Introduction

Kinematics

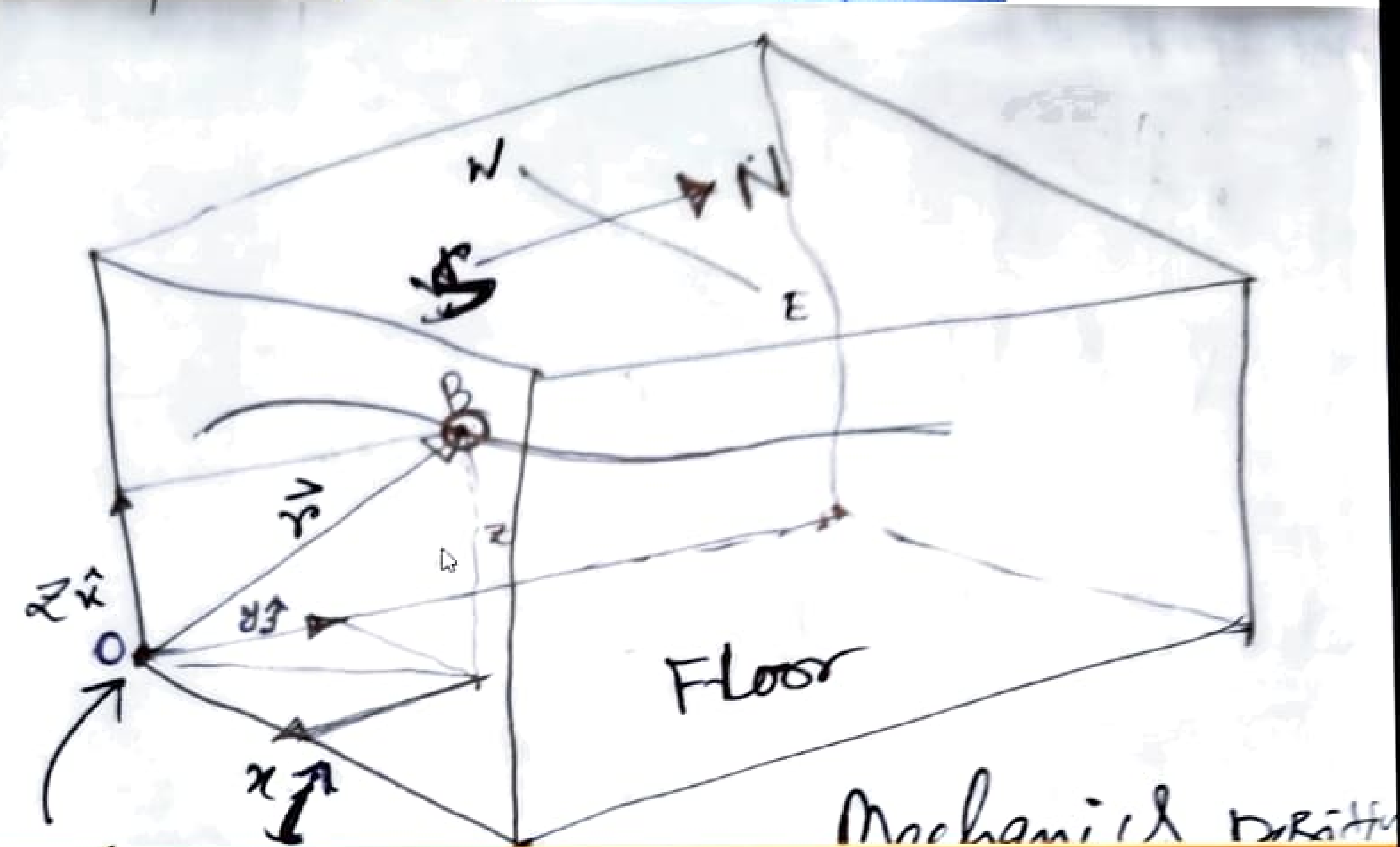
Kinematics. Which is the study of geometry of motion. Kinematics is used to relate displacement, velocity, acceleration and time without reference to cause of motion. Initially Our discussion will be started with kinematics of particle. By saying that the bodies are analyzed as particles, we mean that only their motion as an entire unit will be considered. Any rotation about their mass center will be neglected. Since the position , velocity and acceleration of a particle will be defined as vector quantities, the concept of derivative of vector function need to be introduced.

