cs272: Algorithms

# Run-time Analysis of SelectionSort

| `SelectionSort (A[1..n])` | cost | times |
|---|---|---|
| `for i <- 1 to n-1` | $c_1$ | $n$ |
| `  min <- i` | $c_2$ | $n-1$ |
| `  for j <- i+1 to n` | $c_3$ | $\sum_{i=1}^{n-1}(i+1)$ |
| `    if A[j] < A[min]` | $c_4$ | $\sum_{i=1}^{n-1} i$ |
| `      min <- j` | $c_5$ | $\sum_{i=1}^{n-1} i\, t_{i,j}$ |
| `  swap A[i] <--> A[min]` | $c_6$ | $n-1$ |

Notes

- That $t_{i,j} = 1$ when the if-state is true, 0 otherwise.
- $c_3 \sum_{i=1}^{n-1}(i+1) = c_3 \sum_{i=1}^{n-1} i + c_3 \sum_{i=1}^{n-1} 1 = c_3 \frac{(n-1)n}{2} + c_3(n-1)$
- $c_4 \sum_{i=1}^{n-1} i = c_4 \frac{(n-1)n}{2}$
- $c_5 \sum_{i=1}^{n-1} i t_{i,j} = c_5 \frac{(n-1)n}{2}$ when $t_{i,j} = 1$ and 0 otherwise.

Then for the best-case scenario we have that all $t_{i,j} = 0$ so we have

$$T(n) = n^2\left(\frac{c_3}{2} + \frac{c_4}{2}\right) + n\left(c_1 + c_2 + c_6 - \frac{3c_3}{2} - \frac{c_4}{2}\right) + (-c_6 - c_2 - c_3)$$
$$T(n) = an^2 + bn + c$$

For the worst-case scenario we have that all $t_{i,j} = 1$ so we have

$$\begin{aligned} T(n) = n^2\left(\frac{c_3}{2} + \frac{c_4}{2} + \frac{c_5}{2}\right) + n\left(c_1 + c_2 + c_6 - \frac{3c_3}{2} - \frac{c_4}{2} - \frac{c_5}{2}\right) \\ + (-c_6 - c_2 - c_3) \\ T(n) = an^2 + bn + c \end{aligned}$$

Thus we have that $T(n) = \Theta(n^2)$.