

CS2 RECITATION 5: DYNAMIC PROGRAMMING

1/13/2014

ASSIGNMENT 4

- 7 points of explanation
- 10 points of coding
- 3 “red” points (no coding)
- DNA alignment – Given two strings, find the optimal alignment between them
- Seam Carving - Given an array, find the lowest-weight path from top to bottom

USEFUL CLASSES: `STD::STRING`

- A string classes with helpful functions, similar to string in python
- `.substring(int start, int length)` : returns a copy of the specified substring, using -1 for length goes to end of string
- `.compare(string s)` : returns 0 if two strings are equal
- `.size()` returns the length of the string
- Can be concatenated with `+` and `+=`
- `string(int n, char c)` : constructor that creates a string of c repeated n times

USEFUL CLASSES: `STD::UNORDERED_MAP`

- Similar to a dictionary in python
- Stores values that can be accessed using keys
- Types must be declared
- Values can be accessed and modified using `map[key]`
- `unordered_map<string, align_result>` is aliased to `memo_type` for your convenience

DYNAMIC PROGRAMMING

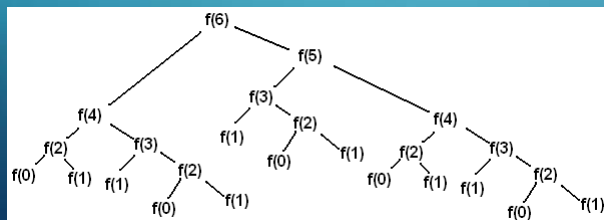
- Basically two things done together
 - Recursion
 - Memoization
- Trades space for time by saving the results of previous function calls
- Makes otherwise unfeasible algorithms possible

SIMPLE EXAMPLE: FIBONACCI SEQUENCE

Fibonacci(n):

If n is 0 or 1 return n

Otherwise return Fibonacci(n-1) + Fibonacci(n-2)



PROGRAMMING
 $O(2^N)$

SIMPLE EXAMPLE: FIBONACCI SEQUENCE

Map<int, int> calculated

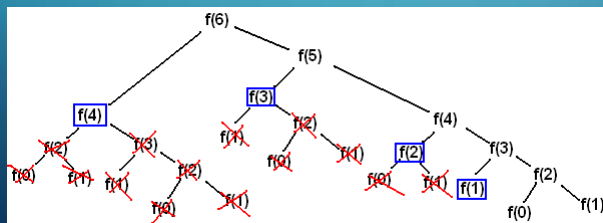
Fibonacci(n):

If n is in calculated return calculated[n]

If n is 0 or 1 return n

calculated[n] = Fibonacci(n-1) + Fibonacci(n-2) //memoize

Otherwise return Fibonacci(n-1) + Fibonacci(n-2)



DYNAMIC
PROGRAMMING
 $O(N)$

DNA ALIGNMENT

- Given two strings of DNA find the optimal way to align them
- We do this by inserting gaps
- For each position in the aligned result, 3 possibilities:
 - Top string character, bottom string gap
 - Top string gap, bottom string character
 - Both strings have a character
 - They can either match or not match
- Instruction string containing {s, t, |, *}

DNA ALIGNMENT

- Scoring: 2 points for a match, -1 points for a mismatch, -5 points for a gap

- Ex. ACTGGCCGT vs. TGACGTAA

S:	A	C	T	G	G	C	C	G	T
T:	T	G	A	C	G	T	A	A	_
Score:	-1	-1	-1	-1	2	-1	-1	-1	-5
Inst:	*	*	*	*		*	*	*	s

