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Group A

(1) b) The given differential eqⁿ is $(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0$

This is of the form $M dx + N dy = 0$

where, $M = \cos y + y\cos x$ &
$N = \sin x - x\sin y$.

For exact: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Here, $\frac{\partial M}{\partial y} = -\sin y + \cos x$

$\frac{\partial N}{\partial x} = \cos x - \sin y$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. So, yes. this equation is exact.

c) Given vectors are : $(1,2,1)$, $(K,1,1)$ and $(1,1,2)$

~~For~~ Here, let, $u = (1,2,1)$, $v = (K,1,1)$ & $w = (1,1,2)$.

Then, $xu + yv + zw = 0$

$$\Rightarrow \left.\begin{aligned} x + Ky + z &= 0 \\ 2x + y + z &= 0 \\ x + y + 2z &= 0 \end{aligned}\right\}$$

~~x + ky + z = 0~~

The corresponding coefficient matrix is :-

$$\begin{bmatrix} 1 & K & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$1(1) - K(3) + 1(1)$
$= 2 - 3K = 0$

$$\sim \begin{bmatrix} 2 & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 1 \\ -1 & K-1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & 1 & 0 \\ -1 & K-1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow \begin{aligned} 3x + y &= 0 \\ -x + (K-1)y &= 0 \\ -x + z &= 0 \end{aligned}$$

$$\left.\begin{aligned} x + ky + z &= 0 \\ 2x + y + z &= 0 \\ x + y + 2z &= 0 \end{aligned}\right\} \text{ or } \quad \begin{aligned} x + ky + z &= 0 \\ (2k-1)y + z &= 0 \\ (k-1)y - z &= 0 \end{aligned}$$

Or,
$$\begin{aligned} x + ky + z &= 0 \\ (2k-1)y + z &= 0 \\ (3k-2)y &= 0 \end{aligned}$$

$\therefore \Rightarrow$ For linearly independent: $k \neq 2/3$ $[\because (3k-2) \neq 0]$

If $k \neq 2/3$, then, $y = 0$, $z = 0$, $x = 0$.

So, $k \in R - \left\{\frac{2}{3}\right\}$.

(f) $f: R^2 \to R^2$, $f(x+y) = (x+3, 2y, x+y)$

~~For linearly independent :-~~

For linear, i) $f(u+w) = f(u) + f(w)$ &

ii) $f(ku) = k\, f(u)$

Here, let, $v = (a, b, c)$ & $w = (a', b', c')$

Then, $f(v+w) = f(a+a', b+b', c+c')$
$= (a+a'+3, 2b+2b', c+c')$

And $f(v) + f(w) = f(a, b, c) + f(a', b', c')$
$= (a+3, 2b, c) + (a'+3, 2b', c')$
$= (a+a'+6, 2b+2b', c+c')$

Thus, $f(v+w) \neq f(v) + f(w)$

Hence, this is not a linear mapping.

(d) $f(s) = \dfrac{1}{s(s+1)^2}$

Now, $\dfrac{1}{s(s+1)^2} = \dfrac{A}{s} + \dfrac{B}{(s+1)} + \dfrac{C}{(s+1)^2}$

On comparing the numerators:

$\Rightarrow \quad 1 = A(s+1)^2 + B(s+1)s + Cs$

$\Rightarrow \quad 1 = A(s^2+2s+1) + B(s^2+s) + Cs$

$$\Rightarrow \left.\begin{array}{l} A+B=0 \\ 2A+B+C=0 \\ A=1 \end{array}\right\} \quad \begin{array}{l} A=1 \\ B=-1 \\ C=-1 \end{array}$$

$\Rightarrow \quad L^{-1}[f(s)] = L^{-1}\left[\dfrac{1}{s(s+1)^2}\right] = L^{-1}\left[\dfrac{1}{s} - \dfrac{1}{(s+1)} - \dfrac{1}{(s+1)^2}\right]$

