Exercise Sheet 8: Matrix Rank and Nullity Cheat Sheet

Introduction

Matrix rank and nullity are important concepts in linear algebra that provide information about the structure of matrices. This cheat sheet presents definitions, properties, and calculations related to matrix rank and nullity.

1 Matrix Rank

1.1 Definition

The rank of a matrix A, denoted $\text{rank}(A)$, is the maximum number of linearly independent rows or columns in A.

1.2 Properties

- The rank of a matrix is equal to the number of non-zero rows in its reduced row echelon form (RREF).
- $\text{rank}(A) \leq \min(m, n)$, where m is the number of rows and n is the number of columns of A.
- If $\text{rank}(A) = \min(m, n)$, A is said to have full rank.

2 Nullity

2.1 Definition

The nullity of a matrix A, denoted $\text{nullity}(A)$, is the dimension of the null space (kernel) of A.

2.2 Properties

- $\text{nullity}(A) = n - \text{rank}(A)$, where n is the number of columns of A.
- If $\text{nullity}(A) = 0$, A is said to have trivial nullity.

2.3 Example

Consider the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$. Calculate the rank and nullity of A.

Solution

The matrix A has rank 1 since its rows are linearly dependent. Since $n = 3$ and $\text{rank}(A) = 1$, the nullity of A is $3 - 1 = 2$.