Inclusive B Decays

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4th Year Project

In support of an M.Sci. in Mathematics and Physics

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April 2013

Abstract

An introduction to gauge symmetries and the Higgs mechanism is presented, before CP violation and its current manifestation in the Standard Model is described. This manifestation, the CKM matrix, is shown to be a direct consequence of generating fermion mass terms through Yukawa couplings between the Higgs and fermion fields. B meson phenomenology is discussed, with particular emphasis on the inclusive $b \to 2c$ quark decay. A proposed measurement strategy for this channel through exclusive $\bar{B}$ meson decays is then examined. Finally, an analysis of these exclusive modes is presented, using a Monte Carlo simulation of 10^6 events in a BaBar environment at the $\Upsilon(4S)$ resonance. The surprising conclusion is that the $\bar{B} \to D^- X$ channel, often neglected due to the CKM suppression of the inclusive $b \to c\bar{c}d$ decay, actually contributes a significant branching ratio of approximately 6 %.

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1 Introduction

For centuries, the enduring mystery of our universe's earliest beginnings has captivated the imagination of scientists, philosophers and the general public alike. With multiple theories being championed at the start of the 20th century, scientific consensus seemed a distant proposition. However, in the wake of Einstein's 1916 paper on General Relativity[1] and Hubble's explanation[2] of the observed galactic redshift, a new model began to prevail: Big Bang cosmology. Whilst there is now a wealth of evidence to support this theory, it is often found to pose more questions than it answers.

There is one particularly troubling anomaly; namely, why do we observe such a large matter-antimatter asymmetry throughout the universe? The Big Bang model predicts equal creation of matter and antimatter, and any initial asymmetry would surely be diluted to insignificance by inflation. Based on current observations, the predominant explanation is that this matter excess was generated some time *after* the Big Bang. This is known as baryogenesis.

In 1967, Russian physicist Andrei Sakharov proposed three minimum conditions necessary to facilitate the process of baryogenesis in the early universe[3]:

1. Baryon number (B) violating processes.
2. CP violating processes.
3. Interactions outside of thermal equilibrium.

The second item here is our main focus. Whilst there is a mechanism to incorporate CP violation within the Standard Model (SM), the theoretically predicted magnitude of this CP violation is insufficient by itself to describe the huge asymmetry we observe[4]. So experimental searches for CP violating processes inconsistent with the predictions of the Standard Model could be the first sign of new physics, as well as provide hints to the early life of our universe.

Two promising results to emerge from Tevatron in early 2011 concern measured excesses in the dimuon production rate from neutral B mesons[5], and in the like-sign dimuon charge asymmetry[6][7]. Although more recent results from LHCb[8] heavily constrain any proposed models, this could still prove to be a fruitful area of research. One proposal centres on a new (unknown) operator coupling to the b quark.

In this paper, a brief summary of one proposed model-independent search strategy for such an operator is presented. We then proceed by examining the experimental difficulties of measuring the necessary inclusive quark decays, such as the $b \to 2\,c$ channel, and discuss the validity of suggested measurement strategies using exclusive decay channels.

2 Theoretical Preliminaries

Within the current framework of the Standard Model, the phenomena of CP (charge-parity) violation can, in theory, be implemented in both the QCD and electroweak sectors; in practice this QCD CP violation has never been observed. Consequently, all CP violation in the Standard Model resides in the electroweak sector and is directly attributed to the structure of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This matrix allows for intergenerational mixing between quarks and is a natural consequence of endowing the quarks with mass through the Higgs mechanism. We will first discuss general gauge symmetries and the Higgs mechanism, before considering some implications of the structure of the CKM matrix.

The simplest example of a gauge symmetry can be found in electromagnetism. Here, all physical quantities ($\vec{E}$ and $\vec{B}$-fields) are described in terms of the electromagnetic field strength tensor, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. We can then represent Maxwell's equations by:

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu \quad \text{and} \quad \partial_\mu \tilde{F}^{\mu\nu} = 0, \tag{2.1}$$

where $\tilde{F}^{\mu\nu} = (1/2)\,\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ defines the dual electromagnetic field strength tensor and $J^\nu = (\rho, \vec{j})$ is the electromagnetic 4-current. This representation also naturally ensures charge conservation:

$$\partial_\nu J^\nu = \frac{1}{\mu_0}\partial_\nu\partial_\mu F^{\mu\nu}. \tag{2.2}$$

Now partials commute, so the first terms are symmetric in μ and ν. But from the definition of $F_{\mu\nu}$, we can see that this is antisymmetric. So the contraction of these two terms vanishes and we are left with the continuity equation, $\partial_\nu J^\nu = 0$. It is also clear that a transformation of the form $A_\mu \to A_\mu + \partial_\mu\lambda$ leaves $F^{\mu\nu}$ (and consequently $\tilde{F}^{\mu\nu}$ and J^ν) invariant:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \to \partial_\mu A_\nu + \cancel{\partial_\mu\partial_\nu\lambda} - \partial_\nu A_\mu - \cancel{\partial_\nu\partial_\mu\lambda} = F_{\mu\nu}. \tag{2.3}$$

So we have a freedom in choosing our unphysical potential, whilst leaving our measurable, physical quantities invariant. This is known as a gauge symmetry, with $A_\mu \to A_\mu + \partial_\mu\lambda$ a gauge transformation.

Now, during the development of early quantum field theory (specifically QED), it was discovered that many of the predictions made by the theory resulted in divergent integrals. This was remedied in the 1940s by Bethe[9], Feynman[10][11], Schwinger[12], Tomonaga[13], Dyson[14] and others with the process of renormalisation. This involves reexpressing fundamental constants such as the electric charge (e) as new, renormalised quantities. Consequently, the notion of renormalisability quickly became a central tenet in constructing new field theories.

In 1971, in collaboration with his thesis advisor Martinus Veltman, Gerard 't Hooft finalised his proof that massless non-abelian gauge theories are renormalisable.[15] It soon became clear that these gauge symmetries were more than a simple theoretical curiosity. Indeed, one of the central principles of the successful modern field theories (Strong, Weak and Electromagnetic) that make up the Standard Model is that they all contain some form of gauge symmetry.

2.1 Abelian Gauge Symmetries

We will start by examining abelian gauge symmetries, where elements in the symmetry group commute. First, consider the Dirac Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi = \bar{\psi}(i\not{\partial} - m)\psi, \tag{2.4}$$

with ψ a Dirac spinor, γ^μ the Dirac gamma matrices defined by the Dirac algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}$ and $\bar{\psi}$ is defined to be the adjoint spinor with $\bar{\psi} = \psi^\dagger\gamma^0$. This is clearly invariant under a transformation of the form:

$$\psi(x) \to e^{i\theta}\psi(x),\ \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\theta}. \tag{2.5}$$

We can see now that this symmetry is abelian; $e^{i\theta}$ is just some complex number and these trivially commute with each other. Consider then a *local* transformation, where θ is a function of x:

$$\psi(x) \to e^{i\theta(x)}\psi(x),\; \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\theta(x)}. \tag{2.6}$$

This dramatically alters the form of the Lagrangian, as we now acquire an extra piece due to the derivative acting on $e^{i\theta(x)}$:

$$\Delta\mathcal{L} = -(\partial_\mu\theta(x))\bar{\psi}\gamma^\mu\psi. \tag{2.7}$$

We require that it is invariant under this new local symmetry; this is known as the gauge principle. In order to make it so, we need to define a new gauge covariant derivative. We do this by defining a new field, A_μ, and introducing a minimal substituting interaction of the form:

$$p_\mu \to p_\mu + eA_\mu, \tag{2.8}$$

with some coupling e. When this expression is quantised, we can replace p_μ by the momentum operator $\hat{p}_\mu \equiv -i\partial_\mu$. This gives:

$$-i\partial_\mu \to -iD_\mu \equiv -i\partial_\mu + eA_\mu \implies D_\mu \equiv \partial_\mu + ieA_\mu. \tag{2.9}$$

Under the gauge transformation, this new vector field must transform as:

$$A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\theta(x). \tag{2.10}$$

This covariant derivative $D_\mu\psi(x)$ will then transform in exactly the same way as $\psi(x)$:

$$\begin{aligned} D_\mu\psi(x) \to& \left[\partial_\mu + ie\left(A_\mu - \frac{1}{e}\partial_\mu\theta(x)\right)\right]e^{i\theta(x)}\psi(x), \\ &= e^{i\theta(x)}\partial_\mu\psi(x) + \cancel{i\psi(x)\partial_\mu\theta(x)e^{i\theta(x)}} + ieA_\mu e^{i\theta(x)}\psi(x) - \cancel{i\partial_\mu\theta(x)e^{i\theta(x)}\psi(x)}, \\ &= e^{i\theta(x)}(\partial_\mu + ieA_\mu)\psi(x) = e^{i\theta(x)}D_\mu\psi(x). \end{aligned} \tag{2.11}$$

Then with the replacement $\partial_\mu \to D_\mu$, the exponentials in the first term will cancel to 1 as they do in the mass term:

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi = i\bar{\psi}\not{D}\psi - m\bar{\psi}\psi \to i\bar{\psi}\cancel{e^{-i\theta(x)}e^{i\theta(x)}}\not{D}\psi - m\bar{\psi}\cancel{e^{-i\theta(x)}e^{i\theta(x)}}\psi = \mathcal{L}, \tag{2.12}$$

and the Lagrangian is invariant under this local gauge transformation. However, for our field to propagate correctly we must also include a free term in the Lagrangian for our vector field A_μ. As this is a vector (spin-1) field, we add in the Proca lagrangian:

$$\mathcal{L}_p = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A_\mu A^\mu. \tag{2.13}$$

The first term is already invariant under the gauge transformation given by (2.10):

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \to \partial_\mu A_\nu - \partial_\nu A_\mu - \cancel{\frac{1}{e}\partial_\mu\partial_\nu\theta(x)} + \cancel{\frac{1}{e}\partial_\nu\partial_\mu\theta(x)} = F_{\mu\nu}. \tag{2.14}$$

As partials commute, these last two terms cancel and we can see that the expression is indeed invariant. Unfortunately, the second term in (2.13) is definitely *not* invariant under this transformation. Hence the only way for the full Lagrangian to remain invariant under the gauge transformation is if we exclude this final term altogether, by taking $m_A = 0$. The full Lagrangian is then given by:

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\gamma^\mu A_\mu\psi. \tag{2.15}$$

We have thus shown how demanding invariance of the Dirac Lagrangian under local gauge transformations requires the existence of a massless gauge field which couples to the Dirac spinors. The expression above is known as the Lagrangian for QED, and represents the interactions of photons with quarks or leptons. Finally, it will become useful later to introduce a quantity known as the gauge commutator, $[D_\mu, D_\nu]$.

Consider this commutator acting on our spinor ψ. Naively we would expect this to remain a differential operator. However, we instead find:

$$\begin{aligned}[D_\mu, D_\nu]\psi &= \cancel{\partial_\mu\partial_\nu\psi} + ie\partial_\mu(A_\nu\psi) + ieA_\mu\partial_\nu\psi - e^2A_\mu A_\nu\psi \\ &\quad - \cancel{\partial_\nu\partial_\mu\psi} - ie\partial_\nu(A_\mu\psi) - ieA_\nu\partial_\mu\psi + e^2A_\nu A_\mu\psi, \\ &= ie\{(\partial_\mu A_\nu)\psi - (\partial_\nu A_\mu)\psi\} - e^2[A_\mu, A_\nu]\psi. \end{aligned} \tag{2.16}$$

For the case of QED, which has an abelian gauge symmetry, this final commutator reduces to zero and we can conclude:

$$[D_\mu, D_\nu] = ieF_{\mu\nu}. \tag{2.17}$$

2.2 Non-abelian Gauge Symmetries

Let us extend this prescription of using gauge symmetries to non-abelian examples, beginning with Yang-Mills theory. In this case, the Lagrangian takes the form:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} + g\bar{\psi}\gamma^\mu A^i_\mu T^i\psi. \tag{2.18}$$

This is clearly in a similar form to (2.15). However, there are some important differences. In the previous segment, we analysed $U(1)$ symmetries of the form $e^{i\theta(x)}$. Here, we have a symmetry:

$$\psi(x) \to V\psi(x),\ \bar{\psi}(x) \to \bar{\psi}(x)V^{-1} \quad \text{with} \quad V = \exp\left(i\alpha^i T^i\right). \tag{2.19}$$

V is then an element of a Lie group G. T^i are known as the generators of this Lie group, and we define the Lie algebra by

$$[T_i, T_j] = ic_{ijk}T_k, \tag{2.20}$$

with $c_{ijk} \in \mathbb{R}$ known as the structure constants of the group. If we now work in $G = SU(2)$ then ψ is given by a doublet of Dirac spinors:

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}. \tag{2.21}$$

The structure constants are $c_{ijk} = \epsilon_{ijk}$ (the 3-dimensional Levi-Civita symbol) and our generators T^i are given by hermitian, unitary 2×2 matrices; specifically, $T^i = \sigma^i/2$ where σ^i are the Pauli matrices. As before, this global symmetry is promoted to a local one, with $V(x) = \exp(i\alpha^i(x)\sigma^i/2)$. Again, we can use the principle of minimal coupling to define a new covariant derivative:

$$D_\mu \equiv \partial_\mu - igA_\mu^i \frac{\sigma^i}{2}, \tag{2.22}$$

for some coupling constant g. This is simply a generalisation of (2.9) with $g = -e$. We now have 3 vector fields A_μ^i corresponding to each of the generators $\sigma^i/2$. It is also possible to calculate how these fields transform under the gauge transformations. Close to the centre ($\mathbb{1}$) of the Lie group we can consider infinitesimal transformations by expanding V as a Taylor series:

$$V(x) = \mathbb{1} + i\alpha^i(x)\frac{\sigma^i}{2} + \mathcal{O}(\alpha^2). \tag{2.23}$$

