Inf620 2025 Lecture Linear, Polynomial and Logistic Regression

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Lesson Plan

- Class Material (click here for the Colab)
- **Today's Lesson**: Regression Techniques
  - Linear Regression
  - Polynomial Regression
  - Logistic Regression
  - Key differences and practical considerations

What is Regression?

- A supervised learning technique to model relationships between input features and output variables
- Used for prediction, trend analysis, and inference
- Types we'll study:
  - Linear Regression
  - Polynomial Regression
  - Logistic Regression

Linear Regression

- Models the relationship as: $y = wx + b$
- Objective: Minimize the Mean Squared Error (MSE)
- Solution via:
  - Analytical method (Normal Equation)
  - **Gradient Descent**
- Output is continuous (Regression)

Polynomial Regression

- Extension of linear regression with polynomial terms:
  $y = w_0 + w_1 x + w_2 x^2 + \cdots + w_d x^d$
- Captures non-linear relationships
- Still a linear model in terms of parameters
- Risk of overfitting with high-degree polynomials

Logistic Regression

- Used for binary classification problems
- Models probability: $P(y = 1|x) = \frac{1}{1+e^{-(\mathbf{wx}+\mathbf{b})}}$
- Output is between 0 and 1 (**Classification**)
- Uses Cross-Entropy (Log Loss) instead of MSE
- Decision boundary at $P = 0.5$

Comparison of Methods

- **Linear Regression:** Predicts continuous values, assumes linearity
- **Polynomial Regression:** Enhances linear regression with non-linearity
- **Logistic Regression:** Predicts class probabilities, used for classification
- Choose based on output type and data pattern

Practical Implementations

- Use `scikit-learn` for all three methods
- Explore in Colab notebook:
  - Linear fit using `LinearRegression()`
  - Polynomial fit using `PolynomialFeatures` + `LinearRegression`
  - Logistic fit using `LogisticRegression()`
- Evaluate using appropriate metrics:
  - MSE/R^2 for regression
  - Accuracy, Precision, Recall for classification

5 years ago.....

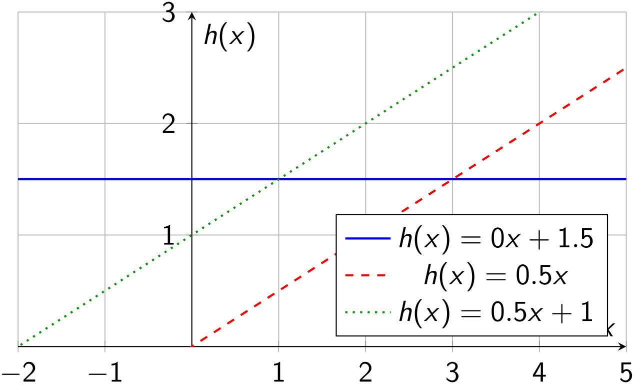
- **Top 10 Machine Learning Models for Data Science**
  1. Linear Regression – the go-to model for prediction and interpretation
  2. Logistic Regression – baseline for binary classification problems
  3. Decision Tree – easy to interpret and fast to train
  4. Random Forest – robust ensemble method improving accuracy
  5. Support Vector Machines (SVM) – powerful for classification with clear margins
  6. Gradient Boosting Machines (GBM) – competitive in Kaggle competitions
  7. XGBoost – efficient and accurate boosting method widely adopted
  8. k-Nearest Neighbors (KNN) – intuitive, lazy learner for classification/regression
  9. Clustering (e.g., K-Means) – unsupervised learning to find natural groupings
  10. Principal Component Analysis (PCA) – key tool for dimensionality reduction

5 years ago.....

