# Introduction to Springs

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Spring 2015





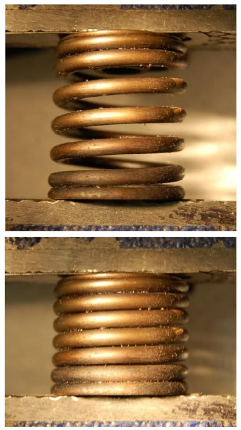
## Spring

- A *spring* is a resilient member that is capable of producing large elastic deformation. Typically springs are made of steels, metals or bronze.
- It is basically an elastic body that gives deformation under the action of the load and returns to its original position when the load is released.
- Application of springs:
  1. **To exert force:** Springs may be used to exert force as to keep cam follower in contact with the cam in an internal combustion engines.
  2. **Source of energy:** Springs are often used in watches, clocks etc., to act as a source of energy.
  3. **To absorb energy/shock/vibration:** Springs are widely used to absorb energy or shock loads in automobile chassis/suspension.





## Elastic Behavior

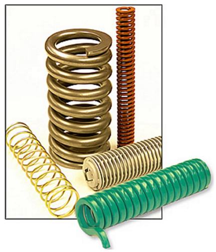


- *Springs change their shape in response to an external force and return to their original shape when the force is removed.*
- *The energy expended in deforming a spring is stored in it and can be recovered when the spring returns to its original shape.*
- *If too large a force is applied, however, the spring will permanently deform and never return to its original shape.*





## Types of spring



*Springs are primarily classified as :*

- **Flat Springs:** *Made up of flat wire and wound in the form of a spiral.*
- **Helical Springs:** *Made up of round wire coiled into helix.*

# Helical Springs

**3 Types**

| Compression | Extension | Torsion |
|---|---|---|
| ( Offers resistance to compressive force) | ( Offers resistance to tensile force) | ( Offers resistance to twisting force) |





# Other springs

Beleville Spring

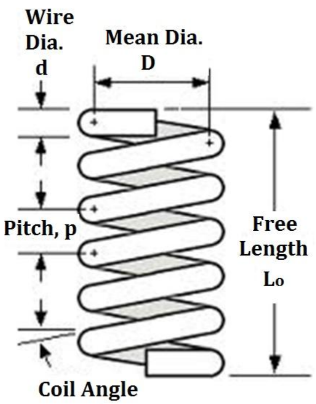
Leaf Spring

- **Bellevile Springs:** *Used as flat spring washers.*
- **Leaf Springs:** *Made of a number of strips of metal curved slightly upwards and clamped together one above the other.*
- **Leaf Springs are widely seen in automobile or rickshaw chassis.**





# Terminology



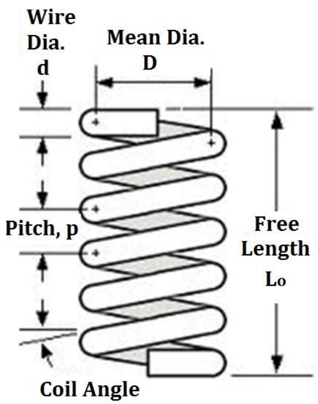

- **Wire diameter ($d$):** *Diameter of the spring wire.*
- **Mean Diameter ($D$):** *Average value of the inner diameter and outer diameter.*
- **Free Length ($L_o$):** *Overall length of the compression spring when no load is applied.*
- **Spring Index ($C$):** *Ratio of the mean diameter of the coil to the diameter of the wire. For most springs* $6 \leq C \leq 12$.

$$C = D/d$$





# Pitch

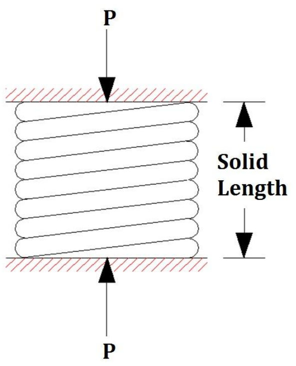



- **Pitch ($p$):** *Distance from center to center of the wire in adjacent active coils.*

$$p = (L_o - L_s)/n + d$$

- **Active coils:** *The coils that deform elastically when subjected to a load.*

## Solid Length



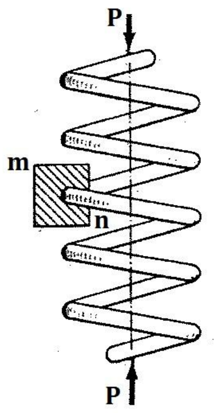

- **Solid Length ($L_s$):** *When a spring is compressed until its coils come in contact with each other, then the spring is said to be solid or shut. The length or height of the spring in this condition is called solid length or shut length or solid height or shut height.*

$$L_s = nd$$





## Expression of Shearing Stress



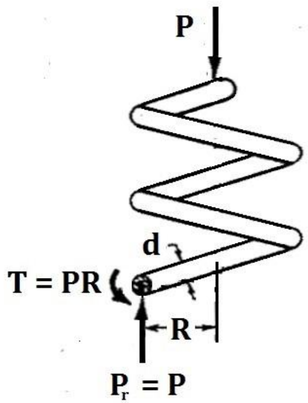

### Derivation

- *Consider, a helical spring of round wire as shown in figure that supports an axial compressive load P.*
- *To determine the stresses produced by P, we pass an exploratory cutting plane m-n through any typical section of the spring as shown in figure and then determine the resisting forces required for equilibrium.*