We now must examine the kinetic terms. We use the gauge transformation in (2.19) to obtain:

$$D_\mu\psi(x) \to D'_\mu V\psi(x). \tag{2.24}$$

As before, the covariant derivative acting on $\psi(x)$, $D_\mu\psi(x)$, needs to transform similarly to $\psi(x)$ itself under the gauge transformation. Then we must have:

$$D'_\mu V\psi(x) = VD_\mu\psi(x) \implies D'_\mu = VD_\mu V^{-1}. \tag{2.25}$$

Using the infinitesimal form of V in (2.23) and ignoring terms of $\mathcal{O}(\alpha^2)$:

$$\begin{aligned}
D'_\mu = \partial_\mu - igA'^i_\mu\frac{\sigma^i}{2} &= VD_\mu V^{-1}, \\
&= \left(\mathbb{1} + i\alpha^i(x)\frac{\sigma^i}{2}\right)\left(\partial_\mu - igA^j_\mu\frac{\sigma^j}{2}\right)\left(\mathbb{1} - i\alpha^k\frac{\sigma^k}{2}\right), \\
\implies -igA'^i_\mu\frac{\sigma^i}{2} &= -igA^j_\mu\frac{\sigma^j}{2} - i\partial_\mu\alpha^k\frac{\sigma^k}{2} + g\alpha^i(x)\frac{\sigma^i}{2}A^j_\mu\frac{\sigma^j}{2} - gA^i_\mu\frac{\sigma^i}{2}\alpha^k(x)\frac{\sigma^k}{2}.
\end{aligned} \tag{2.26}$$

We can reexpress the last terms on the RHS of the equation using a commutator, and with some relabelling of indices are left with the transformation:

$$A^i_\mu\frac{\sigma^i}{2} \to A'^i_\mu\frac{\sigma^i}{2} = A^i_\mu\frac{\sigma^i}{2} + \frac{1}{g}(\partial_\mu\alpha^i(x))\frac{\sigma^i}{2} + i\left[\alpha^i(x)\frac{\sigma^i}{2}, A^j_\mu\frac{\sigma^j}{2}\right]. \tag{2.27}$$

This also looks similar to the abelian case in (2.10). The final term here must be included due to the non-commutativity of the generators (the Pauli matrices).

Finally, we again must include gauge invariant free terms for our fields. Similar to the abelian case, we include the Proca Lagrangian for each gauge field, and it is again trivial to conclude that each of these fields must be massless. As before, we are left with the field strength terms, $F^i_{\mu\nu}F^{i\mu\nu}$. However, there is a subtle difference between the field strength terms here and in the previous abelian case. To see this, we must return to the gauge commutator in (2.17). The field strength derived here is a specific example of a much more general mathematical quantity known as a curvature form. We can generalise this result for any Lie group G with generators T^i:

$$[D_\mu, D_\nu] = -igF^i_{\mu\nu}T^i. \tag{2.28}$$

The field strength is then related to the form of the covariant derivative, which is in turn dependent on the symmetry group the Lagrangian remains invariant under. For the Yang-Mills example, we have:

$$[D_\mu, D_\nu] = -ig\partial_\mu A^i_\nu \frac{\sigma^i}{2} + ig\partial_\nu A^i_\mu \frac{\sigma^i}{2} - g^2 \left[A^i_\mu \frac{\sigma^i}{2}, A^j_\nu \frac{\sigma^j}{2}\right] =: -igF^i_{\mu\nu}\frac{\sigma^i}{2}. \tag{2.29}$$

We can then use the commutation relation for the Lie algebra:

$$\left[\frac{\sigma^i}{2}, \frac{\sigma^j}{2}\right] = i\epsilon^{ijk}\frac{\sigma^k}{2}. \tag{2.30}$$

This simplifies the expression for the field strength to:

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon^{ijk} A^j_\mu A^k_\nu. \tag{2.31}$$

As we saw in (2.14), the electromagnetic field strength tensor was already invariant under the QED gauge transformation: here the field strength is not. We can examine how this field strength transforms by evaluating the transformation of the gauge comparator acting on $\psi(x)$:

$$[D_\mu, D_\nu]\psi(x) \to V(x)[D_\mu, D_\nu]\psi(x). \tag{2.32}$$

We also know that $\psi(x)$ transforms as $\psi(x) \to V(x)\psi(x)$. The gauge comparator must then transform as $[D_\mu, D_\nu] \to V(x)[D_\mu, D_\nu]V^{-1}(x)$. From equation (2.29) we must have:

$$F^i_{\mu\nu}\frac{\sigma^i}{2} \to V(x)F^j_{\mu\nu}\frac{\sigma^j}{2}V^{-1}(x), \tag{2.33}$$

and using the infinitesimal form of V in (2.23):

$$\begin{aligned}
F^i_{\mu\nu}\frac{\sigma^i}{2} &\to V(x)F^j_{\mu\nu}\frac{\sigma^j}{2}V^{-1}(x), \\
&\to \left(\mathbb{1} + i\alpha^i(x)\frac{\sigma^i}{2} + \mathcal{O}(\alpha^2)\right) F^j_{\mu\nu}\frac{\sigma^j}{2}\left(\mathbb{1} - i\alpha^k(x)\frac{\sigma^k}{2} + \mathcal{O}(\alpha^2)\right), \\
&\to F^j_{\mu\nu}\frac{\sigma^j}{2} + i\alpha^i(x)\frac{\sigma^i}{2}F^j_{\mu\nu}\frac{\sigma^j}{2} - iF^j_{\mu\nu}\frac{\sigma^j}{2}\alpha^k(x)\frac{\sigma^k}{2} + \mathcal{O}(\alpha^2).
\end{aligned} \tag{2.34}$$

Now we can relabel some dummy indices and recognise that the last two terms represent a commutator. This leaves us with:

$$F^i_{\mu\nu}\frac{\sigma^i}{2} \to F^i_{\mu\nu}\frac{\sigma^i}{2} + i\left[\alpha^i(x)\frac{\sigma^i}{2}, F^j_{\mu\nu}\frac{\sigma^j}{2}\right]. \tag{2.35}$$

This final commutator does not vanish due to the non-abelian symmetry group, and so we see that $F^i_{\mu\nu}$ is not invariant under the gauge transformation. However, the $F^i_{\mu\nu}F^{i\mu\nu}$ term in the Lagrangian is indeed invariant. To see this, consider $-(^1/_2)\mathrm{Tr}\left[F^i_{\mu\nu}(\sigma^i/2)F^{j\mu\nu}(\sigma^j/2)\right]$. Under a gauge transformation this becomes:

$$-\frac{1}{2}\mathrm{Tr}\left[F^i_{\mu\nu}\frac{\sigma^i}{2}F^{j\mu\nu}\frac{\sigma^j}{2}\right] \to -\frac{1}{2}\mathrm{Tr}\left[V(x)F^i_{\mu\nu}\frac{\sigma^i}{2}V^{-1}(x)V(x)F^{j\mu\nu}\frac{\sigma^j}{2}V^{-1}(x)\right]. \tag{2.36}$$

We can use the cyclicity of the trace operator to rewrite this as:

$$-\frac{1}{2}\mathrm{Tr}\left[\cancel{V^{-1}(x)V(x)}F^i_{\mu\nu}\frac{\sigma^i}{2}\cancel{V^{-1}(x)V(x)}F^{j\mu\nu}\frac{\sigma^j}{2}\right] = -\frac{1}{2}\mathrm{Tr}\left[F^i_{\mu\nu}\frac{\sigma^i}{2}F^{j\mu\nu}\frac{\sigma^j}{2}\right]. \tag{2.37}$$

Then this is invariant under the gauge transformation. We can simplify this quantity further:

$$-\frac{1}{2}\mathrm{Tr}\left[F^i_{\mu\nu}\frac{\sigma^i}{2}F^{j\mu\nu}\frac{\sigma^j}{2}\right] = -\frac{1}{2}\mathrm{Tr}\left[\frac{\sigma^i}{2}\frac{\sigma^j}{2}\right]F^i_{\mu\nu}F^{j\mu\nu} = -\frac{1}{4}\delta_{ij}F^i_{\mu\nu}F^{j\mu\nu} = -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu}, \quad (2.38)$$

and we see that this is indeed the kinetic energy term present in (2.18).

This relatively simple model can be extended to describe both the weak (invariant under an $SU(2)_L$ symmetry) and strong (invariant under $SU(3)_C$) interactions. However, we quickly encounter a problem. Whilst the strong force gauge bosons (gluons) are again massless, the weak ($W^{\pm}, Z^0$) are not. Experimental evidence gives $m_W \approx 80$ GeV and $m_Z \approx 91$ GeV. For comparison, the proton has a mass of $m_p \approx 0.9$ GeV[16]. This completely destroys the gauge invariance, forcing the introduction of additional interactions to generate the masses of these weak force mediators. The most prominent such example is that of the Higgs mechanism[17][18][19].

2.3 Spontaneous Symmetry Breaking and the Higgs Mechanism

To begin, we may explore the idea of spontaneously broken symmetries. A relevant practical example is that of a ferromagnet. At room temperature, iron has a net magnetisation $\vec{M} \neq 0$ in a specific direction. However, above a particular temperature T_c (this is dependent on the material and known as the Curie temperature) this magnetisation $\vec{M} \to 0$. There is now a complete three-dimensional rotation symmetry. However, if the material is once more cooled below T_c then again a net magnetisation $\vec{M} \neq 0$ is measured; this time in a new, random direction. The rotational symmetry is spontaneously broken.

In the context of particle physics, spontaneous symmetry breaking is intimately related to the notion of the vacuum expectation value of a scalar field. We begin by discussing the Klein-Gordon Lagrangian:

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2. \quad (2.39)$$

This has a potential function of the form:

$$V(\phi) = \frac{1}{2}m^2\phi^2. \quad (2.40)$$

Trivially there is a minimum at $\phi = 0$, and so we say that the vacuum expectation value of the field, $\langle\phi\rangle$, is zero. However, we can construct fields whose minimum does not occur at $\phi = 0$. Consider the Lagrangian of a complex scalar field $\phi = {}^1\!/\!\sqrt{2}(\phi_1 + i\phi_2)$ interacting with itself and the electromagnetic field:

$$\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - V(\phi_1, \phi_2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (2.41)$$

with $D_\mu \equiv \partial_\mu + ieA_\mu$ as in the abelian case, and a two-dimensional potential of the form:

$$V(\phi_1, \phi_2) = -\frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) + \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)^2 \text{ with } \mu,\, \lambda \in \mathbb{R}. \quad (2.42)$$

Here, the mass term for the field is $-\mu^2\phi^*\phi = -{}^{\mu^2}\!/\!{}_2(\phi_1^2 + \phi_2^2)$. The form of this term then implies an imaginary mass. Obviously this is not physical. Now this potential is invariant for any ϕ_1 and ϕ_2 as long as the quantity $\phi_1^2 + \phi_2^2$ remains constant: there is a rotational symmetry present. This is easiest to see if we express the fields in polar coordinates:

$$\left.\begin{array}{l} \phi_1 = r\sin\theta \\ \phi_2 = r\cos\theta \end{array}\right\} \implies \phi_1^2 + \phi_2^2 = r^2(\sin^2\theta + \cos^2\theta) = r^2. \tag{2.43}$$

Then at a fixed distance from the origin, r, we can rotate freely by any angle θ about the origin and the potential remains invariant. Now, we can find critical points as normal for multivariable functions; by calculating the partial derivatives:

$$\left.\begin{array}{l} V_{\phi_1} = \phi_1(-\mu^2 + \lambda\phi_1^2 + \lambda\phi_2^2) = 0 \\ V_{\phi_2} = \phi_2(-\mu^2 + \lambda\phi_2^2 + \lambda\phi_1^2) = 0 \end{array}\right\} \implies \left\{\begin{array}{l} \phi_1 = \phi_2 = 0, \\ \phi_1^2 + \phi_2^2 = \dfrac{\mu^2}{\lambda}. \end{array}\right. \tag{2.44}$$

On closer inspection we find that $\phi_1 = \phi_2 = 0$ is in fact a local maximum. Here, the minima of the potential lie on a circle of radius $r = {}^{\mu}/\sqrt{\lambda}$ in the $V = -{}^{\mu^4}/4\lambda$ plane. When used in calculations, the potential in (2.42) must be redefined in terms of fluctuations about the minima, so we are forced to choose a particular ground state. For simplicity, we can choose $\langle\phi_1\rangle = {}^{\mu}/\sqrt{\lambda}$, $\langle\phi_2\rangle = 0$. Then we can define new field varibles Φ_1, Φ_2 such that $\langle\Phi_1\rangle = \langle\Phi_2\rangle = 0$:

$$\Phi_1 \equiv \phi_1 - \frac{\mu}{\sqrt{\lambda}},\ \Phi_2 \equiv \phi_2. \tag{2.45}$$

Reexpressing (2.42) in terms of these new variables yields:

$$V(\Phi_1, \Phi_2) = -\frac{1}{2}\mu^2\left(\Phi_1^2 + 2\Phi_1\frac{\mu}{\sqrt{\lambda}} + \frac{\mu^2}{\lambda} + \Phi_2^2\right) + \frac{1}{4}\lambda^2\left(\Phi_1^2 + 2\Phi_1\frac{\mu}{\sqrt{\lambda}} + \frac{\mu^2}{\lambda} + \Phi_2^2\right)^2. \tag{2.46}$$

Most significantly, the rotational symmetry from (2.42) is lost. If we use polars again, we can see a term in θ survives inside the brackets. This then has an effect on the potential, and rotations about the origin are no longer a symmetry. We can expand this potential out fully and cancel some terms to leave us with:

$$V(\Phi_1, \Phi_2) = \mu^2\Phi_1^2 + \frac{1}{4}\lambda\left(\Phi_1^2 + \Phi_2^2\right)^2 + \mu\sqrt{\lambda}\left(\Phi_1^3 + \Phi_1\Phi_2^2\right) - \frac{\mu^4}{4\lambda^2}. \tag{2.47}$$

Analysing the mass terms, we find Φ_1 has a mass of $m = \sqrt{2}\mu$. This is analagous to our Higgs boson. However, there is no mass term for the field Φ_2. This field is massless, and corresponds to a Goldstone boson, named after Jeffery Goldstone. His theorem[20] states that spontaneous breaking of a continuous global symmetry must be accompanied by the inclusion of one or more massless scalar particles - Goldstone bosons. These do not represent physical particles; they are merely an artifact of our choice of gauge[21]. By working in a particular gauge (the unitarity gauge), we can render our complex field real valued at all points. This removes ϕ_2 from the beginning, and ensures no Goldstone bosons are present in calculations.

