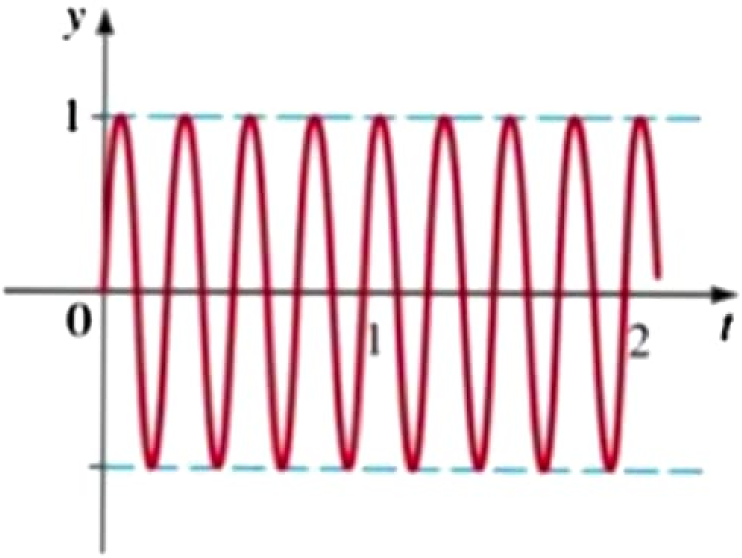
Example 8. *The displacement of a moving particle at any time t is given by :*

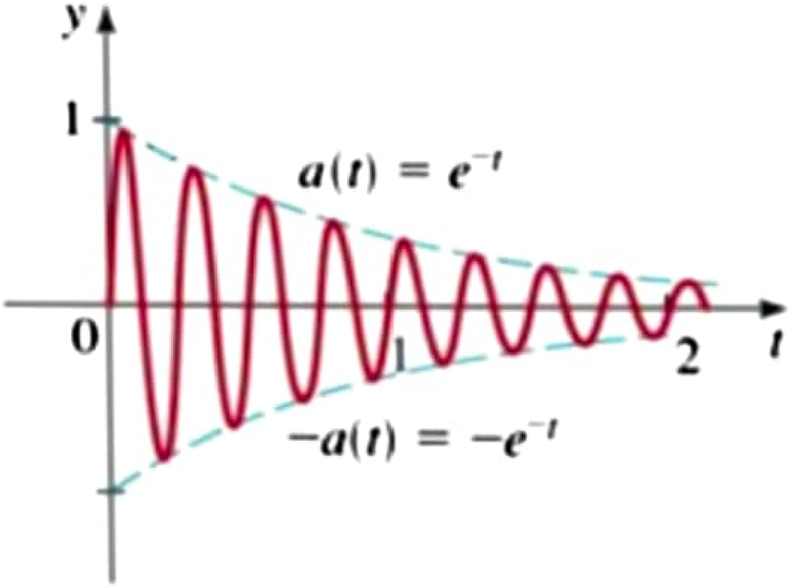
$$y = a\cos\omega t + b\sin\omega t.$$

Show that the motion is simple harmonic. **(Lucknow Univ.)**

Example 6. *A simple harmonic oscillator of mass* 0.2 **g** *vibrates with an amplitude* 4 cm. ***When the displacement is*** *zero, the velocity* **is 1 m/s.** ***Calculate the frequency and the energy of the oscillator.***



SHM



Damped

SHM: Restoring force

Damped: Restoring force + Resistance

Resistance : ??????

http://spiff.rit.edu/classes/phys312/workshops/w5b/damped_theory.html

- is always opposite to the direction of motion (i.e. opposite to the velocity)
- depends LINEARLY on the magnitude of the velocity

$$\vec{F}_{\text{resist}} = -b\vec{v}$$

$$\vec{F} = -k\vec{x} - b\vec{v}$$

Resistances are dependent on velocity

$$\vec{F} = -k\vec{x}$$

$$m\frac{d^2\vec{x}}{dt^2} = -k\vec{x}$$

$$\vec{x}(t) = \vec{A}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$m\frac{d^2\vec{x}}{dt^2} = -k\vec{x} - b\frac{d\vec{x}}{dt}$$

$$m\frac{d^2\vec{x}}{dt^2} + b\frac{d\vec{x}}{dt} + k\vec{x} = 0$$

??????

