Exercise

8. Find the least initial velocity with which a projectile is to be projected so that it clear a wall 4m height located at a distance of 5m, and strikes the ground at a distance of 4 m beyond the wall as shown in Fig-.The point of projection is at the same level as the foot of the wall.

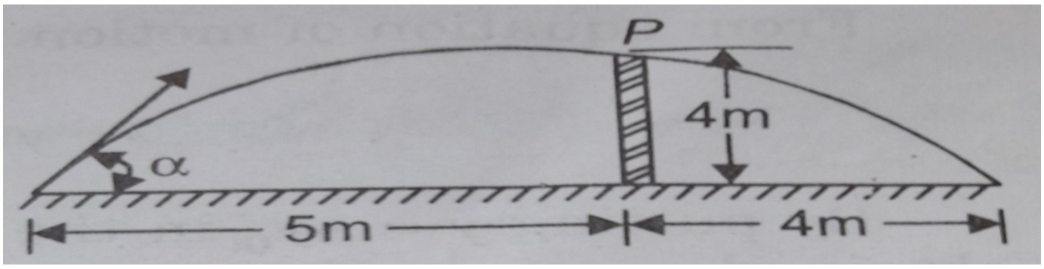


Figure-1

Solution

Solution: Let v_0 be the initial velocity required and α the angle of projection. In this problem, Range = 9 m and at P, the top of wall, $x = 5$ m, $y = 4$ m.

$$\therefore \quad 9 = \frac{v_0^2 \sin 2\alpha}{g}$$

$$\therefore \quad v_0^2 = \frac{9g}{\sin 2\alpha} \quad ..(1)$$

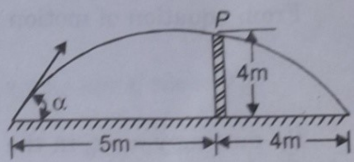


Fig. 10.11

From the equation of trajectory,

$$y = x \tan\alpha - \frac{1}{2}\frac{gx^2}{v_0^2 \cos^2\alpha}$$

$$4 = 5\tan\alpha - \frac{1}{2}\frac{g \times 5^2}{\frac{9g}{\sin 2\alpha}\cos^2\alpha}$$

Substituting $2\sin\alpha\cos\alpha$ for $\sin 2\alpha$, we get

$$4 = 5\tan\alpha - \frac{25}{18\frac{\cos^2\alpha}{2\sin\alpha\cos\alpha}}$$

$$= 5\tan\alpha - \frac{50}{18}\tan\alpha$$

$$= 2.2222\tan\alpha$$

$$\therefore \quad \tan\alpha = 1.8$$

$$\therefore \quad \alpha = 60.95°$$

From (1) $$v_0^2 = \frac{9 \times 9.81}{\sin(2 \times 60.95°)}$$

$$\therefore \quad v_0 = 10.20 \text{ m/sec.}$$

Ans.

9. A bullet is fired from a height of 120m at a velocity of 360km/hr at an angle of 30⁰ upwards. Neglecting air resistance , Find
   (a)Total time of flight
   (b) Horizontal range of the bullet
   (c)Maximum height reached by the bullet, and
   (d) Final velocity of the bullet just before touching the ground.

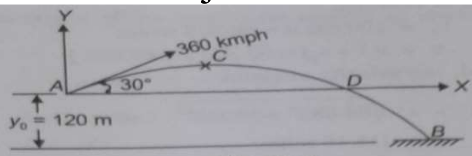


Fig. 10.13

Solution: Velocity of projection

$$v_0 = 360 \text{ kmph}$$

$$= \frac{360 \times 1000}{60 \times 60} = 100 \text{ m/sec.}$$

(a) *Total time of flight*

$$y_0 = -120 \text{ m.}$$

Considering vertical motion

$$y = v_0 \sin \alpha \times t - \frac{1}{2} gt^2,$$

$$-120 = 100 \sin 30° \times t - \frac{1}{2} \times 9.81\, t^2$$

$$t^2 - 10.194\, t - 24.465 = 0$$

or
$$t = \frac{10.194 \pm \sqrt{10.194^2 + 4 \times 1 \times 24.465}}{2}$$

or
$$t = 12.20 \text{ sec.}$$ **Ans.**

(The negative value is neglected, since, it does not give any practical meaning).

