EUR/CHF
Currency exchange rate forecasting

Time Series Analysis

May 2022

Abstract

The goal of this study is to fit an ARIMA model to forecast the EUR/CHF currency exchange rate. Different models are obtained through two trend removal methods: a linear model, and differencing. The analysis reveals that differencing is the most appropriate method, leading to a white noise residual series; consequently, an ARIMA(0,0,0) is fitted. The EUR/CHF exchange rate predictions are obtained back-transforming the residual time series forecast.

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List of Abbreviations

ACF *Autocorrelation Function*
AICc *corrected Akaike Information Criterion*
AR *Autoregressive model*
ARIMA *Autoregressive Integrated Moving Average*
CHF *Swiss Franc*
EUR *Euro*
PACF *Partial Autocorrelation Function*
RMSE *Root Mean Square Error*
SARIMA *Seasonal ARIMA*
WN *White noise*

1 Introduction

The challenge posed by exchange rate forecasting was first illustrated by Meese and Rogoff [1983] comparing time series models to a random walk model. The authors found that none of the models makes more accurate forecasts than a random walk. While finding the ultimate currency exchange rate model is by no means the purpose of this project, it is aimed to present a valid model to forecast the EUR/CHF exchange rate.

The proposed model is an ARIMA(p,d,q) model, a generalization of the class of ARMA(p,q) models. ARMA models are suitable for series that present both autoregressive, and moving average process traits. A series X_t is an ARMA(p,q) process if it can be expressed as:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + Z_t - \theta_1 Z_{t-1} - \theta_2 Z_{t-2} - ... - \theta_q Z_{t-q}$$

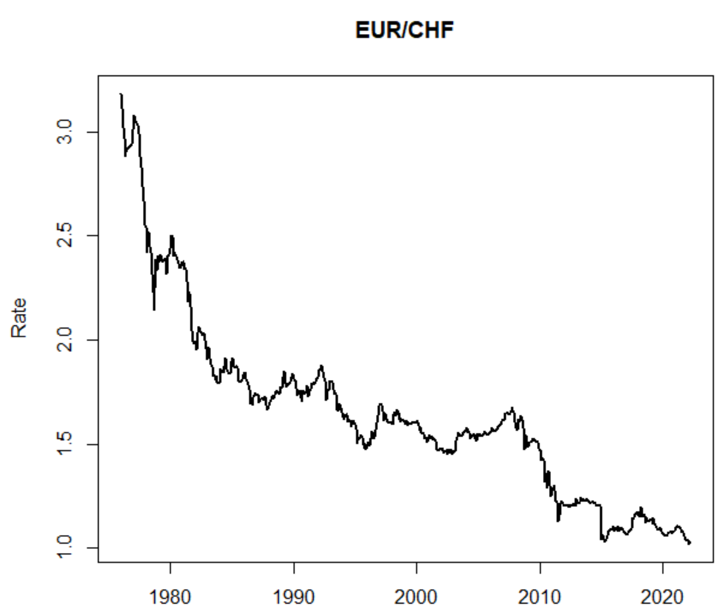
However, ARMA models require the series to be stationary, i.e. constant mean, variance, and autocorrelation over time. If this is not the case, ARIMA models are fitted, which contain an additional component d that uses differencing on the data in order to have a stationary time series. For example, with $\Delta X_t = X_t - X_{t-1}$, an ARIMA(p,1,q) takes the following form:

$$\Delta X_t = \phi_1 \Delta X_{t-1} + \phi_2 \Delta X_{t-2} + ... + \phi_p \Delta X_{t-p} + Z_t - \theta_1 Z_{t-1} - \theta_2 Z_{t-2} - ... - \theta_q Z_{t-q}$$

2 Description

The time series to analyze corresponds to the historical EUR/CHF currency exchange rate. The Swiss Franc was officially introduced as the monetary unit of Switzerland in 1850 [Oanda.com]. The Euro, instead, had been proposed by the European Union in the 1960s, came into existence in 1999, and notes and coins finally began to circulate in 2002 [Wikipedia.com]. The data, obtained from Investing.com, comprises monthly average values of the currency exchange rate from January 1976 to April 2022 (see Figure 1).

Figure 1. EUR/CHF currency exchange rate evolution (January 1976 - April 2022).



