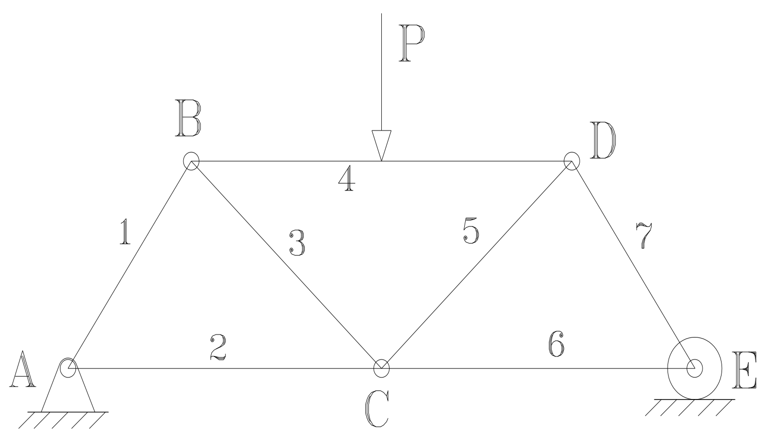
PLANE TRUSSES

1.DEFINITION: A Plane Truss is defined as a structure, made up of several bars, hinged/joined together to form a framed structure to take loads. Bars are angle irons or channel sections and are called members of the frame or framed structure.



The joints are assumed to be hinged or pin-jointed. The determination of force in a frame is an important problem in engineering mechanics, which can be solved by the application of the principles of either statics or graphics. In this chapter, we shall be using the principles of statics for determining the forces in the bars.

2. TYPES OF FRAMES

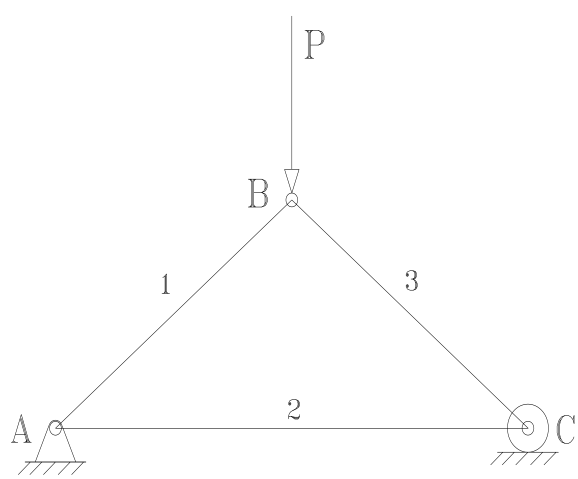
The frames may be classified into the following two groups:

i) Perfect frame. ii) Imperfect frame.

2.1 PERFECT FRAME

A perfect frame is one which is made up of sufficient members just to keep it in equilibrium, when loaded, without any change in its shape.

The simplest perfect frame is a triangle, which contains three members and three joints as shown in Fig. below.



It will be interesting to know that if such a structure is loaded, its shape will not be distorted. Thus, for three jointed frame, there should be three members to prevent any distortion.

The no. of members, in a perfect frame, may also be expressed by the relation:

$n = (2j – 3)$

n = No. of members, and

j = No. of joints.

For triangular frame above, $j=3$ and as per above relation $n = (2x3 – 3) = 3$ and are the no of members. So it is a perfect frame.

