ENGINEERING MECHANICS

INTRODUCTION

The subject Engineering Mechanics is broadly divided into two sections:

1. Statics (body at rest) 2. Dynamics (body in motion)

Dynamics again divided in two sections:

1. Kinematics: which is the study of the motion without referring the force which causes the motion.

2. Kinetics: which is the study of the motion with referring the force which causes the motion.

UNITS

	MKS	SI	FPS
Force	Kg_f	N	Poundal
Work	$m\text{-}Kg_f$	J	Ft-poundal
Power	$m\text{-}Kg_f/s$	W	HP
Pressure	Kg_f/cm^2	$N/m^2(Pa)$	PSi

FUNDAMENTAL PRINCIPLES

The study of elementary mechanics is based on the six fundamental principles. These principles cannot be proved mathematically but can be verified experimentally.

1. Newton's first law of motion: A particle remains at rest or continues to move in a straight line with a constant velocity if there is no unbalanced force acting on it.

2. Newton's second law of motion: The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

$$P = m\,a$$

3. Newton's third law of motion: To every action there is always an equal and opposite reaction.

4. Newton's law of gravitation: Two particles of mass M & m are attracted toward each other by equal and opposite forces P and –P, the magnitude of which is given by the following formula.

$$P = G \times M.m / r^2$$

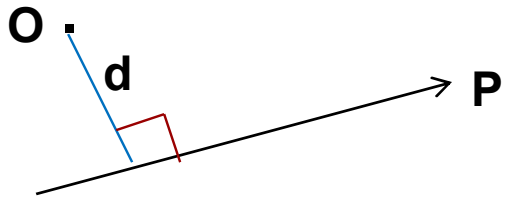
G = Const of gravitation = 6.67×10^{-11} Nm^2/kg^2

r = Distance between two masses.

5. The law of parallelogram of forces: If two forces acting at a point are represented in magnitude and direction by the adjacent sides of a parallelogram then the diagonal passing through the point represents the resultant of the applied forces.

6. The principle of transmissibility: Along the line of action of the forces, the effect of the force on a body is independent of the point of application.

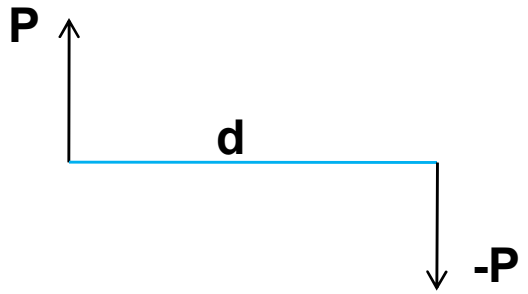
MOMENT: Moment of a force about a point is given by the product of the force and perpendicular distance of the force from the given point. Moment is vector quantity.



$M_o = P \times d$

COUPLES: A couple is a system of two equal and unlike parallel forces. The perpendicular distance between the lines of acting of the constituent forces is called an arm of the couple.

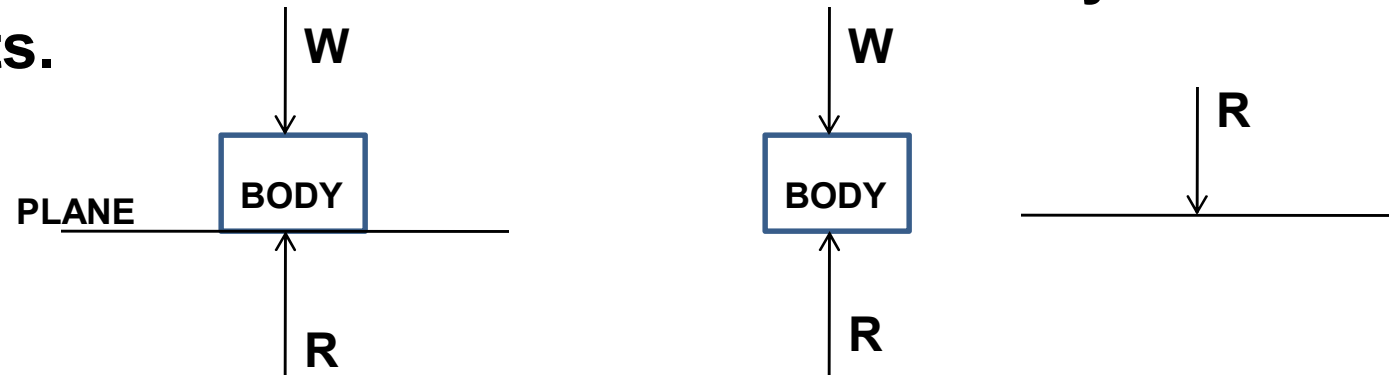
Moment of couple = P x d



EQUILIBRIUM LAW: Two forces can be in equilibrium only if they are equal in magnitude, opposite in direction and collinear in action.

