PS4: RC Circuits

Thursday, March 4, 2021 4:30 PM

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Problem set: RC circuits as signal filters—the Bode plot

See attached worksheet, pp. 11-13

Goal: Build and characterize the filtering behavior of two different filter circuits at a range of frequencies.

Learning objectives

- Convert frequency (*f*, cycles/s or Hz) to angular frequency (ω, 1/s or, equivalently, radians/s);
- Compute the cutoff frequency of an RC circuit in Hz;
- Verify that V_{out}/V_{in} is -3dB at the cutoff frequency;.
- Create and Interpret a Bode plot for a filter circuit.

Why would we want to filter signals?

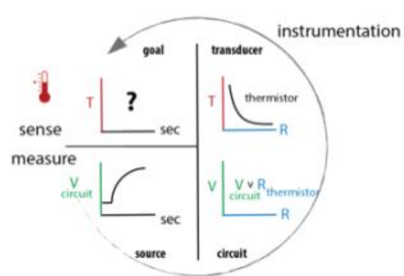
Imagine that you were given a can of mixed red, blue and yellow paint, but you wanted to take out the yellow paint. In the same way, raw sensor signals contain voltage signals that we want and a whole lot more! *Filters* help us to get rid of parts of the raw signal. For example, let's say you were listening to a music piece performed by a piccolo (high pitch, or frequency) and a trombone (low pitch), but you only liked the piccolo part. A filter could isolate and block out the trombone part. To help us, we introduce two tools--the *Fast Fourier Transform*, or **FFT**. and the Bode Plot.

The Upside-down and the FFT Concept

Sketch of a tight-rope walker and a flea, illustrating the premise of an upside-down universe. Source: Stranger Things, "The Upside-Down," Hulu.

Have you ever wondered whether we are living in a multi-dimensional universe with multiple "domains," that perhaps there is an "upside-down" version of our "right-side" up world? Well, wonder no more, because you have been working across the domains of our multi-verse in this very class!

When you measured temperature, you measured it in dimensions of Volts. The electric potential domain (dimension: Volts) is coupled to the thermal domain (dimension: °C) through the instrumentation. Thermal changes show up as changes in voltage and vice versa; you cannot change one without changing the other because it is part of a *whole, interconnected universe!!*

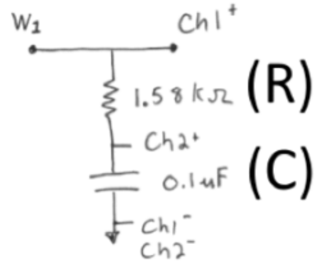


In the same way, there is an upside-down domain to the signals we measure in time.

If the dimensions in a domain are seconds (s), what would be the dimensions of a domain that is its upside, down? 1/s = Hz

We will now build two filtering circuits and test how they each respond to input voltages.

1. Build the **RC** circuit shown; it will *filter* V_{in} (=W1).



2. Connect the O-scope to your circuit.

3. Set Wave generation for 1 V sine wave at the cut-off frequency, centered at 2.5V,

Your circuit has a natural response ("cutoff") when 1=ωRC, or RC= τ ("tau")= $\frac{1}{2\pi f}$. $\tau = RC = 1.58\times10^3 \cdot 1\times10^{-7}$ $= 1.58\times10^{-4}$

$\frac{1}{2\pi f} = \tau \Rightarrow 1 = 2\pi f\tau \Rightarrow f = \frac{1}{2\pi\tau} \longrightarrow f = 1007\,Hz$

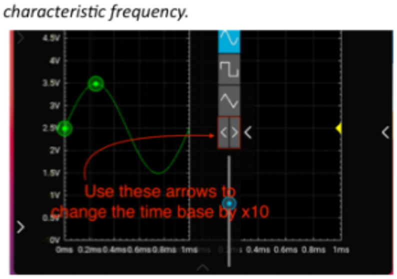
Note: RC is in units of seconds (1 Ω Farad = second). *The units of f are* cycles/second. *And the units of ω (angular frequency) are* radians/sec. *Recall that there are 2π radians per 360 degrees.*

Choose a signal input frequency, $f = \frac{1}{2\pi\tau}$; 2 significant figures are sufficient.