2.1. IMPERFECT FRAME

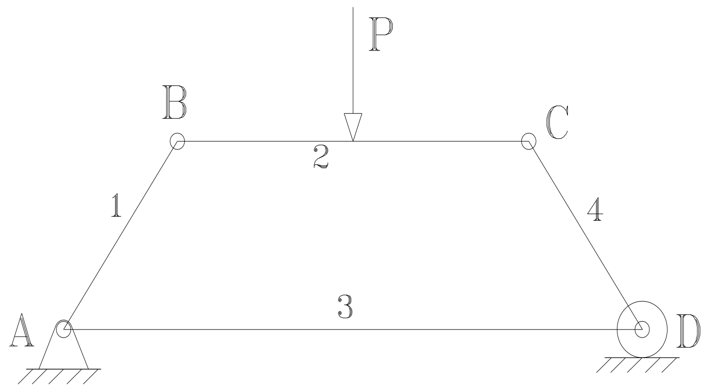
An imperfect frame is that which does not satisfy the equation: $n = (2j – 3)$

or in other words, it is a frame in which the no. of members are more or less than $(2j – 3)$. The imperfect frames may be further classified into the following two types :

i) Deficient frame. ii) Redundant frame.

i) DEFICIENT FRAME

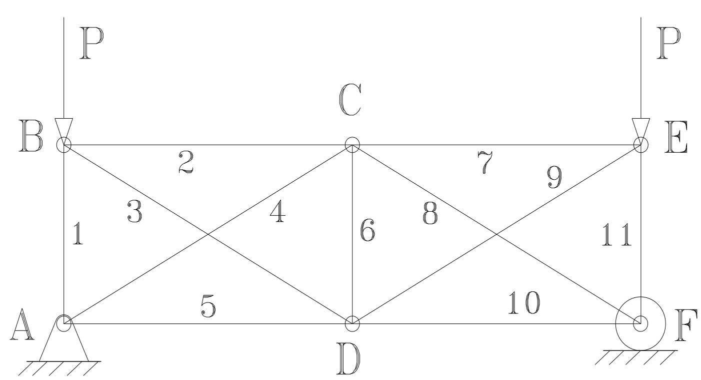
A deficient frame is an imperfect frame, in which the no of members are less than (2j – 3).



m = 4, j = 4, 2j-3 =2x4 -3 = 5 > m, a Deficient Frame.

ii) REDUNDANT FRAME

A redundant frame is an imperfect frame, in which the no of members are more than (2j – 3).



m=11, j=6, 2j-3 = 2x6-3 = 9 < m, Redundant Frame.

In this chapter, we shall discuss only the perfect frames.

3. LOADS

When a frame is acted upon by a force, the internal force which is transmitted through the members is known as load or tension. Following two types of loads are important from the subject point of view:

i) Tensile Load ii) Compressive Load.

i) Tensile Load: Sometimes, a member is pulled outwards by two equal and opposite forces and the body tends to extend, then load induced is called tensile load. (+) sign indicates tensile load.

ii) Compressive Load: Sometimes, a member is pushed inwards by two equal and opposite forces and the body tends to shorten its length. Then load induced is called compressive Load. (-) sign indicates compressive load.

4.ASSUMPTIONS FOR FORCES IN THE MEMBERS OF A PERFECT FRAME

Following assumptions are made, while finding out the forces in the members of a perfect frame:

1. All the members are pin-jointed.

2. The frame is loaded only at the joints.

3. The frame is a perfect one.

4. The weight of the members, unless stated otherwise, is regarded as negligible in comparison with the other external forces or loads acting on the truss.

The forces in the members of a perfect frame may be found out either by analytical method or graphical method. But in this chapter, we shall discuss the analytical method only.

ANALYTICAL METHODS FOR THE FORCES

The following two analytical methods for finding out the forces, in the members of a perfect frame, are important from the subject point of view:

1. Method of joints. 2. Method of sections.

5. METHOD OF JOINTS

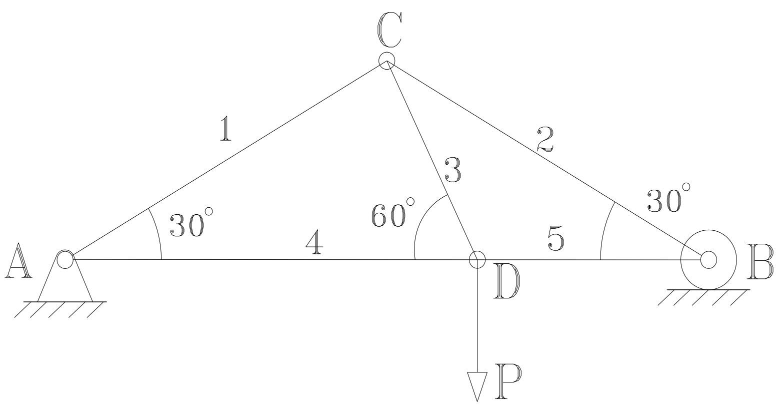
In this method, each and every joint is treated as a free body in equilibrium. The unknown forces are then determined by method of projection / resolution:

$\Sigma X = 0$ and $\Sigma Y = 0$. i.e., Sum of all the horizontal forces and vertical forces are equated to zero.

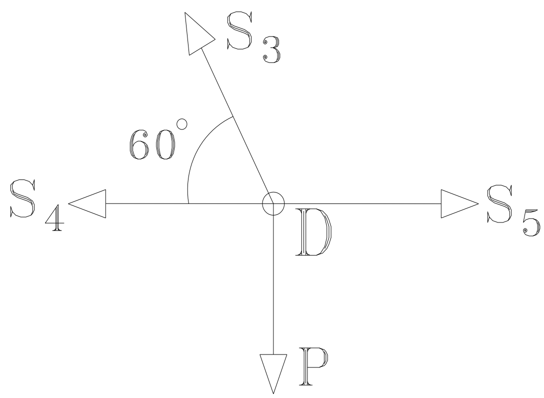
Method of joints procedure:

- **i) Isolating each joints**
- **ii) All reactions are away from joints.**
- **iii) Consider first the joint with external force.**

Prob. 1. Calculate the axial forces S_i (S_1, S_2, S_3, S_4 and S_5) in each bar of the simple truss supported and loaded as shown. P = 5kN



Solution :



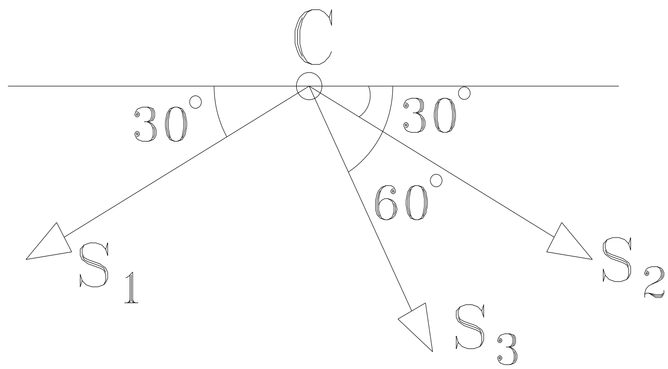
Isolating the joint D as a free body diagram and resolving the forces vertically and horizontally,

By $\Sigma Y=0$, $S_3 \sin 60^o - P = 0$ or $S_3 = 5/\sin 60^o$

or $S_3 = 5.77kN$ (Ans)

$\Sigma X=0$, $S_5 - S_4 - S_3 \cos 60^o = 0$ or $S_4 = S_5 - 5.77 \cos 60^o$

or $S_4 = S_5 - 2.885$ ----- (i)



Isolating the joint C as a free body diagram, and resolving the forces horizontally and vertically,

By $\Sigma X=0$, $S_2 Cos30^o + S_3 Cos60^o – S_1 Cos30^o = 0$

or $0.866S_2 + 0.5x5.77 – 0.866 S_1 = 0$

or $S_1 - S_2 = 3.33$ ----- (ii)

