Dot Product of Two Vectors

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Many mathematical operations are usable on vectors. In this article, we will take a look at the dot product of two vectors.

Let's understand first that vectors can be multiplied by two methods:

- scalar product of vectors or dot product
- vector product of vectors or cross product

The main differentiation between these two methods is the fact that we get a scalar value as resultant through the first method and the resultant obtained is also a vector in nature by using the second technique.

Note that in a two-dimensional Cartesian plane, vectors are denoted in terms of the x -coordinates and y -coordinates of their end points, assuming they start at the origin (x , y) = (0,0).

When it comes to polar coordinates, vectors can be denoted in terms of length (magnitude) and direction. But when they are written in this format, the dot product of two vectors is equal to the product of their lengths, multiplied by the cosine of the angle between them.

Geometric interpretation. **Dot product** of two vectors $\bar{a}$ and b can be defined as a scalar quantity that is equal to the product of magnitudes of vectors multiplied by the cosine of the angle between vectors:

$\bar{a} \cdot b = |\bar{a}| \cdot |b| \cos \alpha$

Algebraic interpretation. **Dot product** of two vectors ā and b is basically a scalar quantity that is equal to the sum of pair-wise products of coordinate vectors a and b.

Dot product is also known as *scalar product* or *inner product*.

Dot Product – Formulae

Dot product formula for plane problems

When it comes to the plane problem, the dot product of vectors $\bar{a} = \{a_x ; a_y\}$ and $b = \{b_x ; b_y\}$ can be determined with the help of the following formula:

$\bar{a} \cdot b = a_x \cdot b_x + a_y \cdot b_y$

Dot product formula for spatial problems

The dot product of vectors $\bar{a} = \{a_x ; a_y ; a_z\}$ and $b = \{b_x ; b_y ; b_z\}$ can be figured out by using the following formula for the spatial problem:

$\bar{a} \cdot b = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$

Dot product formula for n dimensional space problems

Talking about the n dimensional space problem, the dot product of vectors $\bar{a} = \{a_1 ; a_2 ; \dots ; a_n\}$ and $b = \{b_1 ; b_2 ; \dots ; b_n\}$ can be easily determined with the following formula:

$\bar{a} \cdot b = a_1 \cdot b_1 + a_2 \cdot b_2 + \dots + a_n \cdot b_n$

Properties of Dot Product of Vector

1. The dot product is always more than zero or equal to zero when it comes to a vector with itself:

$\bar{a} \cdot \bar{a} \geq 0$

2. The dot product of a vector with itself is zero only if the vector is the zero vector:

$\bar{a} \cdot \bar{a} = 0 \Leftrightarrow \bar{a} = 0$

3. The dot product of a vector with itself is equal to the square of its magnitude:

$\bar{a} \cdot \bar{a} = |\bar{a}|^2$

4. The dot product operation is communicative:

$\bar{a} \cdot b = b \cdot \bar{a}$

5. If the dot product of two not zero vectors is zero, then note that these vectors are orthogonal:

$\bar{a} \neq 0, b \neq 0, \bar{a} \cdot b = 0 \Leftrightarrow \bar{a} \perp$

6. $(\alpha \bar{a}) \cdot b = \alpha(\bar{a} \cdot b)$

7. The dot product operation is distributive:

(ā + b) · c = ā · c + · c

Solved Examples

***Example 1:* Let's assume there are two vectors [6,2,-1] and [5,-8,2]. Now, determine the dot product of the vectors.**

Solution: Given vectors: [6,2,-1] and [5,-8,2] be a and b respectively.

ā. b =(6)(5)+(2)(-8)+(-1)(2)

ā. b =30-16-2

ā. b =12

Example 2: Find the dot product of the two vectors |a|=4 and |b|=2 and Ɵ=60.

Solution: a.b=|a||b|cos 60

a.b=4.2cos60

a.b=4

Example 3. What is the dot product of vectors ā = {1; 2} and b = {4; 8}?

Solution: ā · b = 1 · 4 + 2 · 8 = 4 + 16 = 20.

Example 4. What is the dot product of vectors a and b, if their magnitudes is |a| = 3, |b| = 6, and the angle between the vectors is equal to 60˚.

Solution: ā · b = | ā| · | b | cos α = 3 · 6 · cos 60˚ = 9.

Example 5. If their magnitudes is |a| = 3, |b| = 2, and the angle between the vectors a and b is equal to 60˚, what is the dot product of vectors p = a + 3b and q = 5a – 3 b?

Solution:

p . q = (ā + 3b) · (5 ā – 3b) = 5 ā · ā – 3 ā · b + 15 b · ā – 9 b · b

= 5 | ā|^2 + 12 ā · b – 9 |b |^2 = 5 · 3^2 + 12 · 3 · 2 · cos 60˚ – 9 · 2^2 = 45 +36 -36 = 45.

Example of the dot product of vectors for spatial problems

Example 6. Find the dot product of vectors a = {1; 2; -5} and b = {4; 8; 1}.

Solution: ā · b = 1 · 4 + 2 · 8 + (-5) · 1 = 4 + 16 – 5 = 15.

Example of the dot product of vectors for n dimensional space problems

Example 7. What is the dot product of vectors a = {1; 2; -5; 2} and b = {4; 8; 1; -2}?

Solution: $\bar{a} \cdot b = 1 \cdot 4 + 2 \cdot 8 + (-5) \cdot 1 + 2 \cdot (-2) = 4 + 16 - 5 - 4 = 11$.

Hope you are now thorough with the concept of the dot product of two vectors. To know more about vectors, visit our website Toppr.

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Aridio Silva

I think is missing a graphical visualization of the dot product in both 2-D (plane problems) and 3 dimensions (spatial problems).

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