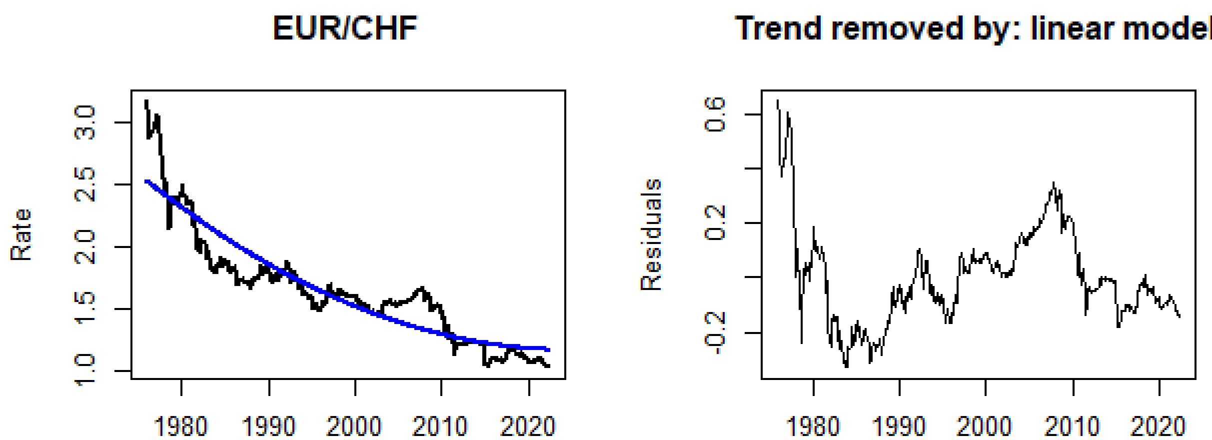
In January 2015, the Swiss National Bank lifted the minimum exchange rate of 1.20 EUR/CHF [swissinfo.ch]. This unexpected decision caused the sudden drop of the exchange rate.

3 Modeling

3.1 Data transformation & trend removal

The ARIMA models are fitted on data with no seasonality nor trend. While no significant seasonality is observed, the latter is clearly visible. Two trend removing methods are studied, the first being a linear model with linear and quadratic components (see Figure 2).

Figure 2. Fitted linear model for the EUR/CHF currency exchange rate (left), and the model residuals (right).



The stationarity of the linear model residuals is debatable, but will still be kept for analysis. In an attempt to find a more suitable trend removing method, differencing is proposed (see Figure 3). Additionally, the aforementioned rate drop of January 2015 could be treated as an outlier. Therefore, two residual series are considered: one where the series is left unchanged, and one where the difference value corresponding to January 2015 is set to zero. Both residual series are centered to zero.

It is important to note that the differencing is not performed on the original series, but in the log transformation of the series. This data transformation is done in order to avoid non-stationarity. Removing the trend by differencing results in a series with constant mean. However, as shown in Figure 4, without the log transformation of the data the variance is significantly larger at the beginning of the series than in the end. A more constant variance is achieved with the data transformation.

To sum up, these are the three different residual series used in the upcoming analysis:

1. Residuals of the fitted linear model.

2. Differencing of the log transformation of the data.

3. Differencing of the log transformation of the data with the value from January 2015 set to zero.

Figure 3. Differencing of the EUR/CHF currency exchange rate with (up) and without (down) the value from January 2015.

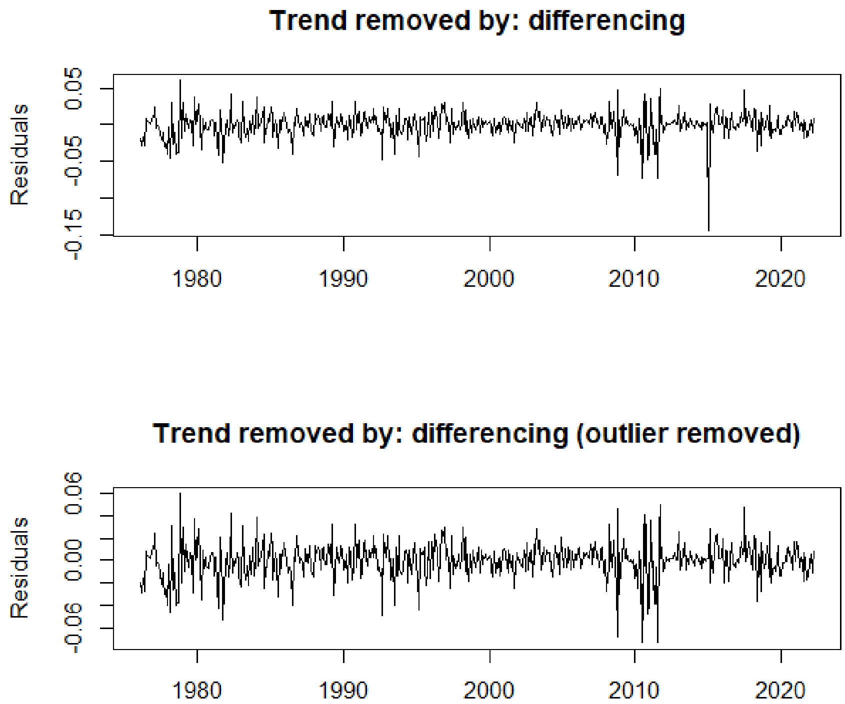
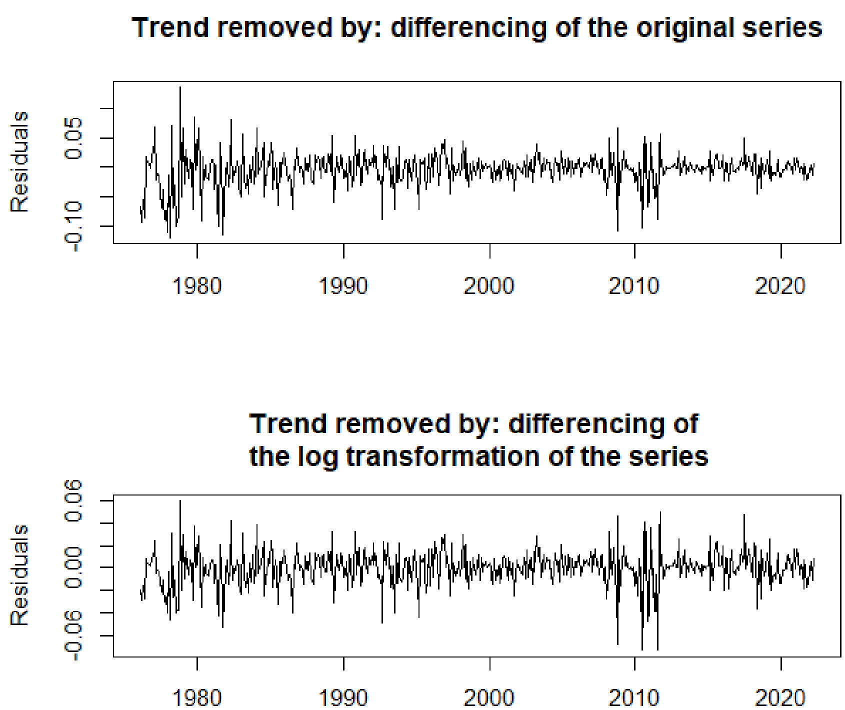


