PS8: Complex Numbers

Thursday, April 22, 2021 4:07 PM

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Problem Set 8: Complex Numbers

<u>Goal</u>: Become familiar with math operations using complex numbers; see how complex numbers can be used to show the frequency response of an RC circuit.

Note: This PSet will be much easier if you have already watched the lectures on complex numbers.

Deliverable: This worksheet and two plots.

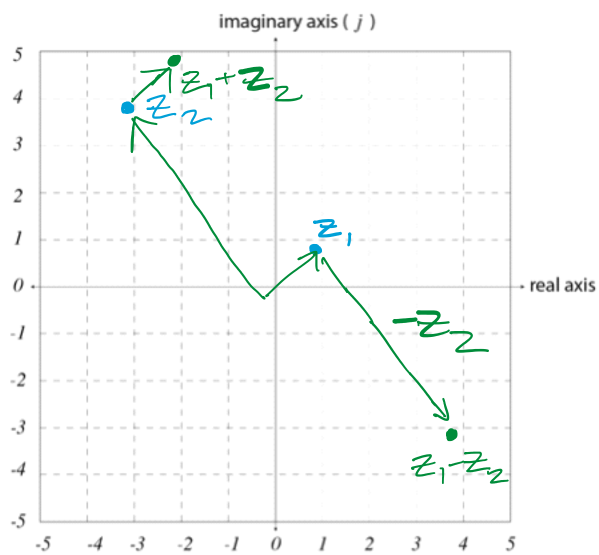
Part I: Basic Operations with complex numbers

For the following, take $z_1 = 1 + j$ and $z_2 = -3 + 4j$.

1. Convert z_1 and z_2 to polar and exponential notation (find r, θ).

$z_1 = \langle 1.41, 45^\circ \rangle = 1.41e^{j45}$ $z_2 = \langle 5, 126.87^\circ \rangle = 5e^{j126.87}$

2. Plot z_1 and z_2 on the complex plane below.



3. Compute $z_1 + z_2$. Show $z_1 + z_2$. graphically on a plot in the complex plane from 2.

$-2+5j$

4. Compute $z_1 - z_2$. Show $z_1 - z_2$. graphically on a plot in the complex plan from 2.

$4-3j$

5. Compute $z_1 z_2$. Repeat the computation using a different notation.

$-3+4j-3j+4 = 1+j = (1.41)(5)e^{j171}$

6. Compute z_1/z_2 using complex notation. Compute z_2/z_1 and compare.

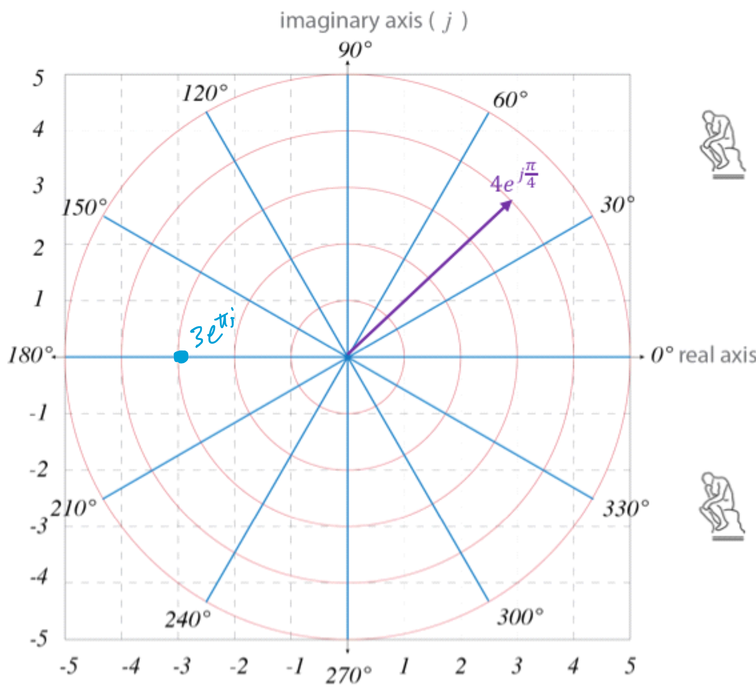
$\frac{1.41}{5}e^{j(45-126)}$ $\frac{5}{1.41}e^{j(126-45)}$

7. Compute z_1^4

$(1.41e^{j45})^4 = 1.41^4 e^{4j45}$

Part II: Plotting complex numbers

Complex numbers using **polar notation** are super useful for illustrating how a circuit responds to time-varying signals.



