

Control Engineering Sessional

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Gratitude

I am immensely grateful to the following person for the preparation and revision of this lecture.

- Mr. Md. Minal Nahin, Graduate Research Assistant , ME, IUPUI
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- Dr. Md. Zahurul Haq, Professor , ME, BUET
- Dr. Sumon Saha, Assistant Professor , ME, BUET

Course Content & Syllabus

Tentative Course Plan ¹

Topic	Week ²
Plotting and Linear Equations	1-2
Polynomials and Curve Fitting	3
Math Operations and Linear Equations in Simulink	4
Differential Equations and System Response	5
Transfer Functions, Zero-Poles and Stability	6
Laplace Transform and Residue	7
System Modelling and Controllers	8
PID Controller Tuning	9
Experiment	10
Experiment	11
Experiment	12
Final Examination	13

¹with Mr. Abu Jafar Rasel

²Each lecture is approximately 120 min.

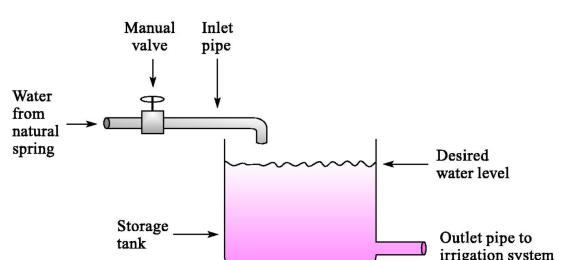
Marks Distribution

- Quizzes 50%
- Classwork 40%
- Assignment 10%
- **Classwork will be held at the beginning of the class.**
- **There will be at least one sudden quiz.**
- **Absence from both quizzes will result in a 'F' grade.**
- **Calculators are not allowed for classwork and quiz.**
- **Rough calculations on the question paper is not allowed either.**
- **For rough calculations you will need to use MATLAB command window.**

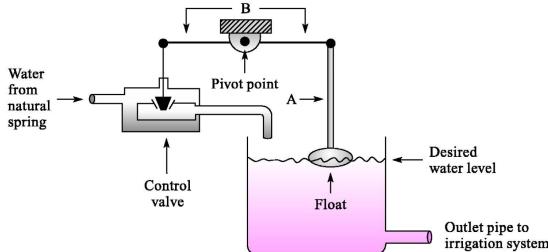
Introduction

- **Control engineering or control systems engineering** is the engineering discipline that applies control theory to design systems with desired behaviors.
- The practice uses sensors to **measure** the output performance of the device being controlled and those measurements can be used to give **feedback** to the input **actuators** that can make corrections toward desired performance.
- When a device is designed to perform without the need of human inputs for correction it is called **automatic control** (such as cruise control for regulating a car's speed).
- Multi-disciplinary in nature, control systems engineering activities focus on implementation of control systems mainly derived by mathematical modeling of systems of a diverse range.

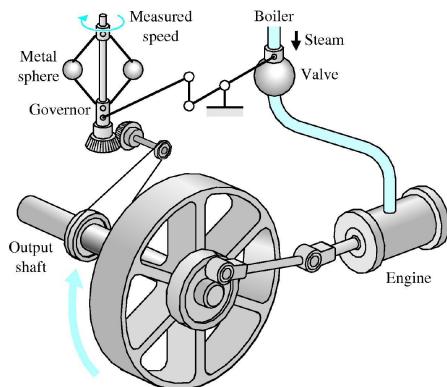
Liquid Level Control



Automatic Control

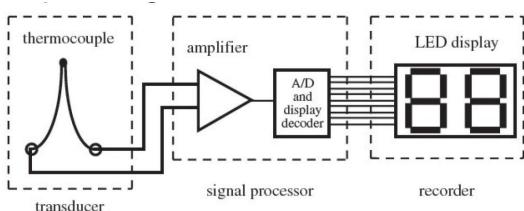


Watt's Flyball Governor



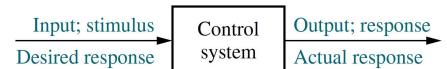
Elements of Measurement System

Consists of 3 basic elements



Control System

- The term **Control** means to regulate, to direct or to command.
- A **control system** is defined as a combination of devices and components connected or related so as to command, direct or regulate itself or another system.



Classification of Control Systems



Toaster

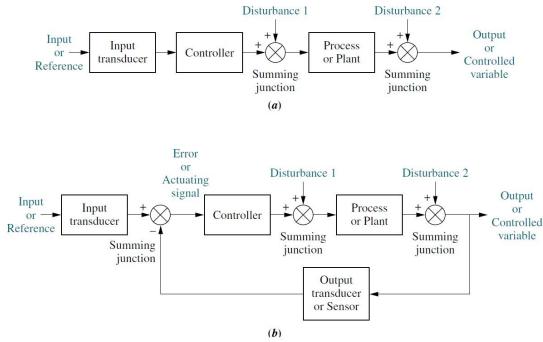


Air Conditioner

Classification of Control Systems

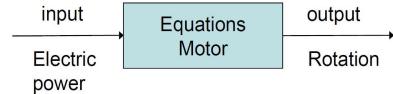
- Open Loop System** : Does not correct for feedback/disturbance. Also called non-feedback system.
- Close Loop System** : Corrects for feedback/disturbance. Also called feedback system.

