# FRICTION

Whenever a body tries to slide over another body, there shall be a resistance which resists the body from sliding, which is called friction.

It is of the following two types:

1. Static friction: It is the friction experienced by a body when it is at rest or when the body tends to move.

2. Dynamic friction: It is the friction experienced by a body when it is in motion. It is also called kinetic friction.

The dynamic friction is of the following two types:

i) Sliding friction: It is the friction, experienced by a body when it slides over another body.

ii) Rolling friction: It is the friction, experienced by a body when it rolls over another body.

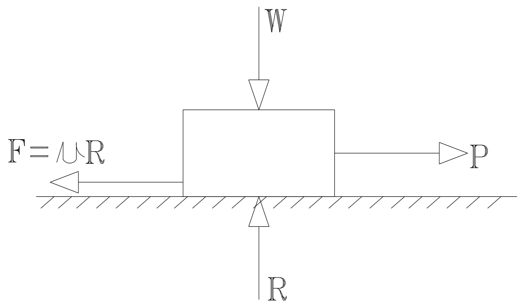
LAWS OF STATIC FRICTION: Followings are the laws of static friction:

i) The force of friction is independent of the area of contact between the two surfaces.

ii) The force of friction depends upon the materials of the contact surfaces.

iii) The force of friction always acts in a direction, opposite to that in which the body tends to move.

iv) The force of friction is proportional to the normal reaction.




Mathematically: $F \propto R$ or $F = \mu R$

Where, F = Force of friction

R = Normal Reaction

μ = Co-efficient of friction

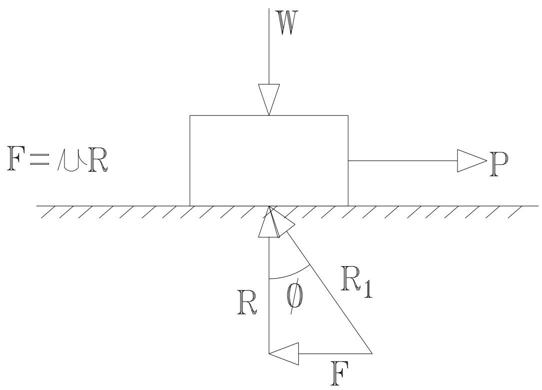
LAWS OF KINETIC FRICTION: Followings are the laws kinetic friction:

i) The force of friction always acts in a direction, opposite to that in which the body is moving.

ii) The magnitude of kinetic force of friction bears a constant ratio to the normal reaction between the two contact surfaces. But this ratio ($\mu_k$) is slightly less than that in case of static friction (μ).

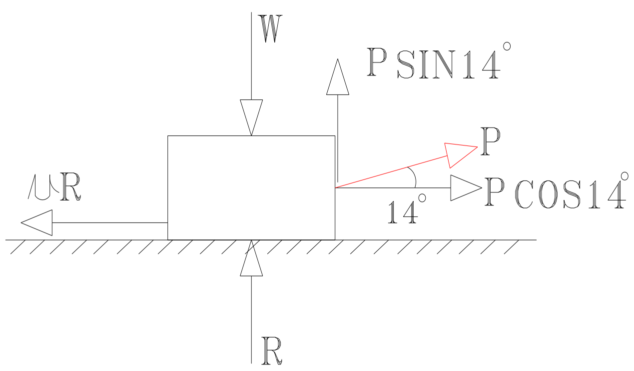
iii) For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

LIMITING ANGLE OF FRICTION: The angle between resultant force of force of friction and normal reaction made with normal reaction is called limiting angle of friction.




In Fig., tan $\phi$ = F / R = $\mu$ R / R = $\mu$, or $\phi = \tan^{-1} \mu$

**PROBLEM 1:** A mass of 50 Kg is pulled along a rough horizontal plane by a pull of 175 N at an angle of 14$^o$ with the horizontal. Find the coefficient of friction.




