Columns

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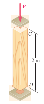

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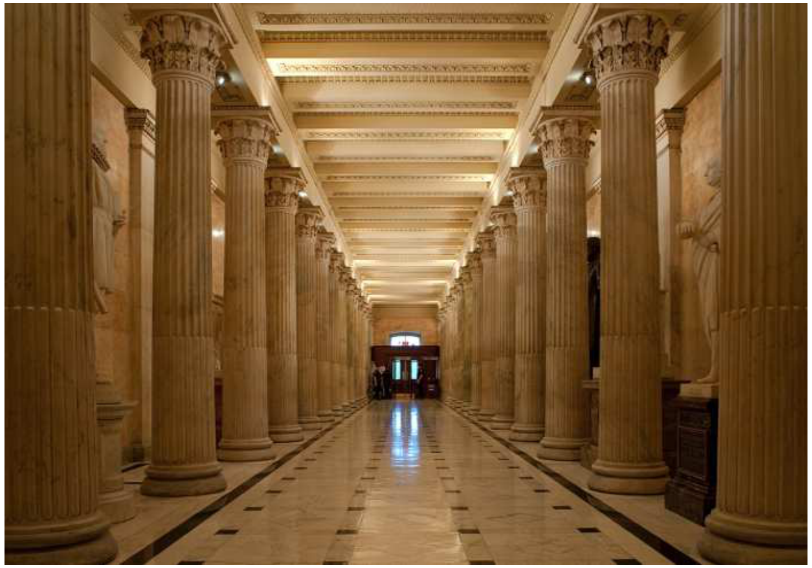
ME 201

Definition



Definition

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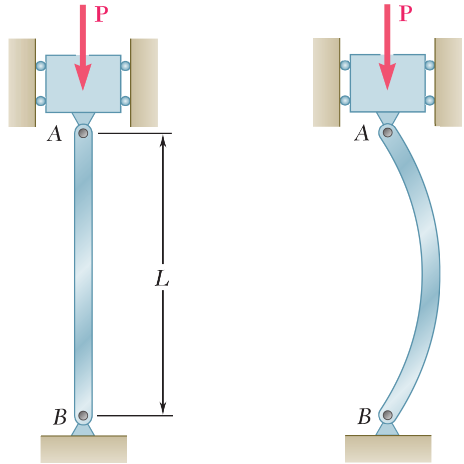
- A column is a compression member that fails by buckling (excessive lateral bending).
- There is no sharp distinction between a compression member and column.
- Roughly when $L/D \geq 10$, then it is a column
- Columns are vertically prismatic members that typically support axial loads.

Buckling

- Buckling refers to excessive lateral bending.
- Long columns fail by buckling (flexural stress).
- Intermediate columns fail by a combination of buckling and crushing.
- Short columns or compression blocks fail by crushing.

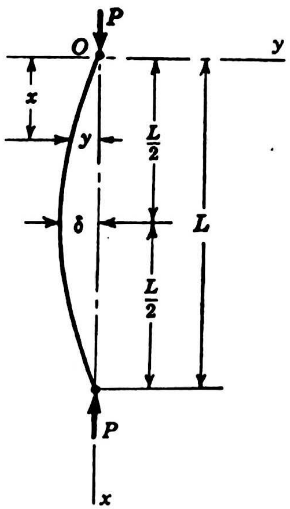
Buckling



Pinned-ended axially loaded column.

Buckled pin-ended column.

Euler's Formula (Critical Load)

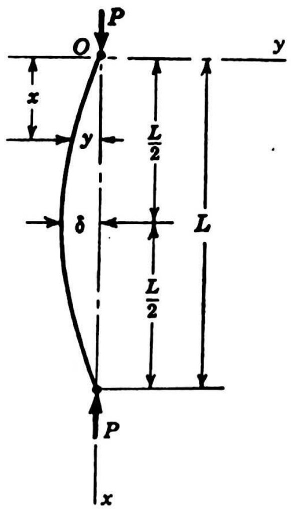


- Elastic curve equation provided by Leonhard Euler is given by,

$$\frac{\frac{d^2y}{dx^2}}{[1+(\frac{dy}{dx})^2]^{\frac{3}{2}}} = \frac{M}{EI} \quad (1)$$

- M = Moment
  E = Modlulus of Elasticity
  I = Moment of Inertia
  L=Length of Column
  δ = Deflection
- Consider the center line of a hinged/pinned/pivoted/rounded end column which is in equilibrium under the action of its critical load, P.
- The ends of the column are restrained against lateral movement.