Classification of Control Systems

Block diagrams of control systems: **a.** open-loop system; **b.** closed-loop system

Definition and Modeling of Systems

- A **system** can be thought of as a box which has an input and an output.
- Equations** are used to describe the relationship between the input and output of a system.
- Response** of a system is a measure of its fidelity to its purpose.
- Modeling** is the process of representing the behavior of a system by a collection of mathematical equations & logics.
- Simulation** is the process of solving the model and it is performed on a computer.
- In this course, we're going to learn to simulate systems using **MATLAB-Simulink**



Introduction

- MATLAB** developed by *The MathWorks Inc.*, stands for Matrix Laboratory.
- It is primarily used to perform scientific computation and visualization.
- Run MATLAB**
- MATLAB Environment** consists of
 - Command Window: *To execute commands.*
 - Command History: *To keep track of previously entered commands*
 - Workspace: *Collection of all variables, their data type and size.*
 - Current Directory: *Shows file path to run a file.*
 - Figure Window: *To display plots or graphics.*
 - Edit Window: *To create or modify a new or existing file respectively.*

To restore default window go to **Menu: Desktop/Desktop Layout/Default**

Basic Plotting (2D & 3D)

Meshgrid & Surf

Plot the function $z = e^{-x^2-y^2}$ over the range $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$

Solution:

```
>> x= linspace(-2,2);
>> y= linspace(-2,2);
>> [X,Y] = meshgrid(x,y);
>> Z= exp(-X.^2-Y.^2);
>> surf(X,Y,Z)
```

Note the use of Plot tools

Try **mesh(X,Y,Z)** instead of **surf(X,Y,Z)**

Practice

Plot the function $c = a^2 - b^2$ over the range $-2 \leq a \leq 2$ and $-2 \leq b \leq 2$

Plot the function $y = \sin x$ over the range $-2\pi \leq x \leq 2\pi$

Plot the function $y = x^3 - x^2 + 3$ over the range $-4 \leq x \leq 4$

Some useful functions for plotting

- figure** ⇒ Creates an empty figure window.
- title('string')** ⇒ Creates title in figure window.
- xlabel('x-axis')** ⇒ Creates x-axis label in figure window.
- ylabel('y-axis')** ⇒ Creates y-axis label in figure window.
- legend('a,b')** ⇒ Creates legend in figure window.
- shading interp** ⇒ Changes shading of plot(faceted, flat and interp).
- hold on** ⇒ holds the current figure window for further plotting.
- ezplot('x')** ⇒ plots the graph of the string input function
- area(x,y)** ⇒ same as **plot(x,y)** but the area under the curves are filled.
- bar(x,y)** ⇒ plots bar graph .
- mesh(x,y,z)** ⇒ same as **surf(x,y,z)** except patches between lines are not filled.

Solving Linear Equations : linsolve function

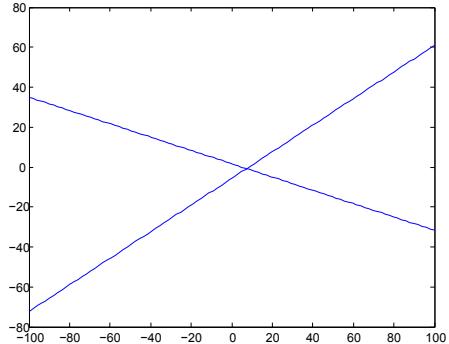
Any linear system of equations must have 0, 1 or infinite number of solutions.

linsolve(A,B) takes coefficient matrix A and constant matrix B as input and solves the system.

Solve the following system.

$$\text{System of Linear equations} \begin{cases} x + 3y + z = 5 \\ 2x - y + 2z = 17 \\ 3x + 4y + 5z = 26 \end{cases}$$

Solution visualization (contd.)



Polynomials

- MATLAB interprets an nth order polynomial as a row vector of $n + 1$
- Consider a polynomial $P(s) = s^4 + 3s^3 - 15s^2 - 2s + 9$
- Enter it in MATLAB as row vector P as follows
- $P = [1 3 -15 -2 9] \leftarrow$
- or $P = [1, 3, -15, -2, 9] \leftarrow$
- To evaluate the value of P at $s = 3$ or $P(3)$ use the following command $\text{polyval}(P,s)$
- $\text{polyval}(P,3) \leftarrow$
- MATLAB will show you the value of $P(3)$ to be 30.
- If there are missing terms, zeros must be entered at the appropriate places.

Solution visualization

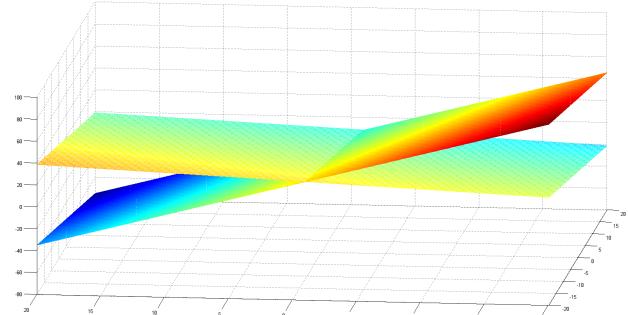
Draw the plot of the following equations in same graph.

$$\text{System of Linear equations with one solution} \begin{cases} x + 3y = 5 \\ 2x - y = 17 \end{cases}$$

Draw the mesh plot of the following equations in same graph.

$$\text{System of Linear equations with infinite solutions} \begin{cases} x + 3y + z = 5 \\ 2x - y + 2z = 17 \end{cases}$$

Solution visualization (contd.)



Practice

**Evaluate the value of
 $y = 2s^2 + 3s + 4$ at $s = 1, -3$**

Answer: $y(1) = 9$ $y(-3) = 13$

Practice

Evaluate the value of
 $y = 2s^3 + 1$ at $s = -4$

Answer: $y(-4) = -127$

Find the Polynomial

- If you know the roots(r) of a polynomial P, **MATLAB** can find the polynomial by the command **poly(r)**
- Represent the roots by a column vector.
- r = [-1; -2] ←**
- p = poly(r) ←**
- The answer is represented in a row vector p = [1 3 2].

Find the polynomial whose roots are -5.5745, 2.5836, -0.7951, 0.7860 .

Answer: $P(s) = s^4 + 3s^3 - 15s^2 - 2s + 9$

Practice

Evaluate the product of $(s + 3)$,
 $(s + 6)$ and $(s + 2)$

Answer: $s^3 + 11s^2 + 36s + 36$

Roots

- MATLAB** can find the roots of a polynomial P by the command **roots(P)**
- or **r = roots(P)**
- The answer is represented in a column vector.

Find the roots of $s^2 + 3s + 2$

Answer: -2, -1

Conv(x,y) Function: Convolution

- MATLAB** can multiply two polynomials by the command **conv(x,y)**
- Define the polynomials separately.
- The product is returned as a row matrix.
- In case there is more than two polynomials perform multiplication two at a time.

