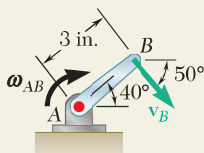


In the engine system shown, the crank  $AB$  has a constant clockwise angular velocity of 2000 rpm. For the crank position indicated, determine (a) the angular velocity of the connecting rod  $BD$ , (b) the velocity of the piston  $P$ .



**Motion of Crank  $AB$ .** The crank  $AB$  rotates about point  $A$ . Expressing  $\omega_{AB}$  in rad/s and writing  $v_B = r\omega_{AB}$ , we obtain

$$\omega_{AB} = \left(2000 \frac{\text{rev}}{\text{min}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 209.4 \text{ rad/s}$$

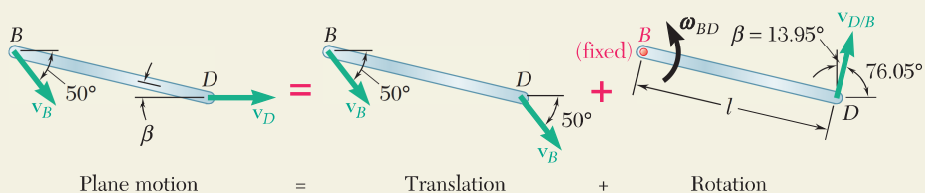
$$v_B = (AB)\omega_{AB} = (3 \text{ in.})(209.4 \text{ rad/s}) = 628.3 \text{ in./s}$$

$$\mathbf{v}_B = 628.3 \text{ in./s} \searrow 50^\circ$$

**Motion of Connecting Rod  $BD$ .** We consider this motion as a general plane motion. Using the law of sines, we compute the angle  $\beta$  between the connecting rod and the horizontal:

$$\frac{\sin 40^\circ}{8 \text{ in.}} = \frac{\sin \beta}{3 \text{ in.}} \quad \beta = 13.95^\circ$$

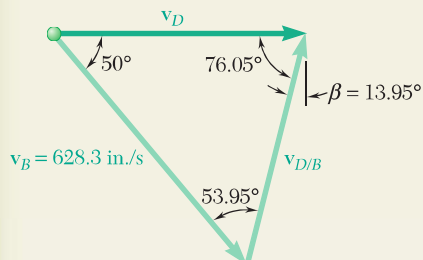
The velocity  $\mathbf{v}_D$  of the point  $D$  where the rod is attached to the piston must be horizontal, while the velocity of point  $B$  is equal to the velocity  $\mathbf{v}_B$  obtained above. Resolving the motion of  $BD$  into a translation with  $B$  and a rotation about  $B$ , we obtain



Expressing the relation between the velocities  $\mathbf{v}_D$ ,  $\mathbf{v}_B$ , and  $\mathbf{v}_{D/B}$ , we write

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

We draw the vector diagram corresponding to this equation. Recalling that  $\beta = 13.95^\circ$ , we determine the angles of the triangle and write



$$\frac{v_D}{\sin 53.95^\circ} = \frac{v_{D/B}}{\sin 50^\circ} = \frac{628.3 \text{ in./s}}{\sin 76.05^\circ}$$

$$v_{D/B} = 495.9 \text{ in./s} \quad \mathbf{v}_{D/B} = 495.9 \text{ in./s} \nearrow 76.05^\circ$$

$$v_D = 523.4 \text{ in./s} = 43.6 \text{ ft/s} \quad \mathbf{v}_D = 43.6 \text{ ft/s} \rightarrow$$

$$\mathbf{v}_P = \mathbf{v}_D = 43.6 \text{ ft/s} \rightarrow$$

Since  $v_{D/B} = l\omega_{BD}$ , we have

$$495.9 \text{ in./s} = (8 \text{ in.})\omega_{BD} \quad \omega_{BD} = 62.0 \text{ rad/s} \curvearrowright$$