We can also study the kinetic terms in the Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{kinetic}} &\equiv (D_\mu\phi)^*(D^\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = (\partial_\mu - ieA_\mu)\phi^*(\partial^\mu + ieA^\mu)\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \\ &= \frac{1}{2}(\partial_\mu\Phi_1)(\partial^\mu\Phi_2) + \frac{1}{2}(\partial_\mu\Phi_2)(\partial^\mu\Phi_2) + \frac{(e\mu)^2}{\sqrt{\lambda}}A_\mu A^\mu + \frac{e\mu}{\sqrt{\lambda}}(\partial_\mu\Phi_2)A^\mu + \cdots, \end{aligned} \tag{2.48}$$

and we can see that the gauge boson corresponding to the gauge field A_μ acquires a mass $m_A = {}^{\sqrt{2}e\mu}/_{\sqrt{\lambda}}$. So by spontaneous symmetry breaking our previously massless gauge boson (the photon) has now acquired a mass. Intuitively, we can consider the photon field to have acquired this mass through "eating" the Goldstone boson. Obviously this doesn't represent a real physical scenario - we know photons are massless - but we can begin to assess non-abelian examples which form the basis of the real-world Higgs mechanism.

One such example is to consider our non-abelian $SU(2)$ gauge field from earlier coupled to a scalar field doublet. Previously we saw how gauge fields acquire mass through "eating" Goldstone bosons. We can then use this to deduce the form of our Higgs term. We need at least 3 Goldstone bosons to give masses to the $W^\pm$ and Z^0 gauge bosons, so the simplest form we are left with is a complex scalar Higgs doublet of the form:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \text{ with } \begin{cases} \phi^+ = \dfrac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \\ \phi^0 = \dfrac{1}{\sqrt{2}}(\phi_3 + i\phi_4). \end{cases} \tag{2.49}$$

Here, the exponents + and 0 refer to the conventional electromagnetic charge, Q, of the fields. We can still use the same covariant derivative we defined in (2.22), and if our scalar doublet Φ has a non-zero vacuum expectation value we can use rotations in $SU(2)$ to express it in the form:

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{2.50}$$

Then as in (2.48), we can deduce gauge boson mass terms by studying the kinetic terms in the Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{kinetic}} \equiv (D_\mu\Phi)^*(D^\mu\Phi) &= \frac{1}{2}\left(\partial_\mu + igA^i_\mu\frac{\sigma^i}{2}\right)\begin{pmatrix} 0 & v \end{pmatrix}\left(\partial^\mu - igA^{j\mu}\frac{\sigma^j}{2}\right)\begin{pmatrix} 0 \\ v \end{pmatrix}, \\ &= \frac{v^2}{2}\left\{\cdots + g^2A^i_\mu\frac{\sigma^i}{2}A^{j\mu}\frac{\sigma^j}{2} + \cdots\right\}. \end{aligned} \tag{2.51}$$

Now $A^{j\mu}$ and ${}^{\sigma^j}/_2$ trivially commute as the indices are summed over, and furthermore we notice this expression is symmetric in i and j. So we can symmetrise it and use the anticommutation relation of Pauli matrices ($\{{}^{\sigma^i}/_2, {}^{\sigma^j}/_2\} = {}^1/_2\delta_{ij}$) to leave gauge boson mass terms of the form:

$$\mathcal{L} = \cdots + \frac{g^2v^2}{8}A^i_\mu A^{i\mu} + \cdots \quad \Longrightarrow \quad m^i_A = \frac{gv}{2}. \tag{2.52}$$

Then all our gauge fields acquire an equal mass through the Higgs mechanism. The main challenge is how to incorporate this with the photon, a massless gauge boson.

2.4 Electroweak Unification

One of the accomplishments of the unified theory of electromagnetism and the weak force was describing the parity violation observed in the Cobalt-60 (or Wu) experiment conducted in 1956[22]. This experiment, proposed by Tsung Dao Lee and Chen Ning Yang[23], showed that during beta decay of Cobalt-60 only left-handed electrons (and consequently right-handed antielectron neutrinos) were emitted; parity was maximally violated. This was subsequently explained by the GWS (Glashow[24]-Weinberg[25]-Salam[26]) or Electroweak theory developed throughout the 1960s.

The weak force mediators ($W^{\pm}, Z^0$ bosons) couple only to left-handed particles and right-handed antiparticles. We therefore assign our left-handed fields to $SU(2)$ doublets, and right-handed fields to $SU(2)$ singlets as follows[27]:

$$\psi_L^l = \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L, \quad \psi_L^{q_i} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \quad \psi_R^l = l_R^-, \;\; \psi_R^{q(u_i)} = (u_i)_R, \;\; \psi_R^{q(d_i)} = (d_i)_R, \tag{2.53}$$

where u_i and d_i refer to an up-type (u, c, t) and down-type (d, s, b) quark of the same generation. We have a single scalar Higgs doublet as before in (2.49), transforming under an $SU(2)_L \times U(1)_Y$ gauge transformation as:

$$\Phi \to \exp\left(\frac{i\alpha^i\sigma^i}{2}\right)\exp(iY_W\theta)\Phi. \tag{2.54}$$

We can deduce the hypercharge, Y_W, of this doublet using the weak analogue of the Gell-Mann-Nishijima formula[28][29]:

$$Q = \frac{\sigma^3}{2} + Y_W. \tag{2.55}$$

We know the form of the Pauli matrices, and the electromagnetic charge of the Higgs field components is given in (2.49). Hence the charge operator must have eigenvalues 1 and 0 implying the weak hypercharge, $Y_W = {}^1\!/_2$. We can again use the rotations in $SU(2)$ to express the non-zero vacuum expectation value of Φ in the form:

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{2.56}$$

Due to the inclusion of the $U(1)$ symmetry, it is now possible to find a particular combination of generators which will leave this vacuum potential invariant. One simple example is to consider $\alpha^1 = \alpha^2 = 0$ and $\alpha^3 = \theta$:

$$\langle\Phi\rangle \to \exp\left(\frac{i\theta\sigma^3}{2}\right)\exp\left(\frac{i\theta}{2}\right)\langle\Phi\rangle. \tag{2.57}$$

The combination ${}^{i\theta\sigma^3}\!/_2$ is nothing but a diagonal matrix, and we can easily exponentiate this:

$$\exp\left(\frac{i\theta\sigma^3}{2}\right) = \begin{pmatrix} e^{i\theta/2} & 0 \\ 0 & e^{-i\theta/2} \end{pmatrix}. \tag{2.58}$$

Then we have:

$$\langle\Phi\rangle \to \begin{pmatrix} e^{i\theta/2} & 0 \\ 0 & e^{-i\theta/2} \end{pmatrix} e^{i\theta/2}\langle\Phi\rangle = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} = \langle\Phi\rangle. \tag{2.59}$$

This combination of generators will then correspond to a single massless gauge boson; in this case, the photon for QED. The others ($W^{\pm}, Z^0$) will gain mass through the Higgs mechanism. The covariant derivative is simply constructed by combining (2.9) and (2.22) with some couplings g and g':

$$D_\mu \equiv \partial_\mu - igA_\mu^i\frac{\sigma^i}{2} - ig'\frac{1}{2}B_\mu. \tag{2.60}$$

Looking at the kinetic terms again:

$$\begin{aligned}\mathcal{L}_{\text{kinetic}} &= \frac{1}{2}\left(\partial_\mu + igA^i_\mu\frac{\sigma^i}{2} + ig'\frac{1}{2}B_\mu\right)\begin{pmatrix}0 & v\end{pmatrix}\left(\partial^\mu - igA^{j\mu}\frac{\sigma^j}{2} - ig'\frac{1}{2}B^\mu\right)\begin{pmatrix}0\\ v\end{pmatrix}, \\ &= \frac{v^2}{8}\left\{g^2A^1_\mu A^{1\mu} + g^2A^2_\mu A^{2\mu} + (-gA^3_\mu + g'B_\mu)(-gA^{3\mu} + g'B^\mu)\right\}. \end{aligned} \quad (2.61)$$

It is difficult to see how this relates to our $W^\pm$ and Z^0 fields. This is because we need to redefine the fields in their mass eigenstate basis. Let:

$$W^\pm_\mu = (A^1_\mu \mp iA^2_\mu), \;\; Z^0_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gA^3_\mu - g'B_\mu), \;\; A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'A^3_\mu + gB_\mu). \quad (2.62)$$

Glashow[24] also defined a quantity, θ_w, known as the Weinberg or weak mixing angle. Let:

$$\cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}} \text{ and } \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}, \quad (2.63)$$

then we can express the Z^0 and A-fields as:

$$\left.\begin{aligned} Z^0 &= \cos\theta_w A^3 - \sin\theta_w B \\ A &= \sin\theta_w A^3 + \cos\theta_w B \end{aligned}\right\} \implies \begin{pmatrix}Z^0\\ A\end{pmatrix} = \begin{pmatrix}\cos\theta_w & -\sin\theta_w\\ \sin\theta_w & \cos\theta_w\end{pmatrix}\begin{pmatrix}A^3\\ B\end{pmatrix}. \quad (2.64)$$

Now this looks very similar to a rotation matrix. We can think of the Z^0 and A fields as rotations of the A^3 and B fields by angle θ_w. Equation (2.61) can then be reexpressed as:

$$\mathcal{L}_{\text{kinetic}} = \frac{v^2}{8}\left\{g^2W^+_\mu W^{-\mu} + \frac{g^2}{\cos^2\theta_w}Z^0_\mu Z^{0\mu}\right\}. \quad (2.65)$$

We can then simply read off the gauge boson masses:

$$m_{W^\pm} = \frac{gv}{2}, \;\; m_Z = \frac{gv}{2\cos\theta_w}, \;\; m_A = 0 \implies m_{W^\pm} = m_Z\cos\theta_w. \quad (2.66)$$

It is also possible to express the mass of our elusive Higgs boson as a function of the scalar potential. Working in the unitarity gauge, we consider a similar potential to that in (2.42):

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2. \quad (2.67)$$

The scalar field $\Phi(x)$ can then be expressed as fluctuations about the vacuum expectation value:

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h(x)\end{pmatrix}, \quad (2.68)$$

with $h(x)$ known as the Higgs field. This leaves us with a potential of the form:

$$V(\Phi) = -\frac{\mu^2}{4}\left(2v^2 + 4vh(x) + 2h(x)^2\right) + \frac{\lambda}{4}\left(v^4 + 4v^3h(x) + 6v^2h(x)^2 + \mathcal{O}\left(h(x)^3\right)\right). \quad (2.69)$$

Now we know that the minimum occur on a circle of radius $v = \sqrt{\mu^2/\lambda}$, which simplifies the potential:

$$V(\Phi) = -\frac{\mu^4}{4\lambda} + \mu^2h(x)^2 + \mu\sqrt{\lambda}h(x)^3 + \frac{\lambda}{4}h(x)^4. \quad (2.70)$$

It is then trivial to conclude that the mass of this scalar Higgs particle is $m_h = \sqrt{2}\mu$. Of course, μ is an input parameter for the potential anyway. All this result does is to parameterise the mass; as we do not know the exact form of the potential, it does not give us a definitive value.