Figure 4. Differencing of the EUR/CHF currency exchange rate with (down) and without (up) a log transformation of the data. The value from January 2015 is set to zero in both results.



3.2 ACF & PACF of the residual series

The ACF and PACF of the three residual series are visualized (see Figure 5) and analyzed (see Table 1).

Figure 5. ACF and PACF of the three residual series.

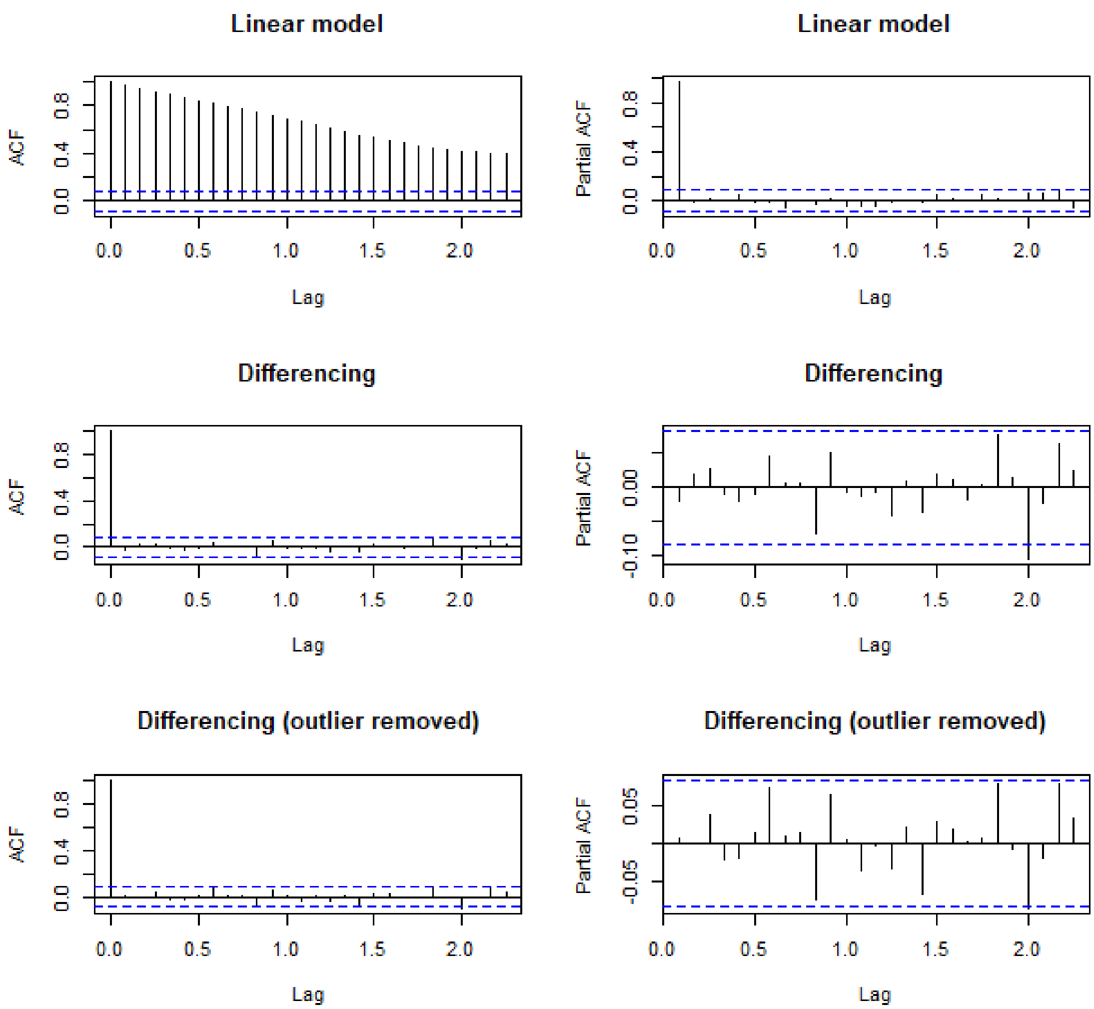


Table 1. Trend removing method, observed behavior of the ACF and PACF, suggested model, and p-value of the Box-Ljung test for the three residual series.
* The spike is scarcely outside the cut-off.