**Solution:** Given,

Mass of weight **(**W) = 50 Kg = (50x9.8) = 490 N

Force applied (P) = 175 N

Let, μ is the coefficient of friction

From the Fig. resolving the forces vertically and horizontally

We get,

$\Sigma Y = 0$

$P \sin 14^o + R - W = 0$

Or, $175 \sin 14^o + R - 490 = 0$

Or, $R = 447.7$ N

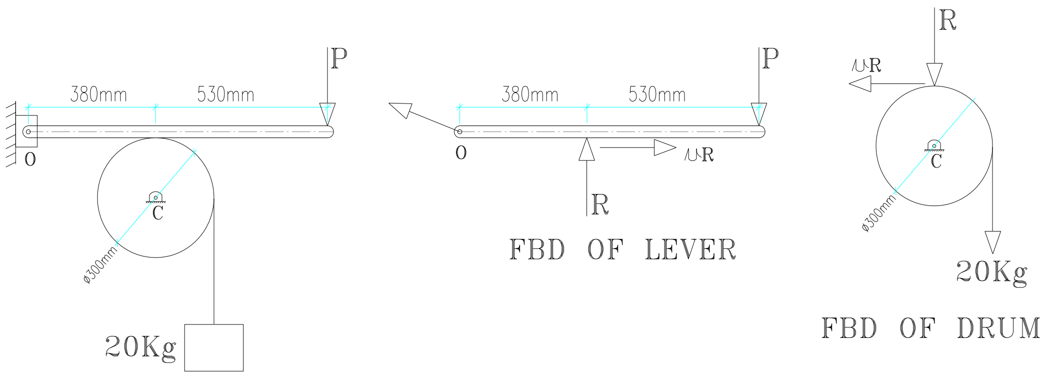
And, $\Sigma X = 0$

$P \cos 14^o - \mu R = 0$

Or, $175 \cos 14^o - \mu \times 447.7 = 0$

**Or, $\mu = 0.379$ (answer)**

PROBLEM 2: A vertical force P exerted on lever AB, which holds the 20 Kg mass (m) on falling as shown in the Fig. The coefficient of friction between the lever and the 300 mm dia. drum is 0.25. Neglect the mass of the drum and lever and determine the force P to hold the drum.




Solution: Given,

The coefficient of friction (μ) between the lever and drum = 0.25

Diameter of the drum = 300 mm

Refer to Fig (i), Free Body Diagram (FBD) of Lever

Taking moment at O, Σ Mo = 0

R X 380 – P (530 + 380) = 0

Or, R = 2.395 P

Again, Refer to Fig (ii), FBD of drum,

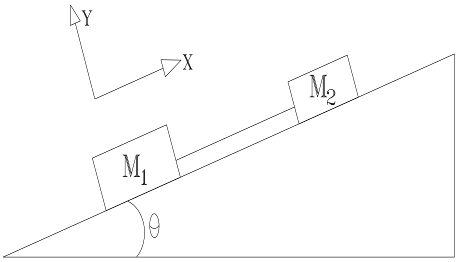
Taking moment at centre of drum C, Σ Mc = 0

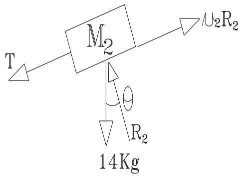
Or, μR X 150 – 20 x 9.8 x 150 = 0

Or, 0.25 X 2.395 P X 150 – 20 x 9.8 x 150 = 0

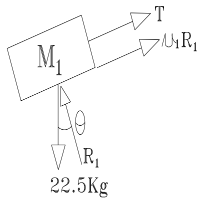
**Or, P = 327 N (answer)**

**PROBLEM 3:** In the Fig. $M_1$ & $M_2$ are two masses of 22.5 Kg and 14.0 Kg respectively. They are tied together by a rope parallel to the plane. The coefficient of friction ($\mu_1$) between the $M_1$ and the plane is 0.25 and the coefficient of friction ($\mu_2$) between the $M_2$ and the plane is 0.5. Determine the value of the angle (θ), at which the sliding occurs. What is the tension in the rope?



FBD OF M2 MASS



FBD OF M1 MASS

Solution: Given,

The coefficient of friction ($\mu_1$) between $m_1$ and the plane = 0.25

The coefficient of friction ($\mu_2$) between $m_2$ and the plane = 0.5

Mass ($m_1$) = 22.5 Kg

Mass ($m_2$) = 14.0 Kg

Considering FBD of ($M_2$) and resolving the forces perpendicular to the plane we get,

$\Sigma Y = 0,$

$R_2 - 14 \times 9.8 \times \cos\theta = 0$

Or, $R_2 = 137.2 \cos\theta$

Again, resolving the forces parallel to the plane we get,

$\Sigma X = 0,$

Or, $\mu_2 R_2 - T - 14 \times 9.8 \times \sin\theta = 0$

Or, $0.5 \times 137.2 \cos\theta - T - 14 \times 9.8 \times \sin\theta = 0$

Or, $T = 68.6 \cos\theta - 137.2 \sin\theta$

Considering FBD of ($M_1$) and resolving the forces perpendicular to the plane we get,

