Dynamics Module-05 of ME-203

Kinetics of Particle

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Introduction

Kinematics: Kinematics is the geometry of motions.

Kinetics:

On the other hand Study of generalized force equilibrium of particles and rigid bodies in motion subjected to external system of force lies within purview of kinetics. So, Kinetics can be defined as the analysis in dynamics associated with laws of motion, inertial effect, work, power, energy, impulse and momentum, simple and complex vibration etc.

- The study of dynamics refers to the motion of bodies under the application of action, i.e., external forces or moments.
- The same laws of dynamics are applicable to the motion of a particle and of the center of mass of any system undergoing translation under the application of forces.
- This is because the mass of a particle is assumed to be concentrated at a point which is also its centre of mass.
- This fact is considered to comprise the translation of the mass centre and a rotation superimposed upon it.

Newton's Law

FIRST LAW:

It states that everybody continues in its states of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it. This leads to the definition of force as the external agency which changes or tends to change the condition of rest or uniform linear motion of the body.

SECOND LAW:

It states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it. Thus according to this Law.

Force ∞ mass $\times$rate of change of velocity(acceleration)

Mathematically F $\infty m \times a$

THIRD LAW :

It states that for every action there is an equal and opposite reaction. Consider the two bodies in contact with each other let one body apply a force F on another. According to this Law of the second body develops a reaction force R which is equal in magnitude to force F and acts in the line same as F but in the opposite direction.

Laws of motion

The dynamics of a mass centre or of a particle is governed by the Newton's law.

$$F = \frac{d}{dt}(m\mathrm{V})$$

➢This is the fundamental equation of motion which governs the interaction of the applied force **F** with the motion of a particle or a mass centre . Problems in dynamics may be concerned with the determination of the motion, i.e accleration, velocity and positions for a prescribed force or vice versa.

➢It may be stated at the outset that the work –energy principle ,impuls-momentum principle and the moment of momentum principle are alternative forms of Newton's Law.

➢One or the other may be preferred under different circumstances . A comparative study of the equivalent dynamical equations is given in table -1.This is indeed a summary of the principles derived and discussed in this chapter.

➢<u>Equation of motion</u>:

➢The equation of motion due to Newton for the centre of mass of any system or a particle of constant mass m may be written as

$$F = m\frac{dv}{dt} = ma = m\frac{d^2 r}{dt^2} \quad .....[\quad 1\ ]$$

Equation of motion

It may be noted that for a particle, The net force, velocity and acceleration refer to the point representation of the particle , while for a rigid body, the net force may be applied anywhere on it but the velocity and acceleration are referred to the mass center only as shown in figure-1.The motion can be determined from a knowledge of the applied force F. Let us first consider some simple cases of rectilinear translation as visualised in figure-2.

- namical equations is given in table -1.This is indeed a summary of the principles derived and discussed in this chapter.

- **<u>Case(a)Constant Force F along s-direction</u>**:
- Then,

$$a = \frac{d^2 s}{d t^2} = F/m$$

On integration $$v = \frac{ds}{dt} = (F/m)t + c_1$$

And $$s = \frac{1}{2}(F/m)t^2 + c_1 t + c_2$$

Explanation with figures

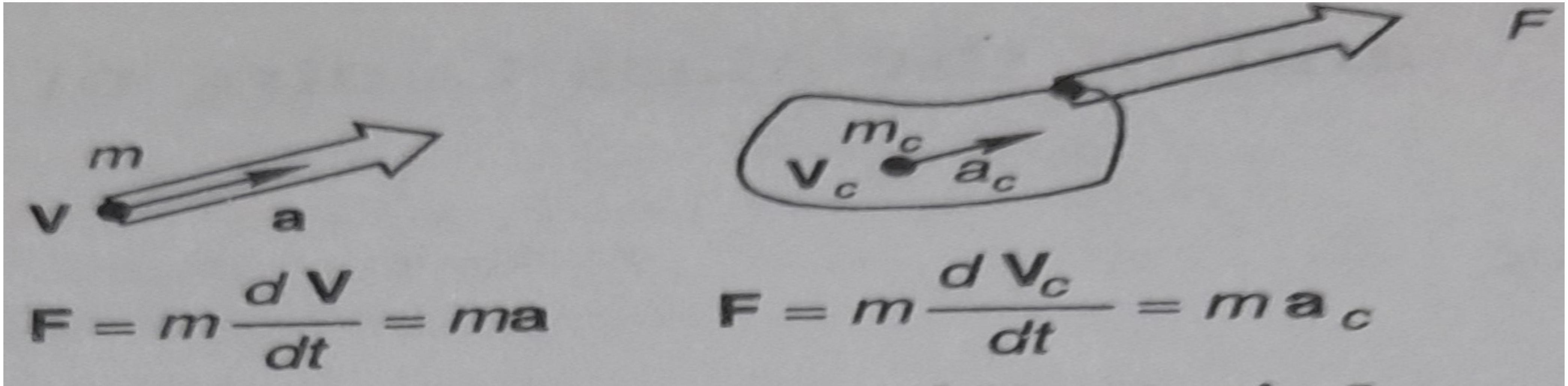


Fig. 6.1 ***Implication of Newton's Law***

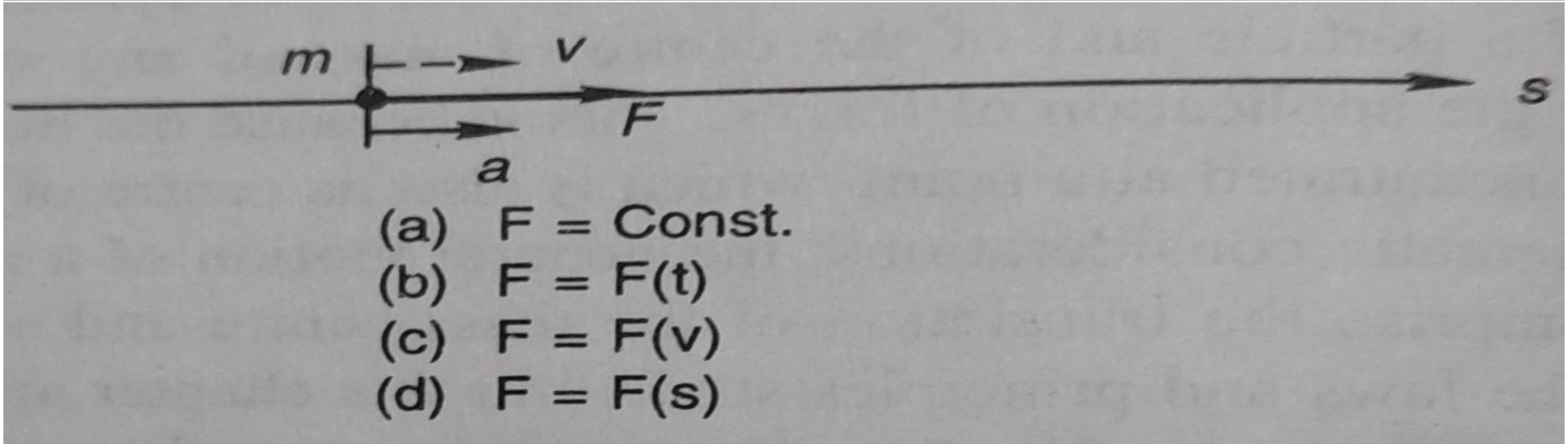


Fig. 6.2 ***Rectilinear Translation***

Equation of motion

It may be noted that for a particle, The net force, velocity and acceleration refer to the point

- **Case(b) Force F(t) is a function of time along s-direction**:
- Then,

$$a = \frac{d^2 s}{dt^2} = F(t)/m$$

On integration

$$v = \frac{ds}{dt} = \int (F(t)/m)\,dt + c_1$$

And

$$s = \int \left(\int (F(t)/m)\,dt + c_1 \right) dt + c_2$$

Where C1 and C2 are again determined from the given conditions.

Equation of motion

- **Case(c) Force F(v) is function of speed along s-direction**:

- Then, $a = \frac{dv}{dt} = F(v)/m$

$$or\ , \frac{dv}{F(v)} = \frac{1}{m}t$$

On integration $$\int \frac{dv}{F(v)} = \frac{1}{m}t + C_1$$

Which provides v=f(t)

And on further integration, yields an expression for the displacement s,the constant being determined from the given conditions.

