Reproduction of the numerical results in *N. Kiyotaki & J. Moore* "Credit Cycles" (1997)

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This Notebook reproduces Figure 3 of "Credit Cycles" (1997), by Kiyotaki and Moore. It uses a shooting algorithm to find the Initial Value Condition that reproduces the cyclicality of the full non-linear model, while converging to the steady state in the long run.

Content

1. Model equations

(1) Land market equilibrium condition

$$q_{t+1} = R(q_t - u\left(K_t\right))$$

(2) Law of motion of the farmers' aggregate landholding

$$K_t = (1-\pi)\lambda K_{t-1} + \frac{\pi}{\phi + q_t - \frac{1}{R}q_{t+1}}\left[\left(a + q_t + \lambda\phi\right)K_{t-1} - RB_{t-1}\right]$$

(3) Law of motion of the farmers' aggregate debt

$$B_t = RB_{t-1} + q_t\left(K_t - K_{t-1}\right) + \phi\left(K_t - \lambda K_{t-1}\right) - aK_{t-1}$$

Where:

- q_t = price of land
- K_t = landholding of farmers
- B_t = aggregate debt of farmers
- $R_t = R$ = constant rate of interest
- $u(.)$ = downpayment required for the purchase of K_t units or land (also opportunity cost of holding land)
- a = constant proportional to the share of tradeable fruit (also "productivity" in the

model)
- $\lambda =$ fraction of trees surviving after one period
- $\pi =$ probability that a new opportunity to plant tree arises
- $\phi =$ amount of trees created with one fruit

2. Numerical solution

I solve model (1)-(2)-(3) numerically, by forward shooting. The steps are:

- 1. Finding the steady state values (q^*, K^*, B^*).
- 2. Defining a one-dimensional grid of initial values $\mathbf{q}_1$ in the neighbourhood of $1.0037 \times q^*$ as the paper suggests. I use a grid of $1,000$ initial guesses in $[q^*(1.0037 - 0.0005), q^*(1.0037 + 0.0005)]$ here, as the paper indicates that the initial increase in q_t is of 0.37%.
- 3. Using a numerical optimisation routine to solve the non-linear sets of equations $\begin{pmatrix} q_{t+1} \\ K_t \\ B_t \end{pmatrix} = f \begin{pmatrix} q_{t+1} \\ K_t \\ B_t \end{pmatrix}, \forall t$. I know $\begin{pmatrix} q_0 \\ K_0 \\ B_0 \end{pmatrix} = f \begin{pmatrix} q^* \\ K^* \\ B^* \end{pmatrix}$ and I guess q_1.
- 4. Finding $q_1^* = \arg\min_{q_1} |q^* - q_1|$, and using this q_1^* to draw the impulse response functions of B_t/B^*, K_t/K^* and q_t/q^*

3. Parameter values

The following parameter values come from the paper (p. 237)

$$R = 1.01$$
$$\lambda = 0.975$$
$$\eta = 0.10$$
$$a = 1$$
$$\Delta a = 0.01$$
$$\pi = 0.1$$
$$\phi = 20$$

I use the functional form $u(K) = K - \nu$ where ν is set to make η, the elasticity of the residual supply of land to farmers, equal to 10 percent in the steady state. So ν is given by:

$$\frac{1}{\eta} = \frac{\mathrm{d}\log(K-\nu)}{\mathrm{d}log(K)}\bigg|_{K=K^*} = \frac{\mathrm{d}(K-\nu)}{\mathrm{d}K}\bigg|_{K=K^*} \frac{K^*}{K^*-\nu} = \frac{K^*}{K^*-\nu}$$

So $\nu = K^*(1-\eta)$ and $K^* = \frac{R-1}{R}\frac{q^*}{\eta}$

4. Steady state values

In steady state:

$$\begin{aligned}
q^* &= \frac{R}{R-1}\frac{\pi a - (1-\lambda)(1-R+\pi R)\phi}{\lambda\pi + (1-\lambda)(1-R+\pi R)} \\
B^* &= \frac{1}{R-1}(a-\phi+\lambda\phi)K^* \\
K^* &= \frac{R-1}{R}\frac{q^*}{\eta}
\end{aligned}$$