Different types of motions will be discussed under following categories

- **Uniform Rectilinear motion or motion with uniform velocity**
- **Uniformly accelerated rectilinear motion or motion with uniform acceleration**
- **Acceleration due to gravity**
- **Motion with varying acceleration**
- **Curvilinear motion(Plane curvilinear motion-2D)**
- **Constrained motion**

<u>EXAMPLE OF CONSTRAINED MOTION</u>: A small rock or stone tied to the end of string and whirled in a circle undergoes constrained motion until the string break. After which instant the motion is unconstrained.

Definition of vector in Dynamics



Position vector

Position Vector in rectangular coordinate system/ Cartesian coordinate system

Let us imagine a Room of transparent walls and roof having a single opening (window) as shown in fig-1. A bat suddenly entered into the room through the opening(window) and starts rooming around the room trying to come out of the room. His motion in every instantanteneous position may be recorded and expressed with the help position vector. Let at any instant of time his position is expressed with the help of vector notation as follows.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Where

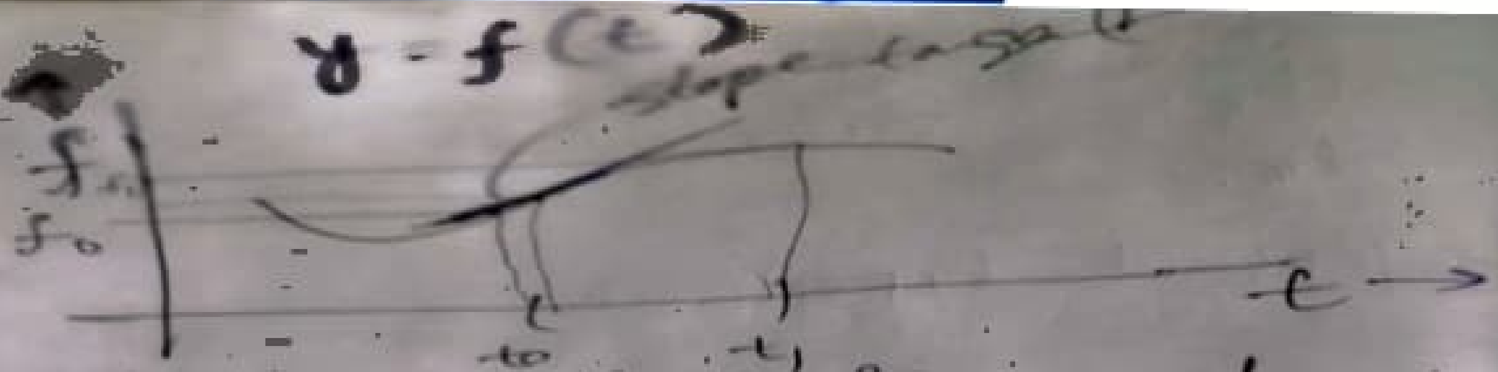
$$|\vec{r}| = \sqrt{(x^2 + y^2 + z^2)}$$

And

$$velocity \quad [v(t)] = \frac{d\vec{r}(t)}{dt}$$

In mechanics the quantities are expressed in terms of mass(M),length(L) and time(T) or some combinations of above .

