Affine Loop Optimization Based On Modulo Unrolling in Chapel

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Abstract

This paper presents modulo unrolling without unrolling (WU), a method for message aggregation for parallel loops in message passing programs that use affine array accesses in Chapel, a Partitioned Global Address Space (PGAS) parallel programming language. Messages incur a non-trivial run time overhead, a significant component of which is independent of the size of the message. Therefore, aggregating messages improves performance. Our optimization for message aggregation is based on a technique known as modulo unrolling, pioneered by Barua [1], whose purpose was to ensure a statically predictable single tile number for each memory reference for tiled architectures, such as the MIT Raw Machine [2]. Modulo unrolling WU applies to data that is distributed in a cyclic or block-cyclic manner. In this paper, we adapt the aforementioned modulo unrolling technique to the difficult problem of efficiently compiling PGAS languages to message passing architectures. When applied to loops and data distributed cyclically or block-cyclically, modulo unrolling WU can decide when to aggregate messages thereby reducing the overall message count and runtime for a particular loop. Compared to other methods, modulo unrolling WU greatly simplifies the complex problem of automatic code generation of message passing code. It also results in substantial performance improvement compared to the non-optimized Chapel compiler.

To implement this optimization in Chapel, we modify the leader and follower iterators in the Cyclic and Block Cyclic data distribution modules. Results were collected that compare the performance of Chapel programs optimized with modulo unrolling WU and Chapel programs using the existing Chapel data distributions. Data collected on a ten-locale cluster show that on average, modulo unrolling WU used with Chapel's Cyclic distribution results in 76 percent fewer messages and a 45 percent decrease in runtime for our suite of benchmarks. Similarly, modulo unrolling WU used with Chapel's Block Cyclic distribution results in 72 percent fewer messages and a 52 percent decrease in runtime.

1 Introduction

Compilation of programs for distributed memory architectures using message passing is a vital task with potential for speedups over existing techniques. The Partitioned Global Address Space (PGAS) parallel programming model automates the production of message passing code from a shared memory programming model and exposes locality of reference information to the programmer, thereby improving programmability and allowing for compile-time performance optimizations. In particular, programs compiled to message passing hardware can improve in performance by aggregating messages and eliminating dynamic locality checks for affine array accesses in the PGAS model.

Message passing code generation is a difficult task for an optimizing compiler targeting a distributed memory architecture. These architectures are comprised of independent units of computation called *locales*. Each locale has its own set of processors, cores, memory, and address space. For programs executed on these architectures, data is distributed across various locales of the system, and the compiler needs to reason about locality in order to determine whether a program data access is *remote* (requiring a message to another locale to request a data element) or *local* (requiring no message and accessing the data element on the locale's own memory). Only a compiler with sufficient knowledge about locality can compile a program in this way with good communication performance.

Without aggregation, each remote data memory access results in a message with some non-trivial run time overhead, which can drastically slow down a program's execution time. This overhead is caused by latency on the interconnection network and locality checks for each data element. Accessing multiple remote data elements individually results in this run time overhead being incurred multiple times, whereas if they are transferred in bulk the overhead is only incurred once. Therefore, aggregating messages improves performance of message passing codes. In order to transfer remote data elements in bulk, the compiler must be sure that all elements in question reside on the same remote locale before the message is sent.

The vast majority of loops in scientific programs access data using *affine array accesses*. An affine array access is one whose indices are linear combinations of the loop's induction variables. For example, for a loop with induction variables i and j, accesses $A[i, j]$ and $A[2i - 3, j + 1]$ are affine, but $A[i^2]$ is not. Loops using affine array accesses are special because they exhibit regular access patterns within a data distribution. Compilers can use this information to decide when message aggregation can take place.

Existing methods for message passing code generation such as [3, 4] all have the following steps:

- **Loop distribution** The loop iteration space for each nested loop is divided into portions to be executed on each locale (message passing node), called iteration space tiles.
- **Data distribution** The data space for each array is distributed according to the directive of the programmer (usually as

block, cyclic, or block-cyclic distributions.)

- **Footprint calculation** For each iteration space tile, the portion of data it accesses for each array reference is calculated as a formula on the symbolic iteration space bounds. This is called the data footprint of that array access.
- **Message aggregation calculation** For each array access, its data footprint is separately intersected with each possible locale's data tile to derive symbolic expressions for the portion of the data footprint on that locale's data tile. This portion of the data tile for locales other than the current locale needs to be communicated remotely from each remote data tile's locale to the current loop tile's locale. Since the entire remote portion is calculated exactly, sending it in a single aggregated message becomes possible.

Unfortunately, of the steps above, the message aggregation calculation is by far the most complex. Loop distribution and data distribution are straightforward. Footprint calculation is of moderate complexity using matrix formulations or the polyhedral model. However, it is the message aggregation calculation that defies easy mathematical characterization for the general case of affine accesses. Instead some very complex research methods [5, 4] have been devised that make many simplifying assumptions on the types of affine accesses supported, and yet remain so complex that they are rarely implemented in production compilers.

Although the steps above are primarily for traditional methods of parallel code generation, polyhedral methods don't fare much better. Polyhedral methods have powerful mathematical formulations for loop transformation discovery, automatic parallelization, and parallelism coarsening. However message aggregation calculation is still needed but not modeled well in polyhedral models, leading to less capable ad-hoc methods for it.

It is our belief that message aggregation using tiling is not used in production quality compilers today because of the complexity of message aggregation calculations, described above. What is needed is a simple, robust, and widely applicable method for message aggregation that leads to improvements in performance.

This paper presents modulo unrolling without unrolling (WU), a loop optimization for message passing code generation based on a technique called modulo unrolling, whose advantage is that it makes the message aggregation calculation above far simpler. Using modulo unrolling WU, the locality of any affine array access can be deduced at compile time if the data is distributed in a cyclic or block-cyclic fashion. The optimization can be performed by a compiler to aggregate messages and reduce a program's execution time and communication.

Modulo unrolling in its original form, pioneered by [1], was meant to target tiled architectures such as the MIT Raw machine. Its purpose for tiled architectures was to allow the use of the compiler-routed static network for accessing array data in unrolled loops. It was not meant for message passing architectures, nor was it used to perform message aggregation. It has since been modified to apply to message passing machines in this work.

Modulo unrolling works as follows. It its basic form, it unrolls each affine loop by a factor equal to the number of locales of the machine being utilized by the program. If the arrays used in the loop are distributed cyclically or block-cyclically, each array access is guaranteed to reside on a single locale across all iterations of the loop. Using this information, the compiler can then aggregate all remote array accesses that reside on a remote locale into a single message before the loop. If remote array elements are written to during the loop, a single message is required to store these elements back to each remote locale after the loop runs.