The system is initialised with these values, and should return to them in the long run.

```
#Necessary modules
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import statsmodels as sm
import scipy.optimize as opt
```

```
#Timeline
periods = 100+1

#Guesses
trials = 1000

#Parameter values
R = 1.01
λ = 0.975
η = 0.10
a = 1.0
Δa = 0.01
π = 0.1
ϕ = 20.0
c = 1.0

#Finding the steady state values
q_star = (R/(R-1))*(π*a - (1-λ)*(1 - R + π*R)*ϕ) / (λ*π + (1-λ)*(1 - R + π*R))
ν = ((R -1)/R)*(q_star/η)
K_star = ((R-1)/R)*q_star + ν
B_star = (1/(R-1))*(a - ϕ + λ*ϕ)*K_star

q_star, K_star, B_star, ν
```

```
(55.1691305437233, 6.008519168128284, 300.42595840641394,
5.462290152843895)
```

```
#Initialising the initial values and productivity shock
B = np.zeros(periods)
q = np.zeros(periods)
K = np.zeros(periods)
Δt = np.zeros(periods)
Δt[1] = Δa
B[0], K[0], q[0] = B_star, K_star, q_star
```

```
#Calculating the values of q, B and K after the productivity shock (need to solv
#a_t = np.ones(periods)
#a_t[1] = a + Δa
#B[0], K[0] = B_star, K_star
#a_t

#Generating guess values for q_1
q_vec = np.linspace(q_star*(1.0037-0.0005), q_star*(1.0037+0.0005), trials)
q_vecSS = np.linspace(q_star, q_star, trials)
q_mat = np.zeros((trials, periods))
q_mat[:,1] = q_vec
q_mat[:,0] = q_vecSS

#Initialising the matrices of B's and K's
B_mat = np.zeros((trials, periods))
K_mat = np.zeros((trials, periods))
B_vec = np.linspace(B_star, B_star, trials)
K_vec = np.linspace(K_star, K_star, trials)
B_mat[:,0] = B_vec
K_mat[:,0] = K_vec


#Solving the model each period, for all guesses of q_1
for s in range(0, trials):
  for t in range(1, periods-1):
    def f(variables):
      (q_tplus1, K_t, B_t) = variables
      first_eq = -(q_tplus1 - R*q_mat[s, t] + R*(K_t - ν))
      second_eq = -(K_t - (1-π)*λ*K_mat[s, t-1] - (π/(ϕ + q_mat[s, t] - (q_tplus
      third_eq = -(B_t - R*B_mat[s, t-1] - q_mat[s, t]*(K_t - K_mat[s, t-1]) - ϕ
      return [first_eq, second_eq, third_eq]

    solution = opt.fsolve(f, (q_star, K_star, B_star))
    q_mat[s, t+1], K_mat[s, t], B_mat[s, t] = solution
```

Selecting the initial value of land price q_1 for which the long term price q_T is closest to q^*.

```
#Defining a new vector of absolute distances to the steady state vale q*
q_vec_abs = np.zeros(trials)
for i in range(0, trials):
  q_vec_abs[i] = abs(1 - q_mat[i, -1]/q_star)

#Smallest absolute deviation
index = np.argwhere(q_vec_abs == np.min(q_vec_abs))[0, 0]
index, q_mat[index, -1], q_vec[index], q_vec_abs[index]
```

```
(827, 55.16912907390393, 55.39134230296437, 2.6642061445869558e-08)
```

