
Assignment 5

20-02-2019

- (1) Let $x, y \in \mathbb{R}$ be such that $x, y > 0$ and $n \in \mathbb{N}$. Show that if $x^n \leq y^n$, then $x \leq y$.
- (2) Show that if $x \in (0, 1)$, then $x \notin \mathbb{Z}$.
- (3) If $r \in \mathbb{R} \setminus \mathbb{Q}$ and $x \in \mathbb{Q} \setminus \{0\}$, show that rx and $r + x$ are elements of $\mathbb{R} \setminus \mathbb{Q}$.
- (4) Show that there is no rational number x such that $x^2 = 3$.
- (5) For all $0 < x \in \mathbb{R}$ and $m \in \mathbb{N}$, define $x^{1/m}$ to be the unique real number $y > 0$ such that $y^m = x$. Show the following:
 - (a) For all $0 < x \in \mathbb{R}, m, n \in \mathbb{N}$, $(x^m)^{1/n} = (x^{1/n})^m$.
 - (b) For $m, n, l, k \in \mathbb{N}$, if $m/n = l/k$, then show that $(x^m)^{1/n} = (x^l)^{1/k}$.
- (6) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = x^2$. Find $f(A)$ for $A =$
 - (i) $\{1, 1/2, 1/3, -1/2\}$ (ii) $[0, 2]$ (iii) $(1, 2]$ (iv) $[-2, 1)$ (v) $[-2, -1)$
- (7) Is the function $f(x) = x^2$ one-one or onto as a function from
 - (i) \mathbb{R} to \mathbb{R} ? (ii) \mathbb{R} to $[0, \infty)$? (iii) $(0, \infty)$ to $(0, \infty)$? (iv) $(0, 1)$ to $(0, 1)$? Can you identify properties of the graph that give the one-one or onto conditions?
- (8) Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be functions.
 - (a) If f and g are one-one, show that $g \circ f$ is one-one.
 - (b) Is the converse true?
 - (c) Answer (a) and (b) with “one-one” being replaced by “onto”.
- (9) Find a bijection from $(0, 1)$ to A , where $A =$
 - (i) $(1, 2)$ (ii) $(0, 2)$ (iii) $(1, 3)$ (iv) Can you find a bijection from $(0, 1)$ to \mathbb{R} ?
- (10) Show that A is countable, where $A =$
 - (i) $\{2, 3, 4, 5, \dots\}$ (ii) $\{2, 4, 6, 8, \dots\}$ (iii) $\{1, 3, 5, 7, \dots\}$ (iv) $2\mathbb{Z}$ (v) $2\mathbb{Z}+1$ (vi) \mathbb{Z} (vii) $\mathbb{N} \times \mathbb{N}$