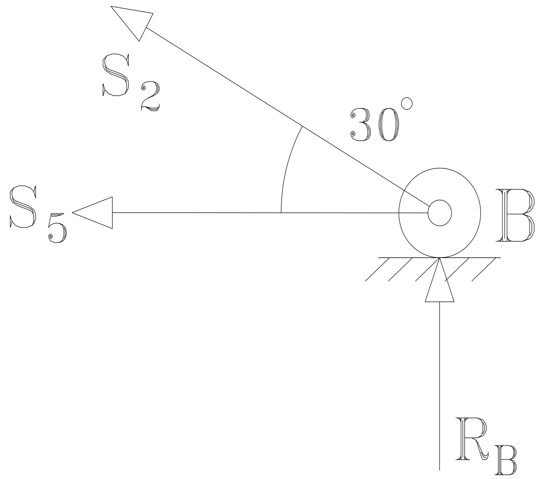
By $\Sigma Y=0$, $-S_1 Sin30^o – S_2 Sin30^o - S_3 Sin60^o = 0$

or $0.5S_1 + 0.5 S_2 = -5.77 x0.866$

or $S_1 + S_2 = -9.99$ ----- (iii)

Solving eqn (ii) & (iii), we get, $2 S_1 = -6.66$

or $S_1 = -3.33kN$ (Ans) and $S_2 = -6.66kN$ (Ans)



Isolating the support B as a free body diagram,

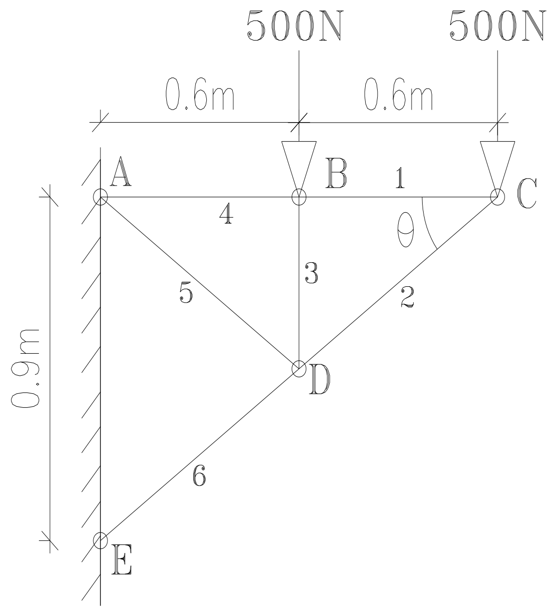
By $\Sigma X=0$, $-S_5 - S_2 \cos 30^o = 0$ or $S_5 = 6.66 \cos 30^o$

or $S_5 = 5.77kN$ (Ans)

From eqn (i) $S_4 = S_5 - 2.885$ or $S_4 = 5.77 - 2.885$

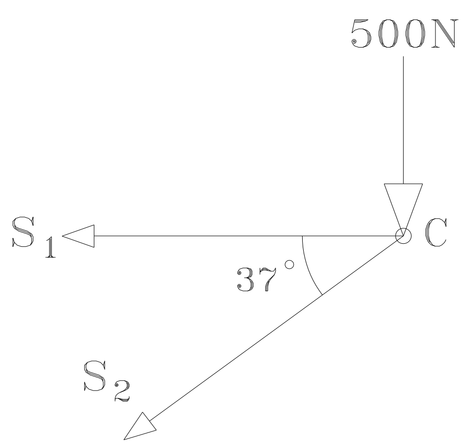
or $S_4 = 2.88kN$ (Ans)

Prob. 2. Calculate the axial forces S_i (S_1, S_2, S_3, S_4, S_5 and S_6) in each bar of the cantilever truss supported and loaded as shown.



