

# Affine Loop Optimization Using Modulo Unrolling in Chapel

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July 16, 2014

## Abstract

This paper presents a method for message aggregation for loops in message passing programs that use affine array accesses in Chapel, a Partitioned Global Address Space (PGAS) parallel programming language. Messages incur non-trivial run time overhead, a significant component of which is independent of the size of the message. Therefore, aggregating messages improves performance. Our optimization for message aggregation is based on a technique known as modulo unrolling, pioneered by Barua [2], whose purpose was to ensure a statically predictable single tile number for each memory reference for tiled architectures, such as the MIT Raw Machine [17]. Modulo unrolling applies to data that is distributed in a cyclic or block cyclic manner. In this paper, we adapt the aforementioned modulo unrolling technique to the difficult problem of efficiently compiling PGAS languages to message passing architectures. When applied to loops and data distributed cyclically or block cyclically, modulo unrolling can decide when to aggregate messages thereby reducing the overall message count and runtime for a particular loop. Compared to other methods, modulo unrolling greatly simplifies the complex problem of automatic code generation of message passing code. It also results in substantial performance improvement compared to the non-optimized Chapel compiler.

To implement this optimization in Chapel, we modify the leader and follower iterators in the Cyclic and Block Cyclic data distribution modules. Results were collected that compare the performance of Chapel programs optimized with modulo unrolling with Chapel programs using the existing Chapel data distributions. Data collected on a ten-locale cluster show that on average, modulo unrolling used with Chapel’s Cyclic distribution results in 76 percent fewer messages and a 45 percent decrease in runtime for our suite of benchmarks. Similarly, modulo unrolling used with Chapel’s Block Cyclic distribution results in 72 percent fewer messages and a 52 percent decrease in runtime.

## 1 Introduction

Compilation of programs for distributed memory architectures using message passing is a vital task with potential for speedups over existing techniques. The partitioned global address space (PGAS) parallel programming model automates the production of message passing code from a shared memory programming model and exposes locality of reference information to the programmer thereby improving programmability and allowing for compile-time performance optimizations. In particular, programs compiled to message passing hardware can improve in performance by aggregating messages and eliminating dynamic locality checks for affine array accesses in the PGAS model.

Message passing code generation is a difficult task for an optimizing compiler targeting a distributed memory architecture. These architectures are comprised of independent units of computation called *locales*. Each locale has its own set of processors, cores, memory, and address space. For programs executed on these architectures, data is distributed across various locales of the system, and the compiler needs to reason about locality in order to determine whether a program data access is remote (requiring a message to another locale to request a data element) or local (requiring no message and accessing the data element on the locale’s own memory). Only a compiler with sufficient knowledge about locality can compile a program in this way with good communication performance.

Without aggregation, each remote data memory access results in a message with some non-trivial run time overhead, which can drastically slow down a program’s execution time. This overhead is caused by latency on the interconnection network and locality checks for each data element. Accessing multiple remote data elements individually results in this run time overhead being incurred multiple times, whereas if they are transferred in bulk the overhead is only incurred once. Therefore, aggregating messages improves performance of message passing codes. In order to transfer remote data elements in bulk, the compiler must be sure that all elements in question reside on the same remote locale before the message is sent.

The vast majority of loops in scientific programs access data using affine array accesses. An affine array access is one whose indices are linear combinations of the loop’s induction variables. For example, for a loop with induction variables  $i$  and  $j$ , accesses  $A[i, j]$  and  $A[2i - 3, j + 1]$  are affine, but  $A[i^2]$  is not. Loops using affine array accesses are special because they exhibit regular access patterns within a data distribution. Compilers can use this information to decide when message aggregation can take place.

Existing methods for message passing code generation such as [9, 20] all have the following steps:

- **Loop distribution** The loop iteration space for each nested loop is divided into portions to be executed on each locale (message passing node), called iteration space tiles.
- **Data distribution** The data space for each array is distributed according to the directive of the programmer (usually as block, cyclic, or block-cyclic distributions.)
- **Footprint calculation** For each iteration space tile, the portion of data it accesses for each array reference is calculated as a formula on the symbolic iteration space bounds. This is called the data footprint of that array access.
- **Message aggregation calculation** For each array access, its data footprint is separately intersected with each possible locale’s data tile to derive symbolic expressions for the portion

of the data footprint on that locale’s data tile. This portion of the data tile for locales other than the current locale needs to be communicated remotely from each remote data tile’s locale to the current loop tile’s locale. Since the entire remote portion is calculated exactly, sending it in a single aggregated message becomes possible.

Unfortunately, of the steps above, the message aggregation calculation is by far the most complex. Loop distribution and data distribution are straightforward. Footprint calculation is of moderate complexity using matrix formulations or the polyhedral model. However, it is the message aggregation calculation that defies easy mathematical characterization for the general case of affine accesses. Instead some very complex research methods [8, 20] have been devised that make many simplifying assumptions on the types of affine accesses supported, and yet remain so complex that they are rarely implemented in production compilers.

Although the steps above are primarily for traditional methods of parallel code generation, polyhedral methods don’t fare much better. Polyhedral methods have powerful mathematical formulations for loop transformation discovery, automatic parallelization, and parallelism coarsening. However message aggregation calculation is still needed but not modeled well in polyhedral models, leading to less capable ad-hoc methods for it.

It is our belief that message aggregation using tiling is not used in production quality compilers today because of the complexity of message aggregation calculations, described above. What is needed is a simple, robust, and widely applicable method for message aggregation that leads to improvements in performance.

This paper presents a loop optimization for message passing code generation based on a technique called modulo unrolling. Using modulo unrolling, the locality of any affine array access can be deduced at compile time if the data is distributed in a cyclic or block cyclic fashion. The optimization can be performed by a compiler to aggregate messages and reduce a program’s execution time and communication.

Modulo unrolling in its original form, pioneered by [2], was meant to target tiled architectures such as the MIT Raw machine. Its purpose for tiled architectures was to allow the use of the compiler-routed static network for accessing array data in unrolled loops. It was not meant for message passing architectures, nor was it used to perform message aggregation. It has since been modified to apply to message passing machines in this work. Modulo unrolling works as follows. In its basic form, it unrolls each affine loop by a factor equal to the number of locales of the machine being utilized by the program. If the arrays used in the loop are distributed cyclically or block cyclically, each array access is guaranteed to reside on a single locale across all iterations of the loop. Using this information, the compiler can then aggregate all remote array accesses that reside on a remote locale into a single message before the loop. If remote array elements are written to during the loop, a single message is required to store these elements back to each remote locale after the loop runs.

We build on the modulo unrolling method to solve the very difficult problem of message aggregation for message passing machines inside Partitioned Global Address Space (PGAS) languages. In the PGAS model, a systems memory is abstracted to a single global address space regardless of the hardware architecture and is then logically divided per locale and thread of execution. By doing so, locality of reference can easily be exploited no matter how the

system architecture is organized.

