PS9: Complex Filters

Thursday, April 22, 2021 5:16 PM

PSet9-Complex Filters

Problem set

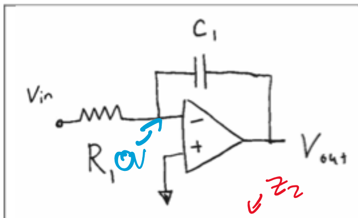
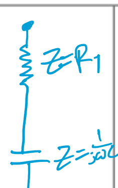
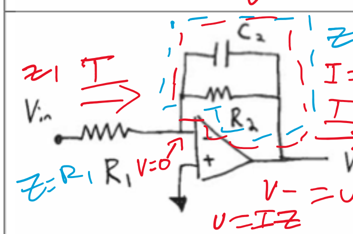
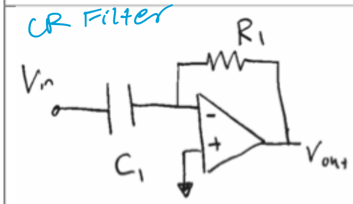
Analysis of filters with complex impedance

For each of the circuits in the table, figure out the complex number that represents the ratio of the output voltage divided by the input voltage. Write your final result in the tables. For each circuit in the table, create a plot of the magnitude of this complex number as a function of frequency. Put both the magnitude and frequency on a logscale. For each circuit use:

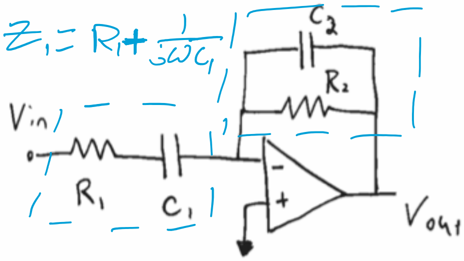
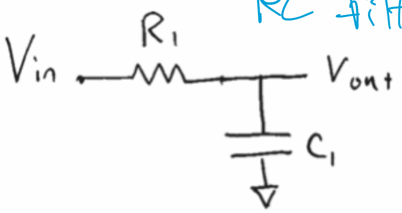
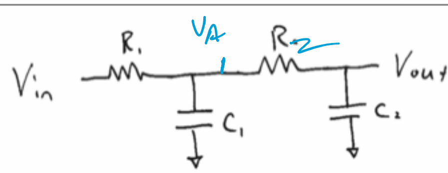
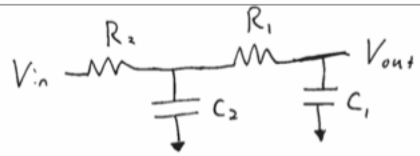
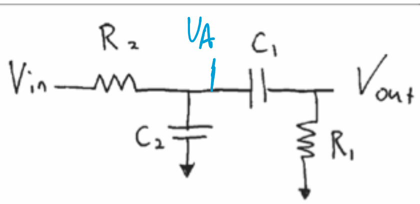
- R1 = 1 kΩ C1 = 1 µF
- R2 = 10 kΩ C2 = 0.1 µF

Remember, the impedance of a resistor and capacitor are

- $Z_R = R$
- $Z_C = \frac{1}{j\,\omega C}$

 	$\frac{V_{out}}{V_{in}} = \frac{\left(\frac{1}{j\omega C_1}\right)}{R_1 + \frac{1}{j\omega C}}$ $= \frac{1}{j\omega C_1\left(R_1 + \frac{1}{j\omega C}\right)}$ $= \frac{1}{j\omega C_1 R_1 + 1}$
 $Z = \frac{1}{1/R_2 + j\omega C_2}$ $I = I_{R_2} + I_{C_2}$ $\frac{1}{Z_2} = \frac{1}{Z_{R_2}} + \frac{1}{Z_{C_2}}$ $\frac{V_{out}}{V_{in}} = \ldots$ $V- = V+$ $V = IZ$ $\frac{V_{out}}{V_{in}} = -Z_1 \cdot Z_2$	$\frac{V_{out}}{V_{in}} = \frac{1/(1/R_2 + j\omega C_2)}{R_1 + 1/(1/R_2 + j\omega C_2)}$ $= \frac{1}{1/(1/R_2 + j\omega C_2)} \cdot \frac{1}{R_1 + 1/(1/R_2 + j\omega C_2)}$ $= \frac{1}{\frac{R_1}{1/R_2 + j\omega C_2} + \frac{1}{\frac{1}{R_2^2} + \frac{2j\omega C_2}{R_2} - \omega^2 C_2^2}}$
CR Filter 	$\frac{V_{out}}{V_{in}} = \frac{j\omega RC}{1 + j\omega RC}$