Euler's Formula (Critical Load)



- If the deflection δ is very small then $\frac{dy}{dx}$ is negligible

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad (2)$$

- Taking moment as negative,

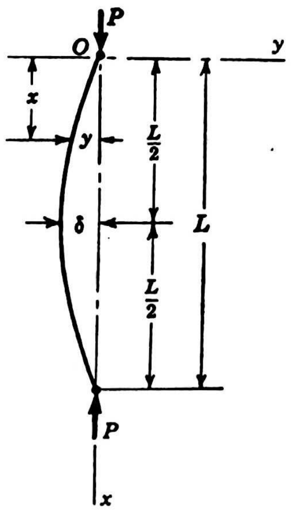
$$M = -Py \quad (3)$$

- Equations 2 & 3 yields,

$$EI\frac{d^2y}{dx^2} = -Py \quad (4)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0 \quad (5)$$

Euler's Formula (Critical Load)



- $\frac{P}{EI} = \lambda^2 \, (say)$

$$\frac{d^2y}{dx^2} + \lambda^2 y = 0 \qquad (6)$$

- Solution of equation 6 is given by

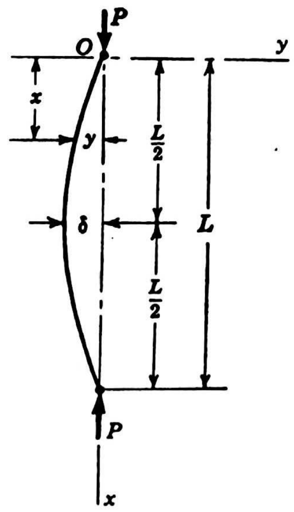
$$y = Asin\lambda x + Bcos\lambda x \qquad (7)$$

where A and B are arbitrary constants

- Apply boundary condition x=0, y=0
  $\therefore B = 0$
- Apply boundary condition again at x=L, y=0

$$0 = Asin\lambda L \qquad (8)$$

Euler's Formula (Critical Load)



- Equation 8 is satisfied if A=0 but in that case there is no deflection or bending of the column. Therefore consider $A \neq 0$

$$sin\lambda L = 0 \qquad (9)$$

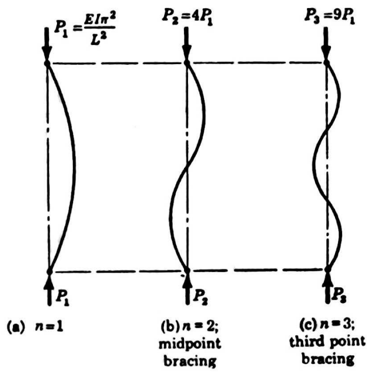
$$\therefore L\sqrt{\frac{P}{EI}} = n\pi \qquad (10)$$

$where, \; n = 0, 1, 2, 3 \ldots$

$$\boxed{\therefore P_{cr} = \frac{n^2\pi^2 EI}{L^2}}$$

- The value of n=0 is meaningless since it implies P=0. For the other values the column bends into different shapes.

Euler's Formula (Critical Load)



Effect of *n* on loads.

Of all the values n=1 is most important since other values of n are possible only when the column is braced at different points.
$\therefore$ Critical load for hinged columns,

$$\boxed{\therefore P_{cr} = \frac{\pi^2 EI}{L^2}}$$

End conditions

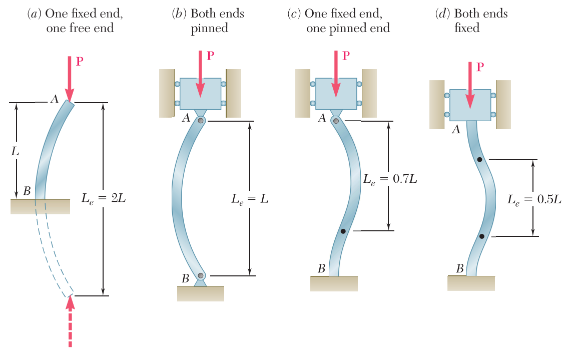


Figure: Different end conditions.

