Belt, Rope and Chain Drives

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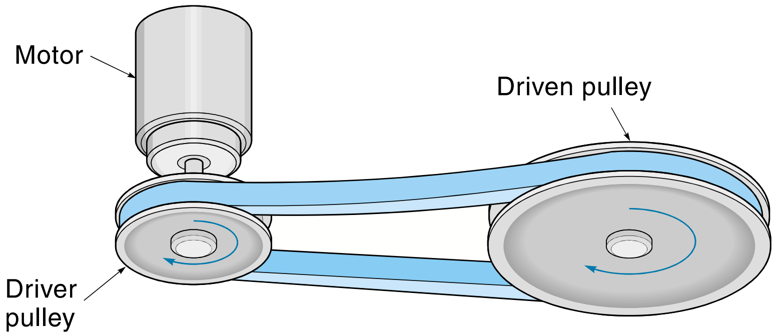
ME 201

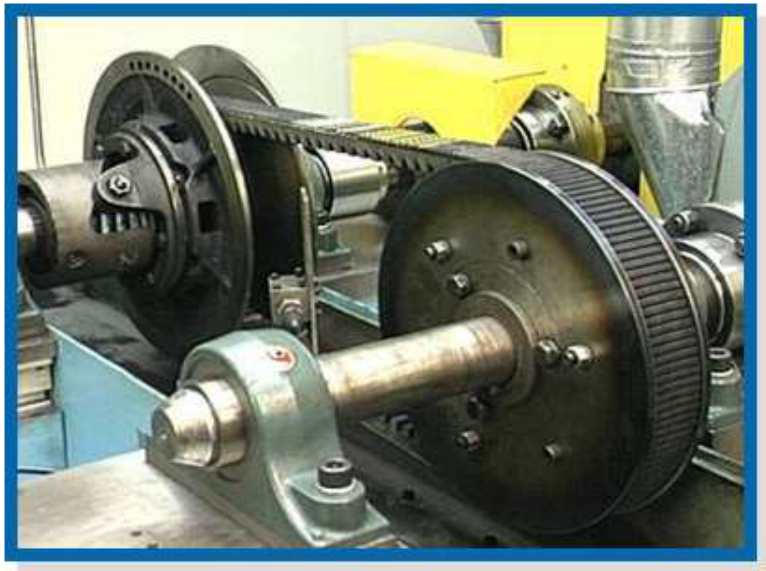
Definition



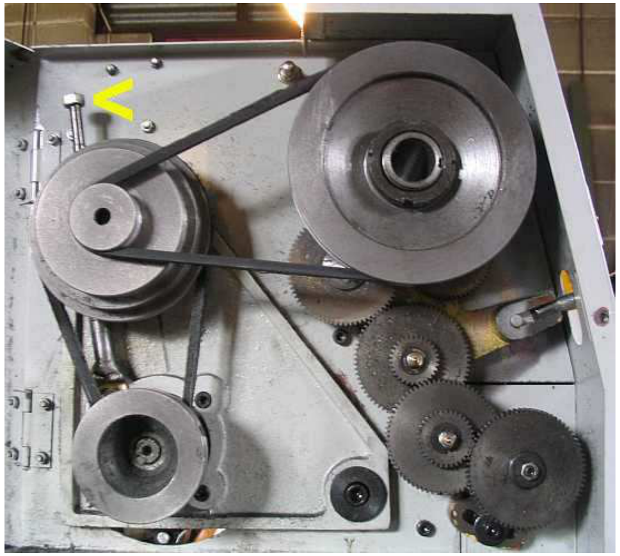
Definition

- **Belts** or **ropes** are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.
- The amount of power transmitted depends upon
  1. Belt velocity
  2. Belt Tension
  3. Contact/lap angle
  4. Operating conditions
- If the distance between shafts are too close, gears or gear trains are preferred for power transmission.
- Limitation: If the distance between shafts are greater than approximately 8m or 25 feet, these drives become poor transmission element.
- Belt Materials.
  1. Leather
  2. Cotton/Fabric
  3. Rubber