$= L^{-1}\left[\dfrac{1}{s}\right] - L^{-1}\left[\dfrac{1}{s+1}\right] - L^{-1}\left[\dfrac{1}{(s+1)^2}\right]$

$= 1 - e^{-t} - \dfrac{e^{-t}.t}{1!}$

$= 1 - e^{-t} - te^{-t}$

$= \underline{1 - e^{-t}[1+t]}$

. This is the required ~~Laplace~~ inverse Laplace transform.

e) $S = \{u+v-2w,\ u-v-w,\ u+w\}$

Given, u, v, w are linearly independent

Thus, let, a, b, c are constents.

Then, $au + bv + cw = 0$

For linearly independent :-

Thus, $a(u+v-2w) + b(u-v-w) + c(u+w) = 0$

we have, the corresponding system of linear eqns. :-

$$\boxed{\begin{aligned} a + b + c &= 0 \\ a - b &= 0 \\ -2a - b + c &= 0 \end{aligned}} \qquad \left.\begin{aligned} u + v - 2w &= 0 \\ u - v - w &= 0 \\ u + w &= 0 \end{aligned}\right\} \text{ or } \begin{aligned} u + v - 2w &= 0 \\ 2v - w &= 0 \\ -3w &= 0 \end{aligned}$$

=) Thus, The only solution is $u=0, v=0, w=0$.

Thus, S is linearly independent.

Group-B (6 ques.)

(2) $x + 2y - 3z + 2t = 2$

$2x + 5y - 8z + 6t = 5$

$3x + 4y - 5z + 2t = 4$

The corresponding augmented metrix is :-

$$M = \begin{bmatrix} 1 & 2 & -3 & 2 & : & 2 \\ 2 & 5 & -8 & 6 & : & 5 \\ 3 & 4 & -5 & 2 & : & 4 \end{bmatrix}$$

The corresponding coefficient metrix is :-

$$M = \begin{bmatrix} 1 & 2 & -3 & 2 \\ 2 & 5 & -8 & 6 \\ 3 & 4 & -5 & 2 \end{bmatrix}$$

Now,

$$\left[\begin{array}{cccc:c} 1 & 2 & -3 & 2 & 2 \\ 2 & 5 & -8 & 6 & 5 \\ 3 & 4 & -5 & 2 & 4 \end{array}\right]$$

$$\xrightarrow[R_3-3R_1]{R_2-2R_1} \left[\begin{array}{cccc:c} 1 & 2 & -3 & 2 & 2 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & -2 & 4 & -4 & -2 \end{array}\right]$$

$$\xrightarrow{R_3+2R_2} \left[\begin{array}{cccc:c} 1 & 2 & -3 & 2 & 2 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Thus, the rank of the given matrix $= \rho(M) = 2$

Here, the no. of variables > no. of equation.

& Rank of augmented matrix is same as the rank of coefficient matrix.

So, the solution exists and is infinitely many.

Because, we have 2 pivot elements and 2 free variables.

→ x and y are pivot elements &

z and t are free elements.

Thus,

$$x + 2y - 3z + 2t = 2$$
$$y - 2z + 2t = 1$$

let, $z=0, t=1$, $y=-1$, $x=2$ → $(2,-1,0,1)$

if $z=1, t=0$, $y=3$, $x=-1$ → $(-1,3,1,0)$

⇒ If $z=a$, $t=b$, then:

$\underline{y = 1+2a-2b}$,

& $x = 2-2(1+2a-2b)-3a+2b$

$\underline{x = -7a+6b}$

3. Given : $y'(t) + 3y(t) + 2\int_0^t y(t)\,dt = t$ given, $y(0) = 1$

$\Rightarrow$ Taking Laplace transform :-

$\Rightarrow L[y'(t)] + 3L[y(t)] + 2L\left\{\int_0^t y(t)\,dt\right\} = L[t]$

$\Rightarrow$ Now, $L[y'(t)] = sL[y(t)] - y(0)$
$= sL[y(t)] - 1$

$\Rightarrow sL[y(t)] - 1 + 3L[y(t)] + 2\frac{L[y(t)]}{s} = \frac{1}{s^2}$

$\Rightarrow \left(s + 3 + \frac{2}{s}\right) L[y(t)] = \frac{1}{s^2} + 1$

$\Rightarrow L[y(t)] = \frac{(1+s^2)s}{s^2(s^2+3s+2)}$

$\Rightarrow y(t) = L^{-1}\left[\frac{1+s^2}{s(s^2+3s+2)}\right]$ —— (1)

$\Rightarrow$ Now, $\frac{1+s^2}{s(s^2+3s+2)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+2)}$

$\Rightarrow$ Comparing the numerators :-

$1+s^2 = A(s+1)(s+2) + B(s)(s+2) + C(s)(s+1)$

$\Rightarrow$
$A + B + C = 1$
$3A + 2B + C = 0$
$2A = 1$

$A = 1/2$

$A = 1/2$
$B = -2$
$C = 5/2$

from (1),

$\Rightarrow y(t) = L^{-1}\left[\frac{1+s^2}{s(s+1)(s+2)}\right]$

$= L^{-1}\left[\frac{1/2}{s} - \frac{2}{(s+1)} + \frac{5/2}{(s+2)}\right]$

$= \frac{1}{2}L^{-1}\left[\frac{1}{s}\right] - 2L^{-1}\left[\frac{1}{s+1}\right] + \frac{5}{2}L^{-1}\left[\frac{1}{s+2}\right]$

$= \frac{1}{2} - 2e^{-t} + \frac{5}{2}e^{-2t}$

(5.) Given differential equation, $p = \log(px - y)$, where $p = \frac{dy}{dx}$.

$\Rightarrow \quad px - y = e^p$

or, $\quad y = px - e^p$ ——— (I)

On differentiating the above equation wrt x, we get;

$\frac{dy}{dx} = p + x\frac{dp}{dx} - e^p\, dp/dx$

or, $\quad p = \cancel{p} + x\frac{dp}{dx} - e^p\frac{dp}{dx} \qquad \{\because \frac{dy}{dx} = p\}$

or, $\quad \frac{dp}{dx}(x - e^p) = 0$

$\Rightarrow \quad \frac{dp}{dx} = 0 \qquad$ or, $(x - e^p) = 0$

$\Rightarrow \quad p = C, \qquad$ or, $x = e^p$

$\qquad\qquad \Rightarrow p = \log_e x$ ——— (II)

Let, putting the value of 'p' from eqn (II) in (I), we get,

$y = px - e^p$

or, $\quad y = pe^p - e^p \qquad [\because x = e^p]$

or, $\quad y = e^p(p-1)$

or, $\quad y = x(\log_e x - 1)$

This is the required singular solution.

(7) a) Given, V is a real vector space & $\{\alpha, \beta, \gamma\}$ is a basis.

To show: $S = \{\alpha+\beta+\gamma, \beta+\gamma, \gamma\}$ is also a basis of V

For a basis, i) It must be indepedent vector.
ii) It must span V.

$\Rightarrow \quad x(\alpha+\beta+\gamma) + y(\beta+\gamma) + z\gamma = 0$ {where x, y, z are constants}

$$\Rightarrow \therefore \left.\begin{aligned} \alpha+\beta+\gamma &= 0 \\ \beta+\gamma &= 0 \\ \gamma &= 0 \end{aligned}\right\}$$

The only solution is $\alpha=0, \beta=0, \gamma=0$.

Thus, this is a set of independent vector.

Also, Every row has a leading 1. &
The pivot element in each row is 1.
So, this spans. V.

