B.Tech. 2nd Semester Examination, 2021

MA 202: ENGINEERING MATHEMATICS- II

Full Marks: 70 **Time: 3 Hours**

Group A (Answer any Five questions)

1. $2\times5=10$

a) Suppose R^3 is a vector space over R with $(a,b,c)+(d,e,f)=(a+d,b+e,c+f)$ and $\alpha*(a,b,c)=(\alpha a,\alpha b,\alpha c)$. Show that for the subset $S=\{(a,b,c): a=2b=3c\}$ is a subspace or not. 2

b) Verify whether the differential equation $(\cos y+y\cos x)dx+(\sin x-x\sin y)dy=0$ is exact or not. 2

c) Determine k so that the vectors $(1,2,1),(k,1,1)$ and $(1,1,2)$ are linearly independent (k ϵ R). 2

d) Find the inverse Laplace transform of the function $\frac{1}{s(s+1)^2}$. 2

e) Suppose u,v,w are linearly independent vectors. Prove that the set S is linearly independent where $S=\{u+v-2w,\ u-v-w,\ u+w\}$. 2

f) Show that the mapping $F: R^2\rightarrow R^3$ defined by $F(x+y)=(x+3,\ 2y,\ x+y)$ is linear or not. 2

Group B (Answer any Six questions) **5×6= 30**

2. Find the co-efficient matrix and augmented matrix and then find their respective ranks and find the solutions (if exist) of the following system of linear equations 1+1+2+1=5

$$x + 2y - 3z + 2t = 2,\ \ 2x + 5y - 8z + 6t = 5,$$
$$3x + 4y - 5z + 2t = 4$$

3. Solve the following integral equation using Laplace Transform. 5

$$y'(t) + 3y(t) + 2\int_0^t y(t)\,dt = t \text{ given } y(0) = 1.$$

4. Find the inverse Laplace transform of the following functions. 2+3=5

i) $f(s) = \frac{s}{(s+1)(s^2-4s+13)}$ ii) $f(s) = \frac{e^{-2s}(s^2+s+1)}{s^2+4s+5}$

5. Solve the general solution of the differential equation $p = \log(px - y)$ and hence find the singular solution where $p = \frac{dy}{dx}$. 2+3=5

6. a) Show that the integral $\int_a^b \frac{dx}{(x-a)\sqrt{b-x}}$ is convergent or divergent. 2+3=5

b) Using Beta and Gamma functions to prove that $\int_0^{\pi/2} sin^4 x\, cos^5 x\, dx = \frac{8}{315}$.

7. a) Let V be a real vector space with $\{\alpha, \beta, \gamma\}$ as a basis. Prove that the set $\{\alpha + \beta + \gamma, \beta + \gamma, \gamma\}$ is also a basis of V. 4+1=5

b) Determine k so that the set $s = \{(1,2,1), (k, 3,1), (2, k, 0)\}$ is linearly dependent in R^3.

8. Let $G: R^3 \to R^2$ be defined by $(x, y, z) = (2x + 3y - z,\ 4x - y + 2z)$. Find the matrix A relative to the bases $S = \{(1,1,0), (1,2,3), (1,3,5)\}$ and $S' = \{(1,2), (2,3)\}$. 5

9. If A=$\begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$, show that $A^2 - 10A + 16I_3 = 0$. Hence obtain A^{-1}. 3+2=5

10 Evaluate $f(22)$ by interpolation formula using the following table: 5

x	20	25	30	35	40	45
f(x)	354	332	291	260	231	204

Group C (Answer any Three Questions) **10×3= 30**

11. a) A mapping $T(x_1, x_2, x_2) = (x_1 + x_2 + x_3,\ 2x_1 + x_2 + 2x_3,\ x_1 + 2x_2 + x_3)$, $(x_1, x_2, x_3) \epsilon R^3$. Show that T is a linear mapping. Find $Ker\ T$ and the dimension of $Ker\ T$.

b) Solve the Linear equation with higher degree $(D^2 + 2D + 5)y = xe^x$ where $D \equiv \frac{d}{dx}$.

c) Solve the following Bernoulli's equation $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y}{x^2}(\log y)^2$

5+3+2=10

12. a) Find the Characteristic polynomial and then Eigen value and the Eigen vectors corresponding to the matrix $A = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix}$.

b) Then show that A satisfy the Cayley Hamilton Theorem and find the matrix $P\ and\ D$ such that P is nonsingular and $D = P^{-1}AP$ is diagonal matrix.

(2+1+2)+(2+3)=10

13. a) Using Newton –Raphson method find the root of the equation $e^x - 3x - \sin x = 0$ correct upto five decimal places.

b) Evaluate the integral $I = \int_0^1 \frac{1}{1+x} dx$ by using i) Trapezoidal rule and ii) Simpson $\frac{1}{3}$rd rule with $h = 0.5$.

5+(2+3)=10

14. a) Discuss the convergence of $\int_0^\infty \frac{\cos x}{\sqrt{1+x^3}}$dx

b) Solve the differential equation $\frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3}$.

e) Solve the differential equation $(x - y + 3)dx = (2x - 2y + 5)dy$.

3+4+3=10

15. a) Find the root of the equation $x^x - 2x + 2 = 0$ in the interval $0 \le x \le 1$ using bisection method correct to three decimal places.

b) Using Laplace Transform solve the following Differential equation
$y'' - 4y' + 4y = x$ given that $y(0) = 0, \quad y'(0) = 1$

c) Find the Order and Degree of the following ordinary differential equation $(y'')^3 + (y'')^4 + y = 0$

4+4+(1+1)=10