Multiply $(s + 2)$ and $s^2 + 4s + 8$

Answer: $s^3 + 6s^2 + 16s + 16$

deconv(z,y) Function

- MATLAB** can divide a polynomial z(s) by another polynomial y(s) by the command **deconv(z,y)**
- The syntax **[x,r] = deconv(z,y)** is preferred.
- Otherwise MATLAB may show the quotient only and skip the remainder.
- The division is shown in two row matrix. One for quotient and one for remainder.

Practice

Evaluate the division of $(s^2 - 1)$ by $(s + 1)$

Answer: $(s - 1) + 0$

Curve fitting: polyfit function

From Data to Equation.

- **MATLAB** can form a polynomial c that fits the given data for vectors x and y with the command `polyfit(x,y,k)`
- The syntax $c = \text{polyfit}(x,y,k)$ is preferred.
- x,y specifies the vectors of the points available for curve fitting.
- k specifies the order of the desired polynomial.
- The result c will be a row vector containing the coefficients of the polynomial.

Practice

Find a second order polynomial to predict the discharge(Q) of a pump after 1.5 seconds from the given data.

t(s)	0	1	2	4
Q(L/s)	1	6	20	100

Answer:

- ➊ $t=[0 \ 1 \ 2 \ 4];$
- ➋ $Q=[1 \ 6 \ 20 \ 100];$
- ➌ $c=\text{polyfit}(t,Q,2)$
- ➍ MATLAB returns, $c = 7.3409 \ -4.8409 \ 1.6818$
- ➎ $\therefore Q = 7.3409t^2 - 4.8409t + 1.6818$ is the equation of discharge.
- ➏ Use `polyval(c,1.5)` to get the answer 10.9375 L/s.

Advanced Polynomial Operations

- **MATLAB** can differentiate a polynomial $y(s)$ analytically by the command `polyder(y)`
- The differentiation is shown in a row matrix.
- **MATLAB** can integrate a polynomial $y(s)$ and integration constant k analytically by the command `polyint(y,k)`
- The integration is shown in a row matrix.

Practice this commands using the polynomial $s^4 + 4s^3 + 8s^2 + 16$

SIMULINK

- **Simulink** is a software package and is a graphical extension of **MATLAB**.
- It is very useful for modeling, simulating and analyzing dynamic systems due to its GUI.
- **Start Simulink**
- **Type Simulink** ←
- It can also be started by clicking its icon.
- Or use the start menu to initiate Simulink.
- Familiarize yourself with Simulink Library Browser

SIMULINK Library Browser

- Note that there are many blocks to conveniently model dynamic systems.
- Frequently used blocks are collected under **Commonly Used Blocks**.
- In this course we'll be using blocks from the following
 - ➊ Continuous
 - ➋ Discontinuous
 - ➌ Math Operations
 - ➍ Sinks
 - ➎ Sources
 - ➏ Signal Routing
 - ➐ Ports and subsystems

Basic Math Operations

Demonstration

Scope Vs. Display

Demonstration

- Add two numbers and check the output in both Scope and Display
- Subtract two numbers and check the output in both Scope and Display
- Multiply two numbers and check the output in Scope and Display
- Divide two numbers and check the output in Scope and Display

Demonstration

View the response of $50 + 60\sin(0.25t + \phi)$

Demonstration

View the response of $50 + 60\sin(0.25t + \phi)$ using 'add' and 'gain' operator.

Mux

Show the three input in a single graph.

- $100 + \sin(0.25t + 2)$
- $50 + \sin(0.5t)$
- $75 + \sin(0.75t + 1)$

System of Equations

- **Simulink** can solve both linear and nonlinear system of equations.

System of equations $\begin{cases} 2x + 3y = 13 \\ 5x - y = 7 \end{cases}$

System of Equations

Write the equations in the following form:

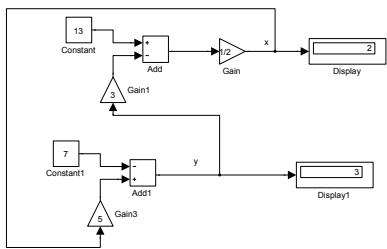
$$2x + 3y = 13$$

$$\Rightarrow x = \frac{13 - 3y}{2}$$

$$5x - y = 7$$

$$\Rightarrow y = 5x - 7$$

System of Equations



Classwork

Solve the following system.

System 1 $\begin{cases} x + 3y + z = 5 \\ 2x - y + 2z = 17 \\ 3x + 4y + 5z = 26 \end{cases}$

Classwork

Solve the following system.

$$\text{System 2} \begin{cases} a + b + c = 6 \\ 2a + 3b + 4c = 15 \\ a + 2b - c = 10 \end{cases}$$

Solution of ODE

Demonstration

If $\frac{dy}{dt} = 0.5t^2$ find y .
 $y(0) = 0$

Demonstration

$\frac{dx}{dt} - 3t^2 = 2t + \sin(0.25t)$; $x(0) = 0$

General System Modelling

A dynamic system can be represented in general by the differential equation:

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \dots + a_2 \frac{d^2 x}{dt^2} + \underbrace{a_1 \frac{dx}{dt}}_{\text{1st Order}} + \underbrace{a_0 x}_{\text{0th Order}} = F(t)$$

$F(t)$ → Forcing Function

$x(t)$ → Output or the response of the system

a's → Constants, System Parameters

Order of a system is designated by order of the differential equation.

Zeroth Order System

$$x = kF(t)$$

- $k = \frac{1}{a_0} \Leftrightarrow$ Static sensitivity or gain. It represents scaling between the input and the output.
- No equilibrium seeking force is present.
- Output follows the input without distortion or time lag.
- System requires no additional dynamic considerations.
- Represents ideal dynamic performance.
- Example: Potentiometer, ideal spring etc.