What does $3 \cdot e^{j\pi}$ look like?

A Wavegen input signal is a sine wave, $V_{peak-peak} = 4Volts$. In r,θ notation, what is its r ?

The **polar coordinates** (above grid of red & blue) make use of a special property of the ***exponential function*** when it operates on $j(=\sqrt{-1})$. You may have seen this function notated (equivalently) as:

$$e^{j\theta}, \exp(j\theta), \text{or } e^{i\theta}$$

where θ represents an angle in radians (Recall that π radians = 180°).

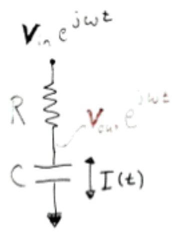
The amazing property of $e^{j\theta}$ is known as Euler's formula (section 6.3 in your book):

$$e^{j\theta} = \cos(\theta) + jsin(\theta)$$

If θ varies with a frequency, $\omega, \theta = \omega \cdot t$, what would $e^{j\omega t}$ look like in time?

Click this link to see. There is more info on page 4 for those who are interested.

Recall from Figure 6.3 that if we represent our cosine voltage input to a **low-pass filter** with polar notation,



$$V_{in}(t) = \boldsymbol{V}_{in} \cdot e^{j\omega t}$$

And $\boldsymbol{V}_{in}$ represents a complex number.

And remember that because the R and C are in series, the time varying current passing through both will be the same, we get,

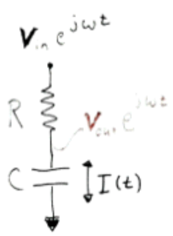
$$\frac{V_{in}(t) - V_{out}(t)}{R} = C\frac{dV_{out}(t)}{dt}$$

and, rearranged a bit,

$$\boldsymbol{V}_{in} \cdot e^{j\omega t} - \boldsymbol{V}_{out} \cdot e^{j\omega t} = RCj\omega \boldsymbol{V}_{out} \cdot e^{j\omega t}.$$

Or

solving for $\frac{\boldsymbol{V}_{out}}{\boldsymbol{V}_{in}}$,



$$\frac{\boldsymbol{V}_{out}}{\boldsymbol{V}_{in}} = \frac{1}{1+j\omega RC}$$

$z_3 = \langle \sqrt{0.5}, \frac{7}{4}\pi \rangle$

Let's let RC=1 second and $z_3 = \frac{1}{1+j\omega}$

And $z_4 = \frac{j\omega}{1+j\omega}$

Convert z_3 and z_4 to r, θ notation.

$z_4 = (\sqrt{0.5}, \frac{1}{4}\pi \rangle$

Plot the magnitude of *r* of z_3 and z_4 as a function of ω on a log-log scale. Let ω* vary from 10^{-3} to 10^3.

Plot θ in degrees for z_3 and z_4 as a function of ω on a semilog scale. Let ω vary from 10^{-3} to 10^3.

*In Matlab, you can use the command:
y= logspace(-3,3)
to generate a logarithmically-spaced vector, y, that spans 10^{-3} to 10^3.

Knowing that z_3 and z_4 represent the $\frac{V_{out}}{V_{in}}$ of low- and high-pass filters, what do you expect the graphs to look like?

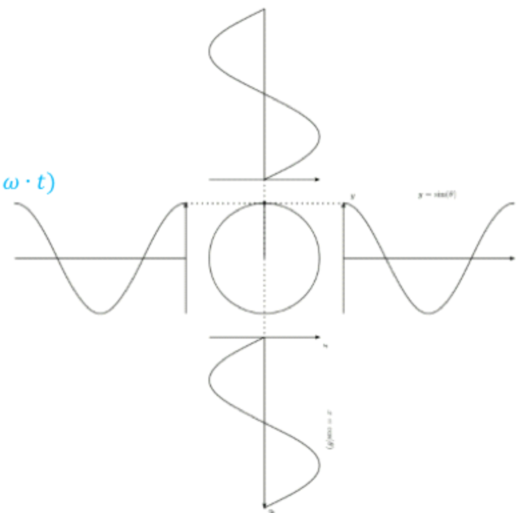
Watch $e^{j\omega t}$ vary as θ varies with a frequency, ω: $\boldsymbol{\theta = \omega \cdot t}$

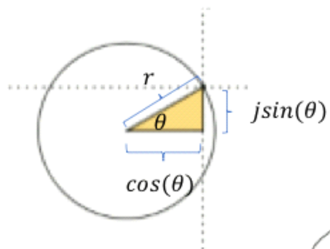
Click this link to see.