Solution :

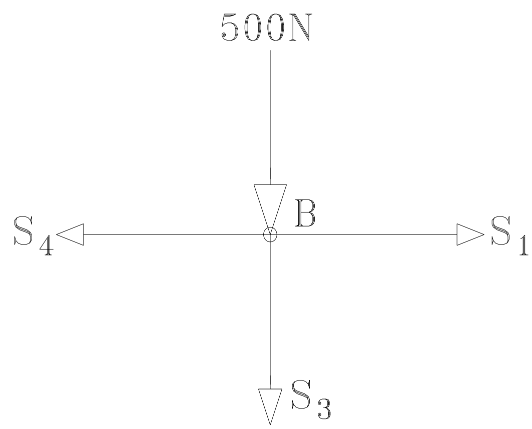
tan θ = AE/AC = 0.9/1.2 = 0.75 or θ=37º



Isolating the joint C as a free body diagram and resolving the forces vertically and horizontally,

By ΣY=0, $-S_2 \sin 37^{o} - 500 = 0$ or $S_2 = -833N$ (Ans)

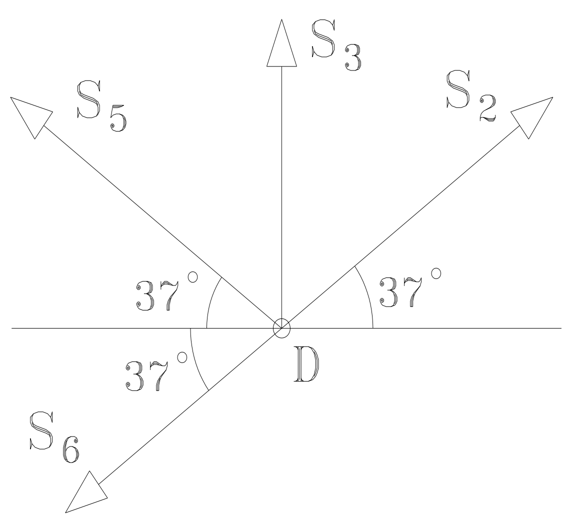
ΣX=0, $-S_1 - S_2 \cos 37^{o} = 0$ or $S_1 = 666N$ (Ans)



Isolating the joint B as a free body diagram and resolving the forces vertically and horizontally,

By $\Sigma Y=0$, $-S_3 - 500 = 0$ or $S_3 = -500N$ (Ans)

By $\Sigma X=0$, $S_1 - S_4 = 0$ or $S_4 = S_1$ or $S_4 = 666N$ (Ans)



Isolating the joint D as a free body diagram and resolving the forces horizontally and vertically,

By $\Sigma X=0$, $S_2 \cos 37^o - S_5 \cos 37^o - S_6 \cos 37^o = 0$

or $S_5 + S_6 = S_2$ or $S_5 + S_6 = -833$ ----- (i)

By $\Sigma Y=0$, $S_3 + S_2 \sin 37^o + S_5 \sin 37^o - S_6 \sin 37^o = 0$

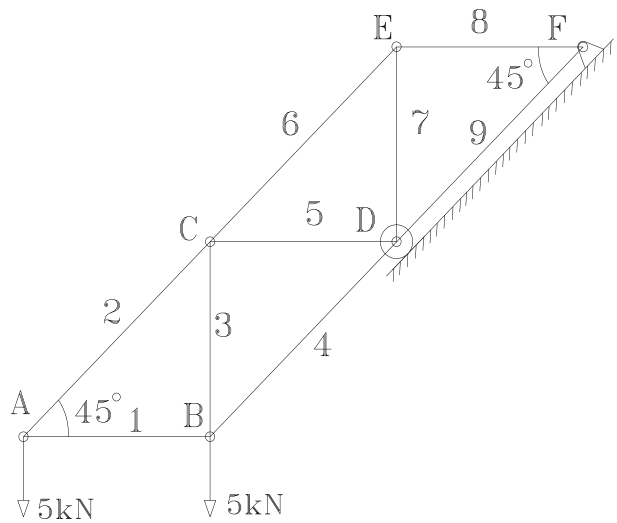
or $-500-833 \sin 37^o + S_5 \sin 37^o - S_6 \sin 37^o = 0$

or $S_5 - S_6 = 1000/ \sin 37^o$ or $S_5 - S_6 = 1666$ ----- (ii)