Thus, set, S is a basis. ~~of vect~~

b) For linearly dependent, the determinant of matrix formed by its cofficient ie, the det. of coefficient matrix $= 0$.

ie, $\begin{vmatrix} 1 & 2 & 1 \\ K & 3 & 1 \\ 2 & K & 0 \end{vmatrix} = 0$

$\xrightarrow{R_2 - R_1} \begin{vmatrix} 1 & 2 & 1 \\ K-1 & 1 & 0 \\ 2 & K & 0 \end{vmatrix} = 0$

Taking determinant along Column 3, we get;

$K(K-1) - 2 = 0$

$K^2 - K - 2 = 0$

$(K-2)(K+1) = 0 \longrightarrow K = 2, -1$ for being linearly dependent.

(8) $G(x, y, z) = (2x + 3y - z, 4x - y + 2z)$

$S = \{(1,1,0), (1,2,3), (1,3,5)\}$

$S' = \{(1,2), (2,3)\}$

Here, $(a, b) = (-3a + 2b)u_1 + (2a - b)u_2$

$\Rightarrow f(w_1) = f(1,1,0) = (5, 3) = -9u_1 + 7u_2$

$f(w_2) = f(1,2,3) = (7,8) = -5u_1 + 6u_2$

$f(w_3) = f(1,3,5) = (6,11) = 4u_1 + u_2$

$$\Rightarrow A = \begin{bmatrix} -9 & -5 & 4 \\ 7 & 6 & 1 \end{bmatrix}$$

Here, it can be also solve by :-

$f(1,1,0) = (5,3) = (1,2)x + (2,3)y$

$\Rightarrow x + 2y = 5$

$2x + 3y = 3$

$y = 7$

& $x = -9$

$x = 9$, $y = 7$

$\Rightarrow f(1,1,0) = (5,3) = \underline{-9u_1 + 7u_2}$.

& $f(1,2,3) = (7,8) = (1,2)x + (2,3)y$

$x + 2y = 7$

$2x + 3y = 8$

$y = 6$

$x = -5$

$\Rightarrow f(1,2,3) = (7,8) = \underline{-5u_1 + 6u_2}$

& $f(1,3,5) = (6,11) = (1,2)x + (2,3)y$

$x + 2y = 6$

$2x + 3y = 11$

$y = 1$

$x = 4$ $\rightarrow$ $f(1,2,3) = \underline{4u_1 + u_2}$ $\rightarrow$ $A = \begin{bmatrix} -9 & -5 & 4 \\ 7 & 6 & 1 \end{bmatrix}$

(9) Given, $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$

To show : $A^2 - 10A + 16 I_3 = 0$

Here, $A^2 = A \cdot A$

$= \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} \cancel{16} \; 24 & 20 & 20 \\ 20 & 24 & 20 \\ 20 & 20 & 24 \end{bmatrix}$

$\Rightarrow 10A = 10 \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 40 & 20 & 20 \\ 20 & 40 & 20 \\ 20 & 20 & 40 \end{bmatrix}$

$\Rightarrow 16 I_3 = 16 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix}$

On putting the above value in the given eq^ns., we get :

$\Rightarrow \begin{bmatrix} 24 & 20 & 20 \\ 20 & 24 & 20 \\ 20 & 20 & 24 \end{bmatrix} - \begin{bmatrix} 40 & 20 & 20 \\ 20 & 40 & 20 \\ 20 & 20 & 40 \end{bmatrix} + \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Thus, $A^2 - 10A + 16 I_3 = 0$

proved.

