University Roll No. - T91/ECE/204058

Subject - Engineering Mechanics

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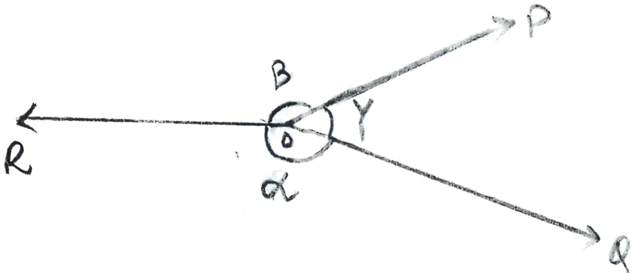
1. a) LAMI'S THEOREM: It states that; if a particle is under the action of three coplanar concurrent forces, all of them acting either towards or away from point of concurrency be in equilibrium, then, each force is proportional to sine of the angle between the other two forces.



let, at point O, 3 coplanar concurrent forces (say) $\vec{P}, \vec{Q}$ and $\vec{R}$ be acting such that the angle formed between P & Q is γ, R & Q is ~~β~~ α, Q & R is β, then,

Lami's Theorem states that :-

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

where, P, Q, R are the megnitudes of the respective forces.

b)

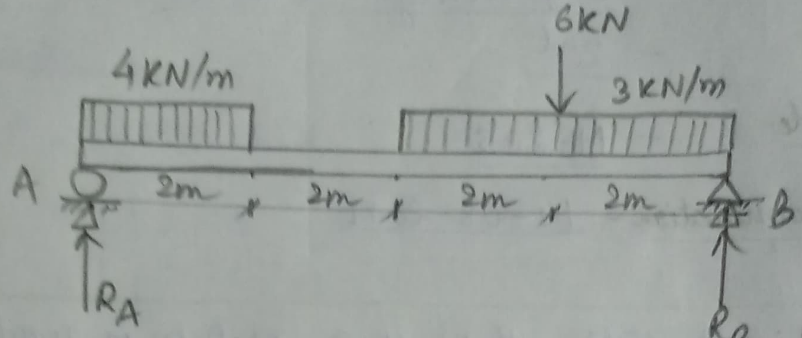
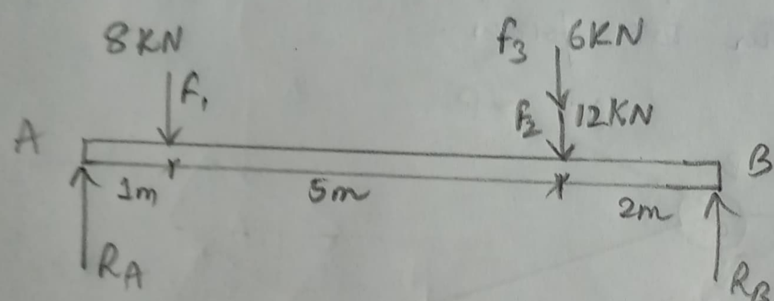


Figure 1

Given, AB is a supported beam of length 8 m.

To find the reaction at A and B.

Sol^n,



Let, R_A and R_B be the reactions at hinge points, A and B respectively

The force, f_1 at point 1m from A $\Rightarrow$ $f_1 = 4KN/m \times 2m$

$= 8KN$

ly, the forces, f_2 at 2m from B $\Rightarrow$ $f_2 = 3KN/m \times 2m$

$= 6KN$

And, force, $f_3 = 6KN$ (given)

Now, By taking the moments at point B;

$\Sigma M_B = 0$

$\Rightarrow -R_A \times 8 + f_1 \times 7 + (f_2 + f_3) \times 2 = 0$

$\Rightarrow -R_A \times 8 + 8 \times 7 + 18 \times 2 = 0$

$\Rightarrow 8 R_A = 56 + 36$

$\Rightarrow R_A = 92/8$ kN

$\Rightarrow \boxed{R_A = 11.5 \text{ kN}}$

Similarly, By taking moment at A; we get,

$\underline{\Sigma M_A = 0}$

$\Rightarrow R_B \times 8 - F_1 \times 1 - (F_2 + F_3) 6 = 0$

$\Rightarrow 8R_B - 8 - 18 \times 6 = 0$

$\Rightarrow 8 R_B = 8 + 108 \longrightarrow R_B = \frac{116}{8} = 14.5$ KN

$\Rightarrow$ ~~$R_B = \frac{182}{8} = 10$ KN~~

Thus the values of reaction forces at A is 11.5 KN vertically upwards & the value of reaction forces at B is 14.5KN, vertically upwards.