Our evaluation is for Chapel, an explicitly parallel programming language developed by Cray Inc. that falls under the Partitioned Global Address Space (PGAS) memory model. The Chapel compiler is an open source project used by many in industry and academic settings. The language contains many high level features such as zippered iteration, leader and follower iterator semantics, and array slicing that greatly simplify the implementation of modulo unrolling into the language.

Although our method is described in the context of Chapel, it is adaptable to any PGAS language. However, for other languages the implementation may differ. For example, if the language does not use leader and follower iterator semantics to implement parallel for loops, the changes to those Chapel modules that we present here will have to be implemented elsewhere in the other PGAS language where **forall** loop functionality is implemented.

## 1.1 Chapel’s Data Distributions

Figures 1 - 2 illustrate the Chapel data distributions that we considered in this work: Block, Cyclic, and Block Cyclic. Each figure shows how a two-dimensional 8 x 8 array can be distributed in Chapel using each distribution. Figure 1 illustrates the Block distribution. Elements of the array are mapped to locales evenly in a dense manner. In Figure 2, the Cyclic distribution, elements of an array are mapped in a round-robin manner across locales. Finally, in Figure 3 the Block Cyclic distribution is shown. Here, a number of elements specified by a block size parameter is allocated to consecutive array indices in a round robin fashion. In Figure 3, the distribution takes in a 2 x 2 block size parameter. Further details about Block, Cyclic, and Block Cyclic distributions in Chapel are described in [16].

The choice of data distribution to use for a program boils down to computation and communication efficiency. Different programs and architectures may require different data distributions. It has been shown that finding an optimal data distribution for parallel processing applications is an NP-complete problem, even for one or two dimensional arrays [12]. Certain program data access patterns will result in fewer communication calls if the data is distributed in a particular way. For example, many loops in stencil programs that contain nearest neighbor computation will have better communication performance if the data is distributed using a Block distribution. This occurs because on a given loop iteration, the elements accessed are near each other in the array and therefore more likely to reside on the same locale block. Accessing elements on the same block does not require a remote data access and can be done faster. However, programs that access array elements far away from each other will have better communication performance if data is distributed using a Cyclic distribution. Here, a Block distribution is almost guaranteed to have poor performance because the farther away accessed elements are the more likely they reside on different locales.

A programmer may choose a particular data distribution for reasons unknown to the compiler. These reasons may not even take communication behavior into account. For example, Cyclic and Block Cyclic distributions provide better load balancing of data across locales than a Block distribution when array sizes may be changed dynamically because in Cyclic and Block Cyclic distributions, the locales of existing array elements do not change when new array elements are added at the end of the array.. In many ap-

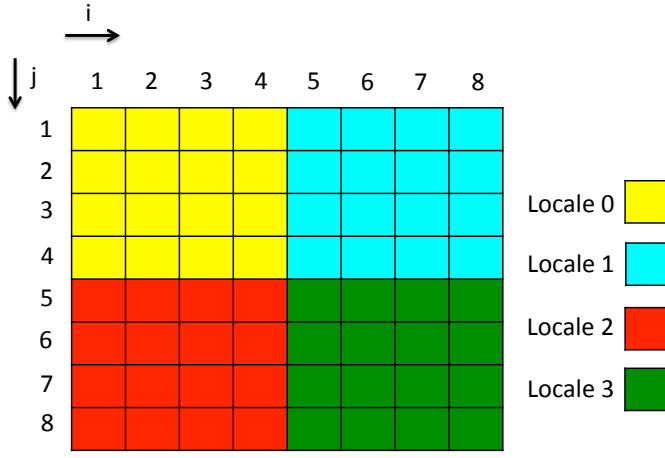


Figure 1: Chapel Block distribution.

plications, data redistribution may be needed if elements of a data set are inserted or deleted at the end of the array. In particular, algorithms to redistribute data using a new block size exist for the Block Cyclic distribution [18, 13]. If an application uses a dynamic data set with elements being appended, a Cyclic or Block Cyclic distribution is superior to Block because new elements are added to the locale that follows the cyclic or block cyclic pattern. For Block, the entire data set would need to be redistributed every time a new element is appended, which can be expensive.

Compatibility with other PGAS languages might be an important consideration for a programmer when selecting a data distribution. Data sets used by Chapel programs and other PGAS programs should use Cyclic or Block Cyclic distributions because other PGAS languages do not support the Block distribution. A programmer would benefit by distributing the same data set using only one scheme so the data would not have to be replicated for different programs. This is an important consideration for vast data sets which have already been distributed on message passing computers, and we want to perform additional computation on them, perhaps with other programs.

Therefore, in this work, it is our view that the compiler should not change the programmer's choice of data distribution in order to achieve better runtime and communication performance. The compiler should attempt to perform optimizations based on the data distribution that the programmer specified. Our optimization is meant to be applied whenever the programmer specifies a Cyclic or Block Cyclic distribution. It is not applied when the programmer specifies a Block distribution.

## 2 Related Work

Compilation for distribution memory machines has two main steps: loop optimizations and message passing code generation. First, the compiler performs loop transformations and optimizations to uncover parallelism, improve the granularity of parallelism, and improve cache performance. These transformations include loop peeling, loop reversal, and loop interchange. Chapel is an explicitly parallel language, so uncovering parallelism is not needed. Other loop optimizations to improve the granularity of parallelism and improve cache performance are orthogonal to this paper. The second step is message passing code generation, which includes mes-

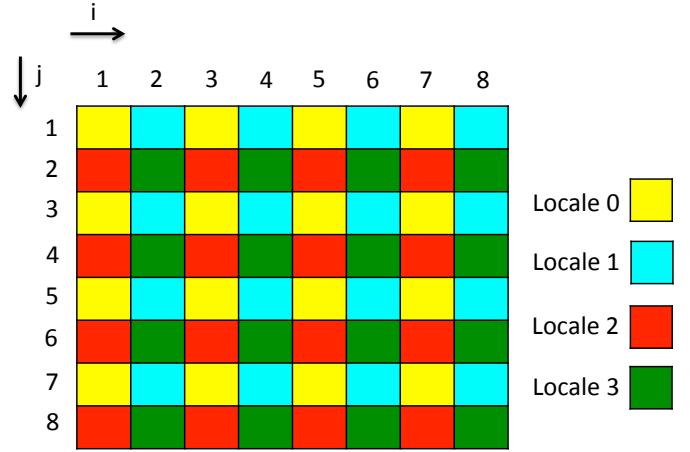


Figure 2: Chapel Cyclic distribution.