2.5 Yukawa Couplings and Fermion Mass Terms

In the previous section we ignored one particularly difficult problem regarding fermion mass terms in the Lagrangian. We can split any generic Dirac fermion into its constituent left and right-handed components using the projection operator:

$$P_{R/L} = \frac{1}{2}(\mathbb{1} \pm \gamma^5) \text{ where } \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3. \tag{2.71}$$

As $P_R + P_L = {}^1\!/\!_2(\mathbb{1} + \cancel{\gamma^5} + \mathbb{1} - \cancel{\gamma^5}) = \mathbb{1}$, we can then act on any Dirac fermion, ψ, with this combination of projection operators and it will remain invariant:

$$\psi = (P_R + P_L)\psi = \psi_R + \psi_L. \tag{2.72}$$

It is useful to highlight some general properties of projection operators and Dirac matrices. Firstly, it follows from the Dirac algebra[1] that:

$$\bar{\psi} P_R = \bar{\psi_L} \text{ and } \bar{\psi} P_L = \bar{\psi_R}. \tag{2.73}$$

The Dirac algebra also shows that:

$$P_R^2 = P_R P_R = \frac{1}{2}(\mathbb{1} + \gamma^5)\frac{1}{2}(\mathbb{1} + \gamma^5) = \frac{1}{4}(\mathbb{1} + 2\gamma^5 + (\gamma^5)^2) = \frac{1}{2}(\mathbb{1} + \gamma^5) = P_R. \tag{2.74}$$

Now, consider a general fermion mass term of the form $m\bar{\psi}\psi$. Using the projection operators, this can be reexpressed as:

$$\begin{aligned} m\bar{\psi}\psi = m\bar{\psi}(P_R + P_L)\psi = m\bar{\psi}(P_R^2 + P_L^2)\psi &= m(\bar{\psi} P_R \psi_R + \bar{\psi} P_L \psi_L), \\ &= m(\bar{\psi_L}\psi_R + \bar{\psi_R}\psi_L). \end{aligned} \tag{2.75}$$

Our problem: the left and right-handed fields transform differently under the gauge transformation. This is not surprising. The GWS model is a chiral theory; we *required* that they must transform differently in order to accommodate the experimental results. But now these fermion mass terms are not gauge invariant. Instead, we must introduce a gauge-invariant Yukawa coupling[31] between the Higgs and fermion fields. For the leptons, this takes the form:

$$\mathcal{L}_{\text{yuk}} = -y_l \bar{\psi}_L^l \Phi \psi_R^l, \tag{2.76}$$

where y_l is the Yukawa coupling of the scalar field Φ to the lepton l^-. After spontaneous symmetry breaking, we can express Φ in terms of fluctuations about its vacuum expectation value, just as in (2.68). Expanding this term out leaves us with:

$$-y_l \begin{pmatrix} \bar{\nu}_l & \bar{l^-} \end{pmatrix}_L \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} l_R^- = -\frac{1}{\sqrt{2}} y_l v \bar{l_L^-} l_R^- - \frac{1}{\sqrt{2}} y_l h(x) \bar{l_L^-} l_R^-. \tag{2.77}$$

We finally have a gauge invariant lepton mass term with mass $m_l = ({}^1\!/\!_{\sqrt{2}})\, y_l v$. It is now clear how the Higgs field generates mass; all fermion mass terms are a function of the vacuum expectation value, v, of the Higgs doublet. We may also analyse the second term in (2.77). This represents a coupling between the Higgs and lepton fields, giving rise to a vertex of the form in Figure 1. It is also possible to reexpress this coupling constant as a function of the lepton mass; $-{}^{y_l}\!/\!_{\sqrt{2}} = -{}^{m_l}\!/\!_v$. So the Higgs boson coupling is proportional

[1] See Appendix A

to the mass of the fermion. This is partially what makes observations of the Higgs particle so tricky; a stronger coupling requires more massive particles, which require a significant amount of energy to produce.

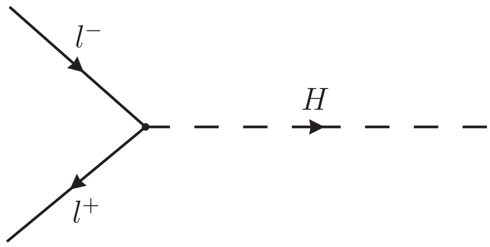


Figure 1: Lepton-Higgs vertex.

Of course, the same can be done in complete analogy for the quark mass terms. Here, the gauge invariant Yukawa coupling takes the form:

$$\mathcal{L}_{\text{yuk}} = -y_{ij}^u \bar{\psi}_L^{q_i} \Phi^c \psi_R^{q(u_j)} - y_{ij}^d \bar{\psi}_L^{q_i} \Phi \psi_R^{q(d_j)}, \tag{2.78}$$

where:

$$\Phi^c = i\sigma^2 \Phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi^- \\ (\phi^0)^* \end{pmatrix} = \begin{pmatrix} (\phi^0)^* \\ -\phi^- \end{pmatrix}. \tag{2.79}$$

Then expressing Φ^c as fluctuations about the vacuum expectation value gives:

$$\Phi^c = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix}. \tag{2.80}$$

We can now use the explicit form of these expressions to expand out this term:

$$\begin{aligned} \mathcal{L}_{\text{yuk}} &= -y_{ij}^u \begin{pmatrix} \bar{u}_i & \bar{d}_i \end{pmatrix}_L \left(\frac{1}{\sqrt{2}}\right) \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix} (u_j)_R - y_{ij}^d \begin{pmatrix} \bar{u}_i & \bar{d}_i \end{pmatrix}_L \left(\frac{1}{\sqrt{2}}\right) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} (d_j)_R, \\ &= -\frac{1}{\sqrt{2}} y_{ij}^u v \bar{u_{iL}} u_{jR} - \frac{1}{\sqrt{2}} y_{ij}^u h(x) \bar{u_{iL}} u_{jR} - \frac{1}{\sqrt{2}} y_{ij}^d v \bar{d_{iL}} d_{jR} - \frac{1}{\sqrt{2}} y_{ij}^d h(x) \bar{d_{iL}} d_{jR}. \end{aligned} \tag{2.81}$$

The first thing to notice is that these do not have the general appearance of quark mass terms. There is a problem; each term mixes the quark generations. We can compare this with the weak gauge boson mass terms in (2.61). We will solve this in a similar way, by redefining the fields in terms of a new basis that diagonalises the Higgs couplings.

Consider the Yukawa couplings to the up-type quarks, y_{ij}^u. These can be represented in matrix form as a general 3×3 complex matrix. Now in order to produce physical results, we must have masses which are positive real numbers. So take the combinations $y^u y^{u\dagger}$ and $y^{u\dagger} y^u$. Both of these are symmetric matrices with positive real values: we know from linear algebra these can always be diagonalised. So we can find unitary transformations U^u and W^u such that:

$$\left.\begin{aligned} y^u y^{u\dagger} &= U^u (Y^u)^2 U^{u\dagger} \\ y^{u\dagger} y^u &= W^u (Y^u)^2 W^{u\dagger} \end{aligned}\right\} \implies y^u = U^u Y^u W^{u\dagger}, \tag{2.82}$$

where Y^u is a diagonalised matrix with positive real values. Note here there is no sum over the u index (or d later); this is merely to distinguish between the transformations. This can be done in complete analogy for y^d to obtain $y^d = U^d Y^d W^{d\dagger}$. Using these unitary transformations, we can write:

$$\begin{aligned} u_{iL} &= U^u_{ij} u'_{jL}, \\ u_{iR} &= W^u_{ij} u'_{jR}, \end{aligned} \quad \text{and} \quad \begin{aligned} d_{iL} &= U^d_{ij} d'_{jL}, \\ d_{iR} &= W^d_{ij} d'_{jR}, \end{aligned} \tag{2.83}$$

where u'_{jL}, u'_{jR}, d'_{jL} and d'_{jR} are the bases which diagonalise the Higgs couplings. To see this, we can substitute these in to (2.81):

$$\mathcal{L}_{\text{yuk}} = -\frac{v}{\sqrt{2}} \left\{ U^u_{ia} Y^u_{ab} W^{u\dagger}_{bj} \bar{u'_{cL}} U^{u\dagger}_{ic} W^u_{jd} u'_{dR} + \cdots \right\}. \tag{2.84}$$

Working in index notation, all terms will freely commute with each other. We can also use the property of unitary matrices, namely $U^u_{ij} U^{u\dagger}_{jk} = U^u_{ik}$, and rearrange to leave us with:

$$\mathcal{L}_{\text{yuk}} = -\frac{v}{\sqrt{2}} \left\{ Y^u_{ab} U^u_{ac} \bar{u'_{cL}} U^u_{bd} u'_{dR} + \cdots \right\} = -\frac{v}{\sqrt{2}} \left\{ Y^u_{ab} \bar{u'_{aL}} u'_{bR} + \cdots \right\}. \tag{2.85}$$

Now by definition Y^u_{ab} is a diagonal matrix. So for $a \neq b$ all terms vanish; there is no longer any terms with mixed quark generations. Finally the quark masses are immediately clear, and each term is manifestly gauge invariant. We conclude:

$$m_{u_i} = \frac{1}{\sqrt{2}} Y^u_{ii} v \quad \text{and} \quad m_{d_i} = \frac{1}{\sqrt{2}} Y^d_{ii} v. \tag{2.86}$$

2.6 The CKM Matrix

We are finally in a position to introduce the CKM matrix. Consider the W-boson currents in the quark sector. Under the unitary transformations outlined in (2.83), these transform as:

$$J^{\mu +} = \frac{1}{\sqrt{2}} \bar{u_{iL}} \gamma^\mu d_{iL} \to \frac{1}{\sqrt{2}} \bar{u'_{jL}} (U^u_{ij})^\dagger \gamma^\mu U^d_{ik} d'_{kL} = \frac{1}{\sqrt{2}} \bar{u'_{jL}} \gamma^\mu (U^{u\dagger} U^d)_{jk} d'_{kL}, \tag{2.87}$$

$$J^{\mu -} = \frac{1}{\sqrt{2}} \bar{d_{iL}} \gamma^\mu u_{iL} \to \frac{1}{\sqrt{2}} \bar{d'_{jL}} (U^d_{ij})^\dagger \gamma^\mu U^u_{ik} u'_{kL} = \frac{1}{\sqrt{2}} \bar{d'_{jL}} \gamma^\mu (U^{d\dagger} U^u)_{jk} u'_{kL}. \tag{2.88}$$

We then define the CKM matrix as the combination of unitary matrices shown in (2.87):

$$V_{\text{CKM}} = U^{u\dagger} U^d \implies V^\dagger_{\text{CKM}} = U^{d\dagger} U^u, \tag{2.89}$$

which simplifies these charged currents to:

$$J^{\mu +} = \frac{1}{\sqrt{2}} \bar{u'_{jL}} \gamma^\mu V_{jk} d'_{kL} \text{ and } J^{\mu -} = \frac{1}{\sqrt{2}} u'_{jL} \gamma^\mu V^*_{jk} \bar{d'_{kL}}; \tag{2.90}$$

in the second expression transposing and relabelling some dummy indices. What then do these expressions represent? In the former case, we have a right-handed up-type antiquark coupled to a left-handed down-type quark, whilst in the latter it is a left-handed up-type quark coupled to a right-handed down-type antiquark. These two couplings are then related exactly by CP symmetry; exchanging particles for antiparticles and left-handed for right-handed gives $J^{\mu +} \leftrightarrow J^{\mu -}$.

Unfortunately, there is one problem. Each coupling is identical, but only up to complex conjugation in the respective CKM element. So (in the electroweak quark sector, at least) CP is a valid symmetry *as long as the CKM matrix elements are real.* When Nicola Cabibbo first postulated his Cabibbo angle in 1963[32], there were was no notion of a quark model and CP was regarded as an unbroken symmetry of nature. His theory is equivalent to a CKM matrix in a two-generational model. Up to an overall (unphysical) phase factor, this is given by:

$$V_{\text{Cabibbo}} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}. \tag{2.91}$$

It is important to note that this matrix is real valued at all points; indeed, it can be shown that a two-generational CKM matrix can always be made real through redefenitions of the quark fields present in the charged currents.[2] Consequently, such a matrix can never contribute any CP violation.

However, during 1964, CP violation within the electroweak sector had been observed in the neutral K meson system[33]. Building on the earlier work of Cabibbo, Makoto Kobayashi and Toshihide Maskawa observed that for a 3×3 CKM matrix there was no way to fully absorb all the complex phase factors into the quark fields. Subsequently, in their 1973 paper[34] they postulated the existence of a third generation of quarks. Even before the confirmation of two generations, they had predicted the existence of a new full generation. This was only confirmed over twenty years later with the observation of the top quark in 1995[35][36]. Quantitatively, the magnitude of the CKM matrix elements are then[37]:

$$|V| = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.97428 & 0.2253 & 0.00347 \\ 0.2252 & 0.97345 & 0.0410 \\ 0.00862 & 0.0403 & 0.999152 \end{pmatrix}. \tag{2.92}$$

Here, each $|V_{ij}|$ term is directly proportional to the transition probability of quark $i \leftrightarrow j$ when coupled to the $W^{\pm}$ boson. Physically, we can think of the down-type quarks propagating as some linear combination of all three down-type generations. Of course, this could be defined such that there was a unitary transformation acting on the up-type quarks. The original choice is merely a standard convention.

We will often use a form of the CKM matrix known as the Wolfenstein parameterisation[3][38]:

$$V = \begin{pmatrix} 1 - {}^{\lambda^2}/_2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - {}^{\lambda^2}/_2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{2.93}$$

which is a perturbative expansion of the standard CKM matrix in powers of $\lambda = \sin\theta_z$, with $\lambda \approx 0.22$, $A \approx 0.81$, $\rho \approx 0.14$ and $\eta \approx 0.35$. This has the advantage that all parameters (A, λ, ρ and η) are of similar order, and all CP-violating processes are restricted to terms of $\mathcal{O}(\lambda^3)$ and higher.

3 B Physics

This section focuses predominantly on the analysis and motivation for study of the B mesons. These mesons consist of a bottom antiquark ($\bar{b}$) paired with either an up (B^+),

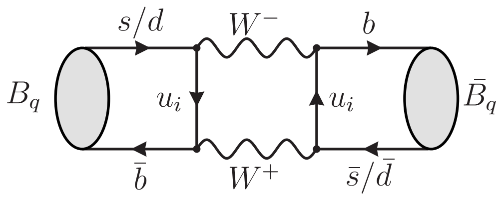
[2]See Appendix B.1.

[3]See Appendix B.4 for a full derivation.

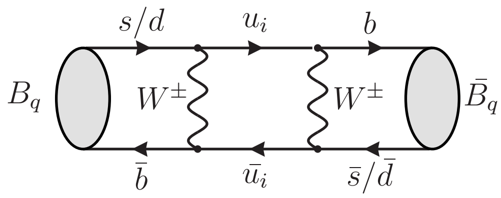
down (B^0), strange (B_s^0), or charm (B_c^+) quark. During the 1990s, a concerted effort was made to study the properties of these B mesons with the development of 2 "B-factory" experiments: the BaBar[39] detector for the PEP-II collider in Stanford, USA, and the Belle[40] detector for the KEKB collider in Tsukuba, Japan. Both have since ceased operations, in 2008 and 2010 respectively, being superseded by the LHCb[41] detector at CERN.