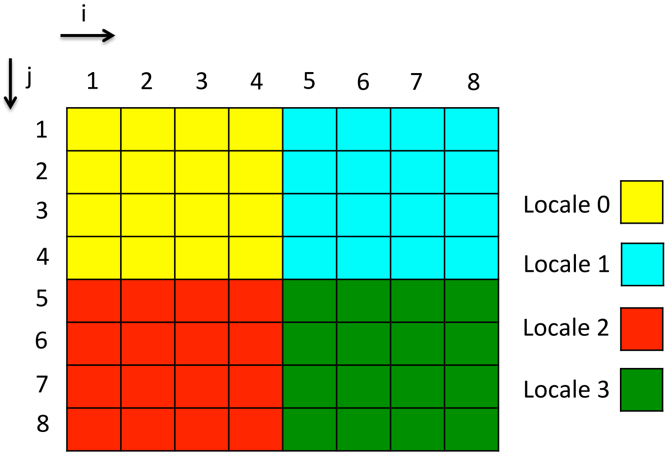


Figure 1: Chapel Block distribution.

We build on the modulo unrolling method to solve the very difficult problem of message aggregation for message passing machines inside PGAS languages. In the PGAS model, a system's memory is abstracted to a single global address space regardless of the hardware architecture and is then logically divided per locale and thread of execution. By doing so, locality of reference can easily be exploited no matter how the system architecture is organized.

Our evaluation is for Chapel, an explicitly parallel programming language developed by Cray Inc. that falls under the PGAS memory model. The Chapel compiler is an open source project used by many in industry and academic settings. The language contains many high level features such as zippered iteration, leader and follower iterator semantics, and array slicing that greatly simplify the implementation of modulo unrolling into the Cyclic and Block Cyclic distribution modules of the language.

Although our method is described in the context of Chapel, it is adaptable to any PGAS language. However, for other languages the implementation may differ. For example, if the language does not use leader and follower iterator semantics to implement parallel for loops, the changes to those Chapel modules that we present here will have to be implemented elsewhere in the other PGAS language where **forall** loop functionality is implemented.

2 Chapel's Data Distributions

Figures 1 - 3 illustrate the Chapel data distributions that we explored in this work: Block, Cyclic, and Block Cyclic. Each figure shows how a two-dimensional 8 x 8 array can be distributed in Chapel using each distribution. Figure 1 illustrates the Block distribution. Elements of the array are mapped to locales evenly in a dense manner. In Figure 2, the Cyclic distribution, elements of the array are mapped in a round-robin manner across locales. Finally, in Figure 3 the Block Cyclic distribution is shown. Here, a number of elements specified by a block size parameter is allocated to consecutive array indices in a round-robin fashion. In Figure 3, the distribution takes in a 2 x 2 block size parameter. Further details about Block, Cyclic, and Block Cyclic distributions in Chapel are

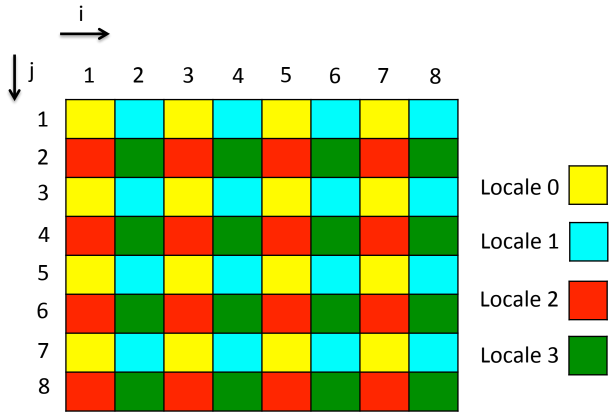


Figure 2: Chapel Cyclic distribution.

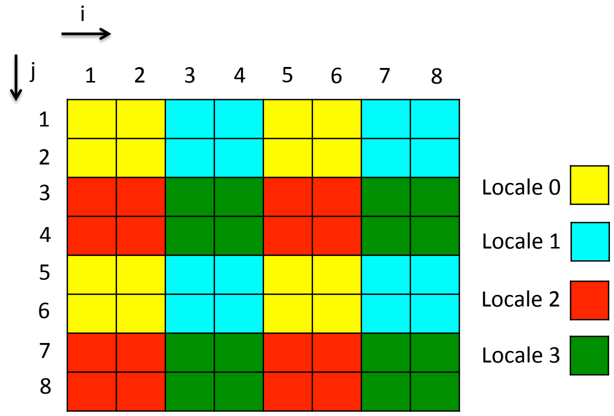


Figure 3: Chapel Block Cyclic distribution with a 2 x 2 block size parameter.

described in [6].

The choice of data distribution to use for a program boils down to computation and communication efficiency. Different programs and architectures may require different data distributions. It has been shown that finding an optimal data distribution for parallel processing applications is an NP-complete problem, even for one- or two-dimensional arrays [7]. Certain program data access patterns will result in fewer communication calls if the data is distributed in a particular way. For example, many loops in stencil programs that contain nearest neighbor computation will have better communication performance if the data is distributed using a Block distribution. This occurs because on a given loop iteration, the elements accessed are near each other in the array and therefore are more likely to reside on the same locale block. Accessing elements on the same block does not require a remote data access and can be done faster. However, programs that access array elements far away from each other will have better communication performance if data is distributed using a Cyclic distribution. Here, a Block distribution is almost guaranteed to have poor performance because the farther away accessed elements are, the more likely they reside on different locales.

A programmer may choose a particular data distribution for reasons unknown to the compiler. These reasons may not even take communication behavior into account. For example, Cyclic and Block Cyclic distributions provide better load balancing of data across locales than a Block distribution when array sizes may be changed dynamically because in Cyclic and Block Cyclic distributions, the locales of existing array elements do not change when new array elements are added at the end of the array. In many applications, data redistribution may be needed if elements of a data set are inserted or deleted at the end of the array. In particular, algorithms to redistribute data using a new block size exist for the Block Cyclic distribution [8, 9]. If an application uses a dynamic data set with elements that are appended, a Cyclic or Block Cyclic distribution is superior to Block because new elements are added to the locale that follows the cyclic or block-cyclic pattern. For Block, the entire data set would need to be redistributed every time a new element is appended, which can be expensive.

Compatibility with other PGAS languages might be an important consideration for a programmer when selecting a data distribution. Data sets used by Chapel programs and other PGAS programs should use Cyclic or Block Cyclic distributions because other PGAS languages may not support the Block distribution. A programmer would benefit by distributing the same data set using only one scheme so the data would not have to be replicated for different programs. This is an important consideration for vast data sets which have already been distributed on message passing computers, and we want to perform additional computation on them, perhaps with other programs.

Therefore in this work, it is our view that the compiler should not change the programmer's choice of data distribution in order to achieve better runtime and communication performance. The compiler should attempt to perform optimizations based on the data distribution that the programmer specified. Our optimization is meant to be applied whenever the programmer specifies a Cyclic or Block Cyclic distribution. It is not applied when the programmer specifies a Block distribution.