$Z_2 = \frac{1}{1/R_2 + j\omega C_2}$

$Z_1 = R_1 + \frac{1}{j\omega C_1}$ 	$\frac{V_{out}}{V_{in}} = \frac{1/R_2 + j\omega C_2}{1/R_2 + j\omega C_2 + R_1 + j\omega C_1}$ $= \frac{1/R_2 + j\omega C_2}{1/R_2 + j\omega(C_1+C_2) + R_1}$
RC filter 	$\frac{V_{out}}{V_{in}} = \frac{1}{1+j\omega RC}$
	$\frac{V_A}{V_{in}} = \frac{1}{1+j\omega R_1 C_1}$, $\frac{V_{out}}{V_A} = \frac{1}{1+j\omega R_2 C_2}$ $\frac{V_{out}}{\frac{V_{in}}{1+j\omega R_1 C_1}} = \frac{1}{1+j\omega R_2 C_2}$ $\frac{V_{out}}{V_{in}} = \frac{1}{1+j\omega(R_1R_2+C_1C_2) - \omega^2 R_1R_2C_1C_2}$
	〃
	$\frac{V_A}{V_{in}} = \frac{1}{1+j\omega R_2 C_2}$, $\frac{V_{out}}{V_A} = \frac{j\omega R_1 C_1}{1+j\omega R_1 C_1}$ $\frac{V_{out}}{\frac{V_{in}}{1+j\omega R_2 C_2}} = \frac{j\omega R_1 C_1}{1+j\omega R_1 C_1}$ $\frac{V_{out}}{V_{in}} = \frac{j\omega R_1 C_1}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}$

ps9

PS9

By Ari Porad

Introduction to Sensors, Instrumentation, and Measurement

Note

As specified by the assignment instructions, the resistors and capacitors have the following values for all circuits:

$R_1 = 1k\Omega$
$R_2 = 10k\Omega$
$C_1 = 1\mu F\ (10^{-6}\,F)$
$C_2 = 0.1\mu F\ (10^{-7}\,F)$

Circuit 1: RC + OpAmp

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V_1 =
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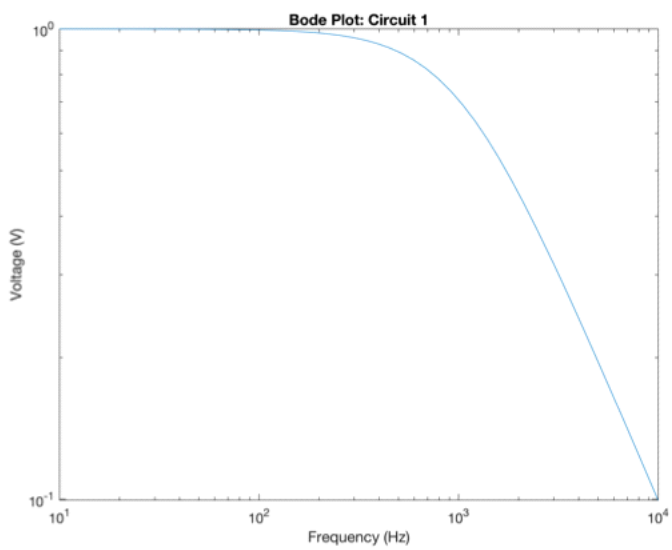
$$-\frac{1\,\mathrm{i}}{C_1\,w\left(R_1 - \frac{1\,\mathrm{i}}{C_1\,w}\right)}$$