$\Sigma Y = 0, \quad R_1 - 22.5 \times 9.8 \times \cos\theta = 0$

Or, $R_1 = 220.5 \cos\theta$

Again, resolving the forces parallel to the plane we get,

$\Sigma X = 0,$

$\mu_1 R_1 + T - 22.5 \times 9.8 \times \sin\theta = 0$

Or, $0.25 \times 220.5 \cos\theta + 68.6 \cos\theta - 137.2 \sin\theta - 22.5 \times 9.8 \times \sin\theta = 0$

Or, $123.7 \cos\theta - 357.7 \sin\theta = 0$

Or, $\tan\theta = 0.346$

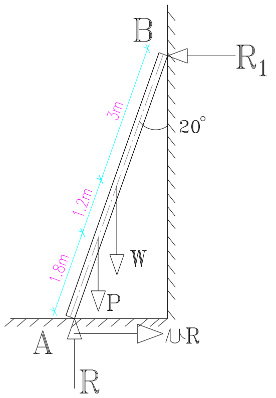
**Or, $\theta = 19^o$ (answer)**

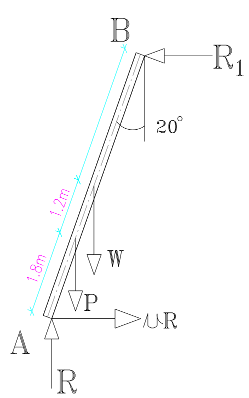
Now, $T = 68.6 \cos\theta - 137.2 \sin\theta$

$= 68.6 \cos 19^o - 137.2 \sin 19^o$

Or, **$T = 19.9$ N (answer)**

**PROBLEM 4:** A ladder 6 m long rests on horizontal ground and leans against a smooth vertical wall at an angle $20^o$ with the vertical. Its own weight is 900 N acting at its middle. It is on the point of sliding when a man weighting 750 N stands on a rung 1.8 m from the foot of the ladder. Calculate the coefficient of friction.




FBD OF LADDER

Solution: Given,

Ladder weight (W) = 900 N

Man weight (P) = 750 N

Length of the ladder = 6.0 m

Ladder inclination ($\theta$) = $20^o$

Considering FBD of the ladder and resolving the forces vertically we get,

$$\Sigma Y = 0,$$

$$R - P - W = 0$$

Or, $$R = 900 + 750$$

Or, $R = 1650$ N

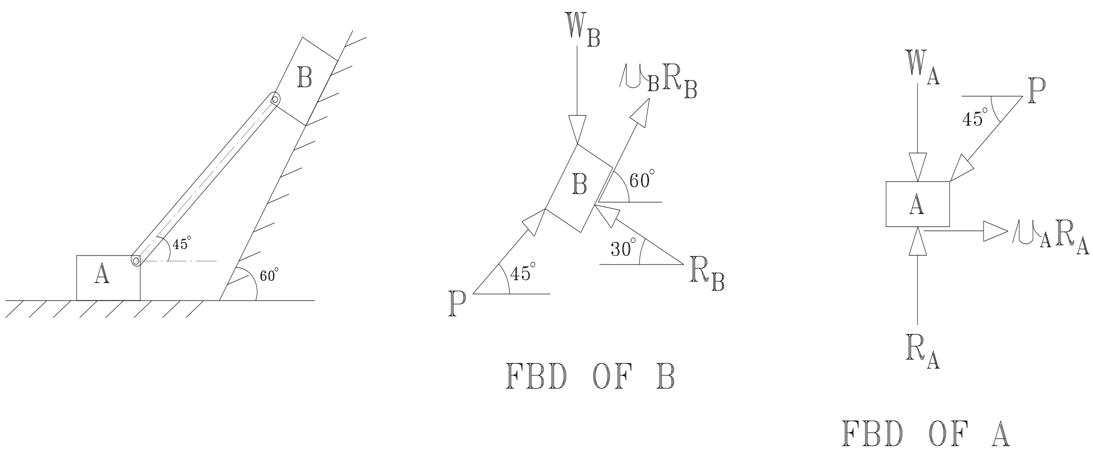
Taking moment on B, $\Sigma M_B = 0$,

$W \times 3 \sin 20^o + P \times 4.2 \sin 20^o - R \times 6 \sin 20^o + \mu R \times 6 \cos 20^o = 0$