Explanation with figure-2



derivative of a function. figure 2

$$\left.\frac{df}{dt}\right|_{t} = \lim_{t_1 \to t_0} \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

change of / change of t

t_1 approaches t_0 to .

velocity : derivative of a position vector

$$v(t_0) = \left.\frac{d\vec{r}(t)}{dt}\right|_{t_0} = \lim_{t_1 \to t_0} \frac{\vec{r}(t_1) - \vec{r}(t_0)}{t_1 - t_0}$$

difference in ti

Figure-2

Explanation with figure-3&4 in a single plane (two dimension)

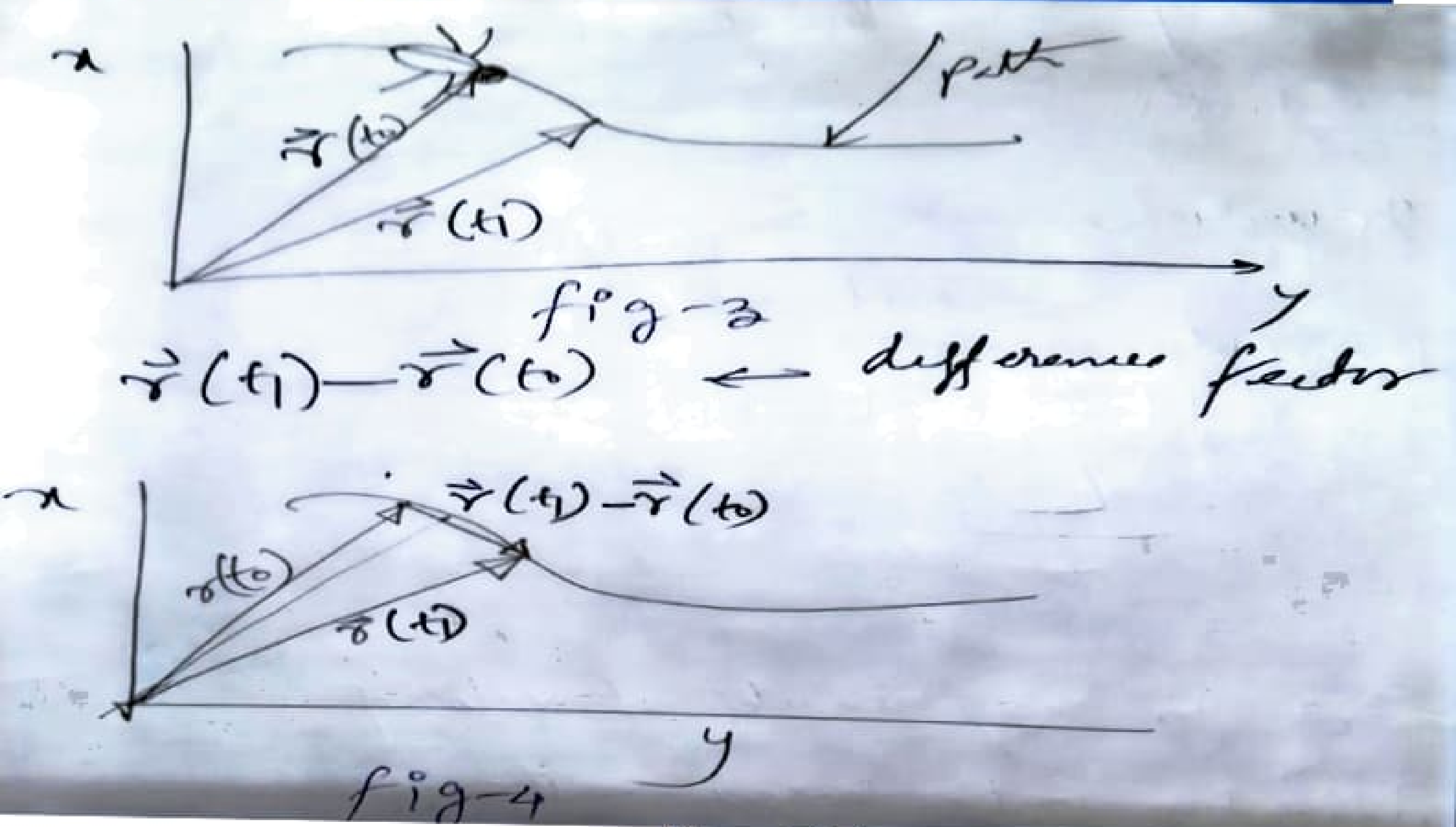


Figure-3 & 4

Defination of velocity and acceleration

Deffination of velocity

It is time derivative of vector.

$$\vec{v} = \frac{d\vec{r}}{dt}$$

acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

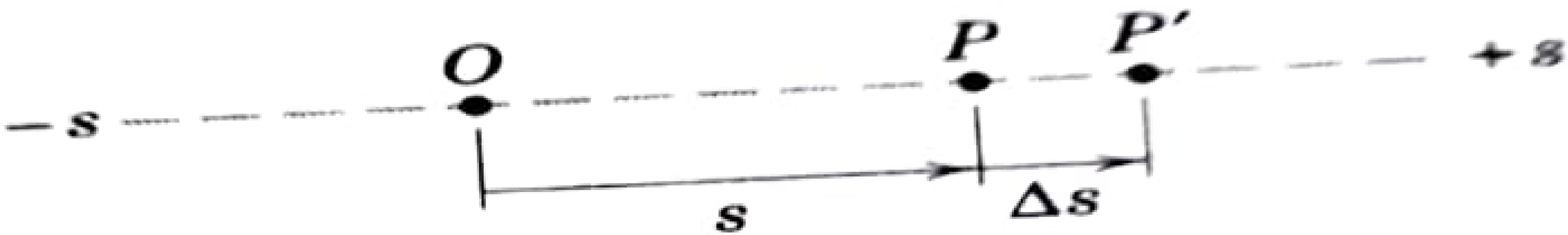
both velocity and acceleration are vector quantity.

RECTILINEAR MOTION OF PARTICLES
Position, velocity and acceleration

2/2 RECTILINEAR MOTION

Consider a particle P moving along a straight line, Fig. 2/2. The position of P at any instant of time t can be specified by its distance s measured from some convenient reference point O fixed on the line. At time $t + \Delta t$ the particle has moved to P' and its coordinate becomes $s + \Delta s$. The change in the position coordinate during the interval Δt is called the *displacement* Δs of the particle. The displacement would be negative if the particle moved in the negative s-direction.

*Often called *Cartesian* coordinates, named after René Descartes (1596–1650), a French mathematician who was one of the inventors of analytic geometry.