Circuit 2: R + (R || C) + OpAmp

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V_2 =
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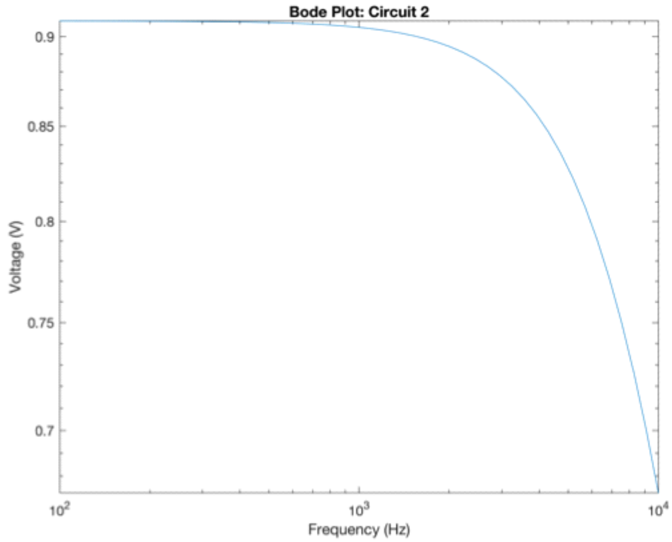
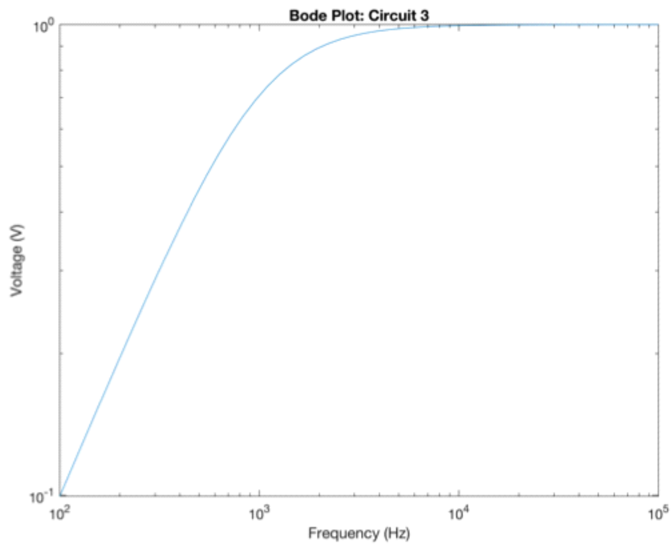
$$\frac{1}{\left(\frac{1}{R_2} + 1\,C_2\,w\,\mathrm{i}\right)\left(R_1 + \frac{1}{\frac{1}{R_2} + 1\,C_2\,w\,\mathrm{i}}\right)}$$



Circuit 3: CR + OpAmp

V_3 =

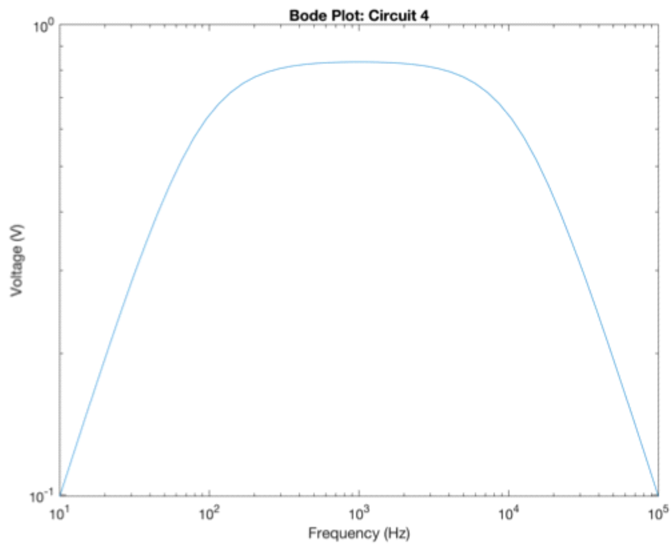
$$\frac{1\,C_1\,R_1\,w\,\mathrm{i}}{1 + 1\,C_1\,R_1\,w\,\mathrm{i}}$$