- **Top 10 Deep Learning Models for Data Science**
  1. Recurrent Neural Networks (RNN) – for time series and sequence data
  2. Convolutional Neural Networks (CNN) – best for image processing
  3. Long Short-Term Memory (LSTM) – solving RNN's vanishing gradient issues
  4. Generative Adversarial Networks (GANs) – realistic image synthesis
  5. AutoEncoders – unsupervised feature learning and anomaly detection
  6. BERT – transformer-based NLP model from Google
  7. GPT-3 – large-scale generative language model revolutionizing NLP

Visualizing $w \cdot x + b$

- Click here - Lecture 3. Linear Regression - Prof. Lucas Nascimento
- Example linear functions in hypothesis space:
  - $h(x) = 0x + 1.5$: horizontal line
  - $h(x) = 0.5x + 0$: passes through origin
  - $h(x) = 0.5x + 1$: same slope, different intercept



Loss Function: Mean Squared Error (MSE)

- To evaluate how well our linear model fits the data, we use a loss function.
- For linear regression, the most common choice is the Mean Squared Error (MSE):

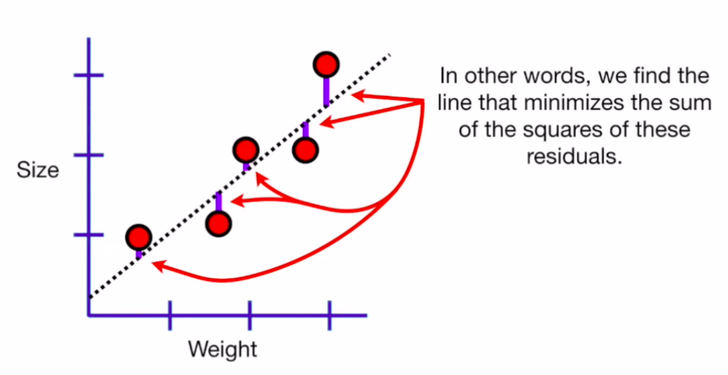
$$L(w, b) = \frac{1}{2m} \sum_{i=1}^{m} (h(x^{(i)}) - y^{(i)})^2$$

- m: number of training examples, $h(x^{(i)})$: predicted value for i
- $y^{(i)}$: true value for example i

Goal

Minimize $L(w, b)$ to make predictions as close as possible to the actual values.

Logistic Regression Statquest - LINEAR REGRESSION



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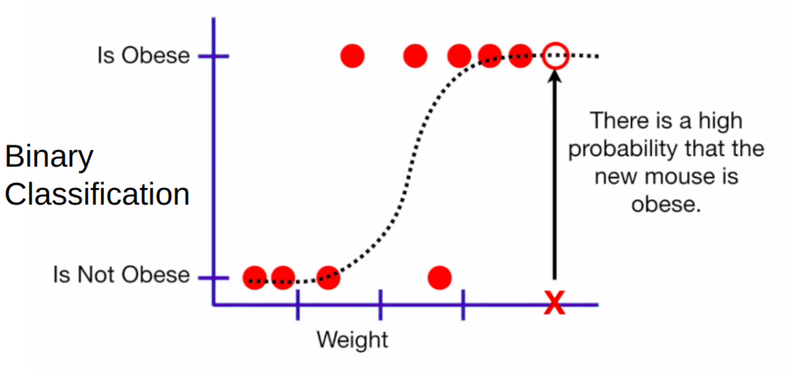
Logistic Regression Statquest



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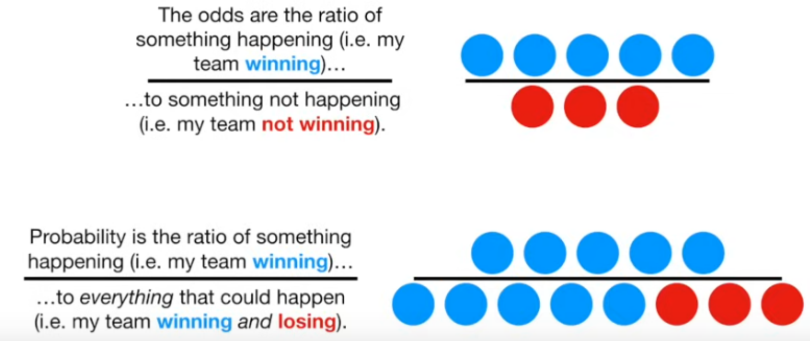