3 Related Work

Compilation for distribution memory machines has two main steps: loop optimizations and message passing code generation. First, the compiler performs loop transformations and optimizations to uncover parallelism, improve the granularity of parallelism, and improve cache performance. These transformations include loop peeling, loop reversal, and loop interchange. Chapel is an explicitly parallel language, so uncovering parallelism is not needed. Other loop optimizations to improve the granularity of parallelism and improve cache performance are orthogonal to this paper. The second step is message passing code generation, which includes message aggregation.

Message passing code generation in the traditional model is exceedingly complex, and practical robust implementations are hard to find. These methods [4, 5, 10, 11] require not only footprint calculations for each tile but also the intersection of footprints with data tiles. As described in detail in Section 1, calculating such intersections is very complex, which explains the complexity and simplifying limitations of many existing methods. Such methods are rarely if ever implemented in production compilers.

The polyhedral method is another branch of compiler optimization that seeks to speed up parallel programs on distributed memory architectures [3, 12, 13, 14, 15, 16]. The primary purpose of the polyhedral method is uncovering parallelism and loop optimization, not code generation. Its strength is that it can find sequences of transformations in one step, without searching the entire space of transformations. However, the method at its core does not compute information for message passing code generation. Message passing code generation does not fit the polyhedral model, so ad-hoc methods for code generation have been devised to work on the output of the polyhedral model. However they are no better than corresponding methods in the traditional model, and suffer from many of the same difficulties.

Similar work to take advantage of communication aggregation on distributed arrays has already been done in Chapel. Like distributed parallel loops in Chapel, whole array assignment suffers from locality checks for every array element, even when the locality of certain elements is known in advance. In [17], aggregation is applied to improve the communication performance of whole array assignments for Chapel's Block and Cyclic distributions. However, [17] does not address communication aggregation that is possible across general affine loops. Whole array assignment and affine loops in Chapel are fundamentally related because every whole array assignment can be written in terms of an equivalent affine **forall** loop. Our method for communication aggregation in parallel loops encompasses more complex affine array accesses than those that are found in whole array assignments and addressed in [17], and our work applies to both Chapel's Cyclic and Block Cyclic data distributions.

One of the contribution's of [17] included two new strided bulk communication primitives for Chapel developers as library calls, `chpl_comm_gets` and `chpl_comm_puts`. They both rely on the GASNet networking layer, a portion of the Chapel runtime. Our optimization uses these new communication primitives in our implementation directly to perform bulk remote data transfer between locales. The methods in [17] are already in the current release of the Chapel compiler.

Work has been done with the UPC compiler (another PGAS language) by [18] to improve on its communication performance. This method targets fine-grained communication and uses techniques such as redundancy elimination, split-phase communication, and communication coalescing (similar to message aggregation) to reduce overall communication. In communication coalescing, small puts and gets are combined into larger messages by the compiler to reduce the number of times the per-message startup overhead is incurred. This work's aggregation scheme is only applicable to programs with many small, individual, and independent remote array accesses. It is not clear whether this method can be used to improve communication performance across more coarse-grained structures, such as distributed parallel loops. Another major limitation to this work's aggregation scheme is that only contiguous data can be sent in bulk. To aggregate data across an entire loop in a single message when data is distributed cyclically, which is done in our work, it must be possible to aggregate data elements that are far apart in memory, separated by a fixed stride. Strided memory accesses are now possible in UPC using the low-level communication calls found in [19], but this was not explored in [18]. Our method handles this by using the strided get and put calls (`chpl_comm_gets` and `chpl_comm_puts`) from [17], described earlier.

4 Background on Modulo Unrolling

Modulo unrolling [1] is a static disambiguation method used in tiled architectures that is applicable to loops with affine array accesses. An affine function of a set of variables is defined as a linear combination of those variables. An affine array access is any array access where each dimension of the array is accessed by an affine function of the loop induction variables. For example, for loop index variables i and j and array A, $A[i + 2j + 3][2j]$ is an affine access, but $A[ij + 4][j^2]$ and $A[2i^2 + 1][ij]$ are not.

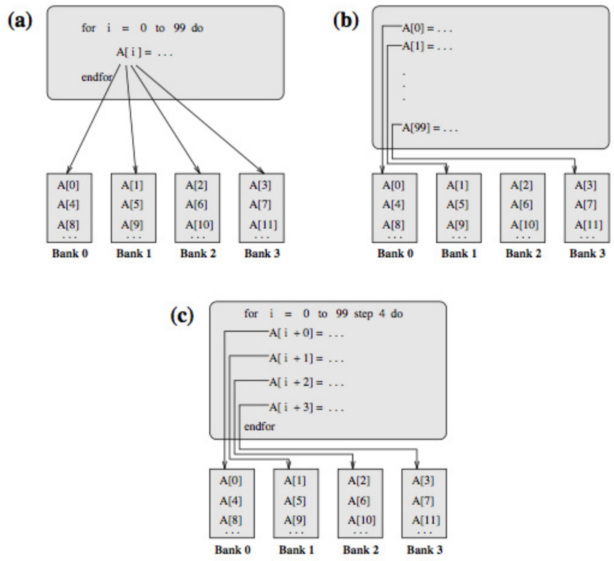


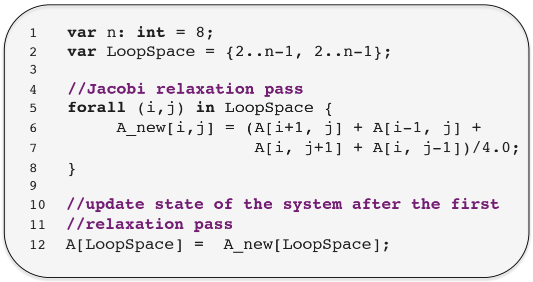
Figure 4: Modulo unrolling example. (a) Original sequential for loop. Array A is distributed using a Cyclic distribution. Each array access maps to a different memory bank on successive loop iterations. (b) Fully unrolled loop. Trivially, each array access maps to a single memory bank because each access only occurs once. This loop dramatically increases the code size for loops traversing through large data sets. (c) Loop transformed using modulo unrolling. The loop is unrolled by a factor equal to the number of memory banks on the architecture. Now each array access is guaranteed to map to a single memory bank for all loop iterations and code size increases only by the loop unroll factor.

Modulo unrolling works by unrolling the loop by a factor equal to the number of memory banks on the architecture. If the arrays accessed within the loop are distributed using low-order interleaving (a Cyclic distribution), then after unrolling, each array access will be *statically disambiguated*, or guaranteed to reside on a single bank for all iterations of the loop. This is achieved with a modest increase of the code size.

To understand modulo unrolling, refer to Figure 4. In Figure 4a there is a code fragment consisting of a sequential **for** loop with a single array access $A[i]$. The array A is distributed over four memory banks using a Cyclic distribution. As is, the array A is not statically disambiguated because accesses of $A[i]$ go to different memory banks on different iterations of the loop. The array access $A[i]$ has bank access patterns 0, 1, 2, 3, 0, 1, 2, 3, ... in successive loop iterations.