3.1 Motivation for Study and Production of B Mesons

The main motivation for this significant interest in B mesons was caused by the phenomena of neutral B oscillations, first observed in 1987 by the ARGUS collaboration at DESY[42]. The relevant leading order Feynman diagrams are shown in Figure 2.



(a) via $W^\pm$ emission.



(b) via $W^\pm$ exchange.

Figure 2: Leading order box diagrams contributing to neutral B meson oscillations.

Although this process can be explained by the Standard Model, it was realised that a thorough investigation into this oscillatory phenomena and B mesons in general could provide important evidence of anomalous CP-violating processes. The B-factories were born.

Now, both the PEP-II and KEKB colliders were electron-positron colliders operating at a centre of mass energy of 10.58 GeV. This energy is significant as it is just over the threshold of the $\Upsilon(4S)$ resonance, which then subsequently decays into B mesons. This production process is shown in Figure 3.

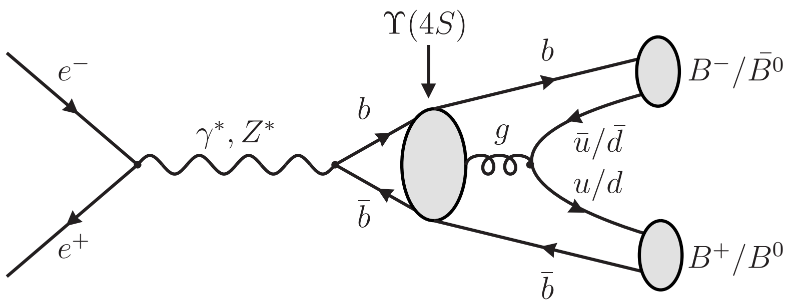


Figure 3: B meson production through electron-positron collision via the $\Upsilon(4S)$ resonance.

Due to the energy of the $\Upsilon(4S)$ resonance (10.5794 ± 0.0012 GeV[16]) being just over twice that of either the $B^\pm$ or B^0 mesons, the decay channels $\mathcal{B}(\Upsilon(4S) \to B^+\, B^-)$ and $\mathcal{B}(\Upsilon(4S) \to B^0\, \bar{B}^0)$ are extremely favourable. Indeed, these are measured as[16]:

$$\mathcal{B}(\Upsilon(4S) \to B^+\, B^-) = (51.3 \pm 0.6)\% \text{ and } \mathcal{B}(\Upsilon(4S) \to B^0\, \bar{B}^0) = (48.7 \pm 0.6)\%. \quad (3.1)$$

Consequently, copious amounts of B mesons are produced with minimal additional products. This provides an unusually clean environment to study their properties.

3.2 An Excess in the Like-Sign Dimuon Charge Asymmetry?

As mentioned in the introduction, some particularly tantalising evidence for new physics emerged from the DØ experiment[6][7] at Tevatron in early 2011. This centred on muon production through B meson decay. Figure 4 has the relevant Feynman diagrams.

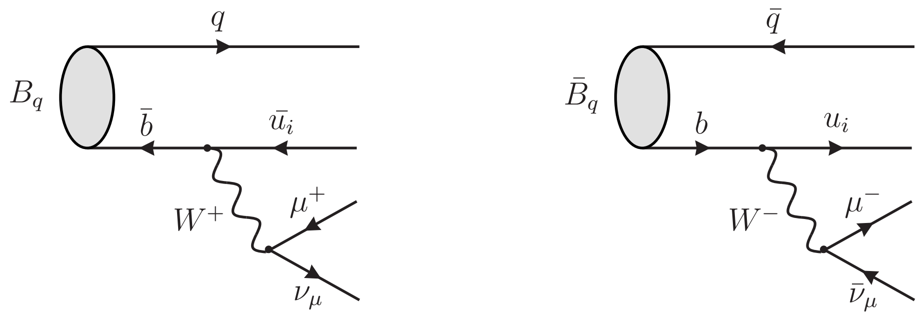


Figure 4: Muon production through semi-leptonic decays of B mesons.

In particular, there was a focus on events producing two like-sign muons (either positive or negative). These events are thought to occur when one of the neutral mesons oscillates into its antiparticle (see Figure 2) before decaying semileptonically.

As shown in Figure 3, each individual event generates a pair of B mesons; a particle, and its corresponding antiparticle partner. For each meson, there is a non-zero probability for oscillation to its own antiparticle. Should the probability of oscillation violate CP-symmetry, i.e. oscillation from meson to antimeson is favoured (or vice versa), we would then observe a slight excess in the number of events producing two muons (or antimuons); there is a like-sign dimuon charge asymmetry.

This effect is predicted by the Standard Model, but the asymmetry is vanishingly small, of the order $\approx 0.01\%$. We would therefore expect, on average, nearly identical numbers of positive and negative like-sign dimuon events. However, after accounting for instrumental effects and excess muons generated by kaon and pion decay, the analysis revealed an asymmetry of $A^b_{\text{sl}} = (-0.787 \pm 0.172\ (\text{stat}) \pm 0.093\ (\text{syst}))\%$, more than 3.9σ outside the Standard Model prediction[43]. How can this be explained?

3.3 Measuring an Unknown Interaction

One proposed suggestion is a sizeable new decay channel in the B_s meson system[44][45][46], with $\tau^+\tau^-$ being a likely candidate. If we consider the most general case, such a coupling would contribute to all b-quark decays:

$$b\,\bar{s} \to X \implies b \to s\,X. \tag{3.2}$$

How can this be examined without having any knowledge of the form of the interaction? One method is to measure the quantity n_c; the average number of charm quarks per bottom quark decay. It is trivial to deduce an expression for this quantity:

$$n_c = \frac{0 \times \Gamma(b \to 0 \text{ charm})}{\Gamma_{\text{total}}} + \frac{1 \times \Gamma(b \to 1 \text{ charm})}{\Gamma_{\text{total}}} + \frac{2 \times \Gamma(b \to 2 \text{ charm})}{\Gamma_{\text{total}}} + \cdots, \tag{3.3}$$

and it is also clear such a series contains infinitely many terms. This expression can be limited by making the approximation:

$$\Gamma_{\text{total}} \approx \Gamma(b \to 0 \text{ charm}) + \Gamma(b \to 1 \text{ charm}) + \Gamma(b \to 2 \text{ charm}). \tag{3.4}$$

A natural question is to ask whether this approximation makes sense; i.e. whether the terms of $\mathcal{O}(\Gamma(b \to 3\,c))$ and higher are small enough to be neglected? The answer is yes. At low energies, we can naively model charm production from bottom quark decay using Fermi's interaction[47]. For any more than two charm quarks, we are required to introduce an extra vertex into the Feynman diagram. This in turn brings an additional factor of Fermi's constant into the amplitude. Fermi's constant has a value of $G_F = 1.166\ldots \times 10^{-5}\ \mathrm{GeV}^{-2}$[16], which means decay widths (which are proportional to the square of the amplitude) for more than two charms are heavily suppressed. So, using this approximation, (3.3) reduces to:

$$n_c = 1 + \frac{\Gamma(b \to 2\ \mathrm{charm})}{\Gamma_{\mathrm{total}}} - \frac{\Gamma(b \to 0\ \mathrm{charm})}{\Gamma_{\mathrm{total}}}. \tag{3.5}$$

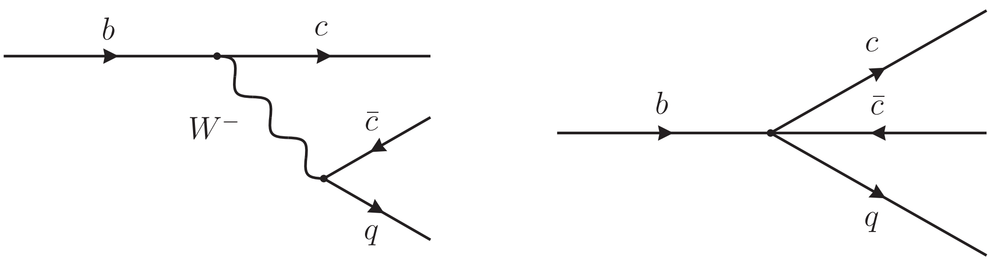
Should there be any sizeable decay mode $\Gamma(b \to s\,X)$, it would contribute to both Γ_{total} and one of either $\Gamma(b \to 0\,c)$, $\Gamma(b \to 1\,c)$ or $\Gamma(b \to 2\,c)$ depending on the form of X. This makes such an expression particularly sensitive. We can rearrage the equation for $\Gamma(b \to s\,X)$ and use this to bound any possible contribution:

$$\begin{aligned} n_c &= 1 + \frac{\Gamma(b \to 2\,c) - \Gamma(b \to 0\,c) - \Gamma(b \to s\,X)}{\Gamma_{\mathrm{total}} + \Gamma(b \to s\,X)}, \\ \implies \Gamma(b \to s\,X) &\leq \frac{1}{n_c}\Big\{\Gamma_{\mathrm{total}} + \Gamma(b \to 2\,c) - \Gamma(b \to 0\,c)\Big\} - \Gamma_{\mathrm{total}}. \end{aligned} \tag{3.6}$$

For such a strategy to provide sensible predictions, it is therefore vital to obtain accurate measurements for these inclusive decays. This is experimentally challenging as we never observe bare quarks.

3.4 Leading Order Calculation for the $b \to c\,\bar{c}\,q$ Decay

Theoretically, however, we can calculate to leading order the decay widths of quarks with relative ease. The dominant decay of the b quark is to a charm via the electroweak interaction. For sufficiently low energies, this can be approximated using Fermi's interaction. This is shown in Figure 5, with q understood to be either an s or d quark.



(a) via the usual W^- boson emission. (b) via the Fermi four-point interaction vertex.

Figure 5: Feynman diagrams at leading order for the inclusive $b \to 2\,c$ decay.

The differential decay width is given by:

$$\mathrm{d}\Gamma^{(0)}_{b\to c\,\bar{c}\,q} = \frac{1}{2m_b}|\overline{\mathcal{M}}|^2(2\pi)^4\delta^4(p_b - p_c - p_{\bar{c}} - p_q)\frac{\mathrm{d}^3\vec{p_c}}{(2\pi)^3 2E_c}\frac{\mathrm{d}^3\vec{p_{\bar{c}}}}{(2\pi)^3 2E_{\bar{c}}}\frac{\mathrm{d}^3\vec{p_q}}{(2\pi)^3 2E_q}. \tag{3.7}$$

The first step is then to calculate the spin-summed squared matrix element, $|\overline{\mathcal{M}}|^2$. Approximating the decay by the Fermi interaction, and using the appropriate Feynman rules, we have:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left\{\bar{u}(p_q)\gamma^\mu(\mathbb{1}-\gamma_5)V_{cq}v(p_{\bar{c}})\right\} \left\{\bar{u}(p_c)\gamma_\mu(\mathbb{1}-\gamma_5)V^*_{cb}u(p_b)\right\} = \frac{G_F}{\sqrt{2}} V_{cq}V^*_{cb}\mathcal{B}_1\mathcal{B}_2. \quad (3.8)$$

V_{cq} and V^*_{cb} can both be pulled out to the front as they are simply numbers and therefore trivially commute with all terms. Initially we will focus on the first bracket, $\mathcal{B}_1$. In component form, the spin-summed squared element is:

$$|\overline{\mathcal{B}_1}|^2 = \sum_{\text{spins}} \bar{v}(p_{\bar{c}})_a(\mathbb{1}+\gamma_5)_{ab}\gamma^\mu_{bc}u(p_q)_c\bar{u}(p_q)_d\gamma^\nu_{de}(\mathbb{1}-\gamma_5)_{ef}v(p_{\bar{c}})_f. \quad (3.9)$$

We now require the identities:

$$\sum_{\text{spins}} v(p_{\bar{c}})_f\bar{v}(p_{\bar{c}})_a = (\not{p}_{\bar{c}} - m_c\mathbb{1})_{fa} \text{ and } \sum_{\text{spins}} u(p_q)_c\bar{u}(p_q)_d = (\not{p}_q + m_q\mathbb{1})_{cd}. \quad (3.10)$$

At this point we can make a slight approximation. The q quark is sufficiently light in comparison to the W^- that we can ignore its mass entirely. So substituting back in to (3.9):

$$\begin{aligned} |\overline{\mathcal{B}_1}|^2 &= (\mathbb{1}+\gamma_5)_{ab}\gamma^\mu_{bc}(\not{p}_q)_{cd}\gamma^\nu_{de}(\mathbb{1}-\gamma_5)_{ef}(\not{p}_{\bar{c}} - m_c\mathbb{1})_{fa}, \\ &= \text{Tr}\left[(\mathbb{1}+\gamma_5)\gamma^\mu(\not{p}_q)\gamma^\nu(\mathbb{1}-\gamma_5)(\not{p}_{\bar{c}} - m_c\mathbb{1})\right]. \quad (3.11)\end{aligned}$$

To proceed, we can consider multiplying out the $(\not{p}_{\bar{c}} - m_c\mathbb{1})$ bracket. The mass term in this case is a diagonal matrix and will commute freely. The Dirac commutation relations then ensure this expression contains $(\mathbb{1}+\gamma_5)(\mathbb{1}-\gamma_5)$. But we know that $(\mathbb{1}+\gamma_5)(\mathbb{1}-\gamma_5) = 0$, so all such terms will vanish:

$$|\overline{\mathcal{B}_1}|^2 = \text{Tr}\left[(\mathbb{1}+\gamma_5)\gamma^\mu\not{p}_q\gamma^\nu(\mathbb{1}-\gamma_5)\not{p}_{\bar{c}}\right] = 2\text{Tr}\left[(\mathbb{1}+\gamma_5)\gamma^\mu\not{p}_q\gamma^\nu\not{p}_{\bar{c}}\right]. \quad (3.12)$$