Now,

$$A^2 - 10A + 16I_3 = 0$$

$$A^2 - 10A + 16 I_3 = 0$$

On premultiplying A^{-1}, we get:

$$\Rightarrow A^{-1}A^2 - 10A^{-1}A + 16A^{-1}I_3 = 0$$

$$\Rightarrow A - 10I_3 + 16A^{-1} = 0$$

$$\Rightarrow A^{-1} = \frac{1}{16}(10I_3 - A)$$

$$\Rightarrow A^{-1} = \frac{1}{16}\left(\begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}\right)$$

Hence, $A^{-1} = \frac{1}{16}\begin{bmatrix} 6 & -2 & -2 \\ -2 & 6 & -2 \\ -2 & -2 & 6 \end{bmatrix}$

Group - C

(12.) a) Given, $A = \begin{bmatrix} 2 & 4 \\ -1 & 6 \end{bmatrix}$

For characteristics polynomial; $\Delta(t) = |tI - A|$

$\Rightarrow \Delta(t) = \begin{bmatrix} t-2 & -4 \\ +1 & t-6 \end{bmatrix}$

$$\Rightarrow \Delta(t) = (t-2)(t-6) - (-4)$$
$$= t^2 - 8t + 12 + 4$$
$$= t^2 - 8t + 16$$
$$= (t-4)^2 = 0 \longrightarrow t = 4$$

So, the eigen value is $\lambda = 4$.

On subtrating $\lambda = 4$ down the diagonel, we get;

$\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \longrightarrow \left.\begin{matrix} 2x - 4y = 0 \ \& \\ x - 2y = 0 \end{matrix}\right\}$ or, $x - 2y = 0$

So, we have one free variable.

If $y = 1$, $x = 2$. $\longrightarrow (2, 1)$ is an eigen vector.

Here, we have repeated eigen values.

If $y = c$, $x = 2c$

$\Rightarrow$ vector, $\vec{v} = \begin{bmatrix} 2c \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} c$

If $y = 2$, $x = 4 \longrightarrow (4, 2)$ is also an eigen vetor.

So, $P = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

(12.) b) Here, Cayley Hamilton ~~eqn~~ Theorem.

$\Rightarrow |A - \lambda I| = 0$

$\Rightarrow$ ~~$|A|^2 - 8$~~

$\Rightarrow |A|^2 - 8|A| + 16 I_3$

$$\Rightarrow \begin{vmatrix} 2 & 4 \\ -1 & 6 \end{vmatrix}\begin{vmatrix} 2 & 4 \\ -1 & 6 \end{vmatrix} - 8\begin{vmatrix} 2 & 4 \\ -1 & 6 \end{vmatrix} + 16\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 4-4 & +8+24 \\ -2-6 & -4+36 \end{vmatrix} - \begin{vmatrix} 16 & 32 \\ -8 & 48 \end{vmatrix} + \begin{vmatrix} 16 & 0 \\ 0 & 16 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 32 \\ -8 & 32 \end{vmatrix} - \begin{vmatrix} 16 & 32 \\ -8 & 48 \end{vmatrix} + \begin{vmatrix} 16 & 0 \\ 0 & 16 \end{vmatrix}$$

$$= \begin{vmatrix} -16+16 & 32-32 \\ -8+8 & 32-48+16 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$\therefore$ it satisfies the Cayley Hamilton Theorem.

Now, eigen values ~~are~~ (calculated in part a) is $\lambda = 4$. (repeated eigen values).

$\Rightarrow$ V_1 and V_2 are same.

$\Rightarrow P = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ (If $y = 1, x = 2$)

Now, $|P| = 0$ so, it's inverse (P^{-1}) cannot be calculated.

Here, P is singular. P^{-1} does not exist.

Here, $D = P^{-1}AP = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

Here, P^{-1} does not exist and hence, the transformation $P^{-1}AP$ is not possible here. Thus, the value of $D = P^{-1}AP$ cannot be found.