FREE BODY: A free body is a body not connected with other bodies and which from any given position can be displaced in any direction in space.

FREE BODY DIAGRAM: Free body diagram is a sketch of the isolated body, which shows the external forces on the body and the reactions exerted on it by the removed elements.



VECTORS

SCALARS: **The Scalar quantities are described by their magnitude only. The scalar is a unidirectional aspect and may be added or subtracted arithmetically.**

Some of the scalar quantities are mass, distance, time, speed, volume, density, pressure, work, energy, power, temp, sp. heat etc.

VECTORS: **The Vector quantities are described by their magnitude and direction. A Vector has two directional aspects. These quantities can be combined by using vector algebra.**

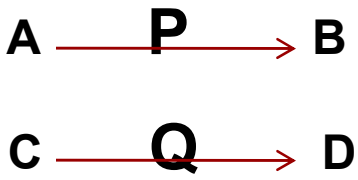
Some of the Vector quantities are position, displacement, velocity, acceleration, force, wt, momentum, impulse etc.

TYPES OF VECTORS

1.Unit-Vector: The vector which has unit magnitude is called unit vector.

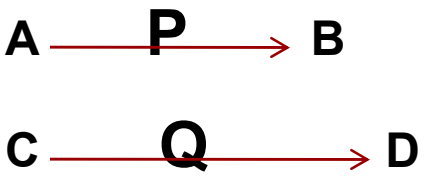
$|\vec{P}| = 1$

2. Equal Vector: The vectors of equal magnitude, same direction and parallel to each other are called equal vectors.



$\vec{P} = \vec{Q}$

3. Like Vector: The vectors of unequal magnitude, same direction and parallel to each other are called like vectors .

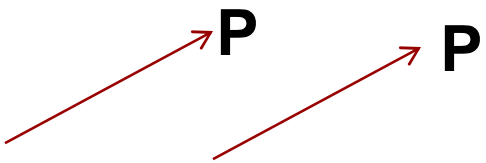


$\vec{P} \neq \vec{Q}$

all the sliding vectors follow rule of transitivity

4. Sliding Vector: The vector which can be moved anywhere but shall have same direction and magnitude is called sliding vector.

5. Free Vector: The vector which can move anywhere in the space without rotation is called free vector.



6. Fixed Vector: The vector which remains at the same point of application is called fixed vector.

7. Zero Vector: Zero vector is obtained when a vector is subtracted from it self. It is a vector of zero magnitude having no specific direction.

$\vec{P} - \vec{P} = 0$

SYSTEM OF FORCES

FORCES

Coplanar Forces

a. Concurrent

b. Non concurrent

c. Co linear - Like , Unlike

d. Parallel - Like , Unlike

e. Non parallel

Non coplanar Forces

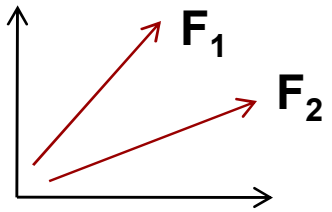
a. Concurrent

b. Non concurrent

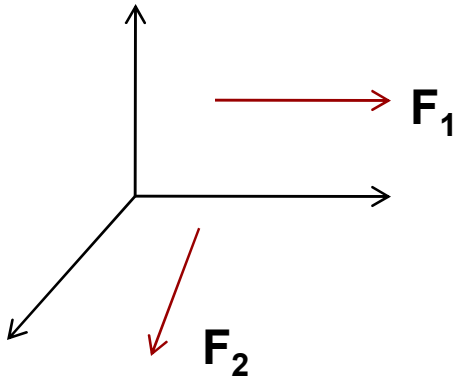
c. Parallel

d. Non parallel

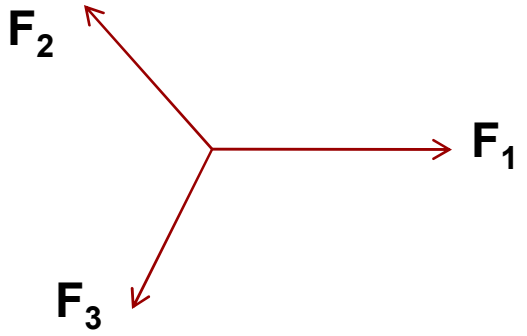
1. Coplanar Forces: Line of action of forces lies on same plane.