4. ▶ Run the scope

5. Pilot test the circuit: Change the input frequency and notice the response

Adjust the frequency of the sine wave to be ~10x lower- and ~10x higher- than $f = \frac{1}{2\pi RC}$, which is the RC filter's *characteristic frequency*.



Observe how the filter behaves qualitatively below its *characteristic f* ("low" *f*) and above *f* ("high" *f*).
You may need to adjust the sampling rate on the scope.

Notice the amplitude of the output as well as the shift in time, Δt, between V_{in} (Ch1+) and the V_{out}(Ch2+).

If your circuit is working as expected, it will allow some frequencies to pass through to V_{out} and it will block others. Check with someone else who has a functioning filter. Are your filters responding similarly?

Record your observations on the worksheet (pp. 11-13).

6. Measure and quantify how this RC circuit *responds* to different frequency inputs at 10Hz and 1 kHz

We will model the input to the RC circuit as a sine wave: $V_{in} = V_{in}\sin(\omega t)$, where ω is the angular frequency in radians/second (also "rads/sec"). The **sine** function takes an argument in units of *radians (which is unitless).*

*How do you convert cycles/second (Hz) to radians/second (ω) for the **sine** function?*

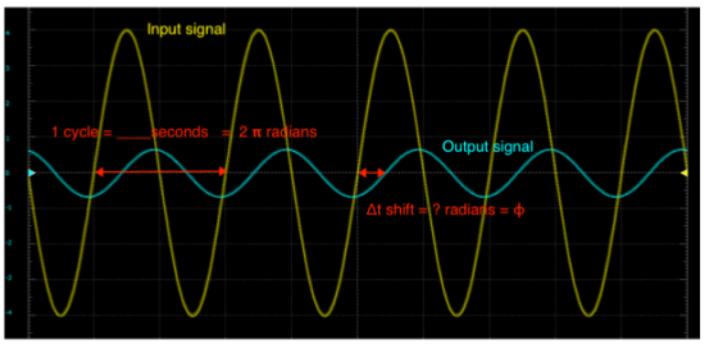
[Hint: This animation illustrates Hz ➡ω: How many **π** radians are in a complete, 360° cycle?]

The output is expected to be $V_{out} = A \cdot V_{in}\sin(\omega t + \varphi)$. From the book, we know that output amplitude, A, will be:

$$A = \frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1+(RC\omega)^2}} = \text{voltage gain}$$

And the **angular shift in phase** is:

$$\varphi = atan(-RC\omega)$$

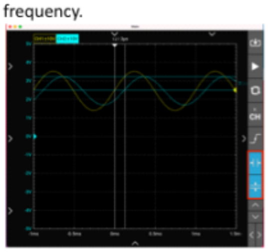


In Waveforms, the data displays in the *time* domain (seconds).

You can compute ϕ by measuring the time shift, Δt:

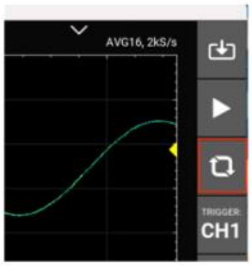
Note, the y-axis in this image is not accurate for this lab.

Use these tools to measure Δt and the amplitude of V_{out} on a single sweep for a 1 V sin wave, V_{in}, at the cutoff frequency.



A = 774 mV
Δt = 120 μS
} 1 KHz

Use the single sweep function of the scope to compute the phase shift at the *characteristic frequency* of this RC filter, 1 kHz, compared to 10 Hz.



$$\left.\begin{array}{l} A = 131\,mV \\ \Delta t = 21.57\,\mu S \end{array}\right\} 10Hz$$

Record your observations and measurements on the worksheet (last page).

7. Analyze the circuit behavior with a Bode plot

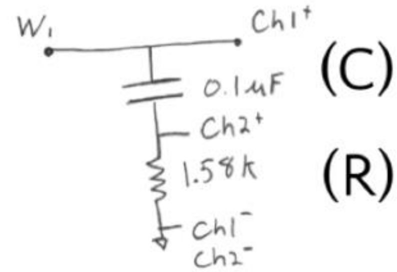
The moment you've been waiting for…the Bode plot will assess the performance of your filter for a range of input frequencies in just a few seconds. There is no need to change your circuit or scope set up from the last step—it will use the same channels.
Use the pull out arrow to reveal the Bode plot* function.