Trend removing method	ACF	PACF	Suggested model	Box-Ljung test
Linear model	Geometric	1 spike	AR(1)	$< 2.2e\text{-}16$
Differencing	0 spikes	0 spikes*	WN	0.622
Differencing (outlier removed)	0 spikes	0 spikes	WN	0.867

The Box-Ljung test indicates whether the data can be considered as white noise. Both series obtained through differencing have p-values>0.05, meaning that the null hypothesis of white noise data cannot be rejected. These results agree with the corresponding ACF and PACF, where no significant spikes are found. On the other hand, the linear model residuals reject the white noise hypothesis. In fact, the geometric behavior of the ACF, and the significant spike found in the PACF, suggest that an AR(1), with ϕ close to 1, is a suitable model for this series.

3.3 Fitted ARIMA models

The model parameters are obtained using the auto.arima() function from the R package 'forecast', which returns the best ARIMA model according to AICc.

For the first residual series, the linear model residuals, the function returns an ARIMA(2,1,2) model with a seasonal AR at the 12^{th} lag of order 2, a SARIMA(2,1,2)(2,0,0)$_{12}$. Nevertheless, no clear seasonal pattern, nor spikes at lags multiple of 12, is observed in the PACF. Besides, the seasonal parameters are relatively small and insignificant when compared to their standard deviation. Therefore, the function is restricted to not look for seasonal components. With the imposed restriction, the function returns an ARIMA(2,1,1). It is not a surprise that the d component of the ARIMA model is non-zero; as previously debated, the residual series does not look stationary, thereby needing a differencing on the data.

As far as the differenced series, with and without the outlier, are concerned, the function returns a SARIMA(1,0,1)(2,0,0)$_{12}$ and a SARIMA(3,0,2)(2,0,0)$_{12}$ model, respectively. Nonetheless, when the model parameters are analyzed, it is found that these models are not far from white noise. For one model, the AR and MA parameters have almost the same value, but with opposite sign, cancelling each other out. For the other, the parameters are small compared to the standard errors, and consequently insignificant. Finally, when the seasonal components are restricted, an ARIMA(0,0,0) is returned for both series, i.e. white noise. This is consistent with the analysis previously done on the ACF, PACF, and Box-Ljung test results.

3.4 Diagnostic plots

Before forecasting, the validity of the models must be assessed. This is done evaluating the standarized residuals of the models, their ACF, and the p-values of the Ljung-Box test for different lags. The diagnostics plots obtained for the second model indicate that the model residuals behave like white noise (see Figure 6); there is no visible pattern on the standarized residuals, the ACF has no significant spikes, and all p-values are larger than 0.05, meaning that the white-noise hypothesis cannot be rejected. Thus, the model is valid. This is the case for all three models.

Moreover, the normality of the model residuals allows to interpret the bounds, which are obtained through the model prediction standard errors, as 95% confidence intervals. The histogram and qq-plot of the residuals reveal a non-normal distribution for the second model residuals (see Figure 7). Additionally, a Shapiro-Wilk test is run. A p-value < 0.05 rejects the null hypothesis of the residuals following a normal distribution. Once again, this is the case for all three models, meaning that none of the prediction bounds will be interpreted as 95% confidence intervals.

3.5 Stationary series predictions

Once the models are considered to be valid, predictions can be made, first for the stationary time series, and then for the initial time series. The predictions given by the first model are shown in Figure 8. The model accurately fits the fed series. However, the forecast does not provide much information. The bounds, which, as previously discussed, cannot be interpreted as 95% confidence intervals, are expected to be wide due to the large standard errors of the model parameters.

The other two models, ARIMA(0,0,0), forecast zero for all times (see Figure 9).

Figure 6. Diagnostics of the model fitted for the series obtained through differencing.