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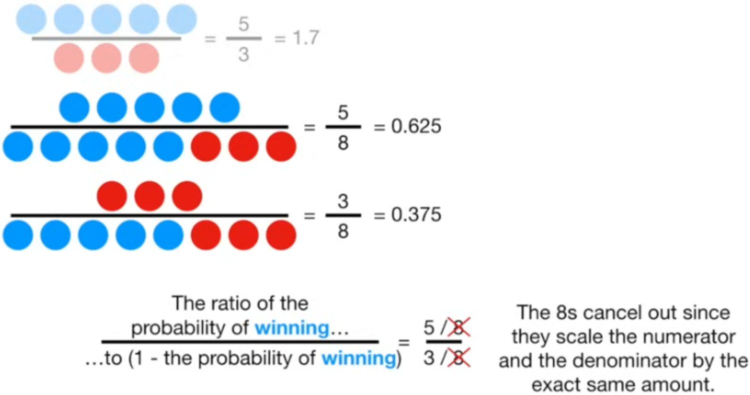

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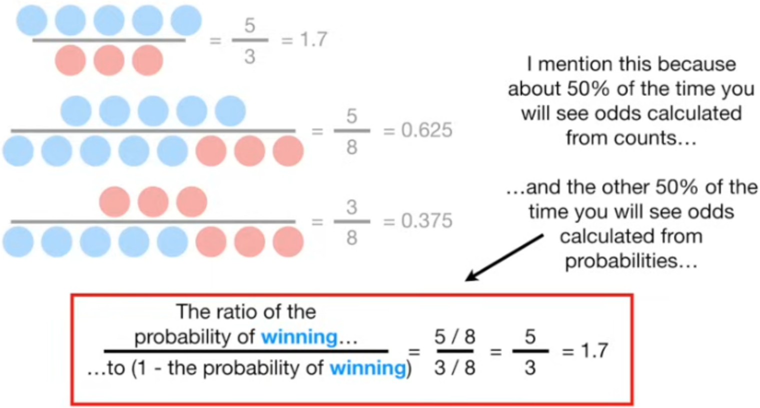

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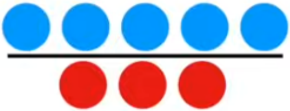
In summary, the odds are just...

...the ratio of something happening (i.e. my team **winning**)...

...to something not happening (i.e. my team **not winning**).



And the log(odds) is just the log of the odds.
It's no big deal!

The log(odds) makes things symmetrical, easier to interpret and easier for fancy statistics.

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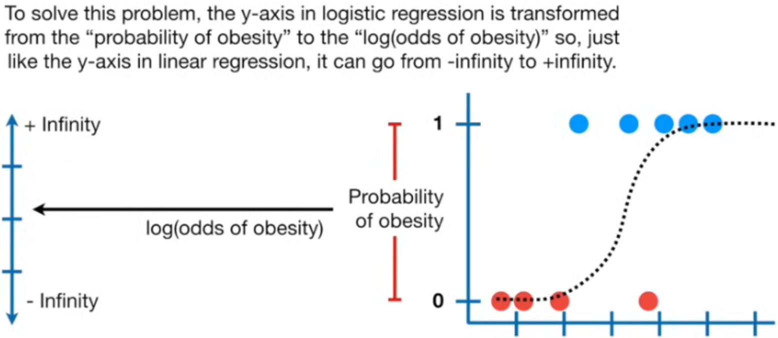

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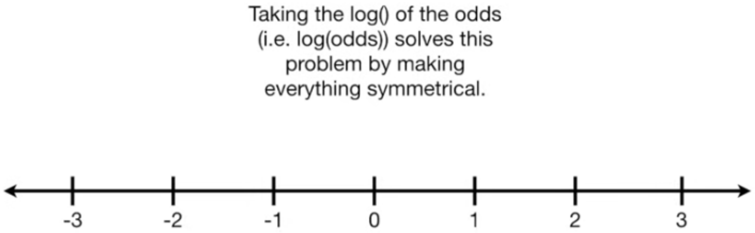