Total score: -10

ANOTHER EXAMPLE

abracadabra vs. avada kedavra

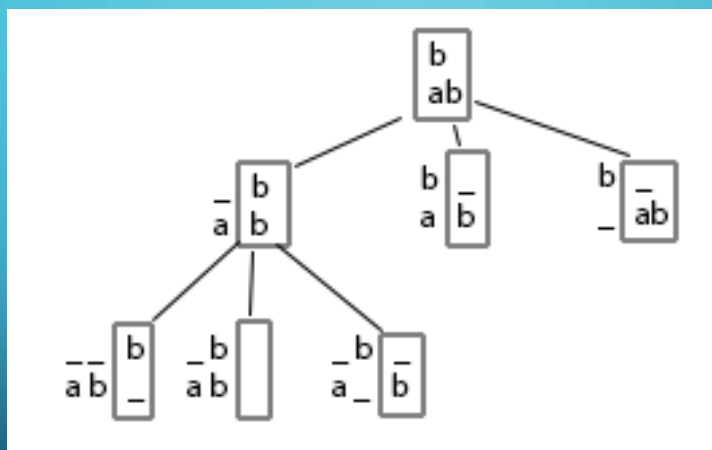
S:	a	b	r	_	a	c	a	_	d	a	b	r	a
T:	a	v	a	d	a	\s	k	e	d	a	v	r	a
Inst:		*	*	t		*	*	t			*		

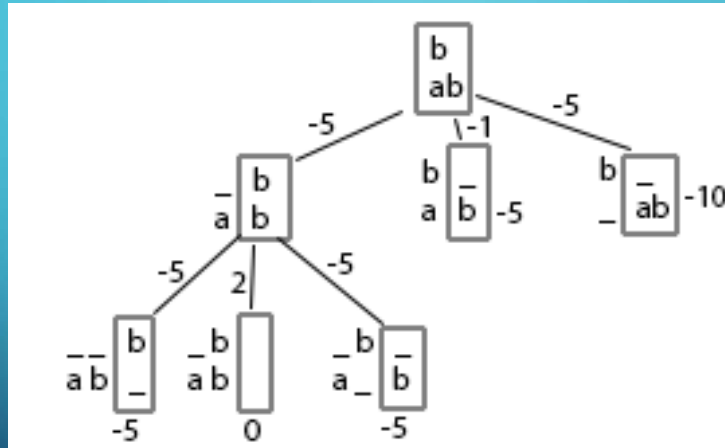
Total Score: -3



ALGORITHM?

- Reduce to a recursive problem
- Make a function call(s) to compare shorter strings
- For each function call, find scores recursively for:
 - A gap in the second string
 - A gap in the first string
 - No gaps
- Remove the first character of one or both strings
- Add the score for the first position and return the highest of 3 scores
- Base Case: aligning two strings, one is empty