## Expression of Shearing Stress



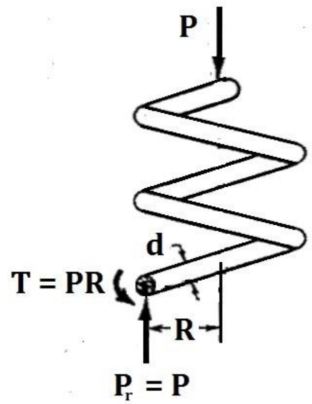

### Derivation (contd.)

- *The free body diagram of the upper half of the spring is shown in figure.*
- *To balance applied load P, the exposed section must provide resistance $P_r = P$.*
- *Therefore the free body is in equilibrium as far as vertical and horizontal summation of forces are concerned.*





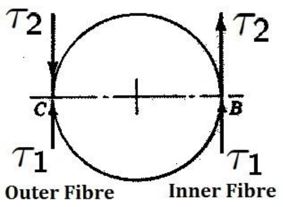
## Expression of Shearing Stress




### Derivation (contd.)

- *To complete equilibrium, summation of moment must also be zero.*
- *Since P and $P_r$ are equal, opposite and parallel, they create a couple of magnitude PR.*
- *To balance this couple a resisting couple $T = PR$ is developed.*
- *This couple is developed due to torsional shearing stress distribution over the cross-section of the spring.*

# Expression of Shearing Stress



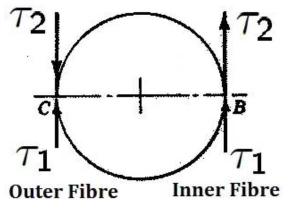

**Derivation (contd.)**

- *The magnified view of the cross section showing the stress distribution is shown in figure.*
- *Two types of stresses are developed.*
  1. *Direct shear stress ($\tau_1$).*
  2. *Torsional shear stress ($\tau_2$).*
- *$\tau_1$ is uniformly distributed over the section and is due to resistive load $P_r$ that passes through the centroid of the section.*
- *$\tau_2$ is due to twisting couple PR and varies in magnitude with radial distance from centroid and it is directed perpendicular to the radius.*
- *The total shearing stress ($\tau$) is therefore summation of direct shear and torsional shear.*
- *From the figure it is evident that maximum shear stress occurs at the inner fibre of the coils.*





# Expression of Shearing Stress



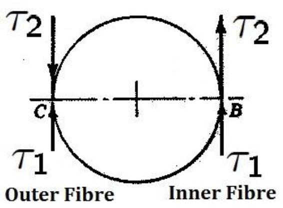

**Derivation (contd.)**

$$\begin{aligned} Now,\ \tau = \tau_1 \pm \tau_2 &= \frac{P}{A} \pm \frac{16T}{\pi d^3} \\ &= \frac{4P}{\pi d^2} \pm \frac{16PR}{\pi d^3} \\ &= \frac{16PR}{\pi d^3} \times (1 \pm \frac{d}{4R}) \\ &= \frac{8PD}{\pi d^3} \times (1 \pm \frac{d}{2D}) \\ &= \frac{8PD}{\pi d^3} \times (1 \pm \frac{0.5}{C}) \end{aligned}$$





# Expression of Shearing Stress

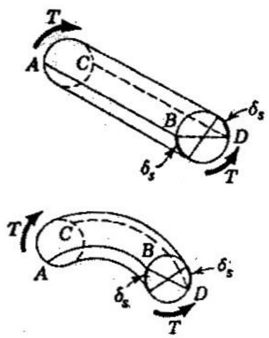



- $\therefore \tau_{max} = \frac{8PD}{\pi d^3} \times (1 + \frac{0.5}{C})$
- $\therefore \tau_{min} = \frac{8PD}{\pi d^3} \times (1 - \frac{0.5}{C})$
- *Now,* $\tau = \frac{8PD}{\pi d^3}(1 + \frac{0.5}{C})$
- *Introducing shear stress concentration factor,* $K_s = 1 + \frac{0.5}{C}$
- *We have,* $\tau = K_s \frac{8PD}{\pi d^3}$
- *This is the expression of shearing stress in a helical spring.*