When the position coordinates of a particle is known for every value of time t, we say that the motion of the particle is known.

Velocity and Acceleration

The average velocity of the particle during the interval Δt is the displacement divided by the time interval or $v_{av} = \Delta s/\Delta t$. As Δt becomes smaller and approaches zero in the limit, the average velocity approaches the *instantaneous velocity* of the particle, which is $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$ or

$$\boxed{v = \frac{ds}{dt} = \dot{s}} \qquad (2/1)$$

Thus, the velocity is the time rate of change of the position coordinate s. The velocity is positive or negative depending on whether the corresponding displacement is positive or negative.

The average acceleration of the particle during the interval Δt is the change in its velocity divided by the time interval or $a_{av} = \Delta v/\Delta t$. As Δt becomes smaller and approaches zero in the limit, the average acceleration approaches the *instantaneous acceleration* of the particle, which is $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$ or

$$\boxed{a = \frac{dv}{dt} = \dot{v}} \quad \text{or} \quad \boxed{a = \frac{d^2s}{dt^2} = \ddot{s}} \qquad (2/2)$$

The acceleration is positive or negative depending on whether the velocity is increasing or decreasing. Note that the acceleration would be positive if the particle had a negative velocity which was becoming less negative. If the particle is slowing down, the particle is said to be *decelerating*.