The trace operator is also linear:

$$|\overline{\mathcal{B}_1}|^2 = 2\left(\text{Tr}\left[\gamma^\mu\gamma^\rho(p_q)_\rho\gamma^\nu\gamma^\sigma(p_{\bar{c}})_\sigma\right] + \text{Tr}\left[\gamma_5\gamma^\mu\gamma^\rho(p_q)_\rho\gamma^\nu\gamma^\sigma(p_{\bar{c}})_\sigma\right]\right), \quad (3.13)$$

and we can then use some basic trace relations:

$$\begin{aligned} \text{Tr}\left[\gamma^\mu\gamma^\nu\right] &= 4g^{\mu\nu}, \quad (3.14)\\ \text{Tr}\left[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right] &= 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}), \quad (3.15)\\ \text{Tr}\left[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\right] &= 4i\epsilon^{\mu\nu\rho\sigma}. \quad (3.16)\end{aligned}$$

Substituting these in reduces (3.13) to:

$$|\overline{\mathcal{B}_1}|^2 = 8\left\{p^\mu_{\bar{c}}p^\nu_q + p^\nu_{\bar{c}}p^\mu_q - g^{\mu\nu}(p_{\bar{c}}\cdot p_q) + i\epsilon^{\mu\nu\rho\sigma}(p_{\bar{c}})_\rho(p_q)_\sigma\right\}, \quad (3.17)$$

where in this case we take the $\cdot$ operator to represent the contraction of the 2 4-vectors. Similarly, we obtain an analagous expression for $|\overline{\mathcal{B}_2}|^2$:

$$|\overline{\mathcal{B}_2}|^2 = 8\left\{(p_b)_\mu(p_c)_\nu + (p_b)_\nu(p_c)_\mu - g_{\mu\nu}(p_b\cdot p_c) + i\epsilon_{\mu\nu\rho\sigma}p^\rho_b p^\sigma_c\right\}. \quad (3.18)$$

The spin-summed squared matrix element is then:

$$|\overline{\mathcal{M}}|^2 = \frac{G_F^2}{2}|V_{cq}|^2|V_{cb}|^2|\overline{\mathcal{B}_1}|^2|\overline{\mathcal{B}_2}|^2. \tag{3.19}$$

Each bracket contains both symmetric and antisymmetric terms in μ and ν. This allows for a greatly simplified expression as the contractions of terms with an opposing symmetry will vanish:

$$|\overline{\mathcal{M}}|^2 = 32G_F^2|V_{cq}|^2|V_{cb}|^2\left\{4(p_b\cdot p_{\bar{c}})(p_c\cdot p_q)\right\}. \tag{3.20}$$

Finally we must average over the initial spin states of the b quark. All quarks are spin-$1/2$ particles, so there are 2 initial spin states. The matrix element is then:

$$|\overline{\mathcal{M}}|^2 = 64G_F^2|V_{cq}|^2|V_{cb}|^2\left\{(p_b\cdot p_{\bar{c}})(p_c\cdot p_q)\right\}. \tag{3.21}$$

Now, in the rest frame of the b quark we have:

$$p_b = (m_b,\vec{0}),\;\; p_c = (E_c,\vec{p_c}),\;\; p_{\bar{c}} = (E_{\bar{c}},\vec{p_{\bar{c}}}),\;\; p_q = (E_q,\vec{p_q}), \tag{3.22}$$

and trivially we conclude that $(p_b\cdot p_{\bar{c}}) = m_bE_{\bar{c}}$. We can also use conservation of 4-momentum:

$$p_b^\mu = p_c^\mu + p_{\bar{c}}^\mu + p_q^\mu, \tag{3.23}$$

to obtain a simplified expression for $(p_c\cdot p_q)$:

$$\begin{aligned}(p_c\cdot p_q) &= \frac{1}{2}\left\{(p_c+p_q)^\mu(p_c+p_q)_\mu - m_c^2\right\},\\ &= \frac{1}{2}\left\{(p_b-p_{\bar{c}})^\mu(p_b-p_{\bar{c}})_\mu - m_c^2\right\},\\ &= \frac{1}{2}\left\{m_b^2 - 2m_bE_{\bar{c}}\right\}.\end{aligned} \tag{3.24}$$

Finally, we must multiply by 3 to account for the 3 different colour-anticolour combinations of the $\bar{c}\,q$ pair. Our final expression for the spin-summed squared matrix element is thus:

$$|\overline{\mathcal{M}}|^2 = 96G_F^2|V_{cq}|^2|V_{cb}|^2(m_bE_{\bar{c}})(m_b^2 - 2m_bE_{\bar{c}}). \tag{3.25}$$

Equation (3.7) can then be integrated to obtain the total decay width, $\Gamma_{b\to 2\,c}$. First, we integrate over $\vec{p_c}$ as there is no dependence other than in the momentum conserving delta function:

$$\int_{-\infty}^{\infty}\delta^4(p_b - p_c - p_{\bar{c}} - p_q)\frac{\mathrm{d}^3\vec{p_c}}{(2\pi)^3 2E_c} = \frac{1}{(2\pi)^3 2E_c}\delta(m_b - E_c - E_{\bar{c}} - E_q). \tag{3.26}$$

Next, integration over $\vec{p_q}$. Using polars, with the polar axis along $\vec{p_{\bar{c}}}$, we can reexpress $\mathrm{d}^3\vec{p_q}$:

$$\mathrm{d}^3\vec{p_q} = 2\pi E_q^2\,\mathrm{d}E_q\,\mathrm{d}\cos\theta, \tag{3.27}$$

and from 3-momentum conservation in the rest frame of the b quark:

$$\begin{aligned}E_c^2 &= |\vec{p_c}|^2 + m_c^2 = |\vec{p_{\bar{c}}}+\vec{p_q}|^2 + m_c^2 = |\vec{p_{\bar{c}}}|^2 + |\vec{p_q}|^2 + 2\vec{p_{\bar{c}}}\cdot\vec{p_q} + m_c^2,\\ &= E_{\bar{c}}^2 + E_q^2 + 2a\cos\theta \quad\text{with}\quad a = E_q\sqrt{E_{\bar{c}}^2 - m_c^2},\end{aligned} \tag{3.28}$$

where θ is the angle subtended by the $\vec{p}_{\bar{c}}$ and $\vec{p}_q$ 3-momentum vectors. Equation (3.7) has been reduced to:

$$\mathrm{d}\Gamma^{(0)}_{b\to c\,\bar{c}\,q} = \frac{|\overline{\mathcal{M}}|^2 E_q\,\mathrm{d}E_q}{16 m_b (2\pi)^4}\frac{\mathrm{d}^3\vec{p}_{\bar{c}}}{E_{\bar{c}}}\int_{-1}^{1}\delta(m_b - E_c - E_{\bar{c}} - E_q)\frac{\mathrm{d}\cos\theta}{E_c}. \tag{3.29}$$

We now use the identity:

$$\delta(f(x)) = |f'(x_0)|^{-1}\delta(x - x_0), \tag{3.30}$$

with $x = \cos\theta$ in this case, to reduce the integral in (3.29) to:

$$\int_{-1}^{1}\delta(m_b - E_c - E_{\bar{c}} - E_q)\frac{\mathrm{d}\cos\theta}{E_c} = \frac{1}{a} = \frac{1}{E_q\sqrt{E_{\bar{c}}^2 - m_c^2}}. \tag{3.31}$$

The differential decay width is simplified to:

$$\mathrm{d}\Gamma^{(0)}_{b\to c\,\bar{c}\,q} = \frac{|\overline{\mathcal{M}}|^2\,\mathrm{d}E_q}{16 m_b (2\pi)^4}\frac{\mathrm{d}^3\vec{p}_{\bar{c}}}{E_{\bar{c}}}\frac{1}{\sqrt{E_{\bar{c}}^2 - m_c^2}}. \tag{3.32}$$

The previous δ-function then implies limits for the integration over E_q:

$$\begin{aligned}
& m_b - E_{\bar{c}} - E_q = E_c^{\pm} = \sqrt{E_{\bar{c}}^2 + E_q^2 \pm 2a}, \\
\implies & m_b^2 + \cancel{E_{\bar{c}}^2} + \cancel{E_q^2} - 2m_bE_{\bar{c}} - 2m_bE_q + 2E_{\bar{c}}E_q = \cancel{E_{\bar{c}}^2} + \cancel{E_q^2} \pm 2E_q\sqrt{E_{\bar{c}}^2 - m_c^2}, \\
\implies & E_q^{\pm} := \frac{m_b^2 - 2m_bE_{\bar{c}}}{2\left(m_b - E_{\bar{c}} \pm \sqrt{E_{\bar{c}}^2 - m_c^2}\right)}.
\end{aligned} \tag{3.33}$$

For the physical scenario of E_q and $E_{\bar{c}}$ strictly positive, these 2 functions will then form the boundary of the integration region. This is shown in Figure 6.

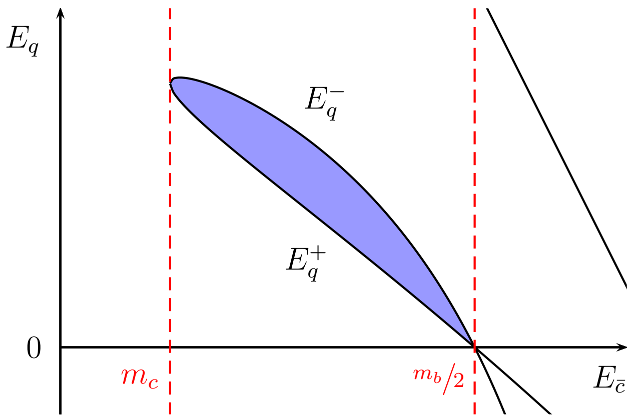


Figure 6: Integration region for E_q and $E_{\bar{c}}$.

Again, there is no dependence on E_q in $\mathrm{d}\Gamma_{b\to 2\,c}$, so we can simply multiply the function by the difference in the upper and lower bounds:

$$\begin{aligned}
\Delta E_q^{\pm} &:= \frac{m_b^2 - 2m_bE_{\bar{c}}}{2\left(m_b - E_{\bar{c}} - \sqrt{E_{\bar{c}}^2 - m_c^2}\right)} - \frac{m_b^2 - 2m_bE_{\bar{c}}}{2\left(m_b - E_{\bar{c}} + \sqrt{E_{\bar{c}}^2 - m_c^2}\right)}, \\
&:= \frac{(m_b^2 - 2m_bE_{\bar{c}})\sqrt{E_{\bar{c}}^2 - m_c^2}}{(m_b^2 - 2m_bE_{\bar{c}} + m_c^2)}.
\end{aligned} \tag{3.34}$$

Substituting in (3.25) for $|\overline{M}|^2$, the decay width reads:

$$\mathrm{d}\Gamma^{(0)}_{b\to c\,\bar{c}\,q} = \frac{6G_F^2|V_{cq}|^2|V_{cb}|^2}{(2\pi)^4} \times \frac{(m_b^2 - 2m_b E_{\bar{c}})^2\, \mathrm{d}^3\vec{p}_{\bar{c}}}{m_b^2 - 2m_b E_{\bar{c}} + m_c^2}. \tag{3.35}$$

We can now reexpress $\mathrm{d}^3\vec{p}_{\bar{c}}$ in spherical polars as $4\pi|\vec{p}_{\bar{c}}|^2\,\mathrm{d}|\vec{p}_{\bar{c}}|$:

$$\mathrm{d}\Gamma^{(0)}_{b\to c\,\bar{c}\,q} = \frac{3G_F^2|V_{cq}|^2|V_{cb}|^2}{2\pi^3} \times \frac{|\vec{p}_{\bar{c}}|^2(m_b^2 - 2m_b E_{\bar{c}})^2\, \mathrm{d}|\vec{p}_{\bar{c}}|}{m_b^2 - 2m_b E_{\bar{c}} + m_c^2}, \tag{3.36}$$

and use the energy-momentum relation to reexpress any remaining energy terms as functions of $|\vec{p}_{\bar{c}}|$. We then integrate over $|\vec{p}_{\bar{c}}|$, the integration limits for $E_{\bar{c}}$ (and hence $|\vec{p}_{\bar{c}}|$) given by Figure 6:

$$\mathrm{d}\Gamma^{(0)}_{b\to c\,\bar{c}\,q} = \frac{3G_F^2|V_{cq}|^2|V_{cb}|^2}{2\pi^3} \int\limits_0^{\sqrt{(m_b^2/4)-m_c^2}} \frac{|\vec{p}_{\bar{c}}|^2\left(m_b^2 - 2m_b\sqrt{|\vec{p}_{\bar{c}}|^2+m_c^2}\right)^2 \mathrm{d}|\vec{p}_{\bar{c}}|}{m_b^2 - 2m_b\sqrt{|\vec{p}_{\bar{c}}|^2+m_c^2} + m_c^2}. \tag{3.37}$$

Using measured values from the PDG[16] and summing over $q = s$ and the CKM suppressed $q = d$, we obtain a numerical value of:

$$\Gamma^{(0)}_{b\to c\,\bar{c}\,q} = 2.547\ldots \times 10^{-14}\ \mathrm{GeV}. \tag{3.38}$$

When including QCD effects and next-to-leading order corrections[48] this rises to:

$$\Gamma_{b\to c\,\bar{c}\,q} = (8.76 \pm 3.32) \times 10^{-14}\ \mathrm{GeV}. \tag{3.39}$$

Combining errors in an appropriate way[49], this corresponds to an overall branching ratio of:

$$\mathcal{B}(b \to c\,\bar{c}\,q) = (24.5 \pm 11.1)\,\%. \tag{3.40}$$

3.5 Measuring Quark Decays Using Exclusive Channels

We would, of course, prefer to rely on experimental measurements rather than theoretical calculations. So the question is, can we find a reliable method to indirectly measure the decays of bare quarks? For the $b \to 2\,c$ channel, one proposed strategy is to measure the branching ratios of “opposite-sign” charm decays in $\bar{B}$ mesons, i.e. decays of the type[50][51]:

$$\begin{aligned}\mathcal{B}(\bar{B} \to \bar{c}\,X) = {} & \mathcal{B}(\bar{B} \to D_s^-\,X) \\ & + \mathcal{B}(\bar{B} \to \bar{D}^0\,X) \\ & + \mathcal{B}(\bar{B} \to \bar{\Lambda}_c\,X) \\ & + \mathcal{B}(\bar{B} \to (c\bar{c})\,X),\end{aligned} \tag{3.41}$$

where $\bar{B}$ denotes any one of $\bar{B}^0$, $\bar{B}_s$, or B^-, with $(c\bar{c})$ any of the charmonium states.