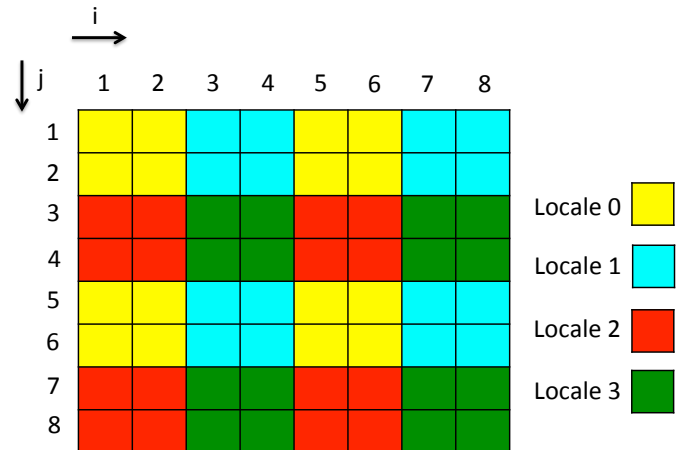


Figure 3: Chapel Block Cyclic distribution with a 2 x 2 blocksize parameter.

sage aggregation.

Message passing code generation in the traditional model is exceedingly complex, and practical robust implementations are hard to find. These methods [8, 20, 3, 14] require not only footprint calculations for each tile but also the intersection of footprints with data tiles. As described in the introduction, calculating such intersections is very complex, and explains the complexity and simplifying limitations of many existing methods. Such methods are rarely if ever implemented in production compilers.

The polyhedral method is another branch of compiler optimization that seeks to speed up parallel programs on distributed memory architectures [5, 7, 9, 10, 11, 19]. The primary purpose of the polyhedral method is uncovering parallelism and loop optimization, not code generation. Its strength is that it can find sequences of transformations in one step, without searching the entire space of transformations. However, the method at its core does not compute information for message passing code generation. Message passing code generation does not fit the polyhedral model, so ad-hoc methods for code generation have been devised to work on the output of the polyhedral model. However they are no better than corresponding methods in the traditional model, and suffer from many of the same difficulties.

Similar work to take advantage of communication aggregation on distributed arrays has already been done in Chapel. Like distributed parallel loops in Chapel, whole array assignment suffers from locality checks for every array element, even when the locality of certain elements is known in advance. In [15], aggregation is applied to improve the communication performance of whole array assignments for Chapel’s Block and Cyclic distributions. Our work goes beyond array assignments and applies aggregation to affine array accesses within parallel loops for Chapel’s Cyclic and Block Cyclic distributions. One of the contribution’s of [15] included two new bulk communication primitives for Chapel developers as library calls, `chpl_comm_gets` and `chpl_comm_puts`. They both rely on the GASNet networking layer, a portion of the Chapel runtime. Our optimization uses these new communication primitives in our implementation directly to perform bulk remote data transfer between locales. The methods in [15] are already in the current release of the Chapel compiler.

Extensive work has been done with the UPC compiler (another PGAS language) by [6] to improve on its communication performance. This method targets fine-grained communication and uses techniques such as redundancy elimination, split-phase communication, and communication coalescing (similar to message aggregation) to reduce overall communication. This work is applicable to programs with many small individual and independent remote array accesses that are not necessarily related to each other. However, it is not clear whether this method can be used to improve communication performance across more coarse-grained structures, such as distributed parallel loops. Another major limitation to this work’s aggregation scheme is that only contiguous data can be sent in bulk. To aggregate data across an entire loop in a single message when data is distributed cyclically, it must be possible to aggregate data elements that are far apart in memory, separated by a fixed stride. Our method handles this by using the strided get and put calls (`chpl_comm_gets` and `chpl_comm_puts`) from [15], described earlier.

### 3 Background on Modulo Unrolling

Modulo unrolling [2] is a bank disambiguation method used in tiled architectures that is applicable to loops with affine array accesses. An affine function of a set of variables is defined as a linear combination of those variables. An affine array access is any array access where each dimension of the array is accessed by an affine function of the loop induction variables. For example, for loop index variables  $i$  and  $j$  and array  $A$ ,  $A[i + 2j + 3][2j]$  is an affine access, but  $A[ij + 4][j^2]$  and  $A[2i^2 + 1][ij]$  are not.

Modulo unrolling works by unrolling the loop by a factor equal to the number of memory banks on the architecture. If the arrays accessed within the loop are distributed using low-order interleaving (a Cyclic distribution), then after unrolling, each array access will be guaranteed to reside on a single bank for all iterations of the loop. This is achieved with a modest increase of the code size.

To understand modulo unrolling, refer to Figure 4. In Figure 4a there is a code fragment consisting of a sequential **for** loop with a single array access  $A[i]$ . The array  $A$  is distributed over four memory banks using a Cyclic distribution. As is, the array  $A$  is not bank disambiguated because accesses of  $A[i]$  go to different memory banks on different iterations of the loop. The array access  $A[i]$  has bank access patterns 0, 1, 2, 3, 0, 1, 2, 3, ... in successive loop iterations.

A naive approach to achieving bank disambiguation is to fully unroll the loop, as shown in Figure 4b. Here, the original loop is unrolled by a factor of 100. Because each array access is independent of the loop induction variable  $i$ , bank disambiguation is achieved trivially. Each array access resides on a single memory bank. However, fully unrolling the loop is not an ideal solution to achieving bank disambiguation because of the large increase in code size. This increase in code size is bounded by the unroll factor, which may be extremely large for loops iterating over large arrays. Fully unrolling the loop may not even be possible for a loop bound that is unknown at compile time.

A more practical approach to achieving bank disambiguation without a dramatic increase in code size is to unroll the loop by a factor equal to the number of banks on the architecture. This is shown in Figure 4c and is known as modulo unrolling. Since we have 4 memory banks in this example, we unroll the loop by a factor of 4. Now every array reference in the loop maps to a single memory bank on all iterations of the loop. Specifically,  $A[i]$  refers to bank 0,  $A[i + 1]$  refers to bank 1,  $A[i + 2]$  refers to bank 2, and  $A[i + 3]$  refers to bank 3. The work in [2] shows that an unroll factor providing this property always exists not only for the code in Figure 4, but for the general case of any affine function loop. The unroll factor may not always equal the number of banks, but a suitable unroll factor can always be computed.

Modulo unrolling, as used in [2] provides bank disambiguation and memory parallelism for tiled architectures. That is, after unrolling, each array access can be done in parallel because array accesses map to a different memory banks. However, as we will show, modulo unrolling can also be used to aggregate messages and reduce communication costs in message passing machines.