Laws of motions(Applicable only where acceleration is constant)

$$v = u + at........[1]$$

$$s = ut + \frac{1}{2} a t^2 ....[2]$$

$$v^2 - u^2 = 2as ....[3]$$

Where
u=initial velocity
V=final Velocity
S=Displacement
a=uniform acceleration

NOTE: Above mentioned equation of motion is applicable only when acceleration is constant. Otherwise problems may be solved considering fundamental concept of displacement , velocity and acceleration .

1. A particle starting from rest moves in a straight line, Where equation of motion is given by the expression

$$s = t^3 - 2t^2 + 3.$$

Find the velocity and acceleration of the particle after 5 s.

Solution :

$$s = t^3 - 2t^2 + 3$$

$$v = \frac{ds}{dt} = 3t^2 - 4t$$

$$v_{5s} = 3 \times 5^2 - 4 \times 5 = 75 - 20 = 55 \text{ m/s}$$

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2} = 6t - 4$$

$$a_{t=5s} = 6 \times 5 - 4 = 26 \text{ m/s}^2$$

[2]The position vector of a particle moving in x-y plane at time t=4s is $5.05\hat{i} + 3.2\hat{j}\ m$.
At t=4.5 s its position vector becomes $6.27\hat{i} + 4.7\hat{j}\ m$.

Determine the magnitude v of it's average velocity during this interval and the angle θ made by v with x- axis.

Solution:

The position vector at t=4s is $\bar{r}_1 = 5.05\hat{i} + 3.2\hat{j}$ and the same at t=4.5s is $\bar{r}_2 = 6.27\hat{i} + 4.7\hat{j}$

Thus, $$\Delta r = \bar{r}_2 - \bar{r}_1 = (6.27\hat{i} + 4.7\hat{j}) - (5.05\hat{i} + 3.2\hat{j}) = 1.22\hat{i} + 1.5\hat{j}$$

$$\therefore v_{avg} = \frac{\Delta\bar{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = \frac{1.22}{0.5}\hat{i} + \frac{1.5}{0.5}\hat{j} = \frac{1.22}{0.5}\hat{i} + \frac{1.5}{0.5}\hat{j} = 2.44\hat{i} + 3\hat{j} = 2.44\hat{i} + 3\hat{j}$$

Therefore the magnitude v of it's average velocity during this interval becomes

$$\sqrt{(2.44^2 + 3^2)}\ \text{m/s} = 3.867\ \text{m/s}$$

And the angle θ made by v with x- axis is $$\theta = \tan^{-1}\left(\frac{3}{2.44}\right) = 50.87^\circ$$

[3]The acceleration of a particle is defined by the relationθ made by v with x- axis $a=t^2-2t+2$, Where a is in m/s^2 and t is in second. The displacement and velocity of the particle at t=1 1s found to be 14.75 m and 6.33 m/s .Find the distance travelled , velocity and acceleration when t=3 s.

Solution :

The acceleration of the particle is $$a=\frac{dv}{dt}=t^2-2t+2$$

$$\int dv=\int a dt=\int (t^2-2t+2)dt$$

$$v=\frac{1}{3}t^3-2\times\frac{1}{2}t^2+2t+c_1$$

When t=1 s ; v=6.33 m/s $\quad$ *Thus*, $6.33=\frac{1}{3}1^3\quad 2\times\frac{1}{2}1^2+2\times 1+c_1 \text{ or } c_1=5$

$$\text{Therefore, } \quad \mathrm{v}=\frac{\mathrm{dx}}{\mathrm{dt}}=\frac{1}{3}\times t^3-2\times\frac{1}{2}t^2+2t+5$$