**Bode plot*: Each Bode (pronounce *boe – dee*) plot is really 2 plots – a plot of Gain (A) v. frequency, and a plot of phase angle (ϕ) v. frequency.

Capture the image of the Bode Plot results from for later reference.
Record your observations in the worksheet.

Remember the experimental output will be in dB. See the book for converting to (or from) dB.

8. Build and test a CR circuit by hand (step 6.) and with a Bode plot(step 7)



The V_{out} in this CR circuit measures the voltage dropped over the 1.58kΩ resistor. How is this different from what V_{out} measures in the RC circuit?

The expected results for amplitude and phase are;

$$A = \frac{RC\omega}{\sqrt{1+(RC\omega)^2}}$$

$$\varphi = atan\left(\frac{1}{RC\omega}\right)$$

10. Generate a Bode plot for the CR circuit

Recreate a Bode plot like you did in step 7 with the RC circuit.

Capture the image of the Bode Plot results.

Record your observations in the worksheet.

Problem set: RC circuits as signal filters—the Bode plot

Worksheet

5. Pilot test the circuit. What do you notice about the input and output signals as you change the wave generation frequency from 0.1 *f* to 10 *f* (*f is the cutoff frequency in cycles/second for this circuit)?*

10f: Vout is basically flat, extrema are @ Vin zeros
$\frac{1}{10}$f: Vout is basically identical to Vin, w/ a very slight delay
1f: Vout starting to flatten, peaks are significantly behind (chronologically) Vin.

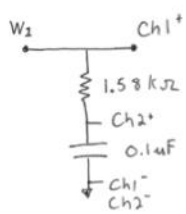
The Bode plot uses a *relative* amplitude of the voltages using the decibel unit (dB_V),

$$dB_V = 20 \cdot log_{10}\left(\frac{V}{V_{ref}}\right) \quad \sim 2.75V$$

For our case, the 1V ("Carrier" signal amplitude) is the V_{ref}. Measure V relative to 2.5V.

Determine the dB_V from the measured signal amplitude at the cutoff frequency? *This should be -3 dB. What is the % error?* It was -2.75db, so ~8% error

Compute the phase shift in degrees at the cutoff frequency for your circuit. *This should be 45º. What is the % error?* 43°, so ~4% error



For the RC circuit, describe what you see happening to the signal amplitude when the input goes from low frequency (~10Hz) to high frequency (100 kHz). The same phase shift start relatively constant, then start sharply decreasing @ the characteristic freq. Phase shift bottoms out & has a lot of noise starting @ ~5KHz characteristic freqs, 1kHz

$\phi = atan(-RC\omega)$

Compute the angular phase shift at the *characteristic frequency,* using the equation, ~~$\varphi = (-RC\omega)$~~

$-8.98°$

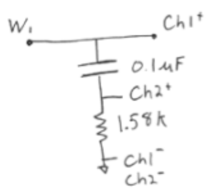
This phase shift should be the same for all filters at the characteristic frequency.

What does the Bode plot tell you about the RC filter's response to low($f < \frac{1}{2\pi\tau}$)and high($f > \frac{1}{2\pi\tau}$) frequencies? (Is this the same as what you've observed?)

It lets lower frequencies thru but not higher ones. Yes, 10Hz was unmodified compared to 1kHz

This signal *attenuation/decade* is an important performance metric for filters.

What does the Bode plot tell you about the performance of the CR circuit at low and high frequencies?

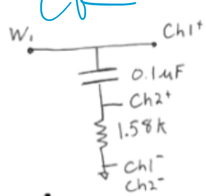


It's mirrored – lets high freqs thru but not low

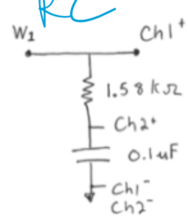
Like the RC filter, this CR filter is a 1[st]-order roll-off filter.

Let's name these circuits! Match the names below with a good name for each circuit:

CR



RC



Page 7/8

Naming