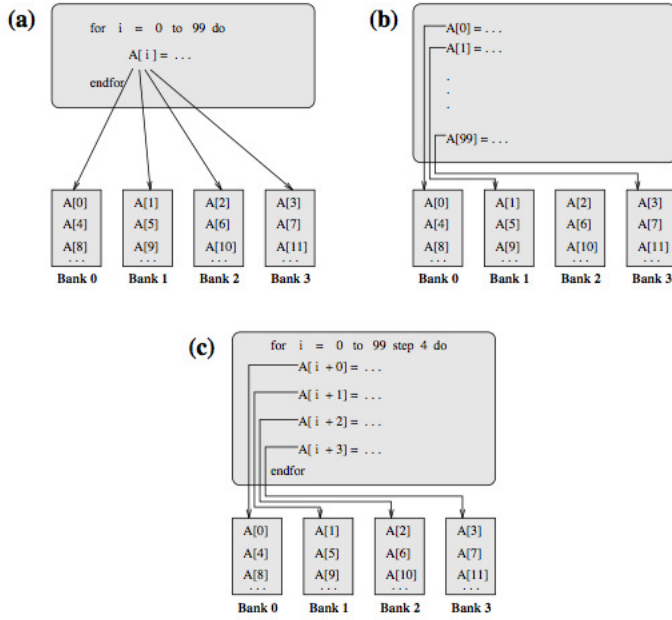


Figure 4: Modulo unrolling example. (a) Original sequential for loop. Array  $A$  is distributed using a Cyclic distribution. Each array access maps to a different memory bank on successive loop iterations. (b) Fully unrolled loop. Trivially, each array access maps to a single memory bank because each access only occurs once. This loop dramatically increases the code size for loops traversing through large data sets. (c) Loop transformed using modulo unrolling. The loop is unrolled by a factor equal to the number of memory banks on the architecture. Now each array access is guaranteed to map to a single memory bank for all loop iterations and code size increases only by the loop unroll factor.

## 4 Chapel Language Features Necessary for Modulo Unrolling

The goal of this section is to provide a basic understanding of zippered iteration and array slicing in Chapel and to show how the modulo unrolling optimization described in Sections 5, 7, and 8 fits in naturally with these language constructs. Any parallel loop with affine array accesses can be written using zippered iterators and array slicing. Therefore, parallel iterators are candidates within the language to implement the optimization.

### 4.1 Zippered Iteration

Iterators are a widely used language feature in the Chapel programming language. Chapel iterators are blocks of code that are similar to functions and methods except that iterators can return multiple values back to the call site with the use of the `yield` keyword instead of `return`. Iterators are commonly used in loops to traverse data structures in a particular fashion. For example, an iterator `fibonacci(n : int)` might be responsible for yielding the first  $n$  Fibonacci numbers. This iterator could then be called in a loop's header to execute iterations 0, 1, 1, 2, 3, and so on. Arrays themselves are iterable in Chapel by default.

Chapel allows multiple iterators of the same size and shape to be iterated through simultaneously. This is known as *zippered iteration* [4]. When zippered iteration is used, corresponding iterations are processed together. On each loop iteration, an  $n$ -tuple is generated, where  $n$  is the number of items in the zippering. The  $d^{th}$  component of the tuple generated on loop iteration  $j$  is the  $j^{th}$  item that would be yielded by iterator  $d$  in the zippering.

Zippered iteration can be used with either sequential `for` loops

or parallel `forall` loops in Chapel. Parallel zippered iteration is implemented in Chapel using leader-follower semantics. That is, a *leader* iterator is responsible for creating tasks and dividing up the work to carry out the parallelism. A *follower* iterator performs the work specified by the leader iterator for each task and generally resembles a serial iterator.

### 4.2 Array Slicing

Chapel supports another useful language feature known as *array slicing*. This feature allows portions of an array to be accessed and modified in a succinct fashion. For example, consider two arrays  $A$  and  $B$  containing indices from 1..10. Suppose we wanted to assign elements  $A[6]$ ,  $A[7]$ , and  $A[8]$  to elements  $B[1]$ ,  $B[2]$ , and  $B[3]$  respectively. We could achieve this in one statement by writing  $B[1..3] = A[6..8]$ . Here,  $A[6..8]$  is a slice of the original array  $A$ , and  $B[1..3]$  is a slice of the original array  $B$ . In Chapel, an array slice can support a range of elements with a stride in some cases. For example, in the previous example, we could have made the assignment  $B[1..3] = A[1..6 \text{ by } 2]$ . This would have assigned elements  $A[1]$ ,  $A[3]$ , and  $A[5]$  to elements  $B[1]$ ,  $B[2]$ , and  $B[3]$  respectively. Since all array slices in Chapel are arrays themselves, array slices are also iterable.

Together, array slicing and parallel zippered iteration can express any parallel affine loop in Chapel that uses affine array accesses. Each affine array access in the loop body is replaced with a corresponding array slice in the loop header, which produces the same elements as the original loop. Consider the code fragment in Figure 5a. There are two affine array accesses  $A[i]$  and  $B[i + 2]$  in Figure 5a. The loop is written in a standard way where the loop induction variable  $i$  takes on values from 1 to 10. Because the loop is a `forall` loop, loop iterations are not guaranteed to complete in a specific order. This loop assigns elements of array  $B$  to  $A$  such that the  $i^{th}$  element of  $A$  is equal to the  $(i + 2)^{th}$  element of  $B$  after the loop finishes. In Figure 5b, the same loop is written using zippered iterators. The loop induction variable  $i$  no longer needs to be specified, and each affine array access has been replaced with an array slice in the zippering of the loop header. It is possible to transform an affine loop in this fashion even when an affine array access has a constant factor multiplied by the loop induction variable. The resulting array slice will contain a stride equal to the constant factor. The two loops in Figure 5 are equivalent and generate the same results, but they differ in their execution.

Because any parallel affine loop can be transformed into an equivalent parallel loop that uses zippered iteration, we observe a natural place in the Chapel programming language in which to implement modulo unrolling: the leader and follower iterators of the Cyclic and Block Cyclic distribution. The leader iterator divides up the loop's iterations according to the locales they are executed on and passes this work to each follower iterator in the zippering. The follower iterator can then perform the aggregation of remote data elements according to the work that has been passed to it.

## 5 Intuition Behind Message Aggregation With An Example

In Chapel, a program's data access patterns and the programmer's choice of data distribution greatly influence the program's runtime and communication behavior. This section presents an example of

```

1  //(a) Parallel loop with affine array accesses
2  forall i in 1..10 {
3      A[i] = B[i+2];
4  }
5
6  //(b) Equivalent loop written using zippered iteration
7  forall (a,b) in zip(A[1..10], B[3..12]) {
8      a = b;
9  }

```

Figure 5: (a) Chapel loop written using a single loop induction variable  $i$  ranging from 1 to 10. The loop contains two affine array accesses. (b) The same loop written using zippered iterators in Chapel. Instead of a loop induction variable and a range of values to denote the loop bounds, two array slices each containing the 10 elements accessed by the loop in (a) are specified.

a Chapel program with affine array accesses that can benefit from message aggregation. It also serves to present the intuition behind the modulo unrolling optimization.