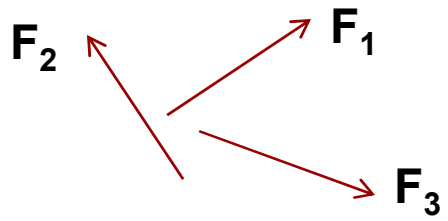
2. Non Coplanar Forces: Line of action of forces are not lying on same plane.



3. Concurrent Forces: Line of action of forces pass through a single point.



4. Non Concurrent Forces: Line of action of forces do not pass through single point.



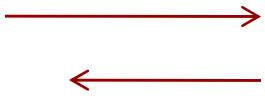
5. Colinear Forces: Line of action of forces pass through a single line of action.

Like **Unlike**

6. Parallel Forces: Line of action of all forces are parallel to each other.

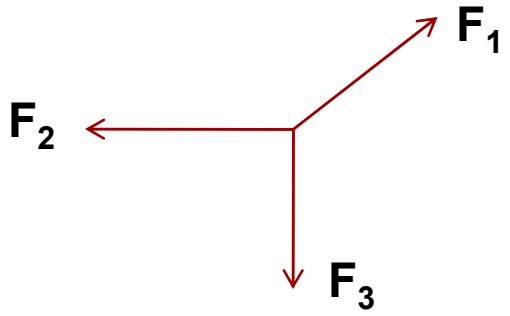
Like

Unlike



7. Non Parallel Forces: Line of action of forces are not parallel to each other.

8. Non Coplanar Concurrent: Line of action of forces are not lying on a single plane but passing through a single point.



Resultant of a Force: Resultant of a force system is a single force which produces the same effect as the force system.

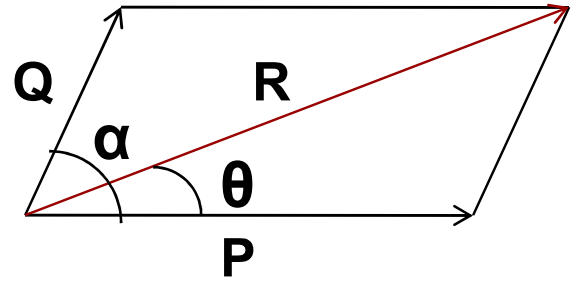
Method of finding the resultant Force: The resultant force of a force system can be determined by various methods.

- **Analytical Method – a. Trigonometric Method.**
  b. Method of Resolution.

- **Graphical Method – a. Triangle law of forces.**
  b. Polygon law of forces.

i)Trigonometric Method: Parallelogram law only for two concurrent forces.

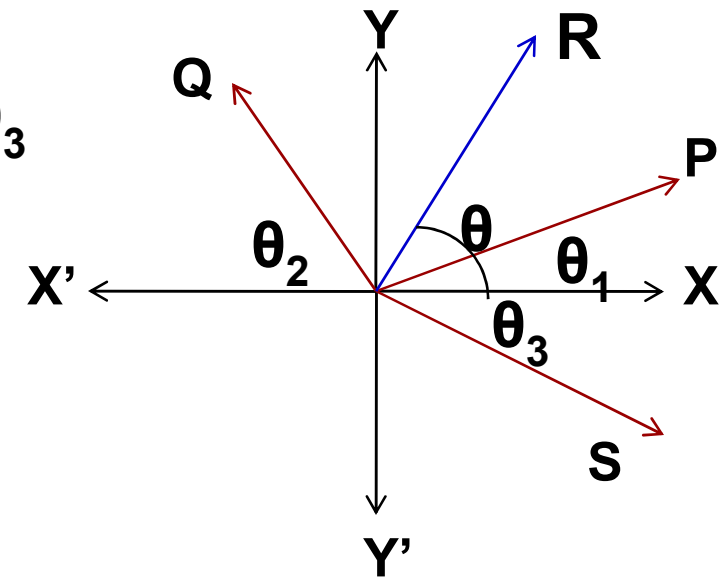
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}, \quad \tan\theta = Q\sin\alpha / (P + Q\cos\alpha)$$