Or, $900 \times 3 \sin 20^o + 750 \times 4.2 \sin 20^o - 1650 \times 6 \sin 20^o + \mu \times 1650 \times 6 \cos 20^o = 0$

**Or, $\mu = 0.149$ (answer)**

**PROBLEM 5:** Two blocks A & B of weights 300 N and 200 N respectively are connected by a uniform strut, the connections are made by frictionless pins. If $\mu_B = 0.3$ under block B. Find the minimum coefficient of friction under block A to prevent the motion as in the Fig. shown.




Solution: Given,

Mass of block A ($W_A$) = 300 N

Mass of block B ($W_B$) = 200 N

Coefficient of friction under block B ($\mu_B$) = 0.3

Coefficient of friction under block A ($\mu_A$) =?

Considering FBD of B and resolving the forces horizontally and vertically we get,

$\Sigma X = 0,$

$\mu_B R_B \cos 60^o - R_B \cos 30^o + P \cos 45^o = 0$

Or, $0.3 \text{ X } R_B \cos 60^o - R_B \cos 30^o + P \cos 45^o = 0$

Or, $0.15 R_B – 0.866 R_B + 0.707 P = 0$

Or, $R_B = 0.987 P$

By $\Sigma Y = 0,$

$\mu_B R_B \sin 60^o + R_B \sin 30^o + P \sin 45^o - W_B = 0$

Or, $0.3 \text{ X} 0.987 P \sin 60^o + 0.987 P \sin 30^o + P \sin 45^o – 200 = 0$ [putting the value of $R_B$]

Or, $0.2564 P + 0.4935 P + 0.707 P = 200$

Or, $P = 137.28$ N

Again, Considering FBD of A and resolving the forces vertically we get,

$\Sigma Y = 0,$

Or, $R_A - W_A - P \sin 45^o = 0$

Or, $R_A - 300 - 137.28 \sin 45^o = 0$

Or, $R_A = 397$ N

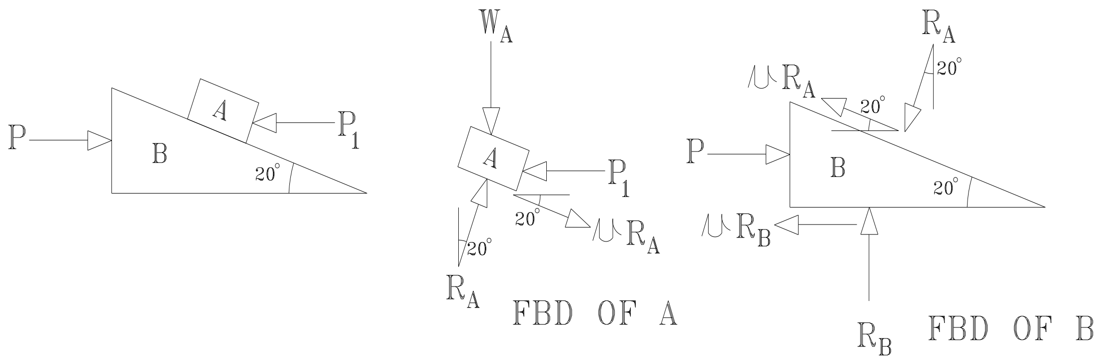
By $\Sigma X = 0$,

$\mu_A R_A - P \cos 45^o = 0$

Or, $\mu_A \times 397 - 137.28 \cos 45^o = 0$

**Or,** $\mu_A = 0.244$ **(answer)**

**PROBLEM 6:** A body "A" of weight 10 kN is to be raised by means of horizontal force P as shown in the Fig. The weight is constrained to move up vertically by the application of a horizontal force $P_1$. What are the magnitude of $P_1$ and P if the coefficient of friction at all the surfaces contact is 0.25.