Consider the Chapel code for the Jacobi-2D computation shown in Figure 6, a common stencil operation that computes elements of a two-dimensional array as an average of that element’s four adjacent neighbors. We assume that arrays  $A$  and  $A_{new}$  have already been distributed using a Cyclic distribution over four locales. On each iteration of the loop, five array elements are accessed in an affine manner: the current array element  $A_{new}[i, j]$  and its four adjacent neighbors  $A[i + 1, j]$ ,  $A[i - 1, j]$ ,  $A[i, j + 1]$ , and  $A[i, j - 1]$ . The computation will take place on the locale of  $A_{new}[i, j]$ , the element being written to. If arrays  $A$  and  $A_{new}$  are distributed with a Cyclic distribution as shown in Figure 2, then it is guaranteed that  $A[i + 1, j]$ ,  $A[i - 1, j]$ ,  $A[i, j + 1]$ , and  $A[i, j - 1]$  will not reside on the same locale as  $A_{new}[i, j]$  **for all iterations of the loop**. Therefore, these remote elements need to be transferred over to  $A_{new}[i, j]$ ’s locale in four separate messages during every loop iteration. For large data sets, transferring four elements individually per loop iteration drastically slows down the program because the message overhead is incurred many times.

We observe that message aggregation of remote data elements is possible over the entire loop for the Jacobi-2D example. Aggregation will reduce the number of times the message overhead is incurred during the loop. When the data is distributed using a Cyclic distribution, all array accesses (including remote accesses) exhibit a predictable pattern of locality.

Figure 7 illustrates this pattern in detail for loop iterations that write to locale 3. During these iterations  $((i, j) = (2, 2), (i, j) = (4, 2), \text{etc.})$ , there are two remote accesses from locale 1 and two remote accesses from locale 2. The remote accesses from locale 1 correspond to the  $A[i, j + 1]$ , and  $A[i, j - 1]$  affine array accesses in Figure 6. If we highlight all the remote data elements corresponding to the  $A[i, j - 1]$  access that occur for loop iterations that write to locale 3, we end up with the array slice  $A[2..7 \text{ by } 2, 1..6 \text{ by } 2]$ , which contains the striped elements in Figure 7. This array slice can be communicated from locale 1 to a buffer on locale 3 before the loop executes in a single message. Then, during the loop, all  $A[i, j - 1]$  accesses can be replaced with accesses to the local buffer on locale 3. We have shown here how aggregation can occur for the  $A[i, j - 1]$  affine array access on loop iterations that write to locale 3. This same procedure applies to the other three remote accesses for locale 3. In addition, this same procedure applies to loop iterations that write to the remaining locales. Finally, this optimization can also be applied to the Block Cyclic distribution, as the data ac-

```

1  var n: int = 8;
2  var LoopSpace = {2..n-1, 2..n-1};
3
4  //Jacobi relaxation pass
5  forall (i,j) in LoopSpace {
6      A_new[i,j] = (A[i+1, j] + A[i-1, j] +
7                  A[i, j+1] + A[i, j-1])/4.0;
8  }
9
10 //update state of the system after the first
11 //relaxation pass
12 A[LoopSpace] = A_new[LoopSpace];

```

Figure 6: Chapel code for the Jacobi-2D computation over an  $8 \times 8$  two dimensional array. Arrays  $A$  and  $A_{new}$  are distributed with a Cyclic distribution and their declarations are not shown. During each iteration of the loop, the current array element  $A_{new}[i, j]$  gets the average of the four adjacent array elements of  $A[i, j]$ .

cess pattern is the same for elements in the same position within a block.

In this example, we chose to perform message aggregation with respect to the element that is written to during the loop. However, this is not always the best choice for all programs. To get better communication performance, we would like to assign loop iterations to locales with the most affine array accesses that are local. The result of this scheme is that elements that are written to during the loop may be the ones that are aggregated before the loop. If so, it is necessary to write these elements from the local buffers back to their remote locales. This is done in a single aggregate message after the loop body has finished.\*

If arrays  $A$  and  $A_{new}$  are instead distributed using Chapel’s Block or Block Cyclic distributions as shown in Figure 1 and Figure 3 respectively, the program will only perform remote data accesses on iterations of the loop where element  $A_{new}[i, j]$  is on the boundary of a block. As the block size increases, the number of remote data accesses for the Jacobi-2D computation decreases. For the Jacobi-2D computation, it is clear that distributing the data using Chapel’s Block distribution is the best choice in terms of communication. Executing the program using a Block distribution will result in fewer remote data accesses than when using a Block Cyclic distribution. Similarly, executing the program using a Block Cyclic distribution will result in fewer remote data accesses than when using a Cyclic distribution.

It is important to note that the Block distribution is not the best choice for all programs using affine array accesses. Programs with strided access patterns that use a Block distribution will have poor communication performance because accessed array elements are more likely to reside outside of a block boundary. For these types of programs, a Cyclic or Block Cyclic distribution will perform better.

## 6 Message Aggregation Loop Optimization using Modulo Unrolling

This section describes our method to transform an affine loop that computes on cyclically or block-cyclically distributed data into an

\*In Chapel, the programmer has some control over assigning loop iterations to locales. Therefore, our optimizations uses the programmer’s assignment of loop iterations to locales when performing message aggregation.

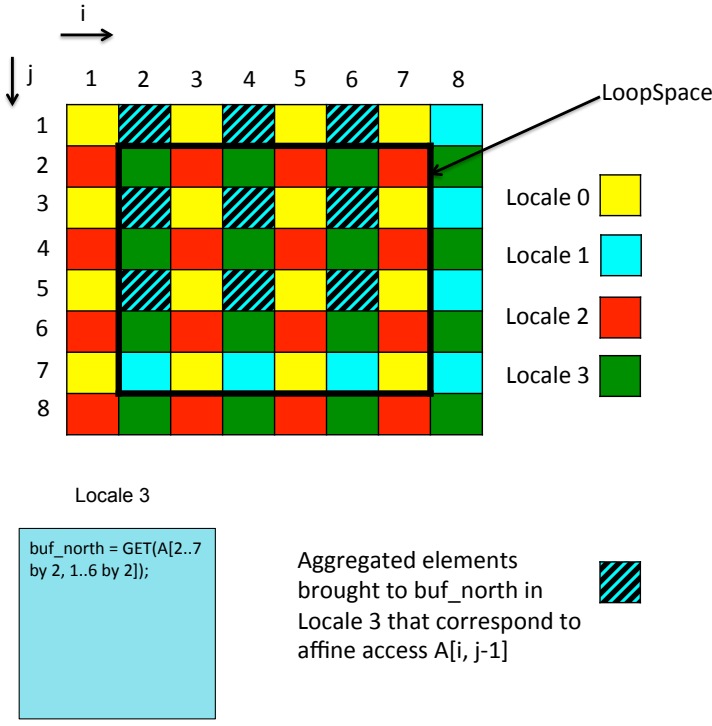


Figure 7: Illustration of message aggregation for the  $A[i, j - 1]$  affine array access of the Jacobi-2D relaxation computation. The region *LoopSpace* follows from Figure 6. The striped elements are the elements of *A* that have been aggregated. This same procedure occurs on each locale for each affine array access that is deemed to be remote for all iterations of the loop.

equivalent loop that performs message aggregation. The proposed method is based on modulo unrolling [2], described in Section 3. Here we describe the method in pseudocode for simplicity and to show that this method is applicable to languages other than Chapel.