We begin by examining the theoretical basis for such a method. The first key assumption made here is to employ the use of a spectator model; naively, these decays of $\bar{B}$ mesons are governed entirely by the decay of the b-quark. It is clear such a model must be used, else the branching ratios of b quarks and $\bar{B}$ mesons would be markedly different, making comparisons meaningless.

Secondly, by considering these "wrong-sign" charm decays, there is an implicit assumption that the b quark will always decay into a c. Is this reasonable? We know that the 2 dominant tree-level possibilities for b quark decay are the $b \to c$ and $b \to u$ transitions. Examining the ratio between these CKM elements:

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} \approx 7.16 \ldots \times 10^{-3}, \tag{3.42}$$

shows that the $b \to u$ channel is around 0.7% the magnitude of the $b \to c$ mode. For our purposes, this is certainly sufficiently small to be considered negligible.

Next we will look specifically at the suggested modes in (3.41). One notable omission from this list is the mode $\bar{B} \to D^- X$. Another is that of the comparatively rare decay to a $\bar{\Xi}_c$ baryon. We will briefly discuss the first of these. At quark level, we can model this as a $b \to c\,\bar{c}\,d$ transition (see Figure 5). Amplitudes for such a transition are therefore suppressed by λ, resulting in a suppression of decay widths to roughly $\lambda^2 \approx 5\%$ of the $b \to c\,\bar{c}\,s$. It would therefore seem reasonable to omit such a mode.

Through Monte Carlo simulation, we may now begin to test whether the relation in (3.41) is an accurate approximation, and whether this is a reliable predictor of the $b \to 2c$ channel. This is accomplished through the event generator SHERPA[52] ver 2.0.β_2, employing the HepMC[53] event record package and Rivet[54] for analysis.[4] We simulate 10^6 events in a BaBar-type[39] environment with asymmetric electron and positron beam energies of 9.0 GeV and 3.7 GeV respectively, producing a centre of mass energy of 10.58 GeV. As with all Monte Carlo simulations, there are associated statistical errors. These can be ignored for our purposes, as with 10^6 events the errors are of the order $\approx 0.01\%$. The dominant errors here then will be associated with the input branching ratios for the Monte Carlo.

We begin by analysing the number of final state charm quarks per $\bar{B}$ meson decay, as shown in Figure 7.

# of c quarks	BR (%)
0	0.67
1	65.76
2	33.56

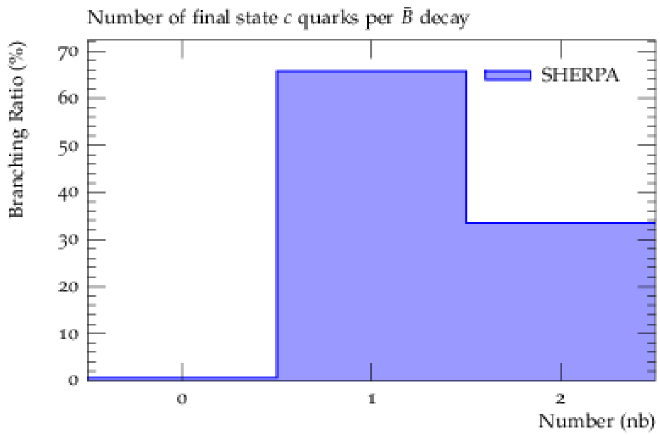


Figure 7: Monte Carlo data for charm (either c or $\bar{c}$) quark abundance in $\bar{B}$ meson final state decay products.

The low $\bar{B} \to 0\,c$ rate is unsurprising; at quark level this represents a $b \to u$ transition. From Equation (3.42) we have that the ratio of the $b \to u$ and $b \to c$ channels is around 0.7%, which is in line with the observations in Figure 7.

[4] See Appendix C for analysis code.

What is surprising is the enhanced $\bar{B} \to 2\,c$ channel. Whilst this is within the theoretically predicted range of $\mathcal{B}(b \to 2\,c) = (24.5 \pm 11.1)\,\%$, it does represent a significant deviation from the central value. It is also inconsistent with some more rigorous measurements of the $b \to c\,\bar{c}\,s$ channel[55]. This is more than likely due to outdaded branching ratio inputs in the Monte Carlo program itself, and should hopefully be corrected with updated values.

Next we will discuss whether the four decays shown in (3.41) are a good approximation to the $\bar{B} \to \bar{c}\,X$ channel. Data for each of these four specified modes, as well as the $\bar{B} \to \bar{\Xi}_c\,X$ and $\bar{B} \to D^-\,X$ channels, is shown in Figure 8.

Bin	Decay Channel	BR (%)
1	$D_s^-\,X$	11.22
2	$\bar{D}^0\,X$	14.39
3	$\bar{\Lambda}_c\,X$	0.13
4	$(c\bar{c})\,X$	1.87
5	$\bar{\Xi}_c\,X$	0.01
6	$D^-\,X$	6.08
	Total	33.70

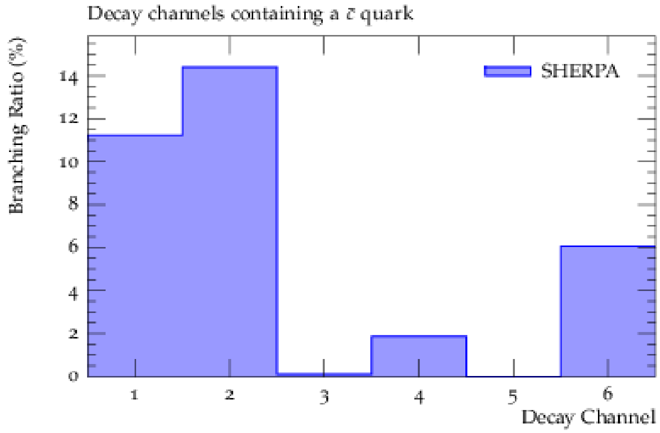


Figure 8: Monte Carlo data for decay channels of $\bar{B}$ mesons containing a $\bar{c}$ quark.

First, we will deal with a minor discrepancy present in the total branching ratio. We would expect this to be equal to (or even less than) the branching ratio for $\bar{B} \to 2\,c$ shown in Figure 7. What we actually observe is a slight excess. This is a direct result of approximating the b quark decay by neglecting the $b \to u$ transition; a small percentage ($\approx 0.7\%$) of these channels actually contribute to the $\bar{B} \to 1\,c$ in Figure 7 rather than $\bar{B} \to 2\,c$.

More significantly, we can also see that sum of the 4 channels in (3.41) undercounts the $\bar{B} \to \bar{c}X$ branching ratio by just over 6%. This is entirely due to the larger than expected D^-X channel (we may discount the vanishingly small $\bar{\Xi}_c$ BR). This shows that the earlier assumption to discount the $\bar{B} \to D^-\,X$ channel simply due to the CKM suppression of the inclusive $b \to c\,\bar{c}\,d$ decay is *not* valid.

Indeed, what we observe here can be attributed instead to the $b \to c\,\bar{c}\,s$ decay with additional quark pairs generated via gluon emission. Due to energy considerations, the predominant mode is the emission of two gluons creating two distinct quark-antiquark pairs: $s\,\bar{s}$ and $d\,\bar{d}$. This new d quark will then pair with the $\bar{c}$ quark from the b decay, and hadronise to form the observed D^- meson.

4 Conclusions

We began by presenting a thorough overview and introduction of current theory relating to gauge symmetries, the Higgs mechanism and flavour physics. This lead naturally through to the development of the CKM matrix and its implementation in B physics.

We highlighted the importance of accurate measurements for inclusive quark decay widths, focusing specifically on the $b \to 2\,c$ channel. The leading order theoretical calculation for this decay width was presented and enhanced through the inclusion of the corresponding NLO and QCD corrections.

A Monte Carlo analysis of $\bar{B}$ meson decay was then performed using the event generator SHERPA. The branching ratio for the decay to two charms was found to be $\mathcal{B}(\bar{B} \to 2\,c) = 33.56\,\%$. This is consistent with the theoretical prediction of $\mathcal{B}(b \to c\,\bar{c}\,q) = (24.5 \pm 11.1)\,\%$ but at odds with other $b \to c\,\bar{c}\,s$ measurements obtained experimentally, suggesting a possible need for updated inputs within the Monte Carlo.

Finally, a method of measuring this mode using "wrong-sign" charm decays was examined. The oft-neglected $\bar{B} \to D^-\,X$ channel was actually found to have a branching ratio of approximately 6%, giving a significant contribution to the total $\bar{B} \to \bar{c}\,X$ branching ratio. Given the previous inconsistency, further investigation is required. Should these results hold, the implication for future $\bar{B} \to 2\,c$ measurements is clear: the $\bar{B} \to D^-\,X$ channel must be included.

5 Acknowledgements

First and foremost I would like to thank my supervisor, Alexander Lenz, for providing help and guidance; as well as his patience with my (often stupid) questions. Special thanks must also be given to Frank Krauss for his significant input developing the project, particularly the initial Rivet analysis code. Similarly, many thanks to Marek Schoenherr and Hendrik Hoeth for their expertise in helping to implement the SHERPA/Rivet analyses. Finally, thanks to my brother Nev and girlfriend Laura for struggling through nearly 40 pages to proofread this report.

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A Dirac Algebra

In this section we will derive some of the identities used in the main paper using the Dirac algebra:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}. \tag{A.1}$$

We begin by considering γ^5, namely the property $(\gamma^5)^2 = \mathbb{1}$. From the defition of γ^5, we have:

$$(\gamma^5)^2 = i\gamma^0\gamma^1\gamma^2\gamma^3 i\gamma^0\gamma^1\gamma^2\gamma^3. \tag{A.2}$$

Now i trivially commutes, and each γ^μ anticommutes with γ^ν for $\mu \neq \nu$.

$$(\gamma^5)^2 = \gamma^0\gamma^0\gamma^1\gamma^2\gamma^3\gamma^1\gamma^2\gamma^3 = \cdots = -(\gamma^0)^2(\gamma^1)^2(\gamma^2)^2(\gamma^3)^2. \tag{A.3}$$

But $(\gamma^0)^2 = \mathbb{1}$ and $(\gamma^j)^2 = -\mathbb{1}$:

$$(\gamma^5)^2 = -(\mathbb{1})(-\mathbb{1})(-\mathbb{1})(-\mathbb{1}) = \mathbb{1}. \tag{A.4}$$

We proceed next by discussing the hermitian conjugate of γ^5:

$$(\gamma^5)^\dagger = (i\gamma^0\gamma^1\gamma^2\gamma^3)^\dagger = i(\gamma^3)^\dagger(\gamma^2)^\dagger(\gamma^1)^\dagger(\gamma^0)^\dagger. \tag{A.5}$$

First, it is well known that $(\gamma^0)^\dagger = \gamma^0$ and $(\gamma^j)^\dagger = -\gamma^j$:

$$(\gamma^5)^\dagger = i(-\gamma^3)(-\gamma^2)(-\gamma^1)(\gamma^0). \tag{A.6}$$

Commuting each γ^μ back to its original place eliminates the negative sign, and we are left to conclude $(\gamma^5)^\dagger = \gamma^5$. Next, we consider the anticommutation relation between γ^5 and the other gamma matrices:

$$\{\gamma^5, \gamma^\mu\} = i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^\mu + \gamma^\mu i\gamma^0\gamma^1\gamma^2\gamma^3. \tag{A.7}$$

Any γ^μ with $\mu \in \{0, 1, 2, 3\}$ will then have to anticommute 3 times with γ^ν for $\mu \neq \nu$ and trivially commute once (with itself!). We thus pick up three factors of (-1):

$$\{\gamma^5, \gamma^\mu\} = i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^\mu - i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^\mu = 0. \tag{A.8}$$

We can now turn our attention to the projection operators, specifically the property outlined in (2.73):

$$\bar{\psi}P_R = \psi^\dagger\gamma^0\frac{1}{2}(\mathbb{1} + \gamma^5). \tag{A.9}$$

We just showed in (A.8) that γ^5 anticommutes with the other Dirac matrices, so we can conclude:

$$\psi^\dagger\gamma^0\frac{1}{2}(\mathbb{1} + \gamma^5) = \psi^\dagger\frac{1}{2}(\mathbb{1} - \gamma^5)\gamma^0 = \psi^\dagger P_L\gamma^0. \tag{A.10}$$

But P_L is built out of two hermitian matrices, $\mathbb{1}$ and γ^5. It is then trivial to conclude that P_L itself is hermitian, and:

$$\bar{\psi}P_R = \psi^\dagger P_L\gamma^0 = (P_L\psi)^\dagger\gamma^0 = (\psi_L)^\dagger\gamma^0 = \bar{\psi}_L. \tag{A.11}$$