Solution: Given,

Mass of body A ($W_A$) = 10 KN = 10000 N

Coefficient of friction under block B (μ) = 0.25

Angle of inclination (θ) = $20^o$

Considering FBD of A and resolving the forces horizontally and vertically we get,

$\Sigma Y = 0,$

$R_A \cos 20^o - \mu R_A \sin 20^o - W_A = 0$

Or, $R_A \cos 20^o - 0.25\, R_A \sin 20^o - W_A = 0$

Or, $R_A = 11707$ N

By $\Sigma X = 0,$

$\mu R_A \cos 20^o + R_A \sin 20^o - P_1 = 0$

Or, $0.25 X 11707 \cos 20^o + 11707 \sin 20^o - P_1 = 0$

**Or, $P_1 = 6.754$ kN (answer)**

Again, considering FBD of B and resolving the forces on vertical and horizontal direction we get,

$\Sigma Y = 0,$

$R_B - R_A \cos 20^o + \mu R_A \sin 20^o = 0$

Or, $R_B - 11707 \cos 20^o + 0.25 \times 11707 \sin 20^o = 0$

Or, $R_B = 10000$ N

By $\Sigma X = 0,$

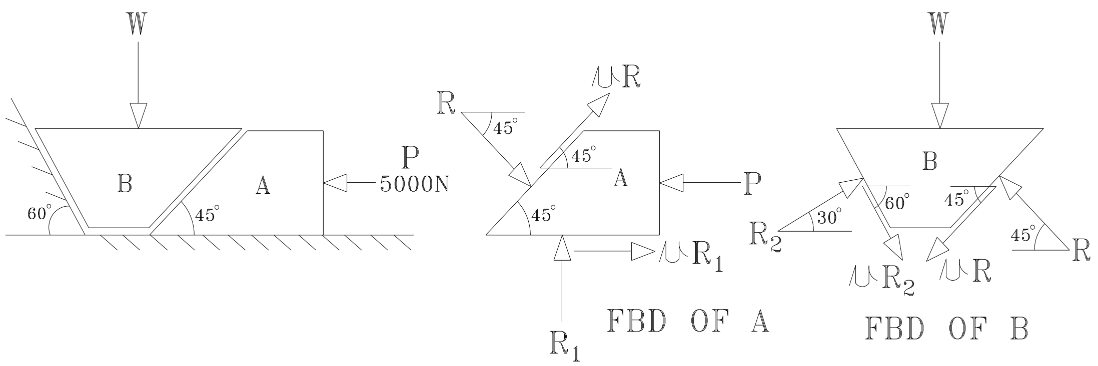
$P - \mu R_A \cos 20^o - R_A \sin 20^o - \mu R_B = 0$

Or, $P - 0.25 \times 11707 \cos 20^o - 11707 \sin 20^o - 0.25 \times 10000 = 0$

Or, $P = 9254$ N

**Or, P = 9.254 KN (answer)**

**PROBLEM 7:** What load 'W' can just be raised on the block 'B' by the application of a horizontal force of 5000 N on the block 'A' shown in the Fig. Coefficient of friction for all the surfaces of contact is 0.25, neglect the weight of 'A' and 'B'.




Solution: Given,

Horizontal force (P) = 5000 N

Coefficient of friction (µ) = 0.25

Considering FBD of A and resolving the forces horizontally and vertically we get,

$\Sigma X = 0$,

$\mu R_1 - P + R \cos 45^o + \mu R \cos 45^o = 0$

Or, $0.25 R_1 - 5000 + R \cos 45^o + 0.25 R \cos 45^o = 0$

Or, $0.25 R_1 + 0.884 R - 5000 = 0$

Or, $R_1 = 20000 - 3.536 R$

By $\Sigma Y = 0$,

$R_1 + \mu R \sin 45^o - R \sin 45^o = 0$

Or, $20000 - 3.536 R + 0.25 R \sin 45^o - R \sin 45^o = 0$

Or, $R = 4918$ N

Again, considering FBD of B and resolving all the forces horizontally and vertically we get,

$\Sigma X = 0$,

$-\mu R \cos 45^o - R \cos 45^o + R_2 \cos 30^o + \mu R_2 \cos 60^o = 0$

Or, $-0.25 X 4918 \cos 45^o - 4918 \cos 45^o + R_2 \cos 30^o + 0.25 R_2 \cos 60^o = 0$

Or, $R_2 = 4386.5$ N

By $\Sigma Y = 0$,

Or, $-\mu R_2 \sin 60^o + R_2 \sin 30^o + R \sin 45^o - \mu R \sin 45^o - W = 0$

Or, $-0.25 X 4386.5 \sin 60^o + 4386.5 \sin 30^o + 4918 \sin 45^o - 0.25 X 4918 \sin 45^o - W = 0$

**Or, W = 3852 N = 3.852 kN (answer)**