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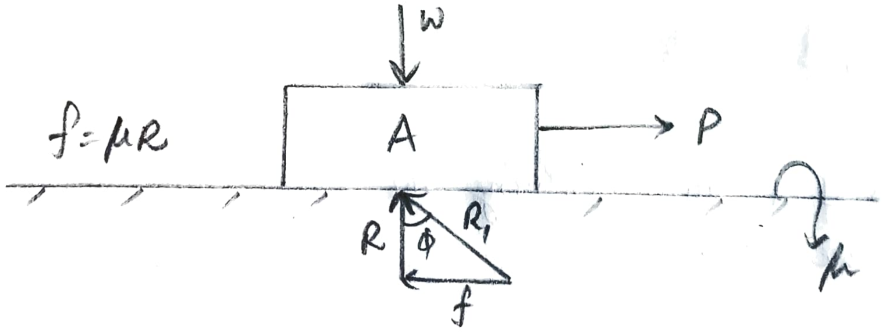
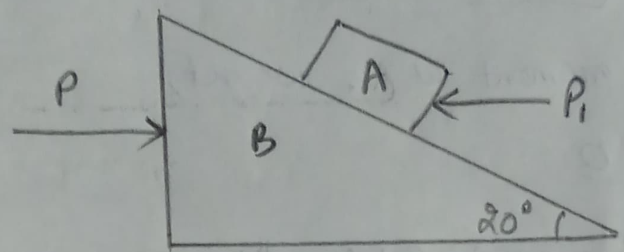
(2) a) LIMITING ANGLE OF FRICTION: This is the angle between resultant force of friction and the normal reaction made with normal reaction.



In the above figure, let, a force, P be acting on a body A of weight W kept on a friction plane with coefficient of friction, μ.
Then, the normal reaction, R acts vertically upwards.
and frictional force, f acts in opp. direction of P; $f = \mu R$

Then, $\tan\phi = \frac{f}{R} = \frac{\mu R}{R} = \mu \longrightarrow \boxed{\phi = \tan^{-1}\mu}$

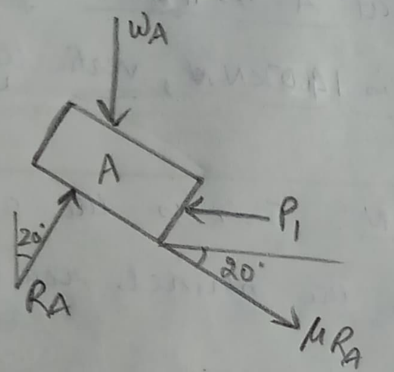
b)



Given; weight of body A ⇒ W_A = 20 KN.
coefficient of friction, μ = 0.25 (for all surfaces).
angle of inclination, θ = 20°

<u>To find</u>; value of P and P_1; for the system to be in equilibrium.

<u>Sol^n</u>;



FBD of body A Fig1.

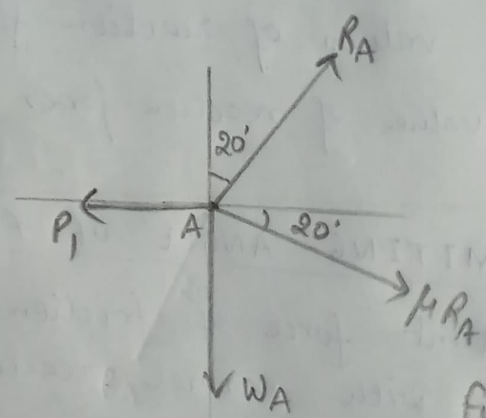


Fig 2.
Resolution of forces at A.

<u>For body A</u>:

Considering the FBD of A and resolving the forces horizontaly and vertically at A, we get;

$$\Sigma Y = 0$$

$$R_A \cos 20° - \mu R_A \sin 20° - W_A = 0$$

or, $R_A \cos 20° - 0.25\, R_A \sin 20° - 20{,}000 = 0$

or, $R_A(\cos 20° - 0.25 \times \sin 20°) = 20{,}000$

or, $R_A(0.93969 - 0.08550) = 20{,}000$

or, $R_A = \dfrac{20{,}000}{0.854185}\text{ N} = 23414.1315\text{ N} \simeq 23414.13\text{ N}$

