Assignment 2

21-01-2019

(1) Let X be a set, $A, B \subset X, x \in X$. Define (i) $A \cap B$ (ii) $A \cup B$ (iii) $A \subset B$ (iv) $A = B$

(2) Write A as a subset of B, where:

(a) $A = \emptyset$; (i) $B = \mathbb{R}$ (ii) $B = \mathbb{Q}$ (iii) $B = \mathbb{Z}$ (iv) $B = \mathbb{N}$

(b) $B = \mathbb{R}$; (i) $A = (0, 1]$ (ii) $A = \mathbb{Z}$ (iii) $A = \mathbb{Q}$

(c) $B = \mathbb{R}^2$; A is (i) the $X-$axis (ii) the unit circle
(iii) the set of solutions of the equation $x + 2y = 0$

(3) Describe the following sets:

(a) (i) $\{x \in \mathbb{R} : x(x-1)(x-2) > 0\}$ (ii) $\{x \in \mathbb{R} : \cos(2\pi x) = 0\}$ (iii) $\{x \in \mathbb{R} : x^2 = 1\}$

(b) (i) $\left\{(x, y) \in \mathbb{R}^2 : \frac{x}{y} + \frac{y}{x} \geq 2\right\}$ (ii) $\{(x^2, x) : x \in \mathbb{R}\}$

(c) $A \times B$, where (i) $A = [0, \infty)$ and $B = [2, 3]$. (ii) $A = [3, 4]$ and $B = \mathbb{N}$.

(4) What is the st $A + 2$, where (i) $A = \mathbb{Z}$ (ii) $A = \{1, 2, 3, 4\}$ (iii) $A = [1, 2)$ (iv) $A = (\infty, 0)$
What is the set $2A$ where A is as above?
What is the set $\frac{\pi}{4}(2\mathbb{Z} + 1)$? Is it related to any of the sets in the previous question?

(5) What is the set $c + \mathbb{Q}$? When does it contain a rational number?
What can you say in the other cases? Answer similar questions about $c\mathbb{Q}$.

(6) Let A and B be non-empty subsets of $\mathbb{R}$, and $c \in \mathbb{R}$
Describe the sets $-A, cA, c + A, A \cap B, A \cup B$ and $A + B$.
How is their lub/glb (if they exist), related to the lub/glb of A and B?

(7) Identify some rational and irrational numbers in $\mathbb{Q} + [0, 1]$? What is this set?

(8) Let $S \in \mathbb{R}$. Is S has a maximum, then S is bounded above and $\max(S) = \text{lub}(S)$.

Note: In (1), the first two sub-parts would require a set description while the last two would be a condition.