First Order System

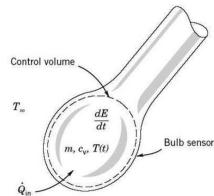
$$a \frac{dq(t)}{dt} + bq(t) = u(t)$$

- a, b are constants or physical system parameters.
- $u(t)$ is the input or forcing function.
- $q(t)$ is the output or response of the system.
- May be written as

$$\tau \frac{dq(t)}{dt} + q(t) = \frac{u(t)}{b}$$

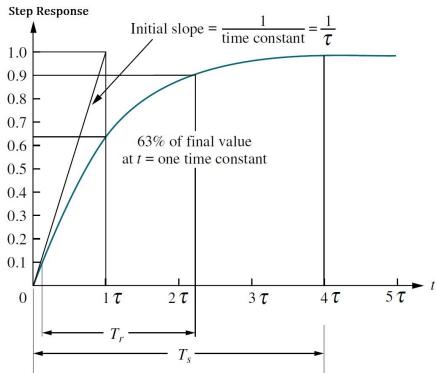
First Order System

Consider, a thermocouple initially at temperature T , exposed to ambient temperature T_α



- ① Convection Heat Transfer : $Q_{in} = hA(T_\alpha - T)$
- ② Heat Transferred to Solid : $Q_{out} = mC_v \frac{dT}{dt}$
- ③ First Law of TD $\Rightarrow Q_{in} = Q_{out}$
- ④ $\Rightarrow hA(T_\alpha - T) = mC_v \frac{dT}{dt}$
- ⑤ $\tau \frac{dT}{dt} + T = T_\alpha$
- ⑥ Where $\tau = \frac{mC_v}{hA}$

Response - 1st Order System: Step Input



First Order System

- **Time Constant, τ** - time required to complete 63.2% of the process.
- **Rise Time, T_r** - time required to achieve response from 10% to 90% of final value.
- For first order system, $T_r = 2.31\tau - 0.11\tau = 2.2\tau$.
- **Settling Time, T_s** - the time for the response to reach, and stay within 2% of its final value.
- For first order system, $T_s = 4\tau$.
- Process is assumed to be completed when $\tau \geq 5\tau$.
- Faster response is associated with shorter τ .

Transfer Function

- **Transfer function** of a system, $G(s)$, is defined as the ratio of the Laplace Transform (LT) of the output variable, $X(s)$, to the LT of the input variable, $F(s)$, with all the initial conditions are assumed to be zero.

$$G(s) = \frac{X(s)}{F(s)}$$

- $s = \sigma + j\omega$
- Amplitude Ratio, $G_a = \text{mod}(G(j\omega))$

Transfer Function of a 1st Order System

- For first order system, $G(s) = \frac{k}{s\tau+1}$.
- **Try the derivation**

$$\tau \frac{dx(t)}{dt} + x(t) = kF(t)$$

Apply Laplace Transform to both sides

$$\begin{aligned} \frac{d^n x(t)}{dt^n} &\Rightarrow s^n X(s) \quad \& F(t) \Rightarrow F(s) \\ &\Rightarrow \tau s X(s) + X(s) = kF(s) \\ &\Rightarrow (\tau s + 1)X(s) = kF(s) \\ &\Rightarrow \frac{X(s)}{F(s)} = G(s) = \frac{k}{(\tau s + 1)} \end{aligned}$$

Second Order System

Write Newton's 2nd Law for the system.

$$\sum F = ma$$

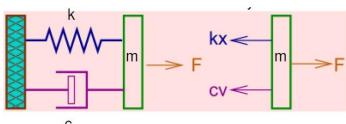
$$\Rightarrow F - cv - kx = ma$$

$$\Rightarrow ma + cv + kx = F$$

$$\Rightarrow m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F$$

Substitute the following:

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{m}} = \text{Undamped Natural Frequency (rad/s)} \\ C_c &= 2\sqrt{mk} = \text{Critical Damping Coefficient} \\ \zeta &= \frac{c}{C_c} = \text{Damping Ratio} \end{aligned}$$



TF of Second Order System

$$\Rightarrow m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F(t)$$

$$\Rightarrow \frac{1}{\omega_n^2} \frac{d^2x}{dt^2} + 2 \frac{\zeta}{\omega_n} \frac{dx}{dt} + x = \frac{F(t)}{k}$$

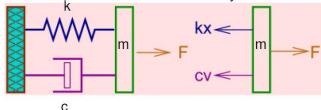
Taking Laplace Transform of both sides.

$$\frac{d^n x(t)}{dt^n} \Rightarrow s^n X(s) \quad \& F(t) \Rightarrow F(s)$$

$$\Rightarrow \frac{1}{\omega_n^2} s^2 X(s) + 2 \frac{\zeta}{\omega_n} s X(s) + X(s) = \frac{F(s)}{k}$$

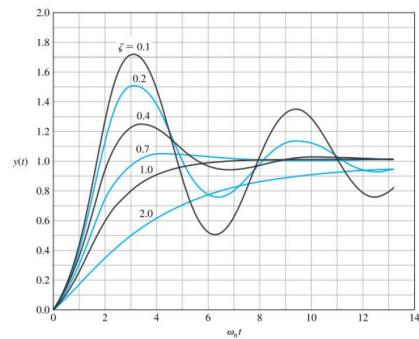
TF of Second Order System(contd.)

$$G(s) = \frac{X(s)}{F(s)} = \frac{\frac{1}{k}}{\omega_n^2 s^2 + 2\zeta\omega_n s + 1}$$



$$\Rightarrow G(s) = \frac{\frac{\omega_n^2}{k}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Response of a 2nd Order System: Step Input



Response of 2nd Order System: Step Input

- Steady state position is obtained after a long period of time.
- Underdamped System ($\zeta < 1$):** response overshoots the steady-state value initially, & then eventually decays to the steady state value. The smaller ζ become , the larger is the overshoot.
- Critically Damped System ($\zeta = 1$):** an exponential rise occurs to approach the steady state value without any overshoot.
- Overdamped System ($\zeta > 1$):** response of the system approaches the steady state value without overshoot, but at a slower rate.