Possible solution

$$x(t) = A\cos(\omega t + \phi)e^{-t/\tau} \qquad ???$$

Due to periodicity

Due to resistance

$$m\frac{d^2\vec{x}}{dt^2} + b\frac{d\vec{x}}{dt} + k\vec{x} = 0$$

$$\frac{dx}{dt} = -\frac{1}{\tau}A\cos(\omega t + \phi)e^{-t/\tau} - \omega A\sin(\omega t + \phi)e^{-t/\tau}$$

$$\begin{aligned}\frac{d^2x}{dt^2} = \quad & \frac{1}{\tau^2}A\cos(\omega t + \phi)e^{-t/\tau} \\ & + \frac{\omega}{\tau}A\sin(\omega t + \phi)e^{-t/\tau} \\ & + \frac{\omega}{\tau}A\sin(\omega t + \phi)e^{-t/\tau} \\ & - \omega^2 A\cos(\omega t + \phi)e^{-t/\tau}\end{aligned}$$

$$\tau = \frac{2m}{b}$$

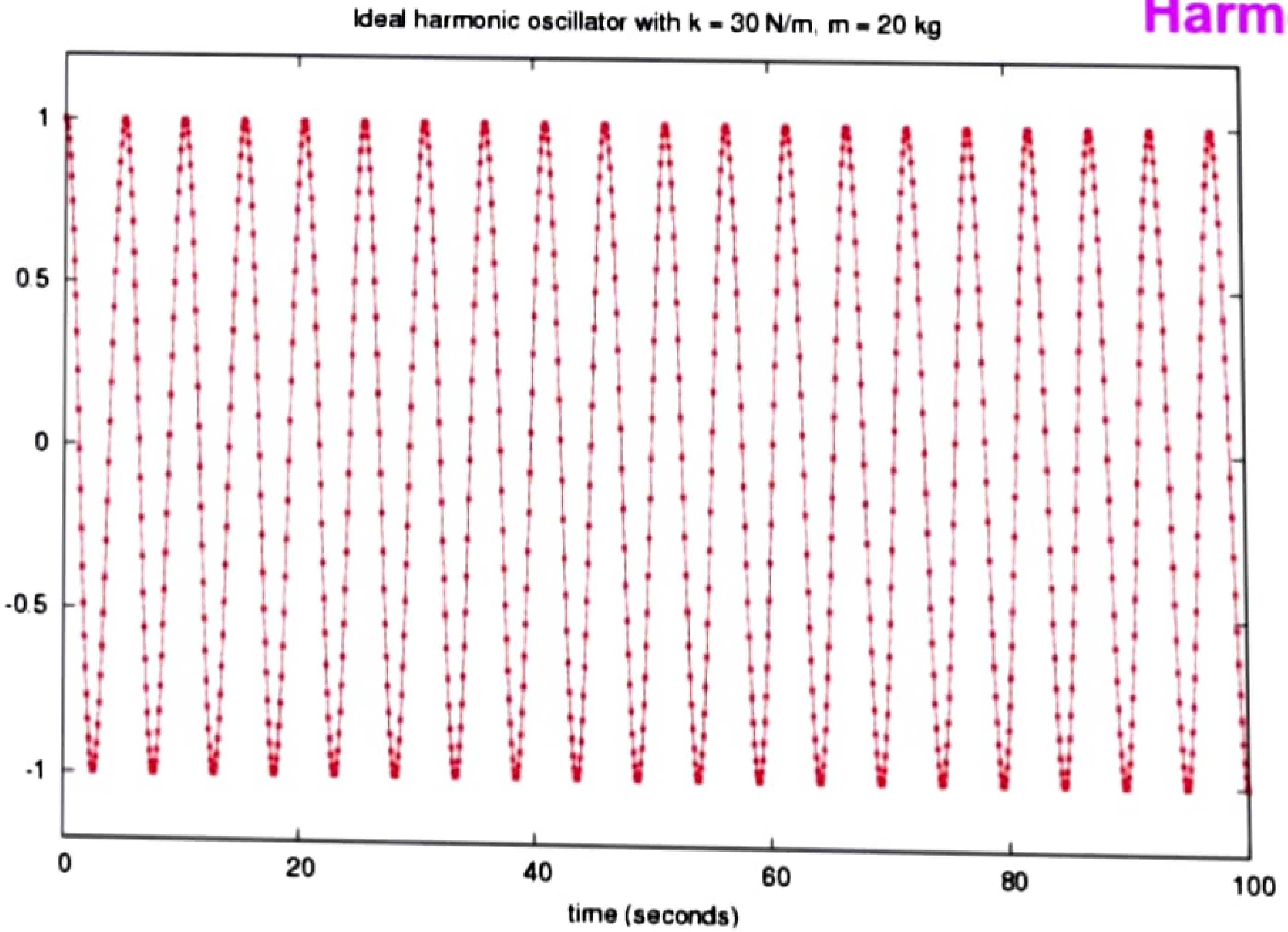
$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$\omega = \sqrt{\frac{4mk - b^2}{4m^2}}$$