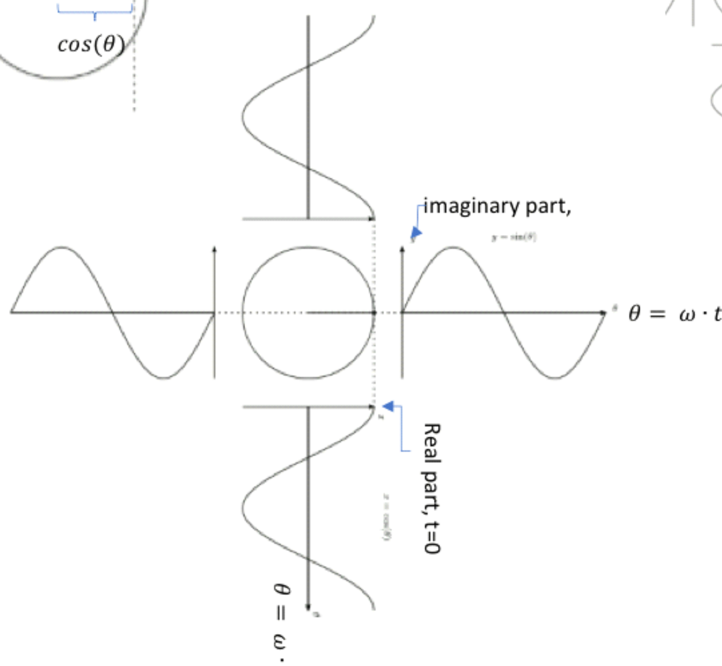
These images illustrate for r=1,

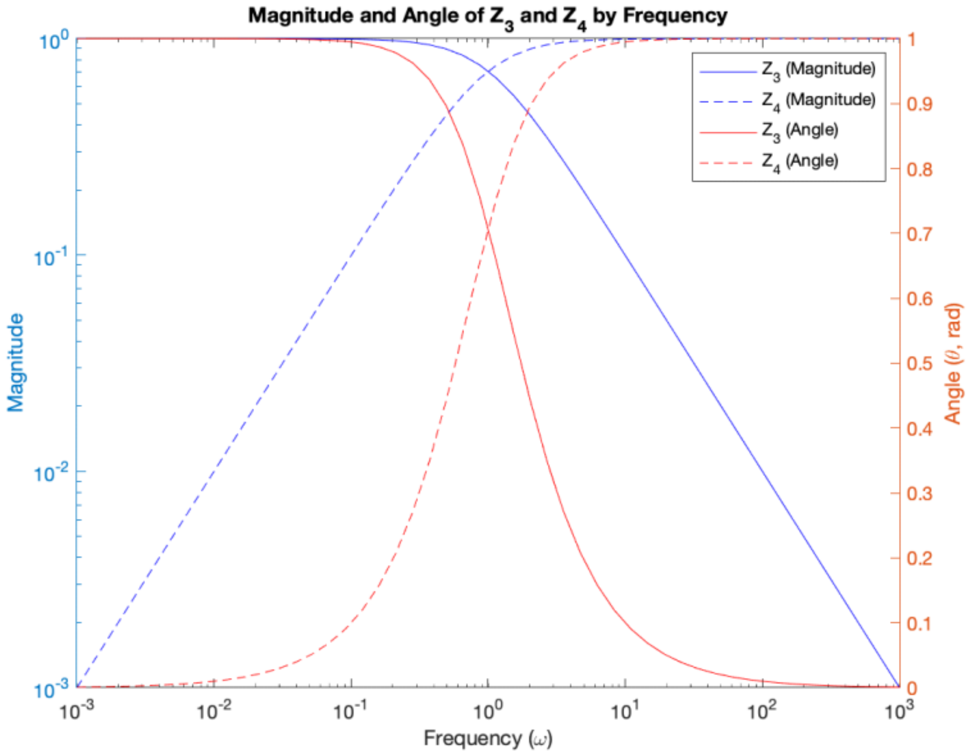
$e^{j\theta} = \cos(\theta) + j\sin(\theta)$

at different points in time ($\theta = \omega \cdot t$)








Magnitude and Angle of Z3 and Z4 by Frequency
Z3 (Magnitude)
Z4 (Magnitude)
Z3 (Angle)
Z4 (Angle)
Magnitude
Angle (θ, rad)
Frequency (ω)
10^0
10^-1
10^-2
10^-3
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0