(b) *Maximum height reached by the bullet*

$$h = \frac{v_0^2 \sin^2 \alpha}{2g} = \frac{100^2 \sin^2 30°}{2 \times 9.81}$$

$$= 127.42 \text{ m above point A.}$$

i.e., $127.42 + 120 = 247.42$ m above the ground. **Ans.**

(c) *Horizontal range*

$$= v_0 \cos \alpha \times t$$

$$= 100 \cos 30° \times 12.2$$

$$= 1056.55 \text{ m}$$

(d) *Velocity of the bullet just before striking the ground*

Vertical component of velocity

$$v_y = v_0 \sin \alpha - gt$$

$$= 100 \sin 30° - 9.81 \times 12.2$$

$$= -69.682 \text{ m/sec}$$

$$= 69.682 \text{ m/sec downward}$$

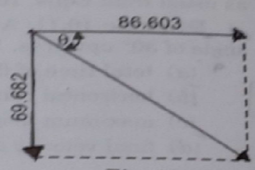


Fig. 10.14

Horizontal component of velocity,

$$v_x = 100 \cos \alpha = 86.603 \text{ m/sec}$$

Thus, $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = 86.603\, \mathbf{i} + 69.682\, \mathbf{j}$

Referring to Fig. 10.14, the velocity at strike

$$v = \sqrt{69.682^2 + 86.603^2}$$

$$v = 111.16 \text{ m/sec}$$ **Ans.**

$$\theta = \tan^{-1} \frac{69.682}{86.603}$$

$$= 38.82° \text{ as shown in Fig. 10.14.}$$ **Ans.**

[12] **Example 10.14:** A soldier fires a bullet with a velocity of 31.32 m/sec at an angle α upwards from the horizontal from his position on a hill to strike a target which is 100 m away and 50 m below his position. Find the angle of projection α. Find also the velocity with which the bullet strikes the object.

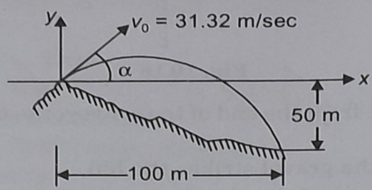


Fig. 10.18

Solution: The equation of the trajectory of bullet is (from Eqn. 10.7)

$$y = x \tan \alpha - \frac{1}{2}\frac{gx^2}{v_0^2}(\tan^2 \alpha + 1)$$

Now, for the point on the ground where bullet strikes,

$$y = -50 \text{ m};\ x = 100 \text{ m}$$

and $\qquad v_0 = 31.32$ m/sec

$$\therefore \quad -50 = 100 \tan \alpha - \frac{1}{2} \times \frac{9.81 \times 100^2}{31.32^2}(\tan^2 \alpha + 1)$$

$$= 100 \tan \alpha - 50 (\tan^2 \alpha + 1)$$

i.e., $\quad \tan^2 \alpha - 2\tan \alpha = 0$

$$\tan \alpha (\tan \alpha - 2) = 0$$

$\therefore \quad \alpha = 0$

or $\quad \alpha = \tan^{-1} 2 = 63.435°$ **Ans.**

$\alpha_0 = 0$ is neglected since α is upward angle as given in the problem.

Horizontal component of velocity

$$v_x = 31.32 \times \cos 63.435°$$

$$= 14.007 \text{ m/sec}$$

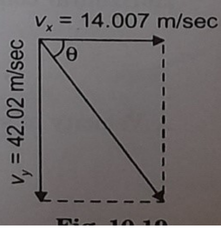
Vertical component of velocity of strike v_y will be downward. The vertical component of initial velocity was 31.32 sin 63.435° = 28.013 m/sec upward.

$$v_y^2 - (-28.013)^2 = 2 \times 9.81 \times 50$$

$$v_y = 42.02 \text{ m/sec downward}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{14.007^2 + 42.02^2}$$

$$= 44.294 \text{ m/sec}$$ **Ans.**



$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \frac{42.02}{14.007}$$

$$= 71.565° \text{ to horizontal, as shown Fig. 10.19.}$$

of β should be taken negative.

Example 10.16: A plane has a slope of 5 in 12. A shot is projected with a velocity of 200 m/sec at an upward angle of 30° to horizontal. Find the range on the plane if:

(*a*) the shot is fired up the plane;

(*b*) the shot is fired down the plane.

Solution: Initial velocity v_0 = 200 m/sec.

Angle of projection $\alpha = 30°$

$$\text{Inclination of the plane} = \tan^{-1}\left(\frac{5}{12}\right) = 22.62°$$

(*a*) When the shot is fired up the plane:

$$\beta = 22.62°$$

$$\text{Range} = \frac{v_0^2}{g\cos^2\beta}[\sin(2\alpha-\beta) - \sin\beta]$$

$$= \frac{200^2}{9.81 \times \cos^2 22.62}[\sin(2 \times 30 - 22.62) - \sin 22.62°]$$

i.e., Range = 1064.65 m **Ans.**

(*b*) When the shot is fired down the plane:

$$\beta = -22.62°$$

$$\text{Range} = \frac{200 \times 200}{9.81\cos^2(22.62)}[\sin(2 \times 30 + 22.62) - \sin(-22.62)]$$

$$= \frac{200 \times 200}{9.81\cos^2 22.62}[\sin 82.62 + \sin 22.62]$$

Range = 6586.27 m **Ans.**