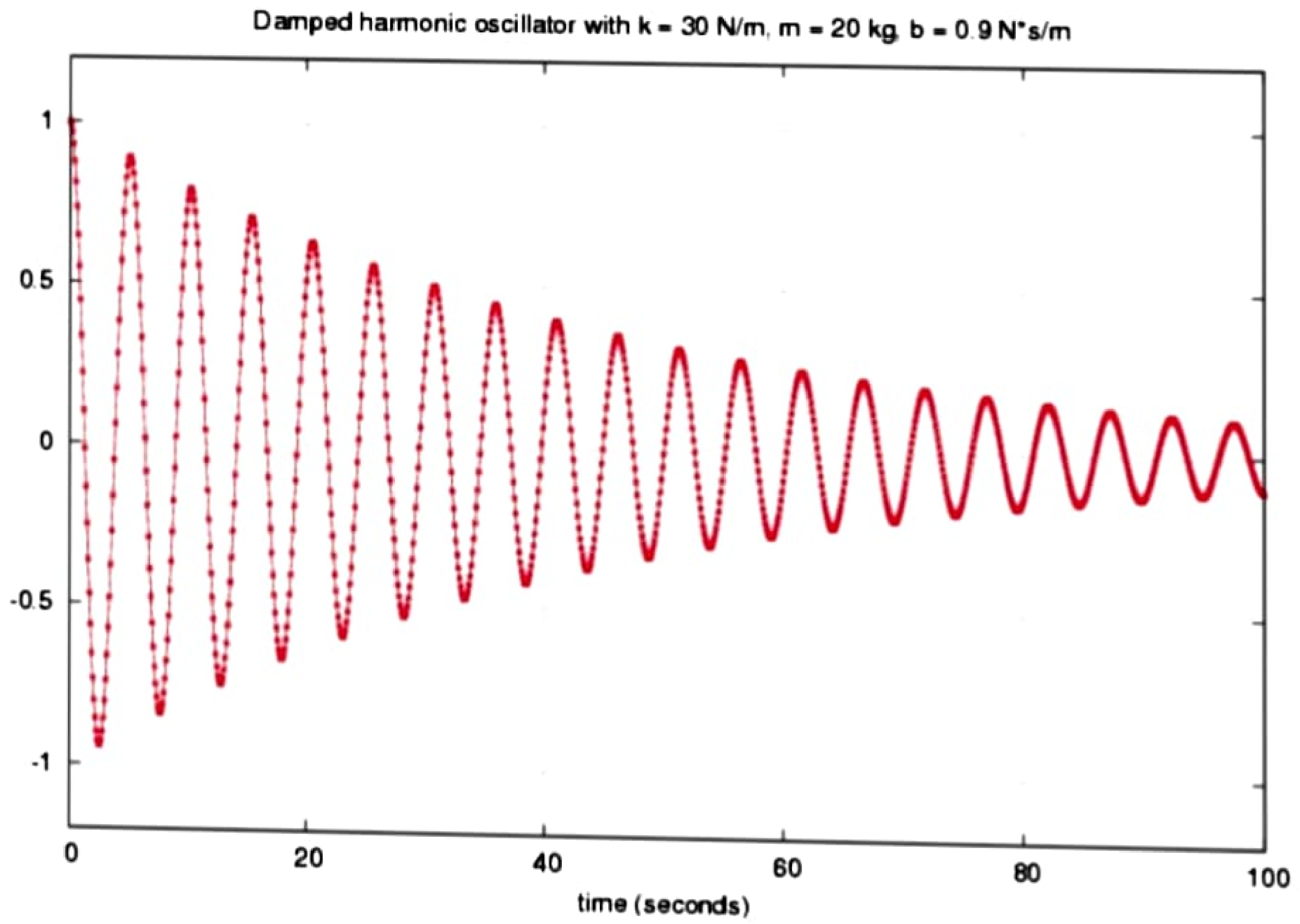
$$\frac{m}{\tau^2} - m\omega^2 - \frac{b}{\tau} + k = 0$$