A naive approach to achieving static disambiguation is to fully unroll the loop, as shown in Figure 4b. Here, the original loop is unrolled by a factor of 100. Because each array access is independent of the loop induction variable i, static disambiguation is achieved trivially. Each array access resides on a single memory bank. However, fully unrolling the loop is not an ideal solution to achieving static disambiguation because of the large increase in code size. This increase in code size is bounded by the unroll factor, which may be extremely large for loops iterating over large arrays. Fully unrolling the loop may not even be possible for a loop bound that is unknown at compile time.

A more practical approach to achieving static disambiguation without a dramatic increase in code size is to unroll the loop by



```
var n: int = 8;
var LoopSpace = {2..n-1, 2..n-1};

//Jacobi relaxation pass
forall (i,j) in LoopSpace {
     A_new[i,j] = (A[i+1, j] + A[i-1, j] +
                   A[i, j+1] + A[i, j-1])/4.0;
}

//update state of the system after the first
//relaxation pass
A[LoopSpace] =  A_new[LoopSpace];
```

Figure 5: Chapel code for the Jacobi-2D computation over an 8 x 8 two dimensional array. Arrays A and A_{new} are distributed with a Cyclic distribution and their declarations are not shown. During each iteration of the loop, the current array element $A_{new}[i,j]$ gets the average of the four adjacent array elements of $A[i,j]$.

a factor equal to the number of banks on the architecture. This is shown in Figure 4c and is known as modulo unrolling. Since we have 4 memory banks in this example, we unroll the loop by a factor of 4. Now every array reference in the loop maps to a single memory bank on all iterations of the loop. Specifically, $A[i]$ refers to bank 0, $A[i+1]$ refers to bank 1, $A[i+2]$ refers to bank 2, and $A[i+3]$ refers to bank 3. The work in [1] shows that an unroll factor providing this property always exists not only for the code in Figure 4, but for the general case of any affine function in a loop. The unroll factor may not always equal the number of banks, but a suitable unroll factor can always be computed.

Modulo unrolling, as used in [1] provides static disambiguation and memory parallelism for tiled architectures. That is, after unrolling, each array access can be done in parallel because array accesses map to a different memory banks.

The intuition behind why modulo unrolling is helpful for message aggregation in message passing machines is as follows. Message aggregation requires knowledge of precisely which elements must be communicated between locales. Doing so requires a statically disambiguated known locale for every array access, even when that array access refers to a varying address. For example, in a loop $A[i]$ refers to different memory addresses during each loop iteration. Modulo unrolling ensures such a known, predictable locale number for each varying array access. This enables such varying accesses to be characterized and aggregated to be sent in a single message. We explain our method of doing so in Sections 6 and 7.

5 Intuition Behind Message Aggregation With An Example

In Chapel, a program's data access patterns and the programmer's choice of data distribution greatly influence the program's runtime and communication behavior. This section presents an example of a Chapel program with affine array accesses that can benefit from message aggregation. It also serves to present the intuition behind how modulo unrolling WU will be used in message aggregation.

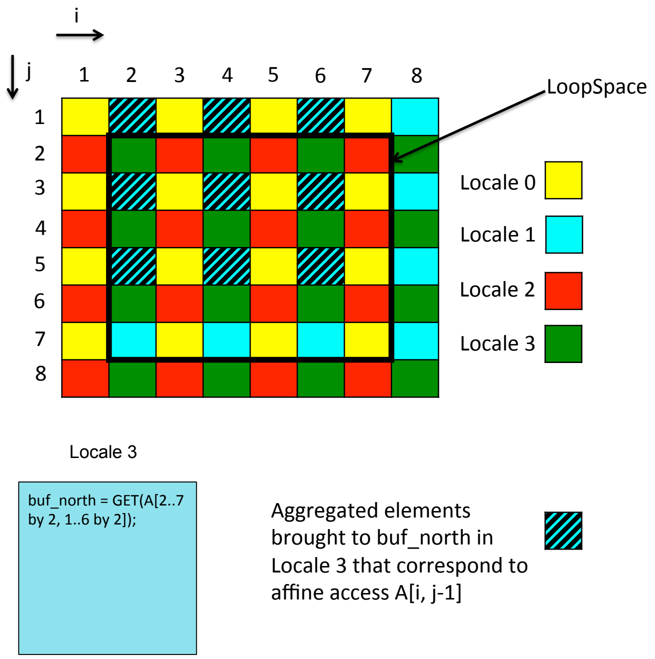

i
j
1
2
3
4
5
6
7
8
LoopSpace
Locale 0
Locale 1
Locale 2
Locale 3
Locale 3
buf_north = GET(A[2..7 by 2, 1..6 by 2]);
Aggregated elements brought to buf_north in Locale 3 that correspond to affine access A[i, j-1]

Figure 6: Illustration of message aggregation for the $A[i,j-1]$ affine array access of the Jacobi-2D relaxation computation with respect to locale 3. The region *LoopSpace* follows from Figure 5. The striped squares are the elements of A that have been aggregated. This same procedure occurs on each locale for each affine array access that is deemed to be remote for all iterations of the loop. For the whole 8 x 8 Jacobi-2D calculation, 144 remote gets containing one element each are necessary without aggregation, but only 16 remote gets containing nine elements each are necessary with aggregation.

Consider the Chapel code for the Jacobi-2D computation shown in Figure 5, a common stencil operation that computes elements of a two-dimensional array as an average of that element's four adjacent neighbors. We assume that arrays A and A_{new} have already been distributed using a Cyclic distribution over four locales. On each iteration of the loop, five array elements are accessed in an affine manner: the current array element $A_{new}[i,j]$ and its four adjacent neighbors $A[i+1,j]$, $A[i-1,j]$, $A[i,j+1]$, and $A[i,j-1]$. The computation will take place on the locale of $A_{new}[i,j]$, the element being written to. If arrays A and A_{new} are distributed with a Cyclic distribution as shown in Figure 2, then it is guaranteed that $A[i+1,j]$, $A[i-1,j]$, $A[i,j+1]$, and $A[i,j-1]$ will not reside on the same locale as $A_{new}[i,j]$ **for all iterations of the loop**. Therefore, these remote elements need to be transferred over to $A_{new}[i,j]$'s locale in four separate messages during every loop iteration. For large data sets, transferring four elements individually per loop iteration drastically slows down the program because the message overhead is incurred many times.

We observe that message aggregation of remote data elements is possible over the entire loop for the Jacobi-2D example. Aggregation will reduce the number of times the message overhead is incurred during the loop. When the data is distributed using a Cyclic distribution, all array accesses (including remote accesses) exhibit a predictable pattern of locality.