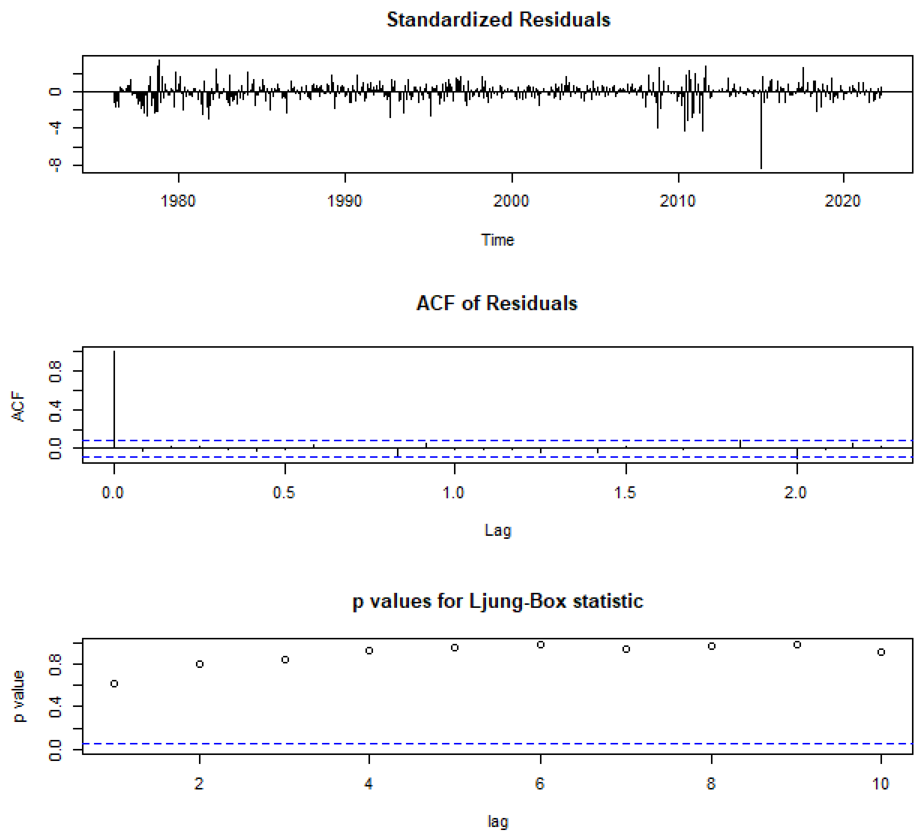


Figure 7. Histogram and qq-plot of the residuals of the model fitted for the series obtained through differencing.

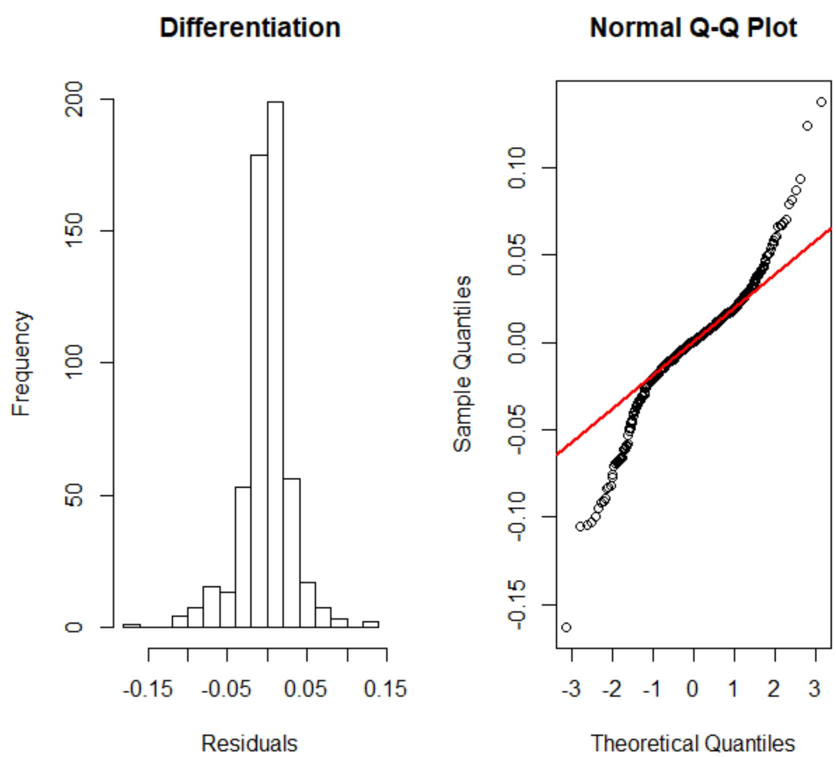


Figure 8. Predictions of the model fitted for the linear model residuals.