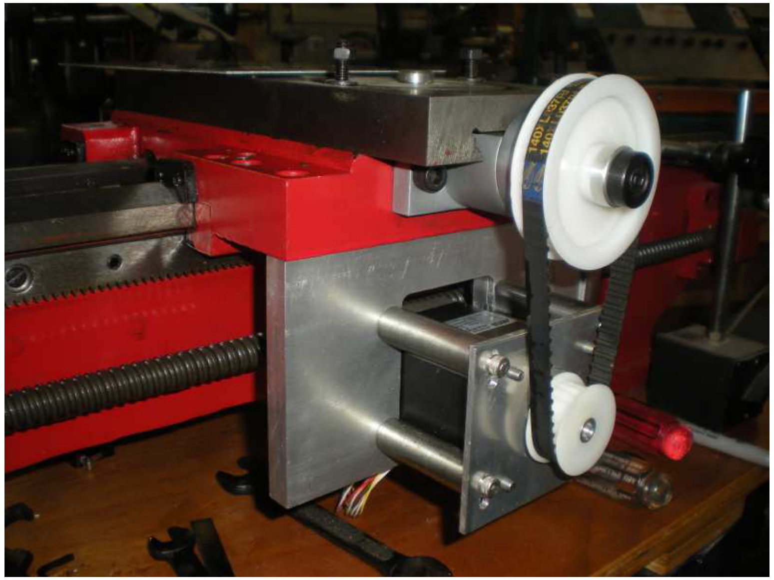
Example



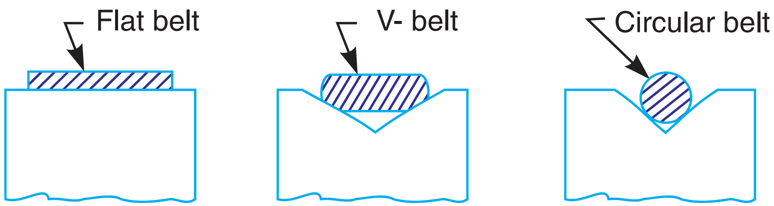
Example



Example



Types of Belt



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

Figure: Types of Belts.

Flat belts are used for distance of 8m or less.
V-belts are used for very close distance pulleys.
Ropes or circular belts are used for 10m or more distance.

Classification of Belt Drive

The power from one pulley to another one or more pulleys may be transmitted by any of the following types of belt drives:

Open Belt Drive

The open belt drive is used with shafts arranged parallel and rotating in the same direction.
Advantages: Simple & spacious
Disadvantages: Doesn't provide enough belt tension.

Cross Belt Drive

The cross belt drive is used with shafts arranged parallel and rotating in the opposite direction.
Advantages: Compact, provides enough tensions.
Disadvantages: Less spacious, rubbing causes excessive wear.

Quarter Turn Belt Drive

The quarter turn belt drive also known as right angle belt drive is used with shafts arranged at right angles and rotating in one definite direction.
Advantages: Compact, provides enough tensions.
Disadvantages: Less spacious, rubbing causes excessive wear.

Classification of Belt Drive (contd.)

Belt Drives with idler pulleys

A belt drive with an idler pulley is used with shafts arranged parallel and when open belt drive can't be used due to small angle of contact on the smaller pulley.
Advantages: Transmission over large distance ,Desired velocity ratio & Flexible design.
Disadvantages: Lots of pulleys & belts required and subsequent complexities.

Compound Belt Drive

If there is more than one pulley in a transmission shaft the arrangement is said to be a compound belt drive.
Advantages: Speed variation, Flexible Design.
Disadvantages: Spacious, Expensive.

Stepped or Coned Pulley Drive

A stepped or cone pulley drive is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.
Advantages: Multi-speed capability.
Disadvantages: Wear and tear of belt.

Open Belt Drive

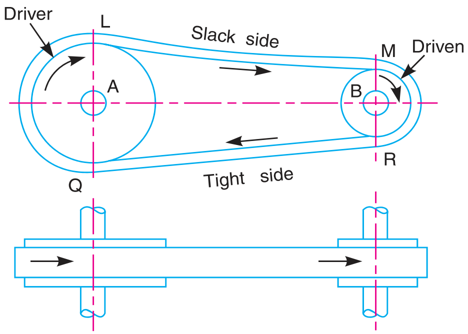


Figure: Open Belt Drive.

Open Belt Drive

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Cross Belt Drive

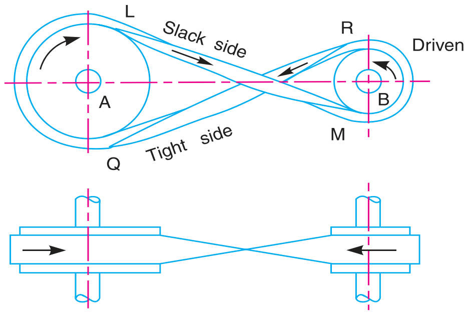

Figure: Cross Belt Drive.

Cross Belt Drive

The cross belt drive is used with shafts arranged parallel and rotating in the opposite direction.
Advantages: Compact, provides enough tensions.
Disadvantages: Less spacious, rubbing causes excessive wear.

Quarter Turn Belt Drive

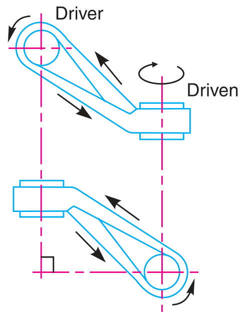

Figure: Quarter Turn Belt Drive.

Quarter Turn Belt Drive

The quarter turn belt drive also known as right angle belt drive is used with shafts arranged at right angles and rotating in one definite direction.
Advantages: Power transmission at right angle.
Disadvantages: Pulley width must be large.

Belt Drive with idler pulleys

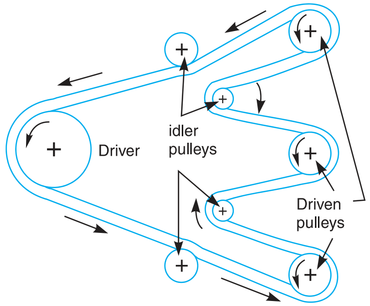

Figure: Belt Drive with idler pulleys.