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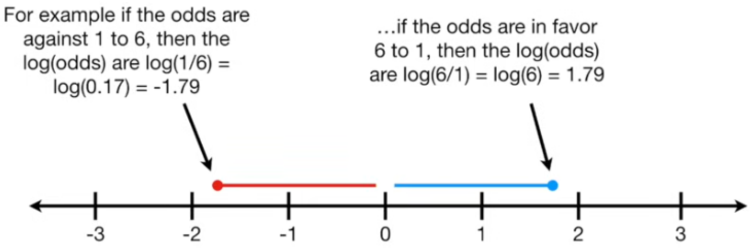

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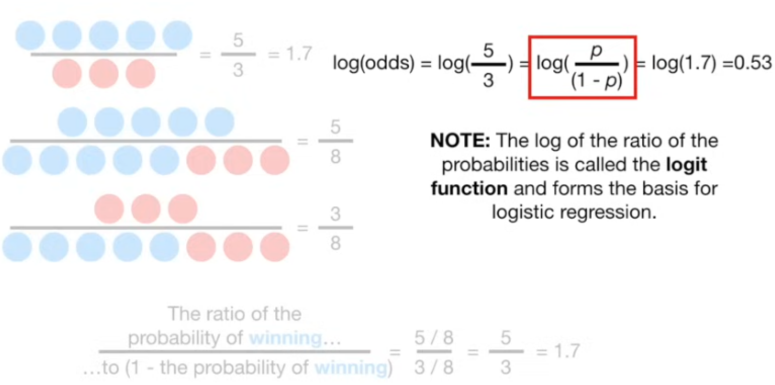

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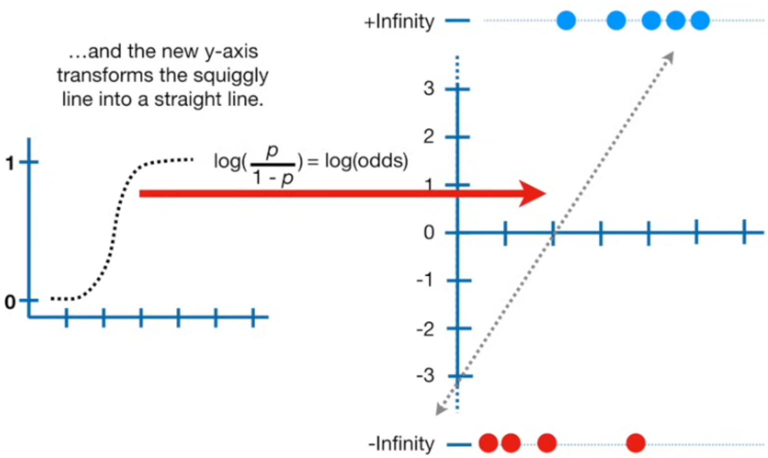

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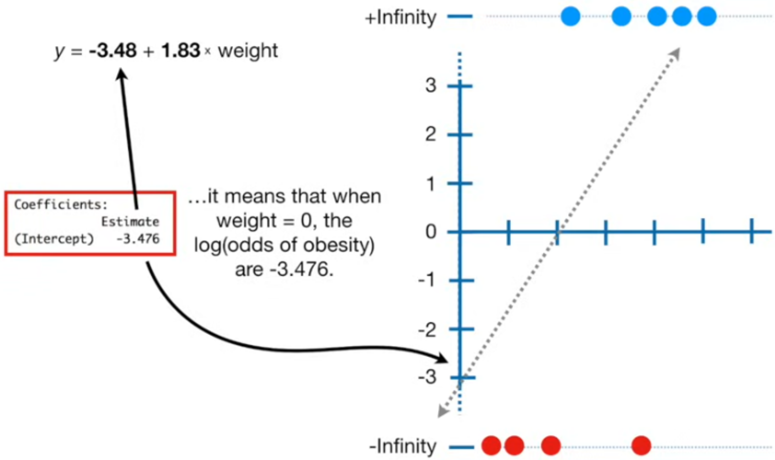

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