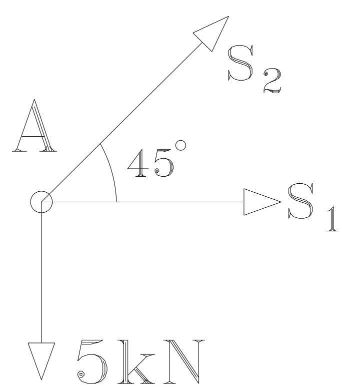
Solving eqn (i) & (ii), we get, $2 S_5 = 833$

or $S_5 = 416N$ (Ans) and $S_6 = -1250N$ (Ans)

Prob. 3. Calculate the axial forces S_i (S_1 to S_8) in each bar of the plane truss supported and loaded as shown.



Solution :



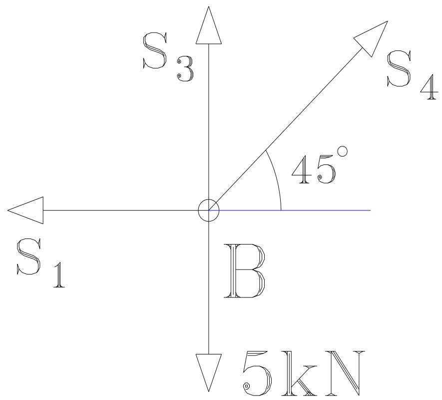
Isolating the joint A as a free body diagram and resolving the forces vertically and horizontally,

By $\Sigma Y=0$, $S_2 \sin 45^o - 5 = 0$ or $S_2 = 5/\sin 45^o$

or $S_2 = 7.07kN$ (Ans)

By $\Sigma X=0$, $S_1 + S_2 \cos 45^o = 0$ or $S_1 = -7.07 \cos 45^o$

or $S_1 = -5kN$ (Ans)



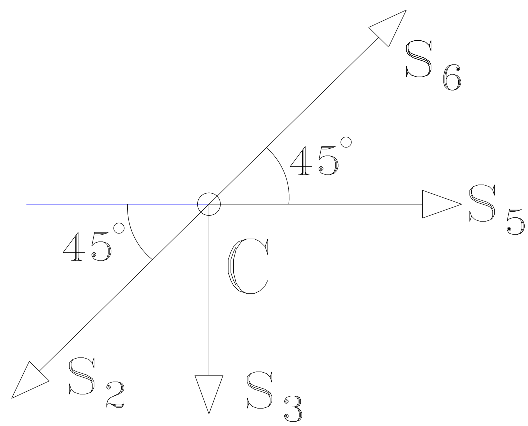
Isolating the joint B as a free body diagram and resolving the forces horizontally and vertically,

By $\Sigma X=0$, $S_4 \cos 45^o - S_1=0$ or $S_4 = -5/\cos 45^o$

or $S_4 = -7.07kN$ (Ans)

By $\Sigma Y=0$, $S_3 + S_4 \sin 45^o - 5 = 0$ or $S_3 = 5+7.07 \sin 45^o$

or $S_3 = 10kN$ (Ans)



Isolating the joint C as a free body diagram and resolving the forces vertically and horizontally,

By $\Sigma Y=0$, $S_6 \, Sin45^o - S_2 \, Sin45^o - S_3 = 0$

or $S_6 \, Sin45^o = 7.07 \, Sin45^o + 10$ or $S_6 = 15/Sin45^o$

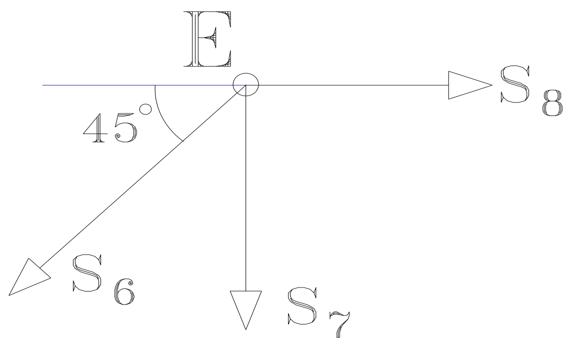
or $S_6 = 21.2$kN (Ans)

