Complex Analysis in Engineering

Your Name

1 Introduction

This document covers the application of complex analysis in engineering.

2 Complex Functions

2.1 Cauchy-Riemann Equations

2.2 Example Problem

Determine if the function $f(z) = z^2$ is analytic and find its derivative.

Solution

The function $f(z) = z^2$ can be written as $f(z) = (x + iy)^2 = x^2 - y^2 + 2ixy$. Separating into real and imaginary parts, we have $u(x, y) = x^2 - y^2$ and $v(x, y) = 2xy$. To be analytic, the Cauchy-Riemann equations must hold:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Calculating the partial derivatives, we get:

$$\frac{\partial u}{\partial x} = 2x, \quad \frac{\partial v}{\partial y} = 2x, \quad \frac{\partial u}{\partial y} = -2y, \quad \frac{\partial v}{\partial x} = 2y.$$

Since $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$, the Cauchy-Riemann equations are satisfied, so $f(z) = z^2$ is analytic. The derivative of $f(z)$ is given by:

$$f'(z) = \frac{d}{dz}(z^2) = 2z.$$