Practice

1. Consider TF of a system , $G(s) = \frac{36}{s^2+4.2s+36}$ with gain k =1. Check the response in Simulink.

$\omega_n \Rightarrow 6$ & $\zeta \Rightarrow 0.35$
Check the underdamped system response.

2. Consider TF of a system , $G(s) = \frac{36}{s^2+42s+36}$ with k =1. Check the response in Simulink.

$\omega_n \Rightarrow 6$ & $\zeta \Rightarrow 3.5$
Check the overdamped system response.

3. Consider TF of a system , $G(s) = \frac{36}{s^2+12s+36}$ with k =1. Check the response in Simulink.

$\omega_n \Rightarrow 6$ & $\zeta \Rightarrow 1$
Check the critically damped system response.

Transfer Function in MATLAB

- MATLAB** defines transfer function using the function **tf(n,d)**
- A variable is used to store the transfer function.
- ∴ Syntax is **variablename= tf(n,d)**
- n and d are polynomials representing the numerator and denominator of the transfer function.

Define a transfer function, $G(s) = \frac{5s^2+15s+10}{s^4+7s^3+20s^2+24s}$

Answer:

- $n = [5 15 10]$
- $d = [1 7 20 24 0]$
- $sys = tf(n,d)$

Transfer Function in MATLAB

- To perform the exact opposite, i.e., separating the numerator and denominator of a given transfer function use the following command.
- [n,d] = tfdata(sys, 'v')** ←
- The numerator and denominator will be returned in 'n' and 'd' variables.
- 'v' is used in syntax to ensure that the polynomials are returned as row vectors and not cell arrays.

Practice separating the numerator and Denominator of $G(s) = \frac{5s^2+15s+10}{s^4+7s^3+20s^2+24s}$

Practice

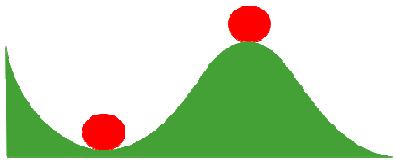
Define a transfer function,

$$G(s) = \frac{s(s+1)(s+2)}{s(s+3)(s^2+4s+8)}$$

Hint: Use poly and conv functions

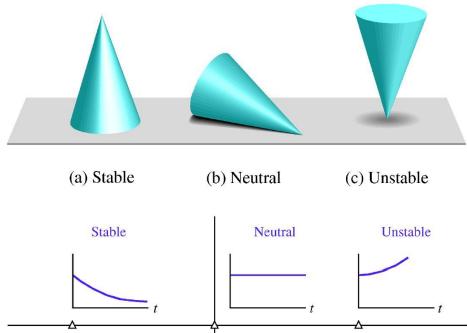
Stability Criteria

A necessary and sufficient condition for a feedback system to be stable is that all the poles of the system transfer function have negative real parts.



Stability

A stable system is a dynamic system with a bounded response to a bounded input. (BIBO)



Transfer Function in MATLAB

- **MATLAB** can also define transfer function using the following code.
- It is helpful when defining TF's with long numerator and denominator.

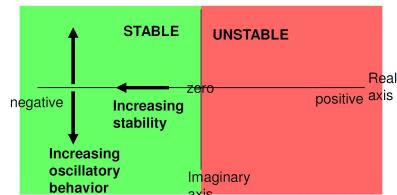
Define a transfer function, $G(s) = \frac{s+1}{s-4}$

Answer:

- ① $s = tf('s')$
- ② $G = (s+1)/(s-4)$

Stability Criteria

A necessary and sufficient condition for a feedback system to be stable is that all the poles of the system transfer function have negative real parts.



Zeros and Poles

- Use **tf2zp** to obtain the zeros z, poles p, and gain k of the transfer function, which is defined as a ratio of two polynomials.
- $[z,p,k] = tf2zp(n,d); \leftarrow$
- To obtain the transfer function, as a ratio of two polynomials when zeros z, poles p and gain k are known use **zp2tf**.
- $[n,d] = zp2tf(z,p,k);$
- The command **pzmap(n,d)** plots the **pole-zero map** of a given transfer function.

Practice

Zero, Pole, Pole-zero map, Stability

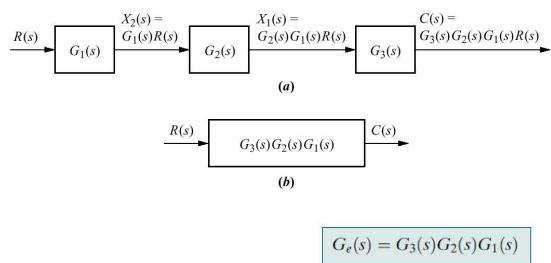
Find the location of zeros and poles and plot the pole-zero map of, $G(s) = \frac{2s^3+8s+6}{s^4+6s^3+12s^2+24s}$
Is the system stable?

Answer: In column matrix form,
 $z = -3, -1, p = 0, -4.5198, -0.7401 \pm 2.1822i, k = 2$. No.

Some useful functions for control systems

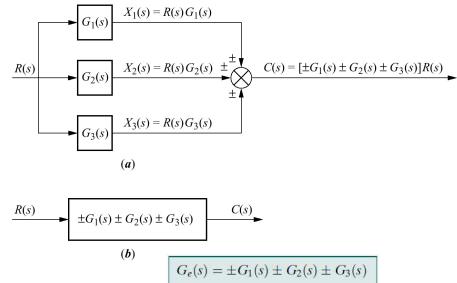
- ➊ **tf(n,d)** ⇒ Defines transfer functions.
- ➋ **pzmap(sys)** ⇒ Draws the pole-zero map.
- ➌ **bode(sys)** ⇒ Draws the bode plot.
- ➍ **rlocus(sys)** ⇒ Draws the root-locus plot.
- ➎ **step(sys)** ⇒ Shows step response.
- ➏ **stepinfo(sys)** ⇒ Rise time, settling time etc.
- ➐ **isstable(sys)** ⇒ Returns 0 if system is unstable.
- ➑ **zpk** ⇒ Creates zero-pole gain model.
- ➒ **impulse(sys)** ⇒ Shows impulse response.
- ➓ **series(sys1,sys2)** ⇒ Equivalent series transfer function.
- ➔ **parallel(sys1,sys2)** ⇒ Equivalent parallel transfer function.
- ➕ **feedback(sys1,sys2)** ⇒ Equivalent feedback transfer function.

