\hat{E}\cdot\psi(x,t) = \frac{1}{2m}\hat{p}_x^2\,\psi(x,t)$$

$$= \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial t}\right)^2\psi(x,t) \qquad \left[\because \hat{p}_x = -i\hbar\frac{\partial}{\partial t}\right)$$

$$= \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial t^2}\cdot\psi(x,t)\right)$$

This is the schrödinger wave equation for 1-D free particle.

$$\psi(x,t) = A e^{-i(kx-wt)}$$

In practice, we cannot have an absolutely free particle :-

The probability of finding a particle, ie, probability density is :-

$$P = \psi^* (x,t)\, \psi(x,t) = |A|^2$$

The probability of finding the particle somewhere in space will be :

$$\int_{-\infty}^{+\infty} \psi^* (x,t) \cdot \psi(x,t)\, dx = |A|^2 \int_{-\infty}^{\infty} dx$$

This results the integral is infinite irrespective of the value of A

But a/c the interpretation of wave fxn, the result of above integral must be 1.

Thus, there is a discrepancy -

So, In practice, no particle is absolutely free.

Hence, it can be said, that the concept of free particle is ideal one.

c) Given, $\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right)$

, which is of the form $A \sin kx = 0$.
whose sol^n is given as $k_n = 2\pi/a$ ~~where~~ . (for n = 2)
This ~~the~~ is the form of final sol^n of schrödinger (time) eq^n.

Hence, for, $k = \frac{2\pi}{a}$ [On applying boundary conditions)

& $k = \frac{\sqrt{2mE}}{\hbar}$

we get, $E = \frac{k^2\hbar^2}{2m} = \frac{4\pi^2\hbar^2}{2ma^2} = \frac{h^2}{2ma^2}$ $\left[\because \hbar = h/2\pi\right]$

9 a) $\lambda = 0.5 A^0$
$\theta = 30^\circ$

i) $KE_{max} = h(\nu - \nu') = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'_{min}}\right)$

$\lambda'_{min} = \lambda + \lambda_c (1 - \cos\theta)$
$= 0.5 + 0.024\left(1 - \frac{\sqrt{3}}{2}\right)$
$= (0.5 + 0.003) A^0 = 0.5003 A^0$

2) $KE_{max} = \frac{6.634 \times 10^{-34} (3 \times 10^8)}{} \left[\frac{10^{10}}{0.5} - \frac{10^{10}}{0.5003}\right]$
$= 19.9 \times 10^{-16} [2 - 1.988]$
$= 19.9 \times 10^{-16} \times 0.0011$
$= 0.0238 \times 10^{-16}$ J
$= 2.38 \times 10^{-18}$ J

ii) % energy lost $= \frac{h\nu - h\nu'}{h\nu} \times 100 = \frac{\lambda' - \lambda}{\lambda'} \times 100$
$= \frac{\Delta\lambda}{\lambda} \times 100 = \frac{0.003}{0.5} \times 100 = 0.6\%$

b). Given : $V = 500\ kV$.

De Broglie wavelength, λ of $e^- = \frac{12.28}{\sqrt{V}}\ A^o$ [This is the formula for wavelength of an e^- acclerated by a potential V]

$$\Rightarrow \lambda = \frac{12.28}{\sqrt{500 \times 10^3}}\ A^o$$

$$\Rightarrow \lambda = \frac{12.28 \times \sqrt{2}}{10^3}\ A^o$$

$$\Rightarrow \lambda = 17.366 \times 10^{-3}\ A^o$$

$$\Rightarrow \lambda = 1.74 \times 10^{-14}\ m.$$

c) Let, the atomic nucleus is 1 fermi.

$\Rightarrow \Delta x = 10^{-15}\ m$.

we have, $\Delta p \geq \frac{\hbar}{2\Delta x}$

$$\Rightarrow \Delta p \geq \frac{1.054 \times 10^{-34}}{(2)(10^{-15})} = 0.527 \times 10^{-19}\ kg\ m/s$$

$$\Rightarrow \Delta p \geq 5.3 \times 10^{-20}\ kg/s.$$

→ If this is the uncertainty in a nucleus, e's momentum p itself must be atleast comparable in magnitude.

→ An e^- with such a momentum has a KE many times grater than its rest mass energy mc^2.

Can let, $KE \approx pc$ Here,

$$KE = pc \geq (5.3 \times 10^{-20})(3 \times 10^8)\ J$$

$$\Rightarrow KE \geq \frac{15.9 \times 10^{-12}}{1.6 \times 10^{-19}}\ eV$$

$$\Rightarrow KE \geq 9.9375 \times 10^7\ eV$$

$$\geq 99.37\ MeV.$$

The KE of an e^- must exceed 99.37 MeV if it is to be in nucleus. Thus, we conclude that nuclei cannot contain e's.