Midsem: MA 107
Spring 2019, IIT Bombay

(1) Write the negation of the following statements in words. Then write both the original statements and its negation mathematically. (While writing the mathematical statements, define the appropriate sets).

   (a) Every library has a book with at least 100 pages.
   (b) Each student is smart or has a smart phone.

(2) For non-empty subsets $A, B \subset \mathbb{R}$, define $A + B = \{x \in \mathbb{R} | \exists y \in A, z \in B (x = y + z)\}$

   (a) For $A = (2, 4)$ and $B = \{3, 4\}$, show that $7 \in A + B$ and $8 \notin A + B$.
   (b) For $A = (2, 4)$ and $B = \{3, 4\}$, find $A + B$ (with proof).
   (c) If A and B are bounded above in $\mathbb{R}$, show that $A + B$ is also bounded above in $\mathbb{R}$.
   (d) Prove or disprove: If A and B are bounded above in $\mathbb{R}$, then $\text{lub}(A) + \text{lub}(B) = \text{lub}(A + B)$

(3) (a) Prove or disprove: Let $x, y \in \mathbb{R}$. If $x + y \geq 0$, then $x \geq 0$ or $y \geq 0$.
   (b) Let $x, y \in (0, \infty)$. If $x^3 > y^3$, prove that $x > y$.

(4) Let $A \subset \mathbb{Z}$ be non-empty and bounded above in $\mathbb{R}$. Show that A contains a maximum element.

(5) Consider the function $f : X \to Y$ defined by $f(x) = x^2$, where $X, Y \subset \mathbb{R}$.

   (a) For $X = \mathbb{R} = Y$ and $A = (0, 2] \subset X$, find $f(A)$ (with proof).
   (b) For $X = \mathbb{R} = Y$, show that f is not (i) one-one (ii) onto.
   (c) Prove or disprove: If $X = (0, \infty) = Y$, then f is a bijection.