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Compound Belt Drive

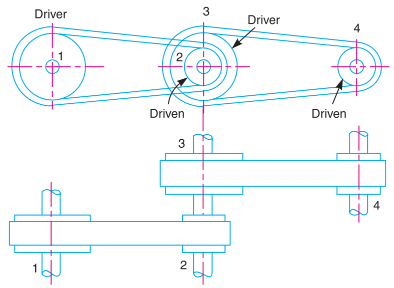

Figure: Compound Belt Drive.

Compound Belt Drive

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Stepped or Cone Pulley Drive

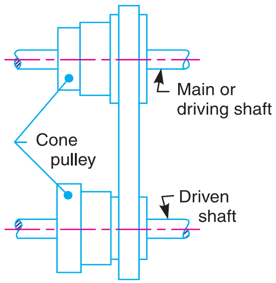

Figure: Stepped or Cone Pulley Drive.

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Advantages: Multi-speed capability.
Disadvantages: Wear and tear of belt.

Speed Ratio

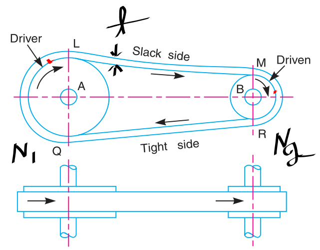


Figure: Speed ratio of belt drives.

Definition

It is the ratio between the velocities of the driver and the follower or driven.

Derivation

- Consider two points on the circular portion of the belt.
- If $v_1 \neq v_2$ then the belt will fail due to tearing or crushing.
- $\therefore v_1 = v_2$
- $\Rightarrow \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$
  $[\because v = \omega r; \omega = \frac{2\pi N}{60}]$
- $\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$
- If thickness of the belt is considered then $r = \frac{d}{2} + \frac{t}{2}$
- $\boxed{\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}}$

Speed Ratio of Compound Belt Drives

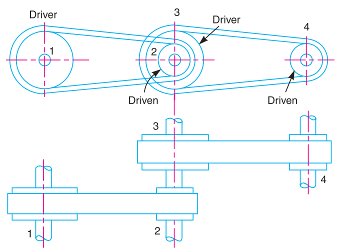


Figure: Compound belt drive.

Derivation

Pulleys	Speed Ratio	Equality
1,2	$\frac{N_1}{N_2} = \frac{D_2}{D_1}$	$N_2 = N_3$
3,4	$\frac{N_3}{N_4} = \frac{D_4}{D_3}$	$N_3 = N_4$
5,6	$\frac{N_5}{N_6} = \frac{D_6}{D_5}$	$N_5 = N_6$
...	...	...
k-1,k	$\frac{N_{k-1}}{N_k} = \frac{D_k}{D_{k-1}}$	$N_{k-1} = N_k$

$$\frac{N_1}{N_n} = \frac{D_2 \times D_4 \times D_6 \times \dots D_n}{D_1 \times D_3 \times D_5 \times \dots D_{n-1}} \quad (1)$$

$$\frac{\text{speed of first drive}}{\text{speed of last drive}} = \text{speed ratio} \quad (2)$$

$$\frac{\text{Product of no. of diameters on drivens}}{\text{Product of no. of diameters on drivers}} = \text{speed ratio} \quad (3)$$

Features

⇒ More than one pulley per shaft.
⇒ Intermediate shafts are available for changing speed & direction of motion.

Speed Ratio of Compound Belt Drives

$$\frac{\text{speed of first drive}}{\text{speed of last drive}} = \frac{\text{Product of no. of diameters on drivens}}{\text{Product of no. of diameters on drivers}} \quad (4)$$

Slip of Belt

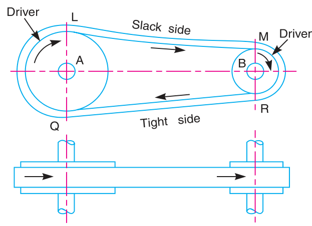


Figure: Slip of belt.

Definition

Slip refers to loss of speed of belt. If the frictional grip becomes insufficient, the driver may move forward without carrying the driven pulley forward. This is called slip of belt.

Derivation: Let,
$s = s_1 + s_2$ = Total Slip
s_1 = % slip between driver and belt
s_2 = % slip between belt and driven/follower
∴ velocity of belt passing over driver,

$$v = \frac{\pi D_1 N_1}{60}\left(1 - \frac{s_1}{100}\right) \quad (5)$$

Again, velocity of belt passing over follower,

$$\frac{\pi D_2 N_2}{60} = v\left(1 - \frac{s_2}{100}\right) \quad (6)$$

Substituting v in the later equation we get,

$$\frac{\pi D_2 N_2}{60} = \frac{\pi D_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{D_1}{D_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \left[\because \frac{s_1 s_2}{1000} \approx 0\right]$$

$$\boxed{\Rightarrow \frac{N_2}{N_1} = \frac{D_1}{D_2}\left(1 - \frac{s}{100}\right)} \quad [\because s = s_1 + s_2]$$

Belt Length (L) & Lap Angle (θ)

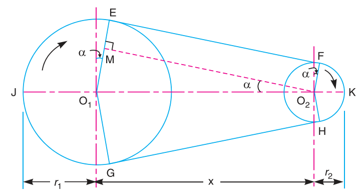


Figure: Length of open belt drive.