$$x=\int dx=\int v dt=\int\frac{1}{3}t^3 dt-\int t^2 dt+2\int t dt+5\int dt$$

$$x=\frac{1}{3\times 4}t^4-\frac{1}{3}t^3+2\times\frac{1}{2}t^2+5t+c_2$$

When t=1 s ; x=14.75 m

$$\textit{Thus}, \quad 14.75=\frac{1}{3\times4}1^4-\frac{1}{3}1^3+2\times\frac{1}{2}1^2+5\times1+c_2$$

$$c_2=9$$

$$\textit{Hence} \quad x=\frac{1}{12}t^4-\frac{1}{3}t^3+t^2+5t+9$$

When t=3s; *Similarly* $v=\frac{1}{3}\times3^3-3^2+2\times3+5=11\ m/s$

$$x=\frac{1}{12}\times3^4-\frac{1}{3}\times3^3+3^2+5\times3+9=30.75\,m$$

$$a=3^2-2\times3+2=5\ m/s^2$$

[4] A particle starting from rest moves in a straight line where acceleration is given by the equation $a = 10 - 0.006\, s^2$ where a is in m/s^2, s is in m. Find the velocity of the particle when it has travelled 50 m and also find the distance travelled by the particle when it comes to rest.

Solution :

$$a = 10 - 0.006\, s^2$$

$$a = \frac{dv}{dt} = 10 - 0.006\, s^2$$

$$a = v\frac{dv}{ds} = 10 - 0.006\, s^2$$

$$\int_0^v v\,dv = \int_0^s (10 - 0.006\, s^2)\,ds$$

$$\frac{v^2}{2} = 10\,s - \left(\frac{0.006}{3}\right) s^3$$

FOR , s=50 m/s we obtain,

V=22.36 m/s

$$\frac{v^2}{2} = 10s - \left(\frac{0.006}{3}\right)s^3$$

Now,

FOR v=0,

Substituting the value v=0 in the above expression We obtain S=70.7 m

[5] The equation of motion of an engine is given by $s = 2t^3 - 6t^2 - 5$

where s is in m, and t is in second . Calculate the displacement and acceleration when velocity is zero. Also find the displacement and velocity , when acceleration is zero.

Solution :

$$s = 2t^3 - 6t^2 - 5$$

$$v = \frac{ds}{dt} = 6t^2 - 12t$$

$$a = \frac{d^2 s}{dt^2} = 12t - 12$$

For v=0,

$$v = 6t^2 - 12t = 0$$

$$v = t^2 - 2t = 0$$

Therefore t=2

Substituting t=2 in the above expression of s and acceleration a we obtain s=-13m and a=12m/s²

respectively.

[6] The position of a particle describing rectilinear motion can be described by the equation $x = t^3 - 9t^2 + 15t + 18$ where x is expressed in m, t is in second. Determine the time, displacement and acceleration of the particle when it's velocity is zero.

Solution :

$$x = t^3 - 9t^2 + 15t + 18 \text{ ....[1]}$$

$$\frac{dx}{dt} = 3t^2 - 18t + 15 \text{ .....[2]}$$

$$a = \frac{d^2x}{dt^2} = 6t - 18 \text{ ......[3]}$$

According to problem $v = \frac{dx}{dt} = 0$

$$3t^2 - 18t + 15 = 0 \text{ ....[4]}$$

After Solving, the roots are , t=1 and t=5 we obtain,

x=25m from equation[1] and a= -12m/s^2 from equation[3] Similarly for t=5,we obtained x=- 7m and a=12 m/s2 respectively.

[7] A particle undergoing a rectilinear motion such that it's displacement from a fixed origin can be expressed by $x = 3t^2 + 2t$ where x is expressed in m, t is in second. Determine the , displacement , velocity and acceleration at the end of 4 s.