Figure 6 illustrates this pattern in detail for loop iterations that write to locale 3. During these iterations ($(i,j) = (2,2), (i,j) = (4,2)$, etc.), there are two remote accesses from locale 1 and two remote accesses from locale 2. The remote accesses from locale 1 correspond to the $A[i,j+1]$, and $A[i,j-1]$ affine array accesses in Figure 5. If we highlight all the remote data elements corresponding to the $A[i,j-1]$ access that occur for loop iterations that write to locale 3, we end up with the array slice $A[2..7\ by\ 2, 1..6\ by\ 2]$, which contains the striped elements in Figure 6. This array slice can be communicated from locale 1 to a buffer on locale 3 before the loop executes in a single message. Then, during the loop, all $A[i,j-1]$ accesses can be replaced with accesses to the local buffer on locale 3.

The previous paragraph showed how aggregation occurs for the

$A[i, j-1]$ affine array access on loop iterations that write to locale 3. This same procedure applies to the other three remote accesses for locale 3. In addition, this same procedure applies to loop iterations that write to the remaining locales. Finally, we claim that this optimization can also be applied to the Block Cyclic distribution, as the data access pattern is the same for elements in the same position within a block.

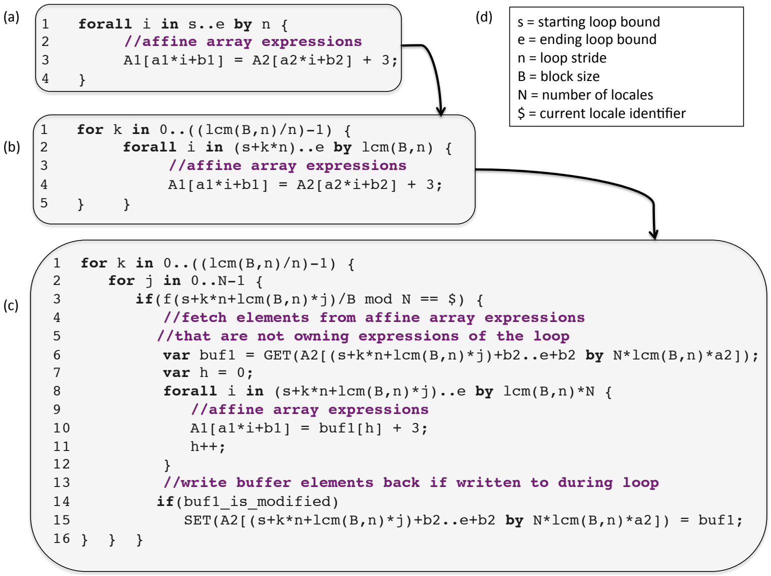
In this example, we chose to perform message aggregation with respect to the element that is written to during the loop. However, this is not always the best choice for all programs. To get better communication performance, we would like to assign loop iterations to locales with the most affine array accessses that are local. The result of this scheme is that elements that are written to during the loop may be the ones that are aggregated before the loop. If so, it is necessary to write these elements from the local buffers back to their remote locales. This is done in a single aggregate message after the loop body has finished.*

If arrays A and A_{new} are instead distributed using Chapel's Block or Block Cyclic distributions as shown in Figure 1 and Figure 3 respectively, the program will only perform remote data accesses on iterations of the loop where element $A_{new}[i, j]$ is on the boundary of a block. As the block size increases, the number of remote data accesses for the Jacobi-2D computation decreases. For the Jacobi-2D computation, it is clear that distributing the data using Chapel's Block distribution is the best choice in terms of communication. Executing the program using a Block distribution will result in fewer remote data accesses than when using a Block Cyclic distribution. Similarly, executing the program using a Block Cyclic distribution will result in fewer remote data accesses than when using a Cyclic distribution.

It is important to note that the Block distribution is not the best choice for all programs using affine array accesses. Programs with strided access patterns that use a Block distribution will have poor communication performance because accessed array elements are more likely to reside outside of a block boundary. For these types of programs, a Cyclic or Block Cyclic distribution will perform better. Section 2 explained several reasons why the programmer may have chosen a Cyclic or Block Cyclic distribution.

6 Message Aggregation Loop Optimization for Parallel Affine Loops

This section describes modulo unrolling without unrolling (WU), our method to transform an affine loop that computes on cyclically or block-cyclically distributed data into an equivalent loop that performs message aggregation. As described in Section 2, our method is not meant for block distributed data. The proposed method is based on modulo unrolling [1], described in Section 4. Here we describe the method in pseudocode for simplicity and to show that this method is applicable to languages other than Chapel.



```
forall i in s..e by n {
    //affine array expressions
    A1[a1*i+b1] = A2[a2*i+b2] + 3;
}
```

```
for k in 0..((lcm(B,n)/n)-1) {
    forall i in (s+k*n)..e by lcm(B,n) {
        //affine array expressions
        A1[a1*i+b1] = A2[a2*i+b2] + 3;
}    }
```

```
for k in 0..((lcm(B,n)/n)-1) {
   for j in 0..N-1 {
      if(f(s+k*n+lcm(B,n)*j)/B mod N == $) {
         //fetch elements from affine array expressions
        //that are not owning expressions of the loop
         var buf1 = GET(A2[(s+k*n+lcm(B,n)*j)+b2..e+b2 by N*lcm(B,n)*a2]);
         var h = 0;
         forall i in (s+k*n+lcm(B,n)*j)..e by lcm(B,n)*N {
            //affine array expressions
            A1[a1*i+b1] = buf1[h] + 3;
            h++;
         }
         //write buffer elements back if written to during loop
        if(buf1_is_modified)
           SET(A2[(s+k*n+lcm(B,n)*j)+b2..e+b2 by N*lcm(B,n)*a2]) = buf1;
} } }
```

s = starting loop bound
e = ending loop bound
n = loop stride
B = block size
N = number of locales
$ = current locale identifier

Figure 7: Steps to transform a parallel affine loop where the data is distributed cyclically or block cyclically into an equivalent loop that performs message aggregation. (a) Original parallel loop with two affine array accesses. (b) Loop after Block Cyclic transformation. After this step, the affine array accesses in loops with data distributed block cyclically will be statically disambiguated.

6.1 Modulo Unrolling Without Unrolling

Modulo unrolling increases code size because it unrolls loops by a factor equal to the number of locales (memory banks) on the system. However modulo unrolling WU for message passing machines does not increase code size. For parallel machines that use message passing, static disambiguation can be achieved by using the locale identifier without increasing the code size. Conceptually, an affine loop written in source code on a message passing machine where data is distributed cyclically among four locales such as:

```
forall i in 0..99 {
    A[i] = B[i+2];
}
```

becomes statically disambiguated using this observation as follows:

```
forall i in 0..99 by 4 {
    A[i+$] = B[i+2+$];
}
```

where $ represents the locale identifier. The above is the code that is run on each locale.