Before presenting the optimization, we point out an important observation. Modulo unrolling statically disambiguates affine array accesses by unrolling the loop by a factor equal to the number of locales (or memory banks) on the system. For tiled architectures, this results in an increase in code size. However, for parallel machines that use message passing, static disambiguation can be achieved by using the locale identifier without increasing the code size. Conceptually, an affine loop on a message passing machine where data is distributed cyclically among four locales such as:

```
forall i in 0..99 {
  A[i] = B[i+2];
}
```

becomes statically disambiguated using this observation as follows:

```
forall i in 0..99 by 4 {
  A[i+$] = B[i+2+$];
}
```

where \$ represents the locale identifier.

The optimization consists of three steps shown in Figure 8: Block Cyclic transformation, owning expression calculation, and message aggregation. The optimization takes as its input a parallel **forall** loop that contains a number of affine array expressions

in its loop body. Non-affine expressions are allowed in the loop body, but they are not optimized. The input loop shown in Figure 8a is defined by three explicit parameters: the starting loop bound *s*, the ending loop bound *e*, and the loop stride *n*. The input loop also contains two implicit parameters based on the data distribution. The number of locales the data is distributed over is *N*, and the block size, the number of consecutive array elements allocated to a single locale, is *B*. All five parameters are elements of  $\mathbb{N}$ . The output of the optimization is an equivalent loop structure that aggregates communication from all of the loop body's remote affine array accesses.

## 6.1 Block Cyclic Transformation

Modulo unrolling as described in [2] guarantees static disambiguation for data distributed cyclically but not for block-cyclically distributed data. However, we can think of a Block Cyclic distribution as *B* adjacent Cyclic distributions, each with a cycle size greater than *N*. In order to achieve static disambiguation for the Block Cyclic distribution, we must transform input loops with  $B > 1$  into an equivalent loop with a loop step size that is a multiple of *B*.

Lines 1 and 2 of Figure 8b show this transformation. We replace the loop step size on line 2 with the *least common multiple* of *B* and *n*. To maintain the original meaning of the input loop, an outer **for** loop is added on line 1 to handle iterations within each block, and the starting loop bound on line 2 is written in terms of the outer loop variable *k*. After this transformation, all affine array accesses in the loop will be statically disambiguated.

Note that the Cyclic and Block Cyclic distributions are closely related. Any Cyclic distribution can be thought of as a Block Cyclic distribution with  $B = 1$ . If we apply the transformation in Figure 8b to a loop with cyclically distributed data, we will end up with the original input loop in Figure 8a, which is already statically disambiguated.

## 6.2 Owning Expression Calculation

There may be many affine array accesses in the input loop, each mapped to single locale after static disambiguation. For the best communication performance, we must determine the *owning expression* for the loop, which is the most common affine array expression in the loop body. More formally, the owning expression is an affine function  $f(i)$ , where *i* is the loop's induction variable, that occurs the most in the loop body. We can then use the owning expression to assign loop iterations to locales.

There are two affine array accesses Figure 8b:  $A[a_1i + b_1]$  and  $B[a_2i + b_2]$ . Each appears once in the loop body, so either expression can be chosen as the owning expression for the loop. For the remainder of Figure 8, we assume that  $a_1i + b_1$  is the owning expression.

## 6.3 Message Aggregation

The final step of the optimization is to aggregate all remote affine array accesses that are not accessed using the loop's owning expression before the loop starts. Figure 8c shows this transformation. In line 6, the array slice of *B* is fetched via a single GET message and brought to a local buffer. Now in lines 8-12,  $B[a_2i + b_2]$  can be replaced with an access to the local buffer. Lines 14-15 handle the



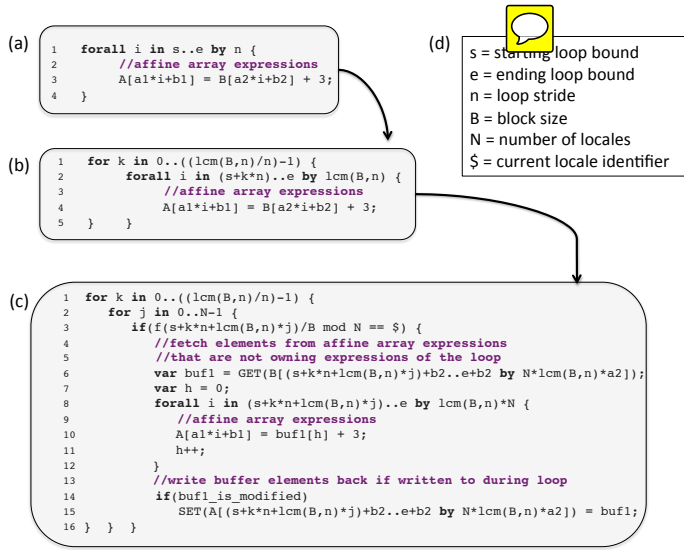


Figure 8: Transformations.

case that elements brought over in bulk need to be written back to their remote locale.

## 7 Cyclic Distribution with Modulo Unrolling

Modulo unrolling is implemented in the follower iterator of the Cyclic distribution. Based on the semantics of parallel zippered iteration, the leader iterator will divide up the iterations of the loop across the locales of the machine according to the first item in the zippering. This could mean that some portions of work will not be local to where the computation is taking place. The follower iterator in the Cyclic distribution recognizes whether or not its chunk of work is local or remote. If remote, all of the remote array elements are brought to the present locale in a local buffer using one `chpl_comm_gets` call. Finally, elements of the local buffer are now yielded back to the loop header. A loop body may modify the elements that are yielded to it via zippered iteration. To account for this, the follower iterator compares the element before it was yielded to the element after it was yielded. If any of the elements in the follower's chunk of work were modified, the entire local buffer is stored back to the remote local via one `chpl_comm_puts` call.

## 8 Block Cyclic Distribution with Modulo Unrolling

For the Chapel Block Cyclic implementation, both the leader and follower iterators have been modified to support the modulo unrolling optimization. Modulo unrolling, in its original form, is not compatible with the Block Cyclic distribution because consecutive array elements can reside on the same locale (this is defined by the block size parameter), which destroys the static locality information that we were able to use in the Cyclic distribution. The Block Cyclic leader iterator is now modified to choose slices of work such that the new "stride" is equal to the product of the block size and the cycle size. This way, when the work is passed to the follower iterator, elements that are in the same position within each block

are guaranteed to be on the same locale. The follower iterator of the Block Cyclic distribution can now perform modulo unrolling in the same way as the Cyclic distribution.