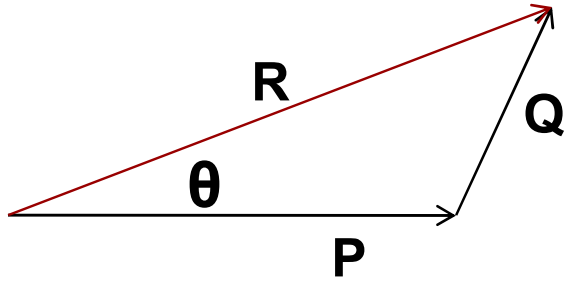
ii) Method of resolution :

$$\Sigma X = P\cos\theta_1 - Q\cos\theta_2 + S\cos\theta_3$$
$$\Sigma Y = P\sin\theta_1 + Q\sin\theta_2 - S\sin\theta_3$$

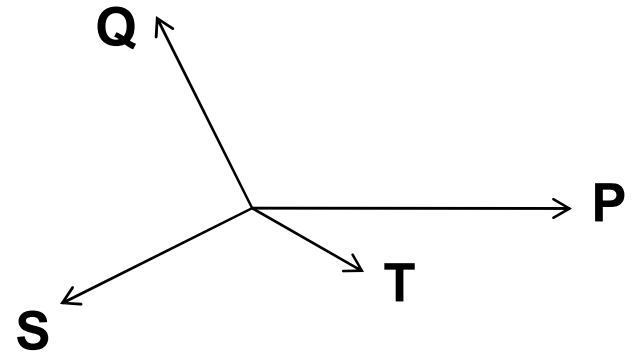
$$R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$$
$$\tan\theta = \Sigma Y / \Sigma X$$

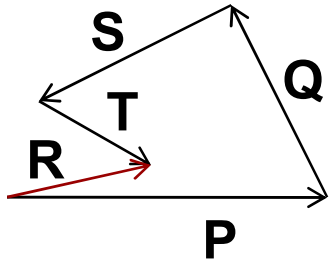


iii) Triangle law of Forces: If two forces acting on a body are represented by the sides of a triangle then their resultant is represented by the third side of the triangle taken in the opposite order.



iv) Polygon law of Forces: If any number of forces are represented in magnitude and direction by the sides of the polygon taken in order, their resultant is represented by the closing side of the polygon take in opposite order.





the system of forces is in eq.

PROB-1: A force of 200N makes angle 50^o, 75^o and 140^o respectively with x, y, z axes. Determine the components of the force and express it as a vector.

Sol^n : F = 200 N, $\theta_x = 50^o$, $\theta_y = 75^o$, $\theta_z = 140^o$

Component of force:

In x direction: $F_x = F \cdot \cos\theta_x = 200 \cdot \cos 50^o = 128.56$ N

In y direction: $F_y = F \cdot \cos\theta_y = 200 \cdot \cos 75^o = 51.76$ N

In z direction: $F_z = F \cdot \cos\theta_z = 200 \cdot \cos 140^o = -153.21$ N

.: Force vector $F = F_x i + F_y j + F_z K$

$F = 128.56\, i + 51.76\, j - 153.21\, K$

PROB-2: Determine the magnitude of force F, whose components are F_X = -15kN, F_y = +27 kN, and F_z = -40 kN. Also calculate the angles θ_x , θ_y and θ_2 , it forms with the axes of co-ordinates.

Sol^n : F_X = -15KN, F_y = +27 KN, F_z = - 40 KN

Magnitude of the force,

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(-15)^2 + (27)^2 + (-40)^2}$$

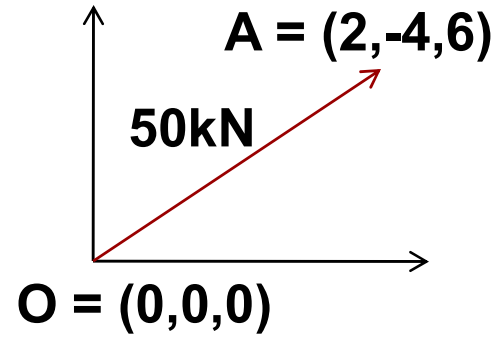
$$= 50.54 \text{ kN}$$

$\cos\theta_x = F_x/F = -15/50.54$ or $\theta_x = 107.26^o$

$\cos\theta_y = F_y/F = 27/50.54$ or $\theta_y = 57.71^o$

$\cos\theta_z = F_z/F = -40/50.54$ or $\theta_z = 142.32^o$

PROB-3: Determine the components and direction of the force of magnitude 50 kN as shown.



Sol^{n} : F = 50 kN

Position of O = (0,0,0)

Position of A = (2,-4,6)

Position vector, $\overrightarrow{OA}$ = (2-0)i + (-4-0)j + (6-0) K

= 2i – 4j + 6K

OA = $\sqrt{2^2 + (-4)^2 + 6^2} = \sqrt{56} = 7.48$

Unit vector $\hat{\lambda} = \overrightarrow{OA} / OA = (2i - 4j + 6K) / 7.48$

.: Force vector $\overrightarrow{F} = F \cdot \hat{\lambda} = 50\,[(2i-4j+6K) / 7.48]$

or $F_x i + F_y j + F_z K = 13.37i - 26.74j + 40.11k$

Hence, $F_x = 13.37$ kN, $F_y = -26.74$ kN, $F_z = 40.11$ kN

PROB-4: Three external forces P_1, P_2, P_3 are acting on an object. Determine the magnitude of P_2 if $P_1 = 10i+20j$ (N) and $P_3 = 25i - 30j$ (N) for equilibrium condition.

Soln : For equilibrium, $P_1 + P_2 + P_3 = 0$

$$(10i + 20j) + P_2 + (25i - 30j) = 0$$

or, $(10 + 25)i + (20 - 30)j + P_2 = 0$

or, $35i - 10j + P_2 = 0$

or, $P_2 = - 35i + 10j$ (N) **(Ans)**

Magnitude of $P_2 = \sqrt{(-35)^2 + 10^2} = \sqrt{1225 + 100}$

$= \sqrt{1325}$

or $|P_2| = 36.40$ N **(Ans)**

PROB-5: A force P = 5i + 3j – 4k acts at point A (2,-1,4). Determine the moment of the force about the point B (4, 3, -2).

Sol^{n} : $\vec{P} = 5i + 3j - 4k$ force vector

$\vec{r}_A = 2i - j + 4k$ Position Vector of A

$\vec{r}_B = 4i + 3j - 2k$ Position vector of B

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = (4i + 3j - 2k) - (2i - j + 4k)$$

$$= 2i + 4j - 6k$$

.: Moment of force P about the point B,

$$\vec{M}_B = \vec{r}_{AB} \times \vec{P} = (2i + 4j - 6k) \times (5i + 3j - 4k)$$

$$= \begin{vmatrix} i & j & k \\ 2 & 4 & -6 \\ 5 & 3 & -4 \end{vmatrix}$$

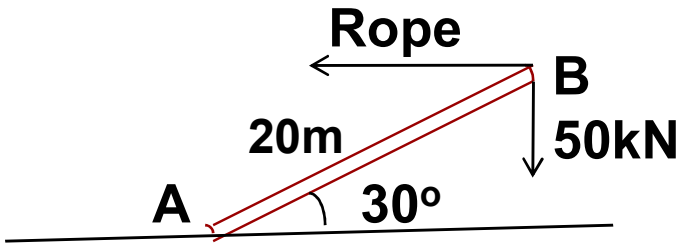
$$= (-16 + 18)i + (-30 + 8)j + (6 - 20)k$$

$$\therefore \vec{M}_B = 2i - 22j - 14k$$

$$\text{Magnitude } |\vec{M}_B| = \sqrt{2^2 + (-22)^2 + (-14)^2}$$

$$= 26.15 \text{ N-m (Ans)}$$

PROB-6: The boom of a crane, 20m long supports a load of 50kN as shown. Determine the moment due to the load about the pivot A of the boom.



Soln : Load $\vec{W} = -50j$ kN

Position vector AB = $\vec{r}_{AB} = (20\cos30^o)i+(20\sin30^o)j$

$= 17.32i + 10j$

Moment about A, $\vec{M}_A = \vec{r}_{AB} \times \vec{W} = (17.32i + 10j) \times (-50j)$

$$= \begin{vmatrix} i & j & k \\ 17.32 & 10 & 0 \\ 0 & -50 & 0 \end{vmatrix}$$

$= i \times 0 + j \times 0 + k \times (-866) = -866k$ kNm. (Ans)

Magnitude of moment $|\vec{M}_A| = \sqrt{(-866)^2} = 866$ kNm (kJ) (Ans)