Figure 7 shows how a generalized affine loop, expressed symbolically, can be transformed by our method in three steps: the Block Cyclic transformation (Figure 7a → Figure 7b), the owning expression calculation (described in Section 6.3), and the message aggregation (Figure 7b → Figure 7c). The optimization takes as its input a parallel **forall** loop that contains a number of affine array expressions in its loop body. Non-affine expressions are allowed in the loop body, but they are not optimized. The input loop shown in Figure 7a is defined by three explicit parameters: the starting loop bound $s$, the ending loop bound $e$, and the loop stride $n$. The input loop also contains two implicit parameters based on the data distribution. The number of locales the data is distributed over is $N$, and the block size, the number of consecutive array elements allocated to a single locale, is $B$. All five parameters are elements of $\mathbb{N}$. The output of the optimization is an equivalent loop structure

*In Chapel, the programmer has some control over assigning loop iterations to locales. Therefore, our optimizations uses the programmer's assignment of loop iterations to locales when performing message aggregation.

that aggregates communication from all of the loop body's remote affine array accesses.

6.2 Block Cyclic Transformation

Modulo unrolling as described in [1] guarantees static disambiguation for data distributed cyclically but not for block-cyclically distributed data. However, we can think of a Block Cyclic distribution as B adjacent Cyclic distributions, each with a cycle size that is greater than N. In order to achieve static disambiguation for the Block Cyclic distribution, we must transform input loops with $B > 1$ into an equivalent loop with a loop step size that is a multiple of B.

Lines 1 and 2 of Figure 7b show this transformation. We replace the loop step size on line 1 of Figure 7a with the *least common multiple* of B and n in line 2 of Figure 7b. The intuition behind this new step size is that two successive loop iterations accessing the same position within a block will always be separated by a fixed stride length that is a multiple of the block size. To maintain the original meaning of the input loop, an outer **for** loop is added on line 1 of Figure 7b to handle iterations within each block, and the starting loop bound on line 2 is written in terms of the outer loop variable k. After this transformation, all affine array accesses in the loop with be statically disambiguated. This transformation is a variant of the well-known strip mining transformation which has been used for many other purposes in the literature.

The Cyclic and Block Cyclic distributions are closely related. Any Cyclic distribution can be thought of as a Block Cyclic distribution with B = 1. If we apply the transformation in Figure 7b to a loop with cyclically distributed data, we will end up with the original input loop in Figure 7a, which is already statically disambiguated.

6.3 Owning Expression Calculation

There may be many affine array accesses in the input loop, each mapped to a single locale after static disambiguation. For the best communication performance, we must determine the *owning expression* for the loop, which is the most common affine array expression in the loop body. More formally, the owning expression is an affine function $f(i)$, where i is the loop's induction variable, that occurs statically the most number of times in the loop body. We can then use the owning expression to assign loop iterations to locales, as shown in line 3 of Figure 7c.

There are two affine array accesses Figure 7b: $A_1[a_1i + b_1]$ and $A_2[a_2i + b_2]$. Each appears once in the loop body, so either expression can be chosen as the owning expression for the loop. For the remainder of Figure 7, we assume that $a_1i + b_1$ is the owning expression.

6.4 Message Aggregation

The final step of the optimization is to aggregate all remote affine array accesses that are not accessed using the loop's owning expression before the loop starts. Figure 7c shows this transformation. The loop nest starting on line 2 symbolically represents which loop iterations are assigned to the N locales on the system based on the owning expression calculation (line 3). In line 6, modulo unrolling WU is directly applied, as the array slice of A_2 is fetched via a single GET message and brought to a local buffer. Modulo unrolling guarantees that all elements in this array slice are remote with respect to a single locale on the loop iterations which they are used. So, they can be brought to the current locale \$ in one message. Now in lines 8-12, the affine array access $A_2[a_2i + b_2]$ can be replaced with an access to the local buffer. Lines 14-15 handle the case that elements brought over in bulk need to be written back to their remote locale.

6.5 Loops with Multi-Dimensional Array Accesses

The series of transformations described in this section and illustrated in Figure 7 all apply to one-dimensional arrays indexed by one loop induction variable. We claim here, but do not prove, that these transformations can also be generalized to apply to certain affine array accesses for multi-dimensional arrays. The intuition for this generalization is as follows. The input affine loop now contains m loop induction variables i_1, i_2, ... , i_m. Similarly, there are now m starting loop bounds, ending loop bounds, loop strides, and block sizes. The p^{th} block size is now the number of consecutive array elements allocated to a single locale in dimension p of the array, where $1 < p < m$. Each affine array access in the loop body now contains m affine array expressions where expression p is an affine function of i_p.

Under these assumptions, the transformations described in this section need only be applied to each loop induction variable independently. The owning expression calculation now produces an m-tuple of affine array expressions.[†]

7 Adaptation in Chapel

The goal of this section is to present our adaptation in Chapel of the modulo unrolling WU optimization presented in Section 6. We also provide a basic understanding of zippered iteration and array slicing, two important features in Chapel used in the optimization's implementation.

7.1 Chapel Zippered Iteration

Iterators are a widely used language feature in the Chapel programming language. Chapel iterators are blocks of code that are similar to functions and methods except that iterators can return multiple values back to the call site with the use of the *yield* keyword instead of *return*. Iterators are commonly used in loops to traverse data structures in a particular fashion. For example, an iterator $fibonacci(n : int)$ might be responsible for yielding the first n Fibonacci numbers. This iterator could then be called in a loop's header to execute iterations 0, 1, 1, 2, 3, and so on. Arrays themselves are iterable in Chapel by default. This is how Chapel can support other important language features such as scalar promotion and whole array assignment.

Chapel allows multiple iterators of the same size and shape to be iterated through simultaneously. This is known as *zippered iteration* [20], and an example is shown in Figure 8b. When zippered iteration is used, corresponding iterations are processed together. On each loop iteration, an n-tuple is generated, where n is the number

[†] In our adaptation of modulo unrolling WU in Chapel, the Cyclic distribution can apply the optimization to loops with multi-dimensional array accesses, but the Block Cyclic distribution is limited to one-dimensional array accesses. This is due to the current lmitations within Chapel's Block Cyclic distribution that remained outside the scope of this work.

```
//(a) Parallel loop with affine array accesses
forall i in 1..10 {
     A[i] = B[i+2];
}

//(b) Equivalent loop written using zippered iteration
forall (a,b) in zip(A[1..10], B[3..12]) {
     a = b;
}
```

Figure 8: (a) Chapel loop written using a single loop induction variable i ranging from 1 to 10. The loop contains two affine array accesses. (b) The same loop written using zippered iterators in Chapel. Instead of a loop induction variable and a range of values to denote the loop bounds, two array slices each containing the 10 elements accessed by the loop in (a) are specified.

of items in the zippering. The d^{th} component of the tuple generated on loop iteration j is the j^{th} item that would be yielded by iterator d in the zippering.

Zippered iteration can be used with either sequential **for** loops or parallel **forall** loops in Chapel. Parallel zippered iteration is implemented in Chapel using leader-follower semantics. That is, a *leader* iterator is responsible for creating tasks and dividing up the work to carry out the parallelism. A *follower* iterator performs the work specified by the leader iterator for each task and generally resembles a serial iterator.

