Assignment 4

04-02-2019

(1) Let X, Y b non-empty sets, $A \subset X, B \subset Y$. Show that $A \times B$ is a subset of $X \subset Y$.

(2) Find $A \times B \subset \mathbb{R}^2$, where
(i) $A = (0, 1)$, $B = \mathbb{R}$ (ii) $A = \{0, 1\}$, $B = \mathbb{R}$ (iii) $A = \mathbb{N}$, $B = \mathbb{R}$
How do your answers change when (i) A and B are interchanged.
(ii) $B = \mathbb{R}$ is replaced by $B = \mathbb{Z}$? (iii) $B = \mathbb{R}$ is replace by $[0, \infty)$

(3) Prove or disprove: Let $Z \subset X \times Y$. Then there are subsets A and B of X and Y respectively such that $Z = A \times B$.

(4) What is the set $c + \mathbb{Q}$ where $c \in \mathbb{R}$? When does it contain a rational number? What can you say in other cases? Answer similar questions about $c\mathbb{Q}$.

(5) Identify some rational and irrational numbers in $\mathbb{Q} + [0, 1]$? What is this set?

(6) Find the lub and glb of the following sets in $\mathbb{R}$ if they exist. Give an argument supporting your answer.

(a) $\{1/2^q \in \mathbb{R} | q \in \mathbb{Z}\}$
(b) $\{x \in \mathbb{R} | x < 1/n \text{ for some } n \in \mathbb{N}\}$
(c) $\{x \in \mathbb{R} | x > 1/n \ \forall n \in \mathbb{N}\}$

(7) Let $S \subset \mathbb{R}$. If $\text{lub}(S) = a$, then show that $\text{glb}\left(\{-s \in \mathbb{R} | s \in S\}\right) = -a$.

(8) Let $S \subset \mathbb{R}$ be a bounded subset of $\mathbb{R}$. Let $T = \{s^2 \in \mathbb{R} | s \in S\}$. Does it follow that $\text{lub}(T) = (\text{lub}(S))^2$? If yes, prove it. If false, give a counterexample, correct the statement and then prove the corrected statement.