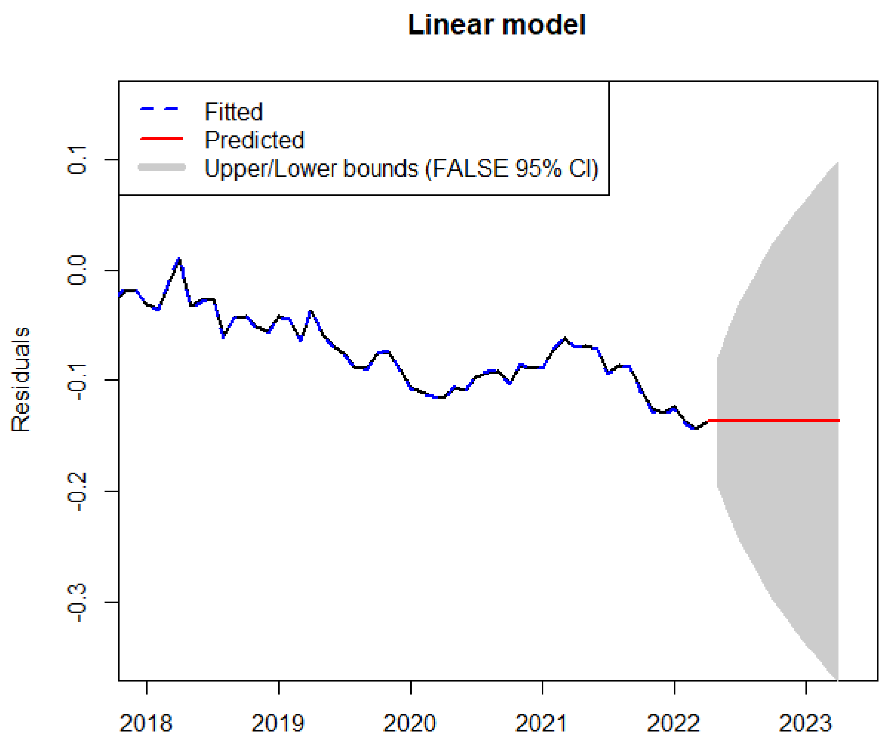
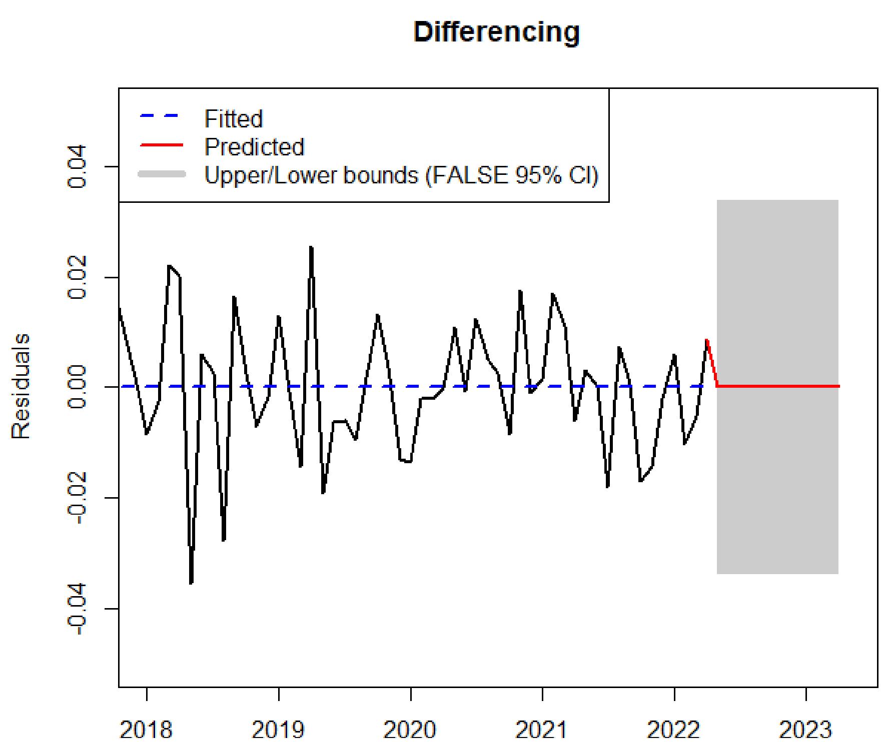