7.2 Chapel Array Slicing

Chapel supports another useful language feature known as *array slicing*. This feature allows portions of an array to be accessed and modified in a succinct fashion. For example, consider two arrays A and B containing indices from $1..10$. Suppose we wanted to assign elements $A[6]$, $A[7]$, and $A[8]$ to elements $B[1]$, $B[2]$, and $B[3]$ respectively. We could achieve this in one statement by writing $B[1..3] = A[6..8]$. Here, $A[6..8]$ is a slice of the original array A, and $B[1..3]$ is a slice of the original array B.

In Chapel, an array slice can support a range of elements with a stride in some cases. For example, in the previous example, we could have made the assignment $B[1..3] = A[1..6\ by\ 2]$. This would have assigned elements $A[1]$, $A[3]$, and $A[5]$ to elements $B[1]$, $B[2]$, and $B[3]$ respectively. Since all array slices in Chapel are arrays themselves, array slices are also iterable.

Together, array slicing and parallel zippered iteration can express any parallel affine loop in Chapel that uses affine array accesses. Each affine array access in the loop body is replaced with a corresponding array slice in the loop header, which produces the same elements as the original loop.

Consider the code fragment in Figure 8a. There are two affine array accesses $A[i]$ and $B[i+2]$ in Figure 8a. The loop is written in a standard way where the loop induction variable i takes on values from 1 to 10. Because the loop is a **forall** loop, loop iterations are not guaranteed to complete in a specific order. This loop assigns elements of array B to A such that the i^{th} element of A is equal to the $(i+2)^{th}$ element of B after the loop finishes. In Figure 8b, the same loop is written using zippered iterators. The loop induction variable i no longer needs to be specified, and each affine array access has been replaced with an array slice in the zippering of the loop header. It is possible to transform an affine loop in this fashion even when an affine array access has a constant factor multiplied by the loop induction variable. The resulting array slice will contain a stride equal to the constant factor. The two loops in Figure 8 are equivalent and generate the same results, but they differ in their execution.

```
1   iter CyclicArr.these(param tag: iterKind, followThis, param fast: bool = false) var
2        where tag == iterKind.follower {
3
4   //check that all elements in chunk are from the same locale
5   for i in 1..rank {
6        if (followThis(i).stride * dom.whole.dim(i).stride %
7            dom.dist.targetLocDom.dim(i).size != 0) {
8             //call original follower iterator helper for nonlocal elements
9   }    }
10  if arrSection.locale.id == here.id then local {
11       //original fast follower iterator helper for local elements
12  } else {
13        //allocate local buffer to hold remote elements, compute source and destination
14       //strides, number of elements to communicate
15         chpl_comm_gets(buf, deststr, arrSection.myElems._value.theData, srcstr, count);
16         var changed = false;
17         for i in buf {
18              var old_i = i;
19             yield i;
20              var new_val = i;
21              if(old_val != new_val) then changed = true;
22         }
23         if changed then
24             chpl_comm_puts(arrSection.myElems._value.theData, srcstr, buf, deststr, count);
25  }    }
```

Figure 9: Pseudocode for the Cyclic distribution follower iterator that has been modified to perform modulo unrolling WU.

Because any parallel affine loop can be transformed into an equivalent parallel loop that uses zippered iteration, we observe a natural place in the Chapel programming language in which to implement modulo unrolling WU: the leader and follower iterators of the Cyclic and Block Cyclic distribution. The leader iterator divides up the loop's iterations according to the locales they are executed on and passes this work to each follower iterator in the zippering. The follower iterator can then perform the aggregation of remote data elements according to the work that has been passed to it.

7.3 Implementation

Modulo unrolling WU is implemented into the Chapel programming language through the Cyclic and Block Cyclic distribution modules, as opposed to being implemented via traditional compiler passes. Specifically, the follower iterator is modified in the Cyclic distribution, and both the leader and follower iterators are modified in the Block Cyclic distribution. Because these modules are written in Chapel, the optimization can be expressed using Chapel's higher-level language constructs, such as zippered iteration and array slicing.

Figure 9 shows a pseudocode representation of the Cyclic follower iterator modified to perform modulo unrolling WU. Some coding details are left out for brevity. The follower iterator is responsible for carrying out the loop iterations that are passed to it by the leader iterator. Because the follower iterator has no knowledge about how the leader iterator divides up the loop iteratrions, this chunk of work can either be entirely local, entirely remote to a single locale, or partially remote and local. Lines 5-9 determine whether all elements of the chunk of work come from the same locale. If not, then we can not aggregate, and the follower iterator calls a helper function responsible for yielding remote elements individually. Lines 10-25 handle the cases where the chunk of work does reside on a single locale. If the chunk is found locally, another helper function responsible for yielding local elements is called, showed in lines 10-12. Finally, if the chunk is entirely remote, we can perform the message aggregation step of modulo unrolling WU that was previously described in Section 6.4.

The entire chunk of work, specified by the `arrSection`

Name	Lines of Code	Input Size	Description
2mm	221	128 x 128	2 matrix multiplications (D=A*B; E=C*D)
fw	153	64 x 64	Floyd-Warshall all-pairs shortest path algorithm
trmm	133	128 x 128	Triangular matrix multiply
correlation	235	512 x 512	Correlation computation
covariance	201	512 x 512	Covariance computation
cholesky	182	256 x 256	Cholesky decomposition
lu	143	128 x 128	LU decomposition
mvt	185	4000	Matrix vector product and transpose
syrk	154	128 x 128	Symmetric rank-k operations
syr2k	160	128 x 128	Symmetric rank-2k operations
fdtd-2d	201	1000 x 1000	2D Finite Different Time Domain Kernel
fdtd-apml	333	64 x 64 x 64	FDTD using Anisotropic Perfectly Matched Layer
jacobi1D	138	10000	1D Jacobi stencil computation
jacobi2D	152	400 x 400	2D Jacobi stencil computation
stencil9†	142	400 x 400	9-point stencil computation
pascal‡	126	100000, 100003	Computation of pascal triangle rows
folding‡	139	50400	Strided sum of consecutive array elements

Figure 10: Benchmark suite. Benchmarks with no symbol after their name were taken from the Polybench suite of benchmarks and translated to Chapel. Benchmarks with † are taken from the Chapel Trunk test directory. Benchmarks with ‡ were developed on our own in order to test specific data access patterns.

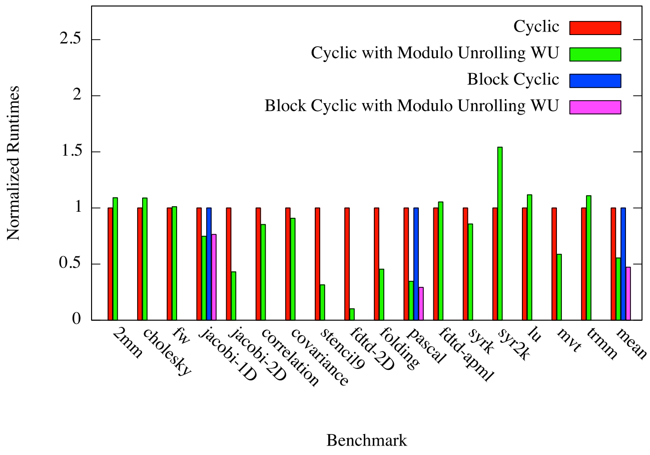


Figure 11: Normalized runtimes.