By $\underline{\Sigma X = 0}$, we get,

$$P_1 - R_A \sin 20' - \mu R_A \cos 20' = 0$$

or, $P_1 = R_A(\sin 20' + \mu \cos 20')$

or, $P_1 = 23414.13 \times (\sin 20' + 0.25 \times \cos 20')$

or, $P_1 = 23414.13 \times (0.342020 + 0.23492)$

or, $P_1 = 23414.13 \times 0.576943$

or, $P_1 = 13508.622\ N$

or, $\underline{P_1 \simeq 13508.62\ N}$

For body B,

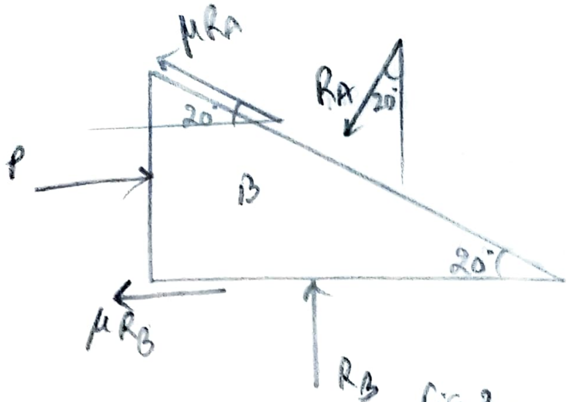


Fig 3.

FBD of B:

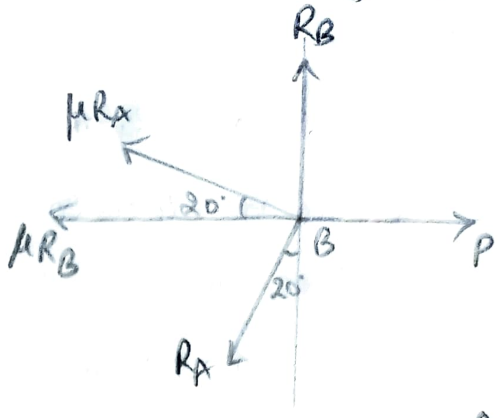


Fig 4.

Resolution of forces on B

Here, friction force acts on 2 surface, one at inclined surface and the other at horizontal surface.

On considering the FBD of B and resolution of forces on body B, we get;

$\Sigma Y = 0$,

$\Rightarrow R_B - R_A \cos 20° + \mu R_A \sin 20° = 0$

$\Rightarrow R_B = R_A(\cos 20° - \mu \sin 20°) = 0$

$\Rightarrow R_B = 23414.13 \times (0.93969 - 0.25 \times 0.342020)$

$\Rightarrow R_B = 23414.13 \times (0.93969 - 0.085505)$

$\Rightarrow R_B = 23414.13 \times 0.854185 \Rightarrow R_B = 19999.9986\ N$

$\Rightarrow$ ~~$R_B = 20,000.0498\ N$~~ N. $\Rightarrow R_B \simeq 20,000\ N$.

By, $\Sigma X = 0$, we get;

$P = \mu R_B + \mu R_A \cos 20° + R_A \sin 20°$

or, $P = \mu R_B + R_A(\sin 20° + \mu \cos 20°)$

or, $P = (0.25 \times 20,000.00 + 13508.62)$

or, $P = (5000.00 + 13508.62)\ N$

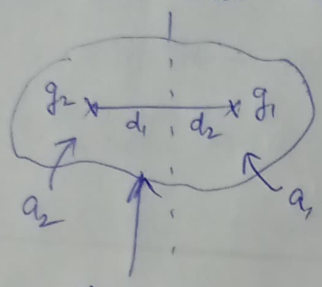
$\Rightarrow P = 18508.62\ N \Rightarrow P = 18,508.62\ N$

Thus the value of $P = 18508.62\ N$ & $P_1 = 13508.62\ N$

(4.) **Centroid**: In mechanics, for plane areas body with negligible thickness, we assume the total area to be concentrated at a single point and such a single point is called the centroid of the plane area.

$$\Rightarrow \bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i}, \quad \bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i}, \quad \bar{z} = \frac{\Sigma A_i z_i}{\Sigma A_i}$$

→ Centroid is a point in a plane area, where the moment of areas about an axis through the centroid is zero.



Plane area, A

Here, $a_1 d_1 - a_2 d_2 = 0$.

→ Centroid is denoted (normally) by G.

Radius of gyration: It is defined as the radial distance to a point which would have a moment of inertia the same as the body's actual distribution of mass, if the total mass of the body were concentrated there. It is denoted by k.

→ Consider an area, A, with moment of inertia, I_x.

Then, $I_x = k_x^2 A$

$$\Rightarrow k_x = \sqrt{\frac{I_x}{A}}$$

(where, k_x is the radius of gyration wrt. x-axis)

4A).