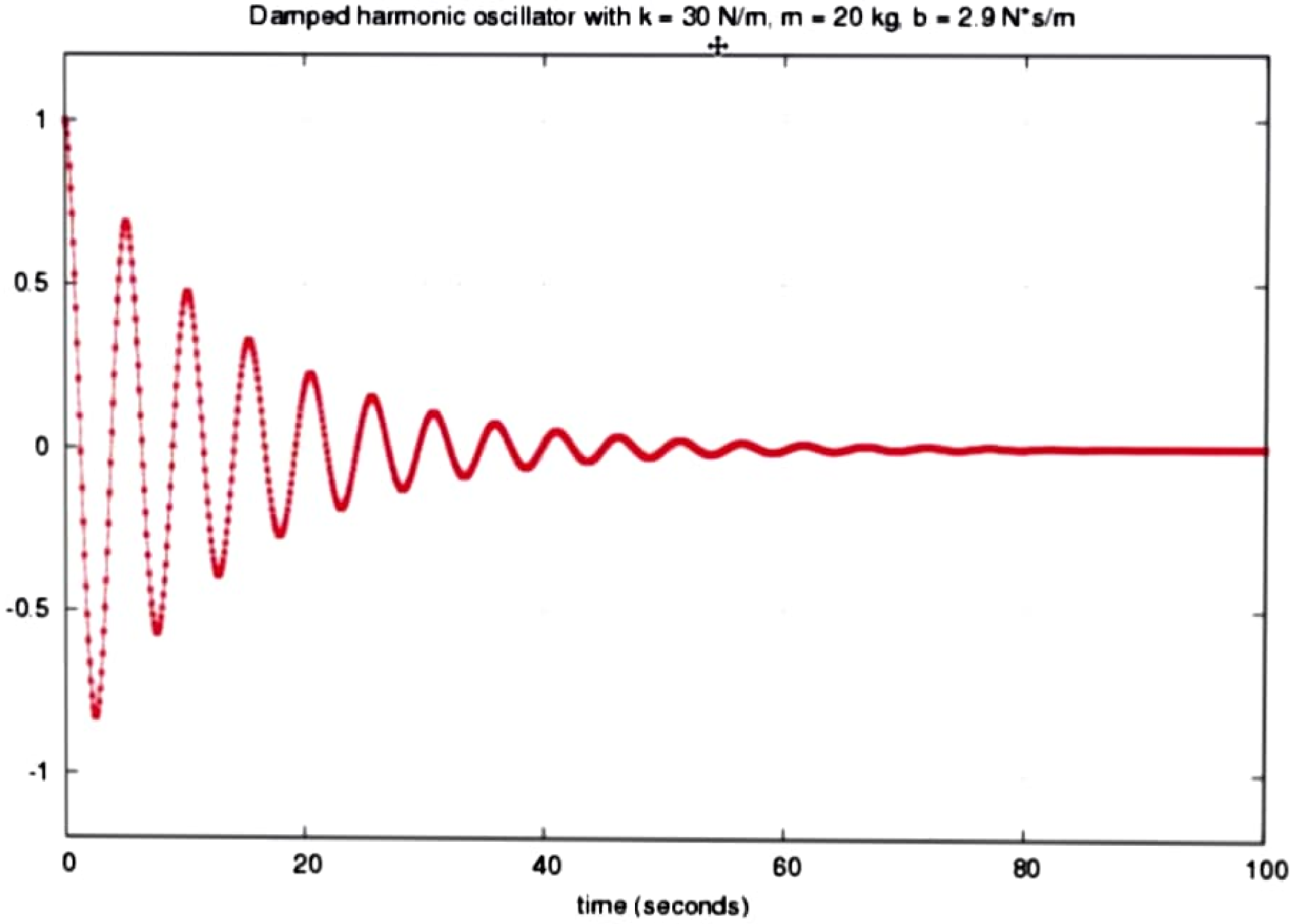
$$\frac{2m\omega}{\tau} - b\omega = 0$$

