Exercise Sheet 10: Linear Independence and Span Cheat Sheet

Introduction

Linear independence and span are fundamental concepts in linear algebra that characterize the properties of vectors and sets of vectors. This cheat sheet presents definitions, properties, and calculations related to linear independence and span.

1 Linear Independence

1.1 Definition

A set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ in a vector space V is linearly independent if the only solution to the equation $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n = \mathbf{0}$ is $c_1 = c_2 = \cdots = c_n = 0$.

1.2 Properties

- If any vector in a set of vectors can be expressed as a linear combination of the others, the set is linearly dependent.
- A set containing the zero vector is linearly dependent.
- Any set of n linearly independent vectors in an n-dimensional vector space forms a basis for that space.

2 Span

2.1 Definition

The span of a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$, denoted $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n)$, is the set of all possible linear combinations of those vectors.

2.2 Properties

- The span of a set of vectors is a subspace of the vector space containing those vectors.
- A set of vectors spans a vector space if and only if every vector in the space can be expressed as a linear combination of the set.

2.3 Example

Consider the vectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Determine if they are linearly independent and find their span.

Solution

The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent since no scalar multiples of one can produce the other. Their span is the set of all vectors in R^2 since they form a basis for R^2.