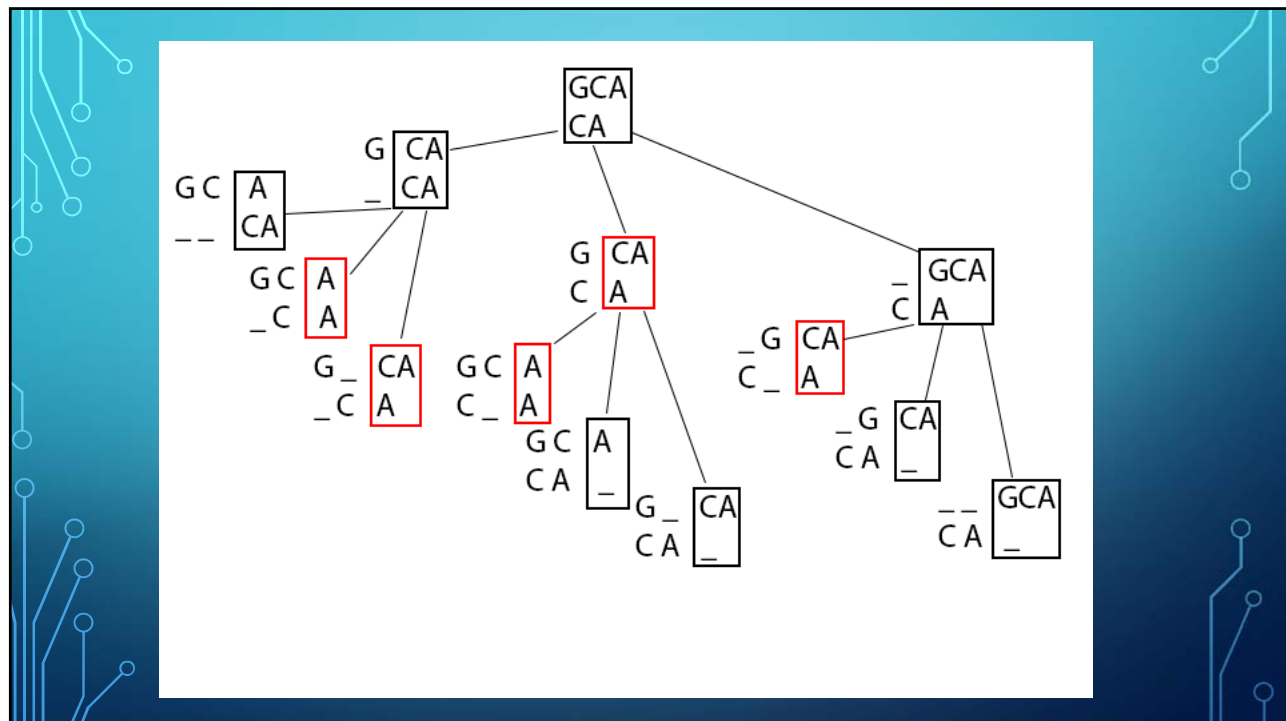
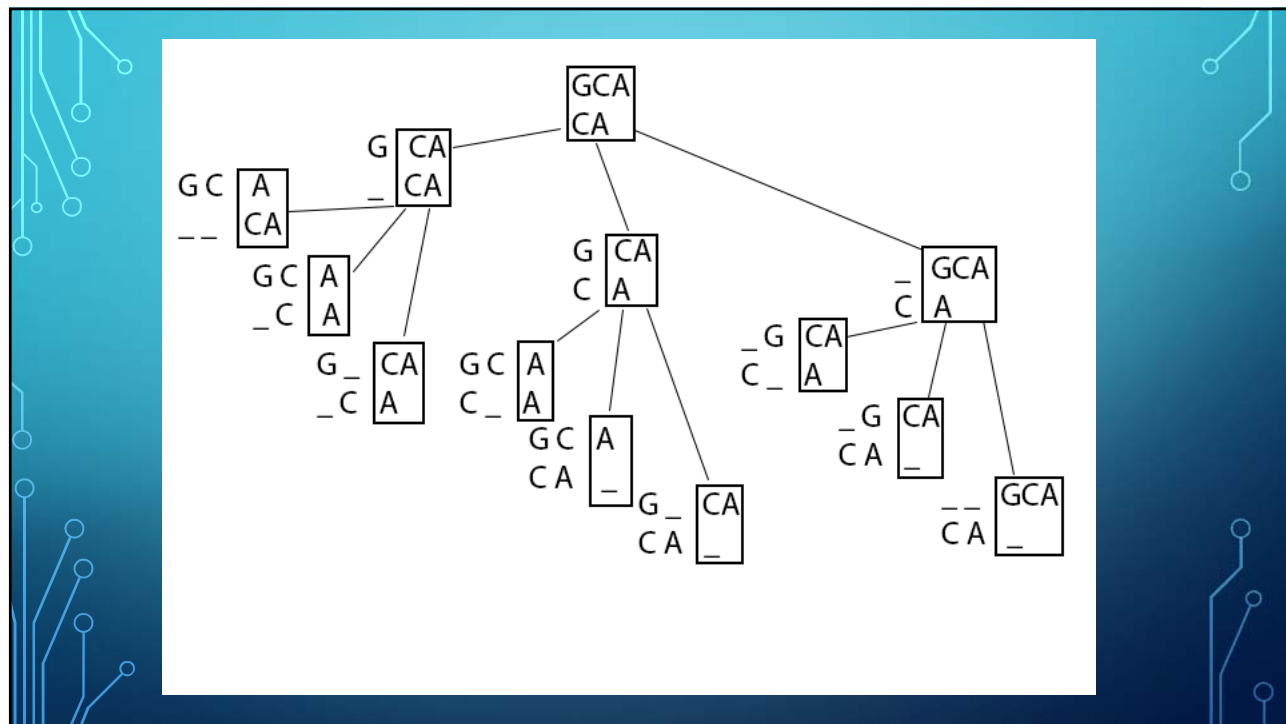