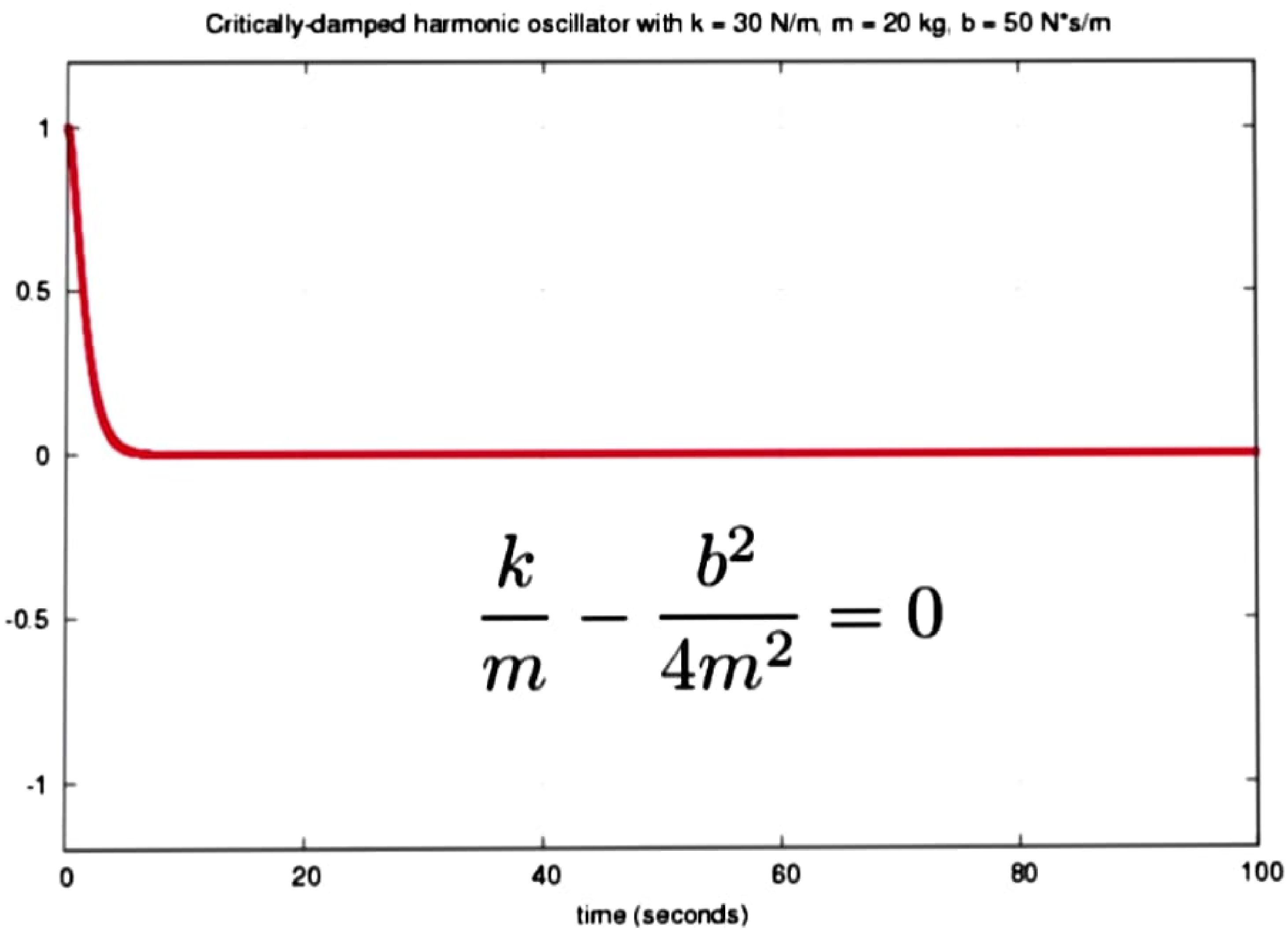
Harmonic

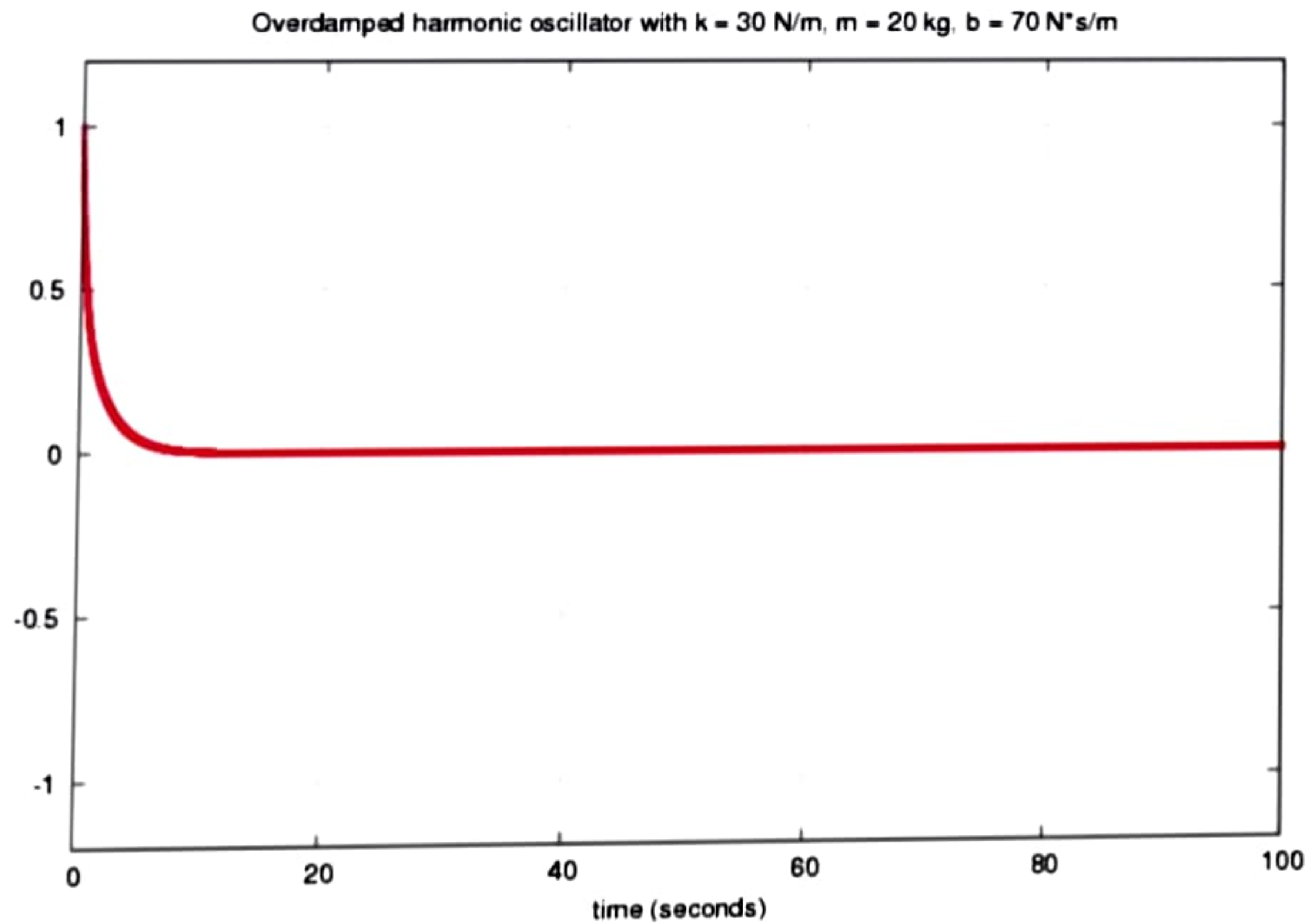


Ideal harmonic oscillator with k = 30 N/m, m = 20 kg
1
0.5
0
-0.5
-1
0
20
40
60
80
100
time (seconds)