Solution :

$$x = 3t^2 + 2t \text{....[1]}$$

$$x_{for\ t=4} = 3 \times 4^2 + 2 \times 4 = 48 + 8 = 56\ m \text{....[2]}$$

$$v = \frac{dx}{dt} = 6t + 2; \qquad v_{for\ t=4} = 6 \times 4 + 2 = 26\ m/s$$

$$a = \frac{dv}{dt} = 6$$

x=56m;v=26m/s and a=6m/s^2

[8] A particle undergoing a rectilinear motion according to the equation $x = t^3 - 3t^2 - 5$ where x is expressed in m, t is in second. Determine the , Change in position , when it's velocity changes from 8m/s to 40 m/s.

Solution :

$$x = t^3 - 3t^2 - 5 \ldots [1]$$

$$v = \frac{dx}{dt} = 3t^2 - 6t \ldots\ldots [\ 2\ ]$$

When v =8m/s, According to problem

$$v = \frac{dx}{dt} = 3t^2 - 6t = 8$$

$$3t^2 - 6t = 8; \quad 3t^2 - 6t - 8 = 0;$$

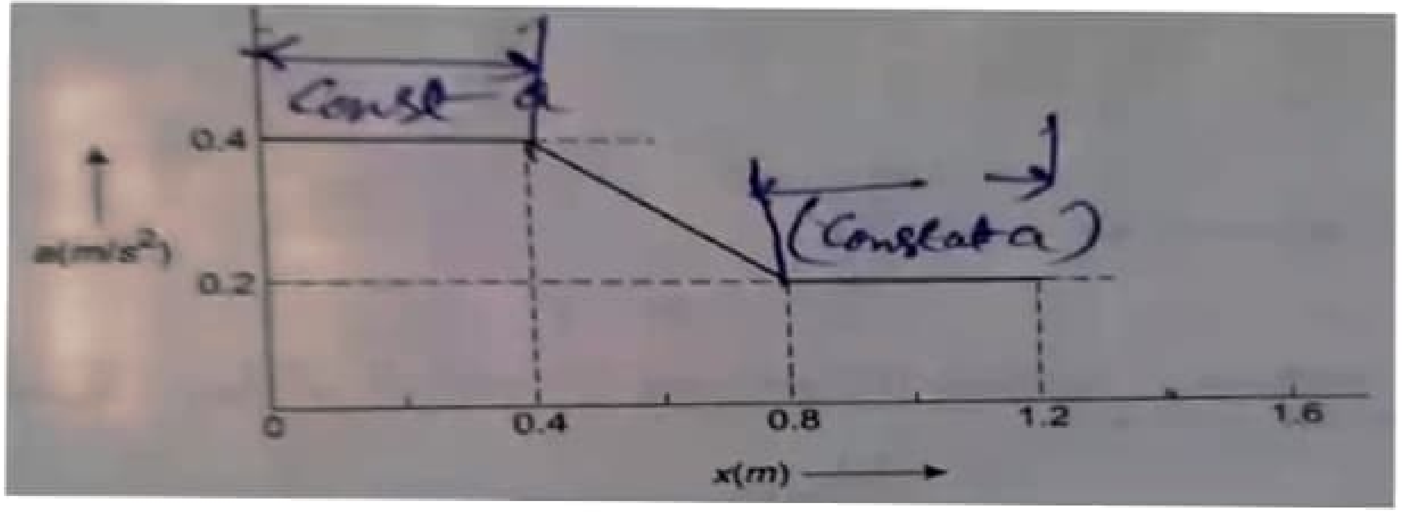
solving t = 2.91485s or t = -0.91485s

$$x = (2.915)^3 - 3 \times (2.915)^2 - 5 = -5.72\ m$$

Similarly for v=40m/s t=4.786 s and corresponding x=35.9 m

Hence Change in position is 41.6 m Ans.41.6 m

[9]The acceleration a of a particle that moves in +x direction varies with it's position as shown in figure below . If the velocity of the particle is 0.8 m/s when x=0. Determine the velocity v, when x=1.4 m .