Circuit 4: R + C + (C || R) + OpAmp

V_4 =

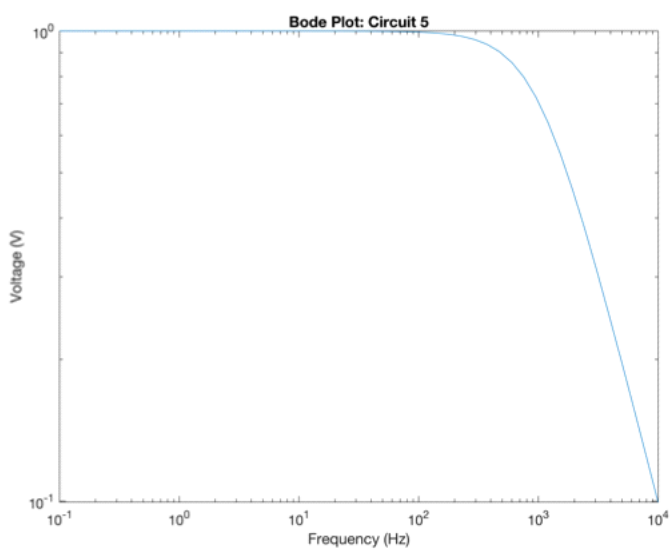
$$\frac{1}{\left(\frac{1}{R_2}+1\,C_2\,w\,\mathrm{i}\right)\left(R_1+\frac{1}{\frac{1}{R_2}+1\,C_2\,w\,\mathrm{i}}-\frac{1\,\mathrm{i}}{C_1\,w}\right)}$$



Circuit 5: RC Voltage Divider

V_5 =

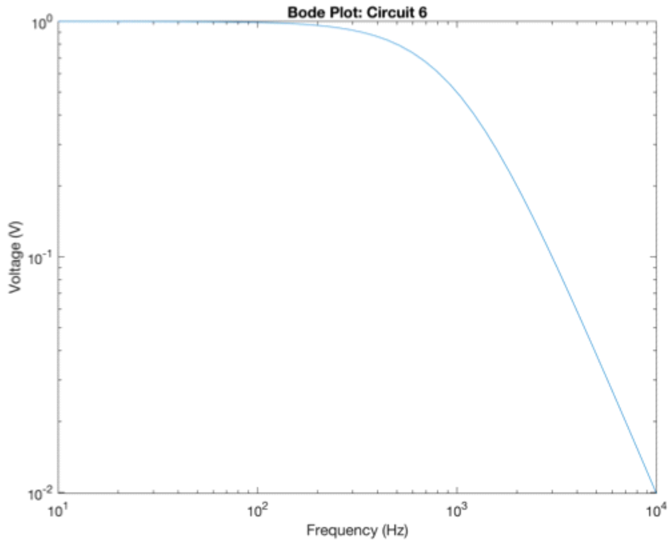
$$\frac{1}{1 + 1\,C_1\,R_1\,w\,\mathrm{i}}$$



Circuit 6: RC + RC

V_6 =

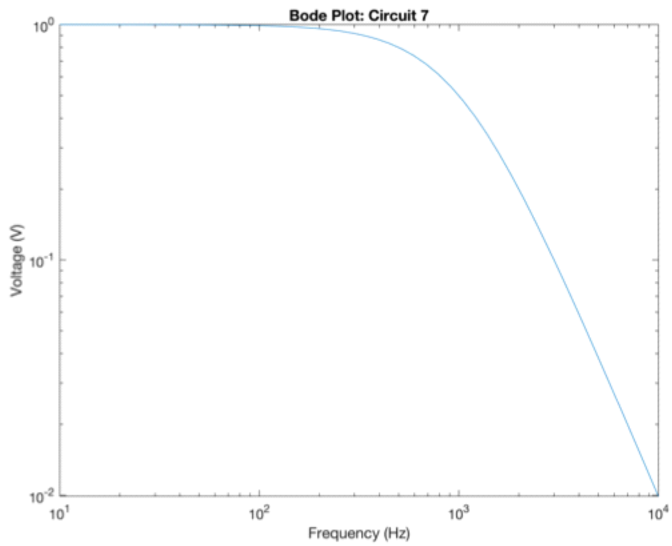
$$\frac{1}{(1 + 1\, C_1\, R_1\, w\, \mathrm{i})\ (1 + 1\, C_2\, R_2\, w\, \mathrm{i})}$$



Circuit 7: RC + RC but reversed

V_7 =

$$\frac{1}{(1 + 1\, C_1\, R_1\, w\, \mathrm{i})\ (1 + 1\, C_2\, R_2\, w\, \mathrm{i})}$$



Circuit 8: RC + CR

V_8 =

$$\frac{1\, C_1\, R_1\, w\, \mathrm{i}}{(1 + 1\, C_1\, R_1\, w\, \mathrm{i})\ (1 + 1\, C_2\, R_2\, w\, \mathrm{i})}$$