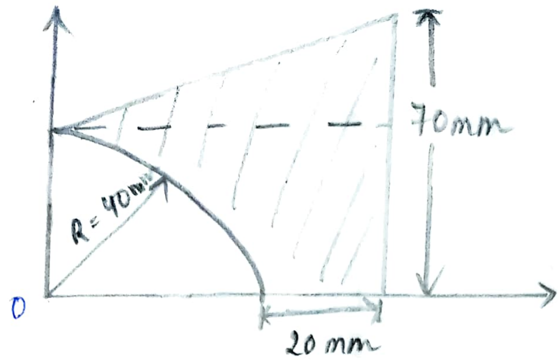


Fig 4A.

Calculation for centroid of the lamina :

This has 3 components (basically): Quarter circle, triangular part & rectangular part; which can be formatted in the below table.

~~For quarter circle :-~~

Here, $x_2 = 40$, $y_2 = 50$

$x_3 = 30$, $y_3 = 20$

$x_1 = \frac{4 \times 40}{3\pi} = \frac{160}{3\pi} = 16.976$.

	Components.	Area (mm^2)	X (mm)	Y (mm)	$a_i x_i$	$a_i y_i$
1.	Quarter Circle	-1256.64	16.97	16.97	-21325.2	-21325.2
2.	Triangle	900	40	50	36000	45000
3.	Rectangle	2400	30	20	72000	48000

Thus, $\bar{x} = \frac{-A_1 x_1 + A_2 x_2 + A_3 x_3}{-A_1 + A_2 + A_3}$

$\Rightarrow \bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i} = \frac{-21325.2 + 36000 + 72000}{-1256.64 + 900 + 2400} = \frac{86674.82}{2043.36}$

$\Rightarrow \bar{x} = 42.42$ mm

Again, $\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i} = \frac{-21325.2 + 45000 + 48000}{2043.36} = \frac{71674.82}{2043.36}$

$\Rightarrow \bar{y} = 35.08$ mm

Thus, $\bar{x} = 42.42$ mm & $\bar{y} = 35.08$ mm

4B)

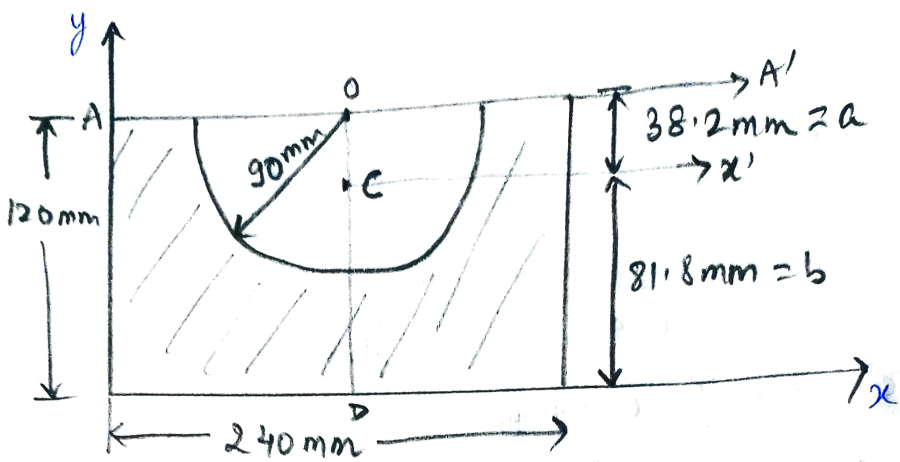
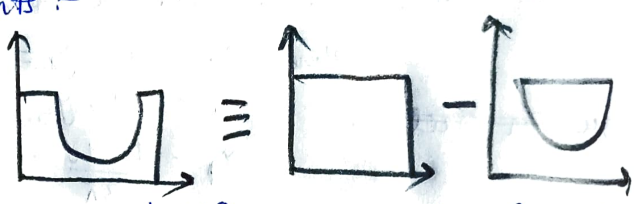


fig 4B

This has basically 2 components :-

i) Rectangular ii) Circular.



For (i):

Moment of inertia along x, $I_{x_1} = \frac{1}{3}bh^3 = \frac{1}{3}(240)(120)^3\ mm^4$.

$\Rightarrow I_{x_1} = 138.2 \times 10^6\ mm^4$ —— (1)

For (ii):

Now, let, C be the centroid of the circular region.

Then, $OC = a\ (say) = \frac{4R}{3\pi} = \frac{4 \times 90}{3\pi} = 38.2\ mm$

& $OD = b\ (say) = 120 - a = 81.8\ mm$

Then, area, $A_c = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(90)^2 = 12.72 \times 10^3\ mm^2$

Now, moment of inertia about AA' is :-

$I_{AA'} = \frac{1}{8}\pi r^4 = \frac{1}{8}\pi(90)^4 = 25.76 \times 10^6\ mm^4$.