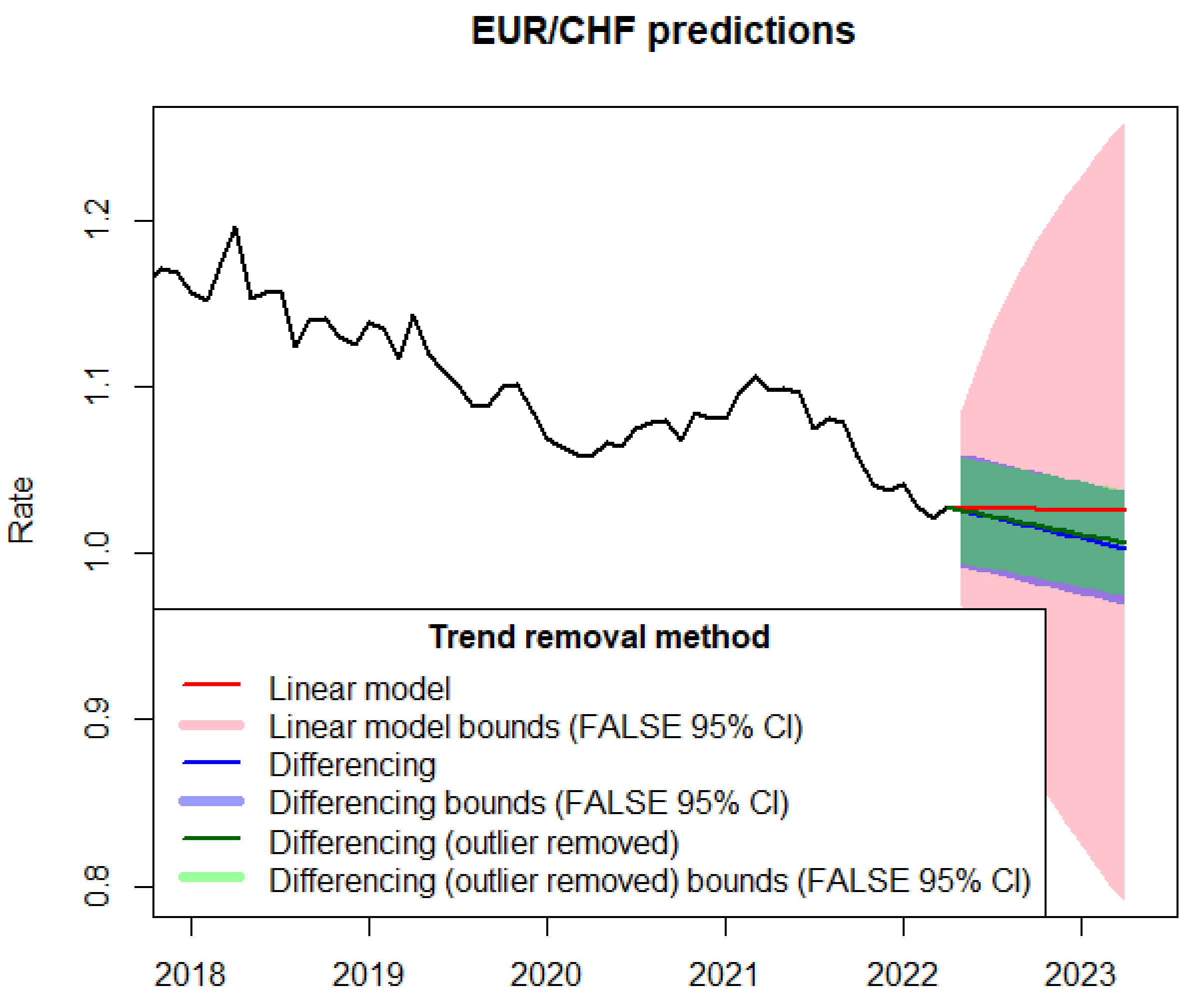
Figure 9. Predictions of the model fitted for the series obtained through differencing.



3.6 Initial series predictions

Finally, the initial time series is forecast, which requires to back-transform the data. This transformation depends on the trend removal method. In the case of the linear model, the initial time series predictions correspond to the sum of the stationary time series predictions and the linear model trend. In the case of differencing, the initial time series is retrieved adding the stationary series mean, and the cumulative sum of the stationary series predictions, to the last value of the initial time series. The final predictions made by the second and third models are virtually identical, while the first model predictions differ more (see Figure 10). Due to the large length of the data, removing the January 2015 outlier has insignificant impact on the final predictions.

Figure 10. EUR/CHF currency exchange rate predictions for the future 12 months, and corresponding bounds, estimated by the three models.



4 Discussion

As previously debated, the stationarity of the linear model residuals is not clear. In fact, currency exchange rates tend to behave more like a random walk, and differencing seems to be more appropriate than a linear model to remove the trend of the series.

On another note, the length of the available data can be a double-edged sword. Although having more data is usually beneficial for statistical analysis, it might be too courageous to claim that the EUR/CHF exchange rate has behaved similarly for 46 years. For instance, one could expect a behavior change when the Euro was launched in 2002, which leads to a thought-provoking question: how does the length of the time series affect future predictions? Moreover, how stable are the proposed models? One would expect predictions to be slightly modified when new data is available. However, if new data drastically changes future predictions, the model will not be reliable. In order to test the stability of the models, three models are fitted for each trend removing method, where each model is fed data starting from a different year. The chosen years are: 1976, 2002 (official launch of the Euro), and 2018 (after the 2015 rate drop). The predictions from the models employing the linear model trend removal method are shown in Figure 11, and the ones from the models employing differencing in Figure 12. Great differences are observed in the predictions of the first, while more consistent predictions are obtained in the latter. If one trend removing method had to be chosen, it would be differencing.

Figure 11. EUR/CHF currency exchange rate predictions for the future 12 months, estimated by three models employing the linear model trend removal method, but using data starting from different years.