# Effect of Curvature

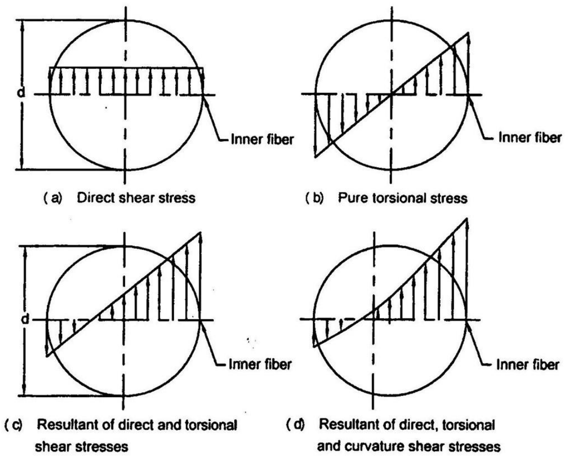



**Torsion of straight and curved segments.**

- *The shear stress calculated by applying torsion formula contains error.*
- *This is due to the fact that torsion formula was derived for straight circular members not for curved members.*
- *When the torsion formula was applied for helical spring , we did not considered the curvature of the coils.*
- *A.M. Wahl corrected this error by introducing Wahl Factor, $K_W$ into the stress equation as follows.*[a]
- $\tau = K_w \frac{8PD}{\pi d^3}$
- *Where,* $K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$
- *Spring Index,* $C = \frac{D}{d}$

[a] A.M. Wahl, Stresses in heavy closely coiled ASME 51, Paper no. APM-51-17

## Stress Distribution over wire section

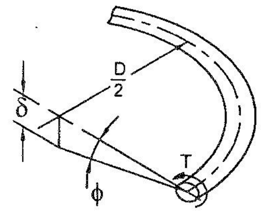



Cross-Section of Wire Showing Distribution of Stresses.





## Spring Deflection ($\delta$) & Spring Rate ($k$)

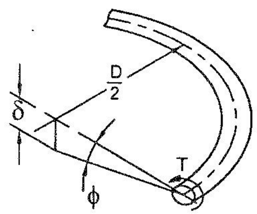



- *Practically, all spring elongation is due to torsional deformation.*
- *Consider a segment of wire of a helical spring under torsional load as shown in figure.*
- *Due to twisting effect, an axial deflection $\delta$ and angle of twist $\phi$ occurs.*
- *Now, Angle of twist,* $\phi = \frac{TL}{GJ}$
- *If* $N_a$ = *No. of active coils and* $L$ = *Active Length of Coil*
- *Then putting,* $L = \pi D N_a$, $T = PR = \frac{PD}{2}$ *and* $J = \frac{\pi d^4}{32}$





## Spring Deflection ($\delta$) & Spring Rate ($k$)




- *We have,* $\phi = \frac{16PD^2N_a}{Gd^4}$
- *From figure ,* $\delta = \frac{\phi D}{2}$
- *Therefore, deflection,*

$$\delta = \frac{8PD^3N_a}{Gd^4}$$

- *Again, Spring Rate or constant,* $k = \frac{P}{\delta}$
- *Therefore, Spring constant,*

$$k = \frac{Gd^4}{8D^3N_a}$$





## Problem

1. A helical compression steel spring is subjected to an axial load of 1 kN. If the spring index is 6, number of turns is 15 and the maximum shearing stress in the spring is not to exceed 120 MPa, find the suitable wire diameter from the available wires of 12 mm to 18 mm with a step of 2 mm. Determine the maximum shearing stress for the selected wire. Determine also the deflection and the spring constant. Modulus of rigidity for steel is 80 GPa. Use Wahl factor for stress concentration effect.

# Solution

Here, Wahl Factor, $K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

$$\implies K_w = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

Maximum shearing stress with Wahl Factor,

$$\tau_{max} = K_w \frac{8PD}{\pi d^3} = \tau_{allowable} = 120$$

$\implies 1.2525 \times \frac{8 \times 1 \times 1000 D}{\pi d^3} = 120$

Since, $\frac{D}{d} = C = 6$

$$\therefore 1.2525 \times \frac{8 \times 1 \times 1000 \times 6}{\pi d^2} = 120$$

Solving for d, we get $d = 12.63$ mm

Since, available wire sizes are 12mm, 14mm , 16mm and 18mm $\therefore d = 14$ mm (Ans.)
For this wire,
Maximum Shearing stress developed,

$$\tau_{max} = K_w \frac{8PD}{\pi d^3} = 1.2525 \frac{8 \times 1 \times 1000 \times 6}{\pi 14^2} = 97.64$$

$\implies \tau_{max} = 97.64$ MPa (Ans.)
Again, Spring Index, $C = \frac{D}{d}$

$$\implies D = C \times d = 6 \times 14 = 84$$

$\implies D = 84$ mm
$\therefore$ *Spring deflection,*
$\delta = \frac{8PD^3 N_a}{Gd^4} = 23.14$ *mm (Ans.)*

$\therefore$ *Spring constant,*
$k = \frac{P}{\delta} = 43.22 \ \frac{kN}{m}$ *(Ans.)*