MEMOIZE

- Each function call will generate 3 function calls
- Time complexity at least $O(3^N)$ where N is len of compared result
- Gets VERY slow very quickly
- Solution?
- Dynamic Programming





MANY REDUNDANT FUNCTION CALLS

- Memoize!
- Store the results of each comparison in a map
- The key is string1 + "," + string2
- The value is a struct containing an int score and a string for alignment instructions

RESULTS

w/
Dynamic Programming

```
Calling DNA align on strings abracadabra, avada kedavra
abr aca dabra
avada kedavra
|**t|**t|*||
Score for this alignment: -3
Number of calls: 168
```

w/o
Dynamic Programming

```
Calling DNA align on strings abracadabra, avada kedavra
abr aca dabra
avada kedavra
|**t|**t|*||
Score for this alignment: -3
Number of calls: 336447346
```

MORE APPLICATIONS OF DYNAMIC PROGRAMMING



BAD



ALSO BAD



SEAM CARVING AKA CONTENT AWARE IMAGE RESIZING



SEAM CARVING ALGORITHM

- Find the paths of “lowest energy” aka the seams
- “Energy” refers to how noticeable each pixel in the seam is with respect to the rest of the image
- Continuously remove seams to shrink the image a pixel at a time
- Energy found using a saliency map
- For each pixel, assign a value based on how different it is from its neighbors
- Already done for you



MORE ALGORITHM

- Cost map represented as 2D array of unsigned ints
- Must find cheapest path from top to bottom
- Brute force?
- Complexity $O(W^L)$ where w and l are width and length of image

memoize



CREATING A COST TABLE

- Records the cheapest cost to each pixel
- First row is identical to first row of saliency map
- Build table row by row by calculating shortest path from above 3 pixels
- Each row memoizes calculations to use in the next row
- At the end, find lowest cost pixel in bottom row and backtrace steps

SALIENCY MAP

5	6	1	3
2	9	0	5
4	8	7	2

COST TABLE

5	6	1	3

SALIENCY MAP

5	6	1	3
2	9	0	5
4	8	7	2

COST TABLE

5	6	1	3
7			

SALIENCY MAP

5	6	1	3
2	9	0	5
4	8	7	2

COST TABLE

5	6	1	3
7	10		

SALIENCY MAP

5	6	1	3
2	9	0	5
4	8	7	2

COST TABLE

5	6	1	3
7	10	1	

SALIENCY MAP

5	6	1	3
2	9	0	5
4	8	7	2

COST TABLE

5	6	1	3
7	10	1	6

SALIENCY MAP

5	6	1	3
2	9	0	5
4	8	7	2

COST TABLE

5	6	1	3
7	10	1	6
11	9	8	3

SALIENCY MAP

5	6	1	3
2	9	0	5
4	8	7	2

COST TABLE

5	6	1	3
7	10	1	6
11	9	8	3

NEW IMAGE WITH SEAM REMOVED

5	6	3
2	9	5
4	8	7

5	6	3
7	12	8
11	15	15

NEW IMAGE WITH SEAM REMOVED

5	6	3
2	9	5
4	8	7

5	6	3
7	12	8
11	15	15

FINAL NOTES ON SEAM CARVING

- Dynamic programming algorithm is approx. $O(W*L)$
- Saliency map represents a 2D array but is stored as 1D array
- We provide a `get(array, x, y)` to automatically translate 2D bounds
- As image shrinks, iteration bounds should change but array size does not
- Return an array containing the x coordinate of seam pixel starting from top

GOOD LUCK; HAVE FUN

OFFICE HOURS ETC.