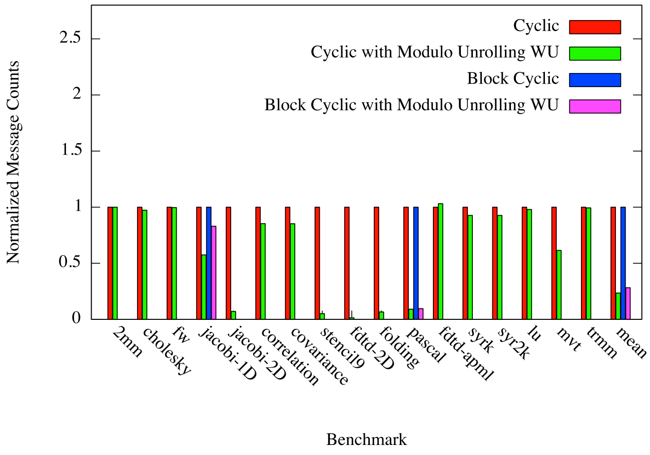


Figure 12: Normalized message counts.

pointer, is communicated to the local `buf` in one message with the `chpl_comm_gets` call on line 15. Then, elements in this buffer are yielded back to the loop following zippered iteration semantics. The values in `buf` are compared before and after they are yielded in order to determine whether or not they were written to in the loop body. If so, a `chpl_comm_puts` call on line 24 is required to write all `buf` elements back to the remote locale.

The implementation of modulo unrolling WU into the Block Cyclic distribution is nearly identical to Figure 9 with one key addition: the Block Cyclic leader iterator is also altered so that chunks of work that the leader creates only contain elements that reside in the same position within a block. This addition ensures static disambiguation for the Block Cyclic distribution, as described in Section 6.2.

8 Results

To demonstrate the effectiveness of modulo unrolling WU in the Chapel Cyclic and Block Cyclic distributions, we present our results. We have composed a suite of seventeen parallel benchmarks shown in Figure 10. Each benchmark is written in Chapel and contains loops with affine array accesses that use zippered iterations, as discussed in 7.2. Our suite of benchmarks contains programs with single, double, and triple nested affine loops. Additionally, our benchmark suite contains programs operating on one, two, and three-dimensional distributed arrays. Fourteen of the seventeen benchmarks are taken from the Polybench suite of benchmarks [21] and are translated from C to Chapel by hand. The *stencil9* benchmark was taken from the Chapel source trunk directory. The remaining two benchmarks, *pascal* and *folding*, were written by our group. *pascal* is an additional benchmark other than *jacobi1D* that is able to test Block Cyclic with modulo unrolling WU. *folding* is the only benchmark in our suite that has strided affine array accesses.

To evaluate improvements due to modulo unrolling WU, we run our benchmarks using the Cyclic and Block Cyclic distributions from the trunk revision 22919 of the Chapel compiler as well as the Cyclic and Block Cyclic distributions that have been modified to perform modulo unrolling WU, as described in Section 7. We measure both runtime and message count for each benchmark. We also compute the geometric means of all normalized runtimes and message count numbers for both distributions to get a sense on average of how much improvement modulo unrolling WU provided.

Data was collected on the eight-locale Golgatha cluster at the Laboratory for Telecommunication Sciences in College Park, Maryland. Each computing node on the cluster is comprised of two 2.93 GHz Intel Xeon X5670 processors with 24 GB of RAM. The nodes are connected via an InfiniBand network communication link.

When evaluating modulo unrolling WU used with the Block Cyclic distribution, we could only run two benchmarks out of our suite of seventeen because of limitations within the original Chapel Block Cyclic distribution. Many of our benchmarks operate on two or three-dimensional arrays and all require array slicing for the modulo unrolling WU optimization to apply. Both array slicing of multi-dimensional arrays and array slicing containing strides for one-dimensional arrays are not yet supported in the Chapel compiler's Block Cyclic distribution. Implementing such features remained outside the scope of this work. There was no limitation when evaluating modulo unrolling WU with the Cyclic distribution, and all seventeen benchmarks were tested. Once these missing features are implemented in the Chapel compiler, then our method will apply to all of our benchmarks.

Figures **??** and **??** compare the normalized runtimes and message counts respectively for the Cyclic distribution and Cyclic distribution with modulo unrolling WU. For 8 of the 11 benchmarks, we see reductions in runtime when the modulo unrolling WU optimization is applied. On average, modulo unrolling WU results in a 45 percent decrease in runtime. For 9 of the 11 benchmarks, we see reductions in message counts when the modulo unrolling WU optimization is applied. On average, modulo unrolling WU results in 76 percent fewer messages.

Some detailed observations on Figures **??** and **??** follow. Two of the benchmarks, *cholesky* and *fw*, showed slight improvements in message count when using modulo unrolling WU but did not show improvements in runtime. For the *2mm* benchmark, both runtime and message count did not improve when using modulo unrolling WU. For these benchmarks, the ratio of the problem size to number of locales is not high enough, leading to an insufficient amount of aggregation possible for the computation to see performance improvements. An increase in the number of locales on a system leads to fewer data elements per locale, which naturally means fewer data elements can be aggregated. When this occurs, the cost of performing bulk transfers of a few data elements is more expensive than transferring elements individually.

Figures **??** and **??** compare the normalized runtimes and message counts respectively for the Block Cyclic distribution and Block Cyclic distribution with modulo unrolling WU. For both benchmarks, we see reductions in runtime when the modulo unrolling WU optimization is applied. On average, modulo unrolling WU results in a 52 percent decrease in runtime. For both benchmarks, we see reductions in message counts when the modulo unrolling WU optimization is applied. On average, modulo unrolling WU results in 72 percent fewer messages.

9 Future Work

As presented, the modulo unrolling WU optimization can be improved upon in a few ways to achieve even better performance in practice. First, there is currently no limit on the number of array elements that an aggregate message may contain. For applications with extremely large data sets, buffers containing remote data elements may become too large and exceed the memory budget of a particular locale. This may slow down other programs running on the system. A naive solution to this problem is to just turn off the optimization when the aggregate message is deemed to large and communicate remote data elements individually. A better solution would be to perform strip mining where the aggregate message is broken down into smaller aggregate messages of a configurable threshold size.

The two forms of bulk communication used in this work (`chpl_comm_gets` and `chpl_comm_puts`) are both blocking communication calls. Our optimization might achieve better performance if it used a non-blocking strided bulk communication scheme. That way, communication and computation may be able to occur in parallel.

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