Here, $P = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

(11) a) $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3,\ 2x_1 + x_2 + 2x_3,\ x_1 + 2x_2 + x_3)$

For linear mapping :-

let, $v = (a, b, c)$ & $w = (a', b', c')$

Then, i) $f(v+w) = f(a+a', b+b', c+c')$

$= (a+a'+b+b'+c+c',\ 2a+2a'+b+b'+2c+2c',\ a+a'+2b+2b'+c+c')$

$= (a+b+c + a'+b'+c',\ 2a+b+2c + 2a'+b'+2c',\ a+2b+c + a'+2b'+c')$

$= f(v) + f(w)$

Also, ii) $f(kv) = f(ka, kb, kc)$

$= (ka + kb + kc,\ 2ka + kb + 2kc,\ ka + 2kb + kc)$

$= k(a+b+c,\ 2a+b+2c,\ a+2b+c)$

$= k f(v)$.

Thus, this is a linear mapping.

Kernel T is given by set of x such that $T(x) = 0$.

$\Rightarrow (x_1 + x_2 + x_3,\ 2x_1 + x_2 + 2x_3,\ x_1 + 2x_2 + x_3) = (0, 0, 0)$

$\Rightarrow \left.\begin{array}{l} x_1 + x_2 + x_3 = 0 \\ 2x_1 + x_2 + 2x_3 = 0 \\ x_1 + 2x_2 + x_3 = 0 \end{array}\right\}$ or $\left.\begin{array}{l} x_1 + x_2 + x_3 = 0 \\ x_2 = 0 \\ 2x_2 = 0 \end{array}\right\}$ or $\begin{array}{l} x_1 + x_2 + x_3 = 0 \\ x_2 = 0 \end{array}$

Thus, here, x_1, x_2 are pivot elements &

x_3 is one free variable.

Thus, $\dim(\text{Ker } T) = \underline{1}$ ~~& |N(A)| = 1~~

& $\ker T = \begin{bmatrix} -c \\ 0 \\ c \end{bmatrix}$ (If $x_3 = c$)

$= \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} c.$

b) $(D^2+2D+5)y = xe^x$, $\left(D = \frac{d}{dx}\right)$

Here, the

$\Rightarrow m^2+2m+5=0$

$\Rightarrow m = \frac{-2 \pm \sqrt{4-20}}{2}$

$\Rightarrow m = -1 \pm 2i$

Thus, $C.f. = e^{-x}\left[C_1 \cos 2x + C_2 \sin 2x\right]$

For, PI :-

$$y = \frac{xe^x}{D^2+2D+5} = \frac{e^x x}{(D+1)^2+2(D+1)+5}$$

$$y = e^x \frac{x}{D^2+4D+8}$$

$$= \frac{e^x}{8}\frac{x}{\left(\frac{D^2+4D}{8}+1\right)} = \frac{e^x}{8}\left(\frac{D^2+4D}{8}+1\right)^{-1}x = \frac{e^x}{8}\left(1+\frac{D^2+4D}{8}\right)^{-1}x$$

$$= \frac{e^x}{8}\left[1-\left(\frac{D^2+4D}{8}\right)-\left(\frac{D^2+4D}{8}\right)^2 \cdots\right]x$$

$$= \frac{e^x}{8}\left[1-\frac{D^2}{8} - \frac{D}{2} - \frac{D^2}{4}\right]x$$

$$= \frac{e^x}{8}\left[x - \frac{3}{8}D^2 x - \frac{D}{2}x\right]$$

$$= \frac{e^x}{8}\left[x - \frac{1}{2} + 0\right]$$

$$= \frac{xe^x}{8} - \frac{e^x}{16}$$

Thus, the general sol^n = Cf + PI

$$= e^{-x}\left[C_1 \cos 2x + C_2 \sin 2x\right] + \frac{e^x}{8}\left[x - \frac{1}{2}\right]$$

c) $\frac{dy}{dx} + \frac{y\log y}{x} = \frac{y}{x^2}(\log y)^2$

$\Rightarrow \frac{1}{y(\log y)^2}\frac{dy}{dx} + \frac{1}{(\log y)x} = \frac{1}{x^2}$

$\Rightarrow$ let, $z = \frac{1}{\log y}$

then, $\frac{dz}{dy} = \frac{-1}{y(\log y)^2}$

$\Rightarrow$ Putting in above eqn, we get;

