

Exercise Sheet 9: Matrix Inverse Cheat Sheet

Introduction

The matrix inverse is a fundamental concept in linear algebra, providing a way to solve systems of linear equations and compute the inverse of linear transformations. This cheat sheet presents definitions, properties, and calculations related to matrix inverses.

1 Matrix Inverse

1.1 Definition

For a square matrix A , its inverse, denoted A^{-1} , is a matrix such that $AA^{-1} = A^{-1}A = I$, where I is the identity matrix.

1.2 Properties

- Not all matrices have inverses. A matrix is invertible if and only if its determinant is non-zero.
- If A is invertible, then A^{-1} is unique.
- $(AB)^{-1} = B^{-1}A^{-1}$ for invertible matrices A and B .

1.3 Calculation

If A is a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with determinant $ad - bc \neq 0$, then its inverse is given by:

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

1.4 Example

Consider the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$. Calculate its inverse.

Solution

The determinant of A is $2 \times 3 - 1 \times 1 = 5$. Since the determinant is non-zero, A is invertible. Using the formula for the inverse of a 2×2 matrix, we have:

$$A^{-1} = \frac{1}{5} \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}.$$