## 9 Results

To demonstrate the effectiveness of modulo unrolling in the Chapel Cyclic and Block Cyclic distributions, we present our results. We have compiled a suite of seventeen parallel benchmarks shown in Figure 9. Each benchmark is written in Chapel and contains loops with affine array accesses that use zippered iterations, as discussed in 4.2. Our suite of benchmarks contains programs with single, double, and triple nested affine loops. Additionally, our benchmark suite contains programs operating on one, two, and three-dimensional distributed arrays. Fourteen of the seventeen benchmarks are taken from the Polybench suite of benchmarks [1] and are translated from C to Chapel by hand. The *stencil9* benchmark was taken from the Chapel source trunk directory. The remaining two benchmarks, *pascal* and *folding*, were written by our group. *pascal* is an additional benchmark other than *jacobi1D* that is able to test Block Cyclic with modulo unrolling. *folding* is the only benchmark in our suite that has strided affine array accesses.

To evaluate improvements due to modulo unrolling, we run our benchmarks using Cyclic and Block Cyclic distributions from the 1.8.0 release of the Chapel compiler as well as Cyclic and Block Cyclic distributions that have been modified to perform modulo unrolling, as described in Sections 7 and 8. We measure both runtime and message count for each benchmark.

When evaluating modulo unrolling used with the Block Cyclic distribution, we could only run two benchmarks out of our suite of seventeen because of limitations within the original Chapel Block Cyclic distribution. Many of our benchmarks operate on two or three-dimensional arrays and all require array slicing for the modulo unrolling optimization to apply. Both array slicing of multi-dimensional arrays and array slicing containing strides for one-dimensional arrays are not yet supported in the Chapel compiler's Block Cyclic distribution. Implementing such features remained outside the scope of this work. There was no limitation when evaluating modulo unrolling with the Cyclic distribution, and all seventeen benchmarks were tested. Once these missing features are implemented in the Chapel compiler, then our method will apply to all of our benchmarks.

Figures 10 and 11 compare the normalized runtimes and message counts respectively for the Cyclic distribution and Cyclic distribution with modulo unrolling. For 8 of the 11 benchmarks, we see reductions in runtime when the modulo unrolling optimization is applied. On average, modulo unrolling results in a 45 percent decrease in runtime. For 9 of the 11 benchmarks, we see reductions in message counts when the modulo unrolling optimization is applied. On average, modulo unrolling results in 76 percent fewer messages.

Some detailed observations on Figures 10 and 11 follow. Two of the benchmarks, *cholesky* and *fw*, showed slight improvements in message count when using modulo unrolling but did not show improvements in runtime. For the *2mm* benchmark, both runtime and message count did not improve when using modulo unrolling. For these benchmarks, the ratio of the problem size to number of locales is not high enough, leading to an insufficient amount of aggregation possible for the computation to see performance im-



Name	Lines of Code	Input Size	Description
2mm	221	128 x 128	2 matrix multiplications ( $D=A*B$ ; $E=C*D$ )
fw	153	64 x 64	Floyd-Warshall all-pairs shortest path algorithm
trmm	133	128 x 128	Triangular matrix multiply
correlation	235	512 x 512	Correlation computation
covariance	201	512 x 512	Covariance computation
cholesky	182	256 x 256	Cholesky decomposition
lu	143	128 x 128	LU decomposition
mvt	185	4000	Matrix vector product and transpose
syrk	154	128 x 128	Symmetric rank-k operations
syr2k	160	128 x 128	Symmetric rank-2k operations
fdtd-2d	201	1000 x 1000	2D Finite Different Time Domain Kernel
fdtd-apml	333	64 x 64 x 64	FDTD using Anisotropic Perfectly Matched Layer
jacobi1D	138	10000	1D Jacobi stencil computation
jacobi2D	152	400 x 400	2D Jacobi stencil computation
stencil9†	142	400 x 400	9-point stencil computation
pascal‡	126	100000, 100003	Computation of pascal triangle rows
folding‡	139	50400	Strided sum of consecutive array elements

Figure 9: Benchmark suite. Benchmarks with no symbol after their name were taken from the Polybench suite of benchmarks and translated to Chapel. Benchmarks with † are taken from the Chapel Trunk test directory. Benchmarks with ‡ were developed on our own in order to test specific data access patterns.

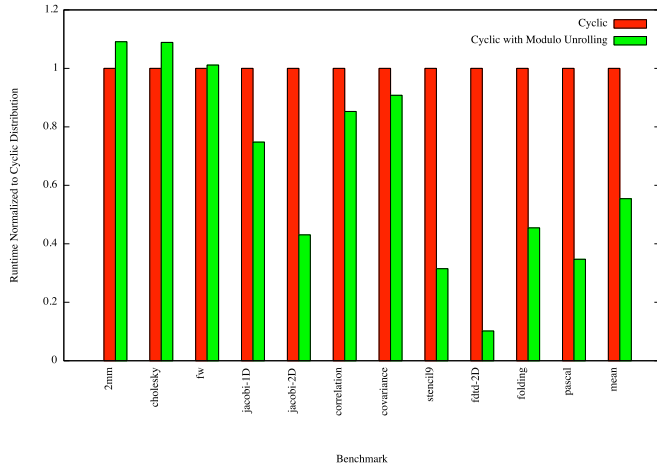


Figure 10: Cyclic runtime.

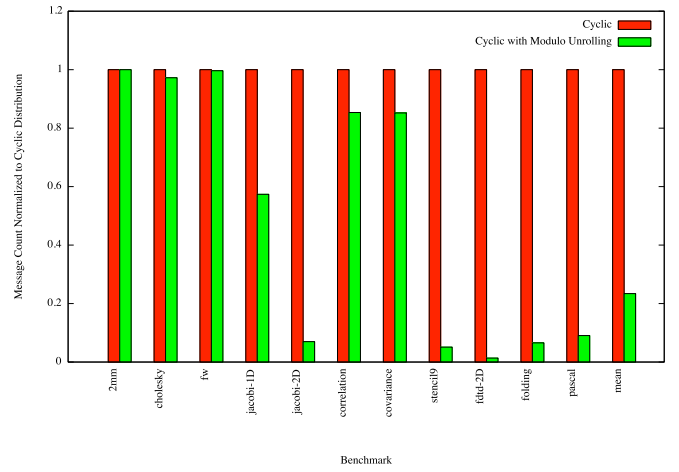


Figure 11: Cyclic message count.