→ moment of inertia wrt x' is :-

$\Rightarrow I_{x'} = I_{AA'} - A_c a^2$ (Parallel axis theorem)

$\Rightarrow I_{x'} = (25.76 \times 10^6) - (12.72 \times 10^3)(38.2)^2$

$= 7.20 \times 10^6\ mm^4$.

Now, moment of inertia about x is :-

$\Rightarrow I_{x_2} = I_{x'} + Ab^2$ (Parallel axis theorem)

$\Rightarrow I_{x_2} = (7.20 \times 10^6) + (12.72 \times 10^3)(81.8)^2$

$\Rightarrow I_{x_2} = 92.3 \times 10^6 \ mm^4$ —— (ii)

Thus, for the given figure ; we have :- (using eq^n (i) & (ii))

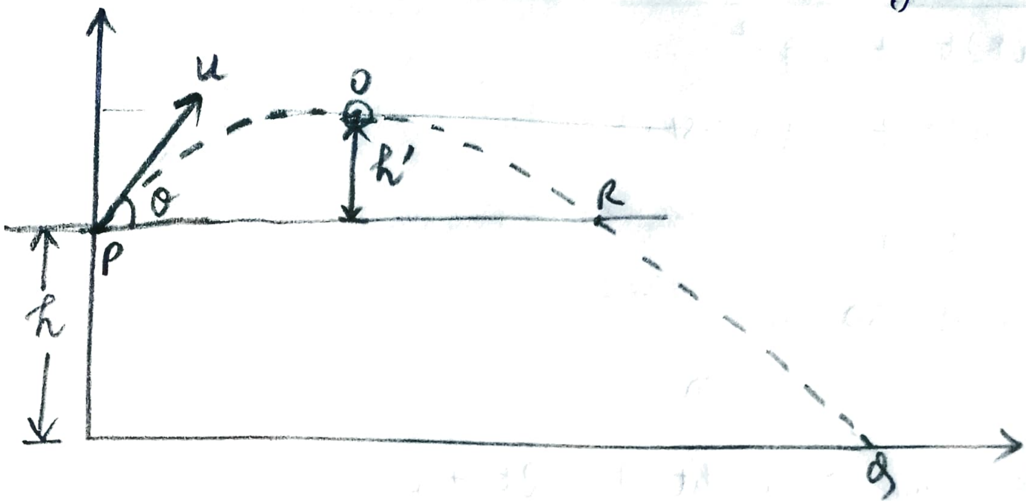
$\Rightarrow I_x = I_{x_1} - I_{x_2}$

$\Rightarrow I_x = 138.2 \times 10^6 \ mm^4 - 92.3 \times 10^6 \ mm^4$

$\Rightarrow I_x = 45.9 \times 10^6 \ mm^4$.

Thus, the moment of inertia of the composite figure is $45.9 \times 10^9 \ mm^4$ about the x-axis.

5. a)



Given, { the initial velocity, $u = 100\ m/s$
angle of projection, $\theta = 30°$ (with horizontal)
~~hig~~ height of tower, $h = 100\ m/s$.

Solⁿ: let, the total time of flight is T sec.

i) Consider, the projectile covers h' m above the tower to reach its peak and the corresponding time is t' and during the fall, it covers h m in t sec.

Thus, we conclude that : $T = 2t' + t$

On applying First equation of motion from pt. P to R, we get:

$v = u + at$.

$\Rightarrow u\sin\theta - gt' = 0 \qquad (\because \cancel{V_R = 0})$.

$\Rightarrow 2t' = 2 \times \frac{u\sin\theta}{g}$

$\Rightarrow 2t' = \frac{2 \times 100 \times \sin 30°}{9.81} = 10.194$ sec. —— (1)

Further, applying second equation of motion from pt. R to Q,

$h = ut + \frac{1}{2}at^2$.