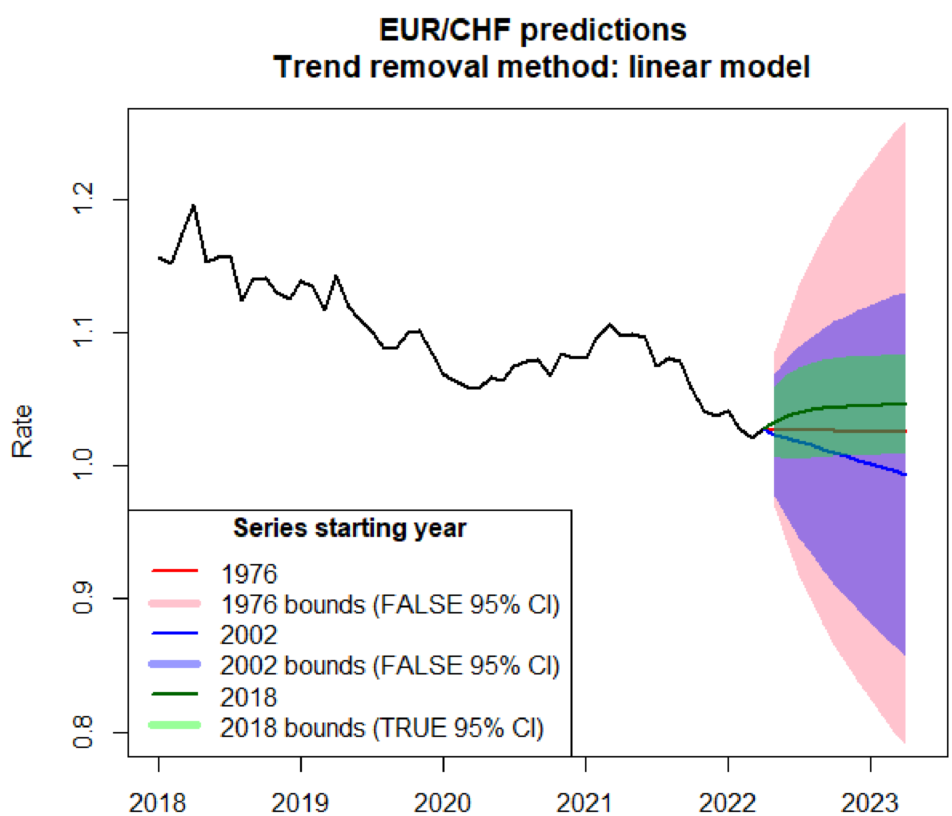
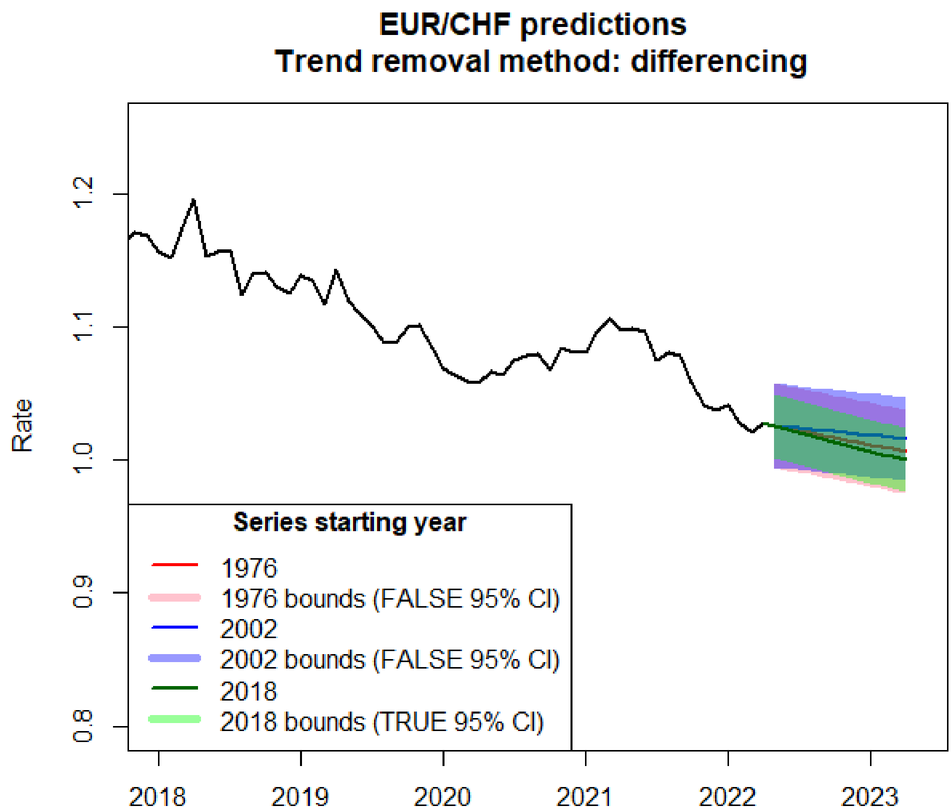


Figure 12. EUR/CHF currency exchange rate predictions for the future 12 months, estimated by three models employing the differencing trend removal method, but using data starting from different years.



5 Conclusion

Three models, using two different trend removing methods, are considered to forecast the EUR/CHF currency exchange rate. Differencing is found to be the most appropriate method to remove the trend. The residual series behaves like white noise. Thus, the most suitable model is an ARIMA(0,0,0), which simply predicts zero for all times. Additionally, the predictions of this model are found to be stable for new data. On the other hand, it is concluded that removing the trend with a linear model is not appropriate for this series.

In future work the models' adequacy could be assessed based on the RMSE, which measures the deviation from the predicted value to the observed value. Furthermore, more research could be done regarding the seasonal component of the SARIMA models returned by the auto.arima() function without restrictions.

References

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