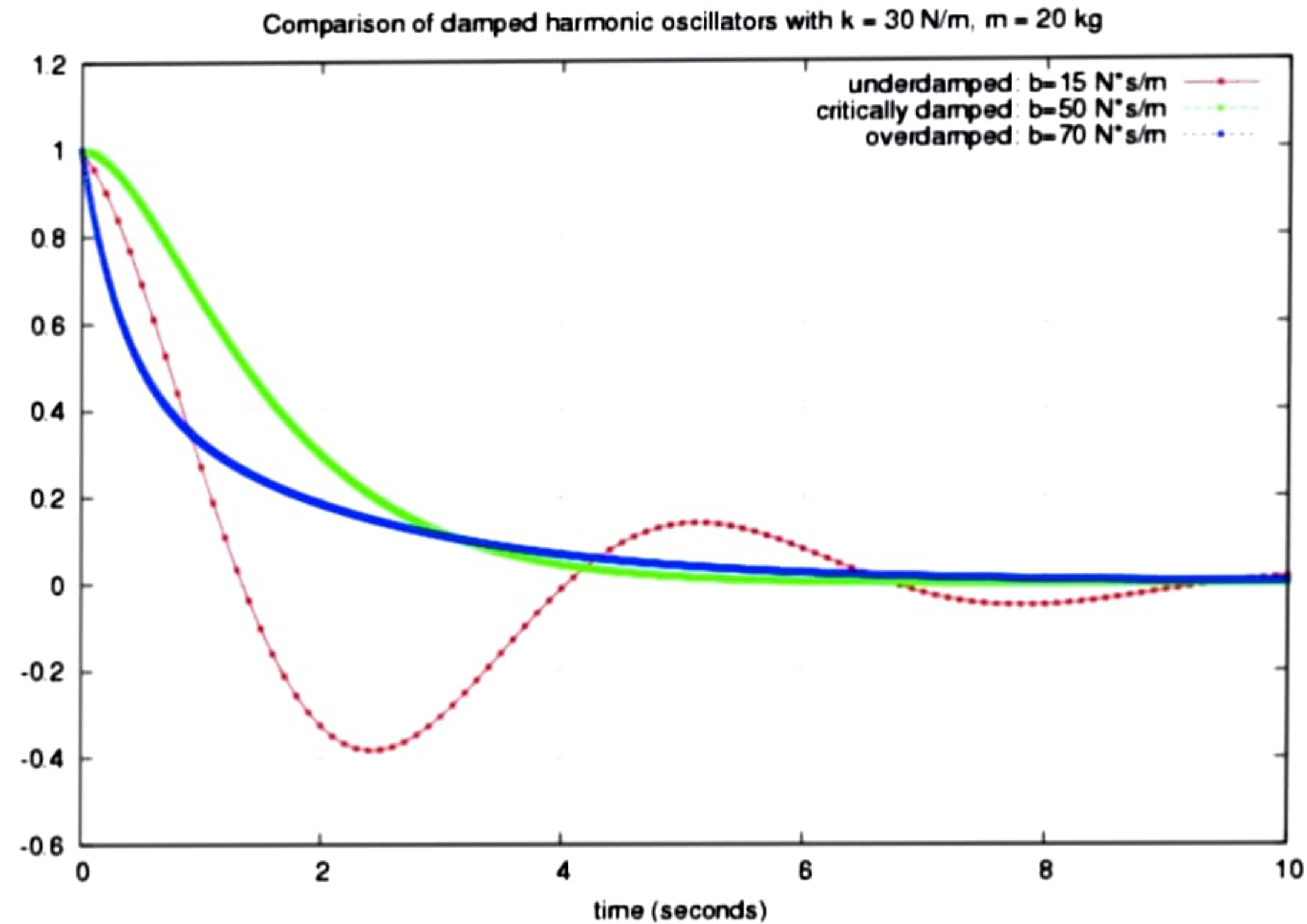

Damped harmonic oscillator with k = 30 N/m, m = 20 kg, b = 0.9 N*s/m
1
0.5
0
-0.5
-1
0
20
40
60
80
100
time (seconds)


Damped harmonic oscillator with k = 30 N/m, m = 20 kg, b = 2.9 N*s/m
1
0.5
0
-0.5
-1
0
20
40
60
80
100
time (seconds)


Critically-damped harmonic oscillator with k = 30 N/m, m = 20 kg, b = 50 N*s/m
1
0.5
0
-0.5
-1
0
20
40
60
80
100
time (seconds)
$$\frac{k}{m} - \frac{b^2}{4m^2} = 0$$



Overdamped harmonic oscillator with k = 30 N/m, m = 20 kg, b = 70 N*s/m
1
0.5
0
-0.5
-1
0
20
40
60
80
100
time (seconds)



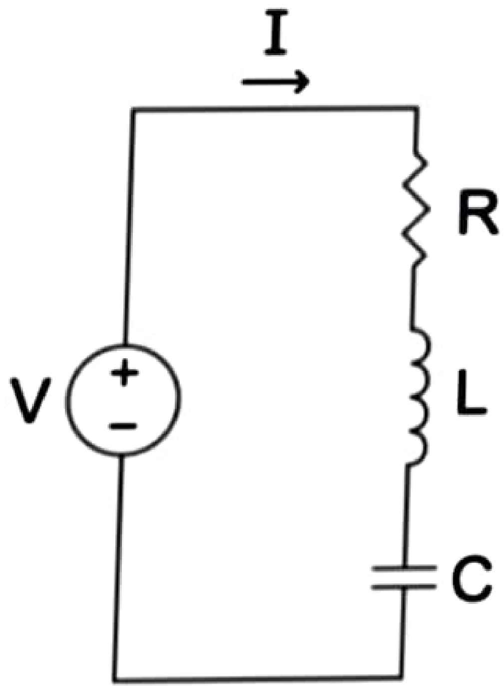
Comparison of damped harmonic oscillators with k = 30 N/m, m = 20 kg
underdamped: b=15 N*s/m
critically damped: b=50 N*s/m
overdamped: b=70 N*s/m
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
2
4
6
8
10
time (seconds)

- An underdamped system moves quickly to equilibrium, but will oscillate about the equilibrium point as it does so.

- An overdamped system moves slowly toward equilibrium.

- A critically damped system moves as quickly as possible toward equilibrium without oscillating about the equilibrium.

https://ocw.mit.edu/courses/mathematics/18-03sc-differential-equations-fall-2011/unit-ii-second-order-constant-coefficient-linear-equations/damped-harmonic-oscillators/MIT18_03SCF11_s13_2text.pdf



I
R
V
+
−
L
C

1. A particle of mass 1 g is displaced from its position of rest and released. If it is acted on by a restoring force 5 dyne/cm and a damping force 1 dyne sec/cm, examine if the motion is aperiodic or oscillatory. If the initial displacement of the particle is 5 cm, what would be the displacement after 5 second?

► **Example 1.** *A mass of 1 kg is suspended from a spring of stiffness constant 25* Nm^{-1}. *If the undamped frequency is* $2/\sqrt{3}$ *times the damped frequency, calculate the damping factor.*

(Himachal Univ.)

► **Example 3.** *In an oscillatory circuit,* $L = 0.5$ H, $C = 1.8\ \mu$F. *What is the maximum value of resistance to be connected so that the circuit may produce oscillations.*

(Himachal Univ.)

15. **Two identical springs, each of force constant k, are connected as shown in Fig. 1.30. In each case, find the value of the effective force constant of the system in terms of k, for the oscillation of body A.**