Partial Differential Equations in Heat Transfer

Your Name

1 Introduction

This document covers the application of partial differential equations in heat transfer.

2 Heat Equation

2.1 One-Dimensional Heat Equation

2.2 Example Problem

Solve the one-dimensional heat equation $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ for a rod of length L with boundary conditions $u(0,t) = 0$, $u(L,t) = 0$, and initial condition $u(x,0) = f(x)$.

Solution

The solution can be obtained using separation of variables:

$$u(x,t) = X(x)T(t).$$

Substituting into the heat equation, we get:

$$X(x)T'(t) = \alpha X''(x)T(t).$$

Dividing both sides by $\alpha X(x)T(t)$, we get:

$$\frac{T'(t)}{\alpha T(t)} = \frac{X''(x)}{X(x)} = -\lambda.$$

Solving the spatial part, we get:

$$X''(x) + \lambda X(x) = 0,$$

with boundary conditions $X(0) = 0$ and $X(L) = 0$. The solution is:

$$X_n(x) = \sin\left(\frac{n\pi x}{L}\right),$$

for $n = 1, 2, 3, \ldots$, and $\lambda_n = \left(\frac{n\pi}{L}\right)^2$. Solving the time part, we get:

$$T_n(t) = e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t}.$$

The general solution is:

$$u(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right) e^{-\alpha\left(\frac{n\pi}{L}\right)^2 t},$$

where the coefficients B_n are determined by the initial condition $u(x, 0) = f(x)$ using Fourier series.