$\Rightarrow h = (u \sin\theta)t + \frac{1}{2} g t^2$

$\Rightarrow 50 = (100 \sin 30)t + \frac{1}{2} \times 9.81 \times t^2$

$\Rightarrow 50 = 50t + 4.905 t^2$

$\Rightarrow 4.9t^2 + 50t - 50 = 0$

$\Rightarrow t = 0.92$ sec. ——— (II)

Then, the total time of flight, $T = 2t' + t$

$= (10.194 + 0.92)$ sec. {using (I) & (II)}

$= \underline{11.114 \text{ sec}}$

Now

ii) horizontal range, $R = (u \cos\theta.) T$

$= (100 \cos 30^\circ) \times 11.114$ m

$= \underline{962.5 \text{ m}}$

iii) Using third eqn of motion from R to Q,

$v^2 - u^2 = 2ah$

$\Rightarrow 0 = (u \sin\theta)^2 - 2gh'$

$\Rightarrow h' = \frac{(100 \sin 30)^2}{2g} = \frac{(100 \times \sin 30)^2}{2 \times 9.81} = 127.42$ m

So, the total height, $H = h' + h$

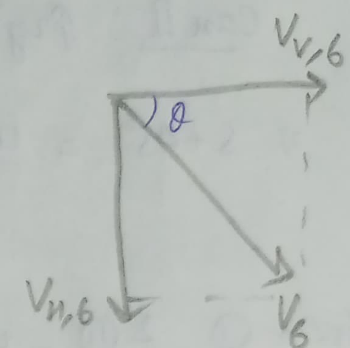
$= (127.42 + 50)$ m

$= \underline{177.42 \text{ m}}$

iv) Since, $t' = 5.097$ sec.

So, after $t = 6$ sec, the projectile is on its downward movement from the peak height.

Thus, the vertically downward component of velocity after 6 sec, is denoted by :-



$$V_{v,6} = 0 + g \times (6 - 5.097)$$
$$= 9.81 \times 0.903$$
$$= 8.858 \text{ m/s}.$$

The horizontal component remains unchanged.

$$\Rightarrow V_{H,6} = u\cos\theta = 100 \cos 30^\circ$$
$$= 86.6 \text{ m/s}.$$

Thus the resultant velocity, V_6 is given by:-

$$\Rightarrow V_6 = \sqrt{(V_{H,6})^2 + (V_{v,6})^2}$$

$$\Rightarrow V_6 = \sqrt{(86.6)^2 + (8.858)^2} = 87.05 \text{ m/s}.$$

and $\tan\theta = \dfrac{V_{H,6}}{V_{v,6}} = \dfrac{86.6}{8.858}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{86.6}{8.858}\right) = 84.16^\circ.$$

(6) ~~Given~~, let, s be the distance of the target from the point of projection and u be the velocity of projection;

then, By using the expression of Range of projection, we get,

$$R = \frac{u^2 \sin 2\alpha}{g}$$

Now, Case I: projectile falls 12 m short at $\alpha = 15°$.

$$\Rightarrow \quad S - 12 = \frac{u^2 \sin(2 \times 15°)}{g} = \frac{u^2}{2g} \quad \text{—— (i)}$$

Case II: Projectile overshoots by 24 m at $\alpha = 45°$.

$$\Rightarrow \quad S + 45° = \frac{u^2 \sin(2 \times 45°)}{g} = \frac{u^2}{g} \quad \text{—— (ii)}$$

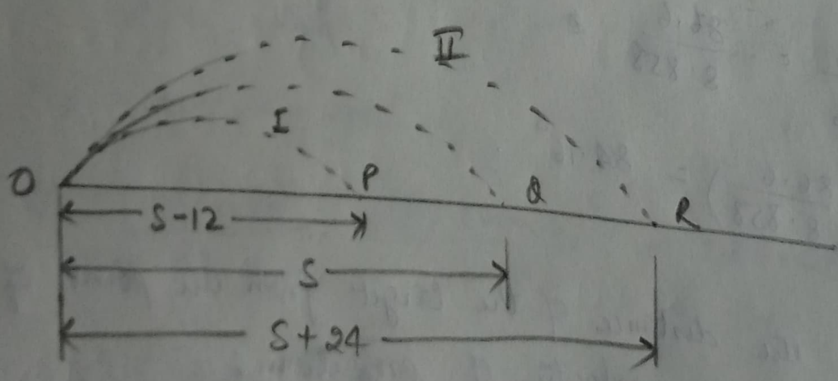
From (i) & (ii), we get,

$\Rightarrow S + 24 = 2(S - 12)$

$\Rightarrow \quad S = 24 + 2 \times 12$

$\Rightarrow \quad \boxed{S = 48 \text{ m}}$

Now, let, the angle of projection be α.