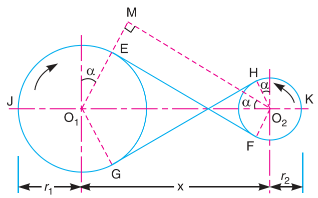


Figure: Length of cross belt drive.

Length of Open Belt Drive

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

Length of Cross Belt Drive

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

Lap/Contact Angle of Open Belt Drive

$$\theta = [180^o - 2sin^{-1}(\frac{r_1 - r_2}{x})] \times \frac{\pi}{180}$$

Lap/Contact Angle of Cross Belt Drive

$$\theta = [180^o + 2sin^{-1}(\frac{r_1 + r_2}{x})] \times \frac{\pi}{180}$$

Power Transmitted

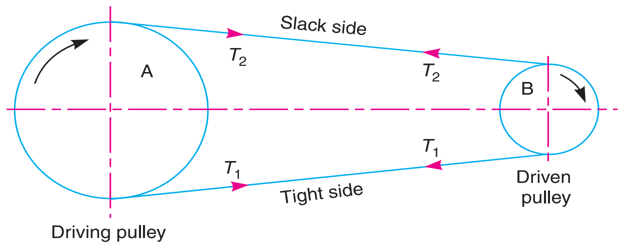


Figure: Power Transmitted by a belt Drive.

Power Transmitted, $\boxed{P = (T_1 - T_2)v}$

T_1 = Tension in tight side of belt (N)
T_2 = Tension in slack side of belt (N)
v = Belt velocity (m/s)

Ratio of Belt Tensions

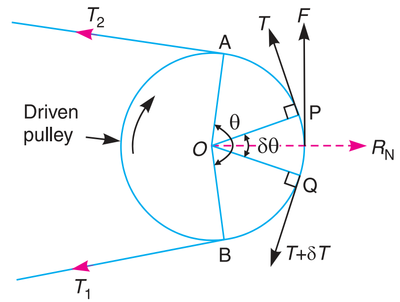


Figure: Driving tensions.

Also consider a small portion of the belt PQ, subtending an angle $\delta\theta$ at the centre of the pulley. The belt PQ is in equilibrium under the following forces :

1. Tension T in the belt at P
2. Tension T + δT in the belt at Q
3. Normal reaction R_N
4. Friction force $F = \mu R_N$ where μ is the coefficient of friction between the belt and the pulley.

Derivation:
Consider a driven pulley with,
T_1 = Tension in tight side of belt (N)
T_2 = Tension in slack side of belt (N)
θ = Angle of contact in radians

Ratio of Belt Tensions(contd.)

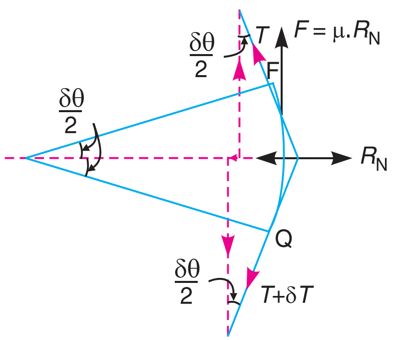


Figure: PQ portion.

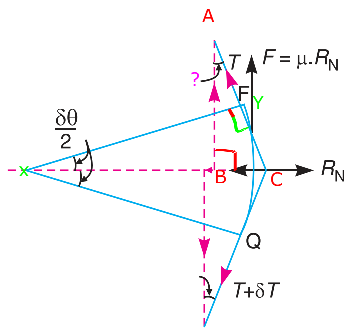


Figure: PQ portion.

Ratio of Belt Tensions(contd.)

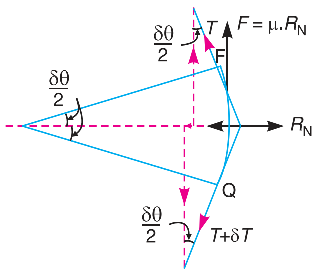


Figure: PQ portion.

Resolving forces in the horizontal direction,

$R_N = (T + \delta T) sin\frac{\delta\theta}{2} + T sin\frac{\delta\theta}{2}$
$\Rightarrow R_N = (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$
$[\because \delta\theta$ is very small $\therefore sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}]$

$\Rightarrow R_N = \frac{T\delta\theta}{2} + \frac{\delta T\delta\theta}{2} + \frac{T\delta\theta}{2}$
$\Rightarrow \boxed{R_N = T\delta\theta}$ $[\because \frac{\delta T\delta\theta}{2} \approx 0]$

Resolving forces in the vertical direction,
$\mu R_N = (T + \delta T) cos\frac{\delta\theta}{2} - T cos\frac{\delta\theta}{2}$

$\delta\theta$ is very small $\therefore cos\frac{\delta\theta}{2} = 1$,
$\Rightarrow \mu R_N = T + \delta T - T$
$\Rightarrow \boxed{R_N = \frac{\delta T}{\mu}}$
Equating the red and blue equations,
$T\delta\theta = \frac{\delta T}{\mu}$
$\Rightarrow \frac{\delta T}{T} = \mu\delta\theta$
$\Rightarrow \int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta$
$\Rightarrow [lnT]_{T_2}^{T_1} = \mu\theta$
$\Rightarrow ln\frac{T_1}{T_2} = \mu\theta$
$\boxed{\therefore \frac{T_1}{T_2} = e^{\mu\theta}}$