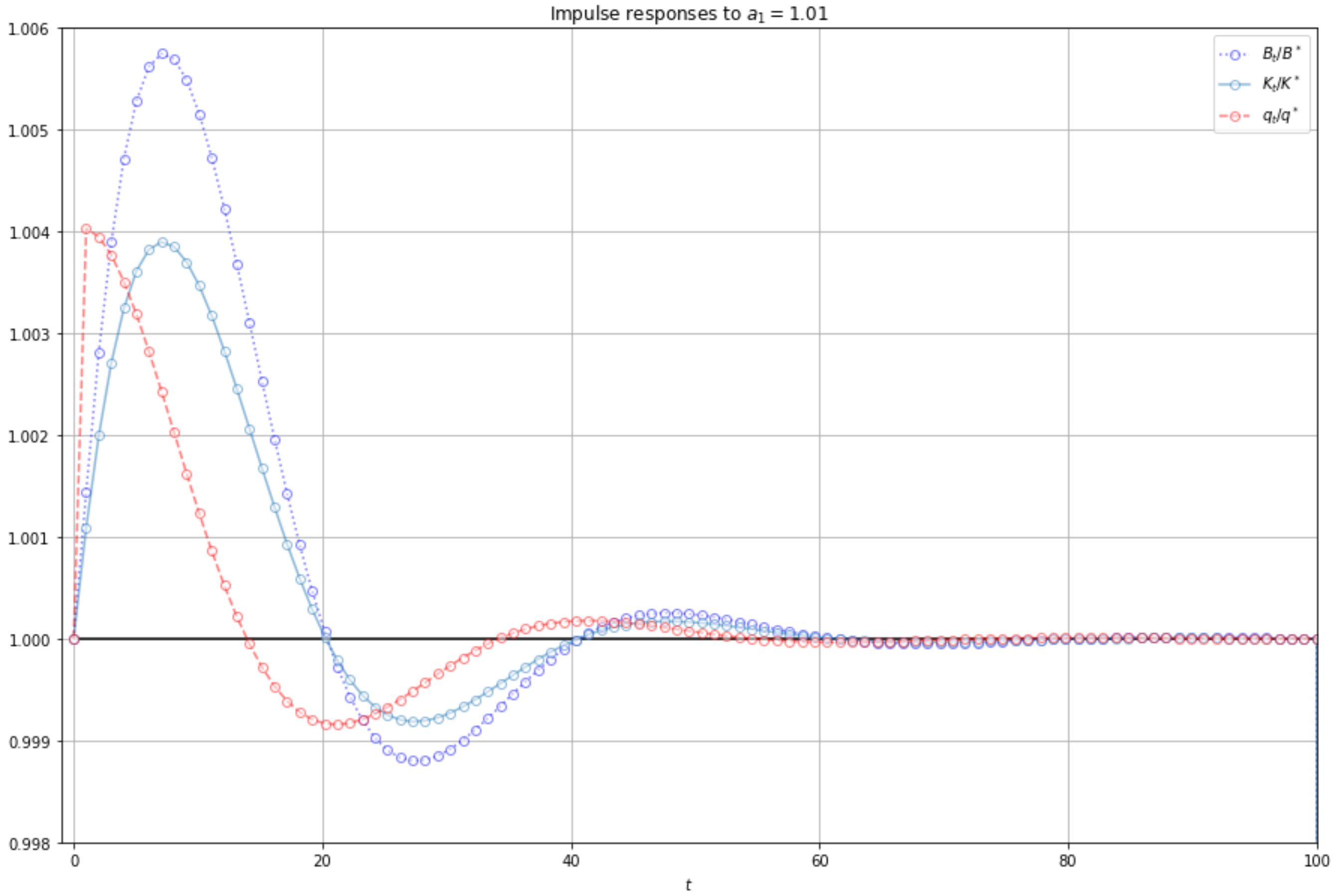
Graphing the impulse functions of the selected initial values:

```
time = np.linspace(0, periods, periods)

B_graph = B_mat[index, :]/B_star
K_graph = K_mat[index, :]/K_star
q_graph = q_mat[index, :]/q_star

plt.figure(1, figsize=(15,10))
plt.title("Impulse responses to $a_1 = 1.01$")
plt.plot(time, B_graph, 'b:', alpha=0.5, marker='o', fillstyle='full', markerfac
plt.plot(time, K_graph, alpha=0.5, marker='o', fillstyle='full', markerfacecolor
plt.plot(time, q_graph, 'r--', alpha=0.5, marker='o', fillstyle='full', markerfa
plt.xlabel("$t$" "")
plt.legend()
plt.grid()
plt.hlines(1, 0, 100)
plt.xlim(-1, 100)
plt.ylim(0.998, 1.006)
```

```
(0.998, 1.006)
```




```
plt.savefig('H:\KM_Fig3.pdf')
plt.close(1)
```

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