By $\Sigma X=0$, $S_5 + S_6 \, Cos45^o - S_2 \, Cos45^o = 0$

or $S_5 = 7.07 \, Cos45^o - 21.2 \, Cos45^o$

or $S_5 = 5 - 15$

or $S_5 = -10$kN (Ans)



Isolating the joint E as a free body diagram and resolving the forces horizontally and vertically,

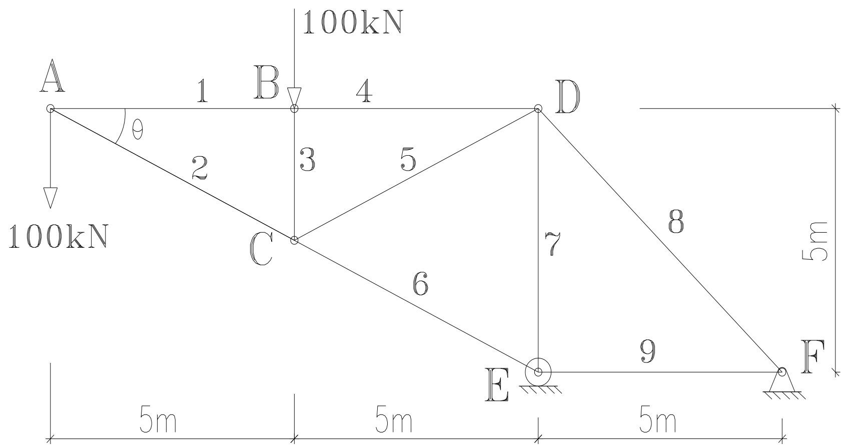
By $\Sigma X=0$, $S_8 - S_6 \cos 45^o = 0$ or $S_8 = 21.2/ \cos 45^o$

or $S_8 =$ 15kN (Ans)

By $\Sigma Y=0$, $-S_7 - S_6 \sin 45^o = 0$ or $S_7 = -21.2 \sin 45^o$

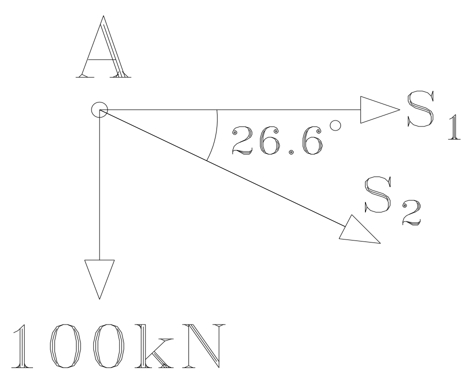
or $S_7 =$ -15kN (Ans)

Prob. 4. Calculate the axial forces S_i (S_1 to S_8) in each bar of the plane truss supported and loaded as shown.



Solution :

$\tan\theta$ = DE/AD=5/10=0.5 or θ=26.6°



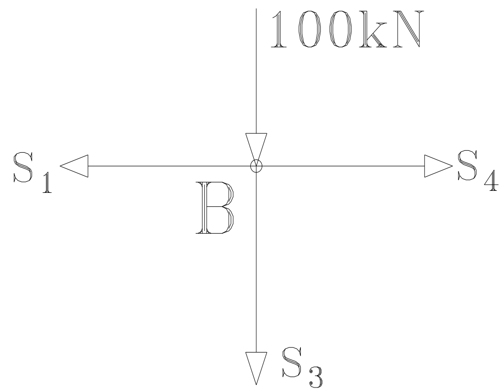
Isolating the joint A as a free body diagram and resolving the forces vertically and horizontally,