Ratio of Belt Tensions for V-belts

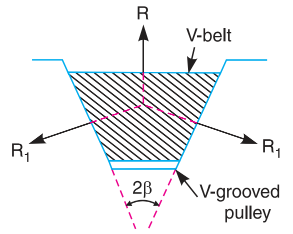


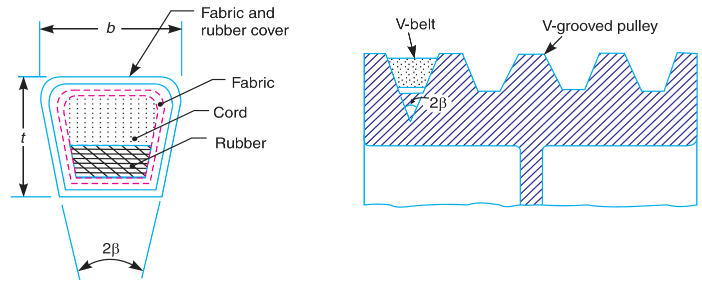
Figure: V-belt(grooved pulley)

Derivation:
Consider a grooved pulley with,
R_1 = Normal Reaction between the belt and sides of the groove (N)
R = Total Reaction in the plane of the groove (N)
μ = Coefficient of friction between the belt and sides of the groove.

Resolving the reactions vertically to the groove,

- $R = R_1 sin\beta + R_1 sin\beta = 2R_1 sin\beta$
- $\Rightarrow R = 2R_1 sin\beta$
- $\Rightarrow R_1 = \frac{R}{2sin\beta}$
- Friction force $F = 2\mu R_N$ [R_N = Normal Reaction to the pulley $= R$]
- $F = 2\mu \frac{R}{2sin\beta} = \frac{\mu R}{sin\beta} = \mu R cosec\beta$
- Rest of the proof is same as before.
- Resolving forces in horizontal direction
- $\boxed{R = T\delta\theta}$
- Resolving forces in vertical
- $\boxed{F = \delta T}$
- $\Rightarrow \mu R cosec\beta = \delta T$
- $\Rightarrow \mu (T\delta\theta) cosec\beta = \delta T$
- Integrate.

Ratio of Belt Tensions(contd.)



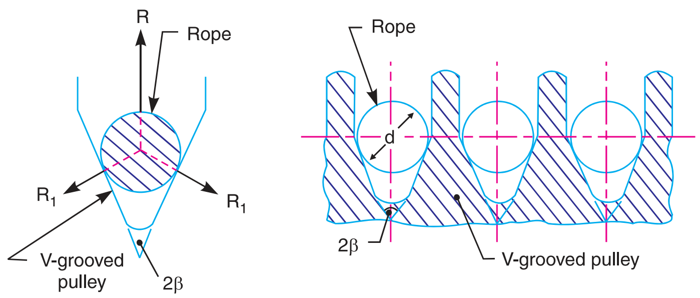
(*a*) Cross-section of a V-belt. (*b*) Cross-section of a V-grooved pulley.

Figure: V-belts

For V-belts and Ropes,

$\boxed{\frac{T_1}{T_2} = e^{\mu\theta cosec\beta}}$; 2β = Angle of groove

Ratio of Belt Tensions(contd.)



(*a*) Cross-section of a rope. (*b*) Sheave (Grooved pulley) for ropes.

Figure: Ropes

$\boxed{\frac{T_1}{T_2} = e^{\mu\theta cosec\beta}}$; 2β = Angle of groove

Centrifugal Tension(T_c)

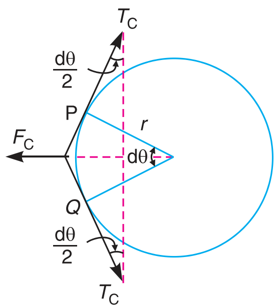


Figure: Centrifugal Tension.

Since the belt continuously runs over the pulleys, therefore some centrifugal force is caused that increase tension on both tight and slack side. However, below 10 m/s belt velocity T_c is negligible.

Consider, a small portion PQ of the belt, subtending angle $d\theta$ and
m = mass of belt per unit length (kg/m)
v= belt velocity (m/s)
r = pulley radius (m)
T_c = Tangential Centrifugal Tension at P,Q (N)

$\therefore$ Length of belt, PQ = r$d\theta$
$\therefore$ Mass of belt, PQ = m(r$d\theta$)

$\therefore$ Centrifugal Force acting on belt PQ,
$F_c = \frac{mrd\theta v^2}{r} = mv^2 d\theta$