TF manipulation: Series



Homework: Take $G = \frac{1}{s+1}$ and $H = \frac{1}{s-1}$ to check **series(G,H)**.

TF manipulation: Parallel



Homework: Take $G = \frac{1}{s+1}$ and $H = \frac{1}{s-1}$ to check **parallel(G,H)**.

TF manipulation: Feedback

Homework: Consider,

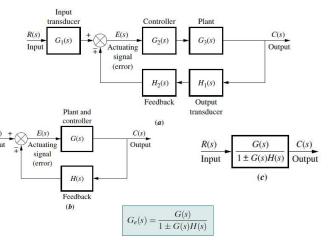


Figure: a) Feedback Control System.
b) Simplified Model.
c) Equivalent Transfer Function.

**Closed Loop System H=1

^asame as feedback(G,H,-1)

Laplace Transform (\mathcal{L}) in MATLAB

- Laplace transform helps to convert differential equations which describes the behavior of a dynamic systems into algebraic equations of a complex variable 's'.
- Differentiation and integration are thus replaced by by algebraic operation in complex plane.
- Both transient and steady-state component of the solution are obtained simultaneously.
- From Definition :

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} [f(t)] dt$$

Practice

- Determine the Laplace Transform of $f(t) = e^{-t}(1 - \sin(t))$

Use the following command:

```
syms t
ft = exp(-t) * (1 - sin(t));
fs = laplace(ft);
The result is shown as
fs = 1/(1+s) - 1/(2 + 2s + s^2)
i.e.,
F(s) = 1/(s+1) - 1/((s+1)^2+1)
```

Inverse Laplace Transform (\mathcal{L}^{-1}) in MATLAB

- Inverse Laplace transform converts the Laplace Transform F(s) to the time function f(t).

- From Definition :

$$\mathcal{L}^{-1}[F(s)] = f(t)$$

- Determine the Inverse Laplace Transform of $F(s) = \frac{1}{s+4}$

```
syms s t
fs = 1/(s+4);
ft = ilaplace(fs);
The result is shown as
ft = exp(-4*t)
i.e.,
F(t) = e^{-4t}
```

Residue Function

Determine the Partial Fraction expansion of $F(s) = \frac{s^3+9s+1}{s^3+s^2+2s+2}$

```
# n = [1 0 9 1];
# d = [1 1 2 2];
# [r, p, k] = residue(n,d)
```

MATLAB outputs the following

```
r =
1.0000 - 1.7678i
1.0000 + 1.7678i
-3.0000
p =
0.0000 - 1.4142i
0.0000 + 1.4142i
-1
k =
1
```

Residue Function

MATLAB output:

```
r =
1.0000 - 1.7678i
1.0000 + 1.7678i
-3.0000
p =
0.0000 - 1.4142i
0.0000 + 1.4142i
-1
k =
1
```

∴ the partial fraction expansion is

$$F(s) = \frac{s^3+9s+1}{s^3+s^2+2s+2} = 1 + \frac{1.0000 - 1.7678i}{s - 1.4142i} + \frac{1.0000 + 1.7678i}{s + 1.4142i} - \frac{3}{s + 1}$$

Practice

Determine the Partial Fraction expansion of $F(s) = \frac{s+1}{s^4+7s^3+16s^2+12s}$

Answer: $F(s) = \frac{0.6667}{s+3} + \frac{-0.75}{s+2} + \frac{0.5}{(s+2)^2} + \frac{0.0833}{s+0}$

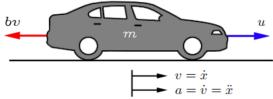
Reverse Residue

$$F(s) = 1 + \frac{1.0000 - 1.7678i}{s - 1.4142i} + \frac{1.0000 + 1.7678i}{s + 1.4142i} - \frac{3}{s + 1}$$

Find the numerator and denominator of F(s).

```
# define r,p and k first.
# [n, d] = residue(r,p,k) =
```

Car Cruise Control



Physical Parameters

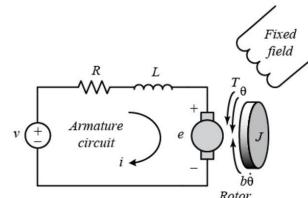
u = force generated between the road/tire interface = 500 N
 b = rolling resistance proportionality constant = 50 Ns/m
 m = mass of the car = 1000 kg

Consider, a simple cruise control system with the assumption that rolling resistance and air drag are proportional to the car's speed (v).

- Newton's 2nd Law : $\sum F = ma$
- u is the force generated between the road/tire interface and can be controlled directly.
- $\Rightarrow ma = u - bv$
- $\Rightarrow m\frac{dv}{dt} = u - bv$
- $\Rightarrow \frac{dv}{dt} = \frac{1}{m} \times (u - bv)$

$$\therefore \frac{dv}{dt} = \frac{1}{m} \times (u - bv)$$

Response of a DC motor

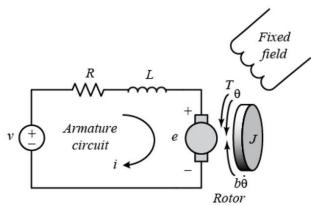


- All physical systems can be modeled using differential equations.
- Response of such systems can be found by simultaneously solving the governing DEQ's.

Physical Parameters

J = moment of inertia of the rotor (kgm^2)
 b = motor viscous friction constant (Nms)
 K_e = electromotive force constant (V/rad/sec)
 K_t = motor torque constant (Nm/Amp)
 R = electric resistance (Ohm)
 L = electric inductance (H)

Response of a DC motor(contd.)



