Assignment 3

28-01-2019

(1) Identify the set $A \subset \mathbb{N}$ given that: if $x \in A$, then $x+1 \in A$. Justify your answer.

(2) The *absolute value* or *mod* function is defined as follows: $\forall x \in \mathbb{R}$, define

$$|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

Identify the following sets:
(i) $\{x \in \mathbb{R} : |x| = 3\}$ (ii) $\{x \in \mathbb{R} : |x| \leq 3\}$ (iii) $\{x \in \mathbb{R} : |x+5| \leq 3\}$

(3) Find lub(A) and glb(A) in B in the following examples, if they exist. If not, explain why they do not exist.

(a) $A = \mathbb{N}, B = \mathbb{N}$
(b) $A = \{x \in \mathbb{Z} : -1 \leq |x+5| < 8\}, B = \mathbb{Z}$. Does your answer change if $B = \mathbb{R}$?
(c) $A = \{x \in \mathbb{Q} : -1 \leq |x+5| < 8\}, B = \mathbb{Q}$. Does your answer change if $B = \mathbb{R}$?
(d) $A = \{x \in \mathbb{Q} : x \neq 0, 1/x \in \mathbb{N}\}, B = \mathbb{Q}$. Does your answer change if $B = \mathbb{R}$?

(4) Let $A \subset \mathbb{R}$ and $\alpha \in \mathbb{R}$ such that α in an upper bound of S in $\mathbb{R}$. Show that the following statements are equivalent.

(a) α is the least upper bound of A in $\mathbb{R}$.
(b) For every $\epsilon > 0 \in \mathbb{R}$, there exist $a \in A$ such that $\alpha - \epsilon < a \leq \alpha$
(c) For every $t \in \mathbb{R}$ with $t < \alpha$, there exists $a \in A$ such that $t < a \leq \alpha$.

(5) Let $c \in \mathbb{R}; A, B \subset \mathbb{R}$ be bounded and non-empty. State whether the following are true or false. If true, prove it. If false, give a counter-example, and state and prove the corrected version.

(a) If $A \subset B$ then $\text{lub}(A) = \text{lub}(B)$.
(b) $\text{lub}(A \cup B) = \min\{\text{lub}(A), \text{lub}(B)\}$
(c) $\text{lub}(A \cap B)$_____$\{\text{lub}(A), \text{lub}(B)\}$
(d) $\text{lub}(cA) = c\,\text{lub}(A)$
(e) $\text{lub}(A+B) = \text{lub}(A) + \text{lub}(B)$

(6) Let X be a set, $A, B \subset X$. Show that the following are equivalent:
(i) $A \subset B$ (ii) $A = A \cap B$ (iii) $A \subset (A \cap B)$
(iv) $B^c \subset A^c$ (v) $B = A \cup B$ (vi) $(A \cup B) \subset B$

(7) Let $a, b, c, d \in \mathbb{N}$. State the following mathematically and write their negations:

(a) c divides a. (Easier to think of: a is a multiple of c).
(b) c is a common divisor of a and b,
(c) d is a greatest common divisor of a and b.