Resolving forces horizontally,

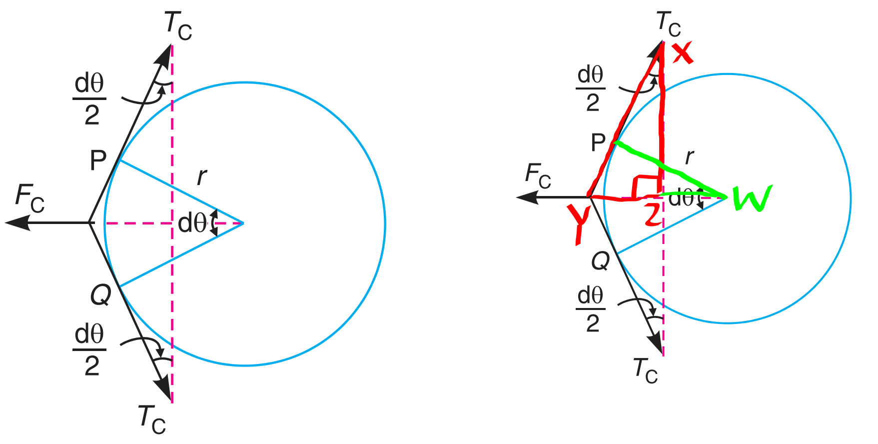
$2T_c sin(\frac{d\theta}{2}) = F_c = mv^2 d\theta$

$\Rightarrow 2T_c(\frac{d\theta}{2}) = mv^2 d\theta$

[$\frac{d\theta}{2}$ is very small $\therefore sin\frac{d\theta}{2} \approx \frac{d\theta}{2}$]

$\therefore \boxed{T_c = mv^2}$

Centrifugal Tension(contd.)



Centrifugal Tension (contd.)

$\therefore$ Tension in,

Tight side , $T_{t_1} = T_1 + T_c$
Slack side , $T_{t_2} = T_2 + T_c$

$\therefore$ Power Transmitted, $P = (T_{t_1} - T_{t_2})v = (T_1 - T_2)v$

$\therefore$ Ratio of belt tension ,

$\frac{T_1}{T_2} = e^{\mu\theta}$

$\Rightarrow \frac{T_{t_1} - T_c}{T_{t_2} - T_c} = e^{\mu\theta}$

Initial Tension, T_0

- The belt is assembled with an initial tension, T_0.
- When power is being to transmitted, the tension in the tight side increases fro T_0 to T_1 and slack side decreases from T_0 to T_2.
- If the belt is assumed to obey Hooke's law and its length remain constant, then the increase in length of the tight side is equal to the decrease in length of the slack side, i.e.,

$T_1 - T_0 = T_0 - T_2$

$\Rightarrow \boxed{T_0 = \frac{T_1 + T_2}{2}}$

If centrifugal tension is considered,

$\Rightarrow \boxed{T_0 = \frac{T_1 + T_2 + 2T_c}{2}}$

Condition for Maximum Power Transmission

Power Transmitted by a belt, $P = (T_1 - T_2)v$

$\Rightarrow P = (T_1 - \frac{T_1}{e^{\mu\theta}})v \; [\because \frac{T_1}{T_2} = e^{\mu\theta}]$

$\Rightarrow P = T_1(1 - \frac{1}{e^{\mu\theta}})v$

$\Rightarrow P = T_1 Cv \; [\because \mu, \theta = \text{Const. } C = 1 - \frac{1}{e^{\mu\theta}}]$

$\Rightarrow P = (T_{t_1} - T_c)Cv \; [\because T_{t_1} = T_1 + T_c]$

$\Rightarrow P = (T_{t_1} - mv^2)Cv \; [\because T_c = mv^2]$

For maximum power transmission, $\frac{dP}{dv} = 0$

$\Rightarrow \frac{d}{dv}[(T_{t_1}v - mv^3)C] = 0$

$\Rightarrow T_{t_1} - 3mv^2 = 0$

$\Rightarrow \boxed{T_{t_1} = 3mv^2 = 3Tc}$

This is the required condition.

$\therefore$ To transmit maximum power, the maximum tension in tight side must equal to three times the centrifugal tension.

Again,
$\Rightarrow T_{t_1} = 3mv^2$

$\Rightarrow \boxed{v_{max} = \sqrt{\frac{T_{t_1}}{3m}} = \sqrt{\frac{T}{3m}}}$

This is the expression for velocity when a belt is transmitting maximum power.

Chain Drives

Chain Drives

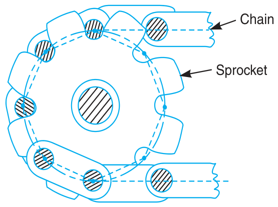


Figure: Sprocket and Chain

Features

⇒ Made of rigid links.

⇒ Links are flexible for warping.

⇒ Wheels have projecting teeth. (called sprockets)

⇒ Wheel and chain are constrained to move together ensuring no slip and perfect velocity ratio.

Classification

1. Hauling or Hoisting Chain: Cranes.
2. Tractive Chain: Conveyor Belts.
3. Power transmitting chain: Bicycles.

Problem 1

Open belt drive

An open belt drive connects two pulleys 1.2m & 0.5m in diameter on shafts 4m apart. Mass of belt is 0.9 kg per metre length & maximum tension should be checked not to exceed 2000 N. Coefficient of friction is 0.3. The 1.2m driver pulley runs at 200 rpm. Due to belt slip on one of the pulleys , the speed in the driven shaft is only 450 rpm. Determine, torque on the two shafts, power transmitted, power lost in friction & drive efficiency.