From Newton's 2nd Law of Motion, $\sum F = ma$
For rotational systems, $\sum T = J\alpha$

$$\Rightarrow T - b\frac{d\theta}{dt} = J\frac{d^2\theta}{dt^2}$$

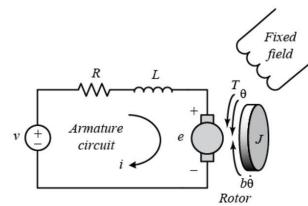
Motor Torque is proportional to Armature Current.

$$\therefore T = K_t i$$

$$\Rightarrow K_t i - b\frac{d\theta}{dt} = J\frac{d^2\theta}{dt^2}$$

$$\therefore \frac{d^2\theta}{dt^2} = \frac{1}{J}(K_t i - b\frac{d\theta}{dt}) \quad (1)$$

Response of a DC motor(contd.)



From Kirchoff's Voltage Law, $\sum V = 0$

$$\Rightarrow V = L\frac{di}{dt} + iR + e$$

Back emf(e) is proportional to the angular velocity of shaft.

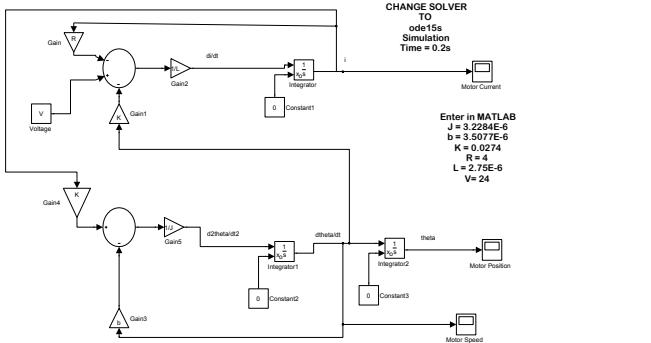
$$\therefore e = K_e \frac{d\theta}{dt}$$

$$\Rightarrow L\frac{di}{dt} = V - iR - K_e \frac{d\theta}{dt}$$

$$\therefore \frac{di}{dt} = \frac{1}{L}(V - iR - K_e \frac{d\theta}{dt}) \quad (2)$$

Solve equation (1) and (2) simultaneously in Simulink for speed/position of motor rotor.

DC Motor Simulink Model



DC Motor Simulink Model

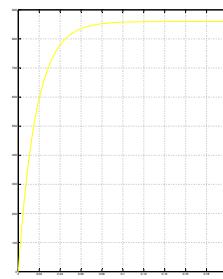


Figure: Motor Speed

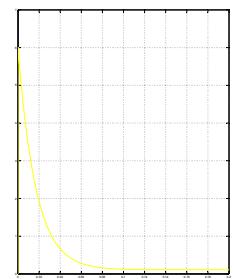


Figure: Motor Current

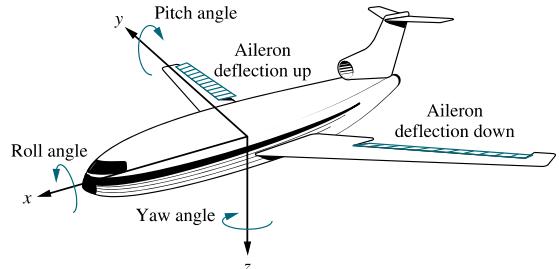
Estimating TF (Motor Speed)

Use the following data to estimate TF of motor (I=Voltage, O=Speed).

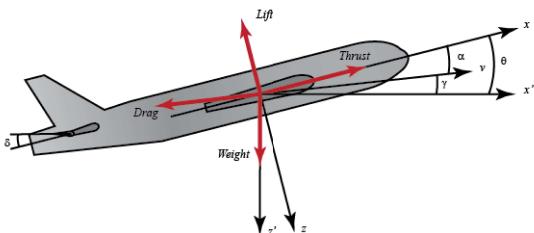
Parameter	Value
Moment of inertia of the rotor	0.01 kgm^2
Motor viscous friction constant	0.1 Nms
Electromotive force constant	0.01 Vradsec
Motor torque constant	0.01 NmAmp
Electric resistance	1 Ohm
Electric inductance	0.5 H

- 1 Declare Subsystem.
- 2 Set Input and output linearization points. Go to Tools/Control Design/Linear Analysis Tool.
- 3 Linearize Model. Export model to workspace from LTI viewer.
- 4 Use the command zpk(modelname).
- 5 $\frac{2}{(s+9.997)(s+2.003)}$ is the approximate transfer function.

Aircraft Pitch Control

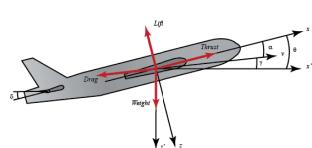


Aircraft Pitch Control



$$\text{Transfer function, } G(s) = \frac{\Theta(s)}{\Delta(s)} = ??$$

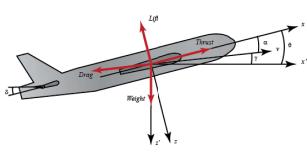
Aircraft Pitch Control



Physical Parameters

- α = Angle of attack
- q = Pitch Rate
- θ = Pitch angle
- δ = Elevator deflection
- ρ = Density of air
- S = Platform area of wing
- \bar{c} = Average chord length
- m = Mass of aircraft
- $\mu = \frac{\rho S c}{4 m}$
- U = Equilibrium flight speed
- C_T = Coefficient of thrust
- C_D = Coefficient of drag
- C_L = Coefficient of lift
- C_W = Coefficient of weight
- C_M = Coefficient of pitch moment
- γ = Flight path angle
- $\Omega = \frac{2U}{\bar{c}}$
- $\sigma = \frac{1}{1 + \mu C_L} = \text{constant}$
- i_{yy} = Normalized moment of inertia
- $\eta = \mu \sigma C_M = \text{constant}$

Aircraft Pitch Control



Under these assumptions, the following 3 governing equations are used by one of Boeing's commercial aircraft.

$$\frac{d\alpha}{dt} = -0.313\alpha + 56.7q + 0.232\delta$$

$$\frac{dq}{dt} = -0.0139\alpha - 4.426q + 0.0203\delta$$

$$\frac{d\theta}{dt} = 56.7q$$

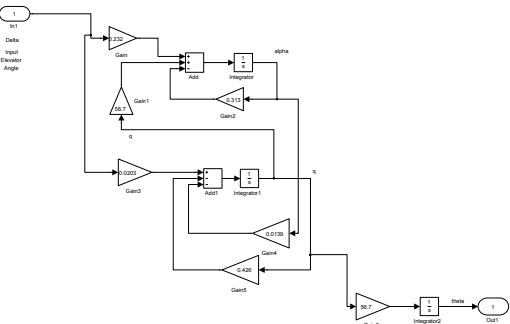
$$\text{Ans: } G(s) = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s}$$

Is the system stable?