Solution :

i. The acceleration remains constant at $a=0.4\ m/s^2$ for $0<x<0.4$.

We know that

$$a = v\frac{dv}{dx}$$

$$or, \quad vdv = adx$$

Integrating both sides, we get.

$$\int vdv = \int adx = a\int dx$$

When x=0; v=0.8 m/s; Let the velocity is v_1 when x=0.4 m . Hence above integration is written as

$$\textit{Hence} \text{ above integration is written as } \int_{0.8}^{v1} v\,dv = a\int_{0}^{0.4} dx \text{ from which } v_1 = 0.98 \text{ m/s.}$$

ii. The acceleration follows straight line for 0.4<x<0.8 . It is now essential to develop acceleration equation considering a=*f*(x).

From the graph, it is found that when x=0.4; a=0.4 and when x=0.8; a=0.2

Let the general equation of the curve is in the form of a=mx +K

$$\textit{Thus } 0.4 = m \times 0.4 + k \text{ and } 0.2 = m \times 0.8 + k$$

Solving these two equation ; the particular equation becomes a= -0.5x+0.6

Let the velocity is v2 =is when x=0.8 m.

Then

$$\int_{0.98}^{v_2} v\,dv = \int_{0.4}^{0.8} (-0.5x + 0.6)\,dx \text{ which yields } v_2 = 1.0956 \text{ m/s}$$

iii. During the period of 0.8<x<1.2 acceleration once again remains constant at $a=0.2 \text{ m/s}^2$.

If the velocity of the particle is v_3 when x=1.2 m , then from the relationship.

$$v_3^2 = v_2^2 + 2ax \; ; \; v_3 = 1.166 \text{ m/s}$$

Since a=0 and when x>1.2; the velocity of the particle remains constant at v_3=1.166 m/s .

This implies velocity $v=v_3$=1.166 m/s, when x=1.4 m

Numerical

[10] The motion of a particle moving in a straight line is given by the expression $s = t^3 - 3t^2 + 2t + 5$ where , s is the displacement in m and t is the time in seconds .

Determine velocity and acceleration after 4 s; Also find maximum or minimum velocity and corresponding displacement ; and further determine time at which velocity is zero.

Solution:

$$s = t^3 - 3t^2 + 2t + 5 \ldots[1]$$

$$v = \frac{ds}{dt} = 3t^2 - 6t + 2 \ldots[2]$$

$$a = \frac{d^2 s}{dt^2} = 6t - 6 \ldots[3]$$

Hence after 4 s

$$v = 3\times 4^2 - 6\times 4 + 2 = 26 m/s \quad \text{Ans}$$

$$a = 6\times 4 - 6 = 18 m/s^2 \quad \text{Ans}$$

The velocity is maximum or minimum when $\frac{dv}{dt} = a = 0$. From Eqn[3]

we get 0=6t-6 or t=1 and corresponding velocity

$$3\times 1^2 - 6\times 1 + 2 = -1 \text{ m/s}$$

Hence, it should be minimum velocity.

(or $\frac{d^2v}{dt^2}$ is positive, Hence minimum velocity.)

Let time t velocity will be zero. Then $0 = 3t^2 - 6t + 2$

$$t = \frac{6 \pm \sqrt{(36 - 4\times 3\times 2)}}{2\times 3} = 1.557 \text{ and } 0.423\text{s.}$$

References:

[1]Engineering Mechanics by SS Bhavikatti.

[2]Engineering Mechanics by P K Nag, Sukumar Pati and T K Jana.

[3]Engineering Mechanics by BB Ghosh, S Chakrabarti and S Ghosh.

[4]Engineering Mechanics DYNAMICS by J.L. Meriam & L.G. KRAIGE.

[5] Vector Mechanics for Engineers DYNAMICS by Ferdinand P. Beer & E. Russell Johnston Jr.

[6] Source : youtube video lecture on topic "The velocity vector-Engineering Dynamics & Notes"

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