Topics to discuss

Loops
Nested loops
Consecutive statements
If - else statements
Logarithm.

1) Loops :

a) for (i=1 ; i ≤ n ; i++) { —— n+1

a = a+2 ; —— n

}

Time complexity = $n+1+n$

$= 2n+1$

$f(n) = 2n+1$

$f(n) = O(n)$

b) for (i=1 ; i ≤ n ; i = i+2) — $\left(\frac{n}{2}+1\right)$ times

{

a = a+2; — $\frac{n}{2}$ times

}

Time complexity $= \frac{n}{2} + 1 + \frac{n}{2}$

$f(n) = n+1$

$$\boxed{f(n) = O(n)}$$

(c) $a = 0$

```
for (i=1 ; a<=n ; i++)
{
    a = a+i;
}
```

i	a
1	1
2	$1+2=3$
3	$1+2+3=6$
⋮	⋮
K	$1+2+3+\dots K$

Terminating condition,

$$a > n$$

$$1+2+3+\dots K > n$$

$$\frac{K(K+1)}{2} > n$$

$$\frac{K^2+K}{2} > n$$

$$\Rightarrow K^2 > n$$

$$\boxed{K > \sqrt{n}}$$

$$\boxed{T.C = O(\sqrt{n})}$$

(d) for (i=1 ; i*i ≤ n ; i++)

```
{
    a = a+2;
}
```

Terminating condition,

$$i^2 > n$$

$$i > \sqrt{n}$$

Time Complexity $= O(\sqrt{n})$

2. Nested Loops :

a)
```
for (i=1 ; i<=n ; i++) {        ———— n+1
    for (j=1 ; j<=n ; j++) {    ———— n × n+1
        a = a+2;                ———— n × n
    }
}
```

Time complexity $= n+1 + n^2 + n + n^2$

$$f(n) = 2n^2 + 2n + 1$$

$$f(n) = O(n^2)$$

b)
```
for (i=1 ; i<=n ; i++){                 ——— n+1
    for (j=1 ; j<=n ; j++) {            ——— n × n+1
        for (K=1 ; K<=n ; K++){         ——— n × n × n+1
            statement ;                 ——— n × n × n
        }
    }
}
```

Time Complexity $= O(n^3)$

(c)

```
for ( i=1 ; i<=n ; i++)
{
    for ( j=1 ; j<=i ; j++)
    {
        statement;
    }
}
```

$$f(n) = \frac{n(n+1)}{2}$$

$$f(n) = \frac{n^2+n}{2}$$

$$f(n) = O(n^2) \checkmark$$

i	j	iteration
1	1 ✓ 2 ✗	1
2	1 ✓ 2 ✓ 3 ✗	2
3	1 ✓ 2 ✓ 3 ✓ 4 ✗	3
⋮		
n	1, 2, 3 ..... n ✓ n+1 ✗	$1+2+3 \dots + n$ $\frac{n(n+1)}{2}$

3) Consecutive statements :

a)
```
for (i=1 ; i<=n ; i++)      ———— n+1
{
    a=a+2;                  ———— n
}
for (j=1 ; j<=n ; j++)      ———— n+1
{
    b=b+2;                  ———— n
}
```

Total time $= n+1+n+n+1+n$

$$f(n) = 4n+2$$

$$\boxed{f(n) = O(n)}$$

④ If-else statement :

```
if ( a==0)
{
    return False;
}
else
{
    for (i=1 ; i<=n ; i++)
    {
        statement;
        return True;
    }
}
```

Best case :

Time Complexity = $\Omega(1)$

Worst case :

Time Complexity = $O(n)$

⑤ Logarithmic complexity :

ⓐ
```
for (i=1 ; i<=n ; i=i*2)
{
    a=a+2;
}
```

i	
1	$= 2^0$
1×2	$= 2^1$
$2^1\times 2$	$= 2^2$
$2^2\times 2$	$= 2^3$
$2^3\times 2$	$= 2^4$
$\vdots$	$\vdots$
	2^K

Terminating Condition,

$$i > n$$

$$2^K > n$$

$$\log 2^K > \log n$$

$$K \log_2 2 > \log_2 n$$

$$K > \log_2 n$$

$$\boxed{T.C = O(\log n)}$$

(b) `for (i = n ; i >= 1 ; i = i/2)`
```
{
    a = a+2;
    statement;
}
```

i
n
$\frac{n}{2}$
$\frac{n}{2^2}$
$\frac{n}{2^3}$
$\vdots$
$\frac{n}{2^k}$

Terminating condition,

$$i < 1$$

$$\frac{n}{2^k} < 1$$

$$n < 2^k$$

$$\log n < k \log_2 2$$

$$\log n < k$$

$$k > \log n$$

$$\boxed{T.C = O(\log n)}$$

Commonly Used Rate of Growth

Time Complexity	Name
1	Constant
$\log n$	Logarithmic
n	Linear
$n \log n$	Linear Logarithmic
n^2	Quadratic
n^3	Cubic
2^n	Exponential
$n!$	Factorial

Commonly Used Logarithms and Summations

$$\rightarrow \log x^{y} = y \log x$$

$$\rightarrow \log xy = \log x + \log y$$

$$\rightarrow a^{\log_b x} = x^{\log_b a}$$

$$\rightarrow \log^{k} n = (\log n)^{k}$$

$$\rightarrow \log \frac{x}{y} = \log x - \log y$$

$$\rightarrow \log_b x = \frac{\log_a x}{\log_a b}$$

Arithmetic series :

$$\sum_{K=1}^{n} K = 1+2+3+\dots\dots+n = \frac{n(n+1)}{2}$$

Geometric series :

$$\sum_{K=0}^{n} x^{K} = 1+x+x^{2}+\dots\dots+x^{n} = \frac{x^{n+1}-1}{x-1} \quad (x \neq 1)$$

Harmonic series :

$$\sum_{K=1}^{n} \frac{1}{K} = 1+\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{n} \approx \log n$$

$$\rightarrow \sum_{K=1}^{n} \log K \approx n \log n$$

$$\rightarrow \sum_{K=1}^{n} K^{p} = 1^{p} + 2^{p} + \cdots + n^{p} \approx \frac{1}{p+1} n^{p+1}$$

Follow Now

Start Practicing

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