Draw step response when elevator deflection is 0.2 rad.

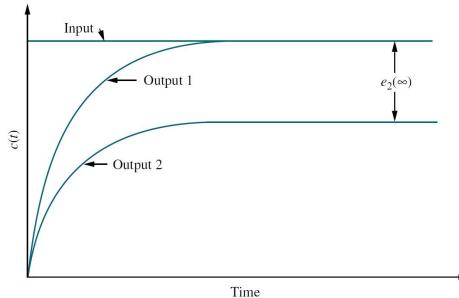
The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. Therefore, in order to simplify, we will assume that the aircraft is in steady-cruise at constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the x- and y-direction and that a change in pitch angle will not change the speed of the aircraft under any circumstance (unrealistic but simplifies the problem a bit).

Aircraft Pitch Control

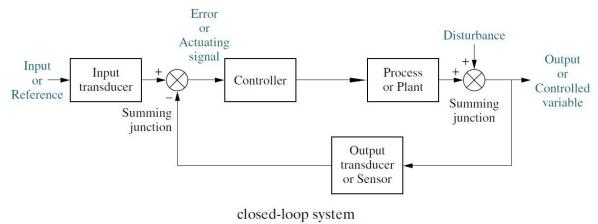


Steady State Error

The difference between the input and output for a prescribed test input as time, t approaches ∞ .



Feedback Control



Feedback Control

- In feedback control , the objective is to reduce error signal to zero.
- $e(t) = y_{sp}(t) - y_m(t)$
- $e(t)$ = Error Signal
- $y_{sp}(t)$ = Set Point
- $y_m(t)$ = Measured value of the controlled or process variable (equivalent sensor signal)
- Although the error signal equation implies that set point is time varying but in many applications it is kept constant over a long period of time.

Different types of feedback control

Proportional Control (P)

- For Proportional Control , the objective is to reduce error signal to zero.
- $p(t) = \bar{p} + K_p e(t)$
- $p(t)$ = Controller output
- \bar{p} = Bias or steady state value
- K_p = Proportional Controller Gain

Integral Control (I)

- For Integral Control , the rate of change of controller output is proportional to the error signal.
- $p(t) = \bar{p} + K_i \int e(t) \text{ i.e., } \frac{dp}{dt} = K_i e(t)$
- $p(t)$ = Controller output
- \bar{p} = Bias or steady state value
- K_i = Integral Controller Gain

Different types of feedback control(contd.)

Derivative Control (D)

- For Derivative Control , the controller output is proportional to the rate of change of error signal.
- $p(t) = \bar{p} + K_d \frac{de(t)}{dt}$
- $p(t)$ = Controller output
- \bar{p} = Bias or steady state value
- K_d = Derivative Controller Gain

How PID affects a process/system

CL Response	Rise Time	Overshoot	Settling Time	S-S Error
K_p	Decrease	Increase	Small change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small change	Decrease	Decrease	No change

- A proportional controller (K_p) reduces the rise time and will reduce but never eliminate the steady-state error.
- An integral control (K_i) eliminates the steady-state error for a constant or step input, but it may make the transient response slower.
- A derivative control (K_d) increases the stability of the system, reducing the overshoot, and improving the transient response.
- Note that these correlations may not be exactly accurate, because K_p , K_i , and K_d are dependent on each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for K_i , K_p and K_d .

PID Controller Design: Step Input, Unit Feedback

```

1 %Controller Design for Unit feedBack
2 clc
3 clear all
4 %----- Controller Design for Unit feedBack
5 disp(' ');
6 Kp = input('Input Kp = ');
7 Ki = input('Input Ki = ');
8 Kd = input('Input Kd = ');
9
10 C= tf([Kp Ki], [1 0]); % Defines the PID Controller
11
12 num = input('Input Plant TF numerator vector = ');
13 den = input('Input Plant TF denominator vector = ');
14
15 P = tf(num,den); % Defines the Process Transfer Function
16
17 step(P); % Draws step response of P
18 hold on; % Uses the same figure for next plot
19
20 T= feedback(C*P,1); % Unit feedback is fed to the controlled system
21
22 t=0:0.01:2; %Sets simulation time
23
24 step(T,t); % Draws step response of closed loop feedBack system T

```

PID Controller Tuning

Problem

Manually tune a PID controller for a plant having $G(s) = \frac{1}{s^2+10s+20}$. Your goal is to achieve -

- fast rise time
- minimum overshoot
- no steady-state error

Open Loop Response

- Put $K_P = 0, K_D = 0, K_I = 0$ in the code.
- The steady-state error is as much as 95%.
- Settling time is 1.5 sec.

P-Controller

- Put $K_P = 300$ in the code.
- Steady-state error is reduced.
- Rise time is reduced.
- Overshoot increased.
- Settling time slightly reduced.

PID Controller Tuning(contd.)

PD- Controller

- Put $K_P = 300 K_D = 10$ in the code.
- Steady-state error is slightly reduced.
- Rise time is slightly reduced.
- Overshoot reduced.
- Settling time reduced.

P,I-Controller

- Put $K_P = 30 K_I = 70$ in the code.
- Proportional gain is reduced because integral controller alone reduces the rise time and increases the overshoot effect.
- If both of them have high values it will create a double effect.
- This controller eliminates the steady-state error.

PID-Controller

- Put $K_P = 350 K_I = 300 K_D = 5500$ in the code.
- Very fast rise time.
- No overshoot.
- No steady-state error.