Quiz 3 MA 107 (Max. Marks 10)
IIT Bombay, 2nd April 2019

(1) Let $S = \{a \in \mathbb{Z} | a \text{ is a multiple of } 3\}$.
Show that (i) $0 \in S$ (ii) If $a \in S$, then $-a \in S$. (iii) If $a, b \in S$, then $a + b \in S$.

(2) Let $S \subset \mathbb{N}$ be such that (1) $1 \in S$ and (2) For $k \in \mathbb{N}$, if $\{1, 2, \cdots, k\} \subset S$, then $k + 1 \in S$.
Show that $S = \mathbb{N}$.

(3) Prove or disprove:
Let (a_n) be a monotonically decreasing sequence of real numbers. Then (a_n) is convergent.

(4) Prove or disprove: The sequence $\left(\frac{(-1)^n}{2}\right)$ is convergent.