By $\Sigma Y=0$, $-S_2 \sin 26.6^{o} - 100 = 0$ or $S_2 = -100/\sin 26.6^{o}$

or S_2 =-223kN (Ans)

By $\Sigma X=0$, $S_1 + S_2 \cos 26.6^o = 0$ or $S_1 = 223\cos 26.6^o$

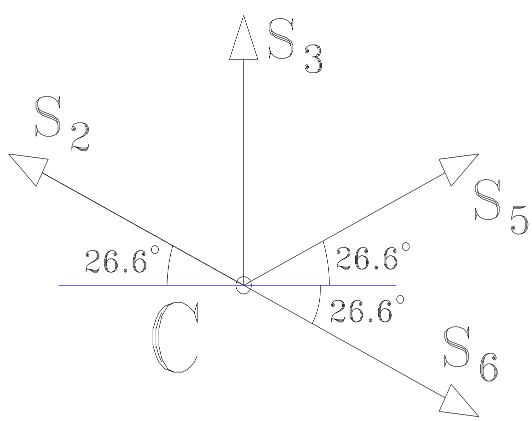
or $S_1 = 200kN$ (Ans)



Isolating the joint B as a free body diagram and resolving the forces vertically and horizontally,

By $\Sigma Y=0$, $-S_3 - 100 = 0$ or $S_3 = -100kN$ (Ans)

By $\Sigma X=0$, $-S_1 + S_4 = 0$ or or $S_4 = 200kN$ (Ans)



Isolating the joint C as a free body diagram and resolving the forces horizontally and vertically,

By $\Sigma X=0$, $S_5 \text{ Cos}26.6^o + S_6 \text{ Cos}26.6^o – S_2 \text{ Cos}26.6^o = 0$

or $S_5 + S_6 - S_2 = 0$ or $S_5 + S_6 = -223$ -----(i)

By $\Sigma Y=0$, $S_3 + S_2 \text{Sin}26.6^o + S_5 \text{ Sin}26.6^o – S_6 \text{ Sin}26.6^o = 0$

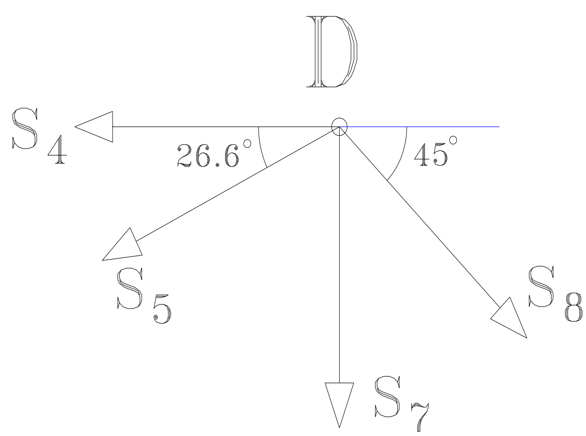
or $S_5 + S_2 – S_6 = -S_3/\text{Sin}26.6^o$

or $S_5 – S_6 = 100/\text{Sin}26.6^o + 223$

or $S_5 – S_6 = 446$ ------(ii)

Solving equation (i) & (ii) we get,

$S_5 = 111.5$kN (Ans) and $S_6 = -334.5$kN (Ans)



Isolating the joint D as a free body diagram and resolving the forces horizontally and vertically,

By $\Sigma X=0$, $S_8 \text{ Cos } 45^o - S_4 – S_5 \text{ Cos}26.6^o = 0$

or S_8 Cos 45^o - 200 – 100 = 0 , or S_8 = 300/ Cos 45^o

or S_8 = 423kN (Ans)

By ΣY=0, – S_7 – S_5 Sin26.6^o – S_8 Sin45^o =0

or S_7 = – 50 – 299 or S_7 = - 349kN (Ans)