Assignment 1

14-01-2019

(1) Let $S \subset \mathbb{R}$ and $a \in \mathbb{R}$. Define what it means to say that

  (a) a is a minimum of the set S.
  (b) the set S has a minimum.

(2) Prove the following statements given that $\mathbb{R}$ is a field. State clearly the axioms you are using.

  (a) For all $a, b, c \in \mathbb{R}$ $a + b = a + c$ implies $b = c$.
  (b) For all $a, b, c \in \mathbb{R}$ and $a \neq 0$, $a \cdot b = a \cdot c$ implies $b = c$
  (c) For all $a \in \mathbb{R}$, $a \cdot 0 = 0$
  (d) If for any $a, b \in \mathbb{R}, a \cdot b = 0$ then $a = 0$ or $b = 0$.
  (e) For all $a \in \mathbb{R}, -a = (-1) \cdot a$
  (f) For all $a, b \in \mathbb{R}, a, b \neq 0$, then $1/(a \cdot b) = (1/a) \cdot (1/b)$

(3) Prove the following statements given that $\mathbb{R}$ is an ordered field. State clearly the axioms you are using.

  (a) For all $a, b \in \mathbb{R}$, if $a < b$ then $(-b) < (-a)$.
  (b) $0 < 1$.
  (c) For all $a, b \in \mathbb{R}$, is $0 < a < b$ then $0 < 1/b < 1/a$.
  (d) $a \in \mathbb{R}, a \neq 0$ then $a^2 > 0$.
  (e) If for some $a, b \in \mathbb{R}, a^2 + b^2 = 0$ then $a = b = 0$.
  (f) For all $a, b \in \mathbb{R}$, if $0 < ab$ then $0 < a, b$ or $a, b < 0$.
  (g) For all $a, b \in \mathbb{R}$, if $ab < 0$ then $a < 0, 0 < b$ or $b < 0, 0 < a$.
  (h) If $a \in \mathbb{R}$ is such that $a \cdot a = a$ then $a = 0$ or $a = 1$.
  (i) For all $a, b \in \mathbb{R}$, if $0 < a < b$ then $a^2 < b^2$
  (j) For all $a, b \in \mathbb{R}$, if $0 < a < b$ then $a < \sqrt{a \cdot b} < b$
  (k) For all $a, b \in \mathbb{R}$, if $a \leq b$ then $a \leq \dfrac{a+b}{2} \leq b$
  (l) For all $a, b \in \mathbb{R}$, if $0 < a < b$ then $\sqrt{a \cdot b} < \dfrac{a+b}{2}$

Notation i) a^2 denotes $a \cdot a$
ii) $\sqrt{x}$ is a number such that $(\sqrt{x})^2 = x$.
As we have not proven that such a number does exist, you may assume for now that it does.