Then, $S = 48 = \dfrac{u^2 \sin 2\alpha}{g}$ —— (iii)

From (ii) & (iii), we get,

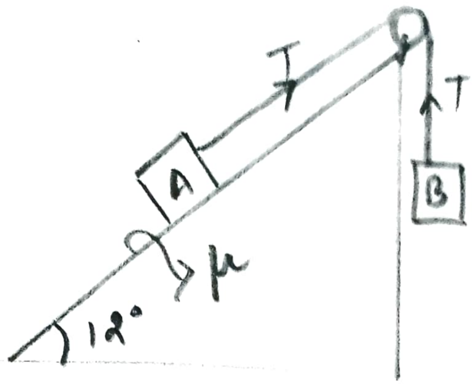
$\Rightarrow 48 = 72 \sin 2\alpha$

$\Rightarrow \quad \sin 2\alpha = \dfrac{48}{72} = 0.67 \longrightarrow 2\alpha = \sin^{-1}(0.67) = 41.810°$

$\Rightarrow \quad \underline{\alpha = 20.905°}$

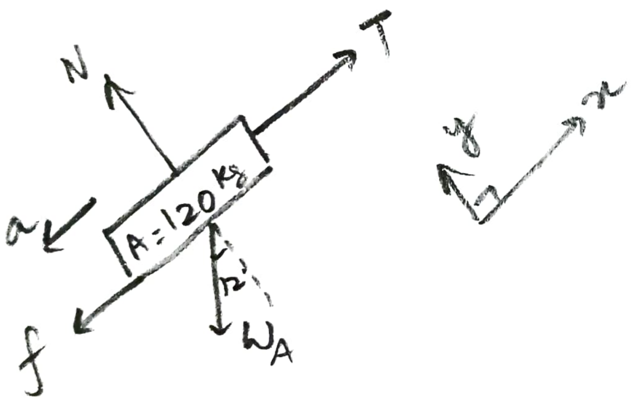
Thus, the angle of projection, to hit the target is $\underline{20.905^\circ}$.

(6)



Given, mass of body, A, $m_A = 125\ kg$.
angle of inclination, $\theta = 12^\circ$
mass of body B, $m_B = 81.5\ kg$.
coefficient of friction, $\mu = 0.25$

Sol^n: i) let, the tension in the rope be T.



FBD of A

let, the normal from surface be N.
& frictional force, $f = \mu N$

Along y axis,

$$N = W_A \cos 12^\circ$$
$$= 125g \times \cos 12^\circ$$
$$= 125g \cos 12^\circ$$

$\Rightarrow N = W_A \cos 12^\circ$

$\Rightarrow N = 125 \times 9.81 \times \cos 12^\circ$

$\Rightarrow N = 1199.4\cancel{53}\ N$

$\Rightarrow N \simeq 1199.45\ N$

$\Rightarrow f = \mu N$

$= 0.25 \times 1199.45$

$= 299.86\ N$

$\Rightarrow \sum F_x = 0$

$\Rightarrow T - f - W \sin 12^\circ - 120 \times a = 0$

$\Rightarrow T - 299.86 - 125 \times 9.81 \times \sin 12^\circ - 120a = 0$

$\Rightarrow T = 120a + (254.95 + 299.86)$

$\Rightarrow T = 120a + 554.812 \quad \text{--- (I)}$

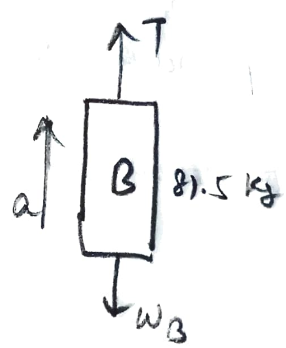
For body B, $m = 81.5$ kg.

$\Rightarrow m_2 g - T - m_2 a = 0$

$\Rightarrow 81.5 \times 9.81 - T - 81.5 \times a = 0$

$\Rightarrow 799.52 - T - 81.5a = 0$

$\Rightarrow T = +799.52 - 81.5a \quad \text{--- (II)}$



FBD of body B

From (I) & (II), we get;

$\Rightarrow 120a + 554.812 = -81.5a + 799.52$

$\Rightarrow 201.5a = 244.708$

$\Rightarrow$ ~~$a = 6.356\ m/s^2$~~

$\Rightarrow a = 1.2144\ m/s^2 \quad \text{--- (III)}$

Putting this value in (II),

~~$T = 120 \times 1.2144 + 554.$~~

$T = 799.52 - 81.5 \times 1.2144$

$T = 700.55\ N$

ii) Thus, from solving (I) & (II), we get from eqn (III),

$$a = 1.2144 \ m/s^2$$

$$\Rightarrow a \approx 1.21 \ m/s^2$$

iii) Let, distance moved by body in 3 sec after starting from rest be s.