B Parameterisations of the CKM Matrix

B.1 2-Generational Model

We will first discuss the simplified theory of the CKM matrix for only 2 generations. Through the restrictions imposed by unitarity and utilising the gauge freedom of the quark fields, it can be shown that such a matrix can always be made real. Consequently, this form of the CKM matrix cannot contribute any CP violation. Consider a general 2×2 complex valued matrix of the form:

$$V = \begin{pmatrix} |c_1|e^{i\phi_1} & |c_2|e^{i\phi_2} \\ -|c_3|e^{i\phi_3} & |c_4|e^{i\phi_4} \end{pmatrix} \implies V^\dagger = \begin{pmatrix} |c_1|e^{-i\phi_1} & -|c_3|e^{-i\phi_3} \\ |c_2|e^{-i\phi_2} & |c_4|e^{-i\phi_4} \end{pmatrix}, \tag{B.1}$$

with $c_1, \cdots, c_4, \phi_1, \cdots, \phi_4 \in \mathbb{R}$ some real valued constants. We know V is unitary, i.e. $VV^\dagger = V^\dagger V = \mathbb{1}$. Using this condition, we obtain 3 equations constraining these parameters. Firstly, consider the diagonal terms:

$$\begin{cases} |c_1|^2 + |c_2|^2 = 1 \\ |c_3|^2 + |c_4|^2 = 1 \\ |c_1|^2 + |c_3|^2 = 1 \\ |c_2|^2 + |c_4|^2 = 1 \end{cases} \implies \begin{cases} |c_1| = |c_4| := \cos\theta_c, \\ |c_2| = |c_3| := \sin\theta_c. \end{cases} \tag{B.2}$$

Now the off-diagonals give only one linearly independent equation:

$$-|c_1||c_3|e^{i(\phi_1-\phi_3)} + |c_2||c_4|e^{i(\phi_2-\phi_4)} = 0 \implies \phi_1 - \phi_3 = \phi_2 - \phi_4. \tag{B.3}$$

We know from (2.5) that each of the quark fields have a $U(1)$ gauge symmetry of the form:

$$u_j \to e^{i\alpha_j} u_j \text{ and } d_k \to e^{i\beta_k} d_k. \tag{B.4}$$

We can choose these phases arbitrarily, so let:

$$\begin{cases} \alpha_j = \arg(V_{j1}) \\ \beta_k = -\arg(V_{1k}) \end{cases} \implies \begin{cases} \alpha_1 = \phi_1, \\ \alpha_2 = \phi_3, \\ \beta_1 = -\phi_1, \\ \beta_2 = -\phi_2. \end{cases} \tag{B.5}$$

The term present in the charged current now looks like:

$$e^{-i\alpha_j} V_{jk} e^{i\beta_k} = \begin{pmatrix} e^{-i\phi_1}\cos\theta_c & e^{-i\phi_1}\sin\theta_c \\ -e^{-i\phi_1}\sin\theta_c & e^{-i(\phi_2+\phi_3-\phi_4)}\cos\theta_c \end{pmatrix} = e^{-i\phi_1}\begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}, \tag{B.6}$$

where we have used (B.3) in the final step. What remains is an overall phase factor $e^{-i\phi_1}$ and a real valued rotation matrix in terms of some angle θ_c, known as the Cabibbo angle.[32] This overall phase factor corresponds to the $U(1)$ gauge symmetry and can be disregarded; we know from quantum mechanics that overall phase factors are unphysical.

B.2 Original Parameterisation

This prescription may be extended to the physical case of 3 generations. Here, a general 3×3 complex valued matrix will have 18 independent parameters. Through the constraints arising from unitarity, this is reduced to 9; 3 real angles and 6 phases. Using redefinitions of the quark fields as in (B.5), we can remove *at most* 5 of these phases. We can no longer fully eliminate all phases, and so for 3 generations there is a possibility for CP-violation.

When Kobayashi and Maskawa first postulated such a form of the matrix, they parameterised it in the following way[34]:

$$V = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}. \tag{B.7}$$

B.3 Standard Parameterisation

As noted previously, for 2 generations the CKM matrix resembles a rotation matrix. This is similarly true in the 3-dimensional case, and we can decompose the CKM matrix into a product of 3 orthogonal rotations. We will denote θ_x, θ_y and θ_z as the real rotation angles about the x, y and z axes respectively, and ϕ as the remaining CP-violating phase. We thus obtain the "standard parameterisation" outlined by Ling-Lie Chau and Wai-Yee Keung[56]:

$$\begin{aligned} V &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{pmatrix} \begin{pmatrix} \cos\theta_y & 0 & \sin\theta_y e^{-i\phi} \\ 0 & 1 & 0 \\ -\sin\theta_y e^{i\phi} & 0 & \cos\theta_y \end{pmatrix} \begin{pmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ &= \begin{pmatrix} c_zc_y & s_zc_y & s_ye^{-i\phi} \\ -s_zc_x - c_zs_xs_ye^{i\phi} & c_zc_x - s_zs_xs_ye^{i\phi} & s_xc_y \\ s_zs_x - c_zc_xs_ye^{i\phi} & -c_zs_x - s_zc_xs_ye^{i\phi} & c_xc_y \end{pmatrix}, \end{aligned} \tag{B.8}$$

where $c_x = \cos\theta_x$ and so on. Analysing the deconstructed form, it is trivial to conclude that here $\theta_z \equiv \theta_c$, the Cabibbo angle.

B.4 Wolfenstein Parameterisation

Finally, we may also consider the Wolfenstein parameterisation[38]. This is an expansion of the standard parameterisation in powers of a small parameter, λ. First, we define:

$$\lambda = s_z, \tag{B.9}$$

$$A\lambda^2 = s_x, \tag{B.10}$$

$$A\lambda^3(\rho - i\eta) = s_ye^{-i\phi}. \tag{B.11}$$

Through the expression for s_z, we can deduce the form of c_z and use a binomial expansion to expand in powers of λ:

$$\begin{aligned} \lambda = s_z \implies c_z &= (1-\lambda^2)^{1/2}, \\ &= 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} - \frac{\lambda^6}{16} + \cdots, \\ &= 1 - \frac{\lambda^2}{2} \text{ to } \mathcal{O}(\lambda^3). \end{aligned} \tag{B.12}$$

The Wolfenstein parameterisation is defined up to $\mathcal{O}(\lambda^3)$, so this is where we will terminate our expansions. We proceed similarly for c_x:

$$\begin{aligned} A\lambda^2 = s_x \implies c_x &= (1 - A^2\lambda^4)^{1/2}, \\ &= 1 - \frac{A^2\lambda^4}{2} - \frac{A^4\lambda^8}{8} + \cdots, \\ &= 1 \text{ to } \mathcal{O}(\lambda^3), \end{aligned} \tag{B.13}$$

and c_y:

$$\begin{aligned} s_y = A\lambda^3 e^{i\phi}(\rho - i\eta) \implies c_y &= \left(1 - A^2\lambda^6 e^{i2\phi}(\rho - i\eta)^2\right)^{1/2}, \\ &= 1 - \frac{1}{2}(A^2\lambda^6 e^{i2\phi}(\rho - i\eta)^2) + \cdots, \\ &= 1 \text{ to } \mathcal{O}(\lambda^3). \end{aligned} \tag{B.14}$$

We then substitute these expressions in to the standard parameterisation in (B.8), neglecting terms of $\mathcal{O}(\lambda^4)$, to leave us with:

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{B.15}$$

C Rivet Analysis Code

The following is the Rivet analysis code used to extract histogram data. All credit must be given to Frank Krauss, whose initial analysis code this is built upon.

```
#include "Rivet/Analysis.hh"
#include "Rivet/Math/LorentzTrans.hh"
#include "Rivet/Math/Constants.hh"
#include "Rivet/Tools/ParticleIdUtils.hh"
#include "Rivet/Projections/Beam.hh"
#include "Rivet/Projections/UnstableFinalState.hh"
#include "Rivet/Projections/FinalState.hh"
#include "Rivet/Projections/IdentifiedFinalState.hh"
#include "Rivet/Projections/VisibleFinalState.hh"
#include "Rivet/Projections/VetoedFinalState.hh"
#include "Rivet/Projections/ChargedLeptons.hh"
#include "Rivet/Projections/MissingMomentum.hh"
#include "Rivet/Projections/FastJets.hh"
#include "Rivet/AnalysisLoader.hh"
#include "Rivet/RivetAIDA.hh"
#include <map>

namespace Rivet {
  class MC_BbarToCharm : public Analysis {
  private:
    long unsigned int nbeauties;
    std::map<std::string,AIDA::IHistogram1D *> m_histos;
    void inithistos() {
      // Init histogram by giving name, number of bins, and range: min, max
      m_histos["cBarParticles"] =
          bookHistogram1D("cBarParticles",6,0.5,6.5);
```

```
  m_histos["charmsPerBbar"] =
      bookHistogram1D("charmsPerBbar",3,-0.5,2.5);
}

bool isCHadron(const PdgId & pid) {
  // Checks whether particle is a hadron containing a c quark
  // pidM for meson, pidB for baryon
  int pidM(int(pid/100)-int(pid/1000)*10);
  int pidB(int(pid/1000)-int(pid/10000)*10);
  return (abs(pidM)==4 || abs(pidB)==4);
}

bool isBHadron(const PdgId & pid) {
  // Checks whether particle is a hadron containing a b quark
  // pidM for meson, pidB for baryon
  int pidM(int(pid/100)-int(pid/1000)*10);
  int pidB(int(pid/1000)-int(pid/10000)*10);
  return (abs(pidM)==5 || abs(pidB)==5);
}

void FindBs(const ParticleVector & FS, std::set<const GenParticle *> &
    beauties) {
  // Finds B hadrons and adds them to the list "beauties"
  beauties.clear();
  foreach (Particle part, FS) {
    CheckForB(part.genParticle(),beauties);
  }
}

void CheckForB(const GenParticle & child, std::set<const GenParticle
    *> & beauties) {
  PdgId pid = child.pdg_id();
  if (isBHadron(pid)) {
    beauties.insert(&child);
    return;
  }
  HepMC::GenVertex* gv = child.production_vertex();
  if (gv) {
    foreach (GenParticle* parent,Rivet::particles(gv, HepMC::parents))
        {
      CheckForB(*parent,beauties);
    }
  }
}

bool SearchForLastCharmsInDecay(const GenParticle * parent,
    std::set<const GenParticle *> & charms) {
  // Searches for last particles in a decay chain that contain charm
      quarks
  // These are the particles observed by the detector
  bool is_charm = isCHadron(parent->pdg_id()), hit = false;
  HepMC::GenVertex* gv = parent->end_vertex();
  if (gv) {
    foreach (GenParticle * child,Rivet::particles(gv,
        HepMC::children)) {
      if (SearchForLastCharmsInDecay(child,charms)) hit = true;
    }
  }
```

```
      if (is_charm && !hit) charms.insert(parent);
        return is_charm;
    }

    void FillBbarHistos(const GenParticle *& beauty,const double & weight)
        {
      // Takes Bbar mesons from the list "beauties" and
      // finds the last particles in the decay chain to contain charm
          quarks.
      // Then counts the number of charm quarks in the final state,
      // and counts the events with a cbar quark in the final state.
      std::set<const GenParticle *> charms;
      charms.clear();
      SearchForLastCharmsInDecay(beauty,charms);
      size_t nBbarCharms(0);
      foreach (const GenParticle * charm,charms) {
        PdgId cpid = charm->pdg_id();
        nBbarCharms++;
        if (abs(int(cpid/10-(cpid/100)*10))==4) {
          nBbarCharms++;
        }
        switch (cpid) {
          case -431:    // D_s- channel
            m_histos["cBarParticles"]->fill(1,weight);
            break;
          case -421:    // Dbar0 channel
            m_histos["cBarParticles"]->fill(2,weight);
            break;
          case -4122:   // Lambdabar_c channel
            m_histos["cBarParticles"]->fill(3,weight);
            break;
          case 441:
          case 443:
          case 445:
          case 10441:
          case 20443:
          case 30443:
          case 100441:
          case 100443:  // Charmonium channels
            m_histos["cBarParticles"]->fill(4,weight);
            break;
          case -4132:
          case -4232:   // Xibar_c channels
            m_histos["cBarParticles"]->fill(5,weight);
            break;
          case -411:    // D- channel
            m_histos["cBarParticles"]->fill(6,weight);
            break;
          default:
            break;
        }
      }
      m_histos["charmsPerBbar"]->fill(nBbarCharms,weight);
    }

  public:
    MC_BbarToCharm() :
      Analysis("MC_BbarToCharm"), nbeauties(0)
```

```
    {}

    void init() {
      addProjection(FinalState(), "FS");
      inithistos();
    }

    void analyze(const Event& event) {
      const double weight = event.weight();
      const ParticleVector & FS =
      applyProjection<FinalState>(event, "FS").particlesByPt();

      std::set<const GenParticle *> beauties;
      FindBs(FS,beauties);
      nbeauties += beauties.size();

      foreach (const GenParticle * beauty,beauties) {
        PdgId bid = beauty->pdg_id();
        switch(bid) {
          case -511:
          case -521:
          // Ensures the Bbar mesons (Bbar0 and B-) are the only
          // particles from "beauties" passed to FillBbarHistos
            FillBbarHistos(beauty,weight);
          break;
          default:
          break;
        }
      }
    }

  void finalize() {
    // Normalisation factors for the histograms.
    // 100* to give percentages.
    scale(m_histos["cBarParticles"] , 100*1./sumOfWeights());
    scale(m_histos["charmsPerBbar"] , 100*1./sumOfWeights());
    }
  };

  // The hook for the plugin system
  DECLARE_RIVET_PLUGIN(MC_BbarToCharm);

}
```