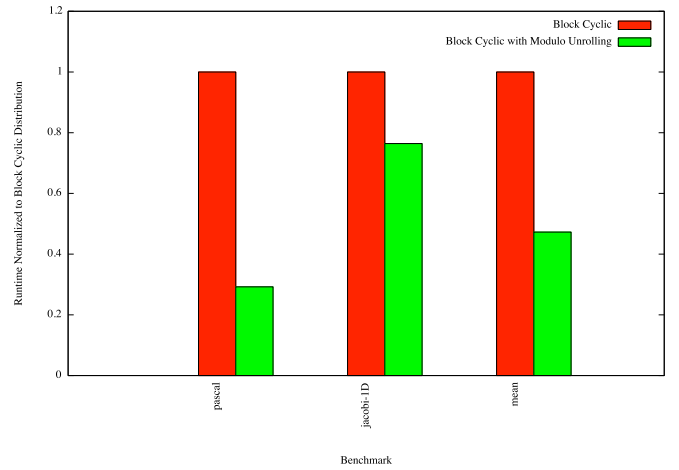


Figure 12: Block Cyclic runtime.

provements. An increase in the number of locales on a system leads to fewer data elements per locale, which naturally means fewer data elements can be aggregated. When this occurs, the cost of performing bulk transfers of a few data elements is more expensive than transferring elements individually.

Figures 12 and 13 compare the normalized runtimes and message counts respectively for the Block Cyclic distribution and Block Cyclic distribution with modulo unrolling. For both benchmarks, we see reductions in runtime when the modulo unrolling optimization is applied. On average, modulo unrolling results in a 52 percent decrease in runtime. For both benchmarks, we see reductions in message counts when the modulo unrolling optimization is applied. On average, modulo unrolling results in 72 percent fewer messages.

## 10 Future Work

As presented, the message aggregation optimization using modulo unrolling can be improved upon in a few ways to achieve even better performance in practice. First, there is currently no limit on the number of array elements that an aggregate message may contain. For applications with extremely large data sets, buffers containing

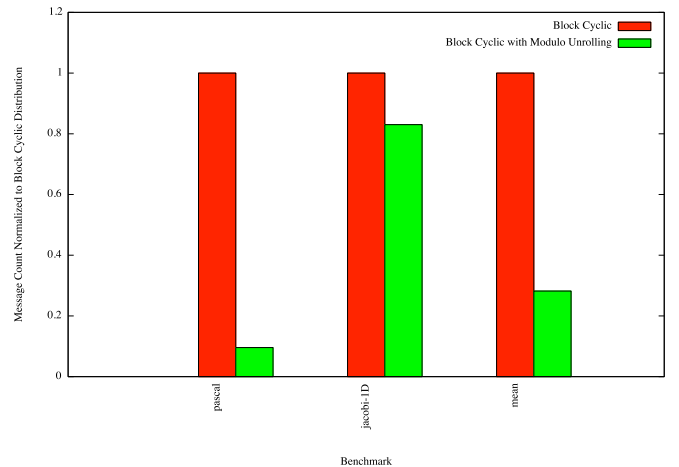



Figure 13: Block Cyclic message count.

remote data elements may become too large and exceed the memory budget of a particular locale. This may slow down other programs running on the system. To solve this, the modulo unrolling optimization should perform strip mining where the aggregate message is broken down into smaller sections if it contains too many elements in order to conserve memory.

The two forms of bulk communication used in this work (`chpl_comm_gets` and `chpl_comm_puts`) are both blocking communication calls. Our optimization might achieve better performance if it used a non-blocking strided bulk communication scheme. That way, communication and computation may be able to occur in parallel.

## References

- [1] Polybench/c- the polyhedral benchmark suite.
- [2] R. Barua, W. Lee, S. Amarasinghe, and A. Agarwal. Maps: a compiler-managed memory system for raw machines. In *ACM SIGARCH Computer Architecture News*, volume 27, pages 4–15. IEEE Computer Society, 1999.
- [3] D. Callahan and K. Kennedy. Compiling programs for distributed-memory multiprocessors. *The Journal of Supercomputing*, 2(2):151–169, 1988.
- [4] B. L. Chamberlain, S.-E. Choi, S. J. Deitz, and A. Navarro. User-defined parallel zippered iterators in chapel. 2011.
- [5] D. Chavarría-Miranda and J. Mellor-Crummey. Effective communication coalescing for data-parallel applications. In *Proceedings of the tenth ACM SIGPLAN symposium on Principles and practice of parallel programming*, pages 14–25. ACM, 2005.
- [6] W.-Y. Chen, C. Iancu, and K. Yelick. Communication optimizations for fine-grained upc applications. In *Parallel Architectures and Compilation Techniques, 2005. PACT 2005. 14th International Conference on*, pages 267–278. IEEE, 2005.
- [7] C. Germain and F. Delaplace. Automatic vectorization of communications for data-parallel programs. In *EURO-PAR’95 Parallel Processing*, pages 429–440. Springer, 1995.
- [8] G. Goumas, N. Drosinos, M. Athanasaki, and N. Koziris. Message-passing code generation for non-rectangular tiling transformations. *Parallel Computing*, 32(10):711–732, 2006.
- [9] M. Gupta and P. Banerjee. Automatic data partitioning on distributed memory multiprocessors. Technical report, 1991.
- [10] S. K. S. Gupta, S. Kaushik, C.-H. Huang, and P. Sadayappan. Compiling array expressions for efficient execution on distributed-memory machines. *Journal of Parallel and Distributed Computing*, 32(2):155–172, 1996.
- [11] C. Iancu, W. Chen, and K. Yelick. Performance portable optimizations for loops containing communication operations. In *Proceedings of the 22nd annual international conference on Supercomputing*, pages 266–276. ACM, 2008.
- [12] M. E. Mace. *Memory storage patterns in parallel processing*. Kluwer Academic Publishers, 1987.
- [13] L. Prylli and B. Tourancheau. Fast runtime block cyclic data redistribution on multiprocessors. *Journal of Parallel and Distributed Computing*, 45(1):63–72, 1997.
- [14] J. Ramanujam and P. Sadayappan. Compile-time techniques for data distribution in distributed memory machines. *Parallel and Distributed Systems, IEEE Transactions on*, 2(4):472–482, 1991.
- [15] A. Sanz, R. Asenjo, J. López, R. Larrosa, A. Navarro, V. Litvinov, S.-E. Choi, and B. L. Chamberlain. Global data re-allocation via communication aggregation in chapel. In *Computer Architecture and High Performance Computing (SBAC-PAD), 2012 IEEE 24th International Symposium on*, pages 235–242. IEEE, 2012.
- [16] S. D. Sung-Eun Choi. chapel: Distributions and layout .
- [17] E. Waingold, M. Taylor, D. Srikrishna, V. Sarkar, W. Lee, V. Lee, J. Kim, M. Frank, P. Finch, R. Barua, et al. Baring it all to software: Raw machines. *Computer*, 30(9):86–93, 1997.
- [18] D. W. Walker and S. W. Otto. Redistribution of block-cyclic data distributions using mpi. *Concurrency Practice and Experience*, 8(9):707–728, 1996.
- [19] W.-H. Wei, K.-P. Shih, J.-P. Sheu, et al. Compiling array references with affine functions for data-parallel programs. *J. Inf. Sci. Eng.*, 14(4):695–723, 1998.
- [20] J. Xue. Communication-minimal tiling of uniform dependence loops. In *Languages and Compilers for Parallel Computing*, pages 330–349. Springer, 1997.