$\Rightarrow -\frac{dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$

$\Rightarrow \frac{dz}{dx} - \frac{z}{x} = \frac{-1}{x^2}$

$\Rightarrow IF = e^{\int -\frac{1}{x}dx} = e^{-\log x} = 1/x$

$\Rightarrow z \times \frac{1}{x} = \int \frac{1}{x}\left(\frac{-1}{x^2}\right)dx$

$\Rightarrow \frac{z}{x} = \int \frac{-1}{x^3}dx$

$\Rightarrow \frac{z}{x} = \frac{+1}{2x^2} + C$

$\Rightarrow \frac{1}{x\log y} = \frac{1}{2x^3} + C$

This is the required equation

(14) a) $\int_0^\infty \frac{\cos x}{\sqrt{1+x^2}} dx$

Applying μ - test.

$\Rightarrow \lim_{x\to\infty} \int_0^x x^{5/4} \frac{\cos x}{\sqrt{1+x^2}} dx = \lim_{x\to\infty} \int_0^x \frac{\cos x \, x^{5/4}}{x^2\sqrt{1+\frac{1}{x^2}}} dx \quad (\mu = 5/4)$

$\Rightarrow \lim_{x\to\infty} \int_0^x x^{5/4-2} \frac{\cos x}{\sqrt{1+\frac{1}{x^2}}} dx$

$\Rightarrow \lim_{x\to\infty} \int_0^x \frac{\cos x}{x^{3/4}\sqrt{1+\frac{1}{x^2}}} dx = 0 \quad (\because \mu = 5/4 > 1)$

$\Rightarrow$ Since, absolute converges. So, its oridnery convergence.

b) $\frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3}$

Let, $3x-4y = V \longrightarrow 3 - 4\frac{dy}{dx} = \frac{dV}{dx} \longrightarrow \frac{dy}{dx} = \left(3 - \frac{dV}{dx}\right)\frac{1}{4}$

$\Rightarrow \frac{dy}{dx} = \frac{V-2}{V-3}$

$\Rightarrow 3 - 4\frac{dy}{dx} = \frac{dV}{dx}$

$\Rightarrow 3 - 4\left(\frac{V-2}{V-3}\right) = \frac{dV}{dx}$

$\Rightarrow \frac{3(V-3) - 4(V-2)}{V-3} = \frac{dV}{dx}$

$\Rightarrow \frac{-V-1}{V-3} = \frac{dV}{dx}$

On integrating both sides :-

$\Rightarrow \int -dx = \int \left(\frac{V-3}{V+1}\right) dV$

$\Rightarrow -x = \int \left(1 - \frac{4}{V+1}\right) dV \longrightarrow -x = V - 4\ln|V+1| + C$

$\Rightarrow V - 4\ln|V+1| + x = C'$

$\Rightarrow \quad V - 4\ln|V+1| + x = c'$

$\Rightarrow \quad (3x-4y) - 4\ln|3x-4y+1| + x = c'$

c) $(x-y+3)dx = (2x-2y+5)dy$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{x-y+3}{2x-2y+5}$

Let, $V = x-y$

$\dfrac{dV}{dx} = 1 - \dfrac{dy}{dx}$

$\Rightarrow \quad \dfrac{dV}{dx} = 1 - \dfrac{V+3}{2V+5}$

$\Rightarrow \quad \dfrac{dV}{dx} = \dfrac{V+2}{2V+5}$

$\Rightarrow \quad \dfrac{2V+5}{V+2}\, dV = dx$

$\Rightarrow \quad \left(2 + \dfrac{1}{V+2}\right) dV = dx$

On integrating both sides, we get:

$\Rightarrow \quad 2V + \ln|V+2| = x + c$

$\Rightarrow \quad 2(x-y) + \ln|x-y+2| = x + c$