Solution: Here,

$$\text{Belt velocity, } v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 1.2 \times 200}{60} \approx 12.57 \text{ m/s}$$

$$\text{Centrifugal Tension, } T_c = mv^2 = 0.9 \times 12.57^2 \approx 142 \text{ N}$$

Now,Maximum (Allowable) Tension,

$$\begin{aligned} T &= T_1 + T_c \\ \Rightarrow T_1 &= T - T_c \\ &= 2000 - 142 \\ \therefore T_1 &\approx 1858 \text{ N} \end{aligned}$$

Problem 1 (contd.)

Now, for open belt drives ,angle of lap,

$$\begin{aligned} \theta &= 180^\circ - 2sin^{-1}(\frac{r_1 - r_2}{x}) \\ &= 180^\circ - 2sin^{-1}(\frac{0.6 - 0.25}{4}) \\ &= 180^\circ - 2(5.02^\circ) \\ \therefore \theta &= 169.96^\circ \end{aligned}$$

For a flat belt,

$$\begin{aligned} \frac{T_1}{T_2} &= e^{\mu\theta} \\ \Rightarrow T_2 &= \frac{T_1}{e^{\mu\theta}} = \frac{1858}{e^{0.3\times169.96\times\frac{\pi}{180}}} \\ &= \frac{1858}{2.438} \\ \therefore T_2 &= 762 \text{ N} \end{aligned}$$

Torque on shaft with larger pulley,
$T_L = (T_1 - T_2)r_L \approx 657.6$ Nm *(Ans.)*

Torque on shaft with smaller pulley,
$T_S = (T_1 - T_2)r_S \approx 274$ Nm *(Ans.)*

Power transmitted by belt,
$P = (T_1 - T_2)v \approx 13.78$ kW *(Ans.)*

Now, Input power,
$P_I = T_L\omega_L = 657.6 \times 2\pi\frac{200}{60} \approx 13.78$ kW

Again, Output power,
$P_O = T_S\omega_S = 274 \times 2\pi\frac{450}{60} \approx 12.91$ kW

$\therefore$ Power lost in friction
$P_{LOST} = P_I - P_O = 13.78 - 12.91 = 0.87$ kW *(Ans.)*

Now, Drive efficiency,
$\eta = \frac{P_O}{P_I} = \frac{12.91}{13.78} = 0.937 = 93.7\%$ *(Ans.)*

Problem 2

V-belts

Two V-belts construct a belt drive on grooved pulleys of the same size. The angle of groove is 30^o. The cross-sectional area of each belt is 750 mm^2 and coefficient of friction is 0.12.The density of belt material is $1.2Mg/m^3$ and maximum allowable stress in the material is 7 MPa. Determine, power transmitted between pulleys of 300mm diameters running at 1500 rpm and the shaft speed at which maximum power will be transmitted.

Solution: Here, Angle of groove, $2\beta = 30°$

$$\text{Belt velocity, } v = \frac{\pi dN}{60} = \frac{\pi \times 0.3 \times 1500}{60} \approx 23.56 \text{ m/s}$$

$$\text{Mass of belt per unit length, } m = \text{Area} \times \text{Length} \times \text{Density} = 750 \times 10^{-6} \times 1 \times 1200 \approx 0.9 \text{ kg/m}$$

Now,

$$\text{Centrifugal Tension, } T_c = mv^2 = 0.9 \times 23.56^2 \approx 500 \text{ N}$$

Problem 2 (contd.)

$$\text{Maximum Tension in belt, } T = \text{Max. Stress} \times \text{Cross-sectional Area} = 7 \times 10^{-6} \times 750 \times 10^{-6} = 5250 \text{ N}$$

Again, $T_1 = T - T_c = 5250 - 500 \approx 4750$ N

Now, for same sized pulleys ,angle of lap, $\theta = 180° = \pi$ radians

For v-belts,

$$\frac{T_1}{T_2} = e^{\mu\theta cosec\beta}$$

$$\Rightarrow \frac{T_1}{T_2} = e^{0.12\times\pi\times cosec15°}$$

$$\therefore T_2 \approx 1105 \text{ N}$$

Problem 2 (contd.)

Power transmitted, $P = (T_1 - T_2) \times v \times 2$ [$\because$ 2 V-belts]

$\Rightarrow P = (4750 - 1105) \times 23.56 \times 2$

$\therefore P = 171.75$ KW *(Ans.)*

For maximum power transmission,

$$v_{max} = \sqrt{\frac{T_{max}}{3m}} = \sqrt{\frac{5250}{3\times0.9}} = 44.1 \text{ m/s}$$

Again,

$$v_{max} = \frac{\pi DN_{shaft}}{60}$$

$$\Rightarrow 44.1 = \frac{\pi\times0.3N_{shaft}}{60}$$

$\therefore N_{shaft} \approx 2809$ rpm *(Ans.)*

Problem 3

Ropes

A rope drive transmits 600 KW from a 4m pulley running at 90 rpm. Lap angle is 160^o; groove angle is 45^o; coefficient of friction is 0.28;mass of rope is 1.5 kg/m & allowable tension in each rope is 2400 N. Find, how many ropes are required for the drive.