$\boxed{P_{cr} = \frac{1}{4}\frac{\pi^2 EI}{L^2}}$ $\boxed{P_{cr} = \frac{\pi^2 EI}{L^2}}$ $\boxed{P_{cr} = 2\frac{\pi^2 EI}{L^2}}$ $\boxed{P_{cr} = 4\frac{\pi^2 EI}{L^2}}$

Limitations of Euler's Formula

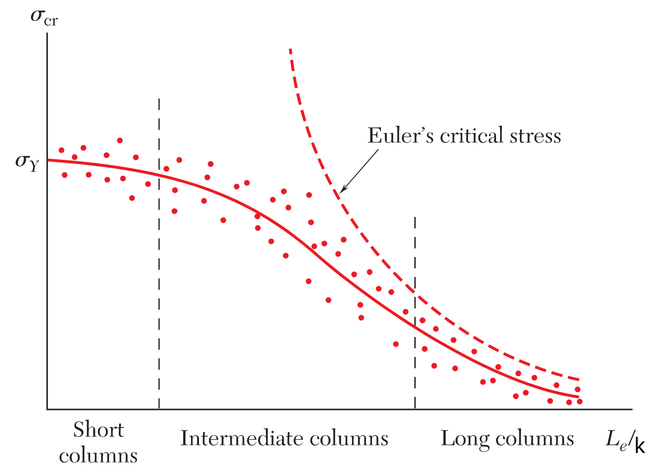
- **Least moment of inertia:** A column always buckles in its most limber direction. Any tendency to buckle occurs about the least axis of inertia of the cross section.
- **Strength of material:** The critical load or buckling load is independent of strength of material. P_{cr} only depends on the columns dimensions and modulus of elasticity. Two identical slender columns, one made of structural steel and the other of ordinary steel, both having the same modulus of elasticity but different strengths will buckle under the same critical load.
- **Proportional Limit:** In order to apply Euler's formula the bending stress due to critical load or buckling load P_{cr} must be less than the proportional limit. The bending stress is calculated as follows.

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EAk^2}{L_e^2} \Rightarrow \frac{P_{cr}}{A} = \frac{\pi^2 E}{\frac{L_e^2}{k^2}} \Rightarrow \sigma_{cr} = \frac{\pi^2 E}{\frac{L_e^2}{k^2}}$$

where, σ_{cr} = Average critical stress, A = Cross-sectional area, k = radius of gyration and L_e = Equivalent length

Note: Euler's formula determine critical loads not working load. Therefore, suitable factor of safety [1.7-2.5] depending on the material is recommended.

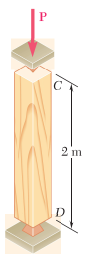
Slenderness Ratio



Plot of test data for steel columns.

Figure: Critical stress σ_{cr} vs Slenderness Ratio $\frac{L_e}{k}$

Problem 1



A 2m long pin-ended column with a square cross section is to be made of wood. Assuming E = 13 GPa, σ_y = 12 MPa and using a factor of safety of 2.5 to calculate Euler's critical load for buckling, determine the size of the cross section if the column is to safely support a 100 kN load.

Solution:

Critical load, $P_{cr} = 2.5 \times 100 = 250\, kN$

Euler's formula, $I = \frac{P_{cr}L^2}{\pi^2 E} = \frac{250 \times 10^3 \times 2^2}{\pi^2 \times 13 \times 10^9}$

$\therefore I = 7.794 \times 10^{-6}\, m^4$

For square cross-section having side length a,
$I = \frac{a^4}{12} \Rightarrow a = 98.3\, mm \approx 100\, mm$

Check: Developed normal stress,
$\sigma = \frac{P}{A} = \frac{100 \times 10^3}{0.1^2} = 10\, MPa$

which is less than yield stress.
$\therefore$ 100 $\times$ 100-mm cross-section is acceptable. **(Ans.)**

Problem 2

A pin ended steel column of circular cross-section having diameter of 100 mm is subjected to an axial load.Determine the buckling load for this column using Euler's formula. The slenderness ratio and radius of gyration is 90.69 and 25 mm respectively. E = 200 GPa

Solution:
Here, $\frac{L_e}{k} = 90.69 \Rightarrow L_e = 90.69 \times 25 \times 10^{-3} = 2.27\, m$

For pin ended columns: $L = L_e = 2.27\, m$

From Euler's column formula,

$$P_{cr} = \frac{EI\pi^2}{L_e^2} = \frac{EAk^2\pi^2}{L_e^2} = \frac{200 \times 10^9 \times \frac{\pi}{4} \times (100 \times 10^{-3})^2 \times (25 \times 10^{-3})^2 \pi^2}{2.27^2}$$

$$\therefore P_{cr} = 1880.39\, kN\, (Ans.)$$

Practice this problem for other end conditions.