Then, $s = ut + \frac{1}{2}at^2$

or, $s = 0 + \frac{1}{2} \times 1.2144 \times 3^2$

or, $s = \frac{1}{2} \times 1.2144 \times 9$

or, $s = 1.2144 \times 4.5$

or, $s = 5.4648 \ m$

or, $\underline{s \simeq 5.465 \ m}$.

3. a) Truss: A plane truss is defined as a structure made up of several bars hinged/joined together to form a framed structure to take loads.

Trusses are classified into 2 different types:-

i) Perfect Frame ($n = 2j - 3$) [n = no. of members] [j = no. of joints]

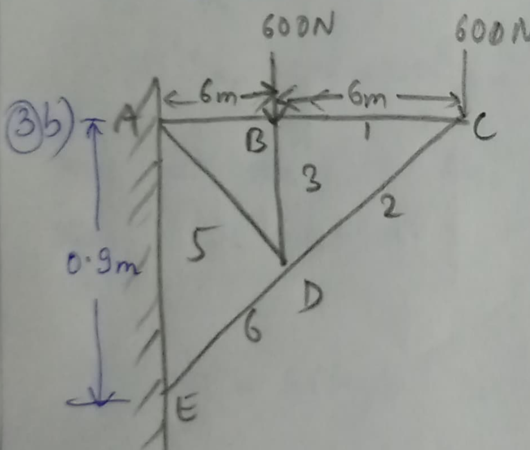
ii) Imperfect Frame ($n \neq 2j - 3$)

Imperfect frames are of 2 types:-

a) Deficient ($n < 2j - 3$)

b) Redundant ($n > 2j - 3$)

3b)



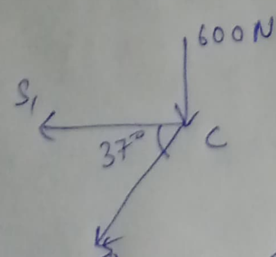
$\Rightarrow \tan\theta = AE/AC$

$= \frac{0.9}{1.2} = 0.75$

$\Rightarrow \theta = \tan^{-1}(0.75)$

$= 37°$.

$\Rightarrow \cos\theta = 0.8$ & $\sin\theta = 0.6$.



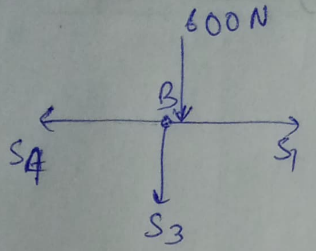
Isolating the joint C as a free body diagram and resolving vertically and horizontally.

$\Sigma Y = 0$, $S_2 \sin 37 = -600$

$S_2 = -1000N$ (Ans)

$\Sigma X = 0$, $-S_1 - S_2 \cos 37° = 0$

$S_1 = 1000 \times 0.8 = 800N$ (Ans)



Isolating the joint B as a FBD and resolving the forces vertically & horizontally, we get;

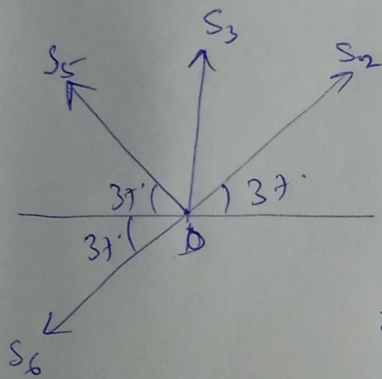
$\Sigma Y = 0$

$\Rightarrow -S_3 - 600 = 0 \rightarrow S_3 = -600\,N = -600N$ (Ans)

$\Sigma X = 0$.

$\Rightarrow S_1 - S_4 = 0$

$\Rightarrow S_4 = S_1 = 800N$ (Ans)



FBD of D.

On resolving forces, we get :-

$\Sigma X = 0$, $S_2 \cos 37° - S_5 \cos 37° - S_6 \cos 37° = 0$

$\Rightarrow S_5 + S_6 = S_2 = -1000\,N$ — (i)

$\Sigma Y = 0$, $S_3 + S_2 \sin 37° + S_5 \sin 37 - S_6 \sin 37° = 0$

$\Rightarrow S_5 - S_6 = \dfrac{600}{\sin 37} + 1000 = 2000\,N$ — (ii)

From (i) & (ii),

$2S_5 = 1000\,N$, $S_5 = 500\,N$ (Ans)

$\therefore S_6 = -1500\,N$ (Ans)

Hence, $\begin{cases} S_1 = 800N, & S_2 = -1000N, & S_3 = -600N, \\ S_4 = 800N, & S_5 = 500N, & S_6 = -1500N \end{cases}$ (Ans)