Solution: Here, Angle of groove, $2\beta = 45°$

$$\text{Rope velocity, } v = \frac{\pi dN}{60} = \frac{\pi \times 4 \times 90}{60} \approx 18.84 \text{ m/s}$$

$$\text{Centrifugal Tension, } T_c = mv^2 = 1.5 \times 18.84^2 \approx 533 \text{ N}$$

Now,Maximum (Allowable) Tension,

$$T = T_1 + T_c$$

$$\Rightarrow T_1 = T - T_c = 2400 - 533$$

$$\therefore T_1 \approx 1867 \text{ N}$$

Problem 3 (contd.)

For ropes,

$$\frac{T_1}{T_2} = e^{\mu\theta cosec\beta}$$
$$\Rightarrow \frac{T_1}{T_2} = e^{0.28\times 160\times\frac{\pi}{180}\times cosec22.5^\circ} = 2.05$$
$$\therefore T_2 \approx 240 \text{ N}$$

Power transmitted by a single rope, $P = (T_1 - T_2) \times v \approx 30.67$ kW

If number of ropes required is n & total power transmitted is P_{Total}, then

$$P_{Total} = P \times n$$
$$\Rightarrow n = \frac{P_{Total}}{P} = \frac{600}{30.67}$$
$$\therefore n \approx 19.56$$

(Ans. : 20 ropes are required)

Problem 4

Design Problem

A flat belt is to be designed to transmit 110kW at a belt speed of 25 m/s between two pulleys of diameters 250mm and 400mm, having a centre distance of 1m. The allowable belt stress is 8.5 MN/m^2 and belts are available having a thickness-to-width ratio of 0.1 and a material density of 1100 kg/m^3. Given that the coefficient of friction is 0.3. Determine the required belt width.
[Hint: Find Angle of lap,Centrifugal Tension, Total (Maximum) Tension and Power transmitted then solve for width]

Try Yourself

Problem 5

Special Problem

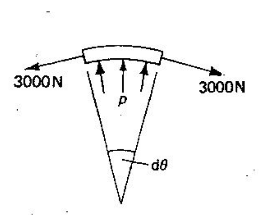
Two parallel horizontal shafts, whose center lines are 4.8m apart, one being vertically above the other, are connected by an open belt drive. The pulley on the upper shaft is 1.05m diameter, that on the lower shaft is 1.5m diameter. The belt is 150mm wide and the initial tension in it when stationary and when no torque is being transmitted is 3kN. The belt has a mass of 1.5 kg/m length; the gravitational force on it may be neglected but centrifugal force must be taken into account. The material of the belt may be assumed to obey Hooke's Law, and the free lengths of the belt between pulleys may be assumed to be straight. The coefficient of friction between the belt and either pulley is 0.3.

Calculate:(a) the pressure in N/m^2 between the belt and the upper pulley when the belt and pulleys are stationary and no torque is being transmitted,

(b) the tension in the belt and the pressure between the belt and the upper pulley if the upper shaft rotates at 400 rev/min and there is no resisting torque on the lower shaft, hence no power being transmitted, and

(c)the greatest tension in the belt if upper shaft rotates at 400 rev/min and the maximum possible power is being transmitted to the lower shaft.

Problem 5 (contd.)



$$p = \frac{Force}{Area}$$
$$= \frac{Force}{length \times width}$$
$$= \frac{2\times 3000\times sin\frac{d\theta}{2}}{0.525d\theta\times 0.15}$$
$$= \frac{2\times 3000\times\frac{d\theta}{2}}{0.525d\theta\times 0.15}$$
$$\therefore p \approx 38100 \ \frac{N}{m^2} \ (Ans.)$$

Solution:
(a): Here, width of belt = 150 mm = 0.15m and initial tension $T_0 = 3000$ N.Consider an element of the belt subtending an angle $d\theta$ at the center. Let, the pressure acting on the element be p. Now by resolving forces(initial tension) in the radial (perpendicular to the element) we have,

Problem 5 (contd.)

Solution: **(b):**

$$\text{Belt velocity, } v = \frac{\pi dN}{60} = \frac{\pi \times 1.05 \times 400}{60} \approx 21.99 \quad \text{m/s}$$

$$\text{Centrifugal Tension, } T_c = mv^2 = 1.5 \times 22^2 \approx 725 \quad \text{N}$$

The total length of the belt remains constant and the material obeys Hooke's law however no power is transmitted, therefore the total tension remains constant at 3000 N but some of this tension are now absorbed as centrifugal tension.

$\therefore$ *Effective tension* $= T - T_c = 3000 - 725 = 2275N$

Since the area remains the same,
$p = \frac{2275}{3000} \times 38100 \approx 28900 \quad \frac{N}{m^2}$ *(Ans.)*

(c): Angle of Lap,
$\theta = 180° - 2sin^{-1}(\frac{r_1 - r_2}{x})$

$= 180° - 2sin^{-1}(\frac{0.75 - 0.525}{4.8})$

$\therefore \theta = 174.6° = 3.05$ rad Now,

$T_{t_1} + T_{t_2} = 2T_0 = 6000 \ldots (1)$

Again,
$\frac{T_{t_1} - T_c}{T_{t_2} - T_c} = e^{\mu\theta}$

$\Rightarrow \frac{T_{t_1} - 725}{T_{t_2} - 725} = e^{0.3 \times 3.05} = 2.5 \ldots (2)$

Solving equations (1) and (2) we get,
$T_{t_1} = 3970$ N *(Ans.)*