Equation of motion

➢ **Case(d) Force F(s) is function of rectilinear displacement s**:

➢ Then,
$$a = \frac{dv}{dt} = \mathrm{F(s)/m} \quad or, \quad \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} = \frac{F(s)}{\mathrm{m}}$$

$$or, \; vdv = \frac{1}{\mathrm{m}} F(s)\, ds$$

On integration $\frac{\mathrm{v}^2}{2} = \frac{1}{\mathrm{m}}\int F(s)\,ds + C_1$

$$\mathrm{v} = \frac{ds}{dt} = \left[\frac{2}{\mathrm{m}}\int F(s)\,ds + C_1\right]^{1/2}$$

Separating the variables and integrating again provides s as a function of t the constant of integration being determined from a knowledge of the given conditions. If the motion of a particle is prescribed, the force required to accomplish it may be determined by employing the equation of motion F=ma and substituting the value of **a** in it.

Summary with Table-1

Summary

Table 6.1 Comparative Study of the Equivalent Dynamical Equations for a Particle or a Mass Centre

	Newton's Law	Work Energy Equation	Impulse Momentum Principle	Moment of Momentum Principle
General form for a particle	$\mathbf{F} = \frac{d}{dt}(m\mathbf{V}) = m\mathbf{a}$	$W_{12} = \int_{r_1}^{r_2} \mathbf{F} \cdot d\mathbf{r} = KE_2 - KE_1$	$\int_{t_1}^{t_2} \mathbf{F} \cdot dt = m(\mathbf{V}_2 - \mathbf{V}_1)$	$\mathbf{M}_0 = \mathbf{r}_0 \times \mathbf{F} = \frac{d}{dt}(\mathbf{r}_0 \times m\mathbf{V}) = \frac{d\mathbf{H}_0}{dt}$
Statement for a particle	Net force acting on a particle equals the rate of change of momentum or the mass times acceleration of the particle	Net work done by the external forces acting on a particle equals the change in kinetic energy possessed by the particle.	Impulse of the net force acting on a particle over a period of time equals the change of momentum of the particle over the same period of time	Moment of the resultant force on a particle about a fixed point O equals the time rate of change of the moment of momentum referred to the point
General form for the mass centre of a body	$\mathbf{F} = \frac{d}{dt}(m\mathbf{V}_c) = m\,\mathbf{a}_c$	$W'_{12} = \int_{r_1}^{r_2} \mathbf{F} \cdot d\mathbf{r}_c = KE_{2c} - KE_{1c}$	$\int_{t_1}^{t_2} \mathbf{F} \cdot dt = m(\mathbf{V}_{2c} - \mathbf{V}_{1c})$	$\mathbf{M}_0 = \mathbf{r} \times \mathbf{F} = \frac{d}{dt}(\mathbf{r} \times m\mathbf{V}_c) = \frac{d\mathbf{H}_0}{dt}$
Statement for the mass centre of a body	Net force acting on a body equals the rate of change of momentum associated with the mass centre or the mass times acceleration of the centre of mass of the body	Net work done by the external forces, as if acting at the mass centre of a body, equals the change in linear kinetic energy associated with the mass centre of the body	Impulse of the net force acting on a body over a period of time equals the change of momentum associated with the mass centre of the body over the same period of time	Moment of the resultant force at the mass centre of a system about a fixed point O equals the time rate of change of moment of momentum about the same point.
Special cases; Associated Principles	When $\mathbf{F} = 0$; in the absence of net external force, a particle or the mass centre of body must continue to move with constant velocity: $\mathbf{V}$ = Const.	When $\mathbf{F} = -\nabla PE$; in a conservative force field, the mechanical energy of a particle or that associated with the mass centre is conserved; $KE + PE$ = Const.	When $\mathbf{F} = 0$; in the absence of an external force, the linear momentum of a particle or that associated with the mass centre is conserved: $\mathbf{p} = \Sigma m\mathbf{V}$ = Const.	When $\mathbf{M}_0 = 0$; in the absence of the moment of the net external force taken about a fixed point, the moment of momentum relative to the same point is conserved; $\mathbf{H} = \Sigma(\mathbf{r} \times m\,\mathbf{V})$ = Const.

Summary with Table-1

Table 6.1 *(Contd)* **Comparative Study of the Equivalent Dynamical Equations for a Particle or a Mass Centre**

	Newton's Law	*Work Energy Equation*	*Impulse Momentum Principle*	*Moment of Momentum Principle*
Circumstances under which preferred	When the external force or acceleration is desired to be evaluated	When the work done may be determined and/or when one of the velocities is to be evaluated; particularly useful for motion in a conservative force field.	When the impulses or application of forces over a period of time is involved and also when the external force or impulse vanishes	When the force on a particle is directed towards or away from a fixed point

D'Alambert's Principle

Newton's 2nd law of motion is applicable to the motion of a particle as well as the motion of a
Body.
If a system of force acting on a body then resultant force is equal to the product of mass and
acceleration in the direction of resultant force, Mathematically

$$R = ma$$

Where R is the resultant of forces acting on the body.
Newton's law may be written in different form as

$$R - ma = 0 \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The term '-ma' may be looked as a force of magnitude m×a applied in opposite direction of
Motion is in dynamic equilibrium with the inertia force of the body.
This is known as the D'Alembert's principle.

<u>Explanation with figure-2</u>

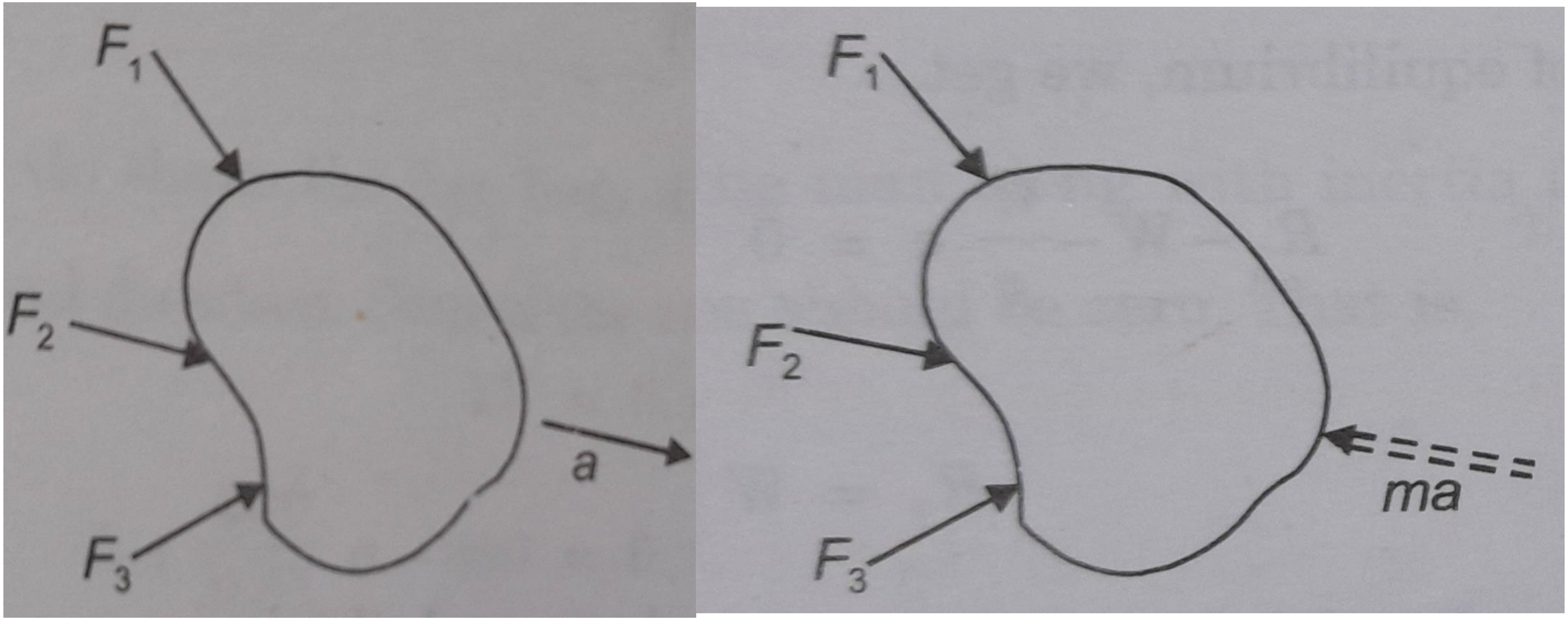


Figure-2

Considering the force balance

$$m_1 g - T = m_1 a \;\ldots\ldots\ldots\ldots\ldots[1]$$

$$T - \mu R = m_2 a$$

$$\text{or } T - \mu m_2 g = m_2 g \;\ldots\ldots\ldots\ldots\ldots[2]$$

$$\text{Or } T = m_2 a + \mu m_2 g$$

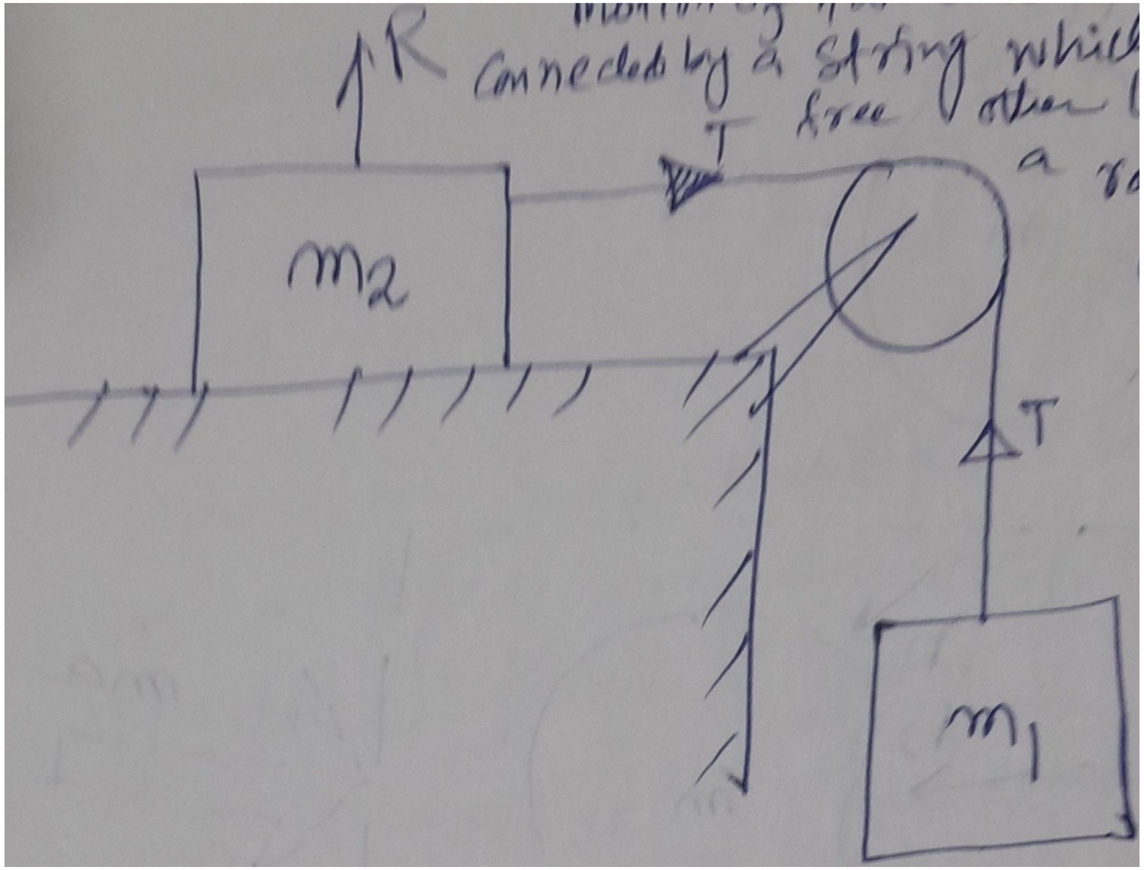
$$T = m_2 \,(\, a + \mu g) \;\ldots\ldots\ldots\ldots[3]$$

Substituting the value of T in eq [1]

$$m_1 g - T = m_1 a$$

$$\text{Or } m_1 g - m_2(a+\mu g) = m_1 a$$

$$(m_1+m_2)a = m_1 g - m_2 \mu g$$

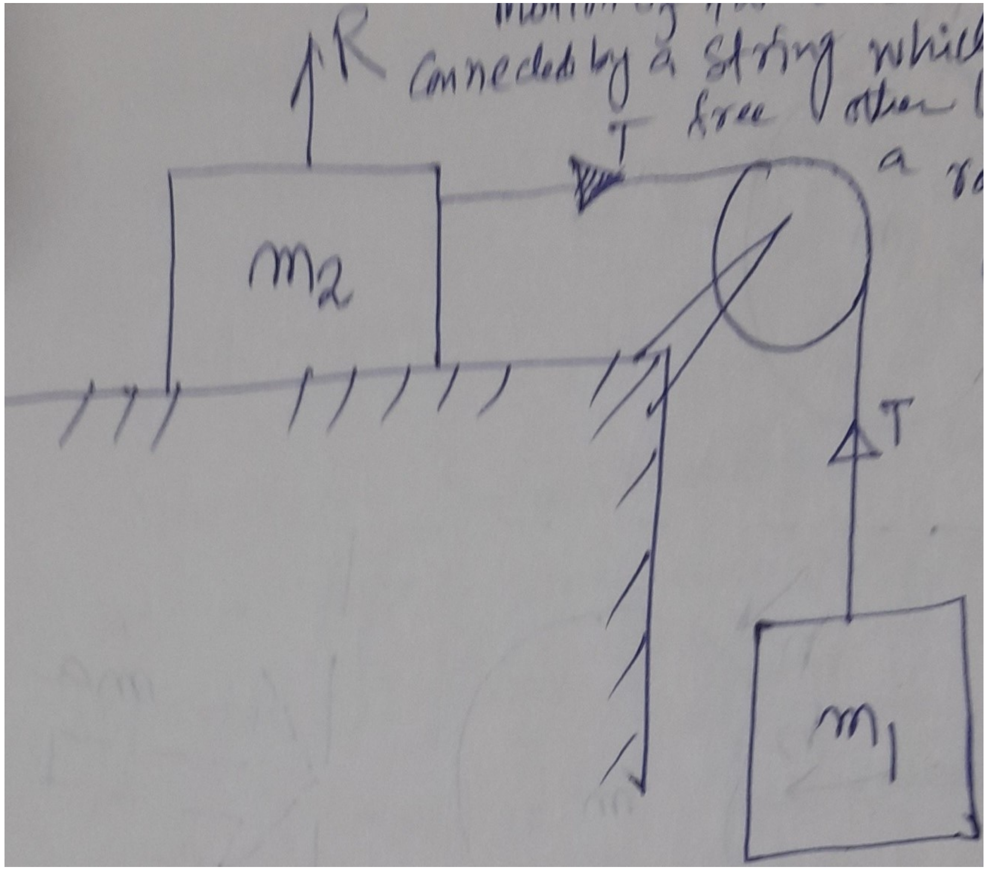


$$a = \frac{g\,(m_1 - \mu\, m_2)}{m_1 + m_2}$$

$$T = m_2 \left[\frac{g\,(m_1 - \mu\, m_2)}{m_1 + m_2} + \mu g \right]$$

$$T = m_2\left[\frac{g\,m_1 - \mu\,m_2\,g + m_1\,\mu g + m_2\,\mu g}{m_1 + m_2}\right]$$

$$T = \frac{g\,m_1 m_2(1+\mu)}{m_1 + m_2}$$



ALTERNATIVELY

$m_1g - T - m_1a = 0$ ....[1]

$T - \mu R = m_2a$

$T - m_2g\mu - m_2a = 0$ ....[2]

$T = m_2(a + \mu g)$ ....[3]

Value of T substituted in Eqn [3]

$m_1g - m_2(a+\mu g) - m_1a = 0$

Or $m_1g - m_2a - m_2\mu g - m_1a = 0$

Or $(m_1 + m_2)a = m_1g - m_2\mu g$

Explanations with figure

$$a = \frac{g(m_1 - \mu m_2)}{m_1 + m_2}$$

$$T = m_2\left[\frac{g(m_1 - \mu m_2)}{m_1 + m_2} + \mu g\right]$$

Work Energy Equation for pure translation motion

Consider a body (as shown in fig A) of weight **W** is subjected to a system of forces $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{F_3}$ & $\mathbf{F_4}$ and moving with an acceleration a in x direction and motion is a pure translation.

Let us consider with certain interval of time **t** the body will reach from position **A** to **B.**

Let initial velocity be u (at point **A**) and final velocity be v m/s(at point **B**).

Then the resultant of system of forces must be in x direction.

$$R = \sum F_x \ldots\ldots\ldots\ldots[1]$$

From Newton's 2nd Law of motion,

$$R = \frac{W}{g}a \quad \Rightarrow \quad Rds = \frac{W}{g}ads \; (multiplying \text{ bothside with elementary distance ds})$$

$$Rds = \frac{W}{g}v\frac{dv}{ds}ds \text{ Since a} = v\frac{dv}{ds}$$

$$Rds = \left(\frac{\mathrm{w}}{\mathrm{g}}\right) v \frac{dv}{ds} ds$$

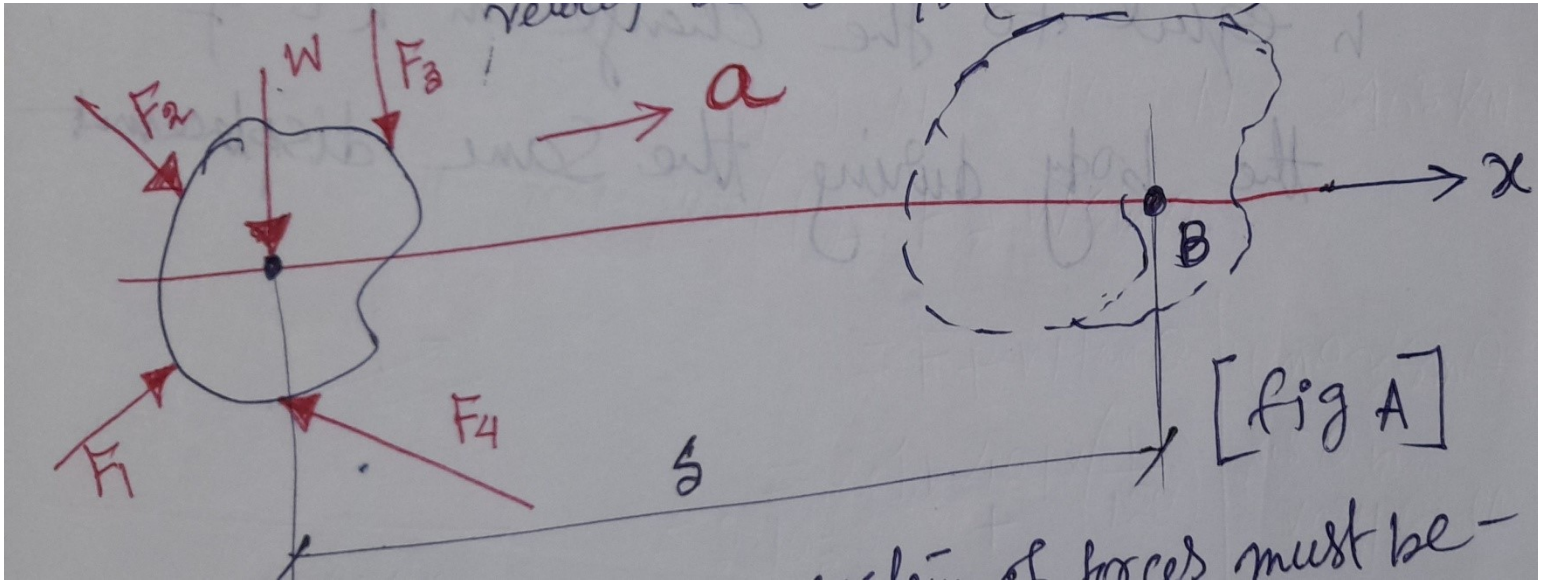
$$= \left(\frac{W}{g}\right) vdv$$

Now integrating both sides for the motion from **A** to **B** we get

$$\int_0^s Rds = \int_u^v \left(\frac{w}{g}\right) vdv$$

$$\Rightarrow Rs = \frac{w}{g}\left[\frac{v^2}{2}\right]_u^v$$

$$\Rightarrow Rs = \frac{1}{2}\frac{w}{g}[v^2 - u^2]$$



$$\Rightarrow Rs = \frac{1}{2} m [v^2 - u^2] \qquad \left\{\frac{w}{g} = m\right\}$$

Work done due to translation motion = change in K.E

$$\Rightarrow work\ \text{done} = \frac{1}{2}mv^2 - \frac{1}{2}u^2$$

$$\Rightarrow work\ \text{done} = \text{Final KE} - \text{Initial KE}$$

This is called Work Energy equation.

FORMAL DEFINITION OF WORK ENERGY EQUATION

It may be stated that as the work done by the system of forces acting on a body during a displacement is equal to the change in Kinetic energy of the body during the same displa cement.

Using this work energy equation anumber of kinetic problems can be solved. This will be found more useful than D’ Alamber’s principle when we are not interested in finding acceleration in the problem, but mainly interested in velocity and distance.

NUMERICAL

1. A body of mass 120kg is rest on a rough plane inclined at 12° to the horizontal. It is pulled up the plane by means of light flexible rope running parallel to the plane and passing over light frictionless pulley at the top of the plane as shown in figure . The portion of the rope beyond the pulley hangs vertically down and carries a mass of 81.5 kg at it's end .

If the co–efficient of friction for the plane and the body is 0.2,Find the followings.

a) Tension in the rope.

b) Acceleration with which the body moves up the plane, and

c) The distance moved by the body in 3sec after staring from rest.

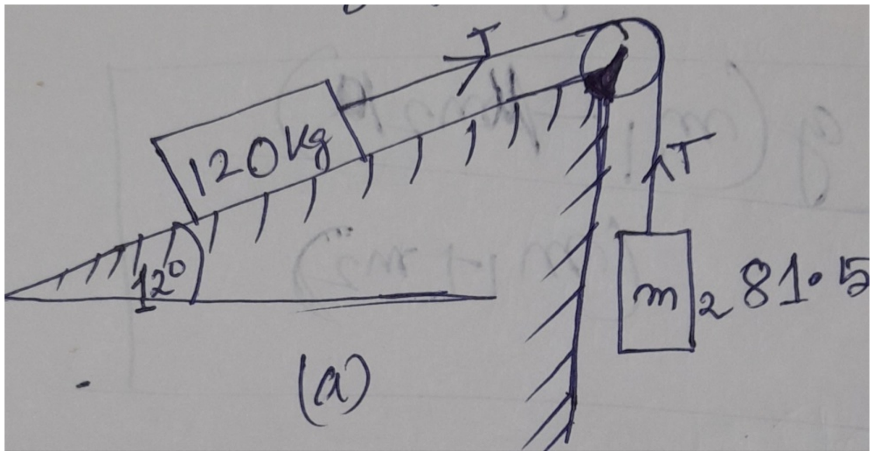
Ans:- Let *a* be the acceleration of the system and force normal to the plane

$$\sum F_y = 0$$

N -120gcos12° = 0

N = 120 cos12°

N = 120×9.81×cos12° or N = 1151.48 N

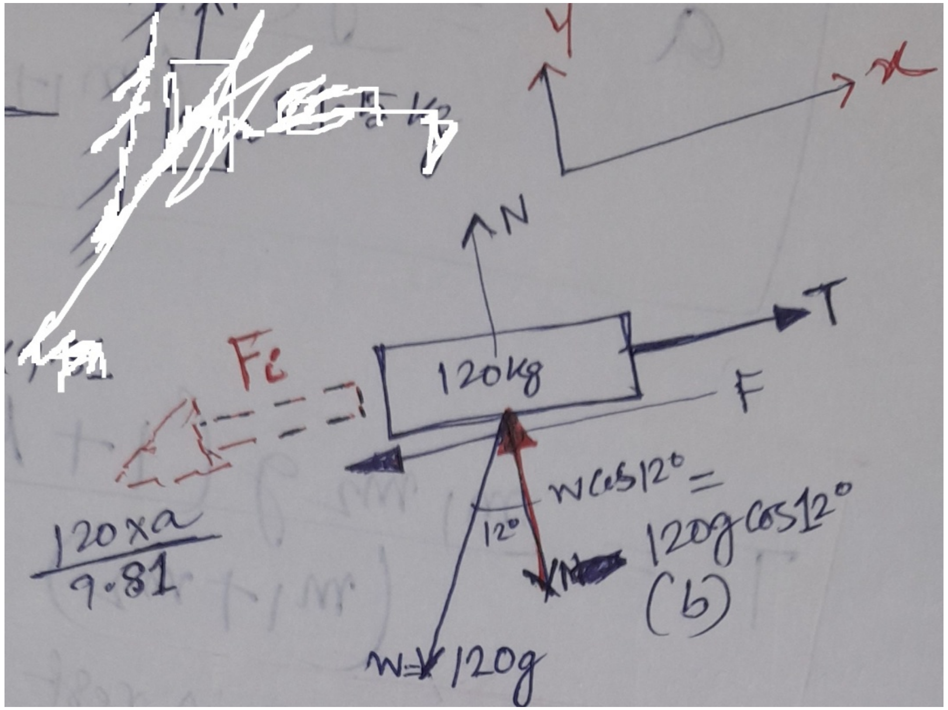


$F = \mu N$

$= 0.2\times 1151.48$ N

$F = 230.3$N

$$\sum F_y = 0$$



$T - F - W\sin 12^o - 120 \times a = 0$……….[2]

Or $T - 230.3 - 120\times 9.81\times \sin 12^o - 120a = 0$

$T - 230.3 - 244.75 - 120a = 0$

$T - 120a - 475.05 = 0$ ……………..[3]

Consider free body diagram of Block, m_2 =81.5 kg

$m_2 g - T - m_2 a = 0$…………..[4]

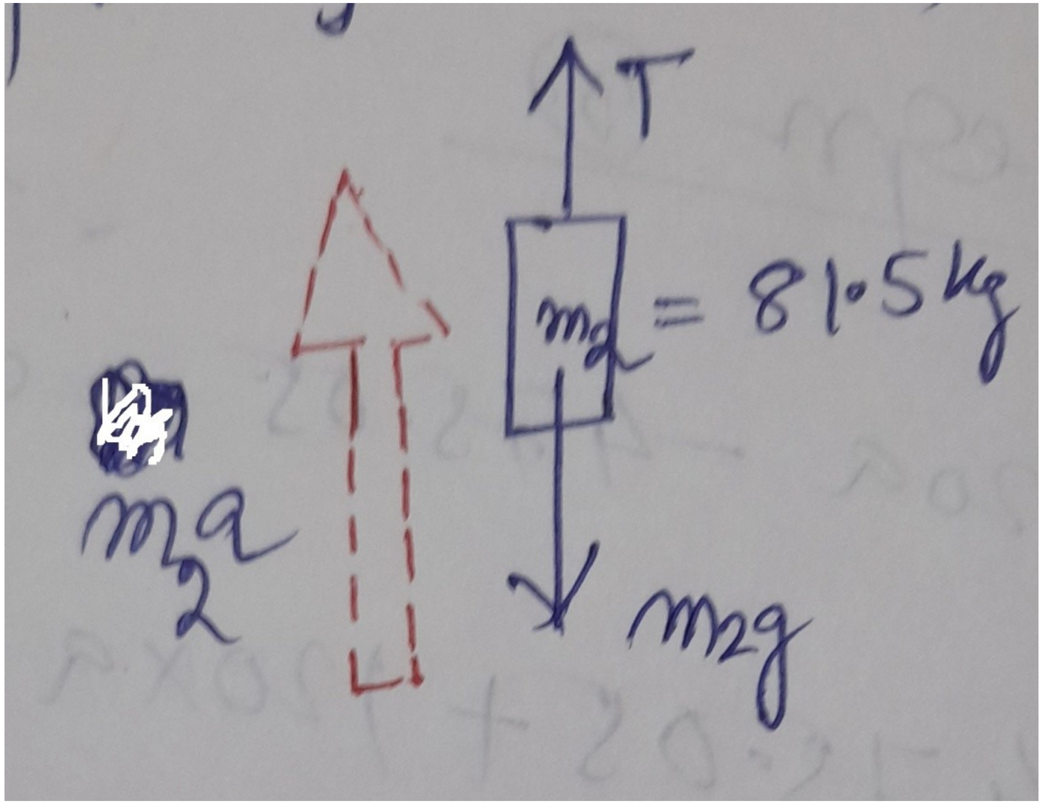
$81.5\times 9.81 - T - 81.5\times a = 0$

$799.52 - T - 81.5 \times a = 0$…………[5]

By adding [3] + [5]

$$799.52 - T - 81.5 \times a = 0 \ldots\ldots\ldots\ldots [5]$$

$$T - 120a - 475.05 = 0 \ \ldots\ldots\ldots\ldots\ldots\ldots [3]$$



$$799.52 - 81.5a - 120a - 475.05 = 0$$

$$201.5a = 799.52 - 475.05$$

$$201.5a = 324.47$$

$$a = 324.47/201.5$$

$$a = 1.61 \text{ m/s}^2$$

From eqn [3]

$$T - 120a - 475.05 = 0$$

$$T = 475.05 + 120 \times 1.61 \text{m/s}^2$$

$$= 475.05 + 193.2$$

$$T = 668.25 \text{ N}$$

Using the eqn

$$S = ut + 1/2at^2$$

$$S = 7.245 \text{ m}$$

2.(a)Find the power of a locomotive drawing a wagon whose weight including engine is 421 KN while it moving up an inclined with inclination 1 in 120 at a steady speed of 56 km/h , the frictional resistance being considered as 5N/kN.

(b) If it is happened that , the motive power is suddenly cut off while it is ascending the incline,then find how far it will move before coming to considering resistance to motion remains the same.

Solution: $v = 56$ Km/h $= 56 \times 10^3 = 15.556$ m/s

Friction force F = 5×421N = 2105 N = 2.105 KN

Effective pull force require (p) = F + Wsinθ =2.105 + 421 × 1/120 = 2.105 +3.50833 KN = 5.6133 KN

Note: tan $\theta \approx$ sin θ

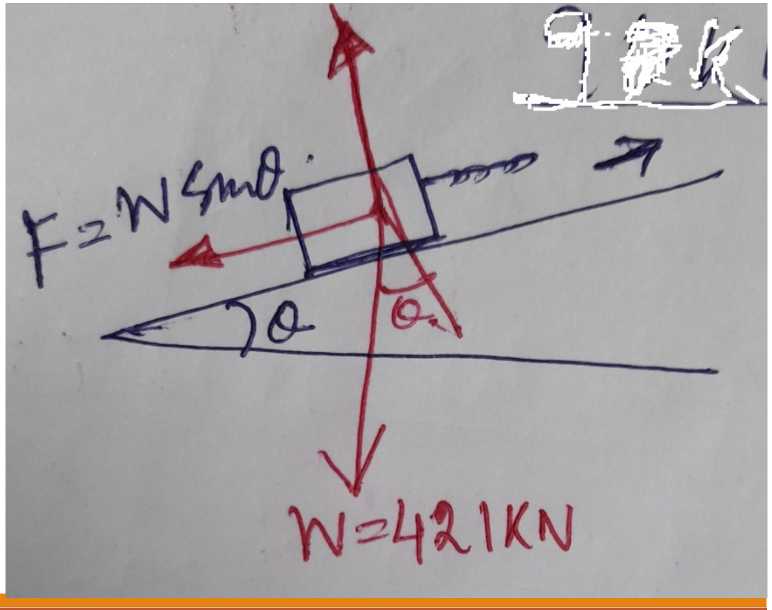
As θ is very small

Power of a locomotive = Work done by force (p)

$= P \times v$

$= 5.6133 \times 15.556$ KN m/s

$= 87.3205$ Kw



When suddenly power in cut off, Let it moves a distance **S** before coming to rest .

Initial Velocity v = 15.556 m/s

Final velocity v = 0 m/s

= F + W sinθ

= 2.105 + 421 × (1/120)

= 5.6133 KN (opposite to the motion of the locomotive down the plane)

Writing work energy equation for translation up to the plane, we have

$$-5.6133\times 10^{3}\times s=\frac{1}{2}\times\frac{421\times 10^{3}}{9.81}(0^{2}-15.556^{2})$$

S=925.04 m. **Ans.**

[3] A Particle moving with a velocity v along a straight line is retarded such that the retardation is (a) proportional to velocity and (b) proportional to square of velocity. Determine the expression for velocity as a function of time and the distance traversed before it comes to rest for both the cases.

Solution For case (a), let the retardation be kv

i.e., $$a = \frac{dv}{dt} = -kv$$

Or $$\frac{dv}{dt} + kv = 0$$

On integration $$\int_u^v \frac{dv}{v} + k\int_0^t dt = 0 \qquad or \qquad \log_e \frac{v}{u} + kt = 0 \qquad v = ue^{-kt}$$

Also,

$$a = \frac{dv}{ds} . \frac{ds}{dt} = v\frac{dv}{ds} = -kv$$

$$or \int_u^v dv + k\int_0^s ds = 0$$

$$v - u + ks = 0$$

$$v = u - ks \qquad or \quad s = \frac{u - v}{k}$$

For case (b), let the retardation be μv^2

i.e., $a = \frac{dv}{dt} = -\mu v^2$

Or $\frac{dv}{dt} + \mu v^2 = 0$

On integration, $\int_u^v \frac{dv}{v^2} + \mu\int_0^t dt = 0$

$$-\frac{1}{v} + \frac{1}{u} + \mu t = 0 \quad or \quad v = \frac{u}{1 + \mu u t}$$

$$Also \quad a = \frac{dv}{ds}.\frac{ds}{dt} = v\frac{dv}{ds} = -\mu v^2 \qquad or \quad v\frac{dv}{ds} + \mu v^2 = 0 \qquad or \int_u^v \frac{dv}{v} + \mu\int_0^s ds = 0$$

$$\log_e \frac{v}{u} + \mu s = 0$$

$$v = u e^{-\mu s}$$

$$and \; s = \frac{1}{\mu}\log_e \frac{u}{v}$$

It may be noticed that the distance traversed before coming to rest in case (a), is finit whereas that in case(b) is infinite

[4] A particle of mass 1 Kg moves in a straight line under the influence of a force which increases linearly with time at the rate of 60 N/s, it being 40 N initially. Determine the position, velocity and acceleration of the particle after a lapse of 5s if it started from rest at the origin.

Solution: From the statement of the problem,

F = 40+60t

Which, by Newton's law should equal mass times acceleration of the particle. Since the mass is 1kg

$$a = \frac{d^2x}{dt^2} = 40 + 60t$$

Integrating the term w.r.t time t $\quad v = \frac{dx}{dt} = 40t + 30t^2 + C_1$

Integrating again, $\quad x = 20t^2 + 10t^3 + C_1 t + C_2$

From the initial condition, v=0 and x=0 at t=0 The constant $C_1 = C_2 = 0$

Hence

$$v = 40t + 30t^2 \quad \Rightarrow x = 20t^2 + 10t^3$$

At the instant t=5s a=340 m/s^2 v=950 m/s and x=1750 m from the origin.

[5]If a body of mass m moves through a liquid at low velocity, the force of resistance due to viscosity is given by F=kv where k is the resistance due to viscosity. Show that the velocity would decress exponentially with the time and linearly with displacement.

Solution:

The motion of the center of mass of the body is given by
Which, by Newton’s law should equal mass times acceleration of the particle.

$$F = ma = m\frac{dv}{dt} = -kv$$

Hence,
$$\frac{dv}{dt} = (k/m)v \Rightarrow \frac{dv}{v} = \frac{k}{m}dt$$

On Integration,
$$\log_e v = -\frac{k}{m}t + C$$

Using the condition
$$v = v_0 \text{ at t } = 0, \text{ and } C = \log_e v_0$$

$$\log_e\left(\frac{v}{v_0}\right) = \frac{k}{m}t \Rightarrow \frac{v}{v_0} = e^{(-k/m)t}dt$$

Which shows that the velocity decreases exponentially with respect to time.

$$v = \frac{dx}{dt} - kv \Rightarrow \frac{dx}{dt} = v_0 \cdot e^{\left(-k/m\right)t}$$

$$dx = v_0 \cdot e^{\left(-k/m\right)t} dt$$

On Integration,

$$x = \frac{v0}{k}\left(1 - e^{\left(-k/m\right)t}\right)$$

Substituting for v_1

$$v = v_0 - kx$$

Which shows that the velocity of the body decreases linearly with displacement.

[6]The angular velocity of a flywheel is observed to decreased by 10% in the first minute. Calculate the decrease in the second minute if the retardation is proportional to the angular velocity.

<u>Solution</u>:

Given that

$$\frac{d\omega}{dt} = k\omega \Rightarrow \frac{d\omega}{dt} + k\omega = 0$$

On Integration,

$$\int_{\omega 0}^{\omega} \frac{d\omega}{\omega} + k\int_{0}^{t} dt = 0 \qquad or, \log\frac{\omega}{\omega 0} + kt = 0 \text{ and } \frac{\omega}{\omega 0} = e^{-kt}$$

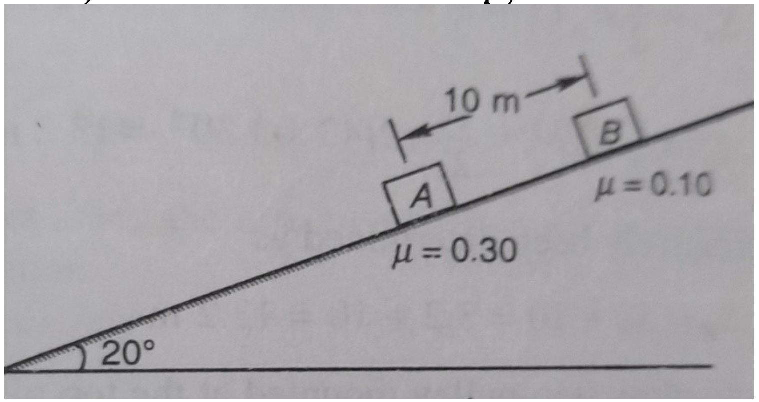
$$\frac{\omega}{\omega 0} = \text{ becomes } 90\%$$

In the first one second,

In the 2nd second, it will becomes 90% of that, i.e.,81% of the original velocity.

[7]Two blocks A and B are held stationary 10m apart on a 20º incline as shown in Fig . The coefficient of dynamic friction between the plane and A is 0.3 whereas between the plane and B is 0.1. If the block are released simultaneously, calculate the time taken and distance travelled by each block before they are at the verge of collision.

solution: From the free body diagrams of the blocks, as shown in Fig for block A



$m_A g \sin 20^o - 0.3R_A = m_A a_A$

$R_A = m_A g \cos 20^o$

Or $m_A g \sin 20^o - 0.3\, m_A g \cos 20^o = m_A a_A$

Or $9.81 \times 0.342 - 0.3 \times 9.81 \times 0.94 = a_A$ or $a_A = 0.59 m/s^2$

similarly, for block B $m_A g \sin 20^o - 0.1 R_B = m_B a_B$

$R_B = m_B g \cos 20^o$ *whence* $a_B = 2.43 m/s^2$

If t is the time taken for block to reach at the same elevation to be at the verge of collision, the distance travelled by A would be 10 m less than that travelled by B over the same time

Solution continuation:

$$\frac{1}{2}a_A t^2 + 10 = \frac{1}{2}a_B t^2$$

or $$\frac{1}{2}0.59t^2 + 10 = \frac{1}{2}2.43t^2$$

Whence $t = 3.30s$

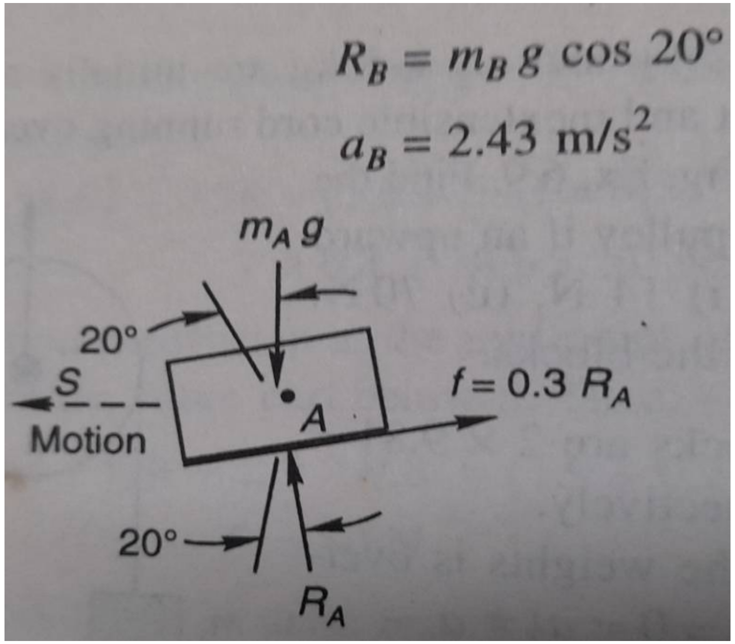
The distance travelled by A and B are

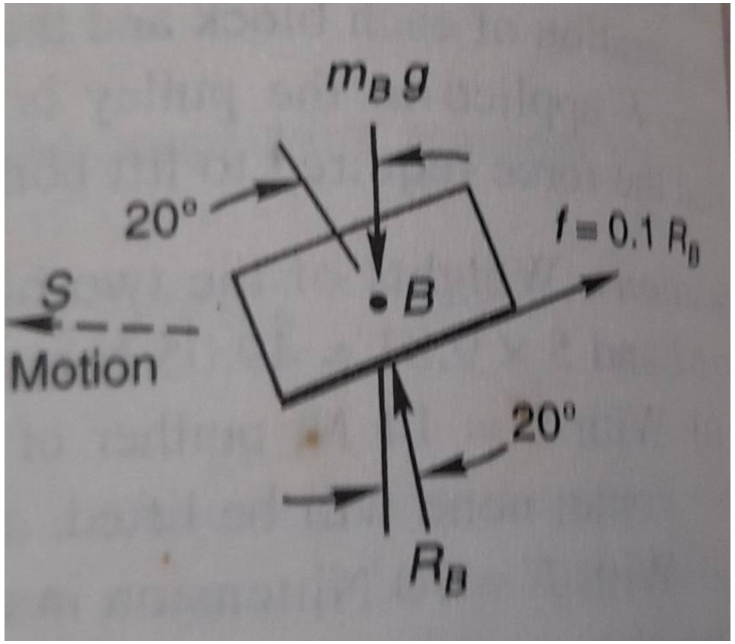
$$s_A = \frac{1}{2}a_A t^2 = \frac{1}{2}\times 0.59\times 3.30^2 = 3.2$$

$$s_B = \frac{1}{2}a_B t^2 = \frac{1}{2}\times 2.43\times 3.30^2 = 13.2m$$

Which could have alternatively been determined as

$S_B = S_A + 10 = 3.2 + 10 = 13.2m$





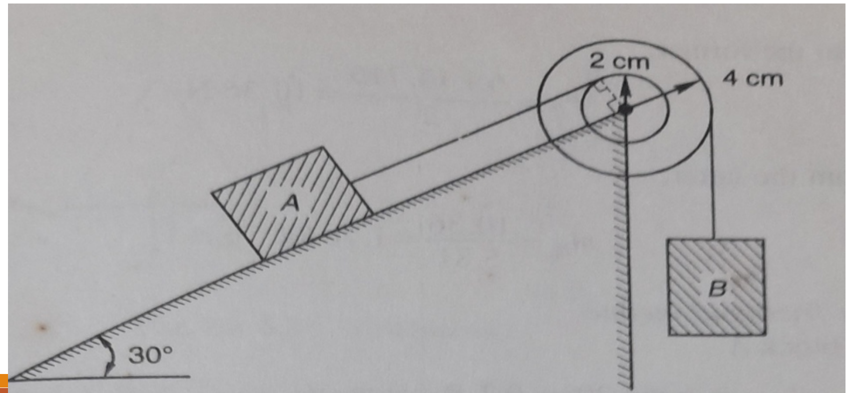
[8]A frictionless step-pulley mounted at the top of an inclined supports a block A restrained to slide over the incline and a block B hanging from it as shown in figure .If the block A of mass 3 kg should accelerate at $2 m/s^2$ up the inclined, determine the mass of block B (i) If the incline is frictionless and (ii)If the coefficient of dynamic friction between the incline and block A is 0.3

solution:

Assuming the step-pulley to be frictionless as well as massless, the tensions in the cords such that the summation of their movements about the centre of rotation vanishes.

$$2T_A - 4T_B = 0 \qquad \Rightarrow T_A = 2T_B \text{...[1]}$$

From the Free-body Diagrams of the blocks as shown in figure the forces along the direction of motion are evaluated and substituted in the equations of motion.



(i) For frictionless incline

For block A,

$$T_A - m_A g \sin 30° = m_A \; a_A \text{..[2]}$$

For block B

$$m_B g - T_B = m_B \quad a_B \; ....[3]$$

From kinematic considerations for the step-pulley,

$$a_B = 2a_A = 2 \times 2 = 4 \,\mathrm{m/s}^2 \; ...[4]$$

Substituting equation (1) and (4) into eqn (2) and (3)

From the former.

$$2T_B - 3 \times 9.81 \times 0.5 = 3 \times 2 \qquad \& \qquad 9.81\, m_B - T_B = 4\, m_B$$

$$T_B = \frac{(6 + 14.715)}{2} = 10.36 \quad \mathrm{N}$$

From the latter, $m_B = \dfrac{10.36}{5.81} = 1.78\ \mathrm{kg}$

(ii)***For frictional incline***

For block A

$$T_A - m_A g \sin 30° - 0.3\, \mathrm{R_A} = m_A \quad a_A$$

Where

$$\mathrm{R_A} = m_A g \cos 30° = 25.49 \;\; \mathrm{N}$$

Solution continued:

Then , $T_A - 3\times9.81\times0.5-0.3\times25.49=3\times2$

Or $T_A - 22.36=6$

Whence $T_A = 28.36$

For block B,

Employing the fact that $m_B g - T_B = m_B a_B$

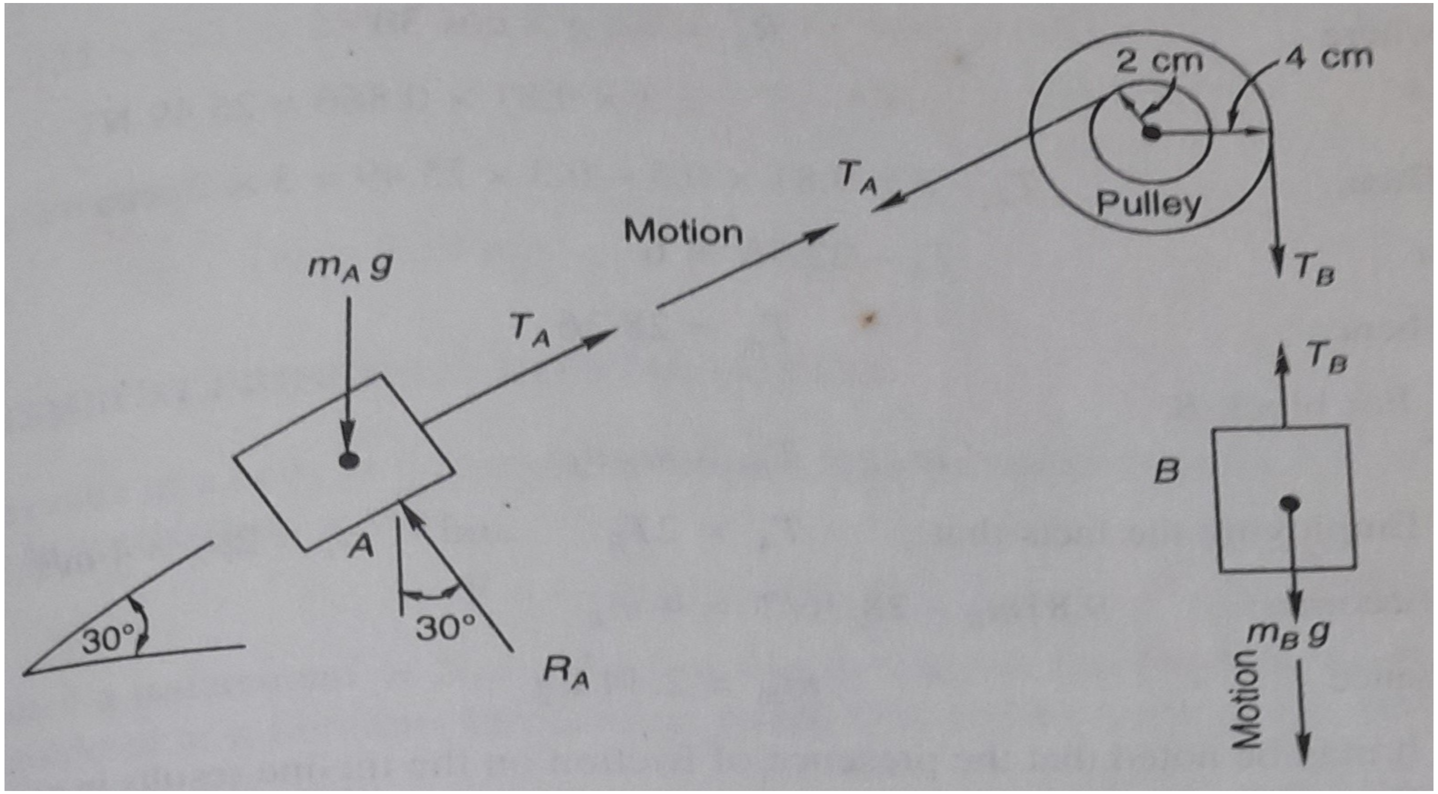
(i) For frictionless incline

For block A, $T_A = 2T_B$ *and* $a_B = 2a_A = 4m/s^2$

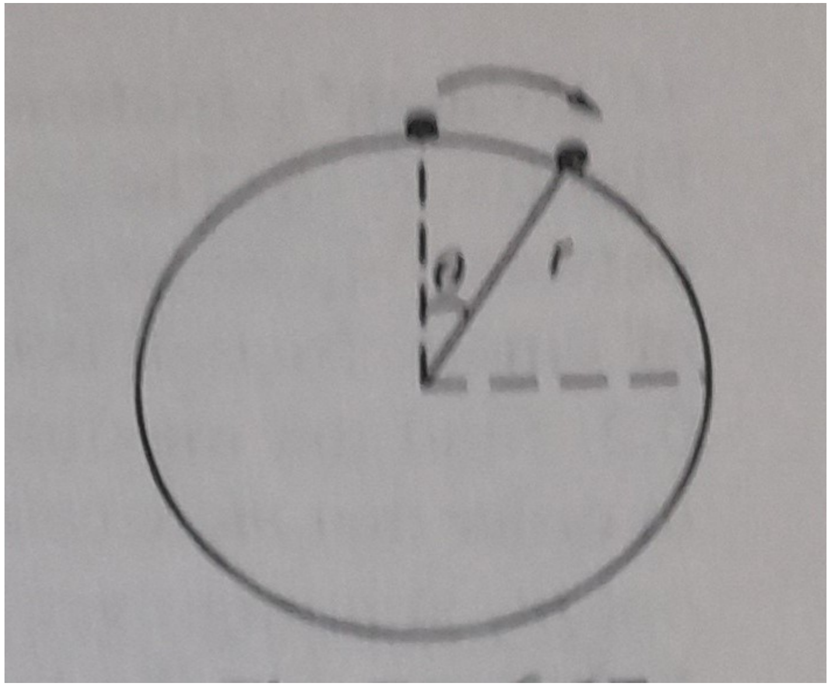
It becomes $9.81 m_B - 28.36/2 = 4 m_B \Rightarrow m_B = 2.44\,\text{kg}$

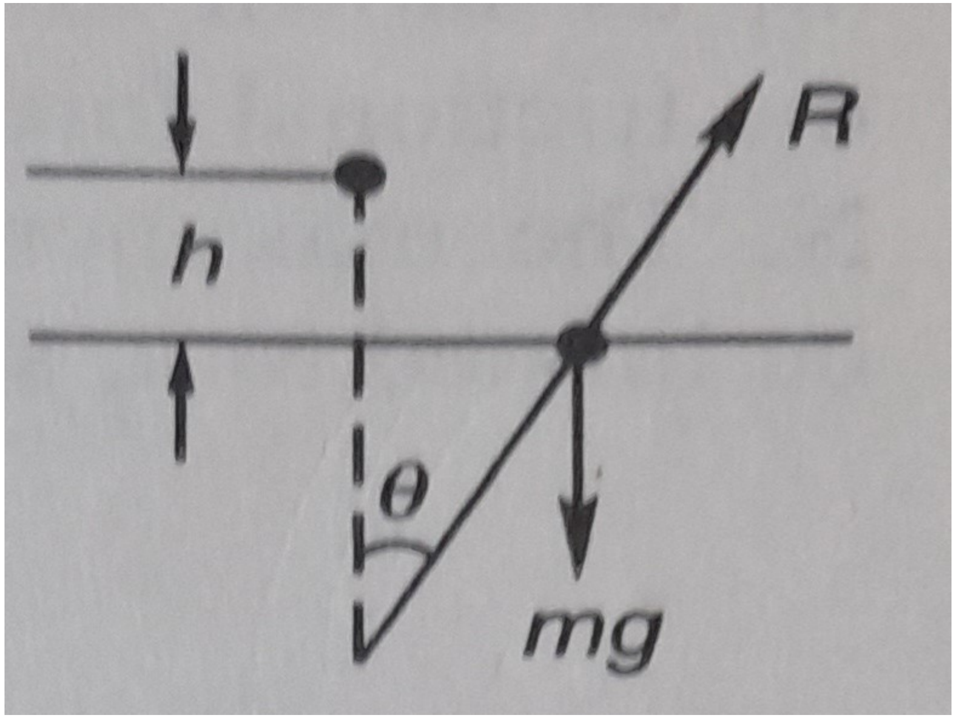
It may be noted that the presence of friction on the inclined results in considerably higher tension in the cords and requires a larger mass of the hanging block to cause the same acceleration of the block on the incline. A little reflection will show that if a single pulley was employed instead of a step pulley, the problem would be lot easier but the mass of block B required for the same purpose would be considerably more.

Figure for problem 8:



[9]A particle of mass 0.05 kg slides down the circular periphery of a horizontal smooth cylinder of radius 12 cm when let go from the top position as shown in figure. Determine the reaction of the cylinder on it when it reaches 30^{o} position.
Will it leave the surface ? If so, where and with what velocity ?





solution:

From the Free-body Diagrams of the mass as shown in figure at an angle θ,

$$mg\cos\theta - R = m\times\frac{v^2}{r} \;..[1]$$

Also, by conservation of energy, $mgh = mg(r - r\cos\theta) = \frac{1}{2}mv^2$

$$or\;, g(1-\cos\theta) = \frac{1}{2}\times\frac{v^2}{r} \;..[2]$$

Eliminating $\frac{v^2}{r}$ between eqn [1] and[2]

$$R = mg\cos\theta - 2\text{mg}(1-\cos\theta)$$

R=0.05×9.81×0.866-2×0.05×9.81×(1-0.866)

R=0.29N

It must leave the circular periphery , latest when it reaches the extremity of the horizontal diameter when the velocity is zero or the mass is zero. With it's finite mass and velocity it will lose contact with the cylinder where R=0, i.e., equating mgcosθ with m.2g.(1-cosθ).

$\cos\theta = 2.(1-\cos\theta)$

$\cos\theta = 2-2\cos\theta$

$3\cos\theta = 2$

$\theta = 48.2^o$

At this location,

$$v^2 = 2gr(1-\cos\theta)$$

$$= 2\times 9.81\times 0.12(1-2/3) = 0.78$$

Whence, $v = 0.886$ m/s

The direction of the velocity must be tangent to the circle at that point,i.e.,at (-48.2^o) with the horizontal direction.

References:

[1]Engineering Mechanics by SS Bhavikatti.
[2]Engineering Mechanics by S. Timoshenko and D.H. Young & JV Rao.
[3]Engineering Mechanics by P K Nag, Sukumar Pati and T K Jana.
[4]Engineering Mechanics by BB Ghosh, S Chakrabarti and S Ghosh.
[5] Engineering Mechanics by K. L Kumar
[6]Engineering Mechanics Dynamics by J.L.Meriam & L.G. Kraige
[7]Vector